Too much of a good incentive? The case of executive stock options

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Abstract

Using a utility-maximization framework, I show that the incentive to increase stock price does not always increase as more options are granted. Keeping the total cost of his compensation fixed, granting more options creates greater incentives to increase stock price only if option wealth does not exceed a certain fraction of total wealth. Beyond this critical level, granting more options actually reduces incentive effects and becomes counterproductive. In addition, stock options also create incentive to reduce (increase) idiosyncratic (systematic) risk. These incentive effects are sensitive to the choice of exercise price.

JEL classification: G11, G13, G30, J33, M52

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1. Introduction

The past decade has witnessed a dramatic rise in the use of executive stock options by U.S. corporations. Hall and Murphy (2000b) estimate that stock options represent approximately 40% of CEO's total compensation for S&P 500 companies in 1998. That is up from only 25% of total pay in 1992. More recently, the 2001 annual survey of executive pay by the *Business Week* estimates that options account for 80% of total CEO pay.[1] Such proliferation of stock options is proclaimed

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[1]See the April15, 2002 *Business Week* special report on executive pay, pp.80-86.

to align the interest of executives with that of shareholders. Is this common practice of executive compensation in the best interest of shareholders?[2]

In this paper, I address this important question by examining the value and incentive effects of stock options. Following prior research (e.g., Lambert et al., 1991; and Hall and Murphy, 2000a,b), I adopt the certainty equivalent valuation framework which stipulates that the value of an option is equal to an immediate cash payment that provides the same expected utility as the option does. Unlike previous research, however, I incorporate and model the executive's investment decisions and portfolio choice. The departure from previous research is two fold. First, the executive may invest his unrestricted wealth (e.g., cash pay) freely between the market portfolio and the risk-free asset. Most previous studies ignore the executive's outside investment opportunities in order to focus on the impact of his risk aversion.[3] A more generalized valuation model is developed where the market portfolio is incorporated to provide additional investment opportunities for the executive's unrestricted wealth. Second, the executive maximizes the expected utility by optimally allocating his unrestricted wealth between the market portfolio and the risk-free asset. Since the executive cannot sell or hedge his stock options, the executive's optimal portfolio choice is expected to be influenced by the size of his stock option grant relative to his unrestricted wealth. The subsequent impact on his portfolio choice may thus alter the executive's value of the option.

Using parameters consistent with previous research and typical stock market conditions in the U.S., I re-examine the value and incentive effects of stock options using the certainty equivalent valuation model developed in this study. I find that a risk-averse executive may value a stock option at a significant discount from its corresponding market value (determined as if it is a traded

[2]Traditional agency theory suggests that the agent (the manager) should be given incentives (ownership) to maximize the principal's wealth (stock price) when his effort cannot be observed or the cost of monitoring is high. See, for example, Berle and Means (1932), Jensen and Meckling (1976), Holmstrom (1979), Grossman and Hart (1983), and Holmstrom and Milgrom (1987).

[3]Independently, two recent studies also incorporate this feature. See Kahl et al. (2001) and Ingersoll (2002).

option). In some severe cases, the executive may discount option value by more than 90%. As expected, the discount is a positive function of the executive's degree of risk aversion, the fraction of total wealth in options, and the volatility of the firm's stock returns. These results are consistent with the findings from previous research (e.g., Lambert et al., 1991; Carpenter, 2000; Hall and Murphy, 2000a,b; and Ingersoll, 2002).

Despite of the well-documented discount, stock options are still considered to be an important way to compensate executives because they create incentives for executives to maximize shareholder value. In other words, stock options have strictly positive delta. An increase in stock price leads to a corresponding increase in the value of stock options. Granting more options to executives is expected to provide greater incentives for them to increase the stock price. However, it also costs the firm more to grant more options if other compensation is kept the same. A more relevant comparison is to examine two compensation packages that cost exactly the same to the firm while one contains more options than the other. Holding the cost of total compensation constant, I find that the incentive to increase stock price is not a monotonically increasing function of the executive's option wealth. It is actually a concave, inverted U-shaped function. Granting more options creates a greater incentive to increase stock price only if option wealth as a fraction of the executive's total wealth does not exceed a critical threshold. Beyond this critical level, granting more options actually reduces incentive effects and becomes counterproductive. This critical level can be as low as 40%, far below the 80% average CEO pay in stock options in 2001 estimated by the *BusinessWeek*. This result implies that stock option grant can be useful in providing the right incentives to executives if it is used in moderation. Excessive use of stock options can be "too much of a good thing" because their cost to the granting firm increases proportionally with the size of the option grant while the incentives to the recipient do not.

Lambert et al. (1991) and Carpenter (2000) find that stock options create incentives to reduce firm risk (measured by the standard deviation of stock returns) while DeFusco et al. (1990) and Williams and Rao (2000) find exactly the opposite. These studies, however, do not distinguish systematic risk from idiosyncratic risk. As shown by Hall an Murphy (2000b) and Meulbroek (2001), executives discount the value of stock options because they are forced to bear risk that they otherwise would have avoided. This unwanted risk is primarily idiosyncratic risk. It is thus important to distinguish systematic risk from idiosyncratic risk and measure their impact on incentive effect separately. I find that stock options create incentive to reduce idiosyncratic risk. As it does not affect the firm's expected return, a lower idiosyncratic risk is beneficial to the risk-averse executive. This is consistent with the findings from Lambert et al. (1991) and Carpenter (2000). More interestingly, I also find that stock options create incentives to increase systematic risk. An increase in systematic risk leads to a greater expected return for the firm. As long as the benefit from greater expected return outweighs the cost of increased risk, stock options create incentives for the executive to increase systematic risk. The incentive to increase systematic risk remains until options are nearly 100% of total pay. Since corporate decisions made by the executive may simultaneously have an impact on both systematic and idiosyncratic risk, the combined risk incentive may depend on the relative strength and interaction between the two effects. Empirical research has indeed found both positive and negative relationships between firm risk and pay-performance sensitivity. For example, Demsetz and Lehn (1985), Smith and Watts (1992), Prendergast (2000a, b), and Core and Guay (2001a) find a positive relationship between firm risk and pay-performance sensitivity while Aggarwal and Samwick (1999), Himmelberg et al. (1999) and Palia (2001) document a negative relationship.

Finally, I find that incentive effects from stock options are strongly influenced by stock price fluctuations and the choice of exercise price. Even if stock options are granted at the money (as is the most common practice), subsequent stock price fluctuations may move the options in or out of the money. I show that the incentive to increase stock price, to increase systematic risk and to reduce idiosyncratic risk becomes stronger if the firm's stock price rises after the grant date. The reverse is true when the firm's stock falls after the grant date. The expected incentives continue to be in place and are actually getting stronger if the firm performs well in the future but diminish if it does not. This dichotomy in incentive strength is frequently used as a rationale for the controversial management practice commonly known as option repricing. Firms often reprice underwater options because the out-of-the-money options no longer provide sufficient incentives. However, such practice remains a contentious issue as it appears to reward poor performance rather than superior performance. In addition, I also find that incentive effects from stock options vary significantly with the choice of exercise price. Holding the cost of the option grant constant, the firm may maximize the incentive to increase stock price by setting the exercise price moderately below the grant-date stock price. The incentive-maximizing exercise price is typically greater than zero but less than the stock price, and its precise value depends on the executive's risk aversion, proportional option wealth, and characteristics of the firm and the market portfolio.

The rest of the paper proceeds as follows. In the next section, I develop a valuation model for executive stock options that incorporate the executive's optimal investment decisions and portfolio choice. Section 3 numerically examines the extent that option value is discounted to its market value. Section 4 investigates the incentive effects created by stock options. The final section summarizes and concludes the paper.

2. The valuation model

Consider the valuation of stock options granted to an executive who cannot sell or exercise the option till maturity. Consistent with prior research, the executive is assumed to be risk averse and has constant relative risk aversion specified by the power utility function:

$$u(w) = \begin{cases} \dfrac{w^{1-a}}{1-a}, & \text{if } a \neq 1, \\ \ln w, & \text{otherwise,} \end{cases} \tag{1}$$

where w is the executive's terminal wealth. The parameter $a > 0$ is a measure of risk aversion, called the *coefficient of relative risk aversion.* A larger a indicates a greater degree of risk aversion.

The executive's total wealth consists of both restricted and unrestricted components. The restricted component is the executive's option grant while the unrestricted component comes from his cash compensation from the firm and his non-firm-related wealth accumulated prior to the current time period. Since the option grant is restricted, the executive's consumption and investments must come from his unrestricted wealth until the options are exercised.[4] To avoid unnecessary complications, I focus on the executive's investment decisions rather than consumption decisions in a single-period utility maximization model. The executive may invest his unrestricted wealth between the risk-free asset (e.g., a money-market fund) and the market portfolio (e.g., an index fund tracking the performance of a market index). Given the option grant, the executive maximizes the expected utility of terminal wealth by optimally allocating his unrestricted wealth between the risk-free asset and the market portfolio.

As the executive cannot sell or exercise the option until maturity, his optimal investment decision may differ from the optimal investment decision of other investors who do not face such restrictions. Consequently, the executive may discount the option value from its market value (determined as if it is traded). As is standard in the literature (see for example, Lambert et al., 1991; and Hall and Murphy, 2000a, b), I use the certainty equivalent framework to determine the executive's private value of stock options. In other words, the option value is determined as the equivalent

[4] For further discussion on this issue, see Kahl et al. (2001).

cash payment that provides the same expected utility as the option would otherwise have provided. Unlike previous research, the executive is assumed to allocate his unrestricted wealth between the market portfolio and the risk-free asset in order to maximize his expected utility. The approach adopted here is consistent with the valuation model of restricted stock by Kahl et al. (2001) and the optimal contracting model of Tian (2001). By allowing the executive to make optimal portfolio decisions, the valuation model developed here avoids the counter intuitive results from earlier studies that stock options are worth more than their market values.[5]

Let W_0 be the executive's total wealth, which consists of N stock options and W_c dollars in unrestricted wealth. This is summarized as:

$$W_0 = N \cdot V_0 + W_c, \tag{2}$$

where V_0 is the cost of the option to the firm. The fraction of wealth tied to options is:

$$\omega = \frac{N \cdot V_0}{W_0}.$$

As the firm does not face restrictions in trading its shares, the cost of its option grant can be approximated by the Black-Scholes model. The executive's terminal wealth at option maturity is influenced by payoffs from his option grant and his investment decision for unrestricted wealth:

$$W(\lambda) = N \cdot V(T) + W_c \left[\lambda \frac{H(T)}{H_0} + (1 - \lambda) \exp(rT) \right], \tag{3}$$

where r is the risk-free rate, T is option maturity, H_0 and $H(T)$ are the unit value of the market portfolio today and at option maturity, respectively, $V(T)$ is the option payoff at maturity defined as:

$$V(T) = \max \left[S(T) - X,\ 0 \right], \tag{4}$$

[5]See Hall and Murphy (2000b) for a discussion on this issue.

$S(T)$ is the stock price at option maturity, X is the exercise price of the option, and λ is the proportion of unrestricted wealth invested in the market portfolio.

As is standard in the literature, the stock price and unit value of the market portfolio are assumed to follow a joint geometric Brownian motion.[6] In particular, the stock price and unit value of the market portfolio at option maturity are joint lognormally distributed and are described by:

$$S(T) = S_0 \exp\left(m_s T + \sigma_s \sqrt{T} Z_s\right), \tag{5}$$

$$H(T) = H_0 \exp\left(m_h T + \sigma_h \sqrt{T} Z_h\right), \tag{6}$$

where S_0 is the firm's initial stock price, $m_s = \mu_s - \frac{1}{2}\sigma_s^2$, $m_h = \mu_h - \frac{1}{2}\sigma_h^2$, μ_s and σ_s are the expected return and volatility of the firm's stock, respectively, μ_h and σ_h are the corresponding terms for the market portfolio, and Z_s and Z_h are correlated standard normal variates with a correlation coefficient of ρ.

To focus on the impact of non-tradeability, lack of diversification and risk aversion, I make the simplifying assumption that the firm's expected return is specified by the Capital Asset Pricing Model:

$$\mu_s = r + \beta(\mu_h - r), \tag{7}$$

where

$$\beta = \frac{\rho\sigma_s}{\sigma_h}.$$

This assumption is consistent with prior research in the related literature (e.g. Hall and Murphy, 2000a, b; Meulbroek, 2001; and Kahl et al., 2001).

[6] See, for example, Johnson and Tian (2000). Dividends are ignored here for simplicity. It is straightforward to incorporate a continuous dividend yield in the model.

The executive makes investment decisions today in order to maximize the expected utility of his terminal wealth:

$$\max_{\lambda} E\left\{u\left[W(\lambda)\right]\right\}. \tag{8}$$

The executive's optimal investment in the market portfolio may depend on the executive's degree of risk aversion, the probability distribution of the firm's stock and the market portfolio, and, in particular, the size of his option grant. With a fraction of his wealth tied to options, the executive may choose to invest a smaller proportion of his unrestricted wealth in the market portfolio than he otherwise would if no such restriction is imposed.

As discussed previously, the value of the stock option (to the executive) is defined as the amount of immediate cash payment the executive receives in exchange for the option grant that leads to the same expected utility of terminal wealth. By exchanging options for an immediate cash payment, the executive no longer has any restricted wealth and is free to invest all his wealth between the risk-free asset and the market portfolio. The value of the option is thus the certainty equivalent value, CE, determined from the following optimization problem:

$$\max_{\lambda_1} E\left\{u\left[W(\lambda_1)\right]\right\} = \max_{\lambda_2} E\left\{u\left[\widehat{W}(\lambda_2)\right]\right\}, \tag{9}$$

where

$$\widehat{W}(\lambda) = (W_c + N \cdot CE)\left[\lambda \frac{H(T)}{H_0} + (1-\lambda)\exp(rT)\right]. \tag{10}$$

The solution to the above optimization problem is denoted by λ_1^*, λ_2^*, and CE.

Note that the determination of CE requires two separate optimization problems. The first optimization problem is the determination of the optimal portfolio decision (λ_1 in the left-hand side of Eq. (9)) when a fraction of the executive's total wealth is tied to options. The second optimization problem is the determination of the optimal portfolio decision (λ_2 in the right-hand side of Eq. (9)) when all options are swapped for a cash payment and the executive is free to invest all his wealth between the market portfolio and the risk-free asset. In addition, the option value (CE) defined in Eq. (9) includes the certainty equivalent value of Lambert et al. (1991) and Hall and Murphy (2000a, b) as a special case. This is when the executive invests his entire unrestricted wealth in the risk-free asset ($\lambda = 0$). In this case, the option value simplifies to:

$$E\left\{u\left[W(0)\right]\right\} = E\left\{u\left[\widehat{W}(0)\right]\right\}. \tag{11}$$

Since a closed-form solution is not available for certainty equivalent value, it is necessary to solve it numerically. This can be done by solving either the maximization problem (9) directly or the first-order conditions. In this study, all numerical results are derived directly from the maximization problem (9).

3. Discounting executive stock options

In this section, I apply the certainty equivalent valuation model developed in Section 2 to examine the extent a risk-averse executive may discount the value of his stock options and the key factors influencing this discount. Without the loss of generality, the executive's total wealth (W_0) is normalized at \$100. The executive's portfolio decisions and option value are not affected by his wealth level because the power utility has constant relative risk aversion. Consistent with typical market conditions in the U.S., the risk-free rate is assumed to be 5% and the market portfolio has an expected return of 10% and a volatility of 20%. To ensure robustness, a range of parameters are considered for the firm's stock return distribution, terms of the option, the percentage of wealth tied to options, and the executive's degree of risk aversion.

3.1. At-the-money options

As the overwhelming majority of options are granted at the money (Murphy, 1999), I first examine the valuation of at-the-money options. Table 1 reports the percentage discount of option value (CE) from its market value (approximated by the Black-Scholes price). If the executive values the option exactly the same as its corresponding market value, this discount is zero. Over a wide range of parameters shown in Table 1, this discount in value can be as low as 13.8% (in **bold** numbers) or as high as 98.6% (in *italic* numbers). This rather large gap between the cost to the firm and the value to the executive is a potential source of inefficiency in compensating executives with stock options.

The large variation in option value also means that it is important for the granting firm to know whether or not the recipient is likely to discount the option value from its market value (i.e., its cost to the firm) and the extent of the discount. It is thus helpful to identify the factors that influence the executive's valuation of stock options. As expected, the executive's discount is positively related to the coefficient of relative risk aversion (a), the fraction of wealth in options (ω), and the firm volatility (σ_s). The coefficient of relative risk aversion determines the executive's degree of risk aversion while the fraction of restricted wealth and firm volatility influence the riskiness of his terminal wealth. The risk-averse executive discounts the option value more when the executive is more risk averse or when his terminal wealth becomes more volatile without commensurate increase in expected payoff. To illustrate the magnitude of this positive relationship, consider the following base-case parameters: $a = 2$, $\omega = 0.5$, $\beta = 1$, $\sigma_s = 30\%$, and $T = 2$. Using this set of base-case parameters, the executive's discount is 50.8% or about half of the option's market value. If the coefficient of relative risk aversion increases from 2 to 4 while other parameters are kept constant, the discount rises from 50.8% to 73.8% (representing an additional 47% loss in option value). Similarly, if the fraction of option wealth increases from 0.5 to 0.9 or the firm volatility increases from 30% to 60%, the discount increases from 50.8% to 87.5% or to 70.4%, respectively. In all three cases, the positive relationship is quite strong and the loss in option value is significant.

In addition, the executive's discount is found to be a negative function of the firm's beta. Consider the same set of base-case parameters, i.e., $a = 2$, $\omega = 0.5$, $\beta = 1$, $\sigma_s = 30\%$, $T = 2$. As shown in Table 1, the discount increases from 50.8% to 61.8% if the firm's beta drops from 1 to 0. In other words, the executive's value of the option declines by 22% if the firm's beta decreases from 1 to 0. This negative relationship is found to be robust over a wide range of parameter values. To further understand this negative relationship, note that beta has both a direct and an indirect impact on option value. From Eq. (7), it is clear that a higher beta leads to a greater expected return on the stock. Because the firm's volatility is kept constant, the greater expected stock return has a positive impact on option value. At the same time, a larger beta also corresponds to a higher correlation between the firm's stock and the market portfolio. The increased correlation allows the executive to use the market portfolio as a hedge against his exposure to firm risk. Therefore, the improved ability of the executive to hedge his firm-related risk has an indirect positive influence on option value.

To provide additional support for the benefit of hedging, Table 2 reports the optimal holdings in the market portfolio with and without the option grant. When the firm's stock is uncorrelated with the market portfolio ($\beta = 0$), the optimal fraction of unrestricted wealth invested in the market portfolio (λ) is not sensitive to proportional option wealth (ω). For example, when $a = 2$, $\beta = 0$, $\sigma_s = 30\%$ and $T = 2$, the optimal fraction of unrestricted wealth in the market portfolio is 0.627 when the executive receives no option grant. This optimal fraction increases slightly to 0.640 when 90% of his total wealth is held in options. In contrast, the optimal fraction in the market portfolio is highly sensitive to the size of option grant if the firm's beta is 1 (implying a 0.67 correlation between the firm and the market portfolio). Using the same set of parameters except for the firm's beta (i.e., $a = 2$, $\beta = 1$, $\sigma_s = 30\%$ and $T = 2$), the optimal fraction in the market portfolio remains to be 0.627 if no options are granted. The optimal fraction drops to 0.392 if 10% of total wealth is tied up in options. In fact, the option holder will start shorting the market portfolio if his option wealth reaches or exceeds 30% of total wealth. As the fraction of option wealth increases to 90%, his short position in the market portfolio rises to 53.7% of his unrestricted wealth. The ability to (partially) hedge the exposure to options when $\beta = 1$ increases option value in comparison with the case when the exposure to options cannot be hedged at all ($\beta = 0$). For example, with 50% of wealth in options, the executive values the option at 49.2% of market value when $\beta = 1$ but only 38.2% when $\beta = 0$ (Table 1). The benefit of hedging is quite robust over the wide range of parameter values used in Tables 1 and 2.

3.2. Option moneyness

Table 3 reports the executive's discount of option value from its market value for various levels of option moneyness. As is standard practice, the ratio of exercise price (X) over stock price (S) is defined as option moneyness (K). The option is in the money, at the money or out of the money when $K < 1$, $K = 1$ or $K > 1$, respectively. Table 3 covers a wide range of option moneyness with K varying from 0 to 2. Since option moneyness defines the payoff of an option, it is expected to have a large impact on option value. A higher exercise price should reduce both the option's private and market value. The executive's discount is affected only if exercise price has a different impact on the two option values. As shown in Table 3, exercise price or option moneyness has a surprisingly large impact on the discount. An increase in the option's exercise price reduces the option private value much more than its market value. When the exercise price is zero ($K = 0$), the discount ranges from 3.0% to 39.5%. In comparison, the corresponding range is from 68.8%

to 99.2% when the exercise price is twice the stock price ($K = 2$). This relationship remains to be true over a wide range of parameters, as shown in Table 3. This result implies that the executive discounts the value of out-of-the-money options much more than in-the-money options. The larger the exercise price, the greater the discount. Another way to interpret this result is that the compensatory component of the option grant declines quickly as the option exercise price increases. Unless the incentive component of the option grant increases in a similar manner as the exercise price increases, in-the-money options are likely to be a more effective way to compensate executives. Incentive effects of stock options are examined next.

4. Incentive effects

Core and Guay (2001b) argue that a compensation contract can be decomposed into two separate components: pure compensation and incentives. Pure compensation is usually in the form of salary or other cash payments. This type of compensation imposes no risk on the executive. On the other hand, the incentive component of a contract ties pay to firm performance and hence exposes the executive to firm-specific risk. Stock option grants have both a compensatory and incentive element. Because stock options impose additional risk to the executive, he discounts its value as demonstrated in the previous section. The certainty equivalent value of a stock option is designed to measure its compensatory value. The incentive component of stock options is examined next.

4.1. Incentive to increase stock price

Proponents of executive stock options argue that stock option grants align the interest of executives with that of the shareholders and provide incentives to increase stock price. As is standard in the literature, I use option delta – the partial derivative of option value with respect to stock price – to examine the incentive to increase stock price. Intuitively, option delta measures the change in option value due to an incremental increase in stock price. Stock option grants create incentives to increase stock price if option delta is positive. This is because the value of the executive's option grant increases as stock price rises, thus aligning the interest of the executive with that of the shareholders.

From Eq. (9), it is straightforward to derive the partial derivative of option value with respect to stock price:

$$\frac{\partial\left(N_o \cdot CE\right)}{\partial S}=\frac{N_o \cdot E\left\{u'\left[W(\lambda_1^*)\right] g(T)\dfrac{S(T)}{S_0}\right\}}{E\left\{u'\left[\widehat{W}(\lambda_2^*)\right] \exp(rT)\right\}}, \tag{12}$$

where

$$g(T)=\begin{cases} 1, & \text{if } S(T) > X, \\ 0, & \text{otherwise.} \end{cases}$$

Note that delta defined in Eq. (12) calculates the change in the value of the executive's entire option grant as the stock price increases by a dollar. If stock option provides an incentive to increase stock price, its delta as defined in Eq. (12) must be positive. From Eq. (12), this is clearly true.

However, such incentives are provided to the executive at a cost to the firm because the risk-averse executive discounts the value of stock options from their market value. If the executive does not discount the value of stock options, it is always efficient to substitute cash pay for stock options in order to create incentives. Of course, the risk-averse executive does discount option value and the extent of the discount increases as a greater proportion of pay is tied to stock options. It is thus reasonable to argue that the incentive effect provided by each additional option weakens as more and more options are granted. The more interesting question is whether the aggregate incentive effect increases monotonically as more options are granted. The answer to this question may depend on how options are granted. If cash pay is fixed (e.g., \$100) while more options are granted (e.g., from 5 to 10 options), there is no doubt that in this case more options will induce greater incentives to increase stock price.[7] Greater incentives are created, however, with a more

[7]This is straightforward to prove mathematically. For brevity, the proof is omitted here.

expensive compensation package. A more relevant comparison is on the incentive strength between two compensation packages that are equally expensive to the firm but with a different mix of cash and option pay. The key question to ask is thus: if the firm keeps the executive's total pay fixed while varies the proportion of option pay (ω), does the aggregate incentive effect of the option grant increase as the proportion of option pay rises?

To answer this question, I again fix the total cost of the executive's compensation package at \$100. I then vary the proportion of option pay between 5% to 95% (i.e., $\omega \in [0.05, 095]$). For each value of ω, I calculate the delta of the executive's entire option grant (using Eq. 12). Fig. 1 plots option delta against ω for the following parameters: $a = 2$ or 4, $\beta = 1$, $\sigma_s = 30\%$, $K = 1$, and $T = 5$. Other parameters used are the same as in Table 1. Panel A illustrates delta per option as option wealth (ω) increases from 5% to 95% of total wealth. The average incentive provided by an option declines quickly as more cash pay is substituted for options. This is not surprising since the executive's value of the option is a negative function of ω. What is more interesting is the aggregate delta of the entire option grant shown in Panel B. It is clear that the aggregate delta is not a monotonically increasing function of option wealth (ω). As more cash pay is substituted for options, the total delta of the entire option grant increases at moderate levels of ω. As the proportion of option wealth exceeds a certain threshold, the total delta actually starts to decline. In other words, the aggregate delta is an inverted U-shaped function of proportional option wealth. The top of the inverted U-shaped curve represents the maximum possible incentive. More options beyond this point no longer provide a greater incentive to increase stock price but actually a smaller incentive. For an executive with a coefficient of relative risk aversion (a) of 2, the total delta reaches the maximum when the option wealth is approximately 60% of the total wealth. For a more risk-averse executive (e.g., $a = 4$), the maximum is reached even more quickly at approximately 40%

of proportional option wealth. Intuitively, as the weight of option wealth increases (while keeping the total cost of compensation fixed), the total number of options granted increases but the value of each option to the executive also falls. At moderate levels of option wealth, the benefit of a greater number of options outweighs the decline in the value of each option. In contrast, at high levels of option wealth, the decline in option value dominates the effect of a larger option grant. This interesting result implies that an excessive use of options reduces not only the compensatory value of the option grant but also its incentive strength. It is a bad idea to grant an ever increasing amount of options to executives. This result is robust over the wide range of parameter values covered in Table 1.

4.2. Risk incentives

Previous research (such as Lambert et al., 1991; and Carpenter, 2000) has demonstrated that stock options may provide incentives for executives to reduce firm volatility. Intuitively, a risk-averse individual increases his expected utility if he can lower the volatility of his wealth while at the same time keeping his expected wealth fixed. This is indeed the case if the executive can reduce firm volatility without affecting its expected return. It is thus conceivable that high risk firms may use stock options (and other forms of incentive pay) less frequently than low risk firms do, holding other things constant. If the option's incentive strength is measured by the executive's pay-performance sensitivity (Jensen and Murphy, 1990), a negative relationship is expected between firm volatility and pay-performance sensitivity. Empirical findings on this relationship are, however, divided. One set of studies, including Demsetz and Lehn (1985), Smith and Watts (1992), Prendergast (2000a, b), and Core and Guay (2001a), find a positive relationship between firm risk and pay-performance sensitivity. These studies argue that the monitoring cost is higher when the firm operates in a more uncertain environment and thus requires a stronger pay-performance sensitivity to motivate the executive. Other studies such as Aggarwal and Samwick (1999), Himmelberg et al. (1999) and Palia (2001), in contrast, find a negative relationship between firm risk and pay-performance sensitivity. This is a possible outcome if the effect of the executive's risk aversion dominates the effect of the monitoring cost.

An important aspect of firm volatility is ignored by these studies, however. A firm's stock return volatility or total risk (σ_s) can be decomposed into two distinct components: systematic risk (β) and idiosyncratic risk (σ_e). This standard decomposition is summarized by the following equation:

$$\sigma_s^2 = (\beta \sigma_h)^2 + \sigma_e^2. \tag{13}$$

An increase in firm volatility can be accomplished by increasing either the systemic risk component or the idiosyncratic risk component or both. This distinction is important because these risks have a different impact on the firm's expected return. Since unrestricted investors (the vast majority of all shareholders) may eliminate idiosyncratic risk through diversification, they do not demand a premium for bearing such risk. The firm's expected return is thus not influenced by idiosyncratic risk. Any increase in idiosyncratic risk will lead to a rise in the volatility of the executive's portfolio without any change in expected return. It is thus expected that stock options provide incentives to reduce idiosyncratic risk. On the other hand, any increase (decrease) in systematic risk will lead to a corresponding increase (decrease) in the firm's expected return. If the increase in expected return more than compensates the executive for the higher risk, stock options may create incentive to increase systematic risk.

To quantify risk incentives, I calculate the partial derivative of option value with respect to both the systematic risk (β) and the idiosyncratic risk (σ_e). From Eq. (9), it is straightforward to derive the partial derivative of total option value with respect to idiosyncratic risk:

$$\frac{\partial (N_o \cdot CE)}{\partial \sigma_e} = \frac{N_o \sigma_e \sqrt{T}}{\sigma_s} \cdot \frac{E\left\{u'\left[W(\lambda_1^*)\right] g(T) S(T)(Z_s - \sigma_s \sqrt{T})\right\}}{\lambda_2^* \exp(\mu_h) + (1-\lambda_2^*)\exp(rT)}. \tag{14}$$

Together with Eqs. (7) and (13), the partial derivative of total option value with respect to systematic risk is:

$$\frac{\partial (N_o \cdot CE)}{\partial \beta} = \frac{N_o \beta \sigma_h^2 \sqrt{T}}{\sigma_s} \cdot \frac{E\left\{u'\left[W(\lambda_1^*)\right] g(T) S(T)(Z_s + \gamma \sigma_s \sqrt{T})\right\}}{\lambda_2^* \exp(\mu_h) + (1-\lambda_2^*)\exp(rT)}, \tag{15}$$

where

$$\gamma = \frac{\mu_h - r}{\beta \sigma_h^2} - 1.$$

Because neither partial derivative can be signed analytically (due to terms involving Z_s), I investigate risk incentives numerically.

Fig. 2 illustrates the incentive to increase idiosyncratic risk using the following parameters: $a = 2$ or 4, $\sigma_s = 0.3$, $\beta = 1$, $K = 1$, and $T = 5$. The partial derivative of total option value with respect to idiosyncratic risk is calculated and plotted against option wealth as a fraction of total wealth (ω). As shown in Fig. 2, this partial derivative is negative implying that stock options create incentives to reduce idiosyncratic risk. In addition, the magnitude (absolute value) of the incentive effect increases as options become a larger proportion of total wealth. The increase continues until proportional option wealth reaches approximately 80% and 65% for executives with a relative risk aversion of 2 and 4, respectively. Any increase in option wealth beyond this point will result in a decline in the incentive to reduce idiosyncratic risk, but the incentive effect does not disappear even if options are 95% of total wealth. In fact, the incentive to reduce idiosyncratic risk is strong for most levels of proportional option wealth. The only exception is when proportional option wealth is less than 25% of total wealth. In this case, the incentive effect is relatively small and can be both positive or negative. The pattern exhibited in Fig. 2 is robust over the wide range of parameters covered by Table 1.

Similarly, Fig. 3 illustrates the incentive to increase systematic risk using the same parameters as in Fig. 2. The partial derivative of total option value with respect to beta is calculated and plotted against option wealth as a fraction of total wealth (ω). As shown in Fig. 3, this partial derivative is mostly positive which means that stock options create incentives to increase systematic risk. The incentive effect is quite strong except when proportional option wealth is either very small or very large. There is also a weak incentive to reduce systematic risk when proportional option wealth exceeds 90% and 70% for executives with a relative risk aversion of 2 and 4, respectively. Similar to the incentive to increase idiosyncratic risk shown in Fig. 2, the incentive to increase systematic risk first gets stronger then declines as more cash pay is replaced with options. This result is found to be robust over the range of parameters covered by Table 1.

The results in Figs. 2 and 3 suggest that risk incentives created by stock options are not unambiguously positive or negative. While stock options provide incentive to reduce idiosyncratic risk, they also create incentive to increase systematic risk. Because decisions made by executives are likely to have an impact on both systematic and idiosyncratic risk, the overall risk incentive may depend on the relative strength and interaction between the two effects. It is thus not surprising that empirical findings are also divided on this important issue. More discussions on this issue follow.

4.3. Stock price fluctuations

So far, I have only examined the incentive effects of at-the-money options ($K = 1$). This is done mainly because the majority of all stock options are granted at the money (Hall, 1999). However, this analysis is only appropriate on or immediately after the grant date when the stock price is close to the option's exercise price. Subsequent fluctuations in stock price may move the options in or out of the money even if they are granted at the money initially. How does future stock price movement impact incentive effects?

To answer this question, I continue to assume that the firm grants at-the-money options to its executives. Immediately after the grant date, a firm-specific shock occurs and the firm's stock price increases or decreases by a certain amount. The market portfolio is assumed to be unaffected because this is a firm-specific shock and the firm's capitalization is immaterial to the market portfolio. In other words, everything else remains unchanged except for an unexpected shock to the firm's stock price. To illustrate the impact of such stock price fluctuations, I consider the following set of parameters: $a = 2$ or 4, $\omega = 0.5$, $\beta = 1$, $X = 50$, and $T = 5$. The grant-date stock price is \$50, identical to the option's exercise price. Immediately after the grant date, a stock price shock occurs causing the stock price to jump to a new level ranging from 0 to \$100. Other parameters used are the same as in Table 1. Fig. 4 illustrates the changes in incentive effect as the stock price fluctuates.

It is clear from Fig. 4 that stock price fluctuations have a strong impact on all incentive effects, especially when stock price increases (making the option in the money). Keeping other things constant, a higher stock price leads to a much stronger incentive to increase stock price (Panel A). Intuitively, a higher stock price makes the option more valuable to the executive. The same dollar increase in stock price thus leads to a greater jump in option value when the stock price is higher, which strengthens the incentive to increase stock price. The opposite is true when the stock price declines. The impact on risk incentives is more intriguing. On the grant date, the at-the-money option creates incentives to increase systematic risk (Panel B) and decrease idiosyncratic risk (Panel C). Subsequent stock price changes may alter both the magnitude and direction of these risk incentives, though the magnitude is affected much more by a stock price increase than by a stock price decline. In both cases, there is a critical stock price such that the incentive to increase systematic or idiosyncratic risk is negative (positive) when the stock price is above (below) this critical stock price. The incentive effects are similar when other parameters are used but may differ somewhat in magnitude.

These results imply that the incentive to increase stock price is much stronger in a bull market than in a bear market (Panel A). This is not necessarily desirable from an incentive point of view because stronger incentive is precisely needed when the stock price is down. The controversial practice of repricing underwater options deals with this problem by lowering the option's exercise price. While option repricing does indeed strengthen the incentive to increase stock price, it also imposes additional cost to the firm because the repriced option is much more expensive than the underwater option before it is repriced. Such cost is often ignored when option repricing is proposed by executives and compensation consultants. There is also an indirect cost of option repricing. The executive may not work as hard to increase the stock price if he expected the firm to reprice the option subsequently. In fact, he might be inclined to let the stock price continue to fall until it is low enough so that the firm reprices the option. It is better to hold back his effort until the option is repriced and then exerts full effort. A better way to rejuvenate the option's incentive effects is the so-called Black-Scholes repricing (Hall and Murphy, 2000b). In this relatively new option repricing scheme, the Black-Scholes value of the underwater options is calculated first. The out-of-the-money options are then replaced with new, at-the-money options of equal Black-Scholes value. The options are repriced at the same cost to the firm (approximated by the Black-Scholes value) while the proper incentive effects are restored. Alternatively, firms should consider granting a series of stock options in small quantities over time than a single mega grant upfront. Using a series of small grants reduces the need to reprice options and thus may avoid the controversy created by option repricing.

In addition, risk incentives (both systematic and idiosyncratic risks) exhibit mean reversion in stock price fluctuations. When stock prices are high (beyond a critical value), options create incentives to reduce both systematic risk and idiosyncratic risk while the opposite is true when stock prices are low. This interesting result implies that stock options are unlikely to create extreme risk incentives. When the stock prices are high, options provide incentives to reduce both systematic and idiosyncratic risk. The executive chooses to hedge the gain in options by reducing risk. Too much reduction in risk may, however, be undesirable for shareholders because of the corresponding diminished expected return. Of course, this is also precisely the situation where the option is deep in the money and the executive is expected to exercise the option, which immediately reduces or eliminates the incentive to reduce risk. On the other hand, options create incentives to increase both systematic and idiosyncratic risks (more so for systematic risk) when stock prices are low. However, the incentive strength is relatively weak compared to the incentive to reduce risk when stock prices are high (Panels B and C). The incentive to increase risk weakens even further if stock price continues to decline.

4.4. Exercise price and incentive effects

The overwhelming majority of all stock options are granted at the money (Hall, 1999). The interesting question is whether there is any underlying economic or institutional reason for this common practice. Current accounting regulations penalize in-the-money option grants (known as discount options), because the grant-date spread between stock price and exercise price on discount options must be charged to income. However, this does not explain why out-of-the-money options (known as premium options) are not granted more frequently. Perhaps, executives dislike premium options since they are initially underwater and may remain so for some time to come. Hall (1999) argues that there seems to be a bias, on the part of executives, toward valuing options according to what they would be worth if exercised immediately. Based on actual interviews with consultants at major compensation consulting firms in the U.S., Hall (1999) finds that many executives view "at the money or out of the money options as near worthless 'because they are not worth anything unless the share price goes up'."[8]

More recently, Hall and Murphy (2000a) argue that setting the exercise price equal to the grant-date stock price is an optimal or near optimal policy. Using the certainty equivalent valuation model of Lambert et al. (1991), they find that the incentive to increase stock price is maximized or nearly maximized when the option is at the money. They conclude that the common practice of granting options at the money is consistent with maximizing incentive effects. However, in deriving the option's certainty equivalent value, they do not incorporate the executive's optimal investment decisions for his unrestricted wealth. Specifically, they assume that the executive must invest his entire unrestricted wealth in the risk-free asset. This rather restrictive assumption may have led to the insensitivity of incentive effects to exercise price. The impact of exercise price on incentive effects is thus not fully explained. In this section, I apply the valuation model developed in this paper to reexamine this issue.

To focus on incentive effects, the analysis below ignores the accounting disadvantages of in-the-money options and assumes that all options are treated equally by accounting and tax rules. Under this assumption, the optimal exercise price is determined entirely by the trade-off between the cost and incentive effects of stock options. To isolate the impact of the exercise price, I fix the total cost of the option grant. The number of options granted changes as the firm selects a different exercise

[8]See Appendix A in Hall (1999).

price. If a lower exercise price is chosen, a smaller number of options is granted, and vice versa. To illustrate the main results, the following set of parameters is used: $a = 2$ or 4, $\omega = 0.5$, $\beta = 1$, $S_0 = 50$, and $T = 5$. The possible exercise prices for the firm to choose from cover a wide range from zero to 200% of grant-date stock price. Other parameters are identical to those used in Table 1. The results are presented in Fig. 5.

Consider first the incentive to increase stock price (Panel A). Measured by aggregate option delta, the incentive to increase stock price has a nonlinear relationship with exercise price. The incentive strength is the strongest at an exercise price that is less than the stock price but greater than zero. The maximum is reached when the exercise price is approximately 70% and 40% of the stock price when the executive's degree of risk aversion (a) is 2 and 4, respectively. From the incentive point of view, these are the optimal exercise price for the firm to select. The incentive strength would be weakened by 5% and 32%, respectively, if the option is granted at the money instead of the optimal exercise price. Similarly, the reduction in incentive strength is 30% and 15%, respectively, if restricted stock ($X = 0$) is granted instead. The loss in incentive strength in either case (restricted stock or at-the-money option) is significant and much greater than reported by Hall and Murphy (2000a). The different assumptions regarding the executive's non-firm-related investment opportunities are the main reason for these differences in the sensitivity of incentive effects to exercise price.

The incentive to increase systematic risk is analyzed next. As shown in Panel B of Fig. 5, options create incentives to increase (reduce) systematic risk when the exercise price is high (low). If the firm chooses to grant the option either at or in the money ($X \leq S_0$), the risk incentive is quite weak and mostly negative. On the other hand, the risk incentive is positive and can be quite strong if the firm grants out-of-the-money options ($X > S_0$). The incentive effects also vary as option wealth and firm beta change, but the general pattern of the risk incentive remains similar to the one illustrated in Panel B.

Finally, I examine the incentive to increase idiosyncratic risk. As shown in Panel C of Fig. 5, stock options create incentives to reduce idiosyncratic risk virtually over the entire range of exercise prices. The only exception is when the exercise price is extremely high (more than 200% of the stock price). As firms are unlikely to grant options with an exercise price more than twice the grant-date stock price, stock options, for all practical purposes and concerns, create incentives to reduce idiosyncratic risk. This is true across a wide range of parameters covered by Table 1. What is more surprising is the variation in the incentive strength across exercise prices. The incentive to reduce idiosyncratic risk is the strongest when the firm grants options at or close to the money but diminishes when the firm grants options farther from the money.

In aggregate, the results in Fig. 5 demonstrate that incentive effects from stock options are sensitive to the choice of exercise price. To maximize the incentive to increase stock price, the firm should set the exercise below the grant-date stock price but greater than zero. The exact exercise price level depends the characteristics of the firm and the executive. Setting the exercise price equal to the grant-date stock price or zero is far from optimal. The incentive to increase stock price is weakened even further if the exercise price is set above the grant-date stock price. Risk incentives are equally sensitive to the choice of exercise price. From a pure incentive point of view, setting the exercise price slightly below stock price might be optimal. However, current accounting regulations penalize firms that grant in-the-money options (including restricted stock). This is because the in-the-money portion of the option grant must be expensed by the firm immediately, thus reducing reported earnings. Firms (and CEOs) concerned about reported earnings are less likely to use discount options (including restricted stock) even though they may be more incentive inducing. Until the FASB and other regulatory bodies implement the necessary reforms in their policies and regulations, firms will continue to grant options at the money. Once all options are treated equally (e.g., charge the cost of all options to earnings at the time of the option grant), firms may begin to consider alternative exercise price policies and focus more on incentive effects from stock options. These needed reforms may play an important role in fostering the best practice of executive compensation.[9]

5. Summary and conclusions

In this study, I have developed a more generalized framework for valuing executive stock options. The certainty equivalent value concept from prior studies (e.g., Lambert et al., 1991; and Hall and Murphy, 2000a,b) is modified to incorporate the executive's optimal investment decisions and portfolio choice. I then apply the new valuation framework to examine the value and incentive effects of stock options. As expected, I find that executives discount stock options significantly from their market value and stock options create incentives to increase stock price. A more striking finding is that the incentive to increase stock price does not increase monotonically as proportional option wealth increases. Keeping the total cost of his compensation fixed, granting more options creates greater incentives to increase stock price when proportional option wealth does not exceed a critical threshold. Beyond this critical level, granting more options actually reduces incentive strength and becomes counterproductive. I also distinguish systematic risk and idiosyncratic risk and find that stock options create incentives to reduce idiosyncratic risk but create incentives to

[9]FASB's previous attempt for such a reform in 1993 met with widespread protests from CEOs and corporate boards nationwide, especially from technology firms (see Fox, 2001 for further details). However, the recent spectacular collapse of Globalcrossing, Enron, and Worldcom, and the ensuing accounting and corporate control scandal may provide the impetus needed to push through such needed reform. In fact, The Ending Double Standards for Stock Options Act, co-sponsored by Senators Carl Levin and John McCain, was re-introduced in the Senate in February, 2002. If passed, the bill would force companies to treat the cost of their stock option plans exactly the same on income statements reported to shareholders and to the IRS. It does not solve the problem entirely, but it is certainly a step in the right direction.

increase systematic risk. The combined risk incentive can be either positive or negative depending on the characteristics of the firm and the executive. Finally, the choice of exercise price has a substantial impact on incentive effects. In particular, the incentive to increase stock price is maximized at an exercise price that is below stock price but greater than zero. These findings provide new insights on the role of stock options in executive compensation.

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References

Aggarwal, R.K., Samwick, A.A., 1999. The other side of the trade-off: The impact of risk on executive compensation. Journal of Political Economy 107, 65-105.

Berle, A., Jr., Means, G., 1932. The Modern Corporation and Private Property. Macmillan, New York.

Black, F., Scholes, M., 1973. The Pricing of options and corporate liabilities. Journal of Political Economy 81, 637-659.

Carpenter, J.N., 2000. Does option compensation increase managerial risk appetite? Journal of Finance 55, 2311-2331.

Core, J., Guay, W., 2001a. The other side of the trade-off: The impact of risk on executive compensation, a comment. Unpublished manuscript, The Wharton School, University of Pennsylvania.

Core, J., Guay, W., 2001b. When contracts require risk-averse executives to hold equity: Implications for option valuation and relative performance evaluation. Unpublished manuscript, The Wharton School, University of Pennsylvania.

Defusco, R.A., Johnson, R.R., Zorn, T.S., 1990. The effect of executive stock option plans on stockholders and bondholders. Journal of Finance 45, 617-627.

Demsetz, H., Lehn, K., 1985. The structure of corporate ownership: Causes and consequences. Journal of Political Economy 93, 1155-1177.

Fox, J., 2001. The amazing stock option sleight of hand. Fortune (June 15), 86-92.

Grossman, S.J., Hart, O.D., 1983. An analysis of the principal-agent problem. Econometrica 51, 7-35.

Hall, B.J., 1999. The pay to performance incentives of executive stock options. Unpublished working paper, Harvard University and National Bureau of Economic Research.

Hall, B.J., Murphy, K.F., 2000a. Optimal exercise prices for executive stock options. American Economic Review 90 (May), 209-214.

Hall, B.J., Murphy, K.F., 2000b. Stock options for undiversified executives. NBER Working Paper #8052.

Himmelberg, C.P., Hubbard, R.G., Palia, D., 1999. Understanding the determinants of managerial ownership and the link between ownership and performance. Journal of Financial Economics 53, 353-384.

Holmstrom, B., 1979. Moral hazard and observability. Bell Journal of Economics 10, 74-91.

Holmstrom, B., Milgrom, P., 1987. Aggregation and linearity in the provision of intertemporal incentives. Econometrica 55, 303-328.

Ingersoll, J., 2002. The subjective and objective evaluation of incentive stock options. Working paper, Yale University.

Jensen, M., Meckling, W., 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. Journal of Financial Economics 3, 305-360.

Jensen, M.C., Murphy, K.J., 1990. Performance pay and top management incentives. Journal of Political Economy 98, 225-264.

Johnson, S.A., Tian, Y.S., 2000. The value and incentive effects of non-traditional executive stock options. Journal of Financial Economics 57, 3-34.

Kahl, M., Liu, J., Longstaff, F.A., 2001. Paper millionaires: How valuable is stock to a stockholder who is restricted from selling it? Working Paper, UCLA.

Lambert, R.A., Larcker, D.F., Verrecchia, R.E., 1991. Portfolio considerations in valuing executive compensation. Journal of Accounting Research 29 (Spring), 129-149.

Merton, R.C., 1973. Theory of rational option pricing. Bell Journal of Economics and Management Science 4, 141-183.

Ballinger Press, Campbridge, MA.

Meulbroek, L.K., 2001. The efficiency of equity-linked compensation: Understanding the full cost of awarding executive stock options. Financial Management (summer), 5-30.

Murphy, K.F., 1999. Executive compensation. In Ashenfelter, O. and Card, D. eds., Handbook of Labor Economics, Vol. III, North Holland.

Palia, D., 2001. The endogeneity of managerial compensation in firm valuation: A solution. Review of Financial Studies 14, 735-764.

Prendergast, C., 2000a. The tenuous trade-off of risk and incentives. Unpublished manuscript, University of Chicago.

Prendergast, C., 2000b. What trade-off of risk and incentives? American Economic Review 90, 421-425.

Rubinstein, M., 1995. On the accounting valuation of employee stock options. Journal of Derivatives 3 (Fall), 8-24.

Smith, C.W., Jr., Watts, R.L., 1982. Incentive and tax effects of executive compensation plans. Australian Journal of Management 7, 139-157.

Tian, Y.S., 2001. Optimal contracting, incentive effects and the valuation of executive stock options. Working Paper, York University.

William, M.A., Rao, R.P., 2000. CEO stock options and equity risk incentives. Working paper, Georgia State University and Texas Tech University.

Table 1
The executive's discount of option value from its market value

a	β	σ_s	T	Fraction of option wealth (ω)				
				0.1	0.3	0.5	0.7	0.9
2	0	0.3	2	20.6	44.8	61.8	76.5	91.2
			5	23.3	47.5	63.7	77.4	91.4
		0.6	2	32.0	57.9	72.9	84.3	94.5
			5	43.3	67.1	79.4	88.3	96.0
	1	0.3	2	**13.8**	33.7	50.8	68.0	87.5
			5	15.1	34.5	50.6	66.9	86.4
		0.6	2	29.1	54.8	70.4	82.6	93.9
			5	39.3	63.5	76.6	86.5	95.2
4	0	0.3	2	34.5	64.4	80.0	90.0	97.1
			5	37.1	65.8	80.5	90.1	97.1
		0.6	2	48.2	75.0	86.6	93.5	98.1
			5	58.7	80.9	89.8	95.0	*98.6*
	1	0.3	2	25.9	55.1	73.8	86.7	96.2
			5	28.0	55.8	73.4	86.1	95.9
		0.6	2	46.0	73.4	85.6	92.9	98.0
			5	56.3	79.3	88.8	94.5	98.4

This table reports the percentage discount in the certainty equivalent value of a non-traded stock option from its market value (approximated by the Black-Scholes value). The executive's coefficient of relative risk aversion is a. The risk-free rate is 5%. The market portfolio has an expected return of 10% and volatility of 20%. The firm's stock return volatility and beta are σ_s and β, respectively. The option is granted at the money and its maturity is T. Option wealth as a fraction of total wealth is ω.

Table 2
Optimal holdings in the market portfolio

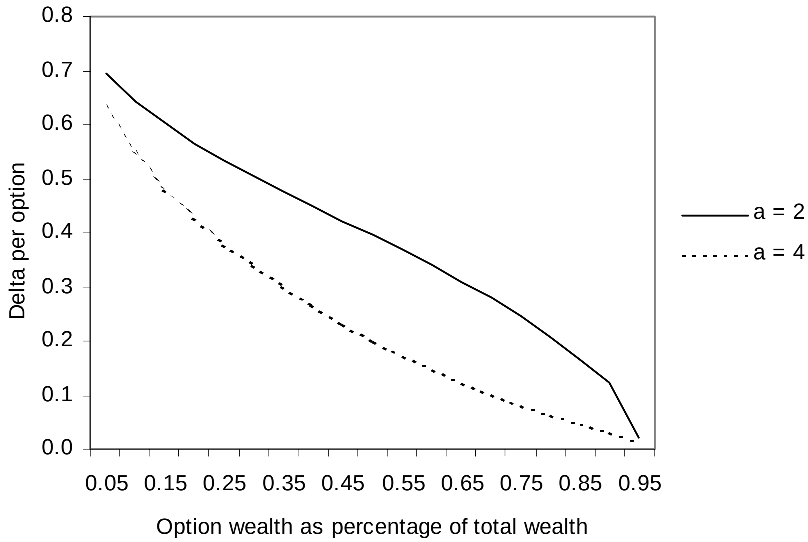
a	β	σ_s	T	No options	Fraction of option wealth (ω) 0.1	0.3	0.5	0.7	0.9
2	0	0.3	2	0.627	0.664	0.681	0.676	0.661	0.640
			5	0.630	0.665	0.680	0.681	0.668	0.646
		0.6	2	0.627	0.654	0.661	0.656	0.647	0.635
			5	0.630	0.650	0.655	0.652	0.647	0.637
	1	0.3	2	0.627	0.392	-0.023	-0.361	-0.459	-0.537
			5	0.630	0.495	0.251	0.025	-0.159	-0.234
		0.6	2	0.627	0.534	0.423	0.347	0.289	0.240
			5	0.630	0.582	0.528	0.488	0.453	0.419
4	0	0.3	2	0.310	0.322	0.322	0.318	0.314	0.311
			5	0.306	0.318	0.319	0.315	0.311	0.307
		0.6	2	0.310	0.318	0.317	0.314	0.312	0.310
			5	0.306	0.312	0.311	0.310	0.308	0.307
	1	0.3	2	0.310	0.075	-0.183	-0.209	-0.214	-0.258
			5	0.306	0.163	-0.007	-0.059	-0.065	-0.076
		0.6	2	0.310	0.224	0.163	0.136	0.121	0.112
			5	0.306	0.260	0.229	0.214	0.204	0.197

This table reports the optimal investment of unrestricted wealth in the market portfolio, with and without non-traded stock options. The executive's coefficient of relative risk aversion is a. The risk-free rate is 5%. The market portfolio has an expected return of 10% and volatility of 20%. The firm's stock return volatility and beta are σ_s and β, respectively. The option is granted at the money and its maturity is T. Option wealth as a fraction of total wealth is ω.

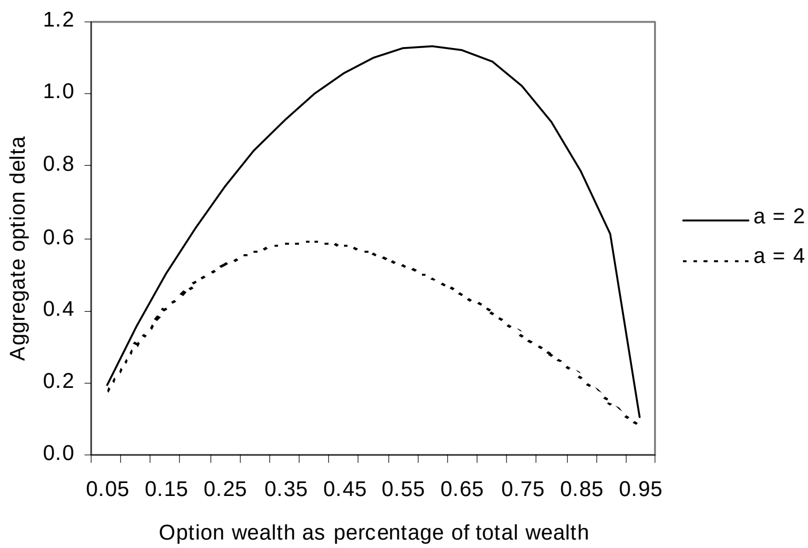
Table 3
Discount in option value and option moneyness

				Exercise price over stock price (K)				
a	β	T	ω	0.0	0.5	1.0	1.5	2.0
2	0	2	0.3	5.3	16.0	44.8	63.9	90.0
			0.5	9.4	25.6	61.8	85.5	95.2
			0.7	11.6	36.9	76.5	92.7	97.8
		5	0.3	12.8	27.5	47.5	64.4	76.5
			0.5	19.2	40.7	63.7	78.4	87.1
			0.7	25.4	54.6	77.4	88.2	93.5
	1	2	0.3	3.0	9.2	33.7	67.6	87.4
			0.5	4.8	15.2	50.8	81.5	93.8
			0.7	6.6	22.8	68.0	90.5	97.1
		5	0.3	7.2	16.8	34.5	53.6	68.8
			0.5	11.3	26.3	50.6	70.2	82.1
			0.7	15.4	37.6	66.9	83.0	90.7
4	0	2	0.3	9.7	28.0	64.4	87.2	95.9
			0.5	15.1	43.5	80.0	93.9	98.2
			0.7	20.5	60.7	90.0	97.3	99.2
		5	0.3	21.0	43.0	65.8	80.2	88.5
			0.5	30.4	59.8	80.5	90.0	94.6
			0.7	39.5	75.2	90.1	95.3	97.5
	1	2	0.3	5.8	17.7	55.1	84.7	95.3
			0.5	9.2	30.5	73.8	92.6	97.9
			0.7	13.0	48.8	86.7	96.7	99.1
		5	0.3	13.5	30.3	55.8	74.9	85.8
			0.5	20.5	46.7	73.4	86.9	93.2
			0.7	28.4	65.2	86.1	93.8	96.9

This table reports the percentage discount in the certainty equivalent value of a non-traded stock option from its market value (approximated by the Black-Scholes value) at various level of option moneyness. The executive's coefficient of relative risk aversion is a. The risk-free rate is 5%. The market portfolio has an expected return of 10% and volatility of 20%. The firm's stock return volatility is $\sigma_s = 0.3$ and its beta is β. The option's maturity is T and its exercise price as a ratio to stock price is K. Option wealth as a fraction of total wealth is ω.



Panel A: Delta per option



Panel B: Aggregate option delta

Fig. 1. Option delta as a function of proportional option wealth. Incentive to increase stock price is measured by option delta which is the partial derivative of option value with respect to stock price. The overall incentive strength is measured by aggregate option delta which is defined as the delta of each option multiplied by the number of options. The parameter values used are: The risk-free rate (r) is 5%. The market portfolio has an expected return (μ_h) of 10% and volatility (σ_h) of 20%. The firm's stock return volatility (σ_s) is 30% and its beta (β) is 1. The option is at the money ($K = 1$) and its maturity (T) is 5 years. The executive's coefficient of relative risk aversion (a) is either 2 or 4.

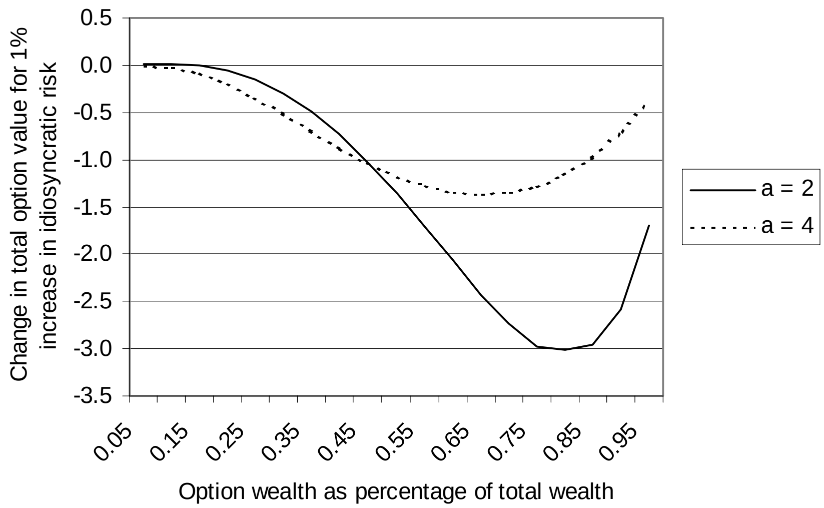


Fig. 2. Incentive to increase idiosyncratic risk. The incentive to increase idiosyncratic risk is defined by the partial derivative of option value with respect to idiosyncratic risk (σ_e). The parameter values used are: The risk-free rate (r) is 5%. The market portfolio has an expected return (μ_h) of 10% and volatility (σ_h) of 20%. The firm's stock return volatility (σ_s) is 30% and its beta (β) is 1. The option is at the money (the ratio of exercise price to stock price $K = 1$) and its maturity (T) is 5 years. The executive's coefficient of relative risk aversion (a) is either 2 or 4.

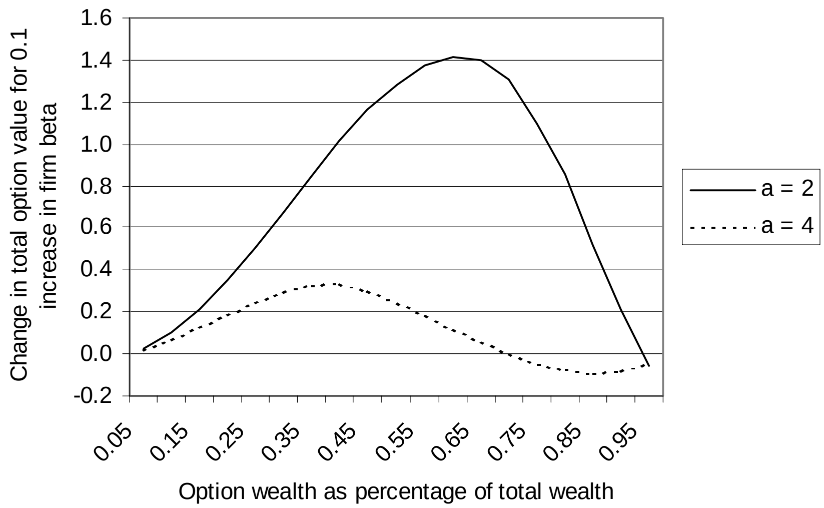
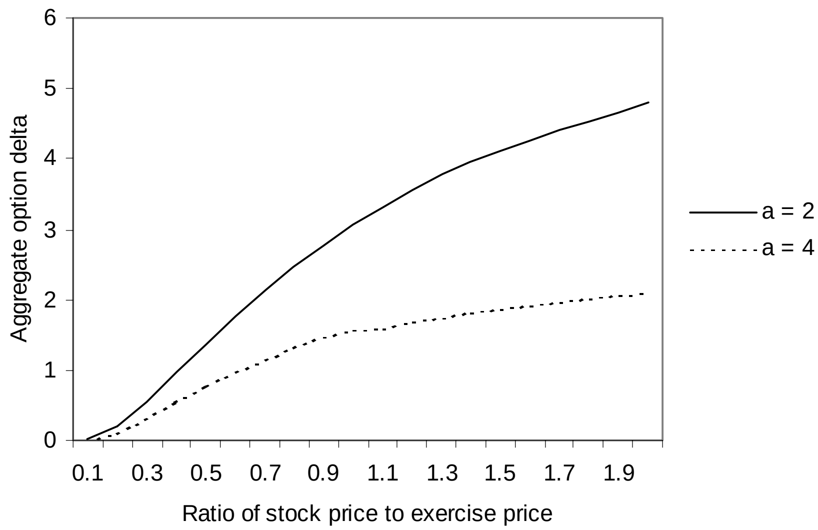
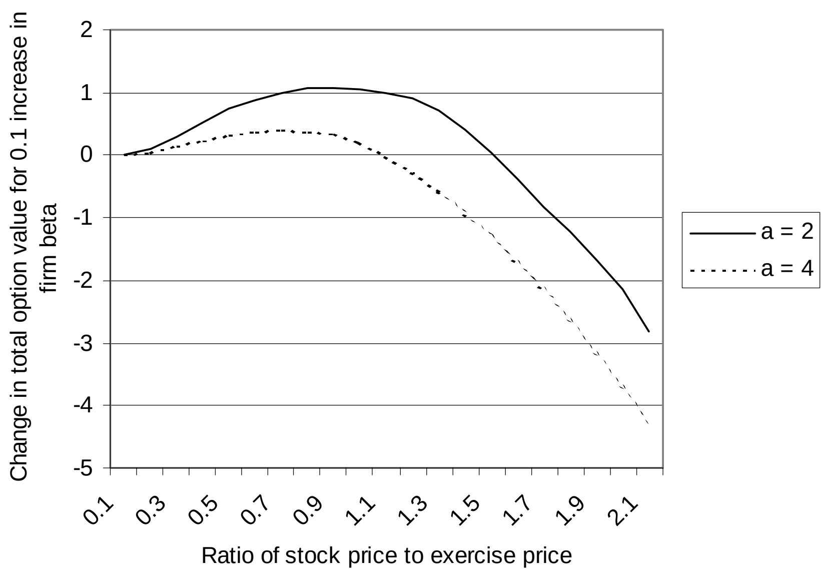


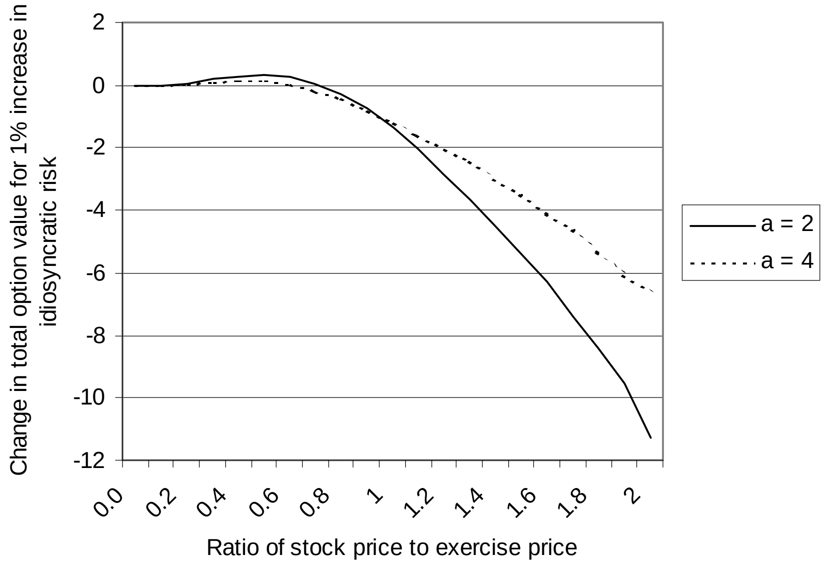
Fig. 3. Incentive to increase systematic risk. The incentive to increase systematic risk is defined by the partial derivative of option value with respect to systematic risk (β). The parameter values used are: The risk-free rate (r) is 5%. The market portfolio has an expected return (μ_h) of 10% and volatility (σ_h) of 20%. The firm's stock return volatility (σ_s) is 30% and its beta (β) is 1. The option is at the money ($K = 1$) and its maturity (T) is 5 years. The executive's coefficient of relative risk aversion (a) is either 2 or 4.



Panel A: Incentive to increase stock price

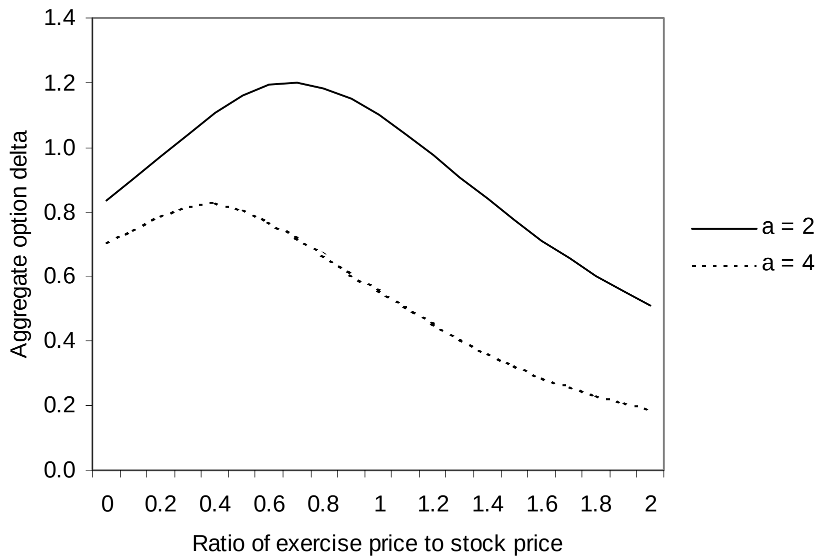


Panel B: Incentive to increase systematic risk

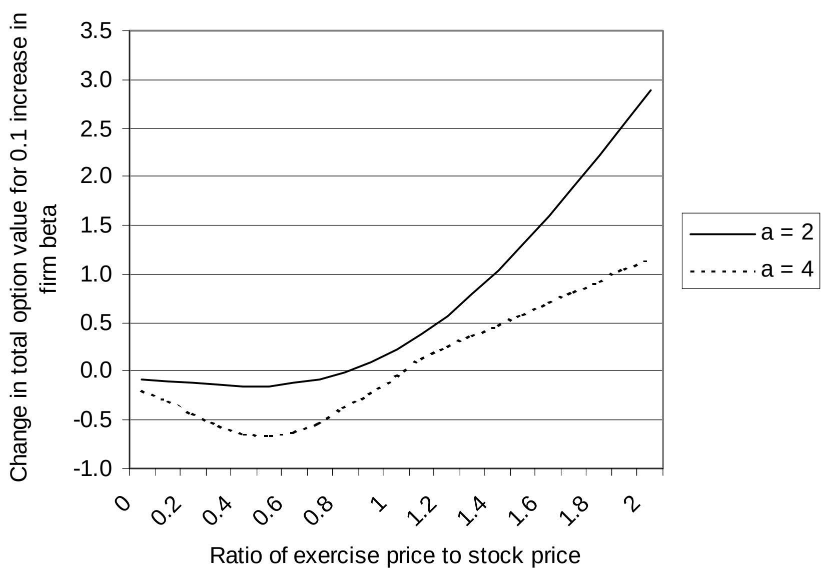


Panel C: Incentive to increase idiosyncratic risk

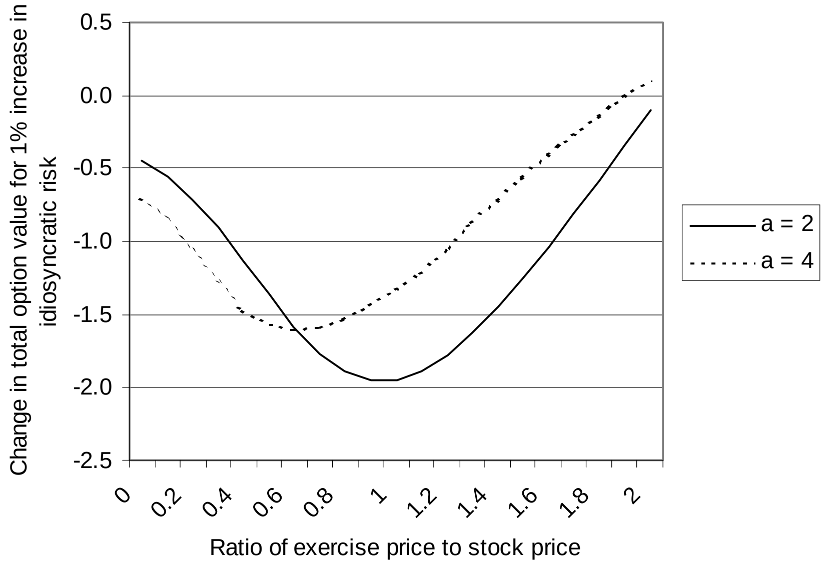
Fig. 4. Stock price fluctuations and incentive effects. The options are granted at the money ($S_0 = X = 50$), but a firm-specific shock occurs immediately after the grant date and causes the stock price to fluctuate between 0 to 100. The parameter values used are: The risk-free rate (r) is 5%. The market portfolio has an expected return (μ_h) of 10% and volatility (σ_h) of 20%. The firm's stock return volatility (σ_s) is 30% and its beta (β) is 1. The option has 5 years left to maturity (T). The executive's option wealth is 50% of total wealth and his coefficient of relative risk aversion (a) is either 2 or 4.



Panel A: Incentive to increase stock price



Panel B: Incentive to increase systematic risk



Panel C: Incentive to increase idiosyncratic risk

Fig. 5. Exercise price and incentive effects. While keeping the total cost of the option grant fixed, these figures illustrate how the choice of exercise price impact incentive effects. The parameter values used are: The risk-free rate (r) is 5%. The market portfolio has an expected return (μ_h) of 10% and volatility (σ_h) of 20%. The firm's stock return volatility (σ_s) is 30% and its beta (β) is 1. The option has 5 years left to maturity (T). The executive's option wealth is 50% of total wealth and his coefficient of relative risk aversion (a) is either 2 or 4.