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SCHOOL OF ECONOMICS AND BUSINESS

MASTER'S THESIS

CHALLENGES IN DETERMINING THE RISK-FREE RATE FOR CORPORATE VALUATION IN ZERO INTEREST RATE ENVIRONMENT

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LIST OF ABBREVIATIONS

CAPM – Capital Asset Pricing Model
CDS – Credit Default Swaps
CRA – Credit Rating Agency
FCFF – Free Cash Flow to Firm
FFM – Fama-French Model
GDP – Gross Domestic Product
IIF – Institute for International Finance
IMF – International Monetary Fond
LSAP – Large Scale Asset Purchases
MBS - Mortgage-backed Securities
OECD - Organization for Economic Co-operation and Development
OTC – Over the Counter
QE – Quantitative Easing
RF – Risk-free
RRB – Real Return Bonds
TV – Terminal Value
WACC – Weighted Average Cost of Capital
YTM – Yield-to-Maturity

INTRODUCTION

When it comes to the field of corporate valuation, finance professionals have developed and used many formulas to calculate necessary financing costs of their businesses. Among the most used formulas to calculate the costs of financing is the Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Lintner, 1965) and its adjusting factors such as Fama-French factor models (2012) that have been added in recent years as a discounting rate for future business earnings.

Arguably the most important component and starting position of the above-mentioned formula and of the overall valuation process is determining the safest asset on the market to which every other investment would be compared. This component is called risk-free rate (RF) in finance industry and it is a component of discount rate (Damodaran, 2013). As a benchmark, investors have explicitly used yields on government bonds in their calculations, since they best presented the nations' financial strength, accompanied by the government's power to tax, print and confiscate properties. Throughout history, debt instruments have proved to be the most certain instruments when investing due to their structure and purpose which best define future obligations of involved parties (BIS, 2013).

However, in recent years, and one could say as a result of the 2008 Great Recession, we are witnessing a new phenomenon in finance where almost every developed, OECD member country is experiencing near zero or negative yielding government bonds (Investing.com, 2020). This comes as a result of an increase in direct participation of developed countries' Central Banks in markets in order to stimulate their fallen economies through lowered interest rates for commercial bank funding, pressuring every other rate in the economy downwards. Additionally, the quantitative easing (QE) programs of these central banks via which they bought government bonds, further depressed the yields (Hausken & Ncube, 2013). Models and finance intuition do not account for negative values of inputs, which explains why the calculations are providing distorted results.

Approaches have been implemented or suggested on how to deal with extraordinary low rates. Duff and Phelps (2019) believe it is reasonable if one decides to "normalize" US rates to their historical average of around 3%, this being based on the assumption that rates are about to reverse from current state towards the mean or above. Problems may arise if it stays at that level for longer. Koller, Goedhart and Wessels (2015), on the other hand, recommend creating a synthetic RF for the US market by taking inflation expectation rates, which they found to be around 2.5%, and adding it to the long-term historical real yields of around 2%. Real yields are nominal yields on government bonds but adjusted to exclude inflation effects to reflect real costs of borrowing. They argue that such move is necessary due to the abnormally low interest rates arising as a by-product of FED's fight with the 2008 recession. Valuations will later have to be updated to reflect new realities when rates increase as the economy gears up again.

Damodaran (2013) provided an argumentation on two rules and a principle which have been developed for investors to think about when they search for RF security. These securities should not have reinvestment risk, meaning that changes of buying price when initial bond matures does not vary compared to before, so it comes as an outcome that longer term bonds are preferred. They also should not have default risk, meaning there should not be uncertainty of necessary payments when matured. The principle, on the other hand, is consistency, and it has to be followed in order to extract precisely the needed information or result. This principle requires every practitioner who is in a quest to find a value of a business to pay special attention to inputs such as currencies, maturities, instruments, etc., to be matched and used throughout the analysis and not changed from one occasion to another. The same is true for the RF rate, since the currency it is denominated in has to be the same as that of cash flows of a firm analyzed.

The specific rate used in the process throughout history has been yield-to-maturity (YTM) of sovereign bonds, which vary depending on the maturity of the bond. Longer-term bonds usually hold higher values due to time preference of money, which explains how people value their choices between present and future consumption, thus driving interest rates up or down and are summed up as a term structure of rates.

Since by definition RF rates must not hold default risk and because only sovereign bonds qualify, the starting position in the search for one should naturally come from those best rated or triple-A rated. But even among them not every is truly considered riskless by markets. Since just few biggest credit rating agencies (CRA's), called the Big Three, consisting of Moody's, Fitch Group and Standard & Poor's, controlled collectively 95% in 2013, there is a growing concern that they are too biased in their credit valuations, thus to some degree increasing risk for those who decide to take their analyses in consideration. As of January 2020, those agencies determined that the best quality government debt rated Triple-A was issued by 10 countries of which 8 are from Europe and N. America: Denmark, Germany, Netherlands, Switzerland, Canada, Norway, Sweden and the USA (Trading Economics, 2020).

Historically, the two most often used benchmark securities in the Western world which satisfy both of these rules are the US 10-year Treasuries and German Bunds, mainly due to their liquidity (Damodaran, 2010b). It is hard to think that the QE process will slow down or even stop since the countries where it is implemented deal with structural issues of persistently low real GDP and inflation growth, while being influenced by big demographic problems (Bloom, 2020).

Another challenge worth discussing deals with RF rates when considering a discount rate for the Terminal Value (TV). The practitioners so far explicitly used spot 10-year yield, practice also backed by academics (Damodaran, 2013). But the QE asset purchase programs squeezed the yields even on European bonds with maturities like 30- and 50-year ones. Investors believe that these bonds will also increase in supply and be of similar liquidity as benchmark sovereigns today.

The purpose of this thesis is to identify and describe all the challenges investors face when they delve in the process of determining RF rates for a company valuation of their choice in today's environment of exceptionally low and/or even negative interest rates in Europe and North America. Since they used to neglect most of them when doing their fundamental analysis and just took spot yields on Treasury bonds, I believe it would be useful to enlist pros and cons of each challenge and approach in dealing with them to better navigate growing uncertainty on markets. The challenges that will be researched among many others are formulation of assumptions, currency of valuation, liquidities and maturities of appropriate benchmark security, rates' normalization and synthetization. The scope of their description will enable more thorough insight on how investors could navigate current economic environment.

The first objective of this thesis is to review the financial theory relevant to this topic and study major developments which helped in fostering current environment. This will make sure that the readers are well equipped to proceed reading the rest of the work with understanding. Then, past data on rates of benchmark securities, their liquidities and maturities in European and N. American markets will be collected and analyzed, so that the theory can be combined with them to create a better explanation of the current environment.

The research questions this master's thesis seeks to answer are:

1. What makes the current interest rate environment really so unique?
2. Why the current sovereign bonds market of Europe and N. America differs from its past?
3. Are Quantitative Easing measures affecting liquidity of government bonds?
4. What should be done in case there is no safe asset?
5. Should analysts normalize, synthetize or just take the current market rate as a proxy for the risk-free rate?
6. Are longer-maturing segments than 10-year one liquid enough to be an alternative to the benchmark used today in valuations?

1 COMPANY VALUATION, INTEREST AND RISK-FREE RATES THEORY

The following text aims to encompass the theory relevant to the topic of the research. It covers the theory on company valuation, more specifically, the fundamental analysis of a business, the capital asset pricing model, its components and recent adjustments, interest rates in general, the term structure of interest rates and the structure of risk-free rates itself. An extensive number of publications dealing with the above-mentioned topics is available today for those wanting to delve into researching them in detail, but only the most comprehensive and relevant ones will be considered here.

1.1 Basics of fundamental company valuation

The development of capitalism we see today has been based on the way individuals organized into companies, where each individual of an organization contributed to the value creation process with their skills and knowledge. As they adapted to capitalistic systems, the accurate determination of value that each company contributes to the system continued to improve with the increasing competition.

Damodaran (2013) outlines several ways to value businesses, grouped into major categories called fundamental analysis, relative valuation and contingent claim, while the use of different technics primarily depends on the type of the business and purpose of the valuation. Among the three, relative valuation is most often used in practice, by which investors find comparable companies in order to determine their price on the market, but fundamental analysis is the base ground to finding value of a business and is much often neglected in the overall process.

Investors like Warren Buffet, who base their investment decision on the long-term potential of a company, highly depend on the fundamental analysis to find the value of a business so they can exploit market mistakes on over- or undervalued companies. Crudely speaking, the process of fundamental analysis depends on the financial statements of companies, market interest rates, and the overall condition of the economy.

The cash-flows estimated for the future periods, over which investors conduct the analysis, determine the value of a company in two ways: 1) as an after-tax earnings generated by the firm's assets before debt payment and including reinvestments, and 2) an after-tax earnings after the payment of debt and reinvestment which can be assigned to the company's equity.

Each of these require its own discount rate with which the value of these future cash-flows will be brought to the present. The cash-flows to the firm require the cost of capital to be calculated as weighted average cost of capital (WACC), taking into account debt and interest payments adjusted for tax effects, while the cost of equity capital, calculated by the capital asset pricing model (CAPM) and its recent additions for Fama-French model (FFM), discount cash-flows only to equity holders. The determination of the latter discount value might be considered as the most delicate issue in the valuation process, since it allows biases to be introduced into the valuation process through some of its components.

Since this thesis focuses on discussing RF rates, which is probably the main component of the CAPM with which investors determine the discount rates and value the company's cash-flows, it is useful to further understand how it found its way into the valuation process. To do this, a guide through the components of the CAPM formulas will be made for better understanding, which will be a base for further writing.

1.2 Capital asset pricing model (CAPM)

Valuing a company based on its fundamentals requires accurate forecasting of cash-flows to be discounted by an appropriate discount value to the present, either to equity or asset holders. The difference between the two is the amount of earnings attributable to debt holders of the company, and what is left after its payment is required to be disposable to equity holders. In that sense, equity is much more risky compared to debt and demands a premium over the safest alternative in the market in order to invest (Berk & DeMarzo, 2017, p. 445).

Investors have always been trying to find the formula that calculates these premiums most accurately. The first written explanation of our understanding today of stock value and market premiums is published by Sharpe (1964) and Lintner (1965), and later expanded by Fama and French (2011) with the addition of factors like size and value. Sharpe summarized his findings as a Securities Market Line (SML) consisting of a pure interest rate and risk-return trade-off.

The SML represents the investor's choice of an investment when an asset exists whose return compensates only for the time during which it locks in the funding, with no uncertainty of repayment. In this case the investor places their wealth proportionally (α) between the pure rate of investment P and the risky alternative A according to their aims and summed up by the following equation:

$$E(\mathrm{Rc}) = \alpha\, Rp + (1 - \alpha) Ra \qquad (1.2.1)$$

In order to value individual investments, it is necessary to come up with the value of that individual investment. To do this, the premium by which investors could demand more above the safest alternative has to be calculated. With the rise of stock indexing, the risky alternative came to be best represented by a stock market index that grouped the biggest companies in the market observed. Thus, if the market index represents a portfolio for the analysis, each investment has to be valued based on its risk relation to the portfolio, also known as beta (β), and calculated as:

$$\beta = Cov\,(Ri\,, Rm) / Var\,(Rm) \qquad (1.2.2)$$

The numerator in the Equation 1.2.2 represents covariance, which is the relationship between movements, of investment and the market index, while the denominator represents the variance of the market index. Adjusting the initial expression (Equation 1.2.1) with the beta coefficient to find the value of an individual investment, investors come up with the following formula which represents CAPM approach to the valuation of stocks over the pure interest rate:

$$Re = RF + \beta\,(Rm - RF) \qquad (1.2.3)$$

In this equation, Re stands for return on equity or individual stock, RF is risk-free rate representing pure interest rate, β is beta coefficient which adjusts the pure rate for equities

risk compared to the Rm, which is return on the market portfolio (Berk & DeMarzo, 2017, p. 440).

From the final equation one can see why riskless security is so important today, as it is a building block of further analysis that adds on to it for the final value. The following text expands more on what the pure rate of interest represents as well as solutions to find such a rate for the valuation process. In order to do that, an explanation of what interest rates are will be given as well as how they relate to investments. Further on, the value of time dimension will be explained so that the connection between RF asset and time could be understood.

1.3 Interest rates

In the finance and investments industry, the core driver of activity is the determination of the return one might receive for parting ways with ownership of certain amount of capital over a period of time. Depending on the type of security being invested in, whether fixed income or variable, it can be called either interest rate or return on equity respectively. However, due to the simplicity of fixed income securities, interest rates, which represent the amount to be received in terms of percentages of the amount of principal landed, have established themselves as building blocks in the calculation of variable types of securities. Their significance to everyday functioning of nations has been evaluated by many economists throughout history and today most of the theories on interest rates can be organized as having two views: classical and neoclassical. Both of them have provided fair amount of explanations on how our societies have conducted financial and investment activities and how these contribute to the overall functioning of countries.

1.3.1 Classical versus neoclassical perspectives

The classical theories on interest rates are best summarized by Adam Smith (1776). In his book, "The Wealth of Nations", he conceptualized the whole process by which nations could create, preserve and lose their wealth. His theories on division of labor, pursuit of self-interest and freedom of trade are most notable. For today's economics enthusiasts this does not seem like a problem, but it used to be at the time of his writing. Other economists, like David Ricardo and Thomas Malthus, later went on to discuss additional topics, but this thesis will focus on Smith's book for the purpose of understanding generalities about interest rates which in my opinion summarizes them best.

Smith (1776) discusses interest rates as a balancing mechanism between saving and investments. He explains how those who wish to save excess of their money do it by landing it as a capital for certain productive purposes. The percentage of lent money is determined as an interest based on the productivity of an investment to which it has been assigned, and

in the process creating new capital to be invested or consumed later. If there was too much capital available for lending, but too little available investments, interest rates would then have to adjust downwards to reflect the market's new positions and would provide cheaper borrowing for existing investments. Inside this mechanism, the rates are argued to be driven solely by the market's demand and supply of productive investment opportunities and no other origin of new wealth was to be established if it was not conducted in this manner. These theories went on to transform the way economics is being taught in schools and how individuals are to conduct their business in order to gain wealth and build their nations.

As opposed to the classical school of thought, John M. Keynes (1936) published his "The General Theory of Employment, Interest and Money", providing several new theories on functioning of economies and markets. A theory important for this thesis is liquidity-preference.

Keynes (1936) strongly disagrees with the classical school of economic thought in terms of self-sustaining equilibrium of interest rates and relation of savings, consumption and investment demand and supply. He believes that markets tend to overreact in times of uncertainty when the demand for liquid assets, such as money, start to eclipse the available supply of money and in the process creating deflationary pressures. He sees the basis for this theory in people's tendency to hoard money in their mattresses in times of perceived increase in possibility of future uncertainty. This tendency, he explains, proves that savings do not always equalize investment for certain degree of interest, but rather comes to create savings which bear no interest and make investments relate to a rate which induces an individual to forgo this liquidity preference for future consumption. He goes on to expand this by establishing that people act according to this theory because of their psychological time-preferences for money, which can be separated in two parts: propensity to consume and savings form. Both of them contributed to economics significantly. The former establishes that in order for an individual to start considering what they might consume, it is important to first establish the degree of income from which consumption might originate. The latter of the two reinforces the meaning of saved money and the need for an interest rate which induces individuals to part ways with current liquidity. Irving Fisher and Milton Friedman, among many others, are notable economists that later went on to expand the teaching of neoclassical economics.

Both of these schools provided great insights into the economics behind the creation of wealth and is irresponsible to choose who won the argument, because the degree of their contribution to how to tackle the issues that stem from the national economies of the world is enormous. Both of them helped in making countries better and more effective on a global stage, depending on the state one country found itself in. Keynes's theories came to be in a time when the Great Depression wracked havoc around the global economies and it was pivotal for the governments to be involved more in the markets to restore the confidence by stimulating demand. The classical economic teachings of Adam Smith, on the other hand, originated at the end of the British imperial powers.

1.3.2 Central banking

Today's interest rates are primarily driven by country central banks and their monetary policies. The first central bank was the Bank of Amsterdam founded in 1609 and operated as a warehouse for citizens' metals on which they issued receipts in a ratio of one-to-one. While at first it did not issue loans, later it started doing so to help the Dutch East India Company and from that point it started earning interest on the coinage difference between issued and deposited metals (Aliber & Kindleberger, 2015). By the end of the 19th century, the central banks have spread throughout the European countries. This happened in the USA in 1913 with the implementation of the Federal Reserve Act, and in Canada in 1934 (Schnidman & MacMillan, 2016, p. 11). They all operated on the same principle of taking deposits of metals, such as gold or silver, and issuing exchangeable notes in predetermined ratios back into these metals.

As the US dollar reached a level of global reserve currency, it started to be much harder to execute dollars convertibility into gold compared to before and the USA's financial system was increasingly being threatened because of this. It all changed when the US president Nixon stopped the convertibility of the dollar into gold in 1973, from which point all global currencies were valued solely in relation to the dollar. This move enabled central banks to start playing greater roles in determining market interest rates and fighting recessions. Their targets of inflation rates, together with the economy's real output growth, drove the determination process of interest rate levels in their monetary policies. As they did not have metals anymore to relate to, their monetary policy and freedom to print money became of much greater interest to governments since their borrowing costs consequently depended on it. Many OECD governments borrowed in excess of their revenues to continue their spending practices, indebtedness among most of them grew to the point of making pressure on fiscal sustainability of these countries. The Central Banks ultimately continued to lower rates all the way to zero percent to avoid financial collapses which might originate due to indebtedness but also to counter recessions occurring in the meantime. Currently, interest rates in many developed countries went into negative territory hoping to spur economic development, but threatening to be counterproductive on the banking industry. It still remains to be seen whether these rates will bring the development they are aiming to achieve or will it end up costing taxpayers huge amounts of money to combat recession like it did the last time when they had to bail out some financial institutions to save the system from collapsing.

1.4 Structure of Risk-free rates

Arguably the most important component and starting position of the above-mentioned asset pricing formula and of the overall valuation process is the determination of the least uncertain asset to which everything else would be compared. This component has in finance industry been called *risk-free rate* (hereinafter RF rate) and it is a component of discount

rates, or better to say the cost of the capital used, by which we bring the estimated future cash flows into the present due to the theory of time value of money and which presents a key point when deciding between investments with different uncertainties associated with it (Berk & DeMarzo, 2017, p. 97). The RF rate is utterly important as an input into the CAPM formula to calculate the total return on equity invested one should receive on any other investment.

As the meaning and function of interest rates overall have just been covered, it would be useful to explain how they relate to RF rates and what constitutes them. By the definition (Damodaran, 2010b, p. 144) the RF rate is supposed to represent an investment whose estimated return is exactly the same as its actual return, meaning no deviations on those returns should occur to be considered safe. Further, it should be free of default and, in case of long-term investment intentions, reinvestment risk. Thus, it is possible to take its return as uncorrelated to other risky investments if we hold on to the definition mentioned before, since its value is affected only by the systemic risk of the entity it represents (BIS, 2013). Like the rates implemented by the central banks, RF rates reflect real economic growth of a country, as well as inflation rate expectations, except certain securities specifically designed to protect against inflation, in which case they represent real RF rates of an economy.

1.4.1 Government as a source of safe securities

Since the RF rate is used as a basis in equity calculations, the only option for individual investors to consider as such are government issued debt securities as they do not hold idiosyncratic risk like individual stocks. Debt instruments in particular have been used as instruments of greater certainty in investing due to their structure and purpose, defining future obligations clearly for involved parties. Government bonds in particular incorporate all needed features to be considered RF due to their representativeness of the whole nation and its overall wealth and, more importantly, they are backed by the government's abilities to print currency, raise taxes or confiscate properties when necessary to meet the obligations stipulated by the contracts.

Unfortunately, not every government issued bonds should be considered riskless. Many are carrying certain degree of possibility to default on their obligations, either as a result of mismanagement of their public finances, refusals to make payments of prior regimes or refusal to pay borrowings made in foreign currencies (BIS, 2013). This automatically rules out the government bond which possesses just a slight risk of default to qualify as RF.

A detailed analysis and risk estimation of all government securities is conducted by credit rating agencies, but also by the markets in a form of tradable securities like credit default swaps (CDS). These agencies aimed to identify and analyze all the relevant factors influencing the real value of sovereigns and providing the results as a ranking. Within those ratings, government bonds were placed in predetermined brackets signifying the quality of debt they offered to the public compared to others, driving the best ones to the top. As of

January 2020, those agencies determined that the best quality government debt rated Triple-A was issued by 10 countries: Australia, Denmark, Germany, the Netherlands, Switzerland, Canada, Norway, Sweden, Singapore and the USA (Trading Economics, 2020). The belief in credit rating agencies and their ratings is ingrained in the investor's assumption when determining the RF rate, something that will be expanded in the following text.

Governments are also offering bonds with different maturities to satisfy market demands for longer-term safe securities, since people like to put their savings in accounts for future consumption when they believe they might need them more. This provides investors with the ability to match the duration of their returns on investment with the duration of bonds and in the process remove the reinvestment risk if holding shorter-term bonds. Damodaran (2010b, p. 149) argues that longer-term government bonds ought to be used in business valuations, because one who is buying a company will most likely hold it for a longer period of time, thus witnessing cash-flows for many years. These future cash-flows have to be valued in present terms and we need the RF rate as an input to asses valuing models matching these cash-flows. He specifically argues the use of 10-year Treasuries for the US investors and companies, but also for outside valuations since they carry the least amount of default risk due to the US government's track record and robustness of the economy, but also reinvestment risk. Companies are assumed to have infinite lives and to live in excess of 10 years, but the difference between 10- and 30-year Treasuries is really small, while the data on additions, like equity risk premium and default spreads, are more available for the former. Both risks mentioned above are of enormous importance when we invest in a firm for a longer period, because it is important to match the duration of the RF instruments with our investment horizons.

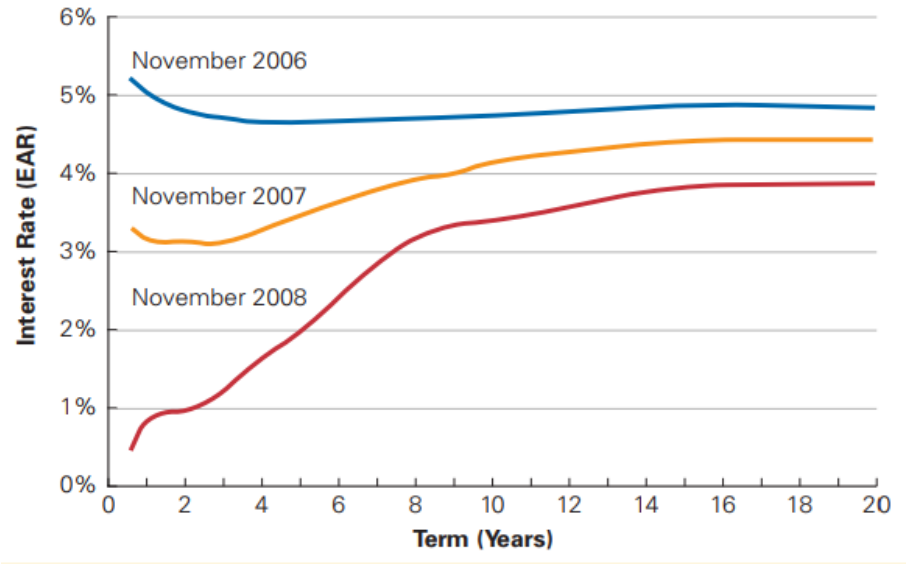
1.4.2 Origins of riskless rates

In the Renaissance age, the 15th and 16th centuries, sovereign bonds were perceived as the least safe and favorable investment one could make. At that time government was perceived as reckless and warmongering so only those who had strong ties to, and had some favors they wanted from the governments in return, were the ones investing and in a way influencing the actions done by them. Really high yields were featuring such bonds in order to attract any investment into the government, while people still opted to invest in private businesses and merchants prior even considering loaning to "dangerous" governments. Unexpected movement that changed this kind of government perception was a "financial revolution" that started in Genoa, Italy, enabling institutions to be more concrete in collecting the revenues. More and better managed revenues translated into more liquid sovereign bonds and these forces pulled yield to their lowest levels, creating a platform for such securities to be considered the .safest assets today (BIS, 2013, p. 19).

1.5 Term structure of Interest rates

Interest rates on outstanding public government bonds are also called yield to maturity (YTM) of the corresponding bond and, in case of riskless ones, are composed of expectations on future inflation and real rates in a market. By plotting together YTM of government bonds of different maturities graphically, one can create *a yield curve* also called *term structure of interest rates* which may later be used to analyze a market's future expectations about the economy and its interest rates. These sentiments take three forms of yield curve: normal/upward slopping, inverted/downward slopping and flat (Berk & DeMarzo, 2017, p. 184).

Figure 1: Yield curves



Source: Berk & DeMarzo, 2017, p. 184

The first yield curve is usually prevalent in the economy, where investors expect higher rate of return the longer they have to hold the bond. The second yield curve, on the other hand, presents more pessimistic expectations in the market about future capacity of the state to meet its obligation. This kind of curve is usually interpreted as the signal of recession expectations in the future and places more value on payments whose maturities are as late as possible. The last one is that of a flat yield said to represent sentiment of uncertainty in the economy with either of the prior mentioned outcomes possible in the future.

1.5.1 Theory behind the term structure

Many theories have been developed in the past to explain why these yield-driving sentiments move in the way they do. The first person said to provide the explanation of these sentiments is Irving Fisher (1896) who formulated a *theory of unbiased expectation* in which

he argues that if investors are rational and unbiased in their expectations about the future, then future short-term rates on average are equal to current long-term rates on those bonds. But this theory produced much discussion and disagreement in the academic community, which ultimately culminated with the formulation of an additional theory called *liquidity premium theory* introduced by John Hicks (1946). He considers investors' preference for current liquidity over future, arguing that the further one decides to determine interest rates, the greater the effect of a change in those interest rates will be and as a consequence the uncertainty. This extra yield necessary to hold longer dated instruments came to be liquidity premium. The theory about the term structure of interest rate expectations, called *market segmentation hypothesis*, proposed by Culbertson (1957), Modigliani and Sutch (1966) tries to blend both of the prior ones, arguing that investors have preferred habitat according to the projects or investment they have considered and by which they measure relevance of interest rates (Maranga, Mwangi & Kaijage, 2018).

2 MAJOR MARKET DEVELOPMENTS DRIVING YIELDS

This part covers the new developments mainly occurring among OECD countries and stemming as a result of the 2008 financial crisis. A decade after the impact of the Lehman collapse, the developed countries find themselves still battling low growth and inflation even after extraordinary measures have been taken by Central Banks such as near-zero interest rates and large asset purchase programs, also called quantitative easing. The following chapters will cover the functioning of the above-mentioned measures in more detail as well as their outcomes during the last decade, such as increased indebtedness and possible joining of debt by Eurozone Member states. This part serves to expand the scope of the latest macroeconomic insight relevant for better navigation around the challenges in the following parts of the thesis.

2.1 Zero and Negative Interest rates

The European sovereign debt crisis, which originated in the EU as a response to the 2008 recession in the USA and intensified as of 2011, challenged the EU project in the greatest extent ever, as sovereign borrowing markets in some of the most prominent countries started showing signs of collapse. These countries, commonly called PIGS (representing Portugal, Ireland, Greece and Spain, with Italy also occasionally added), experienced great public fiscal stress and inability to collect taxes. OECD declared it to be the greatest risk to the global financial system if not properly handled and risk mitigation measures were not implemented (OECD, 2011).

As a response to the crisis in Europe, the central banks in coordination with the governments of many OECD decided to lower the basic funding rates to zero, hoping it will stimulate the

economy, intending also to lower bond yields and increase growth to counter recession. It represents a new phenomenon in finance where many fiscally responsible developed, OECD members consequently started experiencing near zero or negative yielding government bonds as a result of these central bank actions. Such a phenomenon is influencing every corner of the economy, from the stock market and pensions, over housing market and unemployment to inequality and health. This is also an answer to the second research question. Nothing is left unaffected since its values are incorporated in valuations of everything in a form of a RF benchmark.

2.1.1 Sustainability of near-zero rates

An IMF (2017) report provides a detailed analysis of how negative rates might reflect on the financial industry and what could possibly be done to deal with the position these countries found themselves in. They argue that further rate cuts will have mixed results, with the biggest banks, whose clients depend on them heavily for daily transactions, not being able to implement them further due to higher switching costs. To overcome the possibility of a collapse among smaller banks, they advocate consideration of implementing digital currencies, which requires much less storage, handling and transportation costs than hard currencies and would enable facilitation of interest rate cuts further into negative territory (Agarwal & Krogstrup, 2019). In case this suggestion proceeds towards actual implementation, which is more likely to happen with every passing year of low growth and low inflation, some estimates provide banks with the ability to decrease rates even by as much as 600 basis points to counter big future recessions with an estimated limit reaching negative 4 percent without hurting the profitability of the banking industry (Agarwal & Kimball, 2019).

The successful implementation of digital currency would enable deep negative interest rates in banking for longer periods of time and would also translate into further decrease in already negative yields of some governments bonds. Actions to combat negative yield numbers being used in asset pricing formulas, such as rates normalization or synthetization, by which artificial numbers are created that relate to their historical RF yields, might become obsolete. If this happens, it will mean that a part or maybe all of the loan balance companies under the evaluation process, would end up being a source of revenue in the amount of negative levels their rates are in. As of July 2019, two European government bonds rated Triple-A and considered default-free are Germany's Bunds with $1.5 trillion and Dutch with $342.1 billion negative yielding debt out of the global total of $14 trillion negative yielding bond market, with Canada on the other side of the Atlantic holding just $23.7 billion (Ainger, 2019). Further negative yields in these countries would allow them to refinance their existing stock of debts at better terms translating into yet more improvements in their fiscal positions.

2.2 Quantitative Easing

To counter the effect of the 2008 collapse, the central banks started a new and unconventional monetary policy called "Quantitative Easing" (QE). In this way, they supplemented their existing near-zero policy for funding rates targeting inflation and growth. The term itself was first mentioned by the German economist's, Dr. Richard Werner, 1997 paper which outlined the challenges of the Japanese central bank intervention on the state of Japan's economy (Werner, 1997).

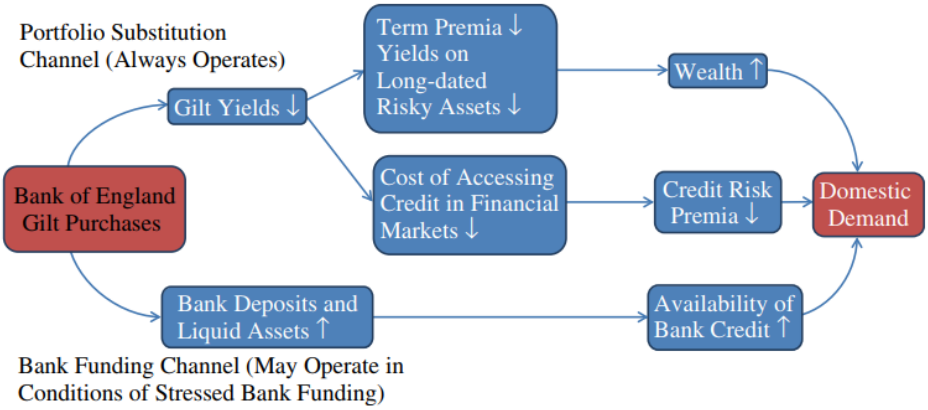
Initially, FED and ECB followed BOJ and started buying short-term government and other collateralized securities such as mortgage-backed securities (MBSs), but later resorted to long-term government securities. The only difference between FED and ECB was in the way they implemented QE. FED went on to buy Treasuries from the market private non-bank entities, while ECB did it through repo market, where bonds served as a collateral for loans (Joyce, Miles, Scott & Vayanos, 2012).

2.2.1 Effective mechanism

Researchers (Joyce, Miles, Scott & Vayanos, 2012) have created a stylized version of prior works on QE mechanism by which they affect market values, among them the government securities of the UK. Such mechanism can also be representable for other major countries implementing QE programs and in that way be relevant to the research of this thesis.

Two channels have been described by which BOE's asset purchases affect prices, namely portfolio substitution and bank funding channel. Through the former it manages sovereigns outstanding on the market by buying them from non-bank institutions such as pension funds and insurance companies, which in this case hold the bulk of them. Many non-bank institutions, in order to compensate for the missing duration on their portfolio due to the sale of sovereigns, resort to more risky long-term assets, such as corporate bonds and prefer equity or even longer maturing bonds to capture a return, which also drives prices for these assets up. An additional demand originating from these institutions decreases yields of all long-term assets, thus affecting the rest of the term premium. Through the latter channel they manage credit availability on the market, that is, the extent of excess reserves it holds.

Figure 2: BoE's Gilt Domestic Demand Impact channels



Source: Joyce, Miles, Scott & Vayanos, 2012

2.2.2 Empirical evidence

The following text provides an answer to the third research question on whether Quantitative Easing measures affect liquidity of government bonds. It also shows by how much central banks programs influenced government bond yields in many different countries.

Several prior findings have been consolidated on the effects of the response to the 2008 recession into what they called large-scale asset purchases (LSAPs) by FED on medium-to-long-term securities such as Treasuries and MBS's. These consolidated findings provide evidence that the program was successful in reducing long-term interest rates, more specifically 10-year term premium (or interest rates), by as much as 30 to 100 basis point. In UK purchase programs, most of the findings listed in the report outlined significant economic effect on Gilt yields, with some stating the range of effect between 35 to 60 basis points as "at the very least", while others reported similar effects to their American counterparts of up to a 100 basis points change (Joyce, Miles, Scott & Vayanos, 2012).

Others have also included the Euro-area in the analysis together with the US, Japan and the UK, and the results they found were similar to those of their colleagues and their UK analysis (Hausken & Ncube, 2013). Their report is about different central bank implementations of the asset purchase program, where ECB and BOJ directly provided liquidity to commercial banks compared to FED and BOE, which bought bonds from the market. They also discuss ineffectiveness of those programs conducted by BOJ and ECB to substantially lower the rates, attributing it to already low rates from the start of the program compared to their counterparts in N. America. Due to the European sovereign debt crisis in 2012, they also record a short rise in rates in long-term bonds attributed to the rise of the sovereigns in Spain, Ireland and Portugal. In the report they stressed the importance of keeping in mind that ECB did not even target lowering the rates, but instead focused mostly

to preserve the status of the Euro. Their conclusion was that by the publication of their 2013 report, the Euro-area long-term bonds were responding to QE programs with overall rise of more than 60 basis points, particularly due to the Euro's preservation.

2.2.3 Future expectations of the quantitative easing program

The sustainability of rate hikes is being tested with the outbreak of the Coronavirus pandemic and corresponding actions needed to preserve the economy, as balance sheets of just 4 biggest Central Banks, namely ECB, FED, BOJ and PBOC, exploded from around 7 trillion during the 2008 recession to a staggering 20 trillion just prior the outbreak (Yardeni Research, 2020). It is hard to assume that the QE process will slow down or even stop anytime soon, since the countries and areas where it is implemented the most deal with structural issues of persistently low real GDP and inflation growth. At the same time they are about to be hit with big demographic challenges of aging population and low birth rates unless they resort to higher immigration (Bloom, 2020).

The notable investment practitioner Ray Dalio, founder and co-chief investment officer of the world's largest hedge fund, Bridgewater Associates, outlined the usual strategies used by the central banks to tackle national crisis. He predicts that the next step the central banks will make is direct payments to households and private individuals in order to stimulate the economy, also known as "Helicopter money", because they already exploited all other options on his list. The inequality created by prior actions is counterproductive, since richest people, including him, do not spend that extra wealth they receive, while household individuals need it the most (Dalio, 2018).

Predicting what the next step of the monetary authorities will be is of great importance, as they affect all the asset prices in the market. So far the safest government bonds are not the exception. But whatever the future step of the central bank is, their involvement will certainly continue to be present, the only question is for how long, tainting the asset prices and making them non-reflective of the true market values.

2.3 Indebtedness

For centuries, governments have used the privilege to borrow money from both domestic and foreign investors to finance their desired undertakings, be it for infrastructure projects, war or some other kind of investment and consumption. Today's society is more prone to taking debt than any time in history, as financial services have developed to the stage where every individual is easily reached all across the globe. The industry provides great advantages for the development of societies, but also poses great dangers if not managed properly for the benefit of the society. Credit has become plentiful in the recent century,

causing several booms and busts during its rise to current levels and never as cheap as it is today.

After the 2008 crisis, rates have been constantly lowering, enabling cheaper refinancing. The latest report by the Institute of International Finance (IIF) on global debt found 2019 global debt-to-GDP ratio to be at 320% (or USD 249.4 trillion), of which big majority was owed by mature, developed countries. The government debt of these countries represented around a third of their share of total debt or USD 51.5 trillion, with the USA leading on the list with half of the debt, compared to government debt being just seventh among EM share of debt owed (Institute of International Finance, 2020).

Table 1: Sectoral Indebtedness

$ trillion	Households		Non-financial corporates		Government		Financial sector		Total	
	Q1 2020	*Q1 2019*	*Q1 2020*	*Q1 2019*	*Q1 2020*	*Q1 2019*	*Q1 2020*	*Q1 2019*	*Q1 2020*	*Q1 2019*
Mature markets	34.7	34.1	44.4	42.6	53.5	51.5	52.7	50.6	185.4	178.8
Emerging markets	13.4	12.7	31.2	30.7	16.4	16.0	11.6	11.2	72.6	70.6
Global	48.1	46.8	75.6	73.3	69.9	67.4	64.3	61.8	258.0	249.4

Source: Institute of International Finance, 2020

OECD (2020) also finds that the governments of these developed, mature countries are projected to refinance 40% of their outstanding marketable debt within the next three years. To the extent they will be able to refinance at more favorable interest rates, depends the sustainability of their debts left to the future generations. The report also finds borrowing costs improvements for Canada, the USA and the UK prior to the coronavirus pandemic, while the new issuances of all OECD governments as an aftermath of coronavirus saw negative rates for 25% with additional 43% being in a range of 0-1%.

Such a ballooning debt in the developed world does not seem to be slowing down anytime soon. The central banks have made it clear that they will not stop their asset purchase program anytime soon due to almost nonexistent growth, ultimately keeping rates low as they are and stimulate borrowing further. In support of this statement, I would like to mention recent announcement of 1.8 trillion euro recovery plan agreed by EU countries, which could be taken as a good step toward full fiscal unity within the EU and the possibility of some kind of future united issuance of government bonds gaining the RF status (European Commission, 2020).

2.4 Eurozone bonds

Deciding on required RF rate for valuation of European Union-wide businesses is much more challenging than for USA-based ones due to the number of different currencies, national market laws and decision-making complexity that have to be taken into account. Since the majority of countries on the European continent participate in a single market

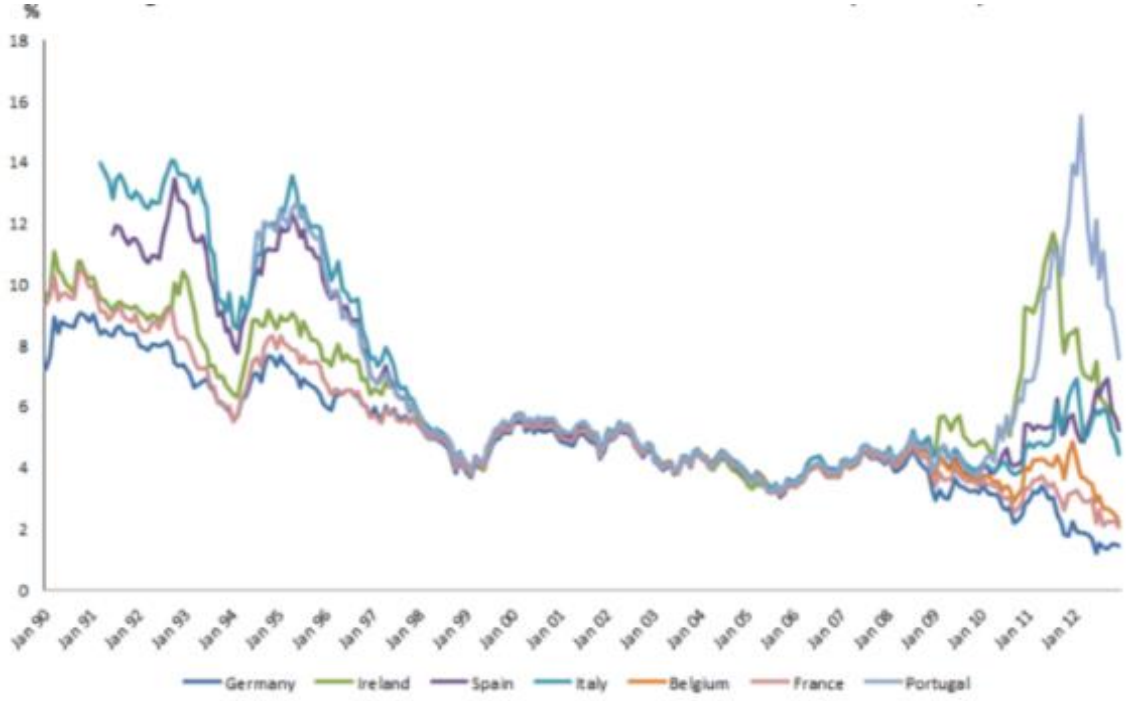
project, which so far has only external borders, big unifying institutions and one currency, while the powers on fiscal policies are left to each country, special kind of attention is required in the process of valuation. Every government still holds the right to issue their own sovereign bonds even though it is not in nationally issued currency, causing these bonds to be of different riskiness due to different country fundamentals (Baldwin & Wyplosz, 2015).

2.4.1 European Union bond yield history

A century before the implementation of the Euro, investors valued bonds of each Member State independently and their prices did not differ substantially, the only exceptions being Italy and Spain, which provided a bit more risky securities, but were still converging to others as they approached the Euro implementation period in 2000s. With the implementation, their sovereign debts decreased to the lowest level and converged with the best performing countries of the Eurozone, as can be seen in Figure 4. Markets found every country to be of a same sovereign risk regardless of the fiscal decentralization. Their behavior might be explained by Maastricht Treaty's prohibition of sovereign defaults, something subsequently understood to mean ensuring financial institutions be bailed out should there be a need to save the system (EUR-Lex, n.d.).

A subsequent sharp divergence between some of these sovereigns after the Recession of 2008 was concluded to stem either from investors' sharp risk appetite decrease for these bonds, even in the case of their smallest debt-to-GDP ratio change, or from fear of accumulating debts in the future. As a result, investors in a quest for safety flew to what emerged as the truly RF security among them, German Bunds, pushing their yields to lowest level ever (BIS, 2013, p. 11).

Figure 3: 10 year sovereign yields divergence pre and post Euro sovereign debt crisis



Source: BIS, 2013

2.4.2 Unification of Euro sovereigns

The divergence among sovereign yields of the Eurozone Member States prompted a social dialog on the topic of joint pan-Euro bond issuance. As a main benefit of such issuance, they identified much higher liquidity as it would occupy around 8 trillion Euros market value, lower average borrowing costs than in non-pooling case, all stemming from the fact that it would rival US Treasuries market as a safe heaven and protect the Euro as a global reserve currency. As a potential drawback they identified the free raider's problem of Southern countries, which they believed to continue on the path of fiscal recklessness, while the more prudent countries of the North were supposed to cover their deficits in a form of fiscal transfers (Bruegel, 2010).

To overcome issues that might originate from the joint bond issuance, Bruegel, a think-tank, proposed to structure the issuance as blue and red bonds. Blue bonds are supposed to pool only up to the limit determined by the Maastricht Treaty of 60% of their national debt-to-GDP, while red bonds would take all the issuances exceeding that percentage. This color themed issuance is intended to use capital markets as a rewarding scheme by which prudent countries would experience additional borrowing cost advantages of bigger pool of safe securities, while other countries would have higher borrowing costs due to their lower fiscal prudence, but would have a clear picture of why it pays to be more fiscally responsible.

The European Commission also published its own report on pooled bonds similar to that of Bruegel's, calling them Stability bonds. The difference of this report, compared to the other

one, is in the way of its implementation. Stability bonds are to facilitate cheaper financing for approved Member States, but they are proposing three options of national issuance through which it is to be implemented: full substitution, partial substitution with joint and several guarantees or partial substitution with several but not joint guarantees. The first is the most ambitious and provides greatest benefits of liquidity and cheaper financing for indebted countries but is also more prone to moral hazards. The other two are similar in their flexibility in issuance through national and Pan-European channels with pre-established limitations, but differ in severance offerings, since the third one also counts on some kind of collateral fund being provided (European Commission, 2011).

So far many projects have been proposed but none of them has seen any further implementation, as frugal countries of the EU led by Germany always opted to stop their progression. In case any joint bond project comes to be implemented, its structure should represent an important source of information for the markets. If they are to be unified fully as of today, they certainly would not satisfy their own Maastricht Treaty requirement and because of that would not qualify as RF securities. But if they opt out to offer unified bonds of those richest and most frugal, as identified by Bruegel's, they may reach that RF status and come to play bigger role in international capital markets.

This part provided an in-depth answer to the first research question about what makes the current interest rate environment really so unique. All the developments mentioned in the text above occurring at the same time are exactly the thing that makes this environment special. The following text will try to explain and tackle some of the most important challenges in the determination of RF rates for corporate valuation processes that are stemming from the above-mentioned developments.

3 CHALLENGES IN RISK-FREE RATE DETERMINATION

The following part outlines the main concerns and approaches to the RF determination process. It covers topics like RF rate assumptions formulation, currencies, credit ratings, liquidity and maturity of the RF securities, the case on non-existence of RF security, RF rates for terminal value discounting and approaches of normalization and synthetization of RF rates, as well as spot RF rates. The findings about these topics will serve in the next part in which they will be applied for the simulation of RF rate determination process.

3.1 Assumptions

As something often not taken seriously as input to valuation, assumptions sure come to haunt investors later for being neglected. According to the Cambridge dictionary (n.d.) and David Oldham from Shoreline College (n.d.) , an assumption is a claim one does not bother to prove, and is usually taken for granted when making an argument. Thus, it serves as a

stepping stone for further analysis and inference about the state of the subject of interest. It should also be clarified that assumptions are not necessarily facts nor speculation, but they are more in line with the former and further they from the latter, resulting in less uncertainty brought into the model. In his books, Damodaran (2013) often reminds readers that valuation is not science nor art, but a mixture of both due to its subjective nature. Since every investor brings a bit of bias into the process, they channel them through the formulation of the assumptions, be it on corporate fundamentals, industry trends or the state of the overall national or global economy. He identifies many assumptions at all three levels, but this thesis will pay most of its attention to the national/global economy as they are ultimately influencing RF rates.

3.1.1 Selection of Assumptions

In order to fully cover the extent of influence macroeconomic factors have on RF rates, several most important assumptions on behavior of investors and markets will be discussed. Many of the most important variables to consider when creating assumptions about RF rates are already being observed in practice, while few new ones will also be considered in the discussion.

The starting point of every assumption is always the way markets operate and how effective they are in adjusting the prices in response to new information. According to the *efficient market hypothesis (EMH)* (Fama, 1970)**,** markets are fully pricing in all the possible information regarding an investment so it is impossible for an individual investor to outperform the market on the long-run. But many investors did succeed to beat the market over the course of their careers by finding business advantages to act upon, challenging the theory by proving the investor biases with the least and most favored companies (Dreman & Berry, 1995).

As a matter of fact, in the process of beating the market, investors establish new grounds and new aims for future investors to chase, improving the economy at the end. Successful events of market outperformance did make the theory in its fullest form incorrect, but still relevant since forces like belief in possibility of beating the market serve to make it more efficient. The sources of inefficiency are numerus but are most often grouped as transaction costs, transparency issues, market illiquidity and cognitive biases (Dreman, 2000). All of them are relevant when dealing with RF sovereigns, especially in today's highly globalized and connected world. Because of that, it pays greatly to do extra research on how these above-mentioned issues could find their way to influence prices from outside of the RF supplier country.

The second assumption is about the continued *involvement and the role of central banks* in the markets. From their establishments, they enjoyed an increasing role in the markets and in the last decade used to dictate the general levels of interest rates in the economy. As they

depressed interest rates to the lowest possible levels, further rate cuts and purchases might negatively affect banking industry, unless they innovate and digitize to drastically lower the costs of banking. Their quantitative easing programs exist to continue providing stimulation to economies which are battling depression and keep interest costs minimal to avoid further pressures. The reasons for persistently low inflation and growth have been identified and range between changing demographics and aging societies, to globalization and lower productivity, to inequality (ECB, 2017). Considering the fact that there will be a continued need for the central bank involvement into the market affects the assumptions behind the RF determination process and appropriateness of the current market values for future analysis.

Another often neglected assumption is on the society's *openness to debt*. Historically, societies have operated according to cultural customs and norms, most of which stem from religious knowledge and practices. Interest used to be forbidden in both the Middle East and Europe during most of history and in modern times. Factors which influence a society's openness to operate and use more debt are not usually discussed until it is too late and debt becomes unsustainable. Ignorance, evident as rejection of consequences established for using debt, followed by heard thinking, occurring when the use of debt becomes common knowledge and starts to be a building block of every business, and spoilage, making future generations on average less willing to engage in creation and productive actions are some of the factors that could change current attitudes to debt. This assumption is hardly ever regarded as relevant since it appears when civilizational shifts occur, taking few centuries to fully develop again (Dalio, 2018).

Another important implicit assumption analysts/investors make is about the *length of the period of peace*, whether globally, locally or regionally. For investors considering long-term investments over more than 10 years, should consider to analyze more the existing geopolitical structures. Of course, in case of the existence of assumptions about the market efficiency, one could find yields on really long-term bonds to fully reflect uncertainty of investing in far future, but the term premium in times of continued failure of credit rating agencies on which we base much of our analysis for long yields, might not fully reflect the likelihood of future conflicts. Peace enables trade, innovation, business optimism and development, and investing to flourish during its times, while an increase in social frictions, valuations come to be much more difficult to properly conduct as inputs change. Some have identified few securities in use today are of use when a war economy emerges or any kind of disaster, making a case for reconsideration of a peace period assumption and how to implement it in the long-term bond yields further (Biggs, 2009).

3.1.2 How should investors deal with assumptions in the future?

In the future investors will depend more on the assumptions they make about most important variables they use for their valuations. Several possible assumptions they could start

thinking about have just been outlined, each affecting the perception of RF rates used for further analyses. The value of RF rates alone to valuation models is about to increase due to increase in market environments volatility and as a result getting it right will be more challenging and important, as opposed to what investors are doing today when just assuming RF status of Treasury securities. Today the practice itself is becoming more of a source of risk added to valuations, challenging the whole idea of RF rates itself. A part of the responsibility could also be attributed to credit rating agencies for the lack of objectivity, transparency and independence in their valuations while another part to investors for not doing their research on the true value of riskless securities.

3.2 Currencies and Credit ratings

The following three chapters cover why investors should take into consideration the currency of an appropriate debt whose RF rate is being used in company valuation, as well as how leading credit rating agencies (CRAs) contribute the problem of rate determination.

3.2.1 Consistency principle

As with any other technique in economics, there are rules, or better say, principles to analysis, which have to be followed in order to extract the needed information or results more precisely. One such principle is also present in the determination of riskless rates and is called *consistency*. This principle requires every practitioner who is in a quest to find a value of some business to pay special attention to inputs to the valuation process to match throughout the analysis. The most important input for company valuation is currency, as one wants to match cash flows of the firm being analyzed to benchmark bonds underlying the RF rate used in the process. As a result of having an option in which currency to value the business, differences between valuation results arise primarily from failure of different interest rates to reflect appropriately inflation expectations. At this point, risk comes from future interest rate adjustments, consequently requiring valuation adjustment for that business (Damodaran, 2013, p. 155).

In case one is not being careful and uses yields and other data on a company from sources providing their analysis in different currencies, this kind of inconsistency will cost him by adding risk arising due to neglecting exchange rates between these currencies. To illustrate, assume an international firm with operations in several countries, earning income in several currencies. Mistakes might arise by taking the data from different analysts which provide them but in different currencies, so that when devaluation of one country's currency happens, the absolute amount earned in that country will lead to decrease in its true value in the firm's original currency. Such decrease in actual earnings will not be accounted for in the valuation due to the mistake done at the beginning of the valuation process by valuing it with different cost of capital than all of these earnings are denominated in.

3.2.2 Investor perception – Icelandic case

Usually when investors think they are in a late economic cycle, they start to focus more on fundamentals behind every investment, because for them it becomes a burning question whether assets are in a bubble or not yet. In terms of sovereign bonds, in good times of market expansions credit ratings become a more prevalent tool of bond analysis and in that sense great liquidity booster through their borrower creditworthiness information. However, the leading issue with credit ratings is that they are perceived to be created in order to facilitate the absolute creditworthiness of borrowers, where in fact they provide just fairly accurate relative creditworthiness among borrowers. This fact is useful in explaining why there are "cliff" events of massive downgrades on ratings coming into the crisis, where some sovereign bonds end up being down by up to 7 notches creating big selloff events on the markets. Some professionals argue that it is the fault of the government's ineffective laws and regulations imposed on credit rating agencies, while others say it is important to move credit rating business models away from the relative valuations more into absolute ratings, which would result in less stable ratings but smoother asset prices in the long-term (BIS, 2013, p. 34).

The Icelandic case of 2008 is one of the best examples of how agencies do a bad job of credit ratings that led to catastrophic outcomes. Prior to the crisis, agencies rated the Icelandic banks and government bonds as triple-A, mainly because of the belief that the government will stand behind their financial institutions totally. The problem arose when it was uncovered that those banks contained off-balance sheet assets around 10 times the whole banking system of Iceland, so believing such a small country with 300,000 people would be willing or capable of taking such losses upon their future generations, was the scandalous to say the least. Luckily, the Icelandic government did a good job with their subsequent decision not to protect risky positions done by their banks so they had to be separated from other parts of the banking and focused on keeping the domestic banking system functioning and continuing the payments on their government bonds. Prioritizing continued payments of their sovereigns now provides an example for other countries how to deal with such problems. Credit rating agencies went on to lower the rating for banks below investment, while thanks to the government actions sovereigns stayed on investment grade rating (BIS, 2013, p. 7).

3.2.3 Credit rating agency business model criticism

Few researchers decided it would be useful to test whether credit rating agencies were biased in their analysis of countries other than OECD ones (Tennant & Tracey, 2016). During their quest, they used several other findings already published about CRAs and their conduct prior a few of the most severe crises in modern history. While they were analyzing CRAs

involvement and actions in relation to developing and emerging markets, they discovered and published several important and relevant issues regarding developed markets as well.

The book provides excessive amount of evidence and arguments on this new kind of business model to be the whole reason behind much of the criticism of CRAs. First, they argue that in order to receive the information they often discuss ratings with issuers in order to secure a deal of doing the rating. Such practice allows issuers to do shopping-rating to find which agency will first provide best ratings of their bonds. Second, they discuss the bad due diligence done by CRAs about the information and data they receive from issuing governments. This problem arose due to high costs of analytical hours spent on the due diligence process behind the respective information, so they simply relied on already existing findings in the market to investigate the receiving information before publishing. Third, and probably the most important one, regards the pro-cyclical nature of the agencies, which used to worsen the euphoria and crisis instead of being corrective arm of the market. Such lag behind the market drew most of the criticism on agencies in the most recent sovereign bond crisis in Europe where CRAs downgraded Greece's bonds only after nine months of their first bond default risk arose.

Reputation-based industry of CRAs enable them to reap enormous profits as they represent the biggest and the most liquid capital markets in the world. As the reputation of these agencies was tested as a result of the most recent developments, it did not diminish in spite of the extreme pressures from the public and the media. Much reformation inside their organizations has been implemented, where they outlined their devotion to isolate their analysts more from the outside influences to increase their efficacy, but the business model still stayed the same and continued to receive a growing number of the proposals for their rankings from all over the world as the need for new debt issues grew significantly in a wake of the crisis. What kept new issuers coming were paradoxically the same regulations implemented throughout the developed world which intended to increase the competition, but ended up increasing the reliance on these agencies since every other financial institution was required to act in accordance with Basel 2 regulations and have their credits be analyzed by credit rating agencies, which by itself allowed incumbents to reap the benefits of their scale and be by far the cheapest provider.

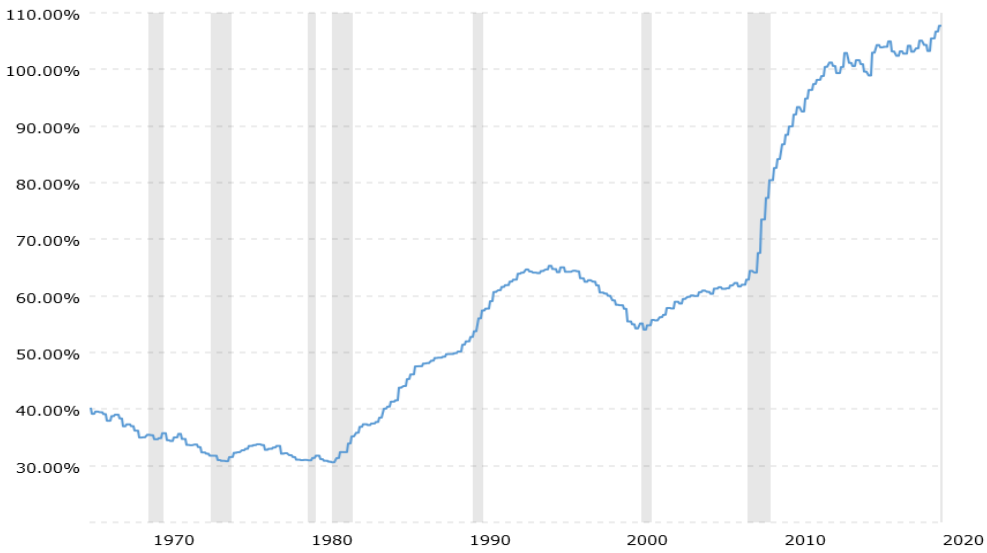
3.3 Nonexistence of Riskless security

According to the theory, a riskless rate is a security whose price at maturity is already predetermined, while its uncertainty is lowest among all the securities. It represents a benchmark based on which every other asset is to be valued. In corporate valuation terms it has been widely accepted to represent a long-term asset that matches cash flows of the firm valued and possessing no reinvestment or default risk. By taking a long enough asset which matches cash flows on a security, one automatically solves the problem of reinvestment risk, but default risk stays to be thoroughly analyzed. Practitioners in valuation agree that there

is no truly risk-free security in existence, but a security whose default risk is minimized almost to the point of it not even existing (Damodaran, 2013).

During the last couple of decades, US Treasuries have been accepted as the benchmark riskless asset, incorporating all of the needed characters: liquidity, maturity and no default risk. Strong fiscal position enabled the government to be free of excessive debt issuance to cover its spending since enough was generated by taxation. All the debt that has been issued to the external landers yield comparatively less than other countries with similar fiscal position. The dollar as a reserve currency based on which all the global trade has been conducted, found its way back to America through Treasuries outstanding and continued to allow the US to keep their fiscal position in controlled and sustainable limits (Macrotrends, 2020).

Figure 4: Debt to GDP Ratio Historical Chart



Source: Macrotrends, 2020

Due to the 2011 US Congressional spat over debt ceiling limit raising, S&P downgraded US Treasuries, a long-term outlook from TRIPLE-A negative to AA+ for the first time in history (Durden, 2011). Egan-Jones, a much smaller credit rating firm, was the only one that followed in the footsteps of S&P with their own downgrade to AA+ (Detrixhe, 2011). The move did not cause much change in the financial markets as many found continued trust in the Treasury's safety to be only as a response to European sovereign crisis which seemed to present much bigger challenge to the long-term safety of the best European sovereigns like France and the UK, the challengers to RF status of its bonds. They ultimately lost their TRIPLE-A status in the same period (Kennedy, Donahue & Deen, 2012). Nothing much changed since then as the problem appears to be much more apparent today as the debt compared to GDP of US, which is still practically used as safest security, is reaching much more unsustainable levels worsened by the Coronavirus pandemic.

As Fitch changed its outlook for the US Treasuries from TRIPLE-A stable to TRIPLE-A negative at the end of the July this year, one might start to think that move resembles that of S&P in 2011 and consider further downgrades AA+ like what S&P did, which this time could lead to much more chance of increase in defaults among the corporate sector as it is more indebted than ever before (Fitch, 2020). All this might lead to worsened standards being accepted in credit valuation, resulting in less obvious differences between those at the top of the scale and those at the bottom and possibly also the non-existence of truly default free entity.

3.3.1 What to do if there is no riskless security?

The following text offers an in-depth answer to the fourth research question. Textbooks have provided us with a way to analyze sovereigns in case of risky government. Damodaran (2010a) provides an argument on how to deal with cases when even the government is risky, meaning there is at least some probability that they might default on a certain part or the whole amount of their bonds outstanding. This possibility of default is quantified as default risk, and is added on the nominal RF rate of the security to come to the total YTM these assets provide. As we know that certain governments offer assets valued by the markets as riskless, meaning no default risk present, times can change where for some countries the default risk starts being reality and the RF rate has to be estimated separately due to lack of choice. These times might occur during depressions, recessions, cold wars or full on wars, and are more relevant today when indebtedness is at an all-time high and central banks are keeping rates artificially low to avoid the recession, making them the biggest buyers of the government's debt. Such a lack of market demand might trigger hyperinflation at some point, resulting in increase in rates, interest payments or default possibilities.

To deal with issues in these occasions, one has to calculate and remove default risk away from the RF rate of a security. Several ways have been used in practice today to calculate default risk of the government bonds, them being: sovereign ratings approach, CDS spreads-based approach or individual analyst reports approach. All of these have their own advantages and drawbacks, and it is important for investors to know them in order to help themselves navigate the murky waters. Today's ratings are focusing on relative valuation of credit risk of sovereigns which might be a challenge to continue doing in times of non-existence of riskless security. By assuming that ratings would still be created in a same relative basis approach, one could follow in the footsteps of Damodaran and his calculation of ratings-based sovereign default spreads over.

Assuming no drastic change in liquidity and maturity is present in the sovereigns market, when there is no RF entity, meaning there exists a chance for potential default by the government on some part of its debt, CDS spreads should be used as an instrument to calculate a part of the YTM that is due to default risk. CDS is an over-the-counter (OTC) financial instrument serving as an insurance on any default possibility by one party on its

debt obligations. From its creation in 1994, through its use in the European Sovereign debt crisis, their market grew to $61.2 trillion, but has fallen to around $9 trillion today (Aldasoro & Ehlers, 2018). They found their way into valuations since analysts use them as a measure of market perception for underlining securities default probabilities. In every one of his valuation classes Damodaran talks why he uses them when he wants to determine RF rates in countries where governments are risky. This kind of sovereign default risk measure, though very useful to analysts when valuing a risky company, is not easily accessible for every country in the world because CDS spreads are available for only 75 out of the global 180 countries (Damodaran, 2020). But even in the case of no RF rate, there would always be few of the government bond issuers deemed safer when compared to others. Their offerings should be evaluated based on CDS spreads on them to arrive at the value one could believe to receive with certainty which represents the RF rate.

3.4 Liquidity and Maturity matching

Two of the most important challenges presented to those finding an RF rate for their valuations in today's environment of quantitative easing measures are finding sufficiently liquid bonds for a benchmark, while at the same time match the time horizon of the intended investment through the choice of appropriate maturity. This part will discuss the most important points on how to deal with these challenges based on the latest market outlook and what to focus on the most when dealing with them.

3.4.1 Liquidity

The liquidity of an asset is a measure by which investors understand how fast and cheap they can sell their investment. An important part in figuring out the liquidity of an investment is how big the market for a particular product is and how diverse it is in its term structure. Another variable taken when researching liquidity in bond markets is liquidity premium demanded for particular bonds, represented by a higher price demanded for bonds considered to be more liquid than others (Ejsing, Grothe & Grothe, 2012).

USA Treasuries have been historically considered the most liquid market for bonds, followed by German Bunds. According to their respective administrative offices, the most of the market turnover is done for 10-year treasuries (U.S. Department of the Treasury, 2016) (Deutsche Financagentur, 2020). Several studies have also been conducted in order to test the value of liquidity premium in these securities, where German securities have proven to possess less liquidity premium than French sovereign bonds, a point used to be considered RF securities prior the 2009 turmoil (Ejsing, Grothe & Grothe, 2012). These studies determined that during difficult times in the global economy, German Bund yields fell together with its corresponding liquidity premium as the considerations of default risk in other safe assets grew, reflecting a belief of the markets in Bunds.

Further studies on the exceptionality of German Bunds as safe heave back these findings, while also providing an argument that their liquidity premiums are likely to fall further as the demand for them is consistently increasing. Additionally, this comes in time when the USA is on the trajectory to increase its issuance of Treasuries, further stimulating the value of Bunds, which has been on the decreasing trend since the government in Berlin decided to keep its policy of balanced budget (Paret & Weber, 2019).

Table 2: Yearly Trading Volume of the German Government Marketable Securities

Year	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009
Trading volume (€ bn)	4,091	4,694	4,790	4,294	4,715	4,869	5,832	5,501	6,184	5,863	4,762

Source: Deutsche Financagentur, 2020

3.4.2 Maturity

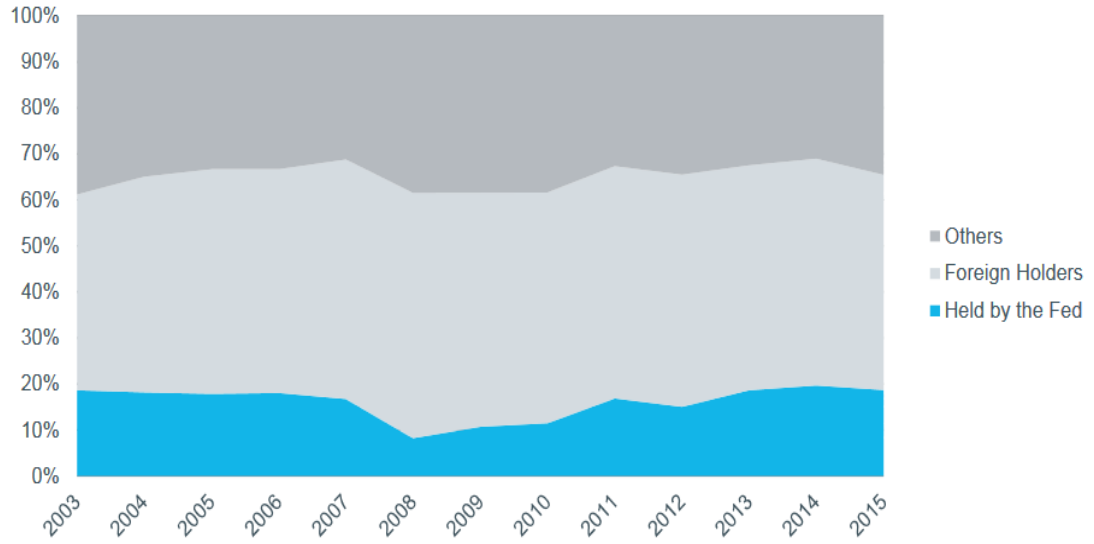
When we determine the value of a company based on its projected future earnings, great attention must be paid to matching maturity of the RF rate used for discount rate with the number of years we project earnings to be valued by discounting. If we use shorter maturities of RF rates then the number of years of earning projecting to discount, we are deducing some of the risk of holding the stock for much longer period than accounted for, while the opposite counts if we choose longer maturities for shorter number of years of projected earnings.

One of the problems today of maturity matching is QE's effect, since bonds don't accurately reflect future expectations of the economy as the Central Banks are focusing on lowering yields, while at the same time balancing what could be diverging rates by the markets. To value a company, analysts in practice valued a company as a going concern most of the time, a concept representing its eternal existence, which relies most on the consistency principle of matching inputs used throughout the analysis. Based on this principle, academics have favored the use of 10-year Treasury rate as an RF benchmark in corporate valuation due to its liquidity and ability to match investments duration, since shorter securities would involve reinvestment risk, which is against the rules on what represents RF security. Longer-term bonds of 20- but also 30-year maturity have been discussed to represent unnecessary risk addition to the valuations, since the lack of liquidity compared to a 10-year bond does not substitute it as a more desirable measure.

But as external forces, such as Central banks and foreigner investors, have added to the demand for safe-haven assets as are US Treasuries and German Bunds, 10-year bonds have dramatically increased in prices and decreased in liquidity together with other short-term ones. The figure below describes the proportional amount of marketable US Treasuries held

by FED and foreign investors, which shows their combined holding to be 65% in 2015 (Duff & Phelps, 2016). Such developments allowed investors to accept previously discouraged longer-term bonds, as they possessed the same properties as their shorter counterpart, while also better reflecting historical returns on those securities.

Figure 5: Relative holdings of Marketable U.S. Treasury Securities Held by the public



Source: Duff & Phelps, 2016

Several developed European and N. American countries have already started issuing bonds with even longer maturity of 50 and 100 years, attributing it to investors' appetite for adequately yielding safe-haven assets, while others are thinking on following them (Rappeport & Phillips, 2019). This move comes as an addition to already growing 30-year segment in these countries, with its market projected to continue growing into the future to capture low interest rates for longer periods, subsequently adding to liquidity of this segment.

3.5 Terminal Value discounting

Another component important to the valuation process is terminal value, a point in the earnings projection process at which an investor stops with thorough analysis of annual earnings and, based on the assumptions he creates for the future from that point in time, comes up with a value to be discounted back today as if he will be forced at that period to exercise his exit strategy for the company (Damodaran, 2013, p. 304).

Two ways in which an investor's exit strategy could be exercised have been identified and based on which we use different terminal value calculation approaches. The first one is to value the company at that point in time as *going concern*, meaning it will be valued either

as if it will continue to generate stable growth earnings in perpetuity or by being given a multiple on its earning, revenues or book-value based on its position on the market compared to others in the same industry or market. In the case of the former, he can serve himself with the assumptions already discussed in the previous chapters about long-term outlook of the global, national and industry economy in order to know at what stage the company is in. If the firm grew rapidly in previous periods and he can with certainty conclude that it cannot keep growing at that pace and that it will start stagnating at some point, it is obvious that by successfully determining when this stagnation will happen might provide the investor with an advantage to better estimate the terminal value. Usually investors take an assumption on the average growth of the national market as a benchmark for the case of earnings perpetual growth.

The second exit strategy is *liquidation value*, as if he is forced to sell the company's assets at an auction to highest bidders. This can be done by either estimating earning potential of assets to be sold or estimate the book value of those assets at that time and adjust it for inflation. The limitation of the latter approach mainly revolves around it not taking into account the earning power of the asset being sold even if earnings better explain assets purpose rather than just its price. But to its defense one could put the state of that asset where in case of great maintenance one could count on greater value of better state than if underestimating earnings power based on the time it spent operating.

3.5.1 Longer-term yield requirements

Near-zero yield of RF bonds of longer-term (10+ year) segments of government bonds like German Bunds and US Treasuries by their nature allow for terminal value to be discounted at much lower rate than usual, increasing the present value of company. The fall in yields of these bonds explains why valuations are at the historical highs, even though the productive potential of advanced countries has been falling during the same period (Bahar, 2017).

To compensate for falling yields and to receive as fairer value as possible when trying to remove the effects of Central Banks, investors resorted to several options such as normalization of yields, synthetization of yields or maturity compensation, all of which will be explained in detail in the subsequent chapters. Regardless of the option investors lean on to find the more appropriate yield for TV discounting, it will be highly influenced by assumptions about both the near and far future. As proposed above, those investors contemplating some degree of friction in the far future will not invest in businesses for the really long-term if it is not appropriately compensated for the length even though the rates are zero for the longest segment of the government bond market due to psychological barriers.

According to Damodaran (2013), some of the assumptions about the terminal value discount rates depend on the consideration of the path of the firm. Thus, the question is whether the

firm will exhibit stable growth from the point of TV calculation, in which case its real return converges to that of the real growths of the economy, or it will battle for survival, which is the case for most of the young companies having negative earnings as the period of distress approaches. The final part of this thesis will test what is the RF rate for TV discount rate taking into consideration both of these cases for targeted countries.

3.6 Normalized rate

Last decade's persistently decreasing yields on developed economies government bonds of all maturities have posed real problems to everyone daring to use them for any kind of valuation activity, from creditors to business analysts. Just taking rates from markets has stopped making logical sense since they did not accurately reflect long-term market conditions. To compensate for such market deficiency, several new approaches which are meant to alleviate some of the problems posed by market rates have been considered, one of them being rates normalization.

The talk about normalization started with the FED in 2015 when some of its officials discussed the need for the potential funding rate increase, which they perceived to be needed in a journey to reach the desired "normal", close to the historical levels of 3% deemed as organic rate in finance (Bullard, 2015). Such an action by the US central bank would be implemented gradually, as done usually in tens of percentage point increases, which consequently spills over into every other rate in the economy by moving them upwards to reflect the rising costs of borrowing. Though their desire to increase rates has gained some success in the period from 2015 up to the 2019, rising them to the maximum of 2.4%, lasted relatively short as they were lowered again to 1.5% due to stagnating macroeconomic conditions and lasting until the end of 2019 (Macrotrends, 2020).

Practitioners at Duff & Phelps (2016) have issued their own analysis on why it is reasonable time to follow up on the FED's idea and determine appropriate approaches to "normalize" government bond yields for the purpose of investment valuations. It is important to state that their thinking was based on the main assumption that FED will be successful in normalizing rates in the near future from the current zero lower boundary sufficiently close to the historical mean or above, and in the process reverting yields on long-term bonds to their historic levels. In their report they advocated two approaches to determining this normalized rate: 1) simple averaging of yields to maturity, and 2) various "build-up" methods.

In the first one, they advocated the use of 20-year Treasury instead of 10-year usually considered as benchmark, due to the convenience it has in valuing companies as going-concern in current environment. This can be understood as a way of playing along the longevity intentions by central banks which have the intentions to stimulate the outlook of countries for long-term, thus making 20-year yields real substitutes for 10-year ones, having in mind increased intentions by the governments to increase issuance of these longer term

bonds. Furthermore, to better capture the most recent trends in its price movements, monthly yields over the last 10 years were used, providing also a reminder on the limitations of using historical data as a method to predict future yields.

The second method, which is also more demanding since it requires a bit more research involved for the analysis of its part. By separating the nominal RF rate into its components, namely, projected long-term inflation, which is the driving determinant of central banks policies, and the real RF rate, which corresponds to real GDP growth of a nation, they are able to determine the range over which the nominal rate might vary more easily, giving them more subjective approach to the path it is been heading. They used estimates of future long-term inflation from several agencies with resulting range of 1.8 - 2.6% and combined them with findings on real RF rates from several academic papers with resulting ranges between 1.2 - 2% to arrive at estimated nominal RF rate range of 3 - 4.6%.

The assumption about rates reverting back to their historic mean is the hardest and baldest one to make, since from the initial talk about normalization, the rates have only been constantly falling. With no major reforms in sight on issues driving productivity decline, economic decline, aging demographics and wealth inequality, long-term GDP growth and inflation outlook are bound to stay as they are, while influence of the governments and central banks might not diminish in counter deflationary forces. Due to such involvements, today's already stained market prices are likely to continue reflecting governments' desires for a couple of decades mainly because of the markets' anti-inflationary capacity. This will be a historical precedent on its own, unless central banks decide to further pursue digital currencies and electronic money in which case deeper negative rates might provide for quicker achievement of target inflation rates.

3.7 Synthetic rates

Another option when dealing with extraordinary rates of today is to create a synthetic rate, composed of a mixture of market expectation on inflation and long-term historic real returns. The difference between this model and the prior one lies in the way of estimating inflation. The creation of a synthetic yield was said to provide more reflective return for the valuation rather than taking spot-yields or normalized rates, because it is focused on more natural average of long-term real interest rates based on historical data and inflation rate expectation (Koller, Goedhart & Wessels, 2015).

In the case of the US analyzed market, Koller, Goedhart and Wessels (2015) suggested average long-term real rate of 2% based on the historic data on return at times of no market "aberrations", as they call low interest rates implemented by monetary institutions. They also suggested 2.5% estimate of long-term inflation based on historic market expectations, giving them nominal dollar risk-free rate of 4.5%. The conclusions about the data was drown based on their long-term analysis on the 2009 post-crisis results in S&P 500 where they found that PE valuations of companies on this index have not followed prior practice stating

that for every 1% decline in cost of equity, index gains 20 to 25%, so they determined to investigate the full extent of the problem. They determined that even though the cost of equity fell by 3%, PE for S&P 500 index stayed in regular trading range. Based on their finding, this unusual market development was attributed to the unnatural decline of government bonds originating from three sources, namely central banks asset purchase program, monetary policy of low interest rates and global flight to safety. The argument is that the synthetic rate will solve these problems and provide a more realistic rate for valuation.

One more important distinction to mention on such calculation versus the others is that the calculation of these rates has explicitly said to be conducted with 10-year US Separate Trading of Registered Interest and Principal of Securities also known as STRIPS. These securities are specifically designed by the US in order to provide zero-coupon bonds after the coupon of US treasuries has been stripped, which will then serve as a principle for other short-term bonds. The reason for this is that any error conducted while calculating parts of the rate itself will not magnify if the coupon values of bonds have not been taken out of the equation. Another important consideration for the whole valuation they identify is in the situation when market expectations about either of the component of the RF rate surpass significantly proposed values, in which case one is supposed to adjust upwards those numbers to reflect better new realities. Downward adjustments have not been suggested.

3.8 Market (spot) rate taking

After all the challenges in RF rate determination and corresponding macroeconomic developments discussed above, it is time to discuss the implications of taking market or spot yields in valuation. The increasing involvement of central banks over the last decade with their zero interest rates and quantitative easing policies stained the market values of government bonds which have been used for corporate valuation as benchmark rates. As discussed before, many of them lost their RF status after the last financial turmoil in the developed world, as their governments resorted to debt financing to save the economy and as a consequence increased their default risk as their debt ratios started to exceed normal levels.

From the several RF securities used initially in valuation practice, only two were left after the dust steeled, these being US Treasuries and German Bunds. Even though they both have structural issues, be it in a form of regional cohesiveness as it is the case with Germany or increased budget deficits like in the US, they provide superior fundamentals like size, depth, liquidity and default-free status, making them the only contenders for RF status today. But spot yields of near zero in Treasuries or negative in Bunds for 10-year bonds make calculations illogical since their share in calculation of the cost of capital make discount rates drastically fall, automatically stimulating the valuations of businesses. To the extent they are successful in increasing their earnings to satisfy the expectations by the market,

these rates might seem to be normal, but it cannot be sustained in perpetuity and eventually might prove to be a bubble as underlying problems of lower productivity and aging society do not allow for rates to rise.

In case of successful implementation of digital currency by central banks this prior perception might change drastically. As discussed before, deep negative rates would mean even greater share of sovereign bond values will come from central bank interventions and market forces would be limited. This might even require new assets to be considered as riskless, like market indexes as they satisfy no-idiosyncratic risk rule determined in a report of BIS (2013, p. 67), which by itself might require a totally new approach to valuations. So far, it might be best to use one of the prior mentioned approaches when calculating RF rates, but keep your mind open to react appropriately in case central banks stick to their current agendas in the next year or two.

4 SIMULATION: PROCESS OF DETERMINING RISK-FREE RATES

The following part intends to combine everything written in the chapters above in order to determine more precisely the appropriate RF rates for the markets in question. It will start by explaining the methodology behind the research, followed by the simulation of the determination process for Denmark, Germany and Canada on different mock companies based in the selected countries and it will end with a summary of the findings from these simulations.

4.1 Simulation setup of the Simulation assumptions

I intend to guide the readers through the practice of RF rate determination when the influences (discussed in the previous chapters) on the most recent market developments and challenges of adequate bonds to the determination process make changes to the riskless rate. The data necessary to do this will be found and used from providers such as Yahoo Finance, Investing.com, Reuters, Damodaran's blog, Eurostat, Bloomberg, and others.

For instance, approaches such as normalization and synthetization of rates will be applied to examine all the differences between the two. Representing a two of the most used approaches in dealing with RF rates in the current environment, finding out what their impacts are on the final outcome matter. The data for the historic values on variables such as government yields, index returns and inflations about the determined segments for analysis will be analyzed according to the methods discussed in each chapter concerning that particular method. The testing through the simulation will be done on countries which are considered safe by institutions and the capital market based on the ratings and instruments in place for that purpose. I have decided to test RF rates for companies in

markets of Denmark, Germany and Canada, since they hold Triple-A status as well as the lowest CDS spreads, which qualifies them for the analysis.

The assumptions discussed before will be taken into consideration when searching data for the DCF analysis, as well as the way by which approaches to RF rates have to be adjusted to better reflect conditions of the particular country. Countries selected for this research are of different sizes and fundamentals, thus requiring investors to think more about the most important variable. The simulations done on these challenges will provide the answers to the fifth and sixth research questions.

4.2 Simulation of risk-free rate determination for businesses in Denmark

4.2.1 Identifying developments and challenges

According to the latest valuations by all three major credit rating agencies, Denmark sovereign bonds hold Triple-A rating, indicating their confidence and safety of securities for domestic and foreign investors. Such sentiment is also shared by the global capital markets expressed through estimated sovereign default likelihood or CDS spreads, which trade at the second lowest level in the world just after Switzerland, allowing the Danish government to enjoy enviously cheap financing conditions from abroad (Damodaran, 2020).

The Danish central bank followed in the steps of ECB and started lowering their funding interest rates from 2009, reaching negative territory for the first time in 2015, a movement also replicated in its government yields with the four year lag (Investing.com, 2020). This historical development, accompanied with the trade surpluses and reduced spending allowed the government to successfully lower its debt ratio in 2019 close to 30% of GDP, something last seen just before the crisis (Trading economics, 2020). Additionally, while the country does not participate in currency union with the other EU Member States but instead participates in ERM 2 mechanism and use its own currency Kroner, it could potentially benefit from its own monetary policy should the unexpected events occur or ECB acted unfavorably.

Denmark as a Member State complies with the EU capital market transparency regulations (European Commission, 2014) and has constantly been ranked among the most transparent countries in the world. From today's point of view, assuming that the Nationalbank of Denmark will hold its role as a lender of the last resort for the investors in Denmark appears as inevitable, as all the major global Central Banks experience increasing pressures to counteract depressionary forces. Even though Denmark possesses low level of debt, and since Nationalbank has not implemented QE programs on a larger scale yet, the Coronavirus experience might change that by creating a need for new borrowing.

Despite its liquidity in the 10-year segment, they still lack the scale compared to the Treasury or Bund market, which possess trillions of dollars of outstanding bonds with

trillions also being traded daily. Besides the already liquid 10-year segment, the government is focusing on its preservation by issuing 2-year and 5-year segments additionally to serve as an alternative for some local investors. Nationalbank decided to start issuing a new 30-year segment as of April in order to replace the existing, much more expansive 20-year one, and would also contribute to this transition with buy-backs and other switching operations to support their liquidity (Danmark's Nationalbank, 2018a). This kind of strategy is a proof of confidence in Danish bond markets, which stands as a standard go to securities for domestic settlements.

Another important consideration arising when considering Danish bonds for an RF rate is that they are denominated in national currency for settlements, Danish Kroner (DKK), and to satisfy the consistency principle, cash flows being discounted are supposed to be in Danish kroner. Fortunately, most of the biggest OMXC 25 stock exchange trading companies offer financial statements in Kroner (NASDAQ, n.d.).

All these forces make the Danish bond yields to be considered RF, but limited its use to domestic valuation as they are insufficient to satisfy the needs of wider investor community. The domestic use of these securities revolves around the banking industry although they can also be applicable in valuations of private and smaller public companies which receive their revenue mostly from the domestic market because in this way a similar currency would allow to avoid the implementation of exchange risks to the valuation process. Table 1 summarizes the important characteristics of the Danish market.

Table 3: Danish market characteristics as of June 2020

Market transparency	High
Central bank involvement	Yes
Currency	Danish Kroner
Credit rating 2019	Aaa/AAA
CDS spreads	0.21%
Government debt (% GDP)	30%
Deposit rate (DKK)	-0.053
10-year segment liquidity	High

Source: Own Work

4.2.2 Challenges to Danish risk-free rate

As an addition to the previous analysis on the Danish market, its bond market traded Danish benchmark 10-year segment in June of 2020 for a -0.3% yield, a level lasting for a year since it first occurred in June of 2019 (Investing.com, n.d.). This rate is well below the historical levels of around 3%, and is even below what used to be the theoretical lower limit

of zero. It artificially lowered discount rates used for valuations as equities continued to break records, leading up to the Coronavirus crisis.

Tough it might seem that these zero or negative yields are here to stay in Denmark for much longer based on spot market sentiment for these bonds, sustainability of such rates for the Danish financial system is questionable as well as for any other economy. To "cleanse" rates from influences of the central banks low rates, approaches such as normalization and synthetization will be applied to determine the Danish RF rate.

Applying two ways to normalize RF rates based on Duff & Phelps (2016), one can construct different values for the Danish RF rate. Instead of 20-year maturity as they proposed, I used the 10-year government bond as it offers the greatest amount of historical data on yields, and calculated their 10 and 15 year averages. Simple-averaging historical yields over both periods comes up to be 1.2% and 2%. Both numbers are far from the "normal" 3% in Treasuries, but closely reflect the real sentiment in the European markets over the past few years.

For a build-up RF rate calculation method, I have decided to use historical inflation numbers and real interest rates for the Danish economy. The inflation rate in the past 10 years averaged out to be 1.2% for a 10-year segment, while over the somewhat longer 15 year period ended up being 1.5%. To get real rates for the Danish market, I subtracted the annual inflation from the average nominal yields on 10-year bond for a period of the last 15 years. I calculated the average real yield of 0.6% over the last 15 years, while over the last 10 years it provided negative 0.3%. What values like this say is that there is a greater trend of real returns falling in the Danish economy. Based on my findings, I would argue taking 15 year period averages as representable for the analysis, but ready to update the components as soon as new data arrive. The combined value of the normalized RF rate base with this approach over the periods of the last 10 and 15 years sums up to 0.9% and 2.1%.

Determining the RF rate via the synthetization approach, I first derived long-term inflation expectations from the Danish central bank yield on inflation-linked bond maturing in 2030 abbreviated as DGBi 2030 (Danmarks Nationalbank, 2018b). In times of positive yields, these securities were used to calculate inflation expectations by taking a difference between nominal bond yields and inflation-linked bond yields. The logic behind it is that investors are willing to pay more for a security that is free of inflation risks and whose bond is only reflective of real market growth. This kind of thinking holds in the environment of negative yields as long as the nominal bond price is lower than that of the inflation-adjusted one, which is a case in normal, peaceful times. Market expectations data on Danish inflation-linked bond have been taken from NASDAQ Nordic exchange on August 2020 (NASDAQ Nordic, n.d.). The bonds price continuously rose up to the point when Coronavirus impacted the Danish economy in March, when it fell from DKK 120 to DKK 109, but then gradually started to come back. This fall can be attributed to market contraction expectations. I am taking yield based on June's market expectations on inflation-linked bond of -1.3% because markets have had enough time to analyze the full extent of the virus impact. Subtracting this

yield from June's average nominal bond yield of -0.3% provided me with market expectations of future inflation of around 1%. Adding this to a previously calculated 10-year real yield of negative 0.3%, results in a nominal synthetized RF yield of 0.7%.

Table 4: Risk-free rate for Danish market per approach as of June 2020

Market rate 10-year (June 2020)	-0.3%
Normalization (historical, last 10 years)	1.2%
Normalization (historical, last 15 years)	2%
Normalization (build-up, last 10 years)	0.9%
Normalization (build-up, last 15 years)	2.1%
Synthetization (last 10 years)	0.7%
Range	**2.40%**

Source: Own Work

4.2.3 Effects on firm value

To best depict the effects of the differently determined RF rates that investors might experience during the corporate valuation process, I have decided to analyze changes to a mock company value stemming from changes in RF rates.

For a Danish market, taking a mock company to represent the majority of publicly traded companies on OMXC 25 index, consisted of mature, slow growing companies. This one operates in the health care sector and industry of pharmaceuticals. Fundamentals like cost of capital (WACC) and cost of equity (CAPM) have been calculated based on inputs from Damodaran's blog corresponding to the observed market. The last five years of cash flows have been taken for a random company listed on the named index to represent projected cash flows for the future, based on which the terminal value was also determined. The assumption behind this thinking is that the mock company will record the exact movement in levels of free cash flows generated to all stakeholders in the company (FCFF) over the period of the next five years like it did in the last five, but afterwards growing at the steady rate of the Danish economy of 0.5%, which I determined to be naturally growing rate of the country based on the analysis of its market developments.

Cash-flows were later discounted by WACC depending on the approach and account for time value of money. Assuming the company succeeds in its policy of preventing substantial changes to cost of capital over the period analyzed originating due to the changes to debt ratio and the RF rate, I will analyze only the effects stemming from RF rate changes to the model of CAPM. Two approaches to discounting are presented in the table below, one based on the standard practice of using one RF rate to discount both cash-flows and terminal value, and the other that is explicitly using normalized, longest-term maturity outstanding on the

market, for a terminal value discount, while using rates from other approaches for cash-flows based on individual preferences.

Table 5: Danish market mock company value changes as of June 2020

RF rate	*0.70%*	*0.90%*	*1.20%*	*2%*	*2.10%*
DCF Value (in mil.; DKK)	609,093	591,163	566,179	508,905	502,557
Two-RF's approach	526,888	523,319	518,026	504,246	/

Source: Own Work

Based on the data provided in Table 3, one can conclude there are drastic changes in the value of the firm corresponding only to the change in the approach to RF rate determination, holding all other inputs to the WACC equal. The normalized yield over a longest period of time provided a value that is lower by around 106 billion DKK compared to the synthetized yield on a 10-year government bond, representing also a range of values for a company based on the approach to RF rate. Two-RF's approach provided lower values of a company then by using just one standard rate for all cash-flows, mainly because of the majority of value that comes from TV. Such range of values for a mock company summarizes the approximate effect public companies are exposed to just because of the change in the approach to the RF rate. .

4.3 Simulation of risk-free rates determination for businesses in Germany

4.3.1 Developments and challenges identified

The German economy today stands in the global financial system as one of the providers of benchmark security, as its good fiscal position and other underlying requirement are among the best in the world, especially in Europe. It complies with the Maastricht Treaty fiscal prudence rules which increases the confidence of both foreign and domestic investors in its economy, something also summarized by the credit rating outlook provided by every credit rating agency in the world which assigned it a Triple-A rating. German Bunds compete with US Treasuries for global supremacy in the field of safest securities, since they provide great liquidity, strong and unified regional currency and diversity in durations.

German Bunds have also experienced substantial fall in yields as a result of the 2009 economic and 2012 euro sovereign debt crisis, which shifted funding from other European securities towards it in a process called flight to safety. This, coupled with negative rates and QE programs throughout the major economies, resulted in a negative government yield phenomena also present in Germany.

The Coronavirus changed the German position on EU debt pooling, evident in its support for the Recovery package, created mostly to help southern European Member States in the frontlines of the battle with the virus. To what degree this package will impact German finances remains to be seen, but it certainly will not provide a substantially different alternative to the current Bunds. German government debt to GDP ratio fell to 59.8% by the beginning of 2020 and before the pandemic (Deutsche Bundesbank, 2020).

The assumptions about market efficiency stay firm on the fact that Germany is considered a leader in the European Union and as such respecting all the EU guidelines on the transparency of capital markets (European Commission, 2014). Germany is also considered to be a leader in the digitalization of financial industry, which contributes to constant decreasing in transaction costs in trading with securities.

When it comes to assumptions about continued future involvement of central banks in the markets, it is important to recall that Germany surrendered its right to sovereign monetary policy over to ECB and adopted the Euro as a unifying currency of the market headed by the main central bank of Europe. In that regard, any future change in ECB intentions towards the market should also be reflected in an investor's assumptions about German Bunds. So far, it is evident that ECB will not suspend its involvement substantially in the markets in the period of consideration and investors should develop strategies according to these assumptions.

Bunds are considered very liquid safe benchmarks by markets, trading even at liquidity premium over other European counterparts. Compared to Denmark, Germany offers a wealth of information on its benchmark 10- and 30-year segments. According to Bundesbank, these segments compose 21% and 44% respectively of the overall government marketable securities, with scheduled new issuances of both to further stimulate their liquidity (Deutsche Finanzagentur, n.d.). Investors can opt to choose much more diverse range of maturities for the valuation purposes than in the case of the Danish market.

Table 6: German market characteristics as of June 2020

Market transparency	High
Central bank involvement	Yes
Currency	Euro
Credit rating 2019	Aaa/AAA
CDS spreads	0.26%
Government debt (% GDP)	59.8%
Deposit rate (Euro)	-0.05
10-year segment liquidity	High
30-year segment liquidity	High

Source: Own Work

4.3.2 Challenges to German risk-free rates

According to market prices on selected Bund segments, they can be classified as trend setters and leaders of the European bond market. Long-term segments of best rated government bond are experiencing negative yields, with 10-year reaching -0.4%, while 30-year segments fell to 0.01% in June 2020. Negative yields on German Bunds certainly pose serious challenge for analysts and investors by bending the valuations of stocks and providing unrealistic picture of a company. If one would like to take a more representative value for an RF variable for a German or European market, it could also be done by normalizing or synthetizing yield on German Bunds.

Normalization was done in two ways by which investors can find a more representative rate: historical and build-up. While both segments are currently experiencing negative yield, the latter one is still closely fluctuating around zero-lower bound.

In the case of the10-year segment, simple averaging of its last ten years provided me with 1% rate. For convenience, simple average of the yield over the last fifteen and twenty year period provided me with 2% and 2.6% rates respectively. The point here is that there is a substantial difference in the practical application of D&F normalization approach for the European markets, where I find that 15 year average provides the most representable yield when interest rates suppression and QE involvement periods are taken into account. For 30-year segment, simple averaging provided me with 1.8% and 2.6% over the last 10 and 15 years.

For a build-up RF determination, simple average of the components of the nominal yield have been calculated, namely historical inflation and real return, for both segments separately. I averaged the last 10 years of data on inflation for the German economy and got 1.4%. For the real return of both segments, the actual annual inflation has been subtracted from their average annual nominal yields over the last 10 years. What I found was that the 10-year segment averaged negative 0.3% real return, while 30-year segment was positive 0.4%. Combining these two components of the yield into a normalized RF rate, provides me with 1.1% and 1.8% for the two segments analyzed.

To calculate the RF rate synthetically, I needed future inflation expectations and historical real rates. By using inflation-linked bonds issued by the Federal Finance Agency with maturity corresponding to that of real rate in the previous approach, I can closely match their durations.

The agency issued inflation-linked bonds with maturities of both 10 and 30 years. In case of the former segment, a yield on the bond maturing in 2030 was -1.3% at the point of writing in June 2020, while for the latter, maturing in 2046, was -0.9%. As done in the simulation for the Denmark RF rate, it is necessary to subtract this number from the nominal yield to get inflation expectations in the market. As the 10-year segment is trading on a -0.4% yield in June 2020 and the 30-year on a 0.01%, I end up getting a long-term market inflation expectation of around 0.9% for them both.

Combining the components above to get a synthetized RF rate for both segments over the future 10 year period ends up providing a yield of 0.6% and 1.3% respectively. Though both might still be considered low compared to pre-2009 numbers, the latter one is above 1% and could be considered as appropriate, especially when real rates are stubbornly below zero and not likely to change much soon.

Table 7: Risk-free rates for German market per approach

Market rate 10-year (June 2020)	-0.4%
Market rate 30-year (June 2020)	0.01%
Normalization (historical, 10-year)	1%
Normalization (historical, 30-year)	1.8%
Normalization (build-up, 10-year)	1.1%
Normalization (build-up, 30-year)	1.8%
Synthetization (10-year)	0.6%
Synthetization (30-year)	1.3%
Range	**2.20%**

Source: Own Work

4.3.3 Effects on firm value

Applying a process similar to the Danish market mock company to that for the German market mock company, investors operating there would get different values only because of the choice of the approach to RF rates. Choosing a company from a DAX index, it would operate in consumer cyclicals sector, an industry of auto and trucks, and known by higher riskiness added to the valuation through a Beta (β) coefficient.

Again, Two-RFs approach is advocated besides standard procedures of one-for-all RF rate in cash-flow discounting. By keeping all inputs to WACC, taken from Damodaran's blog relating to this industry and market, a constant except the RF rate, provides the values of the mock company as listed in Table 6. The assumptions about the growth of the fundamentals of the mock company are important to mention. The first is about the success of the company in preserving constant WACC over the analyzed period, regardless of the debt to ratio adjustments to compensate interest rate changes. The second is about the growth in FCFF, where they are assumed to exhibit the same growth rates in the next five years like they did in the last five years, but afterward to grow at the constant rate of the economy of around 0.5%.

Table 8: German market mock company value changes as of June 2020

RF rate	*0.60%*	*1%*	*1.10%*	*1.30%*	*1.80%*
DCF Value (in mil.; €	639,110	613,741	607,704	595,974	568,502
Two-RF's approach	580,049	576,168	575,202	573,278	/

Source: Own Work

Lowest value occurs by either way normalization of the 30-year Bund was done, compared to the highest value received by doing synthetization. The range of DCF values of a mock company based on the approach to RF rate determination, holding other components of WACC constant, sums up to 70 billion EUR. Two-RFs approach, which explicitly used normalized RF rate for longest maturing liquid security for TV discounting, again provided lower values throughout the range.

4.4 Simulation of risk-free rate determination for businesses in Canada

4.4.1 Developments and challenges identified

Canadian economy performed historically slightly better than its European counterparts, also evident in its government bond yields. The benchmark 10-year segment is now for the first time after the impact of Coronavirus trading below 1%, an event occurring after the announcement of its first QE program, since the Canadian central bank lowering its short-term funding rates after the 2009 crisis all the way to zero lower bound like other OECD peers was not sufficient. Even though they started to increase close to 2% in 2019, they quickly reversed to its historical record lows because of the Coronavirus impact (Investing.com, n.d.).

Both Moody's and S&P credit rating agencies strongly support the safety of the Canadian government bonds by reaffirming their ranking of Triple-A despite the impact on the economy by the virus. Fitch downgraded Canadian debt this June from Triple-A to AA+ with positive outlook, meaning that it gives a great chance of reversing the ranking very soon. Historically, the Canadian government effectively managed its debt ratio at healthy levels. Budget deficits since 2009 have continuously been adding to the debt ratio which now is at 97% of GDP, but still not as destructive as it could be thanks to the low interest rates(Trading Economics, n.d.).

Identifying the assumptions about efficiency of Canadian capital markets should lead the determination process of safe assets in that market. Since Canada is among the global leaders in the fight for transparency in all areas of life, this assumption could be among the most

important challenges to consider in the future. Just recently, it updated its policy on transparency of post-trade information dissemination in debt securities to the public (Ontario Securities Commission, 2020).

Canada is also a NATO participant which enabled it to benefit from the domestic peace established through partnerships with other western countries. The benefits stem from the fact that domestic peace allowed investors to think more favorably about the long-term returns than it is the case in other non-NATO countries, which are experiencing constant territorial disputes. In the world of multi-polarity and shifts in powers towards the East, uncertainty about the long-term expectations might influence Canada mainly because of its proximity to the USA. Thus, it is important to start constantly reevaluating the assumptions of the true value of peace in Canada.

The Canadian government shifted its focus on issuing longer-term bonds in the last five years, something expected to continue into the future in order to capture low interest rates on these securities prevailing in the market (Government of Canada, n.d.). This phenomenon helps to further increase the liquidity of Canadian 10-year segment of safe security, but also to provide alternatives to those willing to invest for longer with 30-year segment, which is about to increase. Increasing the amount of issued sovereign bonds being bought by foreign entities, will serve as a measure of confidence foreigners have in the Canadian securities.

Table 9: Canadian market characteristics as of June 2020

Market transparency	High
Central bank involvement	Yes
Currency	Canadian Dollar (CAD)
Credit rating 2019	Aaa/AAA
CDS spreads	0.37%
Government debt (% GDP)	97%
Deposit rate (CAD)	0.25
10-year segment liquidity	High
30-year segment liquidity	Low

Source: Own Work

4.4.2 Challenges to Canadian risk-free rates

Canadian government bonds have just reached levels below 1% for the first time, indicating a bigger trend like in other OECD countries. At the time of the analysis in June, 10- and 30-year segments traded at 0.5% and 1% yield (Investing.com, n.d.).

To determine a more representative RF rate for the Canadian market, without the impacts of central banks, one could use D&F (2016) approach to normalization of rates as a solution

to rate determination. They provided two approaches for dealing with low yields on government bonds, both discussed in the previous simulations: one that averages historical values of the yield or one that builds desired rates from its parts, namely inflation and real returns.

Averaging monthly yields of a 10- and 30-year Canadian government bond segments over the last 10 years provides the result of 2% and 2.6% for the year of 2020. This is almost 100 basis points higher than that of Germany based on historical data for the same period analyzed.

For the second build-up normalization approach, I had to find historic data on the country's inflation and real rates. In case of the former, a 10 year simple average of the past yearly inflation data and got me a number of 1.7%. Besides this, real yields have been separated from the nominal over the period of the last 10 years for both segments, resulting in an average of 0.2% and 0.7% respectively. Both of these components provide me with a combined value of 1.9% and 2.4%.

To synthetize RF rates for the Canadian market, it is necessary to find market expectations and combine them with the real return of Canadian economy. By establishing what the real rates are in the previous approach, I will add to it the market inflation expectations based on the Real Return Bonds (RRBs) Canadian government is offering with the maturity of 30-years, for those who want to protect themselves from the future inflation.

As explained in the previous parts, these kind of bonds are linked to consumer inflation index and their price differs from the nominal one by the value of inflation expectations. In their June auctions, these bonds traded for 0.08% real yield, providing me with an inflation expectation of around 0.4% and 0.9%, depending on the segment analyzed. The combined value of these two components gives me synthetized RF rates for Canadian market of 0.6% and 1.6%.

Table 10: Risk-free rates for Canadian market per approach as of June 2020

Market rate 10-year (June 2020)	0.5%
Market rate 30-year (June 2020)	1%
Normalization (historical, 10-year)	2%
Normalization (historical, 30-year)	2.6%
Normalization (build-up, 10-year)	1.9%
Normalization (build-up, 30-year)	2.4%
Synthetization (10-year)	0.6%
Synthetization (30-year)	1.6%
Range	**2.10%**

Source: Own Work

4.4.3 Effects on firm value

As for the conclusion of the simulation on the determination of appropriate RF rates for the Canadian market, I prepared Table 9, presenting changes to the value of the mock company stemming exclusively from the changes in the approach to RF rate determination. This company, like the one used for Germany, operates in auto and trucks industry, thus carrying higher risk, while all the inputs to the CAPM and WACC are used from the same Damodaran's page and in the same way, except the RF rate, which is calculated by me. Assuming again that the company will preserve its cost of capital by adjusting debt ratio accordingly to interest rate changes, while its growth of future five year FCFFs exhibit the same movement as the last five, from which point onward it grows at constant 0.5%, its DCF value changes in the amounts enlisted in the table below.

Table 11: Canadian market mock company value changes as of June 2020:

RF rate	*0.60%*	*1.60%*	*1.90%*	*2%*	*2.40%*	*2.60%*
DCF Value (mil.; CAD$)	88,122	77,140	74,345	73,456	70,096	68,524
Two-RF's approach	72,312	71,070	70,703	70,581	/	/

Source: Own Work

Compared to both Germany and Denmark, the Canadian market did not experience low rates for as long period as they did. That allowed for a somewhat higher normalized and synthetized rates to be calculated compared to the others. However, low growth variable used for terminal value is based on the assumption that country should exhibit lower growth rates when all of the developments analyzed by this thesis are present. Two-RFs approach provided lower DCF values throughout the spectrum of RF analysis as the greatest amount of estimated value usually comes from a terminal value for which the longest-maturing RF rate is used exclusively. DCF value range for this mock company sums up to 20 billion CAD.

4.5 Findings

Summarizing all the findings from the simulation exercise provides an answer to the fifth and sixth research questions. Based on the simulation conducted on the RF rate determination process for the countries of Denmark, Germany and Canada, one can conclude that pre-2009 rates are not to be expected coming back anytime soon. European governments are working ever more closely with the central bank in their involvement in the markets, thus making it much harder for investors to decouple the right proportion of

central bank influence from the true market representative values in safe sovereign bonds. For instance, the Danish and German economy experienced similar market dynamics due to ECB's decisions on interest rate policies coupled with similar demographic structure of a society, even though Germany is more than ten times the size of Denmark. Canada, on the other hand, just recently started experiencing demographic trends similar to the European counterparts and fall in interest rates, while its debt ratio is at somewhat higher levels. These forces, coupled with the recent Coronavirus crisis, made governments to issue substantial amounts of new debt, focusing mainly on long-term segments to capture extremely low rates for a longer period of time. All three countries updated their debt management strategies to include increasing the outstanding amount and liquidity in its longest-maturing 30-year segment.

Table 12: Summary of market characteristics for selected countries as of June 2020

Country name / Indicator	**Denmark**	**Germany**	**Canada**
Credit rating	Aaa/AAA	Aaa/AAA	Aaa/AAA
Government debt	30%	59.8%	97%
Currency	Danish Kroner (DKK; kr.)	Euro (EUR; €)	Canadian Dollar (CAD; $)
Benchmark segment	10-year	10-year	10-year
Longest segment status	20-year, new, growing	30-year, established, expanding	30-year; established, expanding

Source: Own work

Normalizing or synthetizing yields of these three countries based on historical average differs greatly for different periods of analysis. I would argue that, for the purposes of valuations of companies in the Danish market, it would be representable to use yield between 1.6% and 2.1% as calculated by these two approaches, as it takes historical values of 10-year yield of the last 15 year period. I compensated the loss of the term premium I would have had with longer-maturing segment, with prolonged period of analysis. The reason for this is increasing geopolitics of growing multi-polarity in the world which continues to penetrate central bank decisions, establishing new realities in capital markets like increased money-printing, the process wonderfully highlighted by hedge fund manager Ray Dalio (2018). This involvement might be evident in Denmark as a spillover from actions done by much bigger ECB, which show signs of tolerance of greater future inflation to avoid greater depression. Secondly, I highly believe the European central banks will succeed in the implementation of digital currency in the next 5 years, at least in combination with the existing hard cash, which will enable them in the medium-to long-term to

implement even deeper negative rates to prop up the economy (Agarwal & Kimball, 2019)). Japan is a prime example that regardless of how much the government spends, inflation will still be well below long-term historical averages mainly because of the deflationary pressures of aging, but also already well developed society. In this case, if inflation starts to increase faster than anticipated, they will have much more space to react while not overburdening the economy with high interest rates.

German Bunds, on the other hand, are much more diverse and liquid compared to the Danish government bonds as they trade in much greater quantities daily. It can continue being considered safest, most liquid, security of the whole Eurozone, also used by many countries all over the world. The diversity in maturities serves to increases the liquidity of the Bund market. I was able to follow the normalization approach by D&F (2016) in full, as the existence of the established 30-year security was present. Because of this, I was able to propose what I called a Two-RFs approach by which an investor could combine two rates based on his perceptions about the market. In my proposition, I explicitly advocated the use of normalized yield on the longest standing maturity of a 30-year segment for the terminal value discount input, while leaving investors with a choice of a RF rate used on projected cash-flows.

Determining RF rates for the Canadian economy proved to be less of a challenge than for both European countries. The reason for it might be economic fundamentals, which missed central bank involvement effects in the degree like in other big economies, but also the political structure of the country which differs highly compared to both Germany and Denmark. To deal with challenges, normalization and synthetization of yields on Canadian long-term bond segments provided a higher rate for an input to discount rate used than both Germany and Denmark. However, with the first-time imposition of the QE program investors should expect Canadian government bond yields to gradually follow the pattern of Germany and Denmark.

In my calculations for a current normalized and synthetized RF rate, I got values of 3% and 3.2% to be representable for the Canadian market when one-for-all discount rate is used, with the expectation of it to be gradually lower as inputs are adjusted accordingly with the emergence of new information. Similar to the simulations for the other two countries, I provided a breakdown of the value when a Two-RFs approach to company valuation is applied. The DCF value again ended up being lower than in the case of one-for-all rate, with the largest difference in value of a firm recorded when combined with spot market rates.

Table 13: Summary of the simulation outcomes per country as of June 2020

Country / Approach	***Denmark***	***Germany***	***Canada***
10-year Market rate	-0.3%	-0.4%	0.5%
30-year market rate	/	0.01%	1%
Normalization (historical; 10-year)	1.2%	1%	2%
Normalization (historical; 10-year; last 15 years)	2%	/	/
Normalization (historical; 30-year)	/	1.8%	2.6%
Normalization (build-up; 10-year)	0.9%	1.1%	1.9%
Normalization (build-up; 10-year; Last 15 year)	2.1%	/	/
Normalization (build-up; 30-year)	/	1.8%	2.4%
Synthetization (10-year)	0.7%	0.6%	0.6%
Synthetization (30-year)	/	1.3%	1.6%
Range	**2.40%**	**2.20%**	**2.10%**
Mean RF rate	**1.10%**	**0.90%**	**1.58%**
Max.-Mean difference	**1.00%**	**0.90%**	**1.03%**
Min.-Mean difference	**1.40%**	**1.30%**	**1.08%**

Source: Own Work

CONCLUSION

The environment investors find themselves in today is much different than those throughout the history in terms of market fundamentals and dynamics. Near-zero central bank funding rates accompanied by QE programs in developed countries pulled yields of government bonds to all-time lows making valuations of companies, as well as government bond prices, artificially high, something that usually does not happen at the same time. The reasons for such low rates are low inflation and GDP growth in developed markets, a trend that seems to continue if not worsen with the occurrence of the Coronavirus pandemic in 2020. Governments of the OECD countries are expected to increase their already highly indebted economies to offset the effects of the virus, with the possibility of a rise in some form of joint debt issuance for Euro-zone countries to exploit these extremely low rates. Additionally, central banks increased their asset purchase programs to keep sovereign yields

at the record lows to avoid substantial increase in interest burdens on governments, with balance sheets of some of the biggest among them reaching a combined value of above USD 20 trillion in 2020. All these developments create the unique environment we are in today, with no end in sight for some of them in the medium-term.

The yields of the majority of European and N. American government bonds are close to zero across the entire term structure. Funding rates set by central banks pulled yields on these bonds down to counter effects of the 2008 crisis, but were unable to bring them nowhere near their long-term average historical highs due to persistent deflationary forces and low GDP growth in the economy. In addition to these rates, QE programs had to be implemented to fight interest expense rising on existing debts by artificially increasing the demand for these bonds. Germany and Denmark, two of the countries used for the simulation analysis, exploited these yields to pay off some of their debt and improving their fiscal position, something not seen in any other OECD country. The former country also experienced substantial capital inflow into its sovereign bonds from other European Member States but also from other countries of the world, pulling the yield on Bunds into negative territory. Such a phenomenon of negative nominal yields is also present in other countries of Europe, with the total market value at the end of 2019 being at USD 14 trillion. The Coronavirus will further increase this total, as central banks in some countries that did not have QE programs up to the occurrence of the virus started executing it now. Zero and negative government yields make part of the money of those who invest in those bonds lost, because they are worth more money now than in they will be in the future at the time of maturing. This environment damaged savings of those who save for the retirement, but they seem to still tolerate it, partly because no other alternative is safe enough to counter markets safe incumbents.

QE programs served as a central bank mechanism by which governments would not experience substantial increase in their interest costs. Initially, only short-term securities have been targeted to reduce immediate effects of debt, but this was later expanded to include longer-term securities to allow for cheaper refinancing of existing debt over much longer period of time. The governments analyzed in this these, but also those from other developed countries, are increasing the issuance of bonds throughout the term structure, providing the diversity to investors and increasing the liquidity, with the explicit future targets being issued mostly in long-term segments of 10- and 30-years.

It is hard to imagine a world where no country issues an asset considered by the markets to be default and reinvestment risk free. Yet, growing geopolitical multi-polarity, in times of lacking productivity growth, growing indebtedness and lacking inflation in countries currently considered as holders of RF bonds, might bring a challenge to that perception. In case that happens, Damodaran's approach to determining RF rates in risky governments based on sovereign credit default swap (CDS) spreads seems the most likely contender to become one of the solutions in the RF rate determination process. They provide a rate that compensates buyers of this security in the amount of a likelihood of default occurring, and

are minimal for safe countries today, but would likely change to reflect new realities when it happens. Investors have to subtract the CDS spread from the nominal yields on a particular segment, to come to the nominal RF rate applicable to further corporate analysis of a country to which it belongs. The current market for sovereign CDS spreads includes only a half of the global countries, and they happen to be only developed and emerging ones.

During the simulation part, I researched for the liquidity and availability of other long-term segments of the government bond market. What I found was that countries used in the simulation focus on offering liquidity in 10-year segment, accompanied by some short-term segments. I also found that German Bunds traded at the liquidity premium in the emergence of the European sovereign debt crisis, meaning they trade at higher prices attributed only to the liquidity they possess in the 10-year segment, particularly compared to other European countries like France. But even if one is to take a bond of an even longer segment, the yield is not much different from the initially taken segment, while its liquidity is. In the analysis I used data on 30-year segment from German and Canadian sovereign bond market, while in case of Denmark only 10-year segment, in order to follow the normalization approach suggested by practitioners at Duff & Phelps. Even in the case of prolonged maturities, I found that they are experiencing negative real rates and that there is deeper trend of rates falling all over the spectrum.

Investors could use any rate from Table 11 they find most appropriate based on their view of the markets. I provided both normalized and synthetized rates for each market in case market rates are deemed excessively low, accompanied by the explanation behind Two-RFs approach to discount rate input determination for terminal value discounting. Normalized rates for Denmark varied between 0.9% and 2.1%, for Germany between 1% and 1.8%, while for Canada it was between 2% and 2.6%. Which rate investors choose for the corresponding market depends on their view of the markets. More conservative or pessimistic investors might choose higher normalized rates in general compared to the optimistic ones to find lower value of company they analyze to be in line with the pessimistic view of the general economy.

REFERENCES

1. Adam Smith (1776). *The wealth of Nations.* W. Strahan and T. Cadell, London, p. 465-477
2. Agarwal, R & Kimball, M (2019). *Enabling Deep Negative Rates to Fight Recessions: A Guide*. Retrieved from: https://www.elibrary.imf.org/view/IMF001/25793-9781484398777/25793-9781484398777/25793-9781484398777_A001.xml?rskey=AGEWsY&result=329&highlight=true&redirect=true

3. Agarwal, R. & Krogstrup, S. (2019). *Cashing In: How to Make Negative Interest Rates Work [Published on blog]*. Retrieved from: https://blogs.imf.org/2019/02/05/cashing-in-how-to-make-negative-interest-rates-work/
4. Ainger, J. (2019). *The Logic behind the Bonds That Eat Your Money*. Retrieved from: https://www.bloomberg.com/graphics/2019-negative-yield-debt/
5. Aldasoro, I. & Ehlers, T. (2018). *The credit default swap market: what a difference a decade makes*. Retrieved from: https://www.bis.org/publ/qtrpdf/r_qt1806b.htm
6. Aliber, Z. R. & Kindleberger, C. P. (2015). *Manias, Panics and Crashes: A History of Financial Crisis* (7th ed.). Palgrave Macmillan
7. Bahar, D. (2017). *Productivity is key to economic growth: Why is it slowing down in advanced economies?*. Retrieved from: https://www.brookings.edu/blog/up-front/2017/09/25/productivity-is-key-to-economic-growth-why-is-it-slowing-down-in-advanced-economies/
8. Baldwin, R. & Wyplosz, C. (2015). *The Economics of European Integration (5th ed.).* McGraw-Hill
9. Berk, J. & DeMarzo, P. (2017). *Corporate Finance* (4th ed.). Pearson Education Limited
10. Biggs, B. (2009). *Wealth, War and Wisdom.* Wiley
11. BIS (2013). *Sovereign risk: a world without risk-free assets?* Retrieved from: https://www.bis.org/publ/bppdf/bispap72.htm
12. Bloom, E. D. (2020). *Population 2020*. Retrieved from: https://www.imf.org/external/pubs/ft/fandd/2020/03/changing-demographics-and-economic-growth-bloom.htm
13. Brealey, A. B., Myers, S. C. & Allen, F. (2016). *Principle of Corporate Finance* (12th ed.). McGraw-Hill Education
14. Bruegel (2010, May 5). *The Blue Bond Proposal*. Retrieved from: https://www.bruegel.org/2010/05/the-blue-bond-proposal/
15. Bullard, J. (2015) *Some Considerations for U.S. Monetary Policy Normalization*. Hyman P. Minsky Conference. Retrieved from: https://www.stlouisfed.org/~/media/files/pdfs/bullard/remarks/bullard-minsky-15-april-2015.pdf
16. Bundesrepublik Deutschland Finanzagentur (2020). *Secondary market*. Retrieved from: https://www.deutsche-finanzagentur.de/en/institutional-investors/secondary-market/
17. Cambridge Dictionary (n.d.). *Assumption*. Retrieved from: https://dictionary.cambridge.org/dictionary/english/assumption
18. Campbell, R. H. (1986). *Recovering expectations of consumption growth from an equilibrium model of the term structure of interest rates* (doctoral dissertation). Chicago: The University of Chicago
19. Cash, D. (2018). *The Role of Credit Rating Agencies in Responsible Finance.* Palgrave Macmillan
20. Countryeconomy.com (n.d.). *Sovereigns Ratings List.* Retrieved from: https://countryeconomy.com/ratings

21. Culbertson (1957). The term structure of interest rates. *The Quarterly Journal of Economics,* 71(4), 485-517.
22. Dalio, R. (2018). *A Template for Understanding Big Debt Crises.* Bridgewater
23. Damodaran, A. (2010a). *Into the Abyss: What if nothing is RF?* SSRN working paper: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1648164
24. Damodaran, A. (2010b). *The Dark Side of Valuation* (2nd ed.). Pearson Education, Inc
25. Damodaran, A. (2013). *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset* (3rd ed.). John Wiley & Sons
26. Damodaran, A. (2014). *Applied Corporate Finance* (4th ed.). John Wiley & Sons
27. Damodaran, A. (2020). *Country Default Spreads and Risk Premiums*. Retrieved from: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html
28. Danmarks Nationalbank (2018a). *Central government borrowing strategy in the 2nd half of 2018*. Retrieved from: https://www.nationalbanken.dk/en/governmentdebt/publications/Documents/Central%20government%20borrowing%20strategy%20in%202nd%20half%202018.pdf#search=20%20year%20bonds
29. Danmarks Nationalbank (2018b). *0.1per cent inflation-linked loan 2030 (DGBi 2030).* Retrieved from: https://www.nationalbanken.dk/en/governmentdebt/IR/Documents/DK0009923724%200.1%20per%20cent%20DGBi%202030.pdf
30. Danmarks Nationalbank (2020). *Update to the central government borrowing strategy 2020*. Retrieved from: https://www.nationalbanken.dk/en/governmentdebt/publications/Pages/Update-to-the-central-government-borrowing-strategy-2020.aspx
31. Detrixhe, J. (2011). *Egan-Jones Cuts U.S. Rating to AA+ on Spending-Cut Concern*. Retrieved from: https://www.bloomberg.com/news/articles/2011-07-18/egan-jones-cuts-u-s-rating-to-aa-on-spending-cut-concern-1-
32. Deutsche Bundesbank (2020, March 31). *German general government debt down in 2019 by €16 billion to €2.05 trillion – debt ratio down from 61.9% to 59.8%*. Retrieved from: https://www.bundesbank.de/en/press/press-releases/german-government-debt-829646
33. Deutsche Finanzagentur (n.d.). *Federal bonds ("Bunds")*. Retrieved from: https://www.deutsche-finanzagentur.de/en/institutional-investors/federal-securities/federal-bonds/
34. Dreman, D. (2000). Investor Overreaction and Contrarian Strategies. *AIMR Conference Proceedings 2000(3):55-64*. Retrieved from: https://www.researchgate.net/publication/240304477_Investor_Overreaction_and_Contrarian_Strategies
35. Dreman, D. N. & Berry, M. A. (1995). "Overreaction, Underreaction, and the Low-P/E Effect". *Financial Analysts Journal. 51 (4): 21–30*
36. Duff & Phelps (2016). *Duff & Phelps Increases Recommended U.S. Equity Risk Premium from 5.0% to 5.5%*. Retrieved from:

https://www.duffandphelps.com/insights/publications/cost-of-capital/duff-phelps-increases-recommended-us-equity-risk-premium-from-5-0-to-5-5

37. Duff & Phelps (2019). *Duff & Phelps' U.S. Normalized Risk–Free Rate Decreased from 3.5% to 3.0% Effective September 30, 2019.* Retrieved from: https://www.duffandphelps.com/insights/publications/valuation/us-normalized-risk-free-effective-september-30-2019
38. Dunne, G. P. (2007). Transparency Proposals for European Sovereign Bond Markets. SSRN working paper: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=956556
39. Dunne, G. P., Moore, J. M. & Portes, R. (2002). Defining Benchmark Status: An Application using Euro-Area Bonds. *NBER Working Paper No. 9087*. Retrieved form: https://www.nber.org/papers/w9087
40. Durden, T. (2011). *S&P Downgrades US to AA+, Outlook Negative - Full Text*. Retrieved from: https://www.zerohedge.com/news/sp-downgrades-us-aa-outlook-negative-full-text
41. ECB (2017). Economic Bulletin: Issue 4 / 2017. Retrieved from: https://www.ecb.europa.eu/pub/pdf/ecbu/eb201704.en.pdf?9863c5b1adfd00fa54dfd1d535657313
42. ECB Statistics (n.d). *Statistics.* Retrieved from: https://www.ecb.europa.eu/stats/html/index.en.html
43. Ejsing, J., Grothe, M. & Grothe, O. (2012). Liquidity and Credit Risk Premia in Government Bond yields. *ECB Working Paper Series No 1440*. Retrieved from: https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1440.pdf
44. EUR-Lex (n.d.). *Consolidated version of the Treaty on the Functioning of the European Union – Part three: Union policies and internal actions – title VIII: Economic and monetary policy - Chapter 1: Economic policy - Article 125 (ex. Article 103 TEC*). Retrieved from: https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:12008E125
45. European Commission (2011). *Green paper on stability bonds.* Retrieved from: https://ec.europa.eu/economy_finance/articles/governance/2011-11-23-green-paper-stability-bonds_en.htm
46. European Commission (2020). *Recovery plan for Europe.* Retrieved from: https://ec.europa.eu/info/live-work-travel-eu/health/coronavirus-response/recovery-plan-europe_en
47. European Commission (n.d.). *Markets in Financial Instruments (MiFID II) - Directive 2014/65/EU.* Retrieved from: https://ec.europa.eu/info/law/markets-financial-instruments-mifid-ii-directive-2014-65-eu_en
48. Eurostat (n.d). Retrieved from: https://ec.europa.eu/eurostat
49. Fama, E. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *Journal of Finance. 25 (2): 383–417*
50. Fama, E. F. & French, K. R. (2011). *Size, Value, and Momentum in International Stock Returns.* Fama-Miller Working Paper; Tuck School of Business Working Paper No.

2011-85; Chicago Booth Research Paper No. 11-10. Retrieved from: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1720139&download=yes

51. Fatas, A., Ghosh, A. R., Penizza, Ugo & Presbitero, A. F. (2019). *The Motives to Borrow*. IMF Working Paper. Retrieved from: https://www.imf.org/en/Publications/WP/Issues/2019/05/10/The-Motives-to-Borrow-46743
52. Fernandez, P., Pershin, V. & Acin, F. I. (2017). Discount Rate (Risk-free rate and Market Risk Premium) used for 41 countries in 2017: A Survey. SSRN working paper: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2954142
53. Fisher, I. (1896). *Appreciation and interest*. New York: Macmillan, 1896
54. Fitch Ratings (2020). *Fitch Revises United States' Outlook to Negative; Affirms at 'AAA'*. Retrieved from: https://www.fitchratings.com/research/sovereigns/fitch-revises-united-states-outlook-to-negative-affirms-at-aaa-31-07-2020
55. Government of Canada (n.d.). *Debt Management Report 2017–2018 - Part 1*. Retrieved from: https://www.canada.ca/en/department-finance/services/publications/debt-management-report/2017-2018/part-1.html
56. Haksar, V. & Kopp, E. (2011). *How Can Interest Rates be Negative?* Retrieved from: https://www.imf.org/external/pubs/ft/fandd/2020/03/what-are-negative-interest-rates-basics.htm
57. Harris, A. & Barrett, E. (2019). *U.S. weight selling 50- and 100-year bonds after yields plummet.* Retrieved from: https://www.bloomberg.com/news/articles/2019-08-16/u-s-treasury-to-do-market-outreach-again-on-ultra-long-bonds-jzejo2qu
58. Hausken, K. & Ncube, M. (2013). *Quantitative Easing and Its Impact in the US, Japan, the UK and Europe.* Springer New York
59. Hicks, J. (1946). *Value and Capital.* Oxford: Clarendon Press.
60. IMF (2017, August 3). *Negative Interest Rate Policies—Initial Experiences and Assessments*. Retrieved from: https://www.imf.org/en/Publications/Policy-Papers/Issues/2017/08/03/pp080317-negative-interest-rate-policies-initial-experiences-and-assessments
61. Institute of International Finance (2020, July 16). *Global Debt Monitor: Sharp spike in debt ratios.* Retrieved from: https://www.iif.com/Research
62. Investing.com (n.d.). *European Financial Markets.* Retrieved from: https://www.investing.com/markets/europe
63. Joyce, M., Miles, D., Scott A. & Vayanos, D. (2012, November). Quantitative Easing and Unconventional Monetary Policy – An Introduction. *The Economic Journal, 122 (November), F271-F288.* Retrieved from: https://www.researchgate.net/publication/256038514_Quantitative_Easing_and_Unconventional_Monetary_Policy_-_An_Introduction
64. Kennedy, S., Donahue, P. & Deen, M. (2012). *France Loses AAA Status as S&P Wields Ax on Debt Ratings*. Retrieved from: https://www.bloomberg.com/news/articles/2012-01-13/france-to-lose-aaa-from-s-p-afp-says-citing-state-official

65. Keynes, M. J. (1936). *The General Theory of Employment, Interest and Money*. Palgrave Macmillan
66. Koller, T., Goedhart, M. & Wessels, D. (2015). *Valuation – Measuring and Managing the Value of Companies* (6th ed.). John Wiley & Sons
67. Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in
Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics, 47*(1), 13-37.
68. Lončarski, I. (2017). Izzivi uporabe netvegane donosnosti v razmerah izjemno nizkih ali negativnih obrestnih mer = The challenges of using a risk-free rate of return in the conditions of extremely low or negative interest rates. *Sir*ius*, 5(3), 39-45.
69. Macrotrends (2020). *Federal Funds Rate - 62 Year Historical Chart*. Retrieved from: https://www.macrotrends.net/2015/fed-funds-rate-historical-chart
70. Macrotrends (n.d.). *Debt to GDP Ratio Historical Chart*. Retrieved from: https://www.macrotrends.net/1381/debt-to-gdp-ratio-historical-chart
71. Maranga, K. C. &. Mwangi, M. (2018). Dynamics of the Term Structure of Interest Rates: A Critical Literature Review. *IOSR Journal of Economics and Finance Volume 9, Issue 1, p. 45-50*
72. Modigliani, F. and Sutch, R. (1966). Innovations in interest-rate policy. *American Economic Review,* 56, 178-197
73. Moosa, I. A. (2014). *Quantitative Easing as a Highway to Hyperinflation.* World Scientific Publishing
74. NASDAQ (n.d.). *OMX Copenhagen 25 Index*. Retrieved from: http://www.nasdaqomxnordic.com/index/index_info?Instrument=DK0060775369
75. OECD (2011, May). *OECD Economic Outlook*. Retrieved from: https://www.oecd-ilibrary.org/economics/oecd-economic-outlook-volume-2011-issue-1-preliminary-version_eco_outlook-v2011-1-en
76. OECD (2020). *Sovereign Borrowing Outlook for OECD Countries 2020: SPECIAL COVID-19 EDITION*. Retrieved from: https://www.oecd-ilibrary.org/governance/oecd-sovereign-borrowing-outlook-2020_dc0b6ada-en
77. Ontario Securities Commission (2020). *CSA Notice of Approval – Mandatory Post-Trade Transparency of Trades in Government Debt Securities, Expanded Transparency of Trades in Corporate Debt Securities and Amendments to National Instrument 21-101 Marketplace Operation and Related Companion Policy*. Retrieved from: https://www.osc.gov.on.ca/en/SecuritiesLaw_csa_20200604_21-101_marketplace-operation-related-cp.htm
78. Paret, A. C. & Weber, A. (2019). *German Bond Yields and Debt Supply: Is There a "Bund Premium"?*. Retrieved from: https://www.imf.org/en/Publications/WP/Issues/2019/11/01/German-Bond-Yields-and-Debt-Supply-Is-There-a-Bund-Premium-48730

79. Rappeport, A. & Phillips, M. (2019). *Treasury Considers 50-Year Bonds as Deficit Tops $1 Trillion*. Retrieved from: https://www.nytimes.com/2019/09/12/us/politics/treasury-50-year-bonds.html
80. Schnidman, E. A. & Macmillan, W. D. (2016). *How the Fed Moves Markets: Central Bank Analysis for the Modern Era.* Palgrave Macmillan
81. Scott, G. & Hayes, A. (2020). *Fama and French Three Factor Model*. Retrieved from: https://www.investopedia.com/terms/f/famaandfrenchthreefactormodel.asp
82. Sharpe, W. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance, 19*(3), 425-442
83. Shoreline Community College (n.d.). *Reasoning and Assumptions.* https://app.shoreline.edu/doldham/Assumptions.htm
84. Smith, A. (1776). *An Inquiry into the Nature and Causes of the Wealth of Nations*. W. Strahan and T. Cadell, London
85. Tennant, D. F. & Tracey, M. R. (2016). *Sovereign Debt and Credit Rating Bias.* Palgrave Macmillen
86. Trading Economics (n.d.). Retrieved from: https://tradingeconomics.com/country-list/government-debt-to-gdp
87. U.S. Department of the Treasury (2016). *A Deeper Look at Liquidity Conditions in the Treasury Market*. Retrieved from: https://www.treasury.gov/connect/blog/Pages/A-Deeper-Look-at-Liquidity-Conditions-in-the-Treasury-Market.aspx
88. US Department of Treasury (n.d.). *Daily Treasury Yield Curve Rates.* Retrieved from: https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yieldAll
89. Werner, R. A. (1997). Towards a New Monetary Paradigm: A Quantity Theorem of Disaggregated Credit, with Evidence from Japan. *Kredit und Kapital, Vol. 30, no. 2*, p. 276-309
90. World Government Bonds (n.d.). *Sovereign CDS.* Retrieved from: http://www.worldgovernmentbonds.com/sovereign-cds/
91. Yardeni Research (2020). *The Great Rebalancing: No Asset Left Behind*. Retrieved from: https://www.yardeni.com/premiumdata/mb_200408.pdf

APPENDICES

Appendix 1: Povzetek (Summary in the Slovene language)

Verjetno najpomembnejši sestavni del in izhodišče formule CAPM ter celotnega postopka vrednotenja je določitev, katera je najvarnejša naložba na trgu, s katerim bi lahko primerjali donosnost drugih naložb. Ta komponenta se v finančni panogi imenuje netvegana donosnost (risk free rate - RF) in je komponenta diskontne stopnje (Damodaran, 2013). Kot merilo so vlagatelji pri svojih izračunih izrecno uporabljali donosnosti državnih obveznic, saj so najboljši približek netveganih naložb. Vendar pa smo v zadnjih letih, predvsem kot posledica velike recesije po letu 2008, priča novemu pojavu v financah, kjer se skoraj vse razvite države članice OECD soočajo z ničelnimi ali celo negativnimi donosnostmi državnih obveznic (Investing.com, 2020). Zgodovinsko gledano sta bila najpogosteje uporabljena referenčna vrednostna papirja v Zahodnem svetu 10-letne ameriške obveznice (Treasuries) in nemške obveznice (Bunds), predvsem zaradi njihove likvidnosti (Damodaran, 2010).

Izziv o katerem je vredno razpravljati je netvegana donosnost katera se uporablja za diskontno stopnjo pri diskontiranju residualne vrednosti (terminal value - TV). Doslej se je v praksi izrecno uporabljal tržni 10-letni donos obveznic, prakso katero podpirajo tudi akademiki (Damodaran, 2013). Program odkupa vrednostnih papirjev (QE) je iztisnili donose tudi pri evropskih obveznicah z 30- in 50-letno zapadlostjo. Vlagatelji verjamejo, da se bodo tudi te obveznice povečale v ponudbi in bodo imele podobno likvidnost kot referenčne danes. Predlagani so bili pristopi kako ravnati z izredno nizkimi stopnjami na današnjem trgu. Duff & Phelps (2019) menijo, da je smiselno, če se nekdo odloči za "normalizacijo" ameriških donosov na njihovo zgodovinsko povprečje okoli 3%. Koller, Goedhart in Wessels (2015) na drugi strani priporočajo ustvarjanje sintetične netvegane donosnosti za ameriški trg, tako da upoštevajo pričakovano stopnjo inflacije, za katere so ugotovili da znaša približno 2.5%, in ji dodajo dolgoročni zgodovinski realni donos od približno 2%.

Raziskoval sem likvidnost in razpoložljivost dolgoročnih segmentov trga državnih obveznic in ugotovil, da se države, uporabljene v simulaciji, osredotočajo na ponujanje likvidnosti predvsem v 10-letnem segmentu. Izkazalo se je tudi, da se je z nemškimi obveznicami ob pojavu evropske državne dolžniške krize trgovalo po likvidnostni premiji, kar pomeni, da so imeli višje cene, ki so bile pripisane le likvidnosti v desetletnem segmentu. Tudi če želimo prevzeti obveznico še daljšega segmenta, se donos ne razlikuje bistveno od prvotno prevzetega segmenta, njegova likvidnost pa da. Pri analizi sem tudi uporabil podatke o 30-letnem segmentu nemškega in kanadskega trga državnih obveznic, medtem ko je bil na Danskem vporabljen le 10-letni segment, da bi sledil pristopu normalizacije, ki so ga predlagali izvajalci Duff & Phelps. Podane so tudi normalizirane in sintetizirane obrestne mere za posamezni trg v primeru ko se tržne obrestne mere štejejo za prenizke. Zraven sem obrazložil pristop uporabe dve netvegane donosnosti (Two-RFs) pri določanju diskontne stopnje za diskontiranje rezidualne vrednosti. Izračunana je bila povprečna stopnja za Dansko 1,1%, za Nemčijo 0,9%, za Kanado pa 1,6%.