UNIVERSITY OF LJUBLJANA
SCHOOL OF ECONOMICS AND BUSINESS

MASTER THESIS

PORTFOLIO DIVERSIFICATION EFFECTS OF THE U.S. EQUITY INDICES

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AUTHORSHIP STATEMENT

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

CAGR – Compounding Annual Growth Rate

CAPM – Capital Asset Pricing Model

ETF – Exchange Traded Fund

HHI – Herfindahl-Hirschmann Index

NAV – Net Asset Value

OECD – Organisation for Economic Co-operation and Development

S&P 500 – Standard and Poor’s 500

SEC – The U.S. Securities and Exchange Commission

SPDR – Standard & Poor's Depositary Receipts

UCITS – Undertakings for Collective Investments in Transferable Securities

UN – United Nations

LIST OF APPENDICES

INTRODUCTION

The development of the Capital Asset Pricing model created a fertile ground for the development of asset strategies that try to emulate the movements of the general market (Goltz & Le Sourd, 2011).

The wide development of index funds is closely related to the theoretical background of the CAPM model. CAPM tells us that all investors will want to hold capital-weighted portfolios of global wealth. At that time, the U.S. was the world's largest market, so a solution like S&P 500 seemed like a fair approximation. Many people recognized S&P 500 as an attractive real-world solution and there was little evidence of active management outperformance. CAPM was the basis for a wide range of index models, and many started emerging. At the beginning of the emergence of passive solutions key indices like the S&P 500 seemed a fair approximation for the development of the index funds. Index funds are funds that match the performance of the S&P (Goetzmann, 1996).

These strategies represent an alternative to existing management practices where the focus lies more on the involvement of the asset managers choosing their strategies rather than on the replication of general market movements (Goltz & Le Sourd, 2011).

Active investing means trying to beat the market or appropriate benchmark. An active manager can add value by either deviating from his benchmark index by forecasting market trends - a so-called market timing or by identifying mispriced market sectors or securities. Such stock or sector selection involves active bets on individual stocks of a defined industry (Hebner, 2007).

After the global financial crisis, the landscape of institutional money management has started to change even more dramatically. More funds have been flowing into passive management forms, such as index mutual funds and exchange-traded funds (ETFs). Such financial assets are referred to as passive index funds. The main goal of passive index funds is to replicate existing stock indices (Hebner, 2007; Fichtner, Heemskerk & Garcia-Bernardo, 2017).

So-called index investment options have been developing from 1973 onwards in the U.S. and slowly spreading through the world, including Europe and Asia-Pacific region. This development has created new ways for retail and institutional investors to obtain their investment objectives and also massively gained acceptance (Goltz & Le Sourd, 2011).

Strategic beta, also called smart beta has been one of the key ETF product development battlegrounds of recent years. New strategic beta ETFs have been introduced in the last years in the last years with a focus mainly on multifactor equity strategies and different themes, including environmental, technological, and others as an area for active differentiation

(Bioy, Garcia-Zarate, Lamont, Boyadzhiev & Kang, 2019; Bender, Briand, Melas & Subramanian, 2013).

At first, ETFs tracked traditional indexes, mainly weighted by market capitalization. As the industry is evolving, index-based ETFs can follow benchmarks that use a range of index construction methodologies (ICI, 2022).

Concerning the portfolio diversification construction process, there are several available options to consider. Some of the options include equating money weights within the portfolio, different risk parity strategies, and mean-variance optimization. Different options allow investors to tailor their portfolio selection process based on their preferences for detailed analysis and input requirements (Hallerbach, 2013).

Risk parity as an investment strategy has gained attraction in recent years. Such a strategy can have a better Sharpe ratio compared to standard approaches like mean-variance optimization (Chaves, Hsu, Li & Shakernia, 2011).

The key difference is that this approach tries to allocate risks. It should deliver true diversification that limits the impact of losses of individual components to the portfolio (Qian, 2005).

Generally, the theory and practice of such a strategy have gained awareness among investors due to reduced equity concentration paired with less tail risk, a more meaningful approach towards diversification, benefits in terms of behavior in a wide variety of economic environments, and new risk/return optimization opportunities (Hurst, Johnson & Ooi, 2010).

It turns out that risk parity is a viable approach to asset allocation. In the absence of the full optimization approach, risk parity appears to provide some alternative to the original Markowitz approach. It focuses on the proper risk allocation of the portfolio, avoiding too concentrated risk allocations to a particular asset, focusing manages to reconsider the marginal risk contribution of a portfolio component (Kazemi, 2011).

Demiguel, Garlappi and Uppal (2009) demonstrate that it is hard to consistently beat the naive 1/N multi-equity portfolio, and Plyahka, Uppal and Vilkov (2012) point out that equal-weighted portfolios outperform capitalization and price-weighted counterparts in terms of total mean return, similar to Taljaard, Mare (2020) who point out that equal-weighted portfolios generally outperform market capitalization counterparts, maintaining better risk-adjusted returns.

The availability of portfolio diversification techniques and thematic schemes raises questions regarding the justification of such high usage of passive capitalization-weighted alternatives as the key passive investment option.

The purpose of this study is to examine the adequacy of the U.S. stock indice being considered as a main passive investment alternative along with an examination of its sufficient diversification. Structural development of the investment ecosystem in the past decades offers a unique development for this research. Besides the analyzing theoretical background for diversification a practical comparison between the main U.S indices and its variation should yield more answers.

Strategies for emulating market movements have appeared to emerge more, however, there is a question if this is superior in comparison with other strategies. Supporting evidence for current market practices should prove current extensive development.

Evidence from this research could be used by retail and institutional investors to potentially rethink their investment policy according to adequate investor goals and preferences.

The Master Thesis aims to examine if the current usage and structure of the main U.S indice sufficiently justify investors' adoption, that is sufficient diversification with beneficial risk-return characteristics.

There are several goals of the thesis:

1. Study and reflect on current theoretical scientific literature about diversification and market portfolio
2. Analyze the systemic development of the passive investment universe as a key catalyst for the use of key index funds.
3. Analyze whether the current use of indices sufficiently coincides with diversification
4. Test portfolio risk-return performance based on diversification and possible alternative portfolios
5. Suggest possible adjustments or potential criticism of current practices.

From what has been defined so far, my main premise is that stock market indices should offer investors favourable risk-return characteristics to justify their clear superior diversification effects.

Diversifying into more not completely correlated securities means further reducing exposure from firm-specific factors and improving risk-return character. These impacts should be seen from the studied indice to confirm the thesis.

Master´s Thesis is divided into theoretical and practical parts. The theoretical part of this study will explore the development of the key U.S. stock indices as being the main category that passive investors use to achieve their investment objectives. Understanding of diversification, theoretical background, and possible risks of current usage will be studied.

The practical part will involve a study of concentration and its development. There will be risk-return characteristics analysis of this indice in different economic periods. Possible alternative portfolios will be examined and compared to create an understanding of portfolio diversification effects. Additional multiple regression analysis will yield a deeper understanding of the asset factor strategies. The goal of the practical part is to offer more insight into examined equity indice (s), offer thorough synthesis, and present meaningful suggestions for investor practice or research questions.

Basis references for the study will involve scientific literature on modern portfolio theory, diversification, and portfolio strategies from books, scientific articles, and websites. Most used databases and programmes include Bloomberg Terminal, Eikon Refinitiv, R Studio, and Microsoft Excel.

1 ASSET ALLOCATION IN MODERN PORTFOLIO THEORY

1.1 Effects of diversification on the risk

Diversification is one of the key elements to consider in the context of portfolio construction. Investment allocation should be made with the diversification principle in mind.

Investors allocating to only one particular investment face risks that would affect this company. For example, an oil company is affected by the drop in the price of oil. Tactically adding a computer company, rising computer prices affect this company positively. The extent that firm-specific risks differ, the greater the impact of diversification, reducing risks in a portfolio. By combining different companies in a portfolio, two outcomes emerge, offsetting and stabilizing portfolio return. Adding multiple companies can be considered to further add new companies to the mixture. With more new companies, diversification increases and company-specific factors get reduced. As the theory would suggest portfolio volatility should be reduced. Even with many of the securities involved, all of the risk can not be eliminated as the companies are impacted by certain common factors. For instance, future business cycles or inflation projections as key macroeconomic variables influence all companies.

Extensive diversification cannot get rid of all risks in a portfolio when shared causes of risk affect numerous enterprises at once. When more companies are added, the portfolio standard deviation is greatly reduced, but some shared risk still exists. Regardless of the firm character, all organizations are impacted by the risk. This risk is called market or systemic also non-diversifiable risk. Another type of risk is a unique risk, firm-specific which is able to be reduced by improved diversification in the portfolio. Such risk can be diversified away with a greater number of securities held. The power of diversification is limited by the systematic sources of risk. The important thing to mention is efficient diversification. This means constructing a risky portfolio with the lowest possible risk for a given level of expected return. In a construction process, covariance, which is a product of two standard deviations and correlation coefficients plays a key role. Where correlation is just positive and one, portfolio standard deviation is just the weighted average of the component standard deviations. The portfolio standard deviation is less than the weighted average of the component standard deviations when the correlation coefficient is not one, creating an effect of diversification (Bodie, Kane & Marcus, 2013).

Reasonable additions to the portfolio components are such that have less-than-perfect correlations. An asset with a negative correlation is particularly effective in reducing risks. When investors decide to add a negative or low-correlated asset to their existing number of holdings, this decision should be applauded. Improvements in efficiency are higher when a component with a small correlation is added. When correlations are lower than 1 between assets, the portfolio's expected return is the weighted average of its component returns, portfolio standard deviation is not the weighted average of the component standard deviations. Risk reduction is always achievable when a portfolio is constructed from imperfectly correlated assets (Statman 2002; Bodie et al., 2013).

1.2 Diversification in equally-weighted portfolios

How risky a portfolio is depends on the weightings of the different stocks, their variances, and their covariances. The portfolio risk profile will alter if some of these variables change. In general terms, when more stocks are chosen at random and included in the evenly weighted portfolio, the risk of the portfolio is reduced (Bodie et al., 2013).

Naive portfolio diversification means equally distributing portfolio weights among constituents. A weight of 1/N is given to each of the N-holdings in the portfolio. Concentrated investments in terms of financial allocation are avoided with this portfolio construction methodology. Such composition implies indirect exposure to small-cap stocks when equity exposure is considered. Therefore, naive diversification includes small-cap stocks and their influence which also means an influence from the size premium. With such a portfolio, positions must be periodically and carefully rebalanced. In contrast to a market cap portfolio, which involves buying and retaining all the assets, this portfolio involves active rebalancing the positions. This is done through the sale of better performing assets. In this context it can be seen as a reversal strategy, meaning buy low and sell high strategy which can turn out to be profitable. When rebalancing happens this also means some portfolio turnover and accompanying transaction costs. Exposure to potential illiquidity can occur depending on the rebalancing frequency. When there is insufficient knowledge to distinguish between various assets, such a naively diversified portfolio is an adequate (best) option (Hellerbach, 2015).

In his study, Demiguel (2009) shows that it is rather difficult to perform better than a general 1/N portfolio. While some could argue that such portfolio building may seem little simplistic, further findings prove it. For those interested in applying mean-variance optimization for allocation problems this is rather bad news. The gain from optimal diversification is offset by the estimation error. Sharpe ratio and certainty-equivalent return prove that none of the 14 tested models across a variety of datasets performed consistently better than the 1/N rule. Based on the analytical findings and simulations used, a portfolio from 25 assets needed an estimation window of about 3000 months for a sample-bases mean-variance to outperform a 1/N benchmark. When testing a portfolio with 50 assets, the needed months were twice as many. This implies that it can take some time before benefits from optimum portfolio selection can appear outside of the samples environment.

Equal-weighted portfolios typically beat their market capitalization counterparts, according to Taljaard and Mare (2020). Equal-weighted stock portfolios in the context of the S&P 500 have, however, underperformed market capitalization-weighted stock portfolios since 2016. Stochastic portfolio theory is used in the study to examine such underperformance. The authors nicely demonstrate that an equal-weighted portfolio generally outperforms the market-weighted alternative over the long term. In the short time period underperformance can occur as well. The market-weighted portfolio has become more concentrated in recent years. The equal-weighted portfolio has shown underperformance, while still maintaining better risk-adjusted characteristics over the whole analyzed period. At the same time, the benefits of diversification from an equal-weighted portfolio have been declining as a result of lower stock volatilities (so diversification is less beneficial) and higher correlations on average from 2009 onward compared to the previous timeframe (higher diversification is impossible to be achieved).The authors also demonstrate how a dynamic market cap or equal weighting portfolio option can be chosen with a help of a linear regression model to improve the performance during the period. While effectively limiting drawdowns, the optimized portfolio also produced greater returns.

Plyahka, Uppal, and Vilkov (2012) analyzed the performance of equal, value, and price-weighted portfolios of stocks in the major U.S. equities. Analysis was performed over the preceding 40 years combined with random equity selection from the SP 500 indices. The authors discover that equal-weighted portfolios beat the value and price-weighted portfolios based on monthly rebalancing and total mean return metric. Portfolios were examined based on the Sharpe, Sortino and Treynor ratios and certainty-equivalent return. With the four-factor model, the equal-weighted alternative had the highest total return, higher return for bearing systemic risk coupled with higher alpha. Based on nonparametric monotonicity relation test factors like size, price, liquidity, and idiosyncratic volatility were all monotonically related to the total return across portfolios. Higher exposure to factors like market, size, and value determined a higher systematic return of the equal-weighted alternative. A higher alpha was due to the monthly rebalancing process for maintaining initial equal weights. Rebalancing exploits reversal and idiosyncratic volatility of equity returns which is defined as a contrarian strategy. The choice of the initial weights did not influence obtained alpha in the analysis. Taking into account transaction costs of 50 basis points, the equal-weighted alternative produced clearly better mean return and four-factor alpha compared to value or price-weighted alternatives.

The marginal decrease in portfolio variance driven by more securities being kept in a portfolio is examined by Evans and Archer (1968). Marginal advantages result from greater variety. They make three primary assumptions that form the basis of their study: the investor is an accidental buyer of common stocks; dividends from securities are not reinvested; and identical amounts of money are invested in each security. The portfolio return and portfolio standard deviation were calculated. They observed securities held within the Standard & Poor index and selected random securities. Multiple runs were performed by the authors. One gathering run generated 40 portfolios with sizes ranging from 1 to 40 securities. There were a total of 60 such runs and 2400 portfolios were produced cumulatively. The analysis's findings revealed that the quantity of securities and the degree of portfolio dispersion have a consistent and predictable relationship. Findings proved the theory that the relationship takes a shape of a fast-declining asymptotic function. Results raised doubts about an increase in the size to more than 10 or so.

Statman (1987) tried to examine the accepted notion that around 10 stocks exhaust nearly all benefits of diversification in the context of an equally weighted portfolio. He showed that a well-diversified portfolio of randomly chosen stocks must include at least 30 stocks. Based on the work of Elton and Gruber, results implied that 51 percent of the portfolio risk was reduced when the portfolio consisted of 10 securities. With additional 10 resulted in additional 5 percentage points in portfolio risk reduction. When the number increased to 30 overall, the standard deviation declined by 2 percentage points. With 75 securities standard deviation was further reduced but still for only 2, whereas an increase to 200, 500, or 700 securities produced an additional 1 percentage point reduction. Such results also indicate falling benefits or adding more securities with asymptotic character.

Marginal analysis is used in mean-variance portfolio theory to identify the optimal level of diversification; As long as its marginal benefits outweigh its marginal costs diversification should be welcomed and enhanced. In the mean-variance portfolio context diversification is beneficial because it reduces risk. However, there are costs related to the increased diversification, namely transaction and holding costs. Marginal gains of diversity improve when there are lower asset correlations involved (Statman, 2002).

As a portfolio holds more components, creating increased diversification effects, the portfolio standard deviation is expected to be reduced. For example, if the correlation between stocks is 0,08, the standard deviation of a portfolio with 20 equities is only 35 percent of a deviation for a 1-stock portfolio. In a reduced setting, with the same correlations, weights, and standard deviations Statman (2004) shows the positive impact of diversification. He increases the leverage of the Total market portfolio with 3444 constituents so that it has the same standard deviation as the equity portfolio with 20 equities. The benefit of expanding diversification from the 20 stock portfolio to the 3444 stock portfolio results in 0,88 percentage points in excess return. This also assumes the correlation between any two stocks is 0,08 with an equity premium of 3,44. Results can offer insight into the needed ability for stock selection. Subtracting 0,06 percent in the net cost of the

Vanguard Total Market Fund from the excess return lowers overall gain from increased diversity to 0,82 percent. Considering this data, investors can offset the drawback of 20 equity portfolio diversification benefits by outperforming the market by at least 0.82 percentage points annually. Reducing risk is, however, welcomed positive news in the mean-variance portfolio approach. At least 300 equities in a portfolio are suggested since the advantages outweigh the costs at this number. Increasing the number further up to 3444 does not produce a meaningful difference. The benefits of risk reduction are reduced to only 0,06 percent, being the same as the 0,06 percent net cost of switching from a 300 stock portfolio to a Total Market Fund (Statman, 2004; Statman, 2002).

Limiting yourself in the portfolio diversification can be highly expensive. Investors holding 3444 stocks in the Vanguard Total Index Stock Market Index fund benefit from increased diversification compared to investors holding only 4 stocks in their portfolios. The difference is equivalent to the 3,3% annual return in favor of a more diversified group (Statman, 2002).

Having a correlation between any two pair stocks at 0,08 combined with equity premium at 3,44 percent indicates that the optimal level of diversity benefits is 120 stocks in a portfolio. At this stage benefits of diversification are equal to the costs related to it, which are the costs of holding and purchasing equities from the Vanguard Total Stock Market Index. When the equity premium is considerably higher, standing at 8,79 percent, this consequently means more equities in a break-even portfolio, more than 290. Diversification benefits are reduced when equity premium decreases. Similarly, diversification's advantages diminish as correlations increase. An optimal number of equities differ with different equity premium and correlation metrics. With an equity premium of 8,79 percent and correlation of 0,08, the ideal level of holdings is 300. With the same equity premium and correlation standing higher at 0,28, the optimal number of equities is 70. The 0,28 is a reported estimate of realized correlation from Campbell, Lettau, Malkiel, and Xu (2001) dating back to the 1960s (Statman, 2002).

Elton and Gruber (1977) acknowledge the importance of diversification. For a total risk of a portfolio to move closer to the minimum total risk possible adding stocks beyond number 15 appears to be important and necessary based on their conclusion from the empirical study.

Tang (2004) looked at how many stocks included in the portfolio affected the amount of diversifiable risk. An analytical examination of the naive diversification concept demonstrated that a portfolio of 20 stocks is needed to, on average, to eliminate 95 percent of the diversifiable risk. Results were based on the infinite population of stocks. To reduce portfolio risk by an additional 4 percent (99 percent total), another 80 equities (a cumulative size of 100) are needed. Results were unaffected by the markets, sampling intervals, or investment horizons analyzed which further strengthens the findings.

Banjelloun (2010) analyzes the conclusions of Evans and Archer. In his study, there are two weighting systems and two risk metrics used. These include time series standard deviation and terminal wealth standard deviation. Market and equal weight weighting algorithms are taken to perform the analysis with data ranging from 1980 to 2000. For every number of stocks (N), 10,000 different portfolios are created. N considers values starting at 1,10,20,30 and up all the way to 100. Each portfolio's two standard deviations are calculated, respectively. The key finding of his article was that a randomly selected portfolio including roughly 40 to 50 stocks can be regarded as well-diversified, regardless of the risk method used or the weighting system applied. This amount of constituents is higher than what Evans and Archer predicted.

Concerning the portfolio construction process there are several available options to consider. The optimal course of action is to blindly naively diversify when there is little evidence of any significant variation in risk premia, standard deviations, and correlations. A 1/N portfolio type is produced by distributing capital with an equal share in each portfolio component. This creates components with the same weigths. By employing volatility weighting investors can produce an inverse volatility portfolio when standard deviation differences are trusted. This allows the desired switch from naive money weight diversification to naive weight diversification (by using standard deviations). On the third level (with volatilities, correlations, and full covariance) portfolios like minimum variance, and full risk parity portfolios are possible. At the top level, where one can indicate meaningful differences between relevant inputs (covariance, risk premia) full-fledged mean-variance optimization is suitable to achieve a maximum Sharpe ratio portfolio. Different options are available investors to tailor their portfolio selection process based on their preferences for detailed analysis (Hallerbach, 2013).

The sensitivity of mean-variance optimization to input parameters is examined by Chopra and Ziemba (1993). They analyze the relative effects of estimate mistakes in means, variances, and covariances in their article. They demonstrate how optimization is affected by the input variables, clearly showing the difference between covariances and variance estimation mistakes. Errors in the parameter estimates are quantified with the framework of mean percentage cash equivalent loss. Risk tolerance impacts the relationship between errors in means, variances, and covariances. Parameter errors are not created equal. The impact of errors in mean estimation is around eleven times greater than that of errors in variances and also more than twenty times the size of errors in covariances at a risk tolerance of 50.

Making a distinction between these two parameters is important due to impact differences. When risk tolerances increase, errors in means hold even greater importance. The relative impact of mean mistakes is multiple times greater than that of variance errors at lower risk tolerances. When risk tolerances decrease estimation differences are more comparable between variance and covariance errors, while mistakes in means still maintain the most impact. Optimal portfolio procedure is the least dependant on the correct estimates of covariances. This suggests appropriate planning of parameter inputs, particularly if investors have a limited budget to acquire estimates of risk and return parameters. Available budget capacity should be optimally spent on activities that would enable the most accurate estimations of the expected returns (Chopra & Ziemba, 1993).

1.3 Risk parity method

There are multiple methods of arranging the composition of a portfolio, risk parity being one of them. A subset of efficient beta portfolios is also risk parity approaches. Such portfolios distribute market risk evenly among asset classes or components that are present in the portfolio (stocks, bonds, and commodities). The key distinction in this approach is that it aims to allocate the risk of the portfolio equally. Based on this it should provide investors real portfolio diversity that lowers the losses from individual components and their negative impact on the whole portfolio. Risk parity portfolios are anticipated to produce attractive risk-return characteristics when applied (Qian, 2005).

Diversification in risky parity is a good thing where the center of the method is to allocate the (equal) amount of risk in each asset, rather than an equal amount of dollars. Empirically, portfolios build on such principles exhibit attractive risk/return characteristics compared to standard 65/35 stock-bond portfolios. While there appear to be some challenges regarding proper risk evaluation of asset class outside the measurment of risk as a standard deviation, expectation about risk premiums of these assets and their consistency is also a wortwhile question (Inker, 2011).

The risk might seem a bit abstract until a loss occurs. When a loss of reasonable size occurs majority is attributable to stocks. This also means that the diversification effect of bonds is very little if not insignificant. This also means that any large loss in stocks will result in a loss of similar size for the whole portfolio. A study of the asset classes that have contributed to losses might be conducted to better understand the parity solution concept. When analysis of the stock and bonds portfolio was conducted, stocks contributed the majority of the losses. When losses were higher than 2 percent, equity contribution to these losses was higher than 95 percent. During losses of more than 3 to 4 percent, contributions from equities were higher, exceeding 100 percent (Qian, 2005).

The risk parity approach is illustrated by Hellerbach (2013). Risk can be further clarified and understood by comparing the component's beta to the market portfolio. The beta relation of the associated asset to the overall portfolio represents the relative marginal contribution. By multiplying the asset's weight and beta component risk contribution can be calculated. While money allocation is given based on weights that components, risk allocation can be way different. So, the difference between these two can be substantial. At first look, a decently diversified portfolio can in reality have underlying risk contributions too concentrated. The market cap portfolio, which spans the years 1926 to 2004, with its allocation of stocks and bonds, appears to be properly diversified. However, reality is not as clear as it seems. Equities in fact account for more than 90% of the portfolio's risk (with a percentage contribution to risk of 90% and a percentage contribution from bonds of 9,7%). A balanced portfolio in terms of capital allocation can be misleading. Balanced capital allocation does not equate to balanced underlying risk allocations. Such findings were presented for conventional 60/40 equity-bond portfolios (Qian, 2006).

Risk-based diversification is a key concept of Risk Parity portfolios, which are designed to produce greater and more reliable returns in different market environments. Compared to alternatives, a typical parity portfolio has reduced exposure to equities when applied with more asset classes. As a result, the risk budget of the portfolio is less concentrated in equities and spread evenly among other asset types. Similarly, diversifying across asset classes that perform differently in dynamic economic circumstances should be the major goal of the strategy. Such balanced exposure can produce more consistent returns regardless of economic development. Balanced exposure among equities, fixed income, and commodities proved to be valuable by AQR Capital Management research. A so-called Simple Risk Parity strategy has generated better returns from 1971 to 2009 compared to traditional 60/40 allocation. It behaved well also in very stressful conditions, including crashes. Similarly, risk-adjusted performance improved, resulting in a 63 percent higher Sharpe ratio. Generally, the theory and practice of such strategy have gained awareness among investors due to reduced equity concentration paired with less tail risk, a more meaningful approach towards diversification, benefits in terms of behavior in a wide variety of economic environments and new risk/return optimization opportunities (Hurst, Johnson & Ooi, 2010).

In recent years, the risk parity portfolio approach has become more prevalent in the investment community. When compared to common approaches like minimum variance or mean-variance optimization, such a strategy may have a higher Sharpe ratio. While this portfolio technique is competitive consistent outperformance (to equal weighted-alternative or 60/40 equity bond structure) is not guaranteed. It has some distinctive traits, mainly a balanced risk distribution. Sharpe ratios of this strategy appear to be more steady over multiple periods. Intriguingly, the authors discover the sensitivity of the inclusion of various assets in the portfolio. There is little guidance on how to approach the asset inclusion problem. While it is not the case that more assets mean better portfolio results, the number of assets to include and which assets to choose to remain a question for further examination. Fixed income asset class inclusion has proven to be valuable in terms of a Sharpe ratio but reasons exist that such a trend would not persist into the future. Authors conclude that research clarifying how to evaluate asset classes for inclusion into risk parity portfolios would provide further benefits (Chaves, Hsu, Li & Shakernia, 2011).

It turns out that risk parity is a viable approach to asset allocation. In the absence of the full optimization approach, risk parity appears to provide some alternative to the original Markowitz approach. It focuses on the proper risk allocation of the portfolio, avoiding too concentrated risk allocations to a particular asset, focusing manages to reconsider the marginal risk contribution of a portfolio component. One of the benefits include lower needed predictive power of the expected returns of an asset class and it always leads to positive weights for the asset classes, therefore not being less suitable for active managers. It could be suitable for institutional investors who do not face significant constraints on their asset allocation guidelines. Other asset classes like alternative investments can offer new opportunities to construct parity portfolios with new desired risk/return profiles (Kazemi, 2011).

1.3.1 Full risk parity-Equal risk contribution portfolio

The foundation of an equal risk contribution portfolio is the idea that the risk profile of any asset should not be bigger or smaller than that of other assets. As a result, risk of the each asset, and it´s contribution to overall portfolio risk is the same. The beta of the asset is represented as the marginal risk contribution of this asset. When this risk-bearing capacity is multiplied by the investment weight, contribution to portfolio risk can be obtained for each of the assets.

$$W_i\beta_i = w_j\beta_j \tag{1}$$

When all pairwise correlations are the same coupled with each component having equal volatility, the equal risk contribution portfolio is a 1/N portfolio. Standard deviation is typically used to quantify the risk. It is possible that some other form of risk is used. The such measure must be linearly homogenous with the portfolio weights. This means that such risk measure must also be multiplied by the same constant just like the portfolio weights. In terms of asset risk contributions, an equal risk contribution portfolio is entirely diversified with no exception. This feature allows it to be not as concentrated as minimum variance can be. An interesting consideration is also the fact that such portfolio construction is more robust i.e is less tilted towards error maximization like standard minimum variance. While a minimum variance portfolio is found through optimization (by equating marginal risk contributions), an equal risk contribution portfolio is found with restrictions on the product of weights and marginal risk contributions. While 1/N focuses on money allocation, this portfolio focuses on equal risk allocation (Hallerbach, 2013).

1.3.2 Naive risk parity

Practitioners have used an inverse volatility portfolio, also known as naive risk parity, to lower portfolio risk and to better understand asset risk. To determine portfolio weights, it ignores correlation data and simply uses standard deviation as an information input. Inverse volatility is the naive diversification strategy when volatilities are uniform. The S&P Low Volatility Index, which consists of the 100 stocks from the S&P 500 Index with the lowest volatility feature, is an example of such a portfolio. To weigh each stock inverse volatility approach is applied. In order to determine the weights of constituents, MSCI Risk Weighted Indices similarly employ inverse volatility rather than volatility. Such a portfolio might remind us of the equal risk contribution portfolio in terms of weight structure. Equal risk contribution portfolio would completely resemble this alternative when correlations are uniform or zero.

$$w_i = \frac{\frac{1}{\sigma_i}}{\sum_j \frac{1}{\sigma_j}} \tag{2}$$

As a beginning stage, if key asset qualities are unknown, such as risk premia and covariance inputs, risk control strategies can be applied. Access to quality information allows meaningful distinction between assets profiles and portfolio construction to be achievable. A lack of trustworthy data with possible estimation error prevents a quality estimation procedure. Assets should be replaced regularly, based on needs, if such issues arise (Hallerbach, 2013; Hallerbach, 2014).

While with a mean-variance technique optimized portfolios offer a great advantage, they are higly influenced by errors which can have significant impact (error-maximizing) in the sense that they offer high sensitivity to risk premia inputs. Small changes in input data can create very different results. Risk control techniques should be applied since the risk of wrong estimations is present; When available information does not offer meaningful identification of assets´ risk premia, it is advisable to treat assets as replacements with attention to their risk profiles (Hallerbach, 2013).

2 PRACTICAL APPROACHES TO ASSET MANAGEMENT

2.1 CAPM background

The development of the CAPM model created an intellectual background for the initial development of index funds. The CAPM incorporates two main theoretical background facts that allow further explanations. The first of these is that the general market portfolio is mean-variance efficient. Put differently, no different portfolio represents a better combination between return and risk. For a targeted risk aversion the investor is compensated with the highest amount of return or an investor achieves a targeted return with minimum risk. Second, a risk premium of an asset is proportionate to its beta. Both of these components are often viewed as being related and interconnected. It can be shown that the second is derived from the first.

The advice that follows from the previously described theory suggests investors to maintain a market portfolio. Whether investors actively follow their preference for risk and return, they should find this alternative as the most attractive. Additionally, raising the investment's beta is the only way to increase the return. With the combination of the risk-free asset and market portfolio, investors can tailor individual specific risk preferences to adjust to target beta and desired risk. A potentially different portfolio than a market portfolio introduces unsystematic risk which is unrewarded, which makes the market portfolio only interest to the investor. Such principles were the basis for index providers to promote that buying a general stock market is a good investment option. Stock market indices must be the same as the market portfolio and CAPM theory must be true to have any practical usage for real investors (Goltz & Le Sourd, 2011; Bodie et al., 2013).

If general stock indices do not closely resemble the market portfolio, they are not efficient portfolios in accordance with the CAPM model. In the context of a CAPM, a market portfolio is a cap-weighted collection of all the accessible assets that represent the total economic wealth available. This also indicates that a variety of other financial assets, including those unlisted on stock exchanges or illiquid ones should be included in the market portfolio. To draw the theoretical conclusion that cap-weighted stock market sufficiently aligns with such a description, it would have to include all possible, various assets in the economic environment. Stock indices, like S&P 500 only include a fraction of stocks listed on the exchange, so the requirement is hardly met (Goltz & Le Sourd, 2011; Bodie et al., 2013).

The CAPM model relies on several assumptions. Regarding investor behavior, investors are risk-averse. This means they take more or less calculated risks. Rationality is what drives investment decisions, and optimization of expected utility is usual. Investors have mean-variance preferences, meaning they consider the first two moments of return distribution-expected return and variance. Also, investors have homogenous expectations about input lists. This means the same investment period with no difference in returns, variances, and covariances of assets. In terms of market structure, there is the assumption about asset tradebility. All assets are tradable on public exchanges with allowed short positions, coupled with the ability to borrow or lend at a risk-free rate. Other key important characteristics involve no operational friction- meaning there is no taxation or transaction costs with assets being endlessly divisible. The model also assumes publicly available information (Bodie, Kane & Marcus, 2013).

Within the theorem, the presence of risk-free asset helps investors to guide their investment decision properly based on their risk preferences. The portfolio with the highest expected return per unit of risk is the called tangency portfolio. Such a portfolio has the highest Sharpe ratio which is an indicator of the optimal return for a unit of risk. Regardless of risk aversion, all investors would prefer a tangency portfolio to another risk-bearing investment alternative, according to theory. The percentage of a person's wealth that is invested in the tangency portfolio will depend on their personal risk tolerance. The capital allocation choice is split between two components. One option is to allocate resources to the tangency portfolio, and another involves allocation to the risk-free asset. The risk-free and market portfolios can be combined in various different ways to satisfy investor individual preferences for risk and return. All these combinations are efficient and located in a straight line. Such a straight line represents optimal line, also called efficient frontier and together with risk-free asset forms a capital allocation line. Personal optimal-best portfolio decision therefore always consists of a linear combination of an allocation to the risk-free asset and allocation to the optimal risky portfolio (Goltz & Le Sourd, 2011; Bodie et al., 2013).

Tangency portfolio, with the highest Sharpe ratio, with different combinations of risk-free asset dominate other naive portfolio choices. Investors should try to find a maximum Sharp ratio portfolio and individually select their leverage based on their risk profile and preferences. CAPM theory extends this perspective into an equilibrium theory. It suggests that if investors behaved in the same way, which also means having the same input characteristics with regards to expected asset returns and covariances, they would end up with the same portfolio of risky assets, only weighted differently in individual allocation policy. The tangency portfolio will be made up of all assets weighted by their market value and held by all investors with assets in equilibrium. This is known as a market portfolio. The market portfolio is efficient and there is a further relationship to the concept of beta as a result of two central emphasises (Goltz & Le Sourd, 2011; Bodie et al., 2013).

The market portfolio's efficiency suggests that each security's beta relation to the market portfolio can be used to effectively determine a security's price. The security´s beta is used to determine the expected return for each security. Variations in the securities' betas capture differences in expected returns for different stocks. Beta is the slope of the regression line between the security's return and the market return. The risk-free asset and the asset's sensitivity, or beta, to the market portfolio, determine the expected return. CAPM provides us with guidance for an optimal investment strategy and unique pricing (beta-pricing relationship). This also implies that holding a market portfolio is the optimal choice given the theory background. The best possible portfolio in risk-return characteristics is simply a market portfolio. Holding a market portfolio reduces the need to analyze expected returns, covariances, and other investment parameters (Bodie et. al., 2013).

The theory of the CAPM model was the basis for the initial start of the discussion about bringing new theoretical knowledge to the average investor. There is a variety of literature supporting practical applications in the form of cap-weighted indices as index funds. Even prominent names like Wells Fargo found support to grow their index support in theoretical concepts of CAPM. One of the key parts of Vanguard's early start was Samuelson's positive view on institutional setup for creating a portfolio that tracks an index (Goltz & Le Sourd, 2011; Bogle, 2016).

The wide development of index funds is closely related to the theoretical background of the CAPM model. CAPM tells us that all investors will want to hold capital-weighted portfolios of global wealth. At that time, the U.S. was the world´s largest market, so a solution like S&P 500 seemed like a fair approximation. Many people recognized S&P 500 as an attractive real-world solution and there was little evidence of active management outperformance. CAPM was the basis for a wide range of index models, and many started emerging. Index funds are funds that match the performance of S&P (Goetzmann, 1996).

2.2 Active investment style

An active portfolio manager can increase the value of the managed portfolio in a few ways. This is done with his portfolio movements being different from his benchmark index by either forecasting market trends; a so-called market timing or by identifying mispriced market sectors or securities. Such stock or sector selection involves active bets on individual stocks of a defined industry (Hebner, 2007).

Market timing, also known as tactical asset allocation involves shifting funds to a particular sector, having a temporary preference for a certain type of equities, or choosing not to allocate available assets at all (Pettajisto, 2013; Bodie, Kane & Marcus, 2013).

The value of these can be substantial. Forecasting market trends or market timing can create value, and active management based on security analysis (mispriced securities) can have even greater. Active investors rely on market movements where they try to pick a winning strategy. Their activities include picking the right managers or adjusting their investment styles. Active investment behavior also means finding the next hot investment style and picking fund managers. It can often include specific investor behavior, including psychological phenomena. Such are overconfidence, familiarity bias, regret avoidance, self-attribution bias, extrapolation, and others. These can make managers believe that they have control over changing environment when in reality they have little. Emotional cycles represent behavior changes and present changing conditions of managers' tastes for risk. Active investment policies are mostly represented in the forms of actively managed mutual funds managed by professional managers (Hebner 2007; Fichtner, Heemskerk & Garcia-Bernardo, 2017).

Fundamental analysis means modeling earnings and future dividend payments of the specific firm to determine the fair value of the stock price. The analysis involves projecting expectations about interest rate movements and adequate risk evaluation of the company operations. Ultimately, the goal is to attempt to accurately project discounted value of the received payments (like dividends) that a stockholder can receive from his ownership.

Fundamental analysis involves the examination of the company´s balance sheet which is further supported by detailed economic analysis. It usually involves the evaluation of the management, and firm´s position within its industry combined with industry prospects as a whole. The key goal is to get insight into future economic developments of key determinants responsible for driving a particular company forward. Managers try to recognize unique value proposition which has not yet been recognized by the rest of the market participants. Actively managed funds use such analysis to evaluate a company´s prospects. While such efforts are done by many, there could be pressure on identifying unique prospects. Analysts rely on publicly available information and much of the analysis could not be so massively more precise than those of other market observers. Companies are often covered by a bunch of analysts, well-informed and well-organized consulting companies that conduct specific or general research. Gaining new insight, that is meaningful and not yet recognized is no easy task. Discovery of well-run firms is in itself not enough when others also recognize such opportunities. When certain knowledge is already common and known to the public, the investor will have to pay a premium or a higher price for such opportunities and less likely to realize the better than average rate of return. The goal is not only to find good but mainly to identify firms that are somehow better analyzed and where better operating insights can be gathered. Fundamental analysis is difficult, and good firm analysis is needed, but having a better one gives managers a chance to find opportunities in the market, not yet being reflected by already available information. Generating attractive analysis brings managers one step closer to generating security returns over and above those predicted by equilibrium models like CAPM (Bodie et al., 2013).

Pettajisto (2013) shows that average actively managed mutual funds performed worse than their benchmark index from 1990 to 2009. Value was added only by the most involved and superior active stock-picking managers. He also shows a high degree of variability in active management practices, measuring it in terms of Active Share and tracking error within 5 categories of the type of active management. These include concentrated stock picks, diversified stock picks, factor bets, closet indexing, pure indexing.

Actively searching and identifying securities that varied from their intrinsic value was traditionally believed to be productive and worthy. However, early research that examined this view proved another fact, namely, that such funds, on average, very hardly outperform the market after taking into account needed fees. A new agreement emerged from several of these research, which is that most managers' value creation is outweighed by associated managers´ costs. Such type of thinking is perhaps well summarized by Carhart (1997) who claimed the existence of skilled or informed mutual fund managers is very hard to support according to the results. Cremers, Fulkerson, and Riley (2019) in the review of the past 20 years of active management unveil a wide range of literature further analyzing the role of active managers and their justification. Some of the key issues include questions regarding average fund underperformance after fees, the persistence of best funds performance, and skill above costs.

Studies suggest Carhart´s claim could still be very valid, however, Cremers et.al claim that the classic literature supported view, which is negatively oriented towards active portfolio management, might be too exaggerated.

2.3 Passive investment style

According to proponents of the efficient market hypothesis, active management should play a smaller role because the costs associated with it are unlikely to be justified. They advocate for a passive investment approach that avoids speculating on future market trends. The goal of passive investing is to build a well-diversified portfolio of securities. There is no need to find under or overvalued securities. The strategy that limits big management swings in holdings, without much adding or reducing positions is a key strategy in this context. The efficient market hypothesis claims that stock prices are generally priced at a reasonable level, most of the time being priced correctly. It becomes ineffective, due to availability of information to purchase and sell assets because of reduced price misalignments between participants, which usually results in high trading expenses and no improvement in expected performance. The creation of an index fund, which is intended to closely track the performance of a broad-based index of equities, is a common passive management method. Vanguard's 500 Index Fund, which owns companies in accordance to their weight in the Standard & Poor's 500 stock price index, is one example of this. Broad diversification with relatively low management fees is available to investors who choose such an investment vehicle (Bodie et al., 2013).

After the global financial crisis, the landscape of institutional money management has started to change even more dramatically. More funds have been flowing into passive management forms, such as index mutual funds and exchange-traded funds (ETFs). Such financial assets are referred to as passive index funds. The main goal of passive index funds is to replicate already existing stock indices. Meanwhile, such products try to be cost-efficient, thus lowering expense ratios (Hebner, 2007; Fichtner, Heemskerk & Garcia-Bernardo, 2017).

In good times before the crises investors tolerated higher expenses due to hopes of increased performance. Beneficial market conditions combined with an active trading strategy attracted positive investors. In recent years many actively managed funds are not that good at consistently generating higher returns than established benchmarks such as S&P 500 (Hebner, 2007; Rizthold, 2015).

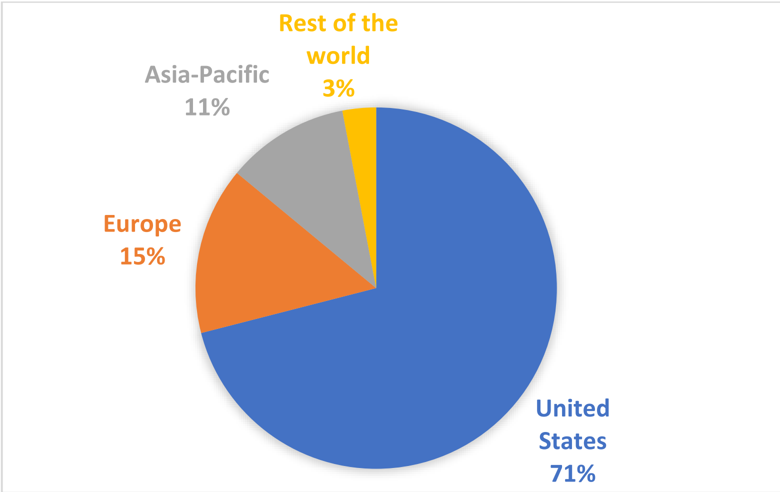
2.4 Overview of the investment fund landscape

An exchange-traded fund is a type of investment product that allows investors to buy and sell shares on the stock market. In the same way that shares of publicly traded companies are regularly traded, investors can purchase or sell ETF shares through a broker or a brokerage account. In the US, ETFs have been a viable investment option for close to 30 years. Most ETFs have a structure resembling mutual funds. Both investment products are governed by regulations. An ETF must post the mark-to-market NAV of its portfolio at the end of the day, much like mutual funds. The Investment Company Act (1940) serves as the legal foundation for investor protection. (ICI, 2022).

At the end of 2021, the US ETF market consisted of 2570 funds with a combined assets of more than 7,2 trillion. The US market still has a dominating market share. The United States holds more than half of the global net assets as presented in Figure 1.

The majority of the assets in US ETFs are regulated by SEC. At the end of 2021, the Investment Company Act from 1940 governs most of the net assets held in the ETFs, with only two percent being excluded. These funds mainly deploy capital in other investment vehicles or try to fund opportunities elsewhere, like commodities, currencies, and futures. At the end of 2021, there was 125 billion in net assets in those investment vehicles managed within 62 ETFs (ICI, 2022).

Figure 1: Global share of net ETF assets



Source: ICI (2022).

United States leads the global share of the net ETF assets, followed by Europe and the Asia-Pacific region.

The Undertakings for Collective Investment in Transferable Securities (UCITS), which are now governed by the EU guidelines, have helped the European ETF market grow. The European ETF market is now the second largest in the world. Two countries, Ireland and Luxembourg, are domicile countries to a majority of European ETFs. Both of these offer English-speaking business environments and follow UCITS Regulations. Favorable taxation terms also attract international investors (Yiannaki, 2015).

Positive regulatory improvements, product innovation, and growing recognition of low-cost investment options have contributed to the positive ETF product development in Europe. Assets of the ETF sector are expected to hit 2 trillion in 2024. Past years have seen new merger and acquisition activities in the marketplace. Many of the biggest asset management companies have moved into the market with an enhanced range of products. New product offerings have emerged to promote unique product features with the continued issuance of plain vanilla equity ETFs. With just 1% of all assets, actively managed ETFs represent a very small portion of the market. The ongoing downward pressure on ETF costs forces sponsors to collaborate with smaller, different index providers. New offerings that are based on environmental solutions, responses to social issues, and responsible governance strategies are becoming more prevalent (Bioy, Garcia-Zarate, Lamont, Boyadzhiev & Kang, 2019).

The European ETF marketplace has enjoyed remarkable growth in the past decade. Assets under management have risen dramatically from around 100 billion in assets in 2018 to almost 760 billion by the end first half of 2019. Similarly, net flows to the exchange-traded products did not turn negative over the last decade. The rising path can be observed, with average net inflows averaging 62 billion from 2104 to 2018.

Double-digit growth in the past years has helped accumulated assets reach new high levels. The only difference involves years 2011 and 2018 where more volatility emerged. While projections for assets show that 2 trillion could be hit by 2024, the European ETF industry outlook similarly remains positively oriented.

Passive funds, namely index mutual funds and ETFs are generally expected to remain attractive to investors. Pressure on the actively managed alternatives is expected to continue. Passive forms could account for 25 percent to 28 percent of the European investment market by 2025, compared to 17 percent at the beginning of 2020. There is a view that ETFs will be the key force to fuel this level of growth. Around 8.6 percent of total assets under management represent ETFs within the European investment funds, standing at 5.5 percent only five years ago. The biggest providers include BlackRock (Ishares), Amundi, UBS, Lyxor, and Xtrackers (Bioy, Garcia-Zarate, Lamont, Boyadzhiev & Kang, 2019).

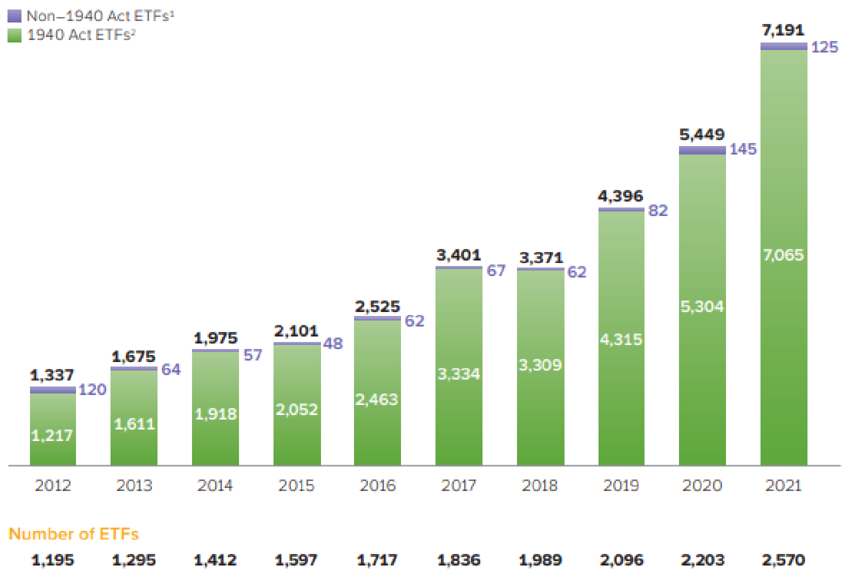
One of the major fronts of ETF product development has been strategic beta, often known as smart beta products. In recent years, new strategic beta ETFs have been released, mostly focusing on multifactor equity type forms. These are promoted as a way to improve the risk-return characteristics of a broadly used capitalization weighted alternative. Innovation in this area is expected. Multifactor ETFs consisting of combinations of factors and geographies allow new offering schemes. Differentiation based on these strategies is possible for the providers. One type of these with a strategic focus are thematic ETFs, which aim to capitalize on long-term changes or society-based, structural developments such as advancing technology, climate change, and others. The launch of ETFs focused on usage of artificial intelligence, implementation of cloud computing, improved digital security, or wide e-commerce adoption has generated some interest (Bioy, Garcia-Zarate, Lamont, Boyadzhiev & Kang, 2019).

Similarly, ETFs with a focus on sustainability have grown from their niche to widely recognized options. New ESG-focused ESG ETFs have been introduced with increased product offerings in last couple of years. The majority of the offerings in the space are related to the equity space, but also fixed-income alternatives emerged (Bioy, Garcia-Zarate, Lamont, Boyadzhiev & Kang, 2019).

Actively managed ETFs represent the area of possible further development, however, it has been largely untapped. As of 2019, there were less than 40 actively managed ETFs with combined money management size of around 8 billion, having a market share of around 1 percent of European ETF classified assets. Just three ETFs are massively used as an investment option, holding around 70 percent of the invested money.

European client base remains institutional. While there is no reliable data on the ETF client base, the view among industry participants is that around 80 percent of the market is held institutionally, among various hedge funds, pension funds, and others. Plain-vanilla market cap-weighting retains most of the ETF´s assets in Europe. Strategic beta alternatives only hold less than a tenth of managing assets despite the recent emergence of new distinct products. The purchase-and-hold ETF strategy is often appealing due to its simplicity, low cost, and absence of actively managed alternatives. Frequent users of such products include money management firms from specialized wealth managers to private banks (Bioy, Garcia-Zarate, Lamont, Boyadzhiev & Kang, 2019).

Figure 2: Total ETF assets in US and number of ETFs



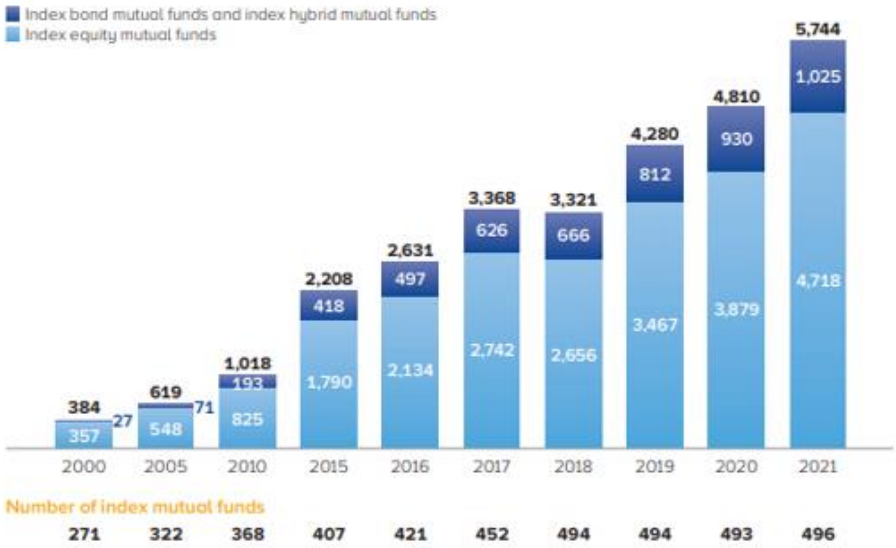
Source: ICI (2022).

The number of ETFs has grown consistently from 2012 onwards as seen from Figure 2. Total assets in the ETFs have grown similarly to the European market. There was a positive increase in almost all years, however, in the year 2018 there was a slight decrease compared to the previous year. This shows the strong positive development of the ETF investment product.

Index-based ETFs closely follow their target underlying indice in a few different ways. Replication can be one viable option. This means investing 100 percent of its assets proportionately in all securities as in the target index. This can be a practical option when facing indices containing thousands of securities, securities with restrictions on ownership, or others that are not as simple to obtain (like fixed-income securities). In the beginning, ETFs mainly followed typical indices that were mostly based on market capitalization. Index-based ETFs can track benchmarks that employ a variety of index-creation approaches as the industry develops, with constructs ranging from market capitalization to fundamental elements, including sales or book value (ICI, 2022).

Between 2008 and 2015 investors placed roughly 600 billion in actively managed mutual funds, while around 1 trillion in net purchases has poured into passively managed index funds. After the wide-spread financial crisis in 2008, index funds have accounted for more than 100 percent of net cash flows into equity mutual funds (Bogle, 2016). Development can be see from Figure 3.

Figure 3: Total Net Assets of Index Mutual Funds in billions



Source: ICI (2022).

Assets managed passively by index funds have increased massively over the last decades from 11 million in 1975 to 511 million in 1985. There were around 55 billion managed in 1995 (Bogle, 2016).

Passive equity funds have enjoyed rapid growth compared to their active counterparts. The market share of passive equity mutual funds has increased from 4 to 16 percent from 1995 to 2005, to 34 percent in 2015. The main reason are lower costs of the passive alternative (Hebner, 2007; Malkiel, 2013).

Combined with ETFs, index equity mutual funds in the US have contributed to another part of the index funds which seek to closely follow the return on a benchmark index. Index mutual fund total net assets increased dramatically in the United States from around 380 billion to $5.7 trillion between 2000 and 2021. As a result, at the end of 2021, the index mutual fund's share of long-term mutual fund net assets will be 25.9 percent. This is much more than 7.5 percent from 2000.

Index equity mutual funds are generally larger than their actively managed counterparts. Economies of scale help to reduce fund expense ratios. At the end of 2021, the average index equity mutual fund was more than 4 times larger than an actively managed equity mutual fund. Expense ratios have declined for both of these two alternatives. The average expense ratio of index equity-type mutual fund decreased from 0.27 percent to around 0.06 percent between 2000 and 2021, meanwhile the average expense ratio of actively managed mutual funds decreased from 1.06 percent to 0.68 percent (ICI, 2022).

As of 2015, there were around 4 trillion passively managed funds invested in equities. The competitive landscape of such growth has also resulted in the concentration of large asset management companies, namely BlackRock, Vanguard, and State Street. These firms have together an ETF market share of around 70 percent. They have more than 90 percent share of Assets under Management in passively managed equity funds. Even regulators have started to talk about the implications of such changes in the asset management space. One of these is Andrew Haldane, chief economist of the monetary analysis at the Bank of England. In his speech from 2014, he highlighted his view about the era of asset management. It was driven by the recent significant expansion in assets under management and the relative threat this could become. This could put pressure on the stability of the banks as well. The main implications of this include the fact that passive investing could support and influence investor herding tendencies and consequently have implications on the movements of financial markets. Possible increased correlated movements can massively influence market sentiment in various ways combined with the cyclical behaviour of financial markets (Fichtner, Heemskerk & Garcia-Bernardo, 2017; Haldane, 2014; Braun, 2016).

Similar observations and statements are made by Sandoval & De Paula (2012), who show that the high volatility of markets is directly linked with strong correlations between them. This means that markets tend to behave very similarly during stressful conditions. Such increased co-movements pose additional pressure on all market participants. Liquidity needs can be significantly increased in such environments.

The second concern regarding risk is represented in security lending activity. To increase their income, passive asset managers lend shares to short sellers. BlackRock and StateStreet have both increased such practices in recent years. Especially BlackRock has grown 40 billion worth of such activities in 2012 to over 130 billion in 2014. Such activities are supposed to be unproblematic in decent market conditions but could put way more pressure on liquidity in stressful market conditions. Even regulators have started to examine the role of leading asset managers, like BlackRock if they need to be categorized as systemically important (financial) institutions. In 2015, Financial Stability Board in Basel decided not to characterize them as such yet, potentially lobbying in their favor (Fichtner, Heemskerk & Garcia-Bernardo, 2017).

3 STATISTICAL MEASURES OF CONCENTRATION

In the following section used statistics are presented to measure the degreee to which indices are diversified. Each statistic has been used in other contexts to measure the extent to which a sample constitution diverges from equal weighting. Different measures are used, since each measure produces a slightly different measure of an index constitution.

A statistical numerical indicator of concentration is the Herfindahl-Hirschman index, sometimes referred to as the Herfindahl index. The two economists Hirchman and Herfindahl separately developed the measure in 1945 and 1950, respectively. Herfindahl's index was presented in his doctoral dissertation, Concentration in the U.S. Steel Industry. Similarly, Hirschman's index was presented in his book, National Power and the Structure of Foreign Trade (Rhoades, 1993).

The Department of Justice and the Federal Reserve have used it to analyze the competitive consequences of mergers and competition analysis, which has led to its level of recognition. Herfindahl index is used to measure concentration in a variety of manners. It can be used to measure income concentration or market concentration, the degree of concentration of the output of the firms in a certain sector. Concentration of companies in a market is one of the key elements of market structure and such examples include analysis of horizontal mergers that affect market concentration, also supported by empirical evidence. In 1982 Department of Justice published formal quantitative guidelines for horizontal mergers based on Herfindahl Index to facilitate the application of the antitrust laws regarding mergers (HHI).

Federal Reserve has similarly used and applied it as the first step when analyzing the effect on competition by bank mergers. The guidelines specify that if after the merger HHI would be less than 1,800 or a HHI would change for less than 200, likely, market structure would not reach a too concentrated level. This would not be enough to maintain market prices above the anti-competitive level for a decisive period (Rhoades, 1993).

The HHI is only one of the available choices when analyzing bank mergers in the competitive context. Due to the importance of market concentration as an indicator of competition and ease of calculating HHI, this index has been used as an efficient screening device for regulators and other market observers. If the post-merger HHI does not exceed numerical limits, it is generally assumed that such a merger would not be dangerously anti-competitive. However, if post-merger HHI exceeds the numerical constraints, a thorough economic analysis of the competition is conducted to better explain HHI's indication.

HHI involves several firms is it´s calculating form, by recognizing all market participants in the specific market, as well as the concentration of these. Numerical conclusion of the HHI is based on using the relative size (market share) of (all) firms in the market (Rhoades, 1993).

HHI values fall between 0 and 1. A value of 0 means a market structure similar to perfect competition with market shares of companies close to 0. Values of 1 indicate a concentrated market with near absolute monopoly. An HHI value below 0.1 is perceived as competitive under US Department of Justice antitrust regulations. Market shares are used as weights in the HHI which is a cumulative concentration indicator. The calculation follows the sum of the squared attribute values-market shares of the companies on the market. Larger businesses receive a larger share of the value of the HHI, which has been criticized from the perspective of the concentration curve. In the event of an unequal distribution of market shares, HHI yields a higher value. This demonstrates the existence of market leaders and dominant companies in a specific market. Practical implications can be limited due to a lack of reliable data, as all market participants´ relative shares are needed (Krivka, 2016).

Herfindahl index is commonly used to measure industry concentration, including by the U.S. department of Justice and in a study by Busse, Green, Baks (2007).

For a chosen fund, Hefindahl index is the sum of squared portfolio weights:

$$H_p = \sum_{i=1}^{N_p} w_{pi}^2 \tag{3}$$

Where fund p has N_p equity holdings each of weight w_{pi}, where all the weights sum to 1. The weight of a holding is the ratio of the value of the holding to the total value of the entire portfolio. Herfindahl index ranges from $1/N_p$ to 1, whereas the normalized Herfindahl index ranges from 0 to 1, regardless of the number of portfolio holdings.

Normalized Herfindahl index is defined as:

$$H_p = \frac{H_p - \frac{1}{N_p}}{1 - \frac{1}{N_p}} \tag{4}$$

The next measure to gauge the concentration of the funds is the coefficient of variation. The coefficient of variation is a relative variability measure. It measures the dispersion of data around the mean.

Coefficient of variation:

$$CV_p = \frac{\sigma(w_{pi)}}{\mu(w_{pi})} \tag{5}$$

It gauges the variation in portfolio weights concerning the average weight of the portfolio. The mean of all stock weights in the portfolio is μ(w), and σ(w) is the standard deviation of all stock weights in a particular portfolio. A larger value of the coefficient of variation indicates a higher portfolio concentration, just like with the normalized Herfindahl Index.

To measure portfolio diversification, Meyer-Bullerdiek (2018) uses diversification ratio (DR) which is defined as the ratio of the weighted average of assets´ volatilies divided by the portfolio volatility:

$$Diversification\, ratio = \frac{\sum w_i \sigma_i}{\sigma_p} \tag{6}$$

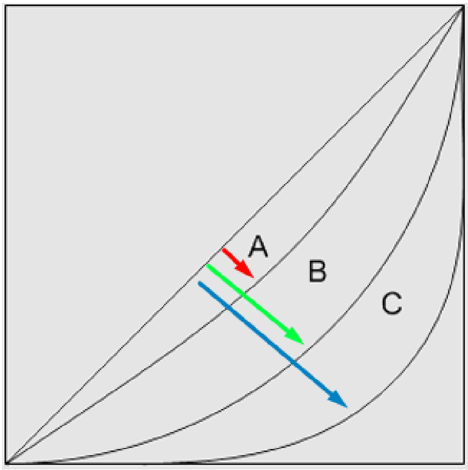
The portfolio weight of asset i (w_i), σ_i is the standard deviation of the asset returns and σ_p is the standard deviation of portfolio returns are all presented in the described formula. According to Choueifaty, Froidure, and Raynier (2013), this measure captures the essence of diversification, wherein a long-only portfolio of assets has volatility lower than or equal to the weighted total of the volatility of the assets.

If at least one investment in the portfolio has a positive standard deviation, the diversification ratio will be larger than or equal to 1. This also assumes no short-selling opportunities. The numerator and denominator can be the same. This can only happen if all correlations between assets were 1. Such occasions are very rare. Normally, due to the diversification effect, the ratio will be higher than 1, which also means a higher numerator than the denominator. The nature of this ratio actually gauges the diversification of investments with imperfect correlation. The denominator is the active risk including diversification, whereas the numerator indicates risk without diversity benefits (Meyer Bullerdiek, 2018).

The Gini index was developed by Corrado Gini around hundred years ago. The purpose of the coefficient is to provide a way of measuring inequality, firstly used in the context of income. It is bounded from 0 to 1, where 1 means complete inequality and 0 complete equality. In the context of income, 0 as perfect equality means everyone has the same income, and 1- perfect inequality (one has all income) (Cobham & Sumners, 2013).

Lorenz Curve describes data graphically. It is a graphical representation of inequality since it illustrates data on cumulative income on the y axis and cumulative population proportions on the x-axis. If there were complete equality everywhere, the Lorenz curve would have a perfect slope of 1, at a 45-degree angle. This would also mean 20 percent of the population has 20 percent of income, 50 percent has 50 percent of income, 73 percent has 73 percent, and so on. The further away the Lorenz curve is from the 45-degree line, the more disparity we have, and the higher inequality. Lorenz curve C represents the biggest disparity as being the furthest away from equal distribution (as the blue arrow indicates) (UNSIAP, n.d).

Figure 4: Perfect equality as a straight line and three different Lorenz curves A, B and C

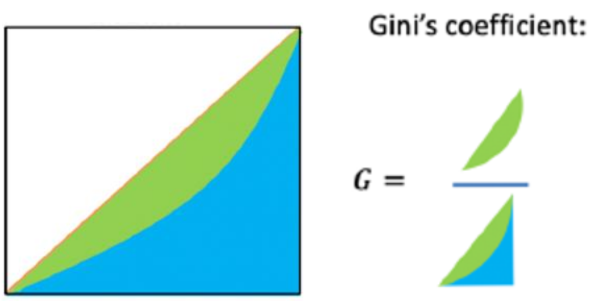


Source: UNSIAP (n.d.)

The Lorenz curve is a basis for the Gini indicator which is a metric used to evaluate the distribution of (income) in a population. This indicator allows observation of the population and its closeness to equality. Description of perfect equality is shown by the equidistribution Lorenz curve on the graph (Bellù & Liberati, 2006).

The main interest should be on the green highlighted area, within a clear straight line, and the curve also called the concentration area. The Gini ratio is the ratio between this area to the entire population. When there is a growing inequality of the observed variable, namely increased concentration in the hands of the few, a green concentration area expands. This also means higher Gini. In the income example, a person possessing all of the income would skew the curve to the extreme and green area would encompass the entire right triangle (be 1) (Bellù & Liberati, 2006).

Figure 5: Graphical representation of Gini calculation



Source: UNSIAP (n.d.)

Calculated as: (according to Busse, Green, Baks 2006)

$$G = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} \left| w_{pi} - w_{pj} \right|}{2N_p^2 \mu(w_{pi})} \tag{7}$$

One of these is that it cannot be further clarified in terms of inequality structure. The global Gini offers between-country contributions but doesn´t distinguish between the individual components of within-country inequality. An alternative measure, the Theil index can be fully decomposed but is less understandable. Theil is more sensitive than Gini in the extreme parts of Lorenz curve distribution, whereas Gini is more sensitive to changes in the distribution's midpoint (Cobham & Sumners, 2013; Cobham, Schlögl & Sumner, 2016).

The fundamental cause of inequality is the rich getting richer and the poor becoming poorer in the case of income. The Gini itself is more responsive to the changes in the middle part of the distribution, in the case of income, middle-class households see less income fluctuations overall. The Palma ratio can capture differences between those in the top and bottom brackets, clearly exposing higher percentiles. The higher the Palma ratio, the greater the inequality. Palma ratio has gained some endorsements from bigger organizations including OECD and UN who include it in their databases (Floyd, 2022).

$$\text{Palma} = \frac{9th\ decile\ \ share\ \ of\ X}{Cumulative\ 1st\ to\ 4th\ decile\ \ share\ of\ X} \tag{8}$$

Given a closer look, less sensitivity of the Gini above a certain level can be observed. Changes can be mainly observed between the upper 10 percent and the lower bottom 40 percent distribution part. When the Palma ratio rises from one to five, increasing by a factor of five, Gini increases from 0.225 to 0.475. In the cases when the Palma ratio doubles, from 5 to 10, Gini only increases from 0.475 to 0.532 as seen in Figure 6. This would suggest an exponential relationship and different behavior in a certain distribution level. Palma is less sensitive to changes in the distribution center compared to the distribution edges (Cobham & Sumners, 2013).

Figure 6: Comparison of Palma and synthetic Gini values

Decile	Income shares (%)									
1	6.25	4.17	3.13	2.50	2.08	1.79	1.56	1.39	1.25	1.14
2	6.25	4.17	3.13	2.50	2.08	1.79	1.56	1.39	1.25	1.14
3	6.25	4.17	3.13	2.50	2.08	1.79	1.56	1.39	1.25	1.14
4	6.25	4.17	3.13	2.50	2.08	1.79	1.56	1.39	1.25	1.14
5	10	10	10	10	10	10	10	10	10	10
6	10	10	10	10	10	10	10	10	10	10
7	10	10	10	10	10	10	10	10	10	10
8	10	10	10	10	10	10	10	10	10	10
9	10	10	10	10	10	10	10	10	10	10
10	25.00	33.33	37.50	40.00	41.67	42.86	43.75	44.44	45.00	45.45
Palma	1	2	3	4	5	6	7	8	9	10
Gini	0.225	0.350	0.413	0.450	0.475	0.493	0.506	0.517	0.525	0.532

Source: Cobham & Sumners (2013).

In 2013, Cobham and Sumner suggested the Palma ratio as an alternative income gauge to the Gini coefficient. It was named after Jose Gabriel Palma, a Chilean economist. Based on Gabriel Palma's observation of the income in the population, the middle classes, which are classified as those in the fifth to ninth income decile (40 percent to 90 percent), tend to gather about 50 percent of the income (Floyd, 2022).

The strength of Palma's thesis and its intuitiveness, according to Cobham and Summer, make a compelling reason for additional investigation. They propose that because it is somehow clearer to understand. Therefore, the Palma ratio might be a better indicator for policymakers and citizens to follow. It might be a more relevant measure of inequality to support different policy-based measures (Cobham & Sumners, 2013).

However, the relationship between Gini and Palma suggests a very close fit. The majority of the information is included in both of the two measures. Between 99 and 100 percent of the Gini variation may be explained by the components of the Palma ratio. The Palma ratio excludes the fifth through ninth decile, that is the key difference to Gini. Although Gini gauges full (income) distribution it does not include any more information in practice (Cobham & Sumners, 2013).

Palma's obvious flaw is that it only takes into account half of the distribution, not the entire distribution. Gini doesn´t collect any fresh additional information, however, it includes all the data. The presentation of the indicator is somewhat in an opaque manner to the average reader. The top decile is actually directly exposed only by Palma, which could be unappreciated, but the ratio's simplicity may be its greatest value. The 0.5 Gini coefficient indicates some inequality but lacks clear further implications. However, the Palma ratio of 5 is available as a clearly understood fact that the richest 10 percent earn five times as much as the poorest 40 percent (Cobham & Sumners, 2013).

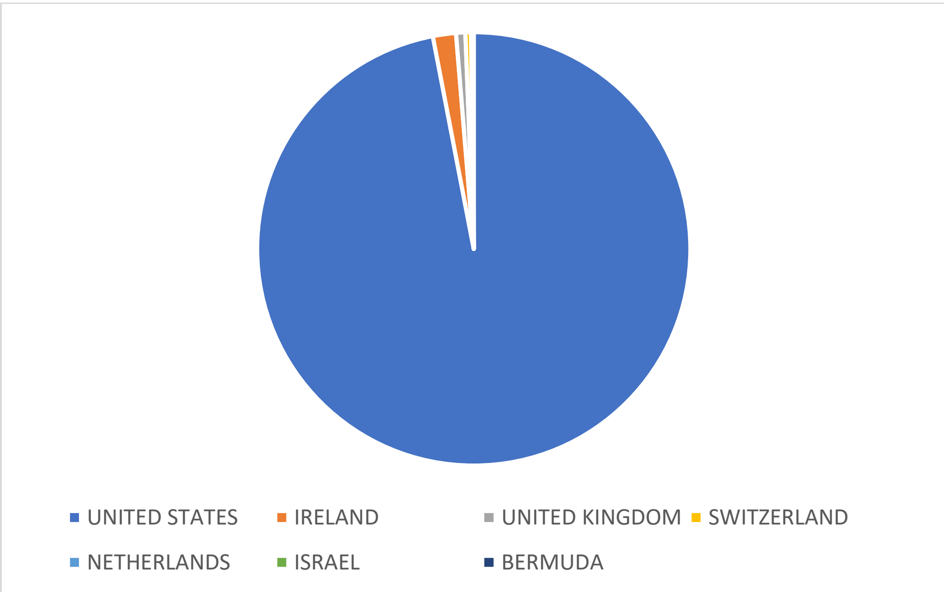
4 EMPIRICAL ANALYSIS

Data for the portfolio concentration calculations were taken from Standard & Poor's Depositary Receipts (SPDR) S&P 500 ETF Trust provided by Thomson's Reuters Refinitiv Eikon database (Reuters, 2023).

The Fund's investment goal is to deliver such investment outcomes that broadly correspond to the price and yield performance of the underlying indice (before fees). Each stock's weight in the Index corresponds to its weight in this Trust, including all companies that are already present in the SP 500. Clear replication of the main underlying indice with minimum deviation is the goal of this Trust, representing the main investment objective (Thomson's Reuters Refinitiv Eikon database) (Reuters, 2023).

The legal structure of the ETF is the Exchange-Traded Unit Investment Trust fund, equity asset type, where income distribution is paid and the custodian being State Street Bank and Trust Company, details in Figure 7.

Figure 7: Invested regions of the fund



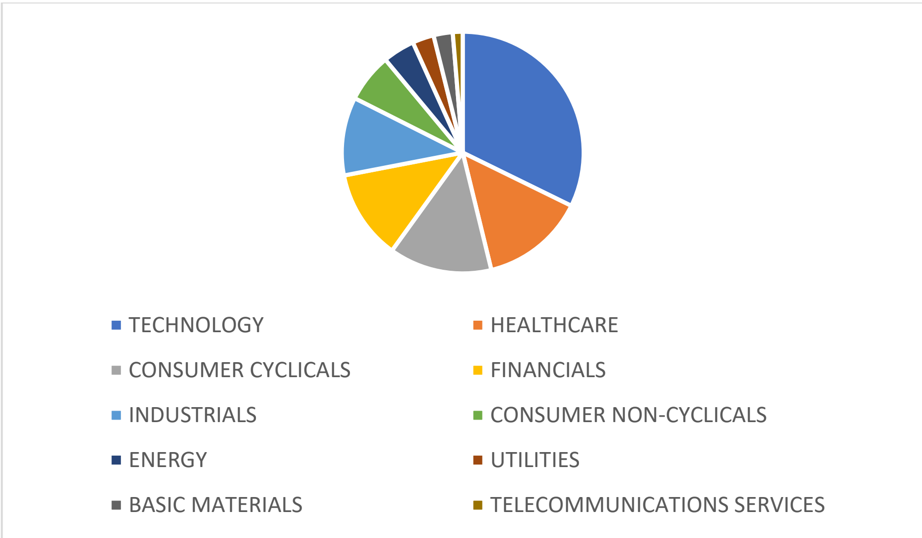
Source: Reuters (2023).

As of April 30, 2022, the fund was mainly invested in the States, at around 96.93 percent of Total net assets. Ireland was second, having a 1.72 percent share, followed by the United Kingdom with 0.69 percent, Switzerland with 0.42 percent, Netherlands with 0.13 percent, Israel with 0.04 percent, and Bermuda with 0.03 percent.

Asset allocation on that date was primarily equity, standing at 99.91 percent, cash at 0.05 percent and other at 0.04 percent.

S&P 500 ETF Trust had 99.91 percent of its assets in equity holdings. While the majority of the assets are invested in the United States, there is no explicit explanation for the involvement of the other invested regions. Potential reasons could include cross-listings of the equities on other exchanges. Based on the 2021 annual report all investments are common stocks based on fair value hierarchy as of September 30., 2021. There are also some investments of the Trust, like State Street Corporation, that are considered an affiliate of State Street Global Advisors Trust Company (the Trustee), and Intercontinental Exchange, Inc., considered an affiliate of PDR Services LLC (the Sponsor). Fund´s sectors can be observed in Figure 8.

Figure 8: Fund´s sectors



Source: Reuters (2023).

Asset allocation in terms of sectors investment follows a similar path to the underlying indice with allocations to technology in the weight at 32.23 percent, healthcare at 13.92 percent, cosumer cyclicals at 13.74 percent, financials at 12.01 percent, industrials at 10.46 percent, consumer non-cyclicals of 6.53 percent, energy of 4.27 percent, utilities of 2.87 percent, basic materials of 2.56 percent and telecommunications services of 1.32 percent weight as of 30. April 2022 (Reuters, 2023).

Launch date of the fund is 22. Jan 1993, domiciled in the USA, where laso the geographical focus lies. Actual annual Management fee is reported at 0.06 percent with a Fund size at around 371 billion. Minimum investment is not given. Promotor is State Street Global Advisors. ETF should be a good approximation to the standard indice due to it´s similarity in the structure (Reuters, 2023).

Technical analysis for the described ETF for the last three years include beta of 1, R squared of 1, a standard deviation of 18.61 percent, Shrape ratio of 0.19, Information ratio of -0.26, a slight tracking error of 0.02, correlation of 1, Return to the risk of 0.23, Maximum drawdown and Treynor ratio of 1.03 as of 30. April 2022. Quick metrics in the Lipper Leader key used to describe the product are Total return, Consistent return, Preservation and Expense so investors can better understand type of the financial product. ETF also has ESG metrics are also added and quantified (Reuters, 2023).

Provided data in the timeframe needed was analyzed so composition was possible to obtain in the periods per three years and described metrics were calculated. The purpose of the data is to gain insight into the development of the key diversification data over time to see structural changes. The above analyzed data will also be graphically presented to give a better understanding to the viewer. Special consideration is given to the Lorenz curve and key statistics.

The analysis's return and risk characteristics are explained in the section below.

Capitalization and equally weighted SP indices are calculated. The data required for the analysis is based on Bloomberg Terminal. Data retrieved from Blomberg Terminal is expressed in monthly data. In the next part portfolio with stocks that had highest weightings from 2007 onwards will be constructed Bloomberg, L. P. (2023).

From the beginning of the period, 20 of the highest weightings from 2007 will be chosen and added by companies in later years that appeared on the list among the highest weightings. The list is completed when the number of companies reaches 40 inspired by the research of Benjelloun (2010). The hypothetical portfolio is equally weighted at the beginning. Each individual holding receives the same beginning value of 25, with a cumulative net value of 1000 at the starting observation date. Values of the individual components are summed on a monthly basis to arrive at the ending portfolio value.

Returns for each component are calculated on a monthly basis as:

$$R = \ln \frac{P_t - P_{t-1}}{P_{t-1}} \tag{9}$$

Where: $R =$ percent change of the stock or indice

P_t = value of the stock of indice at time t

P_{t-1} = value od the stock or indice at time $t\text{-}1$

To investigate the volatility of the returns standard deviation for each indice and portfolio in a given time period is calculated. Since monthly data is used, the standard deviation is multiplied by the square root of 12 to obtain yearly data.

$$STDEV = \sqrt{\frac{1}{N}\sum_{X=1}^{N}(x-\mu)^2} * \sqrt{12} \tag{10}$$

Where: $STDEV$ = Standard deviation

N = the total amount of population

x = return of the stock or indice

μ = mean return of the stock or indice

Further, componud annual growth rate is calculated for all three indices with a given formula:

$$CAGR = \sqrt[t]{\frac{Ending\ value}{Beginning\ value}} - 1 \tag{11}$$

Where: *CAGR* = Compounding Annual Growth Rate

t = total amount of years

Ending value = value of the indice on the last observed date

Beginning value = value of the indice on the first observed date

Further, Sharpe ratios as a risk-adjusted performance metric are calculated as:

$$Sharpe\ ratio = \frac{E(r_p) - r_f}{\sigma_p} \tag{12}$$

Where : $E(r_p)$ = portfolio return

r_f = risk free rate

σ_p = portfolio standard deviation

Returns of the three indices are analyzed with the Fama-French Three-Factor model. It is an asset pricing model developed in 1992 that expands the classic capital asset pricing model (CAPM) by adding two additional risk factors to the market risk factor in CAPM. It adds size risk and value risk to the market risk factor in the CAPM model. This model considers the fact that value and small-cap stocks have outperformed markets based on research concluded by Fama and French. The model has three factors, namely size of the firms (SMB-small minus big), book-to-market (HML-high minus low) values and excess return on the market (Fama & French, 2015; Floyd, 2022).

Smaller capitalization companies have been shown to generate higher returns which are represented by the SMB factor, similar to the HML factor which accounts for value equities with high book-to-market ratios performing better than the market. As a result, sensitivity to the market, sensitivity to size, and sensitivity to the book-to-market ratio (value) are the major factor-regression coefficients influencing predicted returns. There are other elements that have been considered and added to the model by researchers. Some of these factors focus on volatility, momentum, and emphasis on firm quality. The authors themselves have modified their model to include five factors. Along with the main three factors, the authors 'new model adds a profitability factor, meaning that companies reporting larger future earnings have higher returns. The investment factor considering investments and allocation of profit towards major growth projects is added as the fifth (Fama & French, 2015; Floyd, 2022).

Model overview:

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1\left(R_{mt} - R_{f,t}\right) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_{i,t} \tag{13}$$

Where: R_{it} = total return of a stock or portfolio

R_{ft} = risk freee rate of return at time t

R_{mt} = total marker portfolio return at time t

$R_{it} - R_{ft}$ = expected excess return

$R_{mt} - R_{f,t}$ = excess return on the market portfolio

SMB_t = size premium (small minus big)

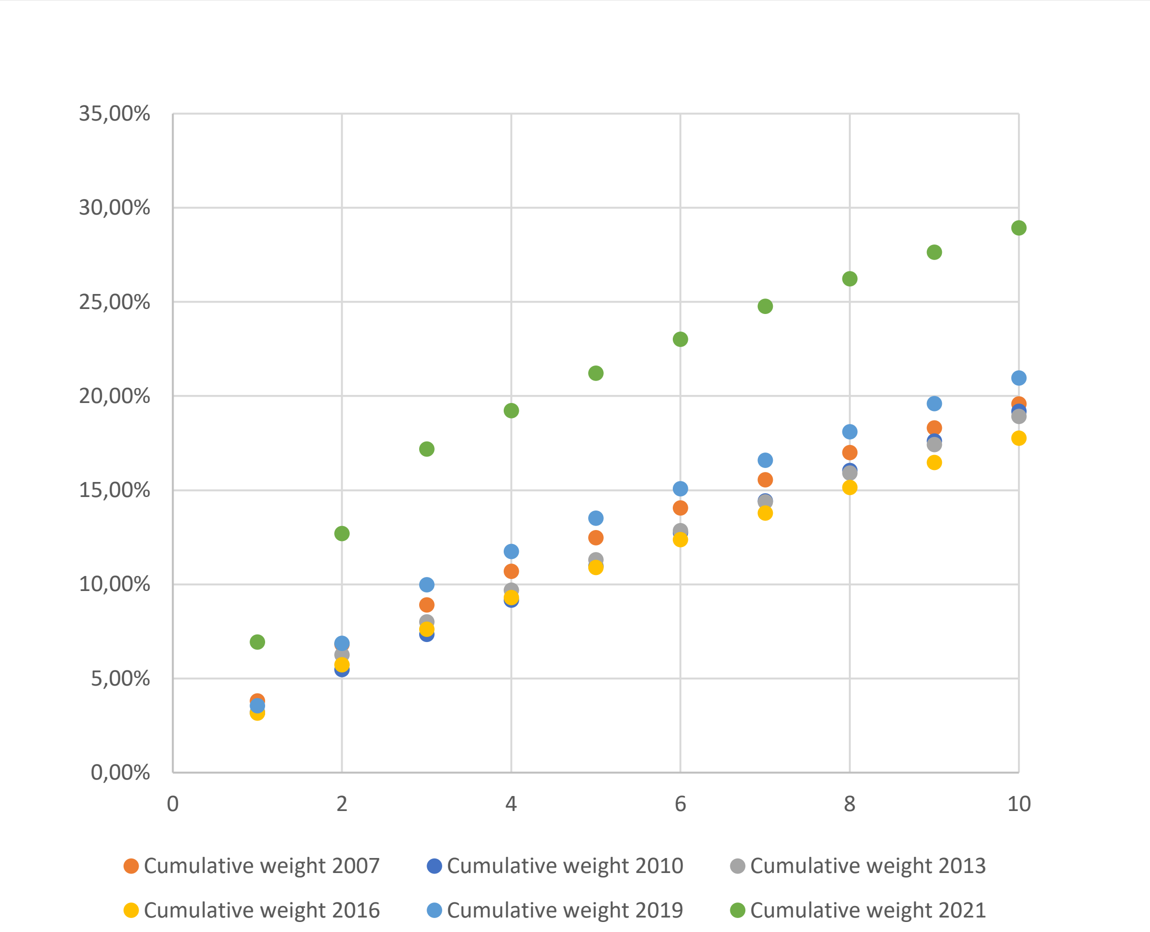
HML_t = value premium (high minus low)

$\beta_{1,2,3}$ = factor coefficients

Other factors can be added to the model and tested for potential explaining power of the excess returns.

5 CONCENTRATION MEASURES

Figure 9: Cumulative weights of biggest holdings

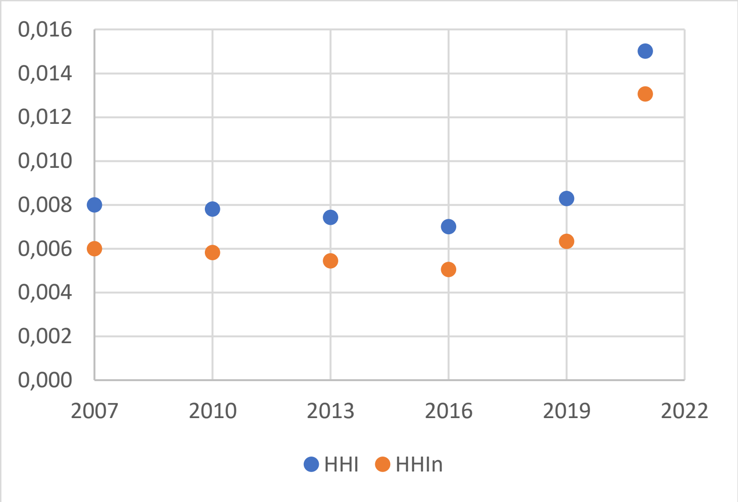


Source: own work

The individual and cumulative weight of the top five companies (Figure 9) has been moving around ten percent up to 2016. Since then, a massive shift has occurred in favor of the concentrated position, leading to the weight of the top 5 companies representing 14 percent of the indice in 2019 up to more than 20 percent in 2021. The biggest companies have gained in weight significantly in the last several years. This has also caused the majority of the described ratios below to increase. Regarding the structure of the highest-weighted companies, these have also seen changes. From 2007 to 2021, only two companies that emerged in the top segment in 2007 remained within the ten highest weighted holdings in 2021.

Taking into account weights from the 30. of April 2022 in the indice and reported 30-day volatilities as of the 19th of May the diversification ratio would be 1.36. This indicates diversification benefits of the indice. The Constitution of the indice positively impacts diversification and shows correlations are less than one.

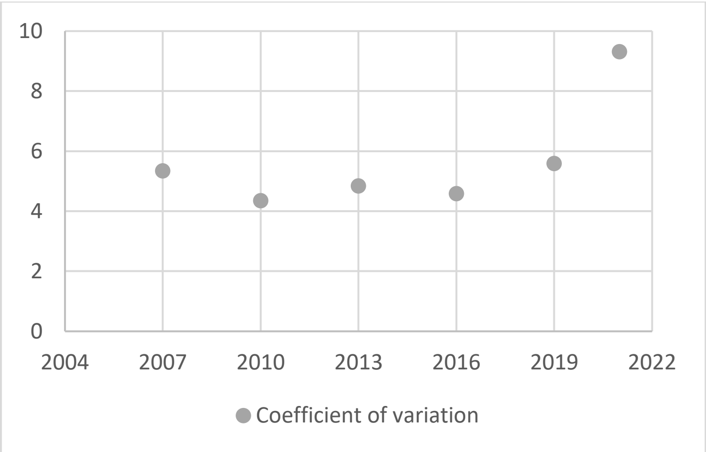
Figure 10: Herfindahl-Hirschmann Index



Source: own work

Changes for Hirschman - Herfindahl index (Figure 10) and its normalized alternative follow similar results. Figures from 2019 to 2007 stand at around 0.8 and 0.6 percent with differences in the second decimal number only. Figures between described years are moving around these two levels at around 0.7 levels. Similarly, to previous statistics, growth has been the highest in recent years.

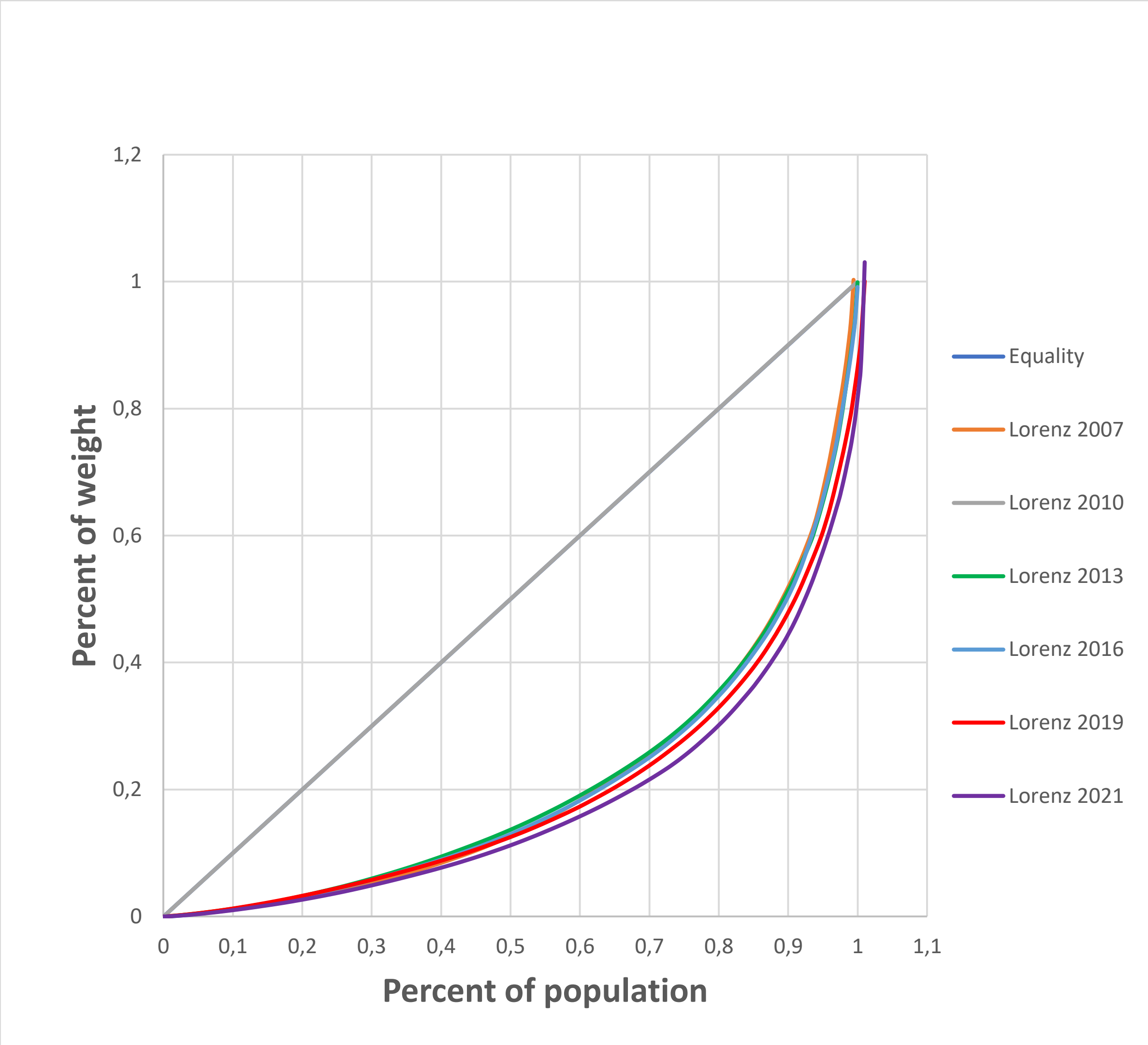
Figure 11: Coefficient of variation



Source: own work

Variability around the mean weight (Figure 11) of the components has not enjoyed big growth, with a value standing at around 5 from years 2007 to 2019. During those years the coefficient has even dropped slightly and remained in the range of 4 to 5. In 2021, this ratio achieved remarkable growth and stands at 9.5, which means an increase of almost 100 percent.

Figure 12: Lorenz curves for different years

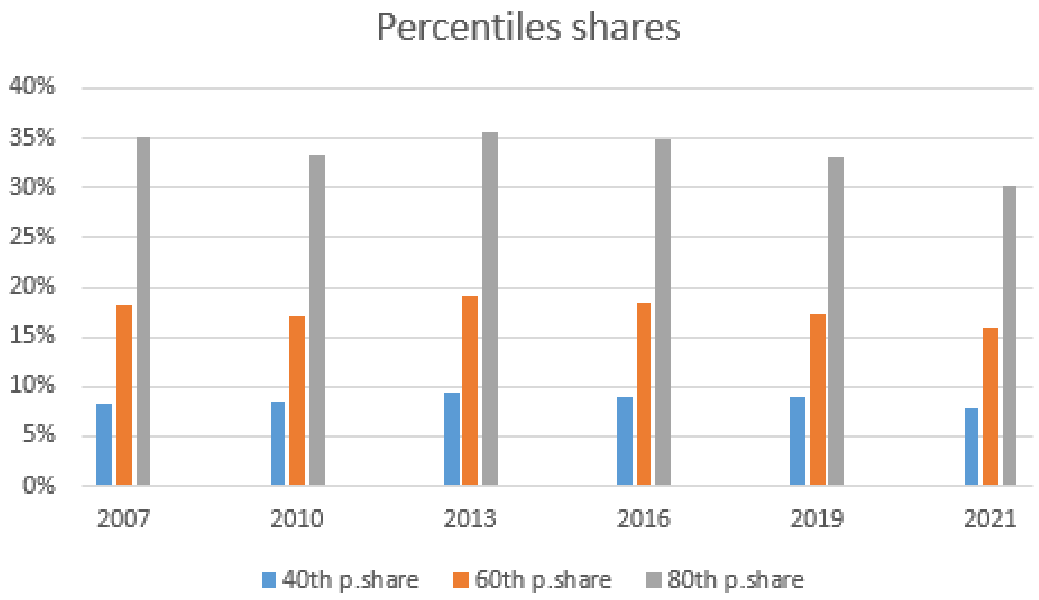


Source: own work

Analysis of the Lorenz curves (Figure 12) offers the first perspective into the structure of distribution. Lorenz's curves show development over the years. There is a tendency to move towards higher concentrations in later years. There has been less difference in the development of Lorenz curves from 2007 to 2016, also observable in Gini coefficients around 0,6, ranging upwards or downwards for about a few percentage points.

Lorenz curves exhibit a similar structure throughout the analysis, however, changes can clearly be observed. Lorenz curves for years 2019 and 2021 show the biggest differences compared to previous years. Similarly, the Gini coefficients of these two show the biggest movements towards more concentration as an obvious consequence. Lorenz curve increased skewness shows a higher concentration of the indice. The Gini index follows a similar path in the observation, pushing higher in the recent years.

Figure 13: Share of 40th, 60th, and 80th percentiles



Source: own work

Percentiles (Figure 13, calculated from lowest to highest weight) of the cumulative weight have been relatively similar over the years up to 2019. Weights for 2019 and 2010 are very similar with cumulative weights standing at 8, 17, and 33 percent in both of these years, indicating the middle part of the distribution has not changed in a dramatic way. Changes in 2021 can be observed in the majority of segments, excluding the 40th percentile share where the percent of the cumulative weight has stayed close to 8 percent. This means 40 percent of the companies hold around 8 percent of the cumulative weighting consistently. The eighty percentile share has dropped the most from 35 percentage points in 2007 to 30 percentage points in 2021. Changes on the bottom part of the distribution have not occurred or at least have been minimal, indicating more of the weighting has shifted toward higher percentiles.

5.1 Concentration effect on returns in last 5 years:

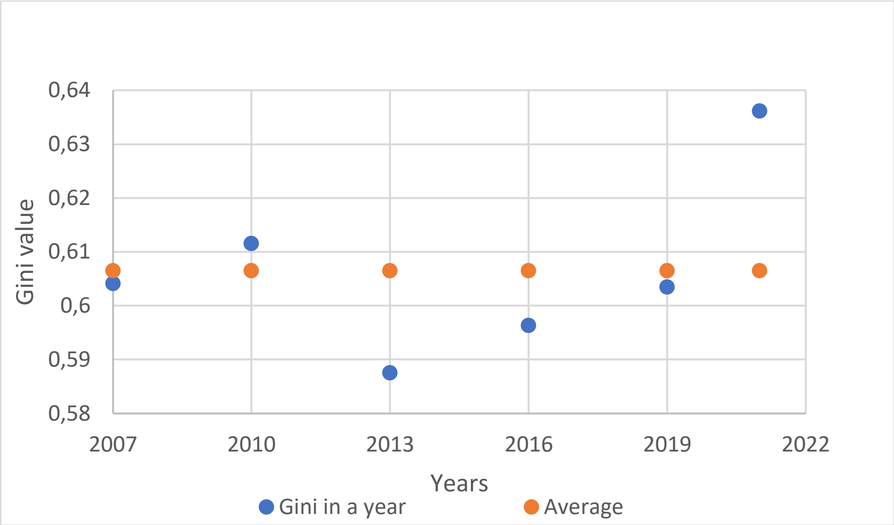
Biggest six companies from 1.1. 2016 to 1.1.2021 represented 12 percent of the indice and contributed 50,4 percent to the indice returns in this time.

Biggest twenty companies represented 28.9 percent of the indice and contributed 72 percent of the indice returns in this time.

Analysis of returns shows us that capitalization weighted indices generated half of the returns from a small amount of its components. This concentration of return generation is one of the characteristics of the capitalization weighted indice.

5.2 Closer look at Gini and Palma ratios

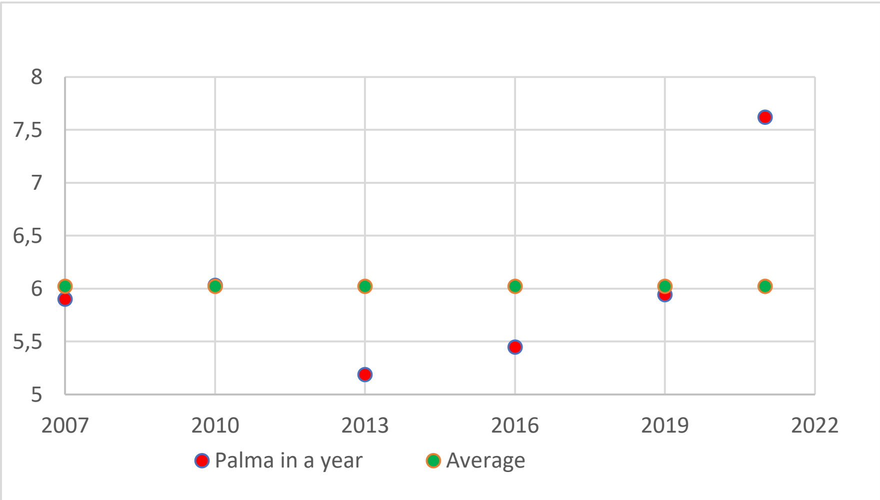
Figure 14: Gini coefficient



Source: own work

While Gini (Figure 14) has dropped from around 2010 to 2013, from 2013 onwards Gini has increased steadily. Biggest increase happened from years 2019 to 2021.

Figure 15: Palma ratios



Source: own work

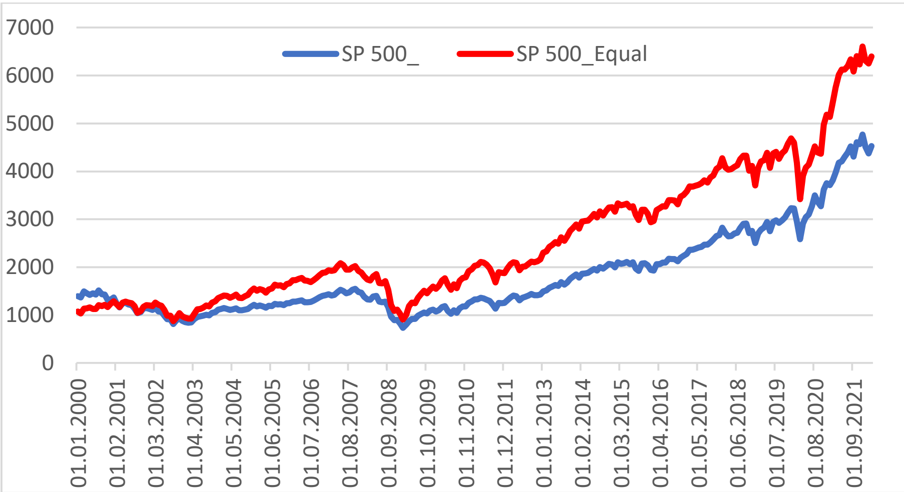
Just like Gini, the Palma ratio (Figure 15) has also seen a big upward shift from 2013 until today. This means companies with the biggest share of the Index have gained importance relative to the share of the bottom 40 percent. This means the biggest companies hold on average 6 times as much weighting in the indice as the bottom 40 percent.

The Palma ratio has remained at similar levels in all described years, except in 2021. There has been a big upwards shift in the ratio. Levels around 6 were present in the years before. On the other hand, the Gini index has been steadily moving around 0,6 after being on a steady rise since 2013. While Gini has only moved up for a few percentage points recently, the Palma ratio at the same time has relatively increased by around 33 percent. This shows how concentration has increased among the biggest companies in the index.

For a palma ratio of 7.5 this would mean that biggest companies in terms of market capitalization carry weight 7.5 times as much bottom 40 percent of companies in the indice.

5.3 Comparison of capitalization weighted and equally weighted indices

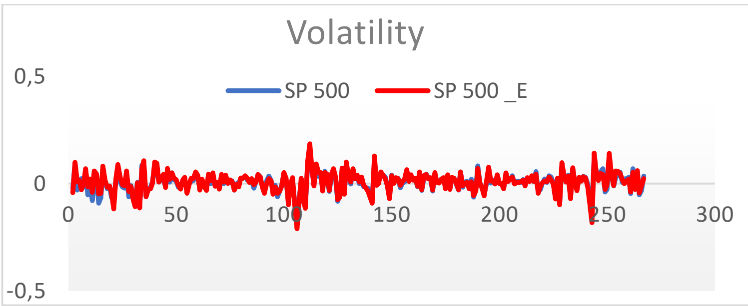
Figure 16: Comparison of indices



Source: own work

SP 500 equal weight has ourperformed it´s capitalization counterpart in terms of return from 2000 to 2021, see Figure 16.

Figure 17: Volatility of indices



Source: own work

Equal weighted indice has enjoyed better returns overall. The compound annual growth rate is 8.18 percent for the indice. Capitalization weighted indice has generated on average a 5.16 percent compound annual growth rate. While these returns are returns before dividends, the picture of performance is only partially complete without exposed risks. Equal weighted indice has achieved a yearly standard deviation of 17 percent compared to 14.9 percent or the capitalization-weighted alternative, detailed in Figure 17.

Overall, based on the average risk-free rate of 3.18 percent (10-year Treasury) in the described period Sharpe ratios are 0.201 and 0.364 in favor of the equally weighted alternative.

This means equal weighted alternative has generated better absolute returns but also better risk-adjusted ones. While this might sound lucrative, further analysis of the return characteristics can better explain the reasons for such results.

Table 1: Cap weighted portfolio statistics

R Square	0.994	
Coefficients		*P-value*
Intercept	-0.150	0.000
Mkt-RF	0.990	0.000
SMB	-0.174	0.000
HML	0.029	0.000

Source: own work

Table 2: Equally weighted portfolio statistics

R Square	0.941	
Coefficients		*P-value*
Intercept	-0.014	0.847
Mkt-RF	1.051	0.000
SMB	-0.022	0.372
HML	0.302	0.000

Source: own work

Performing Fama-French three-factor model to explain the analysis of the underlying reasons for outperformance gives us further insight in Tables 1 and 2.

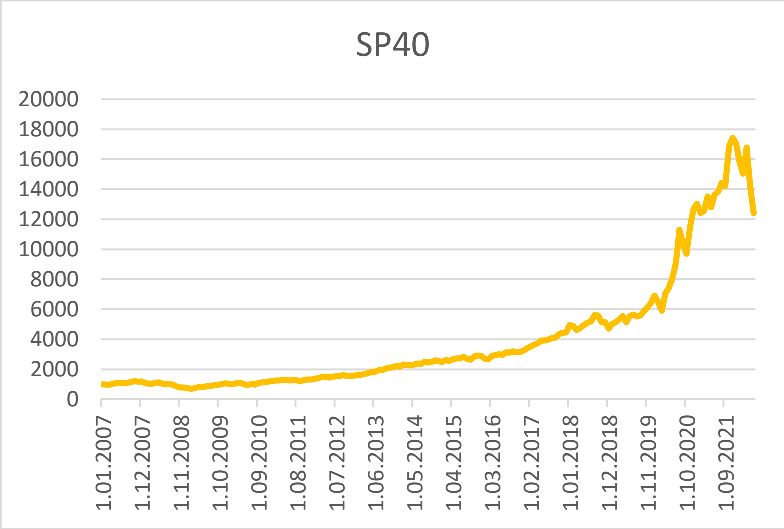
Three-factor model for the capitalization-weighted indice helps to explain the 99.4 percent variability of the data, shown by the R square. The intercept of the multiple regression is negative, a so-called alpha, an abnormal return above what would be predicted by the CAPM or extended model. P value is highly significant so results are reliable. Such results are not that special as capital allocation to the main indice is a widely used investment strategy. All p-values for the coefficients are highly significant at 0.05 and 0.01, indicating the reliability of the coefficients.

The coefficient for the market excess return is close to 1, standing at 0.98. More interesting are coefficients for small-big factor, standing at -0.17. This indicates returns for the indice are negatively related to smaller companies' outperformance. When smaller capitalization equities perform well this would lead the indice to perform worse. For every percentage point outperformance of small capitalization equities over bigger ones, this indice would lose on average 0.17 percentage points. This is reasonable and expected. This is an indice constructed from the highest capitalization companies so a positive coefficient would be surprising. High-Low factor is close to zero, being at 0.028. This means the fund is just slightly positively related to situations when high book-to-market companies outperform lower counterparts.

Multiple regression model for the equally weighted indice was able to predict 94 percent of returns variability with R square of 94 percent. This means a very high percentage of returns variability is explained by the model. The intercept value, representing alpha, is slightly negative, at 1.4 percent. However, this result is not significant, it has a p-value of 0.84. Further interpretations would be biased, and potentially misleading. The coefficient for beta stands at 1.05. SML factor coefficient value is -0.02 but it also has a too high p-value at 0.37 so further conclusion would prove rather misleading. On the contrary, the HML factor coefficient is 0.3 and highly significant at both levels at 0.05 and 0.01. This coefficient gives us insight into the indice character. When high book-to-market companies perform well, this is positive for the return profile of the indice. For every percentage point of outperformance of high book-to-market companies to lower ones, indice increases by 0.3 percentage points.

Both models explain the returns well. The majority of the coefficients are significant, which enables the interpretation. In both cases, alpha values are either negative or not significant, so many investment strategies do not prove to generate abnormal returns for the underlying isks taken. While classic capitalization weighted indice is negatively related to small equity outperformance, the equal-weighted alternative does allow for any reliable conclusions. Alternatively, capitalization-weighted indice does prove to be minimally positively related to the outperformance of so-called value companies, equally weighted alternative shows a much more positive relationship, significant at levels of 5 and 1 percent. Residuals plot randomly around the center line, without any recognizable trends or patterns. This verifies the assumption that residuals are independent from each other.

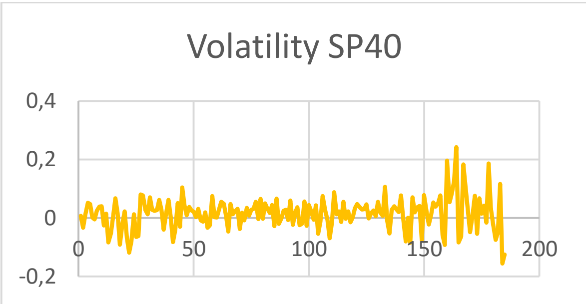
Figure 18: Capitalization weighted indice movement



Source: own work

We can see the development of the capitalization-weighted indice in Figure 18. Such an indice would have experienced growth in the cumulative value. The movement has been more pronounced from 2018 where growth was bigger.

Figure 19: Indice volatility



Source: own work

Indice volatility in seen in Figure 19. While monthly volatility has been fluctuating up or down, it has increased from 150 months onwards. This period is also the period where indice moves more aggressively in both, positive and negative direction.

Table 3: Key indice statistics

R Square	0.786	

Coefficients		*P-value*
Intercept	0.609	0.001
Mkt-RF	1.103	0.000
SMB	-0.223	0.006
HML	-0.346	0.000
M	0.064	0.175

Source: own work

Analysis of the Fama-French 4-factor model was performed on the SP40 (see Figure 18, 19, and Table 3) with added momentum factor. The model explained data better than the classic 3- factor model, as the adjusted R square increased, but only for a percentage point. Overall the model was able to explain 78.5 percent of the variability of the indice, suggesting decent explaining power.

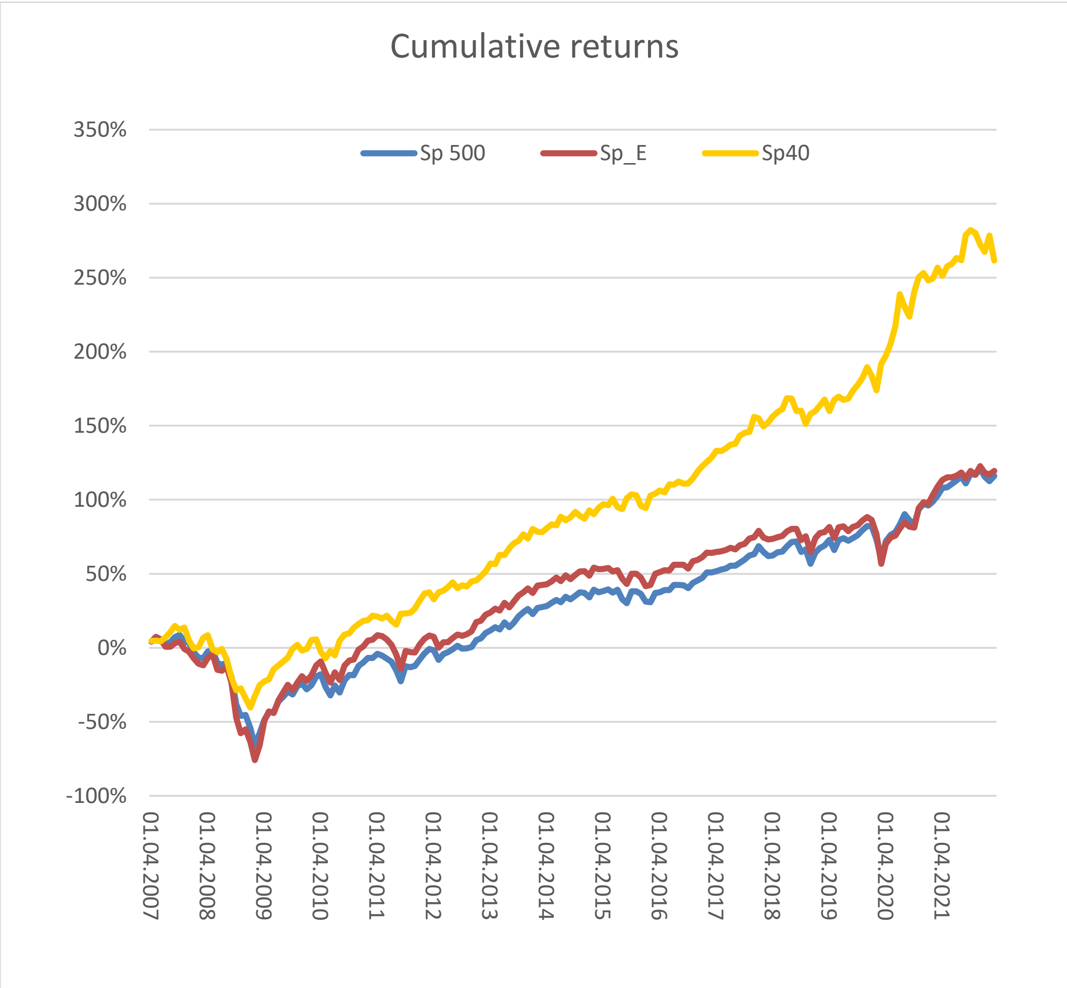
Overall, all coefficients of the model were highly significant at 5 and 1 percent, except the momentum factor with a p-value of 0.17 otherwise being positive. This proves rather sad, as the momentum factor can not be used as a reliable explaining factor. The intercept showed to be rather high, at the value of 0.6. This means such an indice would have produced high positive alpha. The other two factors, SMB and HML proved both highly significant and both well into negative territory. SMB factor resulted in a coefficient of -0.22, suggesting a negative association with the movement. Similarly, HML at -0.34 proved even more reversely associated. The factor for market excess return stands at 1.1, representing the beta of the indice. The beta of the indice is relatively similar to market beta of 1, also highly significant.

Results indicate that such composition was negatively related to the outperformance of smaller stocks, which is logical given the fact that chosen equities were selected from among the highest weighting in the capitalization indice. Perhaps even more surprisingly, the HML factor was even bigger, meaning that the highest weightings that were criteria for selection in this indice proved negatively associated with high book-to-market companies (also called value) outperformance. Lastly, high alpha revealed the value of the companies having the highest weighting in the indice. Such a strategy would have generated positive alpha. However, this also consequently means exceptional selection skills needed within the portfolio department to select companies that would be among the biggest constituents in the future, which is highly unlikely at such high accuracy. The selection capacity of the biggest constituents proved to be valuable after all.

5.4 Comparison of the three indices

Results of the created SP40, consisting of companies that have consisted the highest twenty rankings in terms of weight in weighted capitalization SP 500, show that such an indice would have generated compound annual returns of 19.0 percent and volatility of 19.2 percent in the time frame from 30.3. 2007 to 30.3. 2022 compared to 8.04 and 15.3 percent of return and volatility for capitalization-weighted and 8.29 and 17.76 for equally weighted. On the risk adjusted basis this yields a Sharpe ratio of 0.82 for SP40, 0.42 for capitalization weighted and 0.40 for equally weighted indice.

Figure 20: Indices comparison



Source: own work

Individually created indice from companies that have reached the highest rankings in terms of weights enable us another comparison in cumulative returns (natural logarithms), Figure 20. While such an investment strategy would have performed comparatively in the years during and after the financial crisis, after 2014, these companies would have performed above expectations. Certain companies which were among the biggest during the crises got hit significantly, so idiosyncratic risk proved well alive. Certain constituents like Citigroup created significant losses and did not necessarily recover. After that period biggest companies gained steam, coinciding with increased measures of concentration discussed before.

5.5 Implementation of the Risk parity portfolio approach in R

The following pages are dedicated to the implementation of the parity portfolio concept with R programming language. The analysis is performed with the help of a few libraries within the R language. These include riskParityPortfolio, portfolioBacktest, barplotportfolioRisk, and xts packages to allow further exploration and analysis.

Function riskParityPortfolio designs risk parity portfolios to distribute risk contributions of assets, which is different compared to consideration of the overall volatility as in a mean-variance portfolio. By default, the problem considered is a vanilla risk parity portfolio (with weights being non-negative that sum to 1), with no expected return and no variance term. In such a case, the problem is convex where the optimal solution is guaranteed to be achieved with a perfect risk concentration (R(w)=0). The default option is a formulation by Spinu (2013) while Griveau-Billion, Richard, Roncalli (2013) option can also be selected. In the case of additional box constraints like expected return term or variance term, the problem becomes non-convex. In this case, the global optimal solution cannot be perfectly achieved, just the local optimal. Successive convex optimization (SCA) algorithm is used, proposed by Feng, Palomar (2015), where more risk concentration terms can be chosen through the argument formulation.

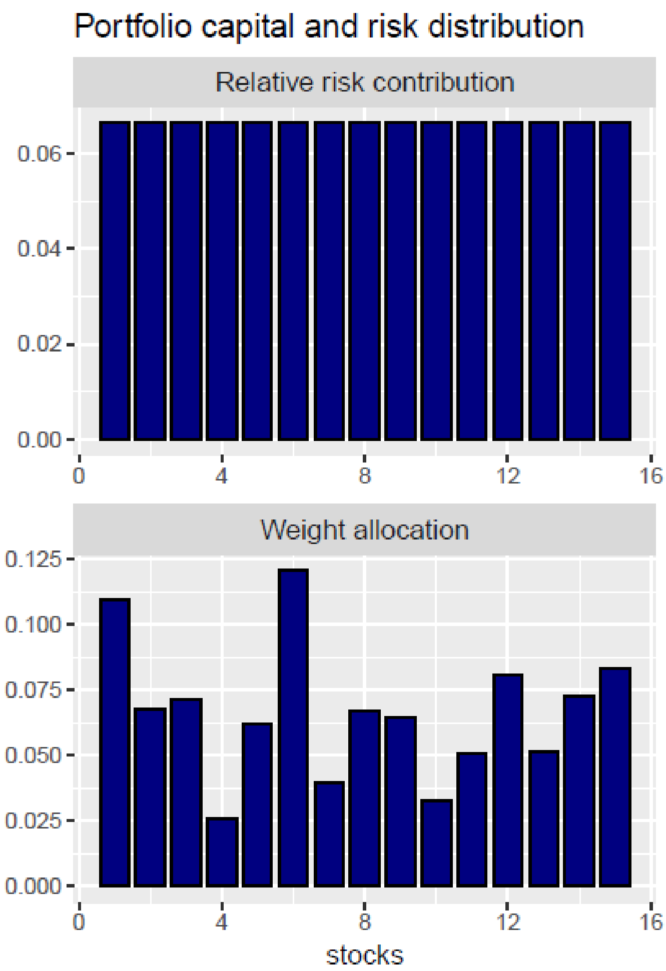
All approaches require an appropriate excel file with corresponding data of the equities. Data should be properly recognized within the R package. Matrix notation is needed to calculate the variance-covariance matrix which is the basis for the work of the riskParityPortfolio library. To obtain the naive diagonal solution, also known as an inverse volatility portfolio, the formulation argument »diag« needs to be used. When calculating the minimum weight constraint, the argument »w_lb« is used which represents the minimum asset weight required for the optimization procedure. Similarly, other potential requirements can be added to the procedure (Vinícius & Palomar, 2019; Palomar, 2020; Griveau-Billon, Richard & Roncalli, 2019).

The further analysis compares parity portfolios to previously analyzed three indices. Both parity portfolios are selected from the universe of 40 companies, with a rebalancing period of 60 days. A comparison of two portfolio approaches, the tangency portfolio and risk parity portfolio follows. There are some useful functions used for his approach like stockdataDownload, backtestChart within the already used libraries. Library quadprog is used to help determine the Sharpe ratio for the tangency portfolio.

On the next page, there is a quick overview of the results obtained for parity portfolios created. Analysis was done using randomly chosen 15 equities from the previously determined universe of 40 from the main indice. Three different allocations for parity portfolios are created within the chosen universe. A classic risk parity portfolio with equal risk contribution of each asset is calculated with corresponding risk and weight characteristics. Then, a naive parity approach is calculated, following a similar principle. Lastly, a specific parity approach with a minimum asset weight constraint is added. The second part involves a comparison between the parity strategy and other indices.

5.5.1 Risk parity portfolio*:*

Figure 21: Risk parity allocations



Source: own work

Table 4: Risk parity allocations

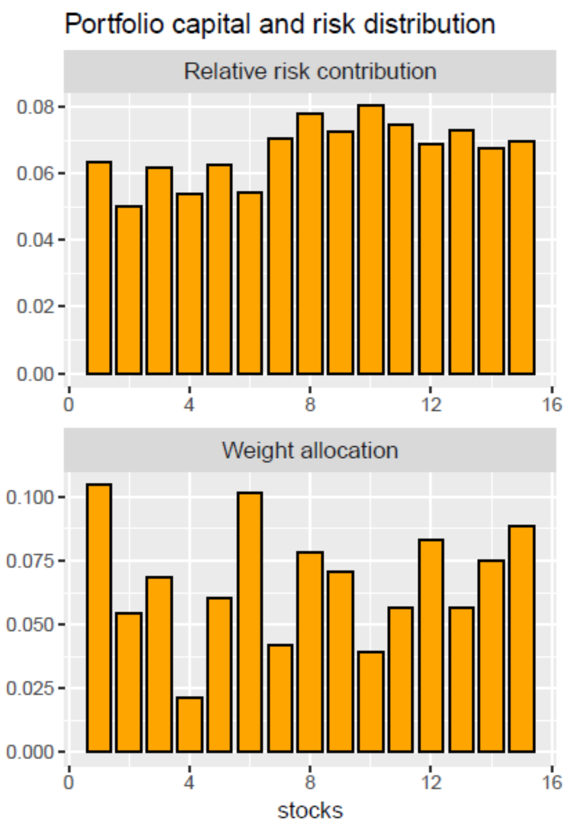
Stock number	Weight allocation	Risk contribution
1	0.109	0.0667
2	0.067	0.0667
3	0.071	0.0667
4	0.025	0.0667
5	0.062	0.0667
6	0.121	0.0667
7	0.040	0.0667
8	0.067	0.0667
9	0.064	0.0667
10	0.032	0.0667
11	0.050	0.0667
12	0.081	0.0667
13	0.051	0.0667
14	0.072	0.0667
15	0.083	0.0667

Source: own work

Generating a classic risk parity portfolio gives perspective into the money and risk allocation. Weight allocation among assets can differ significantly. For example, weight in equity number 6 is allocated at 12.1 percent. The lowest weight is given to equity number 4, standing at 2.5 percent. While weights can differ substantially, this is not the case for risk allocation. Relative risk contribution, risk contribution of each asset is the same, standing at 6.6 percent.

5.5.2 Inverse volatility portfolio*:*

Figure 22: Inverse volatility allocations



Source: own work

Table 5: Inverse volatility allocations

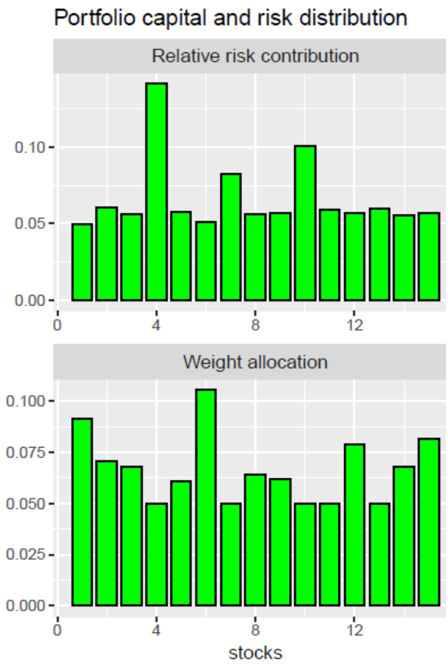
Stock number	Weight allocation	Risk contribution
1	0.104	0.063
2	0.054	0.050
3	0.068	0.061
4	0.021	0.053
5	0.060	0.062
6	0.101	0.054
7	0.041	0.070
8	0.078	0.077
9	0.070	0.072
10	0.039	0.080
11	0.056	0.074
12	0.082	0.068
13	0.056	0.072
14	0.074	0.067
15	0.088	0.069

Source: own work

The inverse volatility approach follows different objectives compared to classic risk parity. This portfolio is constructed using volatilities as a way of creating weight allocations. Weight allocations, in this case, are highest for equity number 1, standing at 10.4 percent, and lowest for equity number 4, standing at 2.1 percent. While in a classic risk parity portfolio relative risk contribution is the same for all assets, in this case, risk contributions range from 5 percent to 8 percent, creating different risk contributions, therefore different diversification.

5.5.3 Parity constraint portfolio

Figure 23: Parity constraint allocations



Source: own work

Table 6: Parity constraint allocations

Stock number	Weight allocation	Risk contribution
1	0.091	0.049
2	0.070	0.060
3	0.068	0.056
4	0.050	0.141
5	0.060	0.057
6	0.105	0.050
7	0.050	0.082
8	0.063	0.056
9	0.061	0.056
10	0.050	0.100
11	0.050	0.058
12	0.078	0.056
13	0.050	0.059
14	0.067	0.055
15	0.081	0.056

Source: own work

In this case, a portfolio of the same equities is calculated, taking into account the minimum weight allocation. The minimum weight is set at 5 percent which was set to multiple equities. Risk contributions differ much more prominently compared to naive parity alternative, ranging from 4.9 percent up to 14.1 percent. Equities 4 and 10 exhibit the most riskiness to the overall portfolio composition, with their risk contributions standing at 14.1 and 10 percent, respectively.

While the classic parity concept gives all allocations an even risk contribution profile, naive parity is open to more differences in risk contributions. Weight allocation limits as seen in the last approach force certain limit weight on the assets and exposes the biggest risk contributors. More concentrated risk positions are caused due to weight constraint objective.

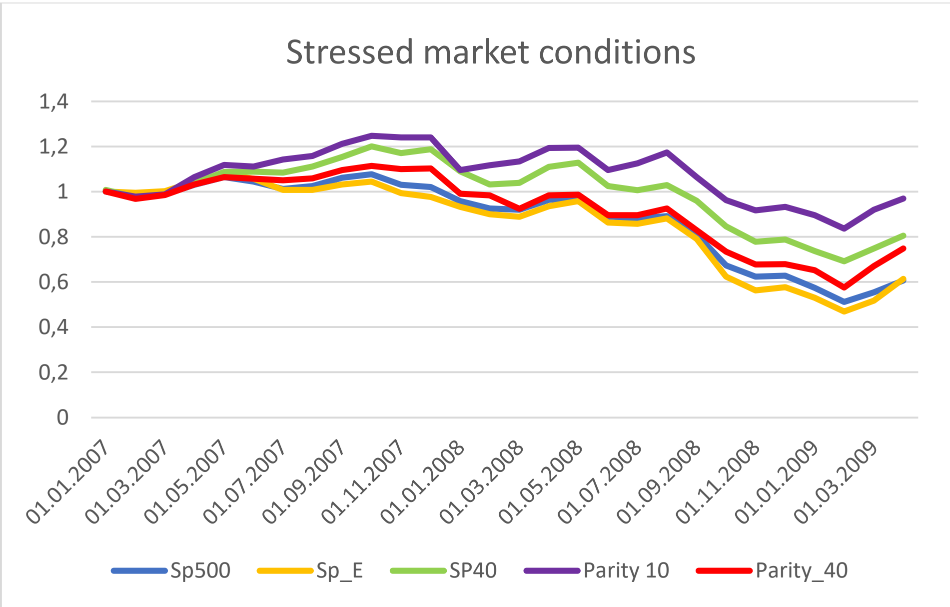
5.6 Indices comparison

In the next section indice comparison is presented. Four portfolios are compared during two different market regimes – pre and post-global financial crisis.

First is stressed market condition. For this thesis, this is the period of the global financial crisis with the extensive wide economic downturn in the real and financial sector. During the global financial crisis, also called The Great Recession, from 2007 to 2009, the global the financial crisis hit several financial institutions. After the Lehman Brothers bank bankrupted, liquidity decreased massively, credit spreads ballooned, stock prices fell and a number of financial institutions were in trouble. The devastating effects of the crisis alarmingly spread to the real economy. The Federal Reserve started with a number of extraordinary steps to ease the spreading of financial difficulties. In late 2007 it established new liquidity measures aiming at providing much-needed liquidity to the institutions and markets. From peak to through gross domestic product fell by 4,3 percent, causing this recession to be the deepest after the second world war. Unemployment also doubled from 5 to 10 percent. During this period GDP of the most developed economies fell, including the US and most Eurozone countries. As Ben Bernanke concluded:" We came very close in October 2008 to Depression 2.0." (Blinder & Zandi, 2010) This period is chosen to represent a more prominent stressed period.

The next part is devoted to the period after the global financial crisis, from 2009 onwards. This period is analyzed to understand indice development during the less difficult economic environment.

Figure 24: Indices movement



Source: own work

Comparing indices can give us more perspective into the development during stressed market conditions. Parity_40 portfolio would have performed similarly to the SP indices. Other metrics show comparable results of the parity strategy to the other portfolio compositions. Strategy parity_10 would have produced decent results for volatility and maximum drawdown as seen in Table 7 and Figure 24.

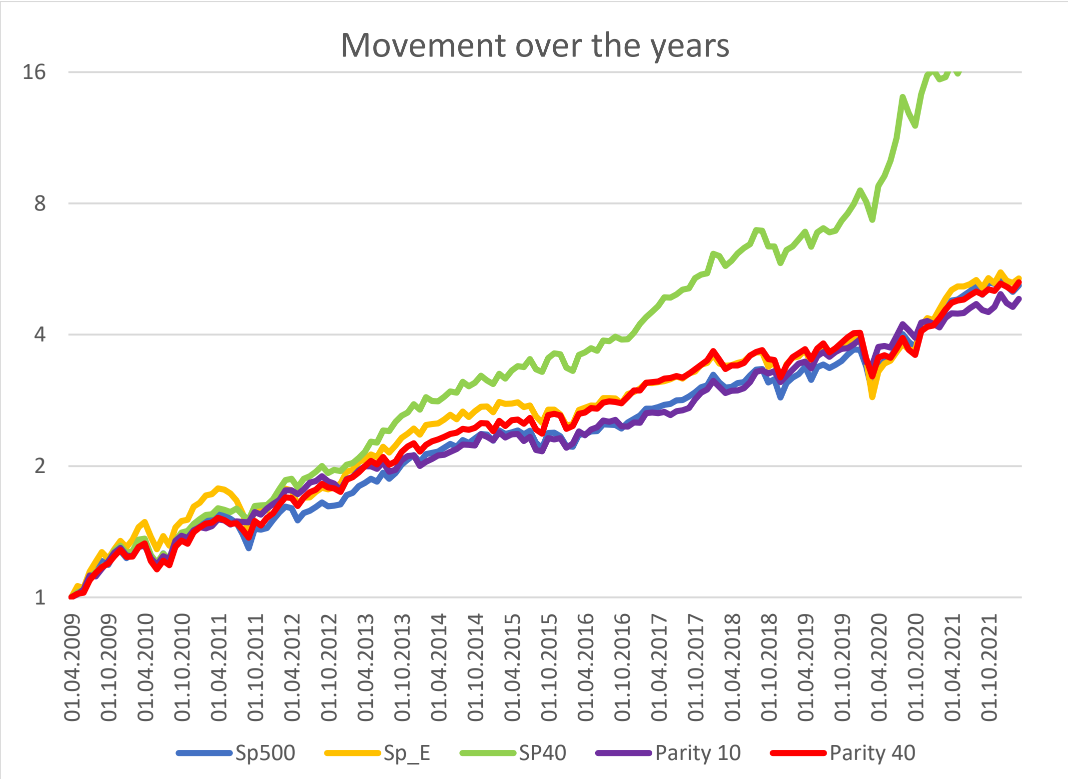
Table 7: Key metrics

	Volatility	Max Loss from base value	Max drawdown
Sp 500	0.177	0.48	-0.52
Sp 500_E	0.189	0.53	-0.56
Sp_40	0.145	0.31	-0.42
Parity_10	0.115	0.16	-0.33
Parity_40	0.157	0.42	-0.48

Source: own work

Portfolio Parity_10 experienced the lowest volatility. It has also experienced the lowest loss from basis value. Similarly, a maximum drawdown is the lowest among all for this strategy.

Figure 25: Indices movement



Source: own work

During less stressed market conditions parity_40 would have performed very similarly to the SP 500 indices in terms of return, generating a compound annual growth rate of 11.8 percent during the period from 30.4. 2009 to 1.4.2022. In terms of risk-adjusted returns, the such portfolio would have generated better risk-adjusted performance with a higher Sharpe ratio of 1.04 but lacking the performance of Sp_40, capitalization-weighted alternative as seen from Table 8 and Figure 25.

Table 8: Key metrics

	CAGR	Sharpe ratio
Sp 500	0.116	0.82
Sp 500_E	0.118	0.76
Sp_40	0.208	1.27
Parity_10	0.094	0.90
Parity_40	0.118	0.86

Source: own work

Both parity portfolios performed reasonably compared to other portfolios. Parity_10 and Parity_40 achieved similar compound returns with 9.8 and 11.8 percent but outperformed Sp 500 and its equal alternative in terms of risk-adjusted returns. Overall Sp_40 generated the best returns on an absolute and risk-adjusted basis. Parity_10 generated the second-best risk-adjusted returns, but the worst absolute returns. It has the best metrics for stressed market conditions combined with the second best risk-adjusted returns. However, this comes at the cost of compounded annual returns. Next section analysis parity factor influences during the period.

Table 9: Parity_40 portfolio statistics

R Square	0.857	
Coefficients		P-value
Intercept	0.041	0.746
Mkt-RF	0.915	0.000
SMB	-0.275	0.000
HML	0.089	0.025

Source: own work

Three-factor model for Parity_40 portfolio indice explains 85.7 percent of the indice movement at this time. This suggests decent explaining power, with the majority of the variability being explained. The intercept of the regression, namely 0.041 could suggest some abnormal return but it is not significant at a big margin, with a p-value standing at 0.74. Market excess return stands at 0.915. This means a high correlation between market movements and dependency on market risk factors. The reliability of this coefficient proves adequate p-value with a significance at 5 percent levels.

The factor for small-big companies' outperformance shows negative relation to smaller companies' outperformance. This is to be understood as indice is constructed from non-smaller companies. The reliability of this metric is not questionable as the p-value is significant at the 5 percent level. The value premium factor (HML) is slightly positive, at 0.089. This shows the small positive tendency in situations with high book-market outperformance. This can be reliably claimed as it is at statistically significant levels. The residual plot shows independent residuals without a recognizable pattern, indicating uncorrelated residuals.

Table 9: Parity_10 portfolio statistics

R Square	0.724	
Coefficients		*P-value*
Intercept	0.203	0.179
Mkt-RF	0.750	0.000
SMB	-0.395	0.000
HML	-0.155	0.001

Source: own work

Three-factor model for the Parity portfolio indice explains around 72.5 percent of the indice movement, shown by the R square. This shows decent explaining power, however, some variability is still unexplained. The intercept of the multiple regression is positive, standing at 0.2. This indicates some abnormal return above what would be predicted by the extended model. However, such a conclusion would be misleading due to insufficient p-value, standing at 17.9 percent, unable to reliably reject the null hypothesis at statistically significant levels.

The coefficient for the market excess return is standing at 0.75 which means a lower market beta compared to previously analyzed indices. This result is also highly statistically significant, at 1 and 5 percent levels. This gives more insight into the indice, having less exposure to market risk compared to other indices.

More interesting are coefficients for small-big factor, standing at -0.39. This indicates returns for the indice are negatively related to smaller companies' outperformance. When smaller capitalization equities perform well this would lead the indice to perform worse. This is quite understandable as indice components are selected from the universe of companies having the highest weightings in cap-weighted indice. The result is statistically significant which supports such a conclusion. The High-Low factor is close to zero, but still negatively associated with this indice. The result is highly significant which enables further conclusions. A value of -0.15 means a slightly negative relation to market situations when high book-to-market companies outperform lower counterparts. The residual plot shows independent residuals showing no recognizable pattern, indicating uncorrelated residuals.

When testing the model with the momentum factor, R square showed minimal improvement, similar to adjusted R square which decreased adjusted for model predictors. The momentum factor was also not significant so any help with additional predictors proved helpless.

Parity portfolios were well explained by the three-factor models, explaining most of the variability in the movements. Both parity portfolios showed dependency on market risk factors, parity_40 showing more dependency. Similarly, both strategies proved to be related to size premium, with the second parity approach showing more exposure. The first parity approach showed a positive value premium factor.

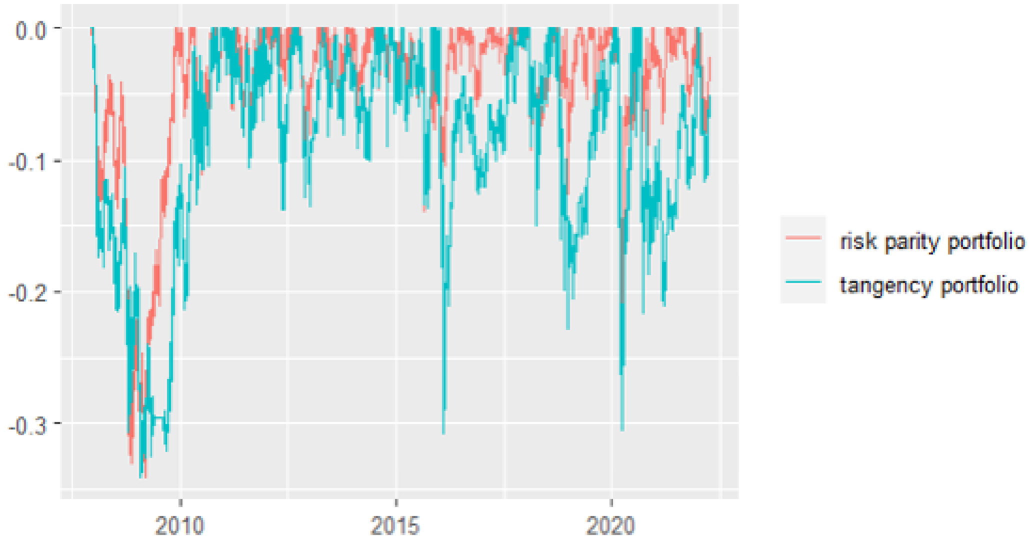
A further example of the risk parity portfolio (Parity_10) compared to the tangency portfolio of the same 10 assets offers additional perspective. This comparison helps in the understanding of the parity portfolio concept. There are the same 10 assets selected to the universe forming a vanilla parity and tangency portfolio from January 2007 to the end of June 2021.

Figure 26: Cumulative return comparison

Source: own work

Comparison between parity and tangency portfolio strategy. The parity alternative generates comparable absolute returns as seen in Figure 26. Such a strategy would have performed well most of the time, as a red colour- risk parity cumulative value is above or at similar is above or at similar levels to the tangency alternative.

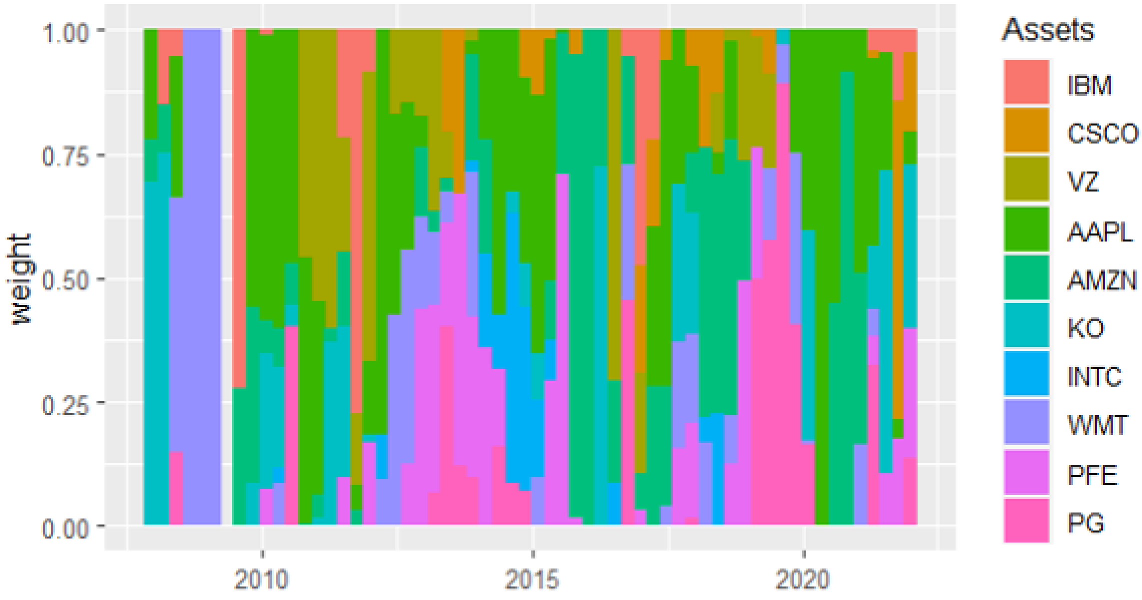
Figure 27: Drawdown comparison



Source: own work

The risk parity portfolio offers comparable or lower drawdown pressure than the tangency alternative as seen in Figure 27.

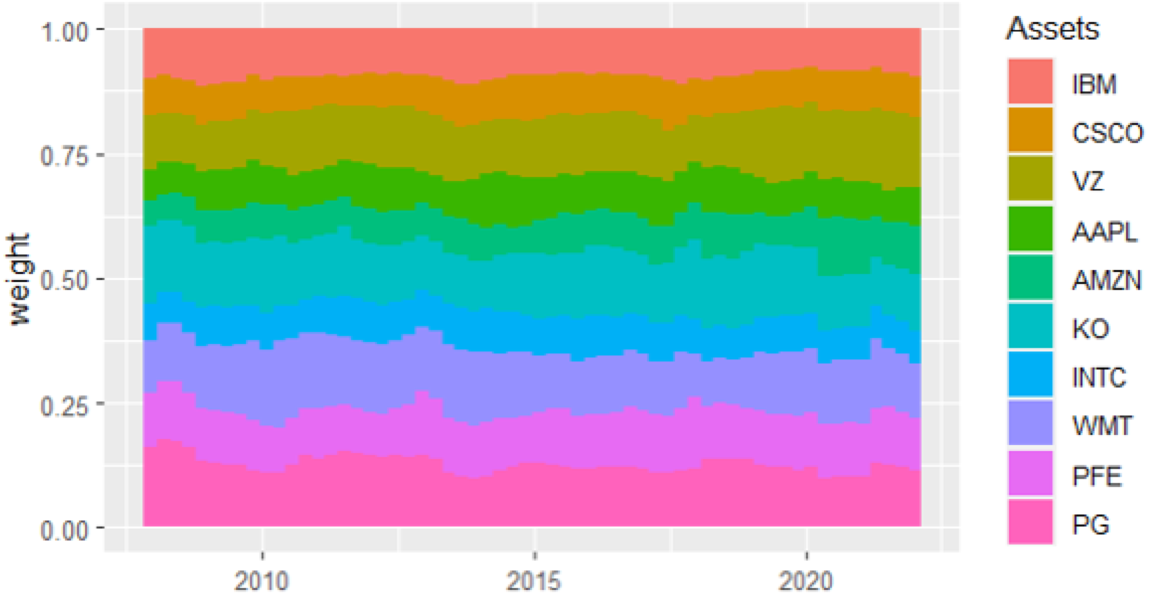
Figure 28: Tangency allocations



Source: own work

Weight allocations for tangency portfolio can be very concentrated as shown from the Figure 28.

Figure 29: Parity allocations



Source: own work

Parity allocations exhibit much smoother differences in Figure 29, keeping components weights at more similar levels compared to tangency alternative. Tangency and Risk parity comparison in Table 9:

Table 9: Portfolio statistics

Risk parity portfolio		Tangency portfolio	
Sharpe ratio	0.651	Sharpe ratio	0.504
Max drawdown	0.341	Max drawdown	0.341
Annual return	0.107	Annual return	0.114
Annual volatility	0.165	Annual volatility	0.226
Sortino ratio	0.915	Sortino ratio	0.719
Downside deviation	0.117	Downside deviation	0.161
Sterling ratio	0.315	Sterling ratio	0.334
Omega ratio	1.131	Omega ratio	1.099
VaR (0.95)	0.014	VaR (0.95)	0.020

Source: own work

Annual return is better for the Tangency portfolio, but at the cost of a much higher volatility and downside drawdown Similarly for the downside deviation, it is higher for the Tangency portfolio. When comparing these two portfolios, the Parity alternative performs mostly better compared to the Tangency alternative in terms of risk-adjusted returns. The Sharpe ratio is higher than the Tangency alternative by around 50 percent. A similar result is shown by the Sortino ratio which focuses on the downside risk-adjusted returns. Results of other metrics perform in favor of the Parity alternative compared to Tangency portfolio construction.

Perhaps even more interesting is the comparison of the asset weights over time. There are huge swings in allocation over time for the Tangency portfolio components. Some assets are massively increased at certain times, while others decreased like before the crisis in 2009 and later developments. Specific assets are presented as the optimal solution, causing extreme concentrations in certain periods. Such swings of these allocations can be well above those in the Parity portfolio. Wild allocation movements of the tangency solution would require sufficient market liquidity combined with additional transaction costs.

CONCLUSION

Multiple quantifiable metrics have shown that the concentration of the capitalization-weighted indice has changed over time. While some metrics have experienced minor changes over time, others have given true insight into the movements of the indice holdings. Capitalization weighted index offers diversification advantages, however, is it prone to significant changes to its structure. Lower percentiles of the distribution show similar weightings over time with key differences within the 9th decile. The majority of the capitalization-weighted indice returns from 2016 were generated from a handful of companies. Twelve percent of the indice weightings in 2016 generated more than half of the indice returns. Equally weighted counterpart offers advantages from 2000 onwards. Its superiority diminishes as the observed timeframe shrinks and the index underperforms capitalization-weighted alternative in terms of risk-adjusted returns in both market regimes. The equally weighted index is positively related to the high book-to-market companies (HML) factor but did not show a significant relationship with the SMB factor. SP40 portfolio has shown some positive alpha intercepts. It also has negative sensitivity to SMB factor together with even more negative sensitivity to HML factor at almost -0.4. The momentum factor was positive but insignificant. Taking into account main regression factor influences, the equal-weighted alternative offered investors a positive value premium, capitalization-weighted a negative size premium and the SP40 portfolio a negative size and negative value premium. Risk concentrations can be highly influenced by asset allocation decisions. Asset weights influence asset risk contributions to the overall risk profile as seen in all Parity examples. Results in the direct comparison of parity and tangency portfolios suggest that the Risk parity method of constructing portfolios can offer additional benefits to investors of all sorts. It offers a more detailed approach towards risk concentration and it can perform well compared to the classic portfolio management techniques. Generated parity portfolios performed similarly or better than capitalization weighted indice during the period of heightened market stress. The parity approach has been shown to decently compare stressed market conditions and offers similar results in absolute and risk-adjusted levels after the global financial crisis. Parity portfolios exhibit unique factor styles. Capitalization weighted indice did not generate better cumulative or risk-adjusted returns from 2009 onwards compared to SP40 or individually created parity alternatives. Capitalization weighted indice offers no clear diversification benefits compared to its equal counterpart or studied alternative portfolio compositions.

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APPENDIX

Appendix 1: Summary (Slovenian)

Namen magistrskega dela je bil raziskati vpliv in gibanje razpršenosti na primeru tujega delniškega indeksa ter primerjati vplive različnih oblik razpršenosti v kontekstu donosov, tveganj ter karakteristik gibanj s pomočjo trifaktorskega regresijskega modela in drugih pokazateljev uspešnosti. Poskušal sem ugotoviti kako se ključne metrike koncentracije spreminjajo v odnosu do izbranega indeksa. V primerjalnem delu sem izbranemu indeksu dodal še dva indeksa, z drugačno obliko razpršenosti, enega izpeljanega iz predhodne analize metrik koncentracije. V drugem delu sem se osredotočil na implementacijo teoretičnih konceptov paritetne metode razpršitve portfeljev in to portfeljsko obliko vključil v primerjavo obstoječih treh indeksov. V zadnjem delu sem paritetno metodo še postavil v kontekst s tangentnim portfeljem.

Rezultati so pokazali spreminjanje razpršenosti in povečevanje koncentracije kapitalizirano uteženega indeksa, s poudarkom predvsem na večanju pomena največjih komponent. V odnosu do enakomerno razpršene alternative se je indeks pokazal kot boljši z vidika tveganju prilagojenih donosov, toda ne v absolutni meri. Kapitalsko utežen indeks izpeljan na podlagi metrik koncentracije se je pokazal kot alternativa obema. Trifaktorski regresijski model je pokazal raznolikost indeksov do izpostavljenosti faktorjem in večinoma statistično značilne sklepe. Manjši paritetni obliki portfeljev sta se v odnosu do ostalih alternativ izkazale kot zanimiva izbira z vidika absolutnih in relativnih mer uspešnosti. V zaključku se je paritetni portfelj pokazal kot boljša alternativa tangentnemu portfelju.

Sklepi so tako pomembni z vidika razumevanja oblik diverzifikacije indeksov in tveganj, ki jih prinašajo različne oblike razpršenosti. Širše znani kapitalsko uteženi indeksi so lahko podvrženi spremembam v svoji razpršenosti, kar spreminja njihovo podobo. Alternativne oblike uteževanj vključno s paritetnimi metodami predstavljajo smiselno dopolnitev tem bolj razširjenim oblikam, kjer lahko investitorji najdejo primerljive ali celo boljše možnosti za izpolnitev svojih preferenc po tveganjih in donosih.