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SCHOOL OF ECONOMICS AND BUSINESS

MASTER'S THESIS

**THE VALUE OF SELL-SIDE ANALYSTS' RECOMMENDATIONS IN
ACTIVE PORTFOLIO MANAGEMENT**

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JAN DRAB

AUTHORSHIP STATEMENT

The undersigned Jan Drab, a student at the University of Ljubljana, School of Economics and Business, (hereafter: SEB LU), author of this written final work of studies with the title 'The value of sell-side analysts' recommendations in active portfolio management', prepared under supervision of prof. dr. Igor Lončarski.

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TABLE OF CONTENTS

INTRODUCTION	1
1 THEORETICAL BACKGROUND.....	4
1.1 The Black-Litterman asset allocation model.....	4
1.1.1 Expected returns and reverse optimization.....	5
1.1.2 The Black-Litterman master formula	6
1.1.3 Specifying the views and the Ω matrix.....	7
1.1.4 Parameter τ	8
1.2 The Treynor-Black asset allocation model.....	9
1.3 Prior research.....	10
1.3.1 Previous studies on the value of analysts' recommendations	10
1.3.2 Studies on combining the Black-Litterman model and analysts' recommendations	11
2 METHODOLOGY AND DATA.....	13
2.1 Investment universe and benchmark selection	13
2.2 Data and historical time horizon	13
2.2.1 Analyst target price consensus data.....	14
2.2.2 Analyst consensus scores data.....	14
2.3 Historical simulation construction	16
2.3.1 Trading strategy 1: Using analyst consensus target prices.....	17
2.3.2 Trading strategy 2: Using analyst consensus scores.....	18
2.4 Introducing constraints	24
2.5 Performance evaluation	25
2.5.1 Benchmark-adjusted rate of return	25
2.5.2 Sharpe ratio.....	26
2.5.3 Sortino ratio	26
2.6 Transaction costs.....	27
3 RESULTS AND DISCUSSION	28
3.1 Strategy 1: Using target price consensus as a view vector	32
3.1.1 The effect of transaction costs	33
3.1.2 τ sensitivity analysis	35
3.1.3 The effect of constraints	37

3.1.4	Comparison to the Treynor-Black model.....	41
3.2	Strategy 2: Adapting weights based on a consensus score.....	43
3.2.1	The effect of constraints	45
3.3	Analyzing the causes of frequently rebalanced portfolios' outperformance. 50	
3.3.1	Limitations and opportunities for further research.....	54
4	CONCLUSION	55
	REFERENCE LIST	58
	APPENDIX	61

LIST OF FIGURES

Figure 1: A step-by-step derivation of the Black-Litterman weights.....	5
Figure 2: The distribution of consensus scores throughout the period 1/1/2013 - 14/2/2023	15
Figure 3: Example of trailing datasets used for portfolio rebalancings	16
Figure 4: Market implied risk aversion coefficient calculated with 2-year rolling data	17
Figure 5: Annualized realized returns over 2-year rolling estimation periods for groups of stocks with different consensus scores	18
Figure 6: Scheme of automated BL portfolio construction for every day (2013-2023)	22
Figure 7: Scheme of automated TB portfolio construction for every day (2013-2023)	23
Figure 8: Unconstrained Black-Litterman portfolio performance ($\tau=0.05$); Daily rebal. ...	32
Figure 9: Portfolio's Sharpe ratio relative to the rebalancing period ($\tau=0.05$)	33
Figure 10: Sharpe ratio relative to the rebalancing period after transaction costs ($\tau=0.05$)	34
Figure 11: P-values of a one-sided two-sample t-test of portfolio's average daily returns vs. benchmark	35
Figure 12: The effect of parameter τ on the portfolio performance; Daily rebal.	36
Figure 13: The effect of parameter τ on Sharpe ratio while adjusting for transaction costs ($\tau = 0.05$ and $\tau = 0.5$)	36
Figure 14: The distribution of daily weights over the simulated period relative to τ	37
Figure 15: The effect of the imposed constraints on a portfolio's performance ($\tau=0.05$) ...	38
Figure 16: The sensitivity of a constrained portfolio's performance to the parameter τ ; Daily rebalancing	38
Figure 17: Sharpe ratio of a constrained portfolio relative to the rebalancing period before and after transaction costs ($\tau=0.5$)	39
Figure 18: P-values of a one-sided two-sample t-test of a constrained portfolio's average daily returns vs. benchmark	40
Figure 19: Calculated daily weights for Apple Inc.'s stock using different values of τ , constraints, and a comparison with the target price consensus implied returns .	40
Figure 20: Treynor-Black portfolio's performance relative to the rebalancing period	42

Figure 21: Constrained Treynor-Black portfolio performance; monthly rebalancing	42
Figure 22: The sensitivity of an unconstrained portfolio's performance to the parameter τ	44
Figure 23: Sharpe ratio relative to the rebalancing period before and after transaction costs ($\tau=0.05$)	44
Figure 24: P-values of a one-sided two-sample t-test of an unconstrained portfolio's average daily returns vs. benchmark.....	45
Figure 25: The effect of a no short-selling constraint on a portfolio's performance ($\tau=0.5$)	46
Figure 26: Sharpe ratio of a constrained portfolio relative to the rebalancing period after transaction costs ($\tau=0.5$)	46
Figure 27: P-values of a one-sided t-test of a constrained portfolio's average returns.....	47
Figure 28: Sharpe ratio of a constrained portfolio with monthly rebalanced ($\tau=0.05$)	47
Figure 29: P-values of a one-sided two-sample t-test of the before-mentioned constrained portfolio's average daily returns vs. benchmark	48
Figure 30: The sensitivity of a constrained portfolio's performance to the parameter τ	49
Figure 31: Comparison of transaction costs incurred for both trading strategies as a function of the rebalancing period (shown in log scale).....	49
Figure 32: Average annualized return after a change in individual asset's consensus	51
Figure 33: Average deviation of a real price from a target price consensus made 12 months prior (left) and average implied returns during the observed period (right)	52
Figure 34: Average target price consensus implied returns vs. actual average returns.....	53

LIST OF TABLES

Table 1: Results of historical simulations (daily rebalancing)	29
Table 2: Results of historical simulations (monthly rebalancing).....	31
Table 3: p-values of one-tailed two-sample-t-tests when testing for a difference in means of returns after a change in consensus vs. all recorded returns during the observed period.....	51

LIST OF ABBREVIATIONS

sl. – Slovene

BL – Black-Litterman

CAPM – (sl. Model določanja cen dolgoročnih naložb) Capital Asset Pricing Model

DJIA – Dow Jones Industrial Average index

TB – Treynor-Black

INTRODUCTION

The value of analysts' recommendations in the stock market has been the subject of much debate in recent years. There are two main groups of security analysts: "sell-side" and "buy-side". Sell-side security analysts who work for investment banks or brokerage firms provide investment reports to institutional and retail investors. When making a stock recommendation, analysts express their opinion about the performance of an individual company relative to the market and attach either a "buy," "hold," or "sell" recommendation, which their clients can then use to make investment decisions. Buy-side analysts work for wealth management firms and provide investment reports to their portfolio managers (Groysberg, Healy, Chapman, Shanthikumar & Gui, 2007).

Previous studies have concluded that sell-side analysts may be subject to various conflicts of interest. These conflicts can arise if a sell-side analyst is rewarded for recommending stocks of current or potential banking clients to maintain good relationships even if their company would not receive a good recommendation otherwise (O'Brien, McNichols & Hsiou-Wei, 2005). An investment banking analyst may also provide optimistic recommendations to encourage clients to buy stocks that would increase brokerage commissions (Cowen, Groysberg & Healy, 2006). Brokerage firm analysts tend to make the most optimistic forecasts (Azzi, Bird, Ghiringhelli & Rossi, 2006). On the contrary, buy-side analysts do not face such conflicts. Their firms do not provide brokerage services, and since their reports remain private to their firm, a negative recommendation would not damage any client relationships. Furthermore, one could argue that buy-side analysts are more valuable to the firm due to their analysis remaining private; hence, a portfolio manager could be trading with superior information to the market. In an efficient market, a publicly accessible and widely distributed sell-side analyst's recommendation should not allow investors to profit from trading on it as stock prices would quickly adjust accordingly.

Surprisingly, many research papers point to the contrary. Groysberg, Healy, Chapman, Shanthikumar & Gui (2007) concluded that buy-side analysts at a large investment firm produce less optimistic stock recommendations than sell-side analysts because they face fewer conflicts of interest. However, returns to their buy recommendations underperform sell-side recommendations, and their earnings forecasts are comparatively overly optimistic and inaccurate. The authors argue that performance differences could be partially explained by the buy-side's higher retention of poor-performing analysts. Jegadeesh and Kim (2006) found that, on average, analyst recommendations are profitable, especially those by U.S.-based analysts. Similarly, Busse, Green & Jegadeesh (2012) find that sell-side analysts' recommendations are profitable while buy-side analysts' are mostly alpha-neutral. Hobbs and Singh (2015) found that portfolios formed based on analyst recommendations have negative contemporaneous excess returns for the buy-side and positive contemporaneous excess returns for the sell-side. Despite the evidence of sell-side analysts' superiority, some publications highlight issues with their recommendations, such as optimism bias or conflicts

of interest. Strategic optimism bias, brought on by conflicts of interest, appears deliberate, pervasive, and economically significant (Malmendier & Shanthikumar, 2014). On the other hand, according to Altinkilic, Balashov & Hansen (2013), investors get little to no value from analysts' recommendations as they are frequently reactive rather than predictive.

Regarding fund managers, various scientific studies have shown that most actively managed funds tend to underperform the market in the short and long term. One study shows that in a 15-year period that ended in December 2016, 92.15% of large-cap, 95.4% of mid-cap, and 93.21% of small-cap active fund managers in the U.S. underperformed their respective benchmarks (Otuteye & Siddiquee, 2019). This finding could suggest that investing in a market index fund, which typically has lower fees than actively managed funds, might be a better option for many investors. As fund managers often rely on buy-side analyst recommendations, I wanted to analyze the effectiveness of using sell-side analysts' recommendations to form an active investment strategy. Active portfolio managers usually track their respective benchmarks to a certain extent. Therefore, the Black-Litterman asset allocation model resembles such an investment approach as it starts with the global CAPM equilibrium and then deviates from it based on subjective views. Academic research on incorporating sell-side analysts' recommendations into the Black-Litterman model's view vector is scarce, and the results of this thesis could provide further insights into the economic value that such recommendations provide to the everyday investor or a portfolio manager with an active investment strategy. A direct comparison of the incorporation of two different kinds of analysts' recommendations was not yet analyzed: their target prices and their recommendation scores. Furthermore, while subjected to the analysts' target prices as input, a direct comparison of the Black-Litterman model to another well-regarded active asset allocation model, the Treynor-Black, is limited.

The purpose of this master's thesis was to determine the economic value of sell-side analysts' recommendations by including them as a view vector in the Black-Litterman asset allocation model. The reason for focusing on the Black-Litterman model as a primary methodology was that in real life, the strategy of taking long positions in favorably recommended and short positions in unfavorably recommended stocks is not feasible. Using the model, I could replicate a portfolio manager's behavior of closely tracking the benchmark while also overweighting and underweighting the stocks, depending on their recommendation. The first goal was to perform historical simulations with the analyst recommendations-based portfolio to determine its performance relative to the benchmark. The performance has been measured as an annualized mean and risk-adjusted return, i.e., the Sharpe ratio. Simultaneously, I wanted to assess the performance of different methods of incorporating analysts' recommendations, such as using the 12-month target price consensus for each stock and sorting stocks into four groups based on their overall analyst consensus score. Furthermore, I wanted to analyze how different input parameters affect portfolio performance. Mainly, I wanted to assess the parameters of the Black-Litterman model, the frequency of portfolio rebalancing, and the effect of transaction fees. Another goal that should contribute to the

aspect of the practicality of the Black-Litterman model was the effect of τ on the volatility of the Sharpe ratios when varying the rebalancing period. Ultimately, I wanted to compare the trading strategy's performance with the Black-Litterman model to another active portfolio strategy, the Treynor-Black model. Analysts' recommended target prices had been incorporated into the model as initial alphas. Based on the goal of this thesis, four research questions were formed:

- What is the economic value of using sell-side analysts' recommendations as a view vector in the Black-Litterman model?
- Which method of incorporating the analyst recommendations in a view vector yields the portfolio with the highest risk-adjusted return before and after transaction costs?
- How do different input parameters affect the portfolio's risk-adjusted return?
- How does the portfolio constructed with the Black-Litterman model compare to the portfolio constructed with the Treynor-Black model in terms of risk-adjusted return given analyst recommendations as an input?

The first chapter of this thesis presents the theoretical background and a literature review of the Black-Litterman and Treynor-Black asset allocation models, as well as an overview of prior research on the value of sell-side analysts' recommendations in general and by incorporating them into the Black-Litterman's view vector. The next chapter describes the methodology and data that was used. I performed historical simulations within a 10-year timeframe from January 1st, 2013, to February 14th, 2023. The portfolios comprised 30 stocks currently included in the Dow Jones Industrial Average (DJIA) index. Analysts' recommendations have been incorporated into the Black-Litterman asset allocation model in two ways. Trading strategy one was based on the analysts' 12-month target price consensus. The expected return has been calculated and incorporated into the view vector concerning the current stock price. Trading strategy two was based on overweighting stocks with favorable and underweighting stocks with unfavorable consensus scores. The expected returns were set as each group's rolling historical average return. The third chapter presents the results of the historical simulations where I assessed the portfolios' performance relative to each other and the benchmark in terms of a mean return and a risk-adjusted return. An important focus was laid on assessing introduced constraints. For a real-life investor, active investment strategies that suggest short-selling and highly concentrated weights are not feasible or even allowed. I also performed a sensitivity analysis by varying the rebalancing frequency, the transaction costs, and the Black-Litterman input variables, such as the parameter τ . Mainly, the focus was on the effect of τ on the volatility of the Sharpe ratios when varying the rebalancing period. Furthermore, I compared the risk-adjusted performance of portfolios constructed with the Black-Litterman and the Treynor-Black model. At last, conclusions are drawn regarding the economic value of incorporating sell-side analysts' recommendations into these models, specifically, which investment strategy produces the highest risk-adjusted returns and is most suitable for an everyday investor.

1 THEORETICAL BACKGROUND

In this section, a description of two well-regarded models in investment theory, the Black-Litterman and the Treynor-Black, is undertaken. Additionally, a comprehensive review of scientific literature on the topic is reviewed, offering a holistic understanding of the current research landscape and setting the foundation for the novel analyses presented in this work.

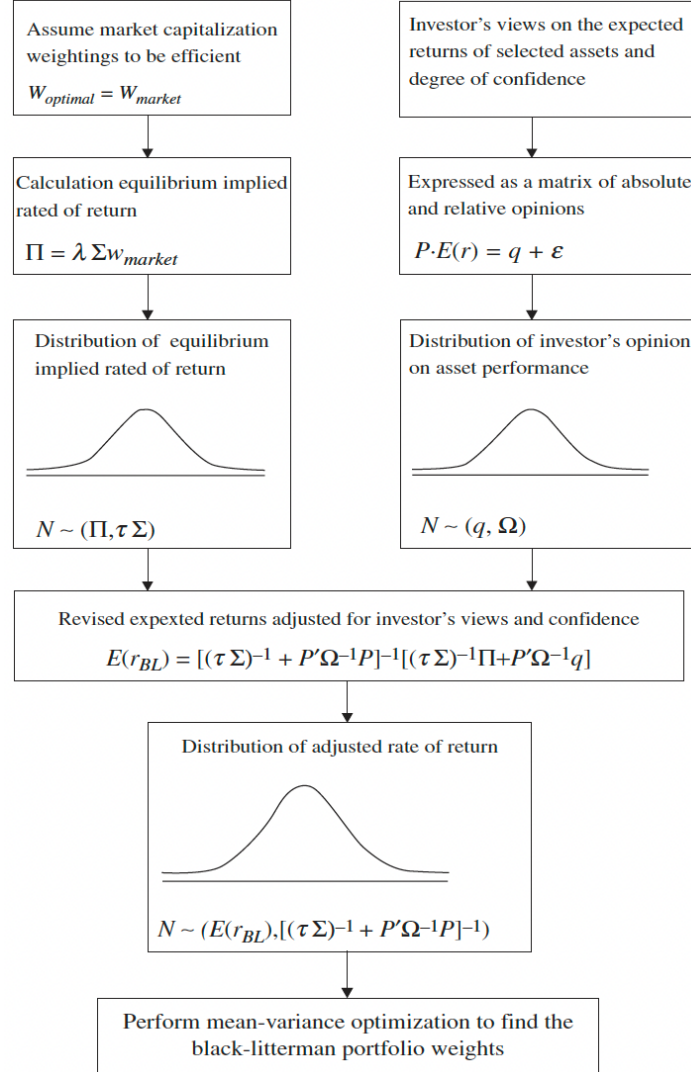
1.1 The Black-Litterman asset allocation model

Fund managers and individual investors have a strong and widely established theoretical foundation to rely on when composing their investment strategy. The introduction of portfolio theory by Markowitz (1952) brought a sophisticated quantification of the two primary goals of investing: maximizing return and minimizing risk. Before his groundbreaking work, the advantages of diversification had not yet been completely understood, and the main takeaway from the mean-variance portfolio optimization procedure was that an appropriately selected collection of assets could form a portfolio that would, through diversification, maximize the expected return and minimize volatility more effectively than any one asset could do on its own. Thus, the foundation for modern portfolio theory was established. While the formulation was exceptional in theory, it proved challenging for investors and fund managers to apply in practice. It is very complex to estimate expected returns and the covariance between assets accurately, and the strategy produces highly concentrated portfolios with only a small number of assets, making it extremely sensitive to changes in input. Even small changes in expected returns can lead to significant portfolio rebalancing (Idzorek, 2007). In addition, the model only relies on historical data and statistical conclusions derived from it. Thus, a fund manager's opinion of the prospect of a specific stock cannot be incorporated into the strategy.

In 1990, Robert Litterman, then an employee at Goldman Sachs, and Fischer Black developed an improved investment strategy, the Black-Litterman asset allocation model. It begins with the global Capital Asset Pricing Model (CAPM) equilibrium as a starting point and mathematically combines quantitative and qualitative research in the optimization. It allows the portfolio managers to express their views on the expected return of stocks and adjust their portfolio from the equilibrium expected returns implied by the value-weighted index benchmark. As views are incorporated in a Bayesian manner, it also requires the expression of the confidence level in each view (Black & Litterman, 1991). The model generates less extreme portfolio weights and is less sensitive to input changes than the mean-variance optimization. The main advantage of this model is the inclusion of investors' views (Meucci, 2006). This can also be its main challenge, as portfolio performance depends on the accuracy of predictions. The model also suggests that if the investor has no views or confidence in their views, there is no reason to deviate from the portfolio generated by the market equilibrium (Cheung, 2009 & Walters, 2013). Despite the model's attractiveness in theory and practice, we still have not seen its wide applications. Cheung suggests that

practical issues such as confidence parameter setting, alternative views, or the curse of dimensionality when applied to large portfolios may hinder its implementation. A step-by-step derivation of the Black-Litterman weights is displayed in Figure 1. The Black-Litterman master formula mathematically combines market and investor-implied returns.

Figure 1: A step-by-step derivation of the Black-Litterman weights



Source: He, Grant & Fabre (2012).

1.1.1 Expected returns and reverse optimization

The Black-Litterman model creates stable, mean-variance efficient portfolios based on an investor's subjective views, which overcome the problem of input sensitivity. As the errors are generally spread along the vector of expected returns, such a portfolio also largely mitigates the problem of estimation error maximization. The market equilibrium returns, a starting point, are derived with a reverse optimization where the vector of implied excess equilibrium returns is calculated using historical data by equation (1) (Idzorek, 2007):

$$\Pi = \lambda \Sigma w_{market}, \quad (1)$$

where Π is the implied excess equilibrium return vector ($N \times 1$), N is the number of assets, λ is the risk aversion coefficient, Σ is the covariance matrix of excess returns ($N \times N$), and w_{market} are the market capitalization weights of the relevant asset universe ($N \times 1$).

The implied risk aversion coefficient λ can be estimated as follows (Grinold & Khan, 1999):

$$\lambda = \frac{E(r_m) - r_f}{\sigma_m^2}, \quad (2)$$

where $E(r_m)$ is the expected market total return, r_f is the risk-free rate, and σ_m^2 is the variance of the market excess returns.

The risk-aversion coefficient (λ) represents a trade-off between risk and the expected return. Hence, more risk-averse investors will require higher expected returns for a given level of risk than those who are risk-taking. In the reverse optimization procedure, this coefficient acts as a scaling factor for estimating the excess returns, as the weighted reverse optimized returns represent the specified market risk premium. Markowitz (1952) assumed a quadratic utility function for an average investor, describing the relationship between risk and return in the following way:

$$U = E[r_p] - \lambda \sigma^2 / 2 = w' \mu - \lambda w' \Sigma w / 2, \quad (3)$$

where $E[r_p]$ and σ^2 are the expected excess return of a portfolio and its variance, and μ is a vector of excess returns.

Equation (4) represents the unconstrained maximization problem.

$$\max_w w' \mu - \lambda w' \Sigma w / 2 \quad (4)$$

The solution is obtained by rearranging equation (1) and substituting vector Π with μ and is described in equation (5) (He & Litterman, 2002).

$$\delta U / \delta w = \mu - \lambda \Sigma w = 0 \xrightarrow{yields} w = (\lambda \Sigma)^{-1} \mu \quad (5)$$

1.1.2 The Black-Litterman master formula

The Black-Litterman master formula combines implied equilibrium returns with the view vector and calculates new expected returns as described in equation (6) (Idzorek, 2007):

$$E[r_{BL}] = [(\tau \Sigma)^{-1} + P' \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q], \quad (6)$$

where $E[r_{BL}]$ is the new combined expected return vector ($N \times 1$), τ is a scalar, Σ is the covariance matrix of excess returns ($N \times N$), P is a matrix specifying the related assets for which a specific view in the view vector is based on ($K \times N$), K is a number of views, Ω is a diagonal covariance matrix of error terms from the expressed views and represents the uncertainty of each view ($K \times K$), Π is the implied equilibrium return vector ($N \times 1$), and Q is the view vector ($K \times 1$).

1.1.3 Specifying the views and the Ω matrix

As mentioned, if a particular investor does now hold any specific views about the future price performance of certain assets, there is no reason to deviate from the global CAPM equilibrium implied returns. Conversely, subjective views can be incorporated into the master formula through the view vector Q . The model does not require that the views on all assets are specified; therefore, the length of a view vector can differ from the number of assets.

As the views are accompanied by uncertainty, it results in a random, unknown, independent, and normally distributed error term vector (ε), the mean of which is 0, and the covariance matrix is Ω , resulting in a view form as shown by equation (7).

$$Q + \varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix} \quad (7)$$

The error term (ε) is not directly incorporated into the master formula; however, it is indirectly applied in the covariance matrix (Ω) as a variance of each error term (ω), which is the absolute difference from the expected value of 0. The covariance matrix is diagonal because the model assumes the independence of the error terms from each other; hence, the non-diagonal elements of the matrix are 0. The uncertainty of the view is greater, the larger the variance of the error term (ω).

Determining the individual variances of the error terms is one of the most controversial and complicated aspects of the Black-Litterman model. Idzorek (2007) developed a variation of the model that allows specifying confidence in each view as a percentage from 0 to 100. For the scope of this master's thesis, I will use the most widely used method described in equation (8):

$$\Omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix} = \begin{bmatrix} p_1 \Sigma p_1' & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & p_k \Sigma p_k' \end{bmatrix}, \quad (8)$$

where p_k is the k -th row of the matrix P , which corresponds to the k -th view ($1 \times N$), and Σ is the covariance matrix of excess returns.

Matrix P ($K \times N$) matches the views with the specific assets that they relate to and is designed as shown in equation (9):

$$P = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{k,1} & \cdots & p_{k,n} \end{bmatrix}, \quad (9)$$

where $p_{k,n}$ relates to the n -th asset specified in the k -th view.

Another advantage of the Black-Litterman asset allocation model is that the subjective views can be expressed in absolute and relative terms (Allaj, 2013). Following is an example of a three-asset portfolio for which an investor has three views. The first row specifies that asset A's subjective expected return is 5%, and the second row specifies that the investor expects asset B's price to decline by 3%. As both views are expressed in an absolute term, the sum of the corresponding rows in the P matrix has to equal 1. The third row specifies that asset C will outperform asset B by 2%, and since this is a relative view, the sum of this row in the P matrix has to equal 0. Consequently, the model will overweight asset C and underweight asset B.

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, Q = \begin{bmatrix} 0.05 \\ -0.03 \\ 0.02 \end{bmatrix} \quad (10)$$

1.1.4 Parameter τ

The Black-Litterman model is a complex, weighted average of the implied equilibrium returns vector and the investor's subjective view vector. The relative weightings are depended upon a scalar τ and the uncertainty of the views. Specifying those two parameters appears to be the model's most abstract and challenging aspect. Generally speaking, if the confidence in the views is greater, the calculated new return vector will be closer to the specified subjective views. On the contrary, if an investor has low confidence in the views, the calculated new return vector will be closer to the implied equilibrium return vector. This aspect is expressed in the covariance matrix (Ω), which is inversely related to the confidence, meaning the diagonal error term ω of 0 corresponds to the 100% confidence in the related view. Idzorek (2007) even developed an extended version of the model's formulation, where an investor could specify the confidence in each view from 0% to 100%, making the model much more intuitive to the user. As Ω is usually mathematically determined, one can fine-tune the model through the scalar (τ). As opposed to the covariance matrix, this scalar is inversely proportional to the relative weight of the implied equilibrium return vector. The larger the τ , the larger the active risk (O'Toole, 2017). Regarding appropriately setting this scalar, the guidance in the literature cannot seem to reach a firm consensus. Black and Litterman (1992) stated that due to uncertainty in the mean being less than that in return, τ should be close to zero. Lee (2000), one of the most experienced practitioners, usually sets

τ between 0.01 and 0.05. After setting the target level of tracking error, the scalar can be backwardly calibrated. According to Meucci (2010), it can be compared to the uncertainty on the sample estimator and set as one divided by the number of samples. On the other hand, Satchell and Scowcroft (2000) prefer to set it to 1, which is a significant departure from Lee.

As the Black-Litterman model vastly depends upon the two mentioned factors, it is possible to fine-tune it. The scalar τ and individual variances of the diagonal error terms in the Ω matrix can be manually adjusted to reflect the wanted risk-return characteristics of the portfolio. One of the reasons I decided to make the Black-Litterman model a primary methodology is also explained in a study by Chen, Da & Schaumburg (2015). They stated that if the long-short portfolio strategy is effective, meaning if the views provide economic value, a lower uncertainty in the views and a higher τ will generate a higher Sharpe ratio. The results of their study confirmed this phenomenon.

1.2 The Treynor-Black asset allocation model

Another well-regarded active asset allocation model was developed by Treynor and Black (1973) aiming to maximize the Sharpe ratio by the mean-variance criterion. Like with the Black-Litterman model, a market index is assumed to be the default efficient or passive strategy. Suppose that some analysts cover a limited number of stocks by providing their assessed alphas. An assumption is made that the stocks that analysts are not covering are thought of as efficiently priced, and one cannot construct an efficient portfolio consisting only of the covered stocks. The goal is to construct an optimal portfolio that mixes the index, i.e., a passive part (P), and an active portfolio of the covered stocks (A). The weights of A and P that maximize the Sharpe ratio ($S_P = \bar{R}_P/\sigma_P$) are calculated by:

$$w_A = \frac{\bar{R}_A\sigma_M^2 - \bar{R}_M\text{Cov}(R_A, R_M)}{\bar{R}_A\sigma_M^2 + \bar{R}_M\sigma_A^2 - (\bar{R}_A + \bar{R}_M)\text{Cov}(R_A, R_M)}; \quad w_M = 1 - w_A, \quad (11)$$

where $\bar{R}_i$ is the expected excess return or $E(r) - r_f$, and r_f the risk free rate.

If the market model is assumed as its diagonal version, the following equations are relevant:

$$R_i = \alpha_i + \beta_i R_M + e_i; \quad E(e_i) = \text{Cov}(R_i, R_j) = 0; \quad \forall i \neq j; \quad \text{Var}(e_i) = \sigma_i^2, \quad (12)$$

where α_i is the abnormal expected return that is determined by the analysts covering the i -th stock, β_i is that stock's beta, e_i is its error term, and σ_i is its residual standard deviation. Combining equations (11) and (12) yields (Arisena, Noviyanti & Zanbar, 2018):

$$w_A = \frac{w_0}{1 + (1 - \beta_A)w_0}; \quad w_0 = \frac{\alpha_A/\sigma_A^2}{R_M/\sigma_M^2}, \quad (13)$$

where $\alpha_A = \sum_{i=1}^n w_i \alpha_i$ is the active alpha, $\beta_A = \sum_{i=1}^n w_i \beta_i$ is the active beta, n is the number of securities covered by analysts, and w_i is the weight of the i -th stock in an active portfolio.

Equation (13) can be interpreted as follows: the larger the systematic risk of the active portfolio, the less effective the diversification with the index would be; thus, w_A should be larger. The risky portfolio's (P) Sharpe ratio is given by equation (14).

$$S_P^2 = \frac{(w_A(\alpha_A + \beta_A R_M) + (1 - w_A)\bar{R}_M)^2}{w_A^2(\beta_A^2 \sigma_M^2 + \sigma_A^2) + (1 - w_A)^2 \sigma_M^2 + 2w_A(1 - w_A)\beta_A \sigma_M^2} = S_M^2 + \frac{\alpha_A^2}{\sigma_A^2} \quad (14)$$

The term α_A/σ_A is the appraisal ratio of the active portfolio and represents its contribution to the passive portfolio's Sharpe ratio. Treynor and Black determined that the appraisal ratio is maximized by setting the i -th stock's weight as:

$$w_i = \frac{\alpha_i/\sigma_i^2}{\sum_{j=1}^n \alpha_j/\sigma_j^2} \quad (15)$$

Combining equations (14) and (15) shows that the marginal contribution of the individual i -th stock to the squared Sharpe ratio of the risky portfolio is its squared appraisal ratio:

$$S_P^2 = S_M^2 + \frac{\alpha_A^2}{\sigma_A^2} = S_M^2 + \sum_{i=1}^n \frac{\alpha_i^2}{\sigma_i^2} \quad (16)$$

Therefore, the Treynor-Black model's performance is, like with the Black-Litterman, determined by the accuracy of the analysts' forecasts.

1.3 Prior research

There are numerous studies on the value of the analysts' recommendations. However, no firm consensus seems to have been reached yet. The same goes for combining these recommendations with the Black-Litterman model to form an investment strategy.

1.3.1 Previous studies on the value of analysts' recommendations

Jegadeesh, Kim, Krische & Lee (2004) found that favorably recommended stocks, on average, outperform the unfavorably recommended ones. On the other hand, they pointed out that sell-side analysts tend to recommend glamour stocks, which may affect the overall performance. Numerous studies also examined the market's reaction to publishing the analysts' recommendations. Aitken, Muthuswamy & Wong (2000) discovered that a sell recommendation has a permanent impact on a stock's price, and a buy recommendation has a positive impact at first, but a partial reversal of the price follows it. Kliger and Kudryavtsev (2013) found that market volatility can also affect the investors' reactions. On the other hand, Chan, Brown & Ho (2006) analyzed analysts' recommendations relevant to the Australian market and found that returns on initiating recommendations were significantly different from zero in the six months following the release. Studies from Boni and Womack (2006) and Barber, Lehavy, McNichols & Trueman (2001) concluded that most of the value

analysts' recommendations provide is due to the post-recommendation price drift and it lasts for a few months. Okada and Azuma (2013) analyzed an extensive database of Japanese analysts' rating information from 2000-2010 derived from Bloomberg. They found that the market reacts significantly to the information contained in analysts' ratings and more to the changes in rating rather than the rating itself. Furthermore, when constructing a dynamic calendar-time long-short equity portfolio based on the ratings, they concluded that investors could capture an abnormal profit with that strategy. After accounting for transaction costs, the investment strategy underperformed. Similarly, Azzi and Bird (2005) analyzed analysts' recommendations from 1994 to 2003. They found that a long position in stocks with a recommendation upgrade and a short position in stocks with a recommendation downgrade would have earned a significantly positive return over the six-month period. Barber, Lehavy, McNichols & Trueman (2001) also obtained similar findings. Green (2006) examined the value of timing the analysts' recommendations when investing. Early access to stock recommendations provided investment value, and abnormal returns were positive and economically meaningful even after adjusting for transaction costs.

Murg, Pachler & Zeitlberger (2014) analyzed a comparison of buy recommendations and target prices. They analyzed the analysts' recommendations for the Austrian market, particularly stocks listed in the Austrian Traded Index from 2000 to 2014. They found that analysts provide additional information regarding the revaluation process and can influence the investors' behavior. The conclusion was that the updates in target prices do not necessarily cause higher abnormal returns than the pure recommendation groups, which cover buy, hold, and sell recommendations; however, changes in these recommendation groups are usually followed by more significant average price impacts. The recommendation changes from sell to buy, or vice versa, produce the most substantial impacts. They also found that if a similar analysis occurred in a row, more significant abnormal returns usually occurred after the first one. Smaller firms and much higher than usual target prices were also accompanied by more significant abnormal returns.

On the contrary, Altinkilic, Balashov & Hansen (2013) dismissed the value of the analysts' recommendations as their intraday returns evidence showed that announcements of analysts' forecast revisions, on average, released little new information. Further cross-sectional evidence of returns around the announcements implied that the revisions were information free. Furthermore, they argue that analysts' forecasts are often issued after significant news reports about the relevant firm. Corbet, Dowling & Cummins (2015) showed that the market state can sometimes influence analysts' recommendations.

1.3.2 Studies on combining the Black-Litterman model and analysts' recommendations

A few studies have been performed on combining the analysts' recommendations with the Black-Litterman model. He, Grant & Fabre (2012) analyzed the analyst recommendations for the Australian stock index S&P/ASX 50 constituents. They found that a portfolio

consisting only of stocks with positive consensus recommendations performed better than one with negative consensus recommendations. Simultaneously, they integrated the consensus score recommendations as a view vector in the Black-Litterman asset allocation model in a way that the model overweighted stocks with good consensus scores and underweighted stocks with bad ones. A daily rebalanced portfolio significantly outperformed the benchmark in terms of raw return and risk-adjusted returns. After adjusting for the transaction costs, no outperformance was achieved. Reducing the rebalancing frequency to monthly rebalancing decreased the turnover and, consequently, the transaction costs incurred. However, the portfolio performance was also reduced, resulting in no outperformance of the benchmark. Arestad and Rahmqvist (2012) constructed the Black-Litterman portfolio from the 36 constituent stocks of the Nordea Sverigefond mutual fund as analyzed from 2008 to 2012. After implementing analysts' recommendations in the form of target prices as a view vector, they did not find any significant difference in the performance of their portfolio compared to the mutual fund. Their results did, however, indicate that the constrained model is more suitable than the non-constrained. The restricted Black-Litterman model also performed better than the mean-variance portfolio, which also resembled the Sharpe ratios and provided better diversification. Gertzell (2013) used the same methodology of target prices in his master's thesis. He analyzed the application of the Black-Litterman model on the OMXS30 index between February 1st, 2012, and January 31st, 2013, and utilized monthly rebalancing. He found that the restricted Black-Litterman portfolio outperformed its unrestricted counterpart, the mean-variance portfolios, and the index. However, no significant differences in daily returns compared to the index were detected, nor in the Sharpe ratios. The effects of transaction costs were not considered. In their study, Chen, Da & Schaumburg (2015) derived an equivalent Black-Litterman formula using the classical multivariate analysis. They implemented that formula with equity analysts' target price forecasts from 1999-2010. Their optimal portfolio outperformed the market, and the outperformance was robust across different periods and model parameter choices. Due to the study from Boni and Womack (2006), which showed that an investment strategy based on stock recommendation revisions within the same industry significantly improved the return when compared to a similar strategy that did not control for industries, they implemented relative views for stocks. The study did not account for transaction costs, and no constraint variations were applied. In relation to that, Da and Schaumburg (2011) found that relative comparison of target price implied returns within a specific industry could be very informative, while the absolute rankings are not.

Becker and Gürtler (2009) assessed the combination of the analysts' forecasts with the Black-Litterman model. They generated views based on a dividend discount model. Their analysts' forecasts integrated Black-Litterman portfolio outperformed all other investment strategies regarding the Sharpe ratio while being constrained and unconstrained. All their applications of the Black-Litterman model achieved good rankings in all performance measurements. They concluded that they would recommend the Black-Litterman model for investment purposes. Jia-long, Bo-wei & Min (2013) constructed and compared the single

index model, Fama-French three-factor model, and Carhart four-factor model for the largest 20 US stocks for a period from 2004 to 2012, the outputs of which were also utilized to construct the Black-Litterman portfolio. Although the four-factor model produced the highest arithmetic and geometric returns, the Black-Litterman portfolio performed best regarding the Sharpe ratio. Another way to construct the views vector is with multivariate models such as EGARCH-M (Duqi, Franci & Torluccio, 2014).

Despite it all, Cheung (2009) concluded that the Black-Litterman model does not have wide applications in real life yet due to a set of parameters, such as confidence in the views and τ , which influence the portfolio performance significantly, must be estimated.

2 METHODOLOGY AND DATA

This section provides a comprehensive overview of the historical simulation approach used to evaluate investment strategies. It outlines the data used, the construction of portfolios, and the statistical methods employed to analyze the simulation results.

2.1 Investment universe and benchmark selection

The investment universe of this thesis is represented by the current members of the Dow Jones Industrial Average Index (DJIA). This index, created in 1896 by Charles Dow and Edward Jones, is a widely followed and established stock market index that tracks the performance of 30 large U.S. companies that are publicly traded. It covers many industries, including technology, finance, energy, healthcare, and consumer goods. A property that makes it unique is that it is price-weighted as opposed to the indices such as S&P 500, which are market-capitalization-weighted. Therefore, a stock with the highest price also has the highest impact on the value of the index (Shoven & Sialm, 2000).

The benchmark for the Black-Litterman generated portfolios was manually constructed to represent a market capitalization-weighted DJIA proxy. It was obtained by setting all the elements of a P matrix to zero, which is the equivalent form of saying that an investor has no views. Therefore, according to the theory, it has no reason to deviate from the market equilibrium portfolio, and the Black-Litterman model sets weights equivalent to the market capitalization weights. The benchmark was rebalanced daily for optimal representation. The risk-free rate was presumed at 1%.

2.2 Data and historical time horizon

The data for this thesis was obtained through the Bloomberg Terminal, a powerful software system and data platform designed for finance professionals, such as investment managers, analysts, and traders (Moreale & Zaynutdinova, 2018). It provides real-time market data, news, and analytics, enabling users to monitor financial markets and analyze securities. It

offers a wide range of features, including stocks, bonds, currencies, and commodities data. It provides in-depth financial analysis, company information, and historical data. Users can also access the analyst forecasts and recommendations for each stock they cover.

The datasets used for performing the historical portfolio performance simulations were historical stock prices and corresponding market capitalizations, historical analyst consensus target prices, and historical analyst consensus scores for each member of the investment universe. The relevant period analyzed was from January 1st, 2013, to February 14th, 2023. Historical data for market capitalizations, analyst target price consensus, and analyst consensus scores had been obtained for this period, while the data for historical stock prices had been obtained from January 1st, 2011, to February 14th, 2023, as two-year rolling calculations of returns and covariances were used for the simulations, which is described more in detail in the following sub-chapters. There were 2745 upgrades and 2744 downgrades of the consensus scores, while there were 11815 upgrades and 7226 downgrades of the consensus target price. On average, a stock's consensus score was changed every 14.78 trading days, and a target price consensus was changed every 4.17 trading days.

2.2.1 Analyst target price consensus data

Bloomberg compiles twelve-month-target price estimates from multiple analysts covering a stock to calculate a consensus target price, a simple average of all inputs. Doing so provides a more balanced view of a stock's potential value, as it averages out individual biases and variations in methodology. The cutoff point for the target prices used to derive the consensus price is three months, meaning that individual analyst target prices submitted more than three months ago are omitted. This way, the consensus target price is kept up-to-date. As the Black-Litterman model's view vector requires specified expected returns as an input, target price implied returns were calculated in the following way using logarithmic returns:

$$\text{Target price implied return}_{i,t} = R_{TP_{i,t}} = \ln\left(\frac{TP_{i,t}}{P_{i,t}}\right), \quad (17)$$

where $R_{TP_{i,t}}$ is the target price implied return for the i -th stock on the t -th day, $TP_{i,t}$ is the consensus 12-month target price on the t -th day, and $P_{i,t}$ is the closing price.

2.2.2 Analyst consensus scores data

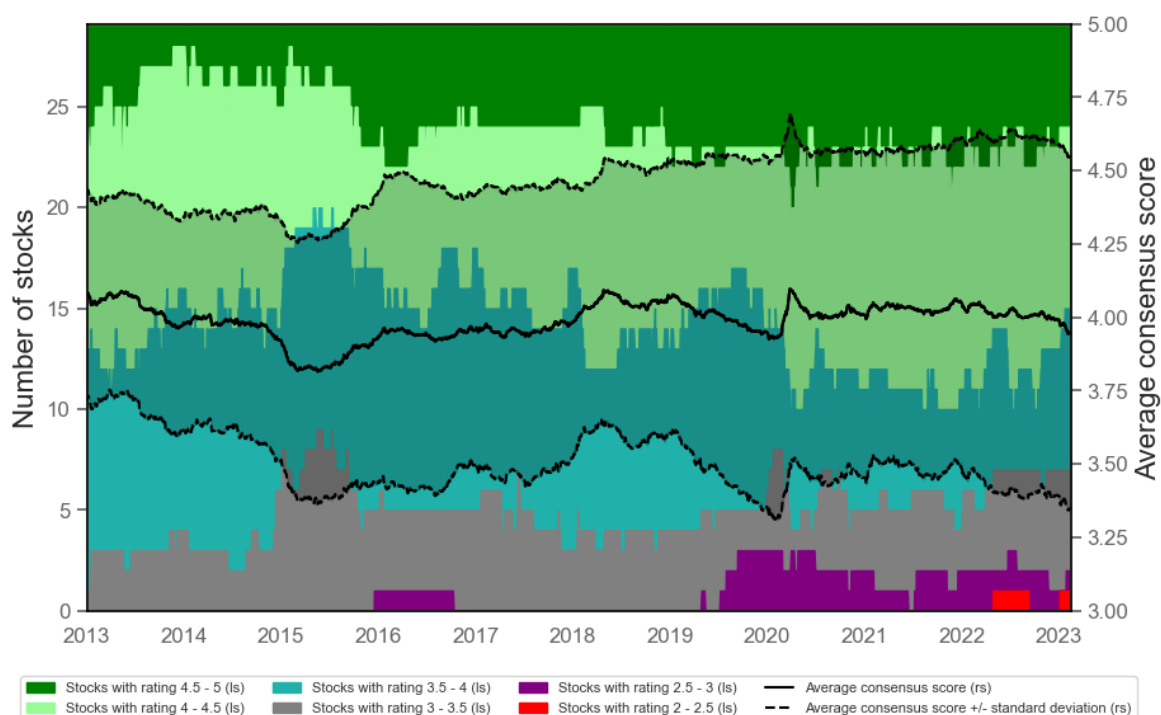
Bloomberg gathers recommendations from various research analysts covering a particular stock. These recommendations typically fall into categories such as 'Buy,' 'Hold,' and 'Sell,' or variations like 'Strong Buy,' 'Outperform,' 'Neutral,' 'Underperform,' and 'Strong Sell.' Each recommendation resembles an analyst's opinion on the stock's potential performance relative to the broader market or industry peers. To make the ratings comparable, Bloomberg standardizes them by converting them into numerical scores by the following criteria:

- Recommendations such as 'Buy/Strong Buy/Outperform': 5
- Recommendations such as 'Hold/Neutral': 3
- Recommendations such as 'Sell/Strong Sell/Underperform': 1

Analyst consensus scores are a simple average of all the analyst scores submitted during the last twelve months. These scores summarize the overall analyst sentiment for a given stock and help investors gauge the prevailing market opinion on its potential performance. The higher the score, the more positive the analyst sentiment, and vice versa.

Figure 2 displays the historical distribution of consensus scores (left axis) and the average consensus score (right axis). It can be observed that there is indeed a systematic overconfidence bias for sell-side analysts' recommendations, which is in accordance with Lin (2005) and Cowen, Groyberg & Healy (2006). The average consensus score for the whole investment universe stands around 4.01 for the whole 10-year period while oscillating between 4.09 and 3.82. In the case of normally distributed scores that range from 1 to 5, the mean should be 3.00. The fact that DJIA members are big successful companies partly justifies higher scores.

Figure 2: The distribution of consensus scores throughout the period 1/1/2013 - 14/2/2023



Source: Own work.

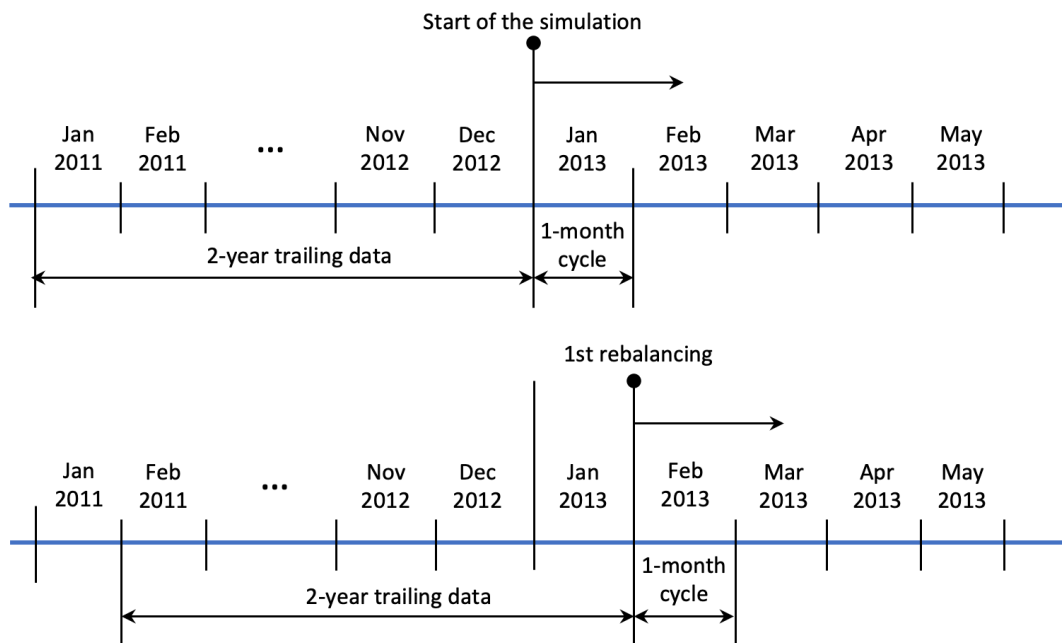
Overall, the average number of analysts covering a stock during the observed period, whose recommendations were revised in the previous 12 months, was 32.36. The highest amount, 34.83, was detected at the end of 2013, and the lowest, 29.97, was detected right at the beginning of the Covid pandemic at the start of 2020. With at least 29.97 analysts covering

a stock each trading day during the observed period, it can be concluded that the consensus recommendations retrieved from Bloomberg are representative.

2.3 Historical simulation construction

For the construction of the simulations, I used logarithmic returns over the normal returns because the former are more stable and less sensitive to outliers, which can help produce more reliable estimates of the covariance matrix and more accurate estimates of the expected returns. The objective was to form and test two trading strategies. One was formed by implementing target price implied returns as a view vector, and the other was a trading strategy formed with analyst consensus scores. Both strategies utilize two-year trailing datasets for calculating inputs required at every portfolio rebalancing, such as average historical returns of stocks, covariance matrices, betas, average historical market returns, and market variances. An example of the trailing datasets used for the first rebalancing of a trading strategy with monthly rebalancings is shown in Figure 3.

Figure 3: Example of trailing datasets used for portfolio rebalancings



Source: Own work.

Figure 4 displays the market-implied risk aversion coefficient calculated during the simulations according to equation (2) with a two-year rolling window of inputted data. The coefficient was volatile during the investment horizon, ranging from 1.35 at the beginning of 2023 to 14.11 at the end of 2014. The most significant change occurred during the onset of the Covid pandemic in March of 2020. The two-year market-implied risk aversion coefficient dropped from 9.06 to as low as 1.85 due to decreased returns and increased volatility.

Figure 4: Market implied risk aversion coefficient calculated with 2-year rolling data



Source: Own work.

2.3.1 Trading strategy 1: Using analyst consensus target prices

The first trading strategy was based on indirectly implementing analyst target price consensus in a view vector for each stock in the investment universe. This way, target price implied returns had to be specified for every stock. Increasing the parameter tau (τ) puts more weight on the view vector versus the market equilibrium returns vector. After performing the sensitivity analysis for the input parameter τ , a goal was to determine the trend of average portfolio returns and Sharpe ratios, thus, assessing the economic value of the consensus target prices formed by the sell-side analysts.

Each row in the constructed P matrix stated the corresponding element in the view vector in absolute terms, resulting in the diagonal matrix. An example of the P matrix with views expressed in absolute terms and the view vector Q with target price implied returns on a t -th day for n portfolio members is shown in equation (18).

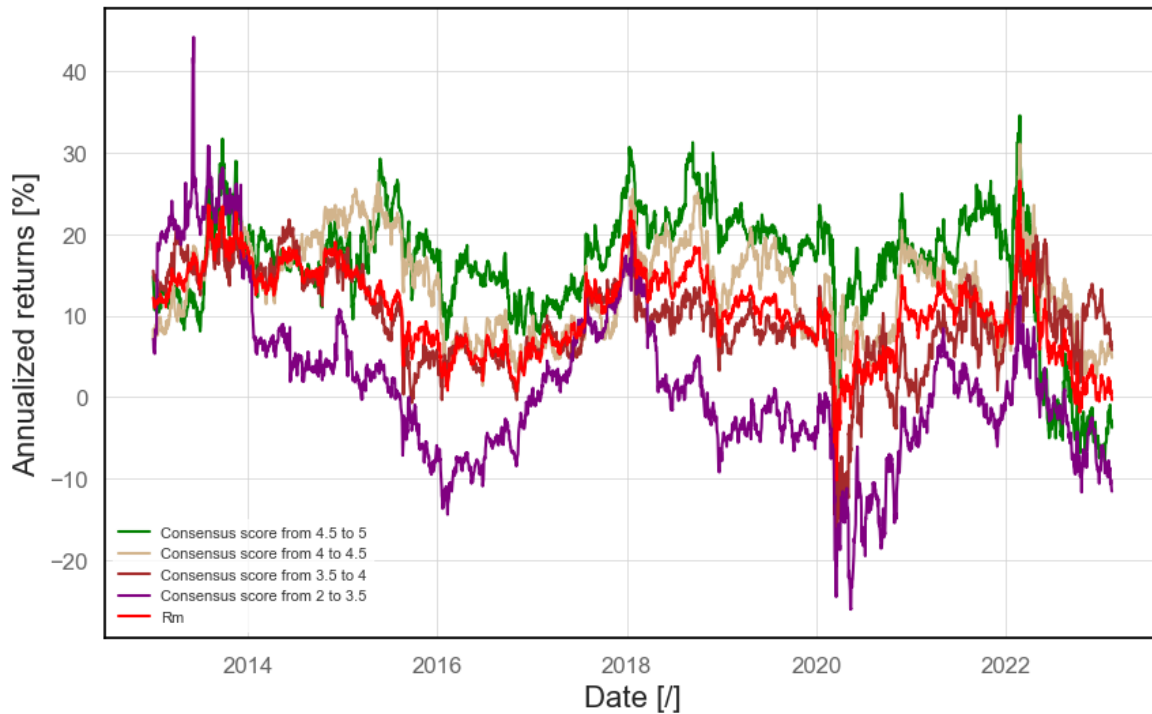
$$P_t = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}_{[n \times n]}, Q_t = \begin{bmatrix} R_{TP_{1,t}} \\ R_{TP_{2,t}} \\ \vdots \\ R_{TP_{n-1,t}} \\ R_{TP_{n,t}} \end{bmatrix}_{[n \times 1]} \quad (18)$$

2.3.2 Trading strategy 2: Using analyst consensus scores

In contrast with Strategy 1, we cannot directly calculate implied returns from analysts' recommendations. To quantify analysts' stock purchase recommendations and integrate them into the Black-Litterman model, I chose a strategy inspired by the ones formed by Drobetz (2001) and He, Grant & Fabre (2012). To convert the analysts' consensus scores into relative views suitable for the view vector, we must establish the proportionate outperformance or underperformance of stocks with different consensus scores relative to the market, in this case, the DJIA index. I segmentized the investment universe each trading day into four groups: stocks with a consensus score from 4.5 to 5 (g_1), stocks with a consensus score from 4 to 4.5 (g_2), stocks with a consensus score from 3.5 to 4 (g_3), and stocks with a consensus score from 2 to 3.5 (g_4). Each trading day trailing two-year average returns were calculated for each group of stocks to estimate the relative performance against the market. This relative performance was then incorporated each trading day for all the members of each group in a view vector and acted as a subjective expectation for each stock's return.

For example, suppose we want to rebalance the Black-Litterman portfolio on 2/5/2015. First, stocks must be segmentized into four groups with respect to their consensus scores on that day. Average returns for each group are calculated from 1/5/2013 until 1/5/2015, and relative outperformance (underperformance) against the market of each group is assigned to the stocks that are their corresponding members on 2/5/2015.

Figure 5: Annualized realized returns over 2-year rolling estimation periods for groups of stocks with different consensus scores



Source: Own work.

Annualized realized returns over two-year rolling estimation windows for each of these four groups are shown in Figure 5. The average performances of these groups are aligned with logical expectations and are significant. Over the ten-year window, the average annualized market (DJIA) return was $\bar{R}_m = 9,19\%$. On average, stocks with consensus scores of 4.5 – 5 outperformed the market by 6,02 percentage points ($\bar{R}_{g_1} = 15,21\%$), stocks with consensus scores of 4 – 4.5 outperformed the market by 2,07 percentage points ($\bar{R}_{g_2} = 11,26\%$), while stocks with consensus scores of 3.5 – 4 underperformed the market by -1,89 percentage points ($\bar{R}_{g_3} = 7,30\%$). Stocks with the lowest consensus scores, 2 – 3.5, performed the worst. On average, this group of stocks underperformed the market by a staggering -7.31 percentage points ($\bar{R}_{g_4} = 1,88\%$). It is also important to note that the members of these groups were not constant throughout the whole period as analysts frequently updated their recommendations; therefore, some stocks migrated from one group to another, which was taken into account.

This result is in accordance with findings by Jegadeesh, Kim, Krische & Lee (2004) and He, Grant & Fabre (2012), who found that, on average, favorably recommended stocks outperform unfavorably recommended ones. These observations indicate that the Black-Litterman portfolio should, on average, overweight members of g_1 and g_2 , while underweight members of g_3 and g_4 . However, there were occasions when a group of stocks with a lower consensus score momentarily outperformed a group with a larger consensus score. It can be observed in 2013, where g_4 momentarily outperformed other groups, in 2014-2016, where g_2 momentarily outperformed g_1 , and especially from 1/1/2022 onwards, where g_1 members underperformed the market. The period from 1/1/2022 onwards was accompanied by substantial economic uncertainty due to rising inflation which required interest rate hikes by central banks and various geopolitical tensions. One reason for the underperformance of g_1 members could be that analysts prefer to assign better recommendations to growth stocks, which exhibit larger potential for future development (Jegadeesh, Kim, Krische & Lee, 2004 & Moshirian, Ng & Wu, 2009). Also, during that period, stocks from g_3 performed the best.

I utilized the relative performance of groups and their corresponding members to create an optimized portfolio that integrated long-term abnormal returns as a view vector within the Black-Litterman model. The assumption was that the average outperformance and underperformance of groups and their corresponding members, compared to the market, would continue throughout the subsequent rebalancing period. Views for the investment universe on t -th day were stated in the following way:

- each member of g_1 will outperform (underperform) the market by $Q_{1,t}$ basis points,
- each member of g_2 will outperform (underperform) the market by $Q_{2,t}$ basis points,
- each member of g_3 will outperform (underperform) the market by $Q_{3,t}$ basis points,
- each member of g_4 will outperform (underperform) the market by $Q_{4,t}$ basis points.

Elements $Q_{i,t}$ represent the relative performance of the i -th group against the market on the t -th day, calculated by the two-year trailing data up to the t -th day. Constructed view vector Q_t , along with the Ω_t matrix, is shown in equation (19):

$$Q_t = \begin{bmatrix} Q_{1,t} \\ Q_{2,t} \\ Q_{3,t} \\ Q_{4,t} \end{bmatrix} = \begin{bmatrix} R_{g1,t} - R_{m,t} \\ R_{g2,t} - R_{m,t} \\ R_{g3,t} - R_{m,t} \\ R_{g4,t} - R_{m,t} \end{bmatrix}, \Omega_t = \begin{bmatrix} \omega_{1,t} & 0 & 0 & 0 \\ 0 & \omega_{2,t} & 0 & 0 \\ 0 & 0 & \omega_{3,t} & 0 \\ 0 & 0 & 0 & \omega_{4,t} \end{bmatrix}, \quad (19)$$

where $R_{g1,t}, R_{g2,t}, R_{g3,t}, R_{g4,t}$ are the average trailing two-year returns of each group on the t -th day, $R_{m,t}$ is the average trailing two-year market return on the t -th day, $\omega_{i,t}$ is a diagonal element of Ω_t calculated as $p_{i,t}\Sigma p_{i,t}'$, p_i is the i -th row of the P_t matrix, and i ranges from 1 to 4, which is the number of different groups.

P_t matrix was constructed the following way each t -th day. Suppose that on this day, stocks 1 and 2 are members of g_1 , stocks 3 and 4 are members of g_2 , stock 5 is a member of g_3 , and stock 6 is a member of g_4 . For active members of a specific group, their weight in a corresponding row is equal to the percentage that their market capitalization weight represents of the total market capitalization of their group. In row 1, it is specified that members of g_1 will outperform the market by $Q_{1,t}$ basis points. $W_{g1,t}$ is derived as the market capitalization of stock 1 on the t -th day divided by the sum of the market capitalizations of all the g_1 members on the t -th day, in this case, stocks 1 and 2. As the views are stated in relative terms, the sum of each row must equal 0. As we compare the performance of each group member against the market, each member of the row gets assigned a negative value $W_{m,t}$, which is its market capitalization weight, i.e., its market capitalization divided by the sum of market capitalizations of all DJIA members. The example of the first three rows of P_t matrix construction is shown in equation (20).

$$P_t = \begin{bmatrix} W_{g1,t} - W_{m1,t} & W_{g2,t} - W_{m2,t} & -W_{m3,t} & -W_{m4,t} & -W_{m5,t} & \dots & -W_{m30,t} \\ -W_{m1,t} & -W_{m2,t} & W_{g3,t} - W_{m3,t} & W_{g4,t} - W_{m4,t} & -W_{m5,t} & \dots & -W_{m30,t} \\ -W_{m1,t} & -W_{m2,t} & -W_{m3,t} & -W_{m4,t} & W_{g5,t} - W_{m5,t} & \dots & -W_{m30,t} \\ -W_{m1,t} & -W_{m2,t} & -W_{m3,t} & -W_{m4,t} & -W_{m5,t} & \dots & -W_{m30,t} \end{bmatrix} \quad (20)$$

As we cannot quantify the confidence in these views, the uncertainty matrix consisted of diagonal elements, which were variances of the normally distributed error terms, ε , with the assumed mean of 0. Each diagonal element was calculated on the t -th day as $p_{i,t}\Sigma p_{i,t}'$, where $p_{i,t}$ is the i -th row of the P_t matrix on the t -th day and $i \in (1,2,3,4)$.

The main reason for not specifying the confidence is that, besides being very hard to assess, by specifying it as a variance of the error term, the confidence is correlated to the market's volatility. In periods with high price volatility, the Black-Litterman model would assign a smaller weight to the views and depart from market weights less. In periods with low volatility, the Q_t diagonal elements will be closer to zero, implying lower variance and higher confidence in the views. Consequently, the Black-Litterman model will deviate from

the market weights more and give a larger relative weight to the view vector as opposed to the market-implied returns vector.

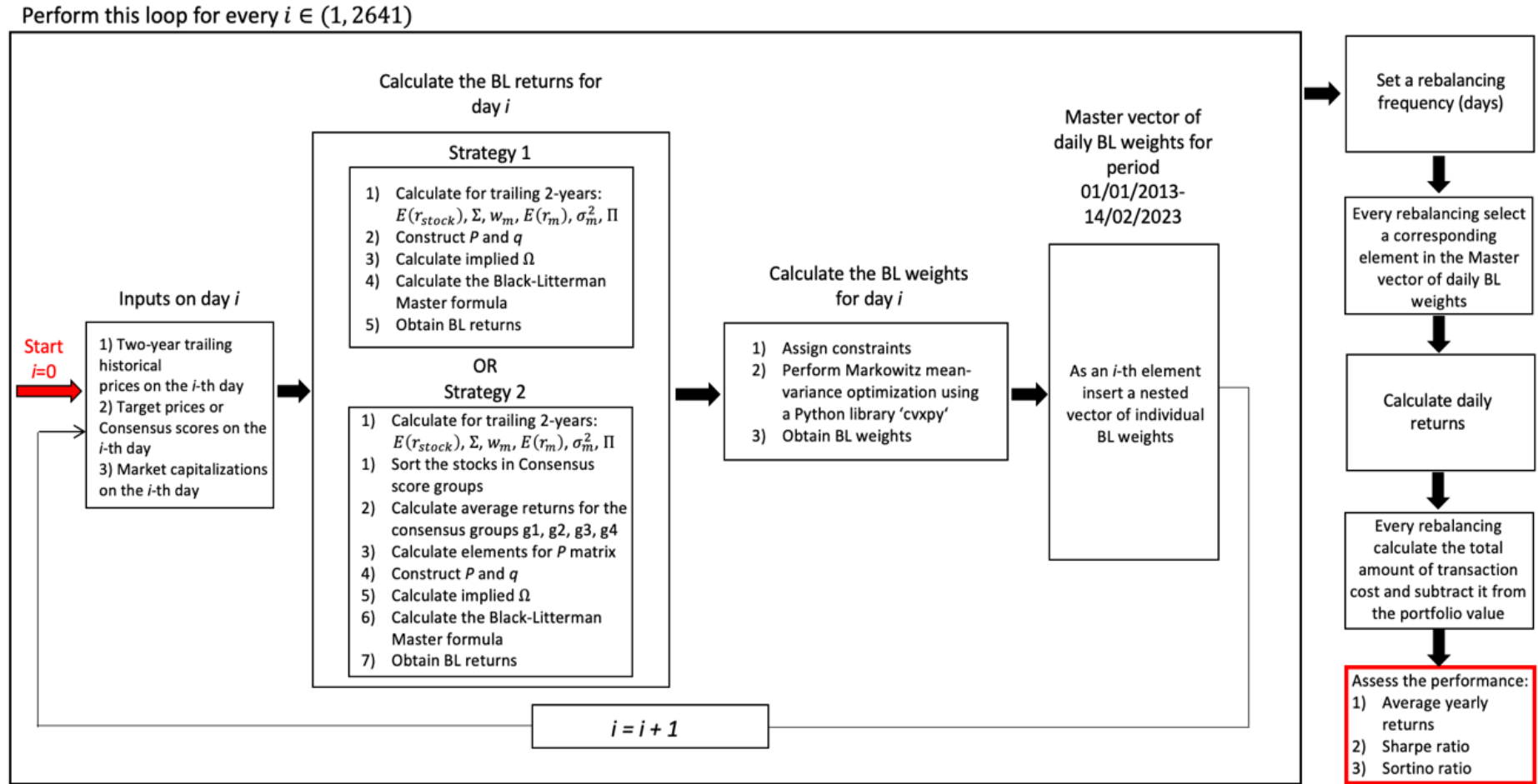
A scheme describing every step in the automated portfolio construction for both strategies using the Black-Litterman model is shown in Figure 6. The simulated period (1/1/2013-14/2/2023) was divided into 2641 trading days. As I wanted to derive the relationship between the portfolios' Sharpe ratios, mean returns, and the rebalancing period while also assessing the effect of τ , I automated the whole portfolio construction in a way that it would calculate the corresponding Black-Litterman weights for every single trading day. By doing so, I could select a desired rebalancing period and looped through the master vector of daily Black-Litterman weights, selecting the vector element that corresponded to the day on which a specific rebalancing occurred. These weights were then held constant until the next rebalancing act, and so on. Doing so allowed me to analyze the volatility of the Sharpe ratio for multiple values of τ in relation to rebalancing frequency, ranging from 1 to 250 trading days (one calendar year).

This kind of analysis was not found in the literature review; therefore, I intended to get further insight into the practical properties of the Black-Litterman model while having integrated analysts' recommendations. The largest square shape in Figure 6 represents a 'for loop,' which had looped through every trading day, ranging from 1 to 2641. Ultimately, I obtained a master vector of daily Black-Litterman calculated weights.

On the i -th day, I assigned every individual stock's trailing 2-year returns, target prices or consensus scores on the $(i-1)$ -th day, and market capitalizations on the i -th day as input data. The next step calculated the Black-Litterman expected returns for either Strategy 1 or Strategy 2, as described earlier. After obtaining the new expected returns, I performed a Markowitz mean-variance optimization using a Python library, 'cvxpy,' where I assigned a desired batch of constraints and maximized equation (4). The next step appended the calculated vector of individual weights to the master vector. Then the index i was increased by one, and the whole procedure was repeated until ' i ' equaled 2641. After that, the loop exited, and the master vector was constructed. Next, I could specify a desired rebalancing period and iterate through the master vector, picking only the corresponding daily weights relevant to each rebalancing act. These corresponding weights were then held constant until the next rebalancing act, and so on.

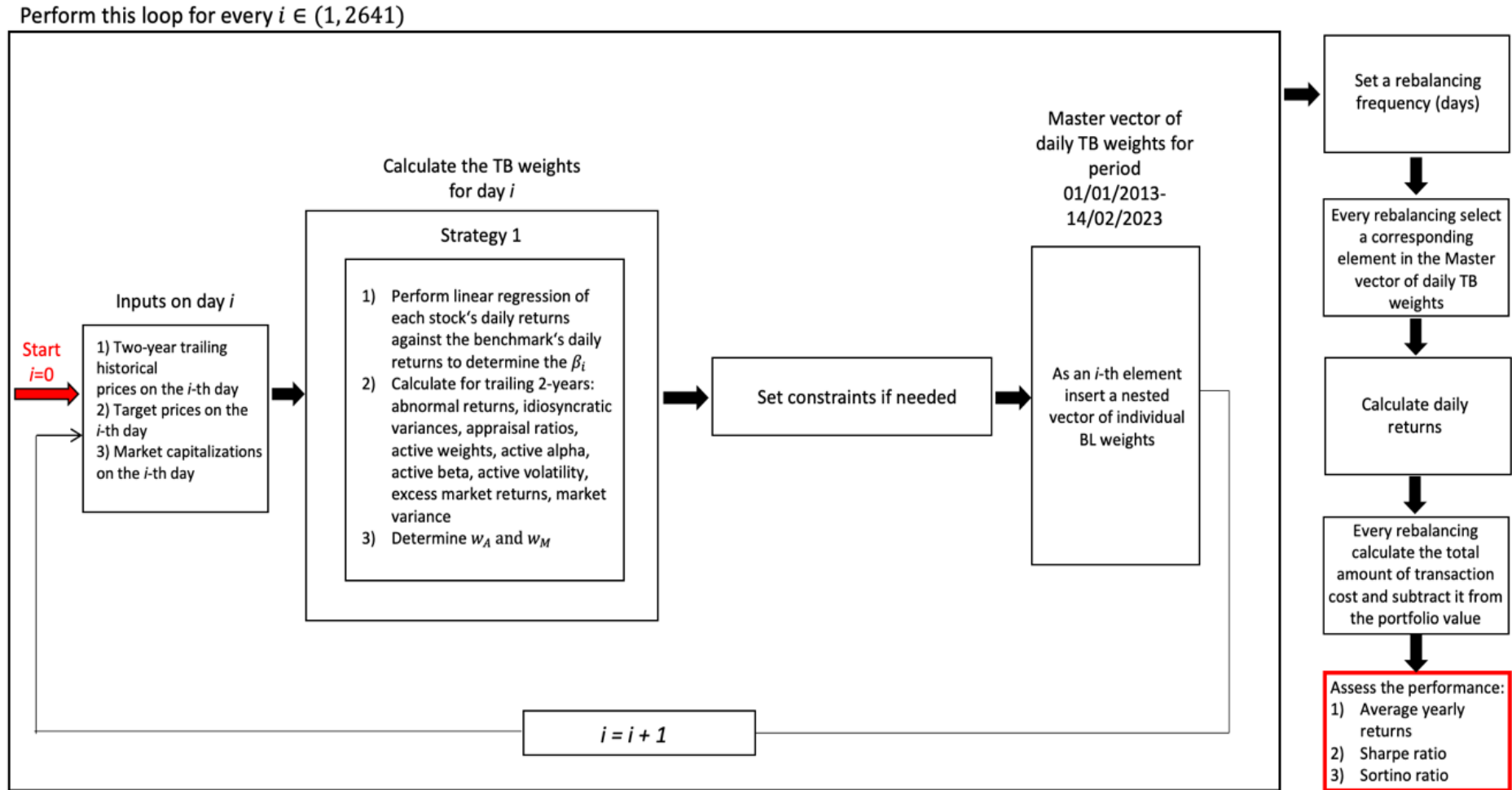
Doing so allowed me also to automate the rebalancing period specification when analyzing its relation to the Sharpe ratio, which ranged from 1 to 250 trading days. After setting a desired rebalancing period, daily portfolio returns were calculated. The total amount of stocks bought and sold was calculated during each rebalancing, and the transaction costs were subtracted from the whole portfolio value. For every generated portfolio, the final step included the performance assessment by calculating the mean yearly returns, Sharpe ratio, and Sortino ratio.

Figure 6: Scheme of automated BL portfolio construction for every day (2013-2023)



Source: Own work.

Figure 7: Scheme of automated TB portfolio construction for every day (2013-2023)



Source: Own work.

A similar procedure was applied to constructing the portfolios based on the Treynor-Black model (Figure 7). All steps were the same except for the derivation of weights and setting constraints. To calculate the individual daily weights, a linear regression of each stock's daily returns against the benchmark's daily returns was performed to obtain the β_i on the two-year trailing data. Doing so allowed me to calculate abnormal returns, idiosyncratic variances, appraisal ratios, active weights, active alpha, active beta, market returns, and market volatility. In the end, w_A and w_M could be derived.

According to equation (15), the formula for calculating the individual weights is itself a form that maximizes the appraisal ratio. As a result, the constraints were not inputted directly in the mean-variance optimization procedure. The constraints were applied to the already calculated individual weights in the same manner as with the Black-Litterman model. No short-selling was allowed, and the maximum value of the individual weight in an active portfolio was 40% of the active portfolio value. Additional constraints were applied to the w_A and w_M stating that the value had to be between 0 and 1. The individual weights were then normalized for their sum to equal one. Other than that, the procedure was the same as with the Black-Litterman model.

2.4 Introducing constraints

The simulated portfolios were unconstrained as a starting point, meaning that short-selling and 'wild' individual weights were allowed. That way, the absolute economic value of sell-side analysts' recommendations could be assessed. The only constraint the Black-Litterman model requires in the mean-variance optimization process is that the sum of all individual weights equals one. However, short-selling and wild portfolio weights are not feasible for an everyday investor and an average mutual fund asset manager.

I introduced two additional constraints to test the economic value of analysts' recommendations for investors, who must meet specific regulatory requirements, ethical considerations, or risk management preferences, and separately measured newly formed portfolio performances. First, I introduced a no short-selling constraint, which specified that all the time, all individual weights had to be equal or larger than zero. Another constraint I introduced was meant to avoid wild individual weights or extreme portfolio concentration in a few stocks while not including the whole investment universe, consequently not diversifying enough. Therefore, all individual weights had to be equal to or smaller than 0.4 or 40% of the portfolio value. Equation (21) specifies these two constraints:

$$\text{Constraint 1: } w_i \geq 0; \text{ Constraint 2: } w_i \leq 0.4 \quad (21)$$

Introducing a no short-selling constraint also helps maintain a focus on long-term investment strategies, which may be better aligned with some investors' investment objectives. It is important to note that when comparing the performance of a constrained portfolio to an unconstrained one, the constrained portfolio may exhibit lower volatility and reduced

potential for extreme losses. However, it also limits the potential for higher returns, as short-selling can provide additional opportunities for gains when short-sold stock's price declines.

Ultimately, choosing between a constrained and unconstrained portfolio depends on the investor's risk tolerance, investment horizon, and overall objectives. Constrained portfolios are the primary investment sub-strategy that I had focused on as the majority of investors aspire to invest long-term and do not want to take unnecessary risks that occur when a portfolio is overly concentrated, or short-selling is introduced. The latter allows one to obtain funds received by selling a borrowed stock and investing them into the long portion of the portfolio, thus, increasing the possibility for even more extreme individual weights. The more extreme the values of individual weights are, the higher the chances for a significant profit as well as a significant loss.

2.5 Performance evaluation

The performance of the Black-Litterman and Treynor-Black portfolios based on analyst recommendations has been measured in three different ways. I wanted to measure the overall mean return and risk-adjusted return to assess different kinds of portfolio performance. The performance was therefore assessed as a mean market-adjusted or benchmark-adjusted rate of return, Sharpe ratio, and Sortino ratio.

2.5.1 Benchmark-adjusted rate of return

The market was represented by the DJIA index. A benchmark represented a Black-Litterman portfolio with no views, which served as a market-capitalization-weighted DJIA proxy. The market-adjusted or, in this case, the benchmark-adjusted average rate of return was calculated by equation (22):

$$\text{Benchmark adjusted average rate of return} = \bar{R}_{p,T} - \bar{R}_{B,T}, \quad (22)$$

where $\bar{R}_{p,T}$ is an average rate of return of the portfolio during the certain period T and $\bar{R}_{B,T}$ is an average rate of return of the benchmark in the same period.

The average rate of return of the portfolio during the certain period T was calculated by equation (23) while using logarithmic daily returns:

$$R_{p,T} = \frac{\sum_{t=1}^T (\sum_{i=1}^N w_{i,t-1} r_{i,t})}{N}, \quad (23)$$

where $w_{i,t-1}$ is the i -th stock's weight assigned on day $t - 1$, $r_{i,t}$ is the i -th stock daily return on day t , N is the number of the stocks included in the portfolio, and T is the number of trading days during the observed period.

2.5.2 Sharpe ratio

A crucial aspect of portfolio performance is captured by the Sharpe ratio, developed by William F. Sharpe. It is a widely used financial metric that measures the risk-adjusted performance of a portfolio and can be calculated by dividing the excess return by the portfolio's standard deviation, which represents its volatility or risk (Sharpe, 1964). The standard deviation accounts for positive and negative deviations from the average return. The higher the Sharpe Ratio, the better the risk-adjusted performance of the investment. The calculation of the Sharpe ratio is shown in equation (24):

$$\text{Sharpe ratio} = \frac{\bar{R}_{p,T} - r_{f,T}}{\sigma_{p,T}}, \quad (24)$$

where $\bar{R}_{p,T}$ is the portfolio's average rate of return during the observed period T , $r_{f,T}$ is the risk-free interest rate during the observed period T , and $\sigma_{p,T}$ is the portfolio's standard deviation during the observed period T .

The Sharpe ratio has been the primary measure of portfolio performance. The main reason is that I wanted to compare the unconstrained portfolios to the constrained ones, where short-selling was not allowed. Doing so, the Sortino ratio would have provided an inadequate assessment of the performance. If both portfolios, in comparison, would have had a long-only trading strategy, then the Sortino ratio would be superior to Sharpe as there would only be a downside risk. However, as the unconstrained portfolio can include short positions, the risk for such positions occurs when the asset price increases. For such a portfolio, the Sharpe ratio is a much superior risk assessment as it includes upside and downside volatility.

2.5.3 Sortino ratio

Another financial metric that measures the risk-adjusted performance of an investment or portfolio is the Sortino ratio. It is very similar to the Sharpe Ratio; however, it focuses specifically on the downside risk, thus, making it more relevant for investors primarily concerned with the potential for losses.

It is calculated by dividing the excess return of an investment by the investment's downside deviation, which represents the volatility of negative returns (Sortino & Price, 1994). It is important to note that since it only accounts for the downside volatility, the Sortino ratio usually produces higher values than Sharpe for the same portfolio. It is calculated by equation (25):

$$\text{Sortino ratio} = \frac{\bar{R}_{p,T} - r_{f,T}}{\text{Downside risk}_{p,T}}, \quad (25)$$

where $\bar{R}_{p,T}$ is the portfolio's average rate of return during the observed period T , $r_{f,T}$ is the risk-free interest rate during the observed period T , and $Downside\ risk_{p,T}$ is the downside risk factor of the portfolio during period T calculated by equation (26).

$$Downside\ risk_{p,T} = \sqrt{\frac{1}{N} \sum_{n=1}^N (R_{p,n} - \bar{R}_{p,T})^2 I(R_{p,n} \leq \bar{R}_{p,T})} \quad (26)$$

$\bar{R}_{p,T}$ is the portfolio's average rate of return during the observed period T , $R_{p,n}$ is the portfolio's daily return on day n , N is the number of days during the period T , and I is the indicator function.

2.6 Transaction costs

Another crucial aspect for investors seeking to optimize their portfolio performance is the incorporation of transaction fees during portfolio rebalancing. Transaction fees, which include brokerage commissions, and other related costs, can significantly impact the portfolio's performance. These costs may completely erode gains, particularly in cases where the rebalancing strategy involves frequent buying or selling of stocks. Adopting a cost-efficient rebalancing approach can ultimately enhance the portfolio's long-term performance and help investors achieve their financial goals.

To mitigate the adverse effects of transaction fees, I carefully evaluated the incurred costs in relation to the rebalancing frequency and considered the trade-off between possible advantages of frequent rebalancing and the corresponding increase in transaction costs. Another aspect that I assessed is the relationship between the parameter τ and the transaction costs; all else held equal. If increasing the parameter τ increased inter-daily portfolio weights volatility, this would substantially affect the portfolio's performance as it would increase the transaction costs.

The type of transaction costs that I modeled is proportional, meaning that on each rebalancing day, the total amount of stocks sold has been calculated, and a specified percentage of this amount has been subtracted from the whole portfolio value. It is important to note that usually, an investor can only sell or buy a whole stock or multiple stocks; however, in this case, I assumed that the calculated amount sold could be sold or bought in whole without adjusting it to be a multiple of the closing price of the stock.

Transaction fees while purchasing stocks were also taken into account. As the whole investment universe has most of the time formed the portfolio, and the sum of all weights had to equal 1, the amount sold was invested in different stocks, meaning that the amount bought equaled the amount sold. Additionally, I assumed the same transaction fee amount for buying and selling the stock. Overall, 1% seems to be the most frequently used

transaction fee by brokers in the real world, and it was the primary value used in the simulations.

3 RESULTS AND DISCUSSION

In this chapter, I will present and discuss the results of the comparative analysis between the Black-Litterman portfolios, formed by different investment strategies using sell-side analysts' recommendations, the Treynor-Black portfolio, and the designated benchmark. The primary objective of this study was to evaluate the performance of the Black-Litterman model equipped with analysts' recommendations in generating optimized portfolio allocations and, consequently, to determine whether it could outperform the benchmark in terms of mean return and risk-adjusted return. The simulation was conducted using historical data from 1/1/2013 to 14/2/2023 and is assessed by various performance metrics. It allowed me to draw insightful conclusions regarding the effectiveness of using sell-side analysts' recommendations and the Black-Litterman model in portfolio management. Throughout this chapter, I will delve into the findings of the analysis, highlighting the areas where the Black-Litterman portfolio outperformed the benchmark while discussing the possible explanations behind its superior performance. I will also consider the limitations of this study and identify potential areas for future research.

The results of the simulations with daily rebalancings are shown in Table 1. Except for the Treynor-Black model, all variations of the two formed trading strategies outperform the benchmark in terms of annualized mean return. Despite that, the same does not hold for the risk-adjusted measures.

Most variations produced larger Sharpe and Sortino ratios than the respective benchmark—however, Treynor-Black portfolios and some of the variations within Strategy 2 underperformed (unconstrained portfolios with $\tau = 0.25$, $\tau = 0.5$, and $\tau = 1$). The main reason is the enlarged volatility of daily returns which is, at least for the Black-Litterman model, the consequence of the larger scaling parameter τ . After performing two-sampled t-tests, the results showed that only the mean returns of unconstrained Strategy 1 portfolios with $\tau = 0.5$ and $\tau = 1$ were statistically significant with a p-value of less than 5% and 1%, respectively. After accounting for the transaction costs of 1%, the analysis concluded that the initial outperformance was diminished with daily rebalancing. None of the portfolios outperformed the benchmark, not in terms of mean benchmark-adjusted nor risk-adjusted measure. This is, to some extent, in accordance with He et al. (2012) and Gertzell (2013). The former found that the Black-Litterman portfolio with analysts' recommendations tends to significantly outperform the respective benchmark with daily rebalancing; however, no positive statistical significance is achieved after transaction costs. The latter found that the Black-Litterman portfolio tends to outperform the benchmark; however, no statistically significant differences in mean returns were found.

Table 1: Results of historical simulations (daily rebalancing)

Portfolio	Constraint	Yearly mean return	Yearly mean benchmark-adjusted return	Sharpe ratio	Sortino ratio	Yearly turnover	Yearly transaction costs	Yearly return after transaction costs	Yearly benchmark-adjusted return after transaction costs	Sharpe after TC	Sortino ratio after TC
Strategy 1: BL, $\tau = 0.05$	Non-constr.	13.118%	+3.624% (0.451)	0.676	0.963	478%	9.579%	3.539%	-5.955% (-0.563)	0.267	0.369
	Constr.	11.585%	+ 2.090% (0.264)	0.654	0.921	377%	7.547%	4.038%	-5.456% (-0.578)	0.297	0.409
Strategy 1: BL, $\tau = 0.25$	Non-constr.	21.989%	+12.494% (1.239)	0.785	1.161	1571%	31.420%	-9.431%	-18.926% (-1.382)	-0.155	-0.218
	Constr.	14.332%	+ 4.837% (0.547)	0.727	1.042	845%	16.900%	-2.568%	-12.063% (-1.253)	-0.017	-0.023
Strategy 1: BL, $\tau = 0.5$	Non-constr.	29.074%	+19.579% (1.736)*	0.845	1.268	2522%	50.457%	-21.383%	-30.878% (-1.910)**	-0.383	-0.536
	Constr.	15.853%	+ 6.358% (0.763)	0.769	1.104	1060%	21.206%	-5.353%	-14.848% (-1.535)	-0.139	-0.192
Strategy 1: BL, $\tau = 1.0$	Non-constr.	35.784%	+26.289% (2.171)**	0.903	1.372	3736%	74.727%	-38.943%	-48.438% (-2.511)***	-0.617	-0.859
	Constr.	16.478%	+ 6.983% (0.801)	0.775	1.111	1226%	24.520%	-8.042%	-17.537% (-1.810)*	-0.253	-0.347
Strategy 1: Treyner-Black	Non-constr.	-46.076%	-55.571% (-3.239)***	-0.467	-0.677	1568%	31.367%	-77.443%	-86.938% (-3.877)***	-1.466	-1.947
	Constr.	-0.475%	-9.970% (-1.271)	0.059	0.081	572%	11.451%	-11.926%	-21.421% (-1.983)**	-0.623	-0.821
Strategy 2: BL, $\tau = 0.05$	Non-constr.	9.934%	+0.439% (0.095)	0.595	0.821	831%	16.621%	-6.687%	-16.182% (-1.174)	-0.184	-0.221
	Constr.	10.419%	+0.924% (0.116)	0.632	0.872	395%	7.903%	2.516%	-6.979% (-0.813)	0.225	0.304
Strategy 2: BL, $\tau = 0.25$	Non-constr.	9.848%	+0.353% (0.078)	0.509	0.691	1119%	22.397%	-12.549%	-22.044% (-2.040)**	-0.402	-0.504
	Constr.	12.081%	+2.586% (0.313)	0.684	0.951	608%	12.171%	-0.092%	-9.585% (-1.080)	-0.089	-0.121
Strategy 2: BL, $\tau = 0.5$	Non-constr.	10.033%	+0.538% (0.104)	0.465	0.626	1322%	26.446%	-16.413%	-25.908% (-2.268)**	-0.417	-0.529
	Constr.	12.761%	+3.266% (0.385)	0.702	0.981	673%	13.471%	-0.713%	-10.205% (-1.136)	-0.061	-0.082
Strategy 2: BL, $\tau = 1.0$	Non-constr.	12.902%	+3.407% (0.392)	0.511	0.696	1523%	30.477%	-17.575%	-27.070% (2.336)**	-0.319	-0.415
	Constr.	12.696%	+3.201% (0.383)	0.689	0.964	717%	14.349%	-1.653%	-11.148% (-1.124)	-0.016	-0.021
Benchmark	/	9.495%	/	0.589	0.816	/	/	/	/	0.589	0.816

Note: *, **, and *** indicate the statistical significance with p-values of 10%, 5%, and 1%, respectively.

Source: Own work.

Looking at the results of Strategy 1, it is evident that non-constrained portfolios outperform the constrained ones in terms of the annualized mean return as well as the Sharpe and Sortino ratios. By increasing the parameter τ , all the performance measures also increase. This occurrence is explained by Chen, Da & Schaumburg (2015), who stated that if the long/short strategy is effective, meaning that the views provide economic value, an increase in τ should generate a higher Sharpe ratio. However, this increase is more present in the non-constrained portfolios as the Black-Litterman model can freely enlarge the individual weights without constraints. The improved performance of constrained portfolios is also present; however, much more sublime. Increasing τ also significantly enhances the portfolio turnover and the transaction costs incurred. The turnover and transaction costs increased more for the non-constrained portfolios due to a larger inter-day volatility of the weights. After accounting for the transaction costs, the best-performing variation of the Strategy 1 was a portfolio with $\tau = 0.05$ as the transaction costs increased faster relative to the increase in the mean yearly

return when increasing τ . Variation with $\tau = 1$, therefore, performed the worst after transaction costs, despite being by far the best performer prior to it. When utilizing the Treynor-Black model for trading Strategy 1, the performance of the portfolios was the worst. The mean annualized return before transaction costs was already negative. In this case, the constrained portfolio performed better than the non-constrained one in all aspects. Overall, this model produced absurd individual weights.

Variations of Strategy 2 portfolio performed similarly to Strategy 1. However, in all the occasions, except where $\tau = 1$, the constrained portfolio outperformed the non-constrained one even before the transaction costs by all three performance measures. The no short-selling constraint and the upper limit of 40% on each weight suited this strategy better and was even beneficial for the performance. After accounting for transaction costs, all the variations underperformed the benchmark. Due to the lowest turnover and transaction costs, like with Strategy 1, the portfolio with $\tau = 0.05$ performed the best even though it was the worst performer before transaction costs amongst its peers with higher τ 's. When directly comparing the results of both trading strategies, it can be concluded that, with daily rebalancing, Strategy 1 performed better than Strategy 2 before transaction costs in terms of all three performance measures. However, due to Strategy 2's turnover increasing much less when increasing τ , Strategy 2's portfolios outperformed Strategy 1's portfolios regarding mean returns after adjusting for the transaction costs in variations with $\tau = 0.5$ and $\tau = 1$. When $\tau = 0.25$, the constrained portfolio also outperformed its Strategy 1 counterpart.

In columns “Yearly mean benchmark-adjusted return” and “Yearly benchmark-adjusted return after transaction costs,” each cell contains the t-statistic of a two-sample two-sided t-test where the differences in mean returns of the portfolios and the benchmark were tested. Statistically significant positive differences in mean returns were found for two non-constrained portfolios as part of strategy 1 ($\tau = 0.5$ and $\tau = 1$). However, no statistically significant positive differences were found after accounting for the transaction costs.

The effect of less frequent rebalancing is shown in Table 2, where monthly rebalancing results are displayed. The performance trend of the portfolios relative to the benchmark is, before transaction costs, comparable with that of daily rebalancing. Except for the Treynor-Black model, most variations of both trading strategies outperform the benchmark in terms of mean return, Sharpe ratio, and Sortino ratio. Two exceptions are Strategy 2's constrained portfolios with $\tau = 0.5$ and $\tau = 1$, where Sharpe and Sortino ratios were slightly lower than the benchmark's ratios. An important finding is that the Strategy 1 portfolios produce lower mean and risk-adjusted returns when rebalanced monthly instead of daily. Strategy 2 portfolios' performance in terms of mean returns was, on average, increased for the non-constrained portfolios, while the constrained portfolios performed slightly worse. Regarding risk-adjusted returns, all variations, on average, performed worse than while being rebalanced daily except for constrained Strategy 2 portfolios, which performed better.

The most important effect of less frequent rebalancing is the exponentially decreased portfolio turnover and the transaction costs incurred. After accounting for the transaction costs of 1%, the following outperformance relative to the benchmark was detected:

- Outperformed by mean returns only: Constrained Strategy 1 portfolio with $\tau = 0.05$ and non-constrained Strategy 2 portfolios with $\tau = 0.05$, $\tau = 0.5$, and $\tau = 1$.
- Outperformed by mean returns and Sortino ratio: Non-constrained Strategy 1 portfolios with $\tau = 0.5$ and $\tau = 1$.
- Outperformed in all three measures: Non-constrained Strategy 1 portfolios with $\tau = 0.05$, $\tau = 0.25$, and constrained Strategy 2 portfolio with $\tau = 0.05$.

Table 2: Results of historical simulations (monthly rebalancing)

Portfolio	Constraint	Yearly mean return	Yearly mean benchmark-adjusted return	Sharpe ratio	Sortino ratio	Yearly turnover	Yearly transaction costs	Yearly return after transaction costs	Yearly benchmark-adjusted return after transaction costs	Sharpe ratio after TC	Sortino ratio after TC
Strategy 1: BL, $\tau = 0.05$	Non-constr.	10.911%	+1.416% (0.175)	0.639	0.896	36.0%	0.720%	10.191%	+0.696% (0.096)	0.605	0.847
	Constr.	11.044%	+1.549% (0.092)	0.659	0.922	71.1%	1.423%	9.621%	+0.126% (0.026)	0.582	0.809
Strategy 1: BL, $\tau = 0.25$	Non-constr.	14.575%	+5.080% (0.655)	0.709	1.017	131.6%	2.633%	11.942%	+2.447% (0.401)	0.607	0.866
	Constr.	11.405%	+1.910% (0.214)	0.649	0.905	98.3%	1.967%	9.438%	-0.057% (-0.006)	0.551	0.765
Strategy 1: BL, $\tau = 0.5$	Non-constr.	17.914%	+8.419% (1.025)	0.737	1.073	227.0%	4.541%	13.373%	+3.878% (0.642)	0.593	0.857
	Constr.	11.291%	+1.796% (0.233)	0.624	0.868	118.9%	2.378%	8.913%	-0.582% (-0.043)	0.512	0.708
Strategy 1: BL, $\tau = 1.0$	Non-constr.	22.342%	+12.847% (1.415)	0.751	1.105	379.8%	7.596%	14.746%	+5.251% (0.889)	0.564	0.824
	Constr.	10.963%	+1.468% (0.219)	0.592	0.821	139.2%	2.785%	8.18%	-1.317% (-0.098)	0.464	0.641
Strategy 1: Treynor-Black	Non-constr.	-47.76%	-57.261% (-3.279)***	-0.21	-0.295	132.35%	2.647%	-50.413%	-59.908% (-3.303)***	-0.271	-0.379
	Constr.	9.089%	-0.406% (-0.082)	0.584	0.806	47.55%	0.951%	8.138%	-1.357% (-0.194)	0.533	0.735
Strategy 2: BL, $\tau = 0.05$	Non-constr.	10.398%	+0.903% (0.166)	0.622	0.867	45.0%	0.900%	9.498%	+0.003% (0.003)	0.579	0.804
	Constr.	10.445%	+0.950% (0.144)	0.633	0.873	26.7%	0.534%	9.911%	+0.416% (0.092)	0.602	0.836
Strategy 2: BL, $\tau = 0.25$	Non-constr.	12.46%	+2.965% (0.309)	0.619	0.872	158.8%	3.176%	9.284%	-0.211% (-0.034)	0.496	0.691
	Constr.	10.219%	+0.724% (0.083)	0.59	0.823	51.8%	1.036%	9.183%	-0.312% (-0.030)	0.543	0.754
Strategy 2: BL, $\tau = 0.5$	Non-constr.	15.128%	+5.633% (0.677)	0.634	0.904	229.5%	4.591%	10.537%	+1.042% (0.140)	0.490	0.692
	Constr.	10.304%	+0.809% (0.113)	0.583	0.815	60.9%	1.218%	9.086%	-0.409% (-0.039)	0.529	0.736
Strategy 2: BL, $\tau = 1.0$	Non-constr.	19.641%	+10.146% (1.291)	0.686	0.999	300.1%	6.003%	13.638%	+4.143% (0.591)	0.538	0.773
	Constr.	9.953%	+0.458% (0.088)	0.561	0.785	69.7%	1.395%	8.558%	-0.937% (-0.127)	0.499	0.696
Benchmark	/	9.495%	/	0.589	0.816	/	/	/	/	0.589	0.816

Note: *, **, and *** indicate the statistical significance with p-values of 10%, 5%, and 1%, respectively.

Source: Own work.

For a real-life investor who does not intend to short-sell or have concentrated weights in only a few stocks, the result of a constrained Strategy 2 portfolio with $\tau = 0.05$ is very important. As opposed to the reviewed literature, where the initial outperformance by similar trading

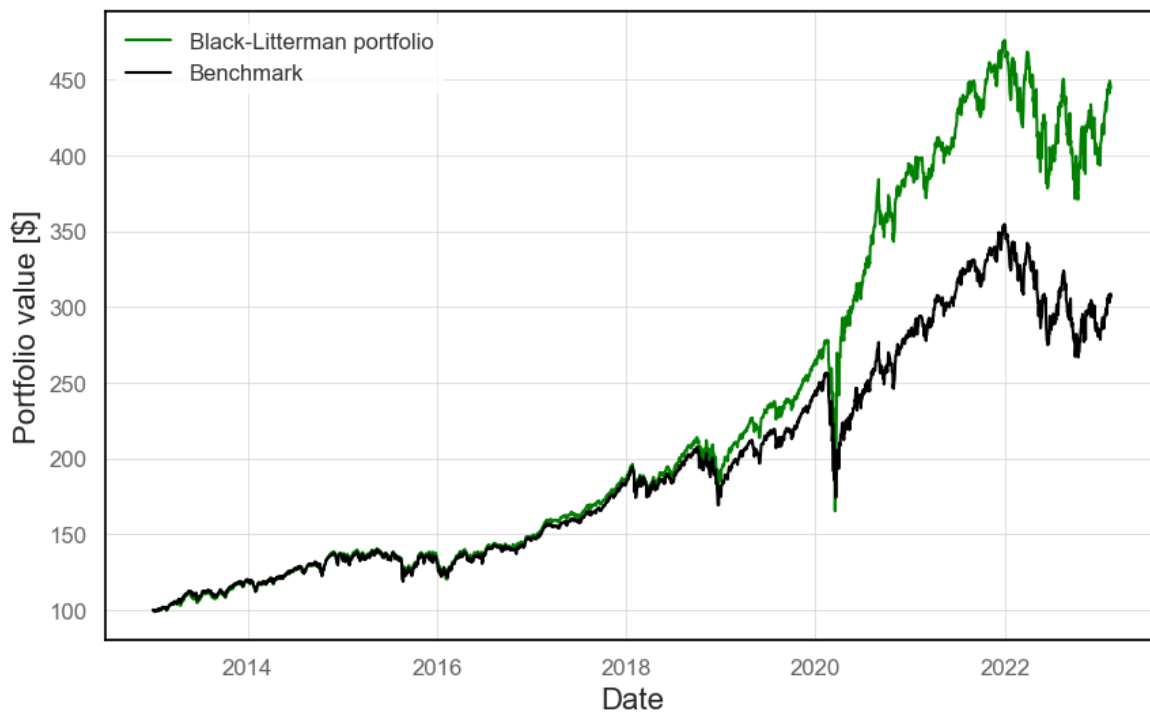
strategies was diminished once accounting for transaction costs, this result states that if an investor started trading on 1/1/2013, only relied on the analysts' recommendations, and was trading according to the Strategy 2, with transaction costs of 1%, a monthly rebalancing would, on average, produce 0.416% larger annualized returns than the benchmark, while also larger risk-adjusted Sharpe and Sortino ratios by 0.013 and 0.02, respectively.

Further in-depth analysis of both trading strategies is presented in the following sub-chapters. It is important to note that no statistically significant differences in mean returns were found for Black-Litterman portfolios with monthly rebalancings before and after accounting for transaction costs.

3.1 Strategy 1: Using target price consensus as a view vector

As part of trading Strategy 1, I first constructed a non-constrained Black-Litterman portfolio utilizing sell-side analysts' target prices as a view vector to assess its performance relative to the benchmark. To ensure a fair comparison, I normalized the data and simulated an investment of \$100 in both the Black-Litterman portfolio and the benchmark. As retrieved from the literature, I started with the controversial τ parameter set to 0.05, the most utilized value. The visual result of the simulation with daily rebalancings and τ set to 0.05 is displayed in Figure 8.

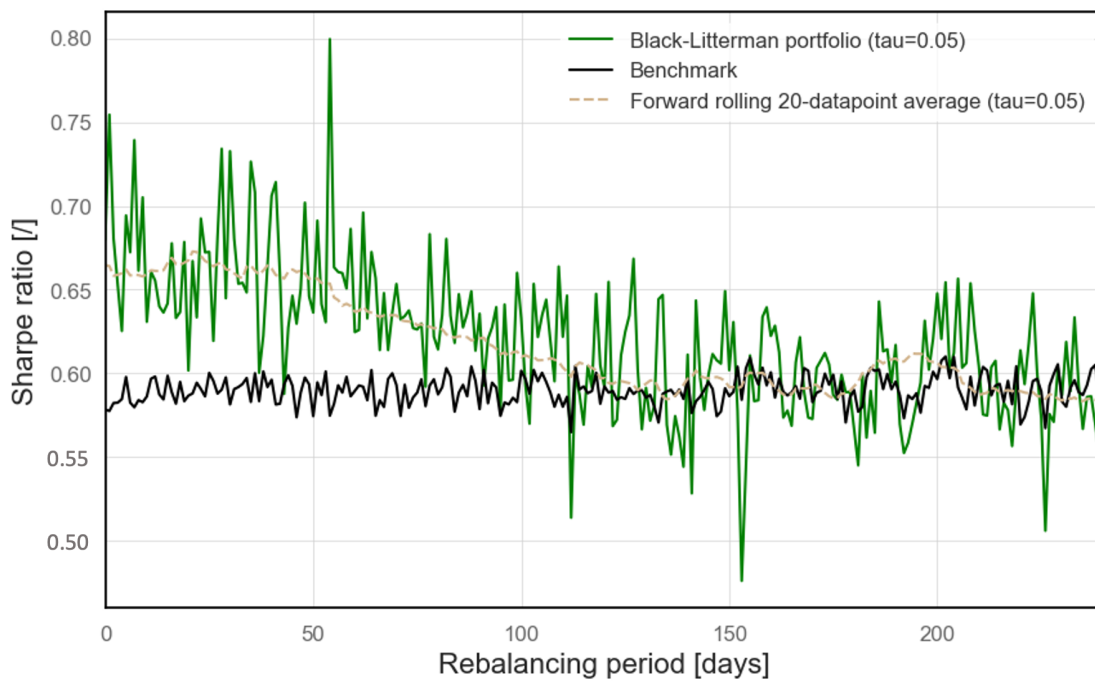
Figure 8: Unconstrained Black-Litterman portfolio performance ($\tau = 0.05$); Daily rebal.



Source: Own work.

The results indicate that the Black-Litterman portfolio with target price implied returns as a view vector outperformed the benchmark regarding outright return when being rebalanced daily. The outperformance is most vivid after the onset of the Covid-19 pandemic in early 2020. After a significant slump in the market value in March 2020, the stock market rebounded quite heavily and continued with steep growth until the start of 2022. The reasons for such an outperformance during this period will be discussed later. The performance in terms of risk-adjusted returns and the effect of the rebalancing period are shown in Figure 9. An important finding suggests that, for trading Strategy 1, the overall Sharpe ratio increases with the frequency of rebalancing. Lowering the frequency, the value of the Sharpe ratio starts decreasing until rebalancing approximately every 115 days. The simulation has shown that the benefits of frequent rebalancing are present up until rebalancing with approximately 115 trading days. The Sharpe ratio is quite volatile in relation to the rebalancing period. This is because, in this case, the daily weights produced for each stock are very volatile inter-day. Therefore, even a one-day difference in the rebalancing period produced vastly different weights, and, as the future stock movements were unknown, the portfolio performed quite differently. This is, of course, reflected in the Sharpe ratio.

Figure 9: Portfolio's Sharpe ratio relative to the rebalancing period ($\tau=0.05$)



Source: Own work.

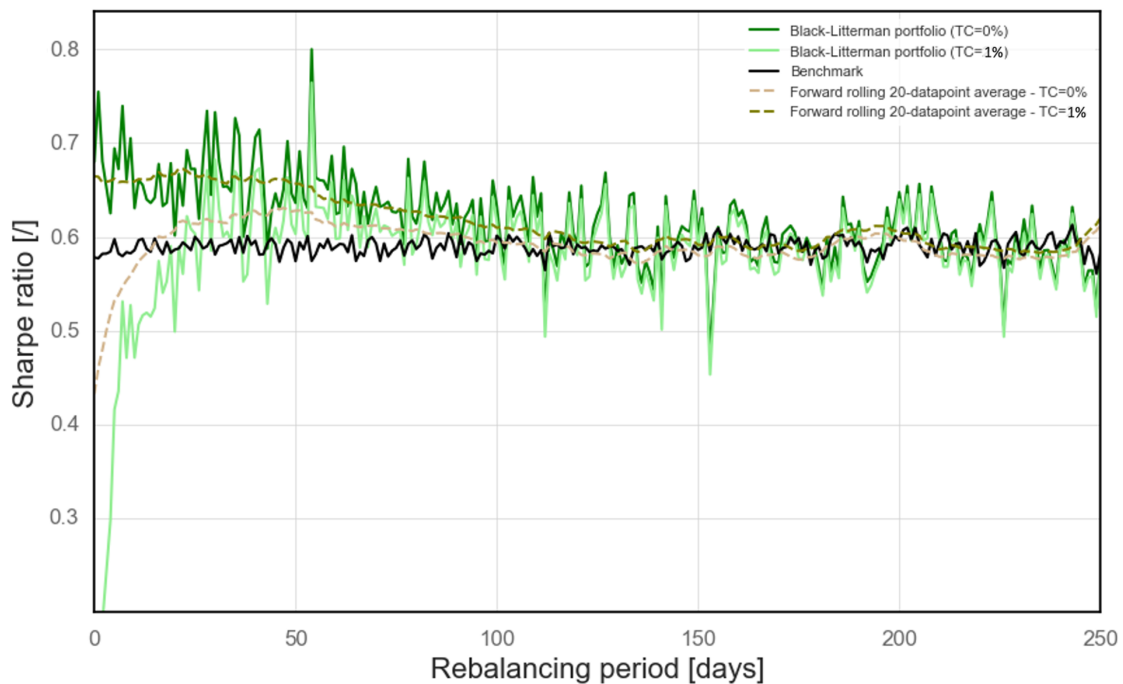
3.1.1 The effect of transaction costs

Incorporating transaction costs into trading simulations is crucial as it ensures a more accurate representation of real-world trading conditions because costs such as commissions

and fees directly impact the net returns of an investment. Therefore, accounting for transaction costs allows for a better evaluation of trading strategies, as high-frequency trading strategies may appear profitable without considering these costs, especially without penalizing the portfolio's performance because of an enlarged portfolio turnover which increases exponentially.

I simulated a 1% transaction fee when selling and purchasing a stock, the results of which are shown in Figure 10. It can be observed that a daily rebalanced portfolio's risk-adjusted performance plummets after transaction costs. As the overall transaction costs rose exponentially with the rebalancing frequency, the portfolio's performance was highly affected by the transaction costs until the rebalancing period of approximately 50 trading days. Despite considering the transaction costs, a trend of slight outperformance in terms of the Sharpe ratio can be seen when rebalancing with a rebalancing period from approximately 25 to 75 trading days which is an important finding.

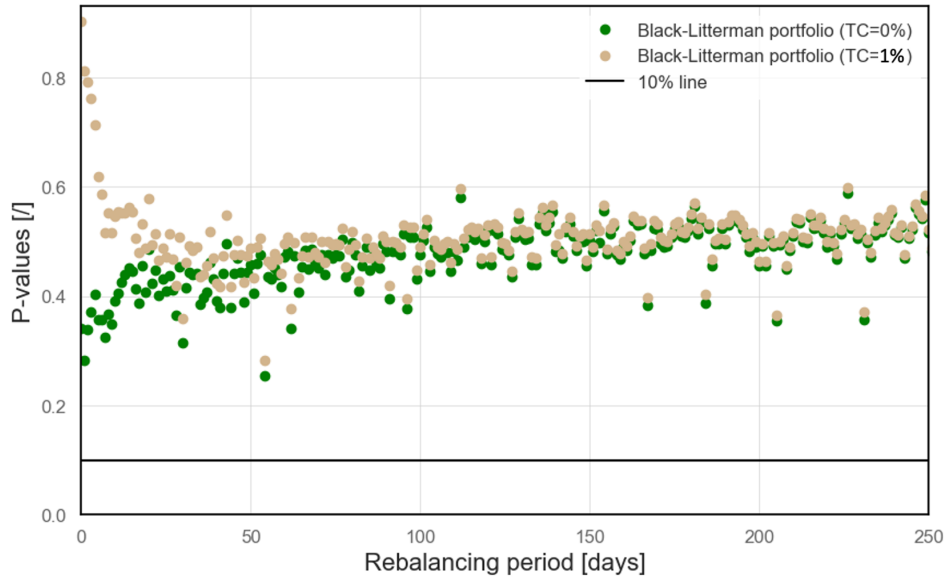
Figure 10: Sharpe ratio relative to the rebalancing period after transaction costs ($\tau = 0.05$)



Source: Own work.

Figure 11 presents the p-values obtained after performing a one-sided two-sample t-test where I tested if the average daily return generated by the portfolio is statistically significantly larger than the benchmark's average return. For a conservative value of 0.05 for τ , the average daily returns are not statistically significantly larger than the benchmark. However, the higher the rebalancing frequency, the closer the p-value is to the threshold line of 10%. When including transaction costs, the p-values are even farther from being significant.

Figure 11: P-values of a one-sided two-sample t-test of portfolio's average daily returns vs. benchmark



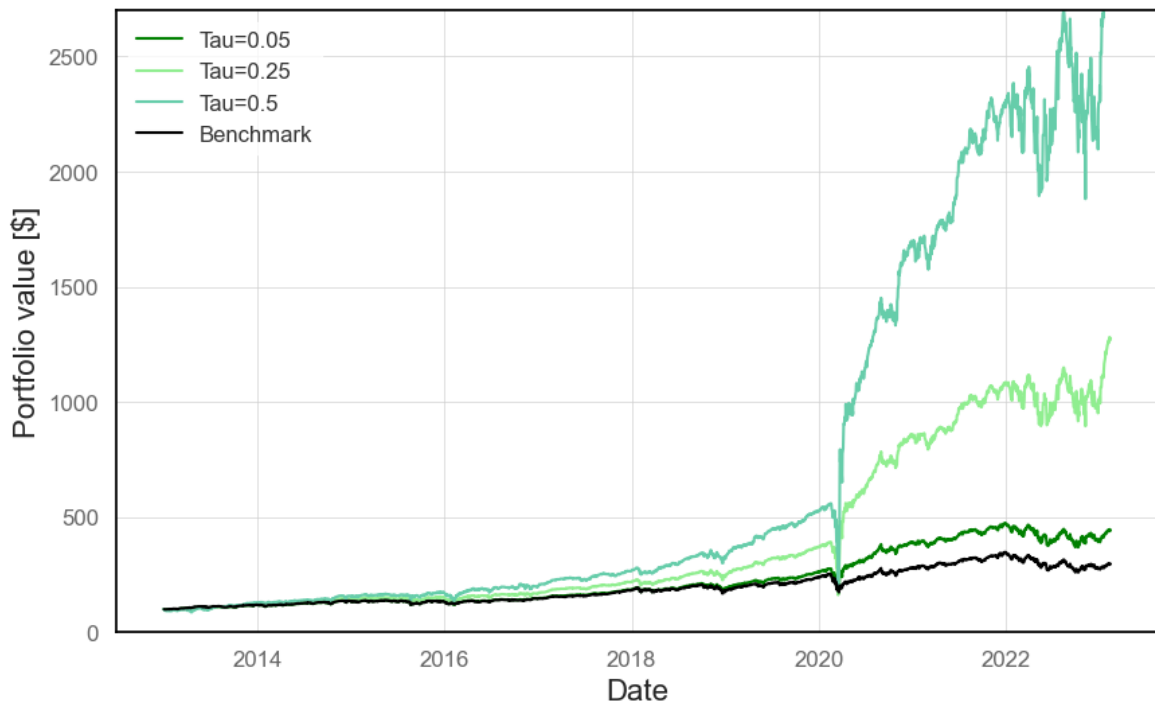
Source: Own work.

3.1.2 τ sensitivity analysis

An essential aspect of the strategy I intended to test was the effect of the parameter τ on the performance. According to theory, τ is a scaling factor and allows one to fine-tune the Black-Litterman model. As the literature cannot find a consensus regarding its value, I set it to 0.05, as recommended by a few papers, and then scaled it up to 0.5 for the unconstrained portfolio. The results with daily rebalancing are presented in Figure 12. We can see that maximizing the parameter τ increases the portfolio's outright return. The reason for this occurrence is the following: by incorporating the sell-side analysts' recommendations as a view vector and daily rebalancing the portfolio, there seems to be a trend of outperforming the market on most occasions. The reason for this will be discussed later. As the portfolio with $\tau = 0.05$ outperformed the benchmark, especially from March 2020 onwards, increasing τ further enhanced the portfolio performance. Another reason for such an enormous outperformance is that the portfolio is unconstrained. Therefore, the Black-Litterman model scaled the long and short positions, which resulted in a more considerable outperformance.

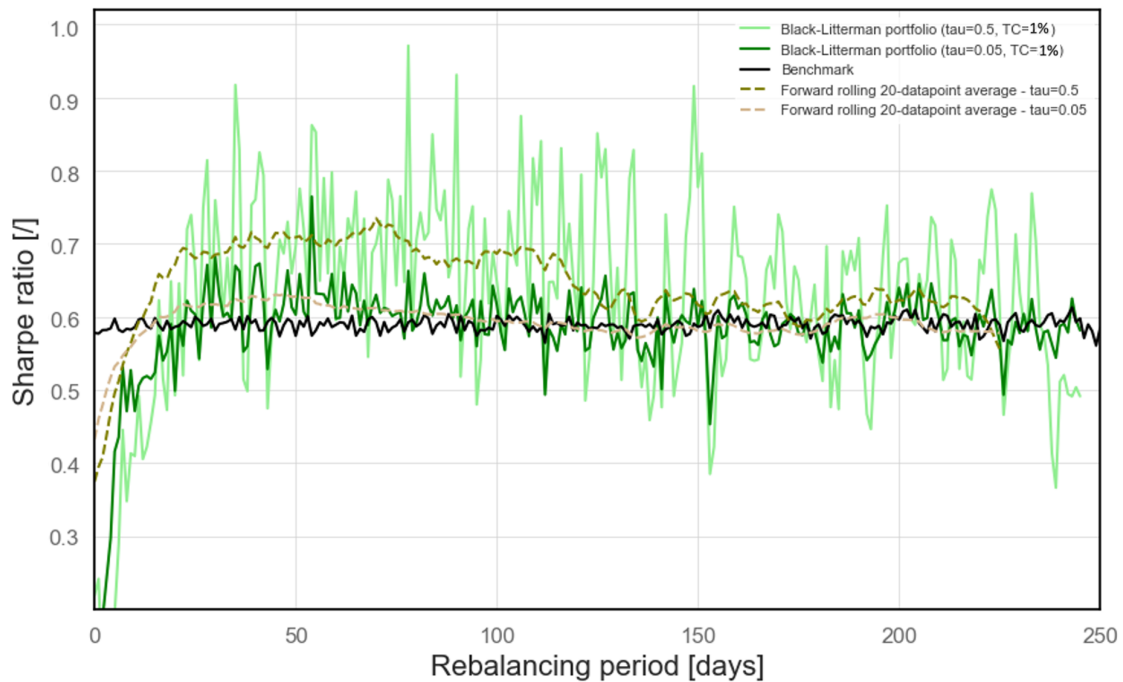
Figure 13 presents the Sharpe ratio as a function of the rebalancing period for a portfolio with $\tau = 0.5$ after adjusting for the transaction costs. Even after accounting for transaction costs, the trend of higher Sharpe ratios relative to the benchmark persists. An important observation is also the increased volatility of the Sharpe ratio when comparing it to the portfolio with $\tau = 0.05$. The reason is similar to before. By increasing τ , the inter-day volatility of daily weights increases drastically. The risk of rebalancing on a day the wrong stocks are overweight increases, which poses a concern about the model's practical use.

Figure 12: The effect of parameter τ on the portfolio performance; Daily rebal.



Source: Own work.

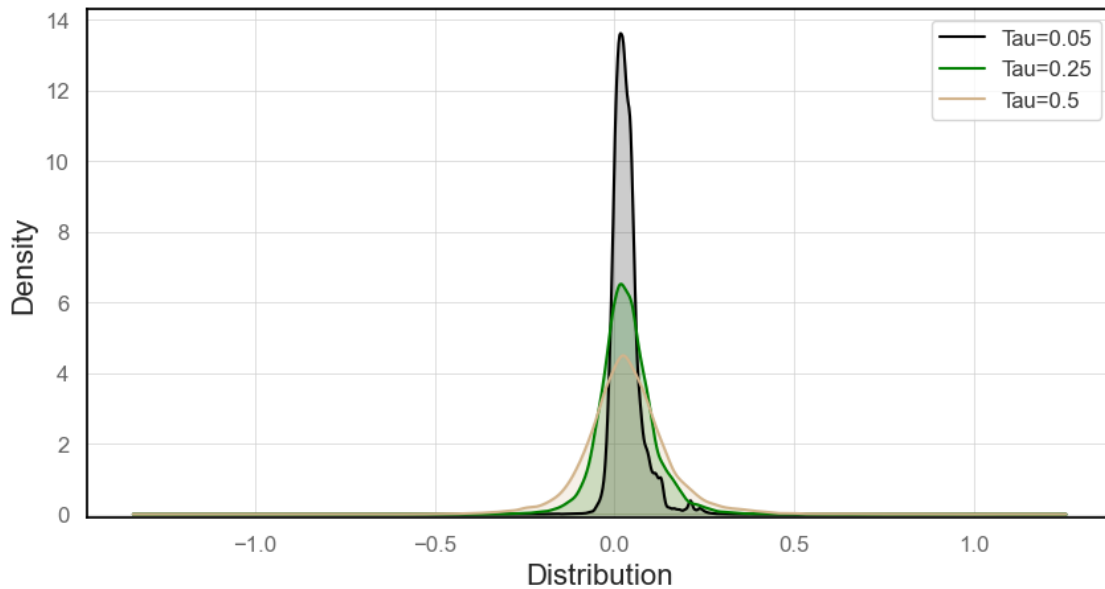
Figure 13: The effect of parameter τ on Sharpe ratio while adjusting for transaction costs ($\tau = 0.05$ and $\tau = 0.5$)



Source: Own work.

Figure 14 displays the distribution of daily weights over the simulated period relative to τ . The larger the τ , the flatter and wider the distribution curve becomes, which indicates that the inter-day individual weights are much more »wild« for most stocks. In extreme cases, on some trading days, the unconstrained portfolio with $\tau = 0.5$ included individual weights which represented +128% and -138% of the whole portfolio value.

Figure 14: The distribution of daily weights over the simulated period relative to τ



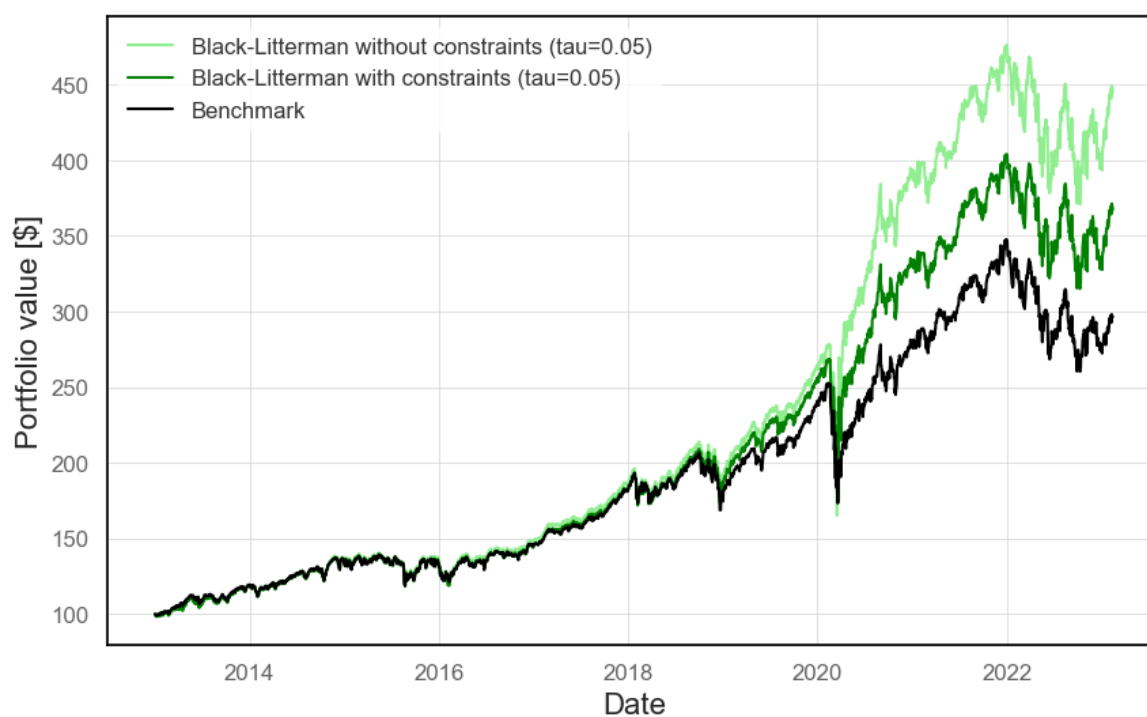
Source: Own work.

3.1.3 The effect of constraints

The non-constrained nature of the portfolio contributes to its performance. Without any imposed constraints, the model could freely allocate stocks based on the optimal risk-return trade-off, thus potentially enhancing overall performance, given the correct forecasts. This results in wild long and short positions, which is not feasible in the real world. To replicate an average investor, who is prohibited from short-selling stocks and does not want to concentrate most of the portfolio value on only a few stocks, I performed a simulation under the same conditions as before, only with two more constraints: short-selling was prohibited and the highest weight an individual stock could have on any given trading day was 40%. The results of a daily rebalancing and $\tau = 0.05$ are shown in Figure 15. In line with the expectations, the constrained portfolio underperformed the unconstrained one regarding the outright return while still outperforming the benchmark.

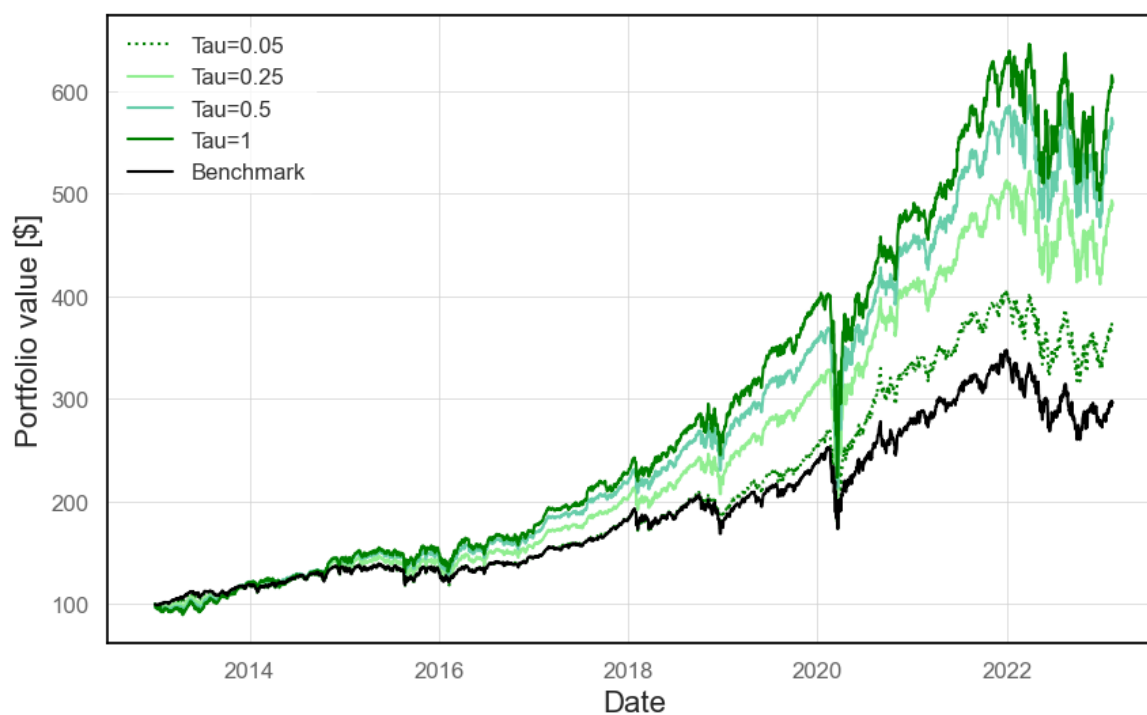
Another observation is that the overall and risk-adjusted returns increase by increasing the parameter τ . However, as opposed to the unconstrained portfolios, the effect of this action has a limit and slowly converges to a certain level. This can be seen in Figure 16 (daily rebalancing).

Figure 15: The effect of the imposed constraints on a portfolio's performance ($\tau=0.05$)



Source: Own work.

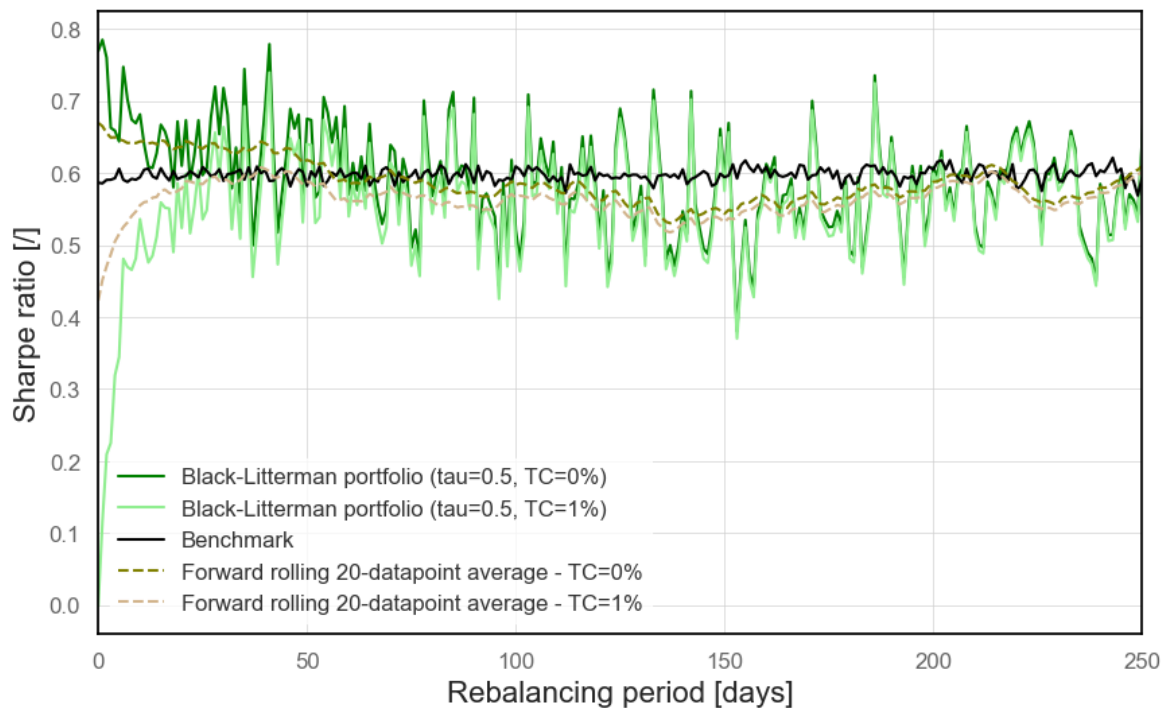
Figure 16: The sensitivity of a constrained portfolio's performance to the parameter τ ;
Daily rebalancing



Source: Own work.

According to Figure 16, it is apparent that the first incremental τ increase (from $\tau = 0.05$ to $\tau = 0.25$) has the highest effect on the performance, while further increases, although larger than the first one, have a diminishing contribution to the overall portfolio performance. Setting the τ to 0.5 and 1 produces similar performance, and since weights are less volatile the smaller the parameter value is, I focused on the Sharpe ratio of a portfolio with $\tau = 0.5$ with and without considering transaction costs. Results are presented in Figure 17. The daily rebalanced portfolio still outperformed the benchmark regarding the Sharpe ratio. However, the 20-day average line is lower than it was with an unconstrained portfolio. While accounting for transaction fees of 1% per the amount sold and bought, the portfolio, on average, underperformed the market with all analyzed rebalancing periods, from 1 trading day to 250. This finding is mainly in line with He, Grant & Fabre (2012). There were some intervals where the constrained portfolio, while adjusted for transaction costs, performed similarly to the benchmark regarding the Sharpe ratio. The most apparent is the rebalancing period from 39 to 46 trading days.

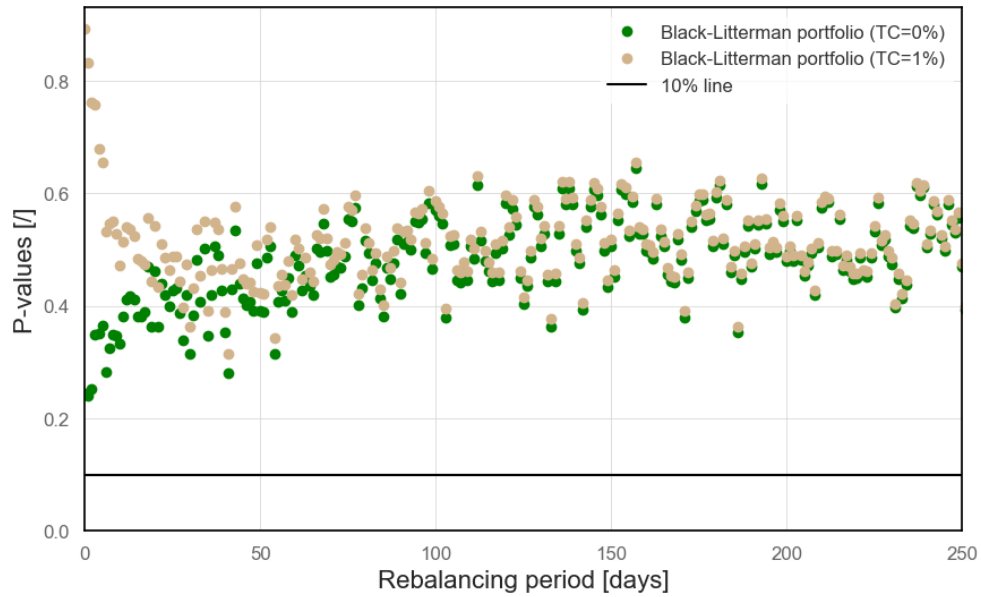
Figure 17: Sharpe ratio of a constrained portfolio relative to the rebalancing period before and after transaction costs ($\tau = 0.5$)



Source: Own work.

Figure 18 presents the p-values obtained after performing a one-sided two-sample t-test of the average daily returns generated by the portfolio and the benchmark. For a value of 0.5 for τ , the average daily returns are not statistically significantly larger than the benchmark. However, the higher the rebalancing frequency, the closer the p-value is to the threshold of 10%. When including transaction costs, the p-values are even farther from being significant.

Figure 18: P-values of a one-sided two-sample t-test of a constrained portfolio's average daily returns vs. benchmark



Source: Own work.

Figure 19: Calculated daily weights for Apple Inc.'s stock using different values of τ , constraints, and a comparison with the target price consensus implied returns



Source: Own work.

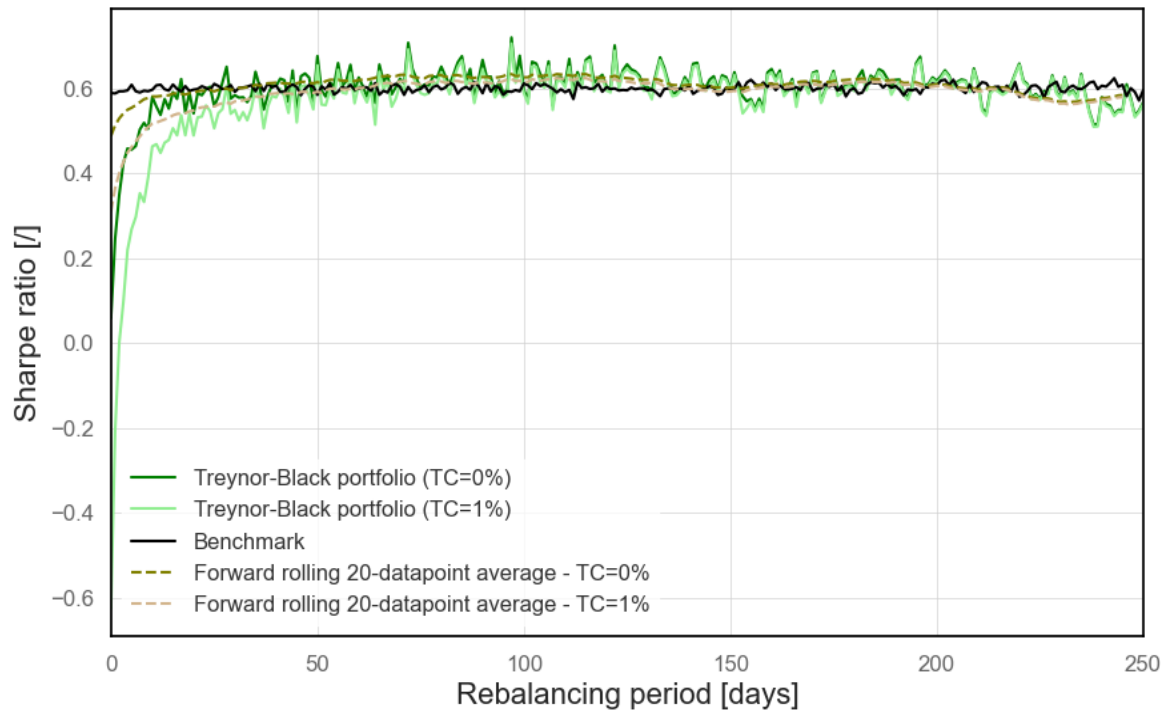
For a better visual presentation of the effect of τ on individual weights, an example of Apple Inc.'s stock is presented in Figure 19. Setting τ to 0.05, the calculated weights closely track the real-life market-capitalization equivalent weights. The deviation is correlated to the movement of the rollingly-calculated target price consensus implied returns for this particular asset. It is much more apparent when increasing τ to 0.5. As this is quite a considerable value for this parameter, the historical weight distribution becomes highly correlated to the historical distribution of the target price consensus implied returns. In this case, the amplitudes adjust too. It is important to note that this is due to a large market capitalization of Apple's stock relative to other members of the investment universe. The correlation also becomes more prominent for other stocks with increasing τ ; however, the amplitude differs. When introducing constraints such as no short-selling and the upper limit of 40% of the portfolio value, the weights are still reasonably correlated with implied returns. All this observation is in accordance with the theory, as increasing the τ assigns a lower relative weight to the CAPM equilibrium implied returns vector and more so to the subjective view vector, which includes target price consensus implied returns.

3.1.4 Comparison to the Treynor-Black model

When utilizing the Treynor-Black model for Strategy 1, I discovered a different behavior of portfolio performance relative to the rebalancing period. Interestingly, when not applying any constraints, the results were inversed relative to using the Black-Litterman model. The unconstrained portfolio underperformed the constrained one regarding the Sharpe and Sortino ratio. The mean yearly return was very volatile regarding the rebalancing period. The reason for this occurrence was the extreme volatility of the active weights and the absurdly large weights for active and passive parts. The amplitude reached even ± 65 , meaning that there were some occasions where the Treynor-Black model suggested a short position in the benchmark that equaled approximately 65x of the whole portfolio value and investing this amount into the active part, which included analysts' recommendations in the form of individual stock alphas. The result of this was the extreme unreliability of the performance as there were some rebalancing periods where the portfolio extremely outperformed and when it extremely underperformed, thus, not enabling me to form any conclusions about the value of analysts' recommendations.

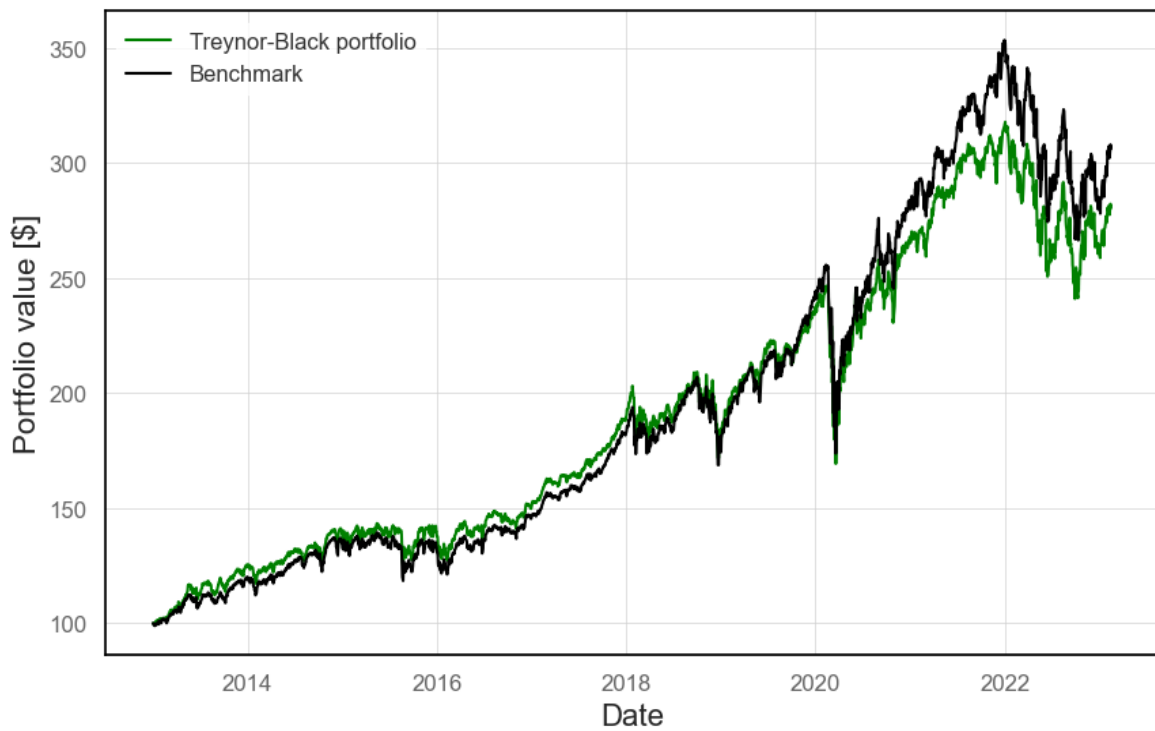
I decided to focus on the constrained portfolio, which appeared to have been much more stable regarding mean and risk-adjusted returns. I imposed the following constraints: the upper weight limit on the active and passive part equaled 1, while the lower limit equaled 0, which dictated the no short-selling rule. Furthermore, all the individual active weights were assigned between 0 and 1. Those weights were then normalized so that the sum equaled 1. It resulted in the active weight equalling 1 most of the trading days while investing in a passive portfolio was omitted (only 12.8% of trading days the model prescribed investing in a passive part). Furthermore, there was no trading day where the active-to-passive weight ratio was not equal to either 0 or 1. The risk-adjusted results are presented in Figure 20.

Figure 20: Treynor-Black portfolio's performance relative to the rebalancing period



Source: Own work.

Figure 21: Constrained Treynor-Black portfolio performance; monthly rebalancing



Source: Own work.

The frequent rebalancing was detrimental to the portfolio performance, as seen in the derived relationship between the Sharpe ratio and the rebalancing frequency. Table 1 states that the mean annualized return before transaction costs for a daily rebalanced portfolio was -0.475%. The main reason, I believe, was the extreme interday volatility of individual active weights, hence resulting in a distribution of daily returns that significantly underperformed the benchmark. With increasing the rebalancing period to one month, the Sharpe ratio increases and stays on a similar level than the benchmark.

I concluded that, for a given investment universe, analyzed period, and analysts' recommendations, the effectiveness of using the Treynor-Black model was inferior to the Black-Litterman in terms of risk-adjusted performance and reliability of results. The performance of the constrained Treynor-Black portfolio with monthly rebalancing is displayed in Figure 21. The portfolio closely tracks the benchmark yet slightly underperforms from 2020 onwards.

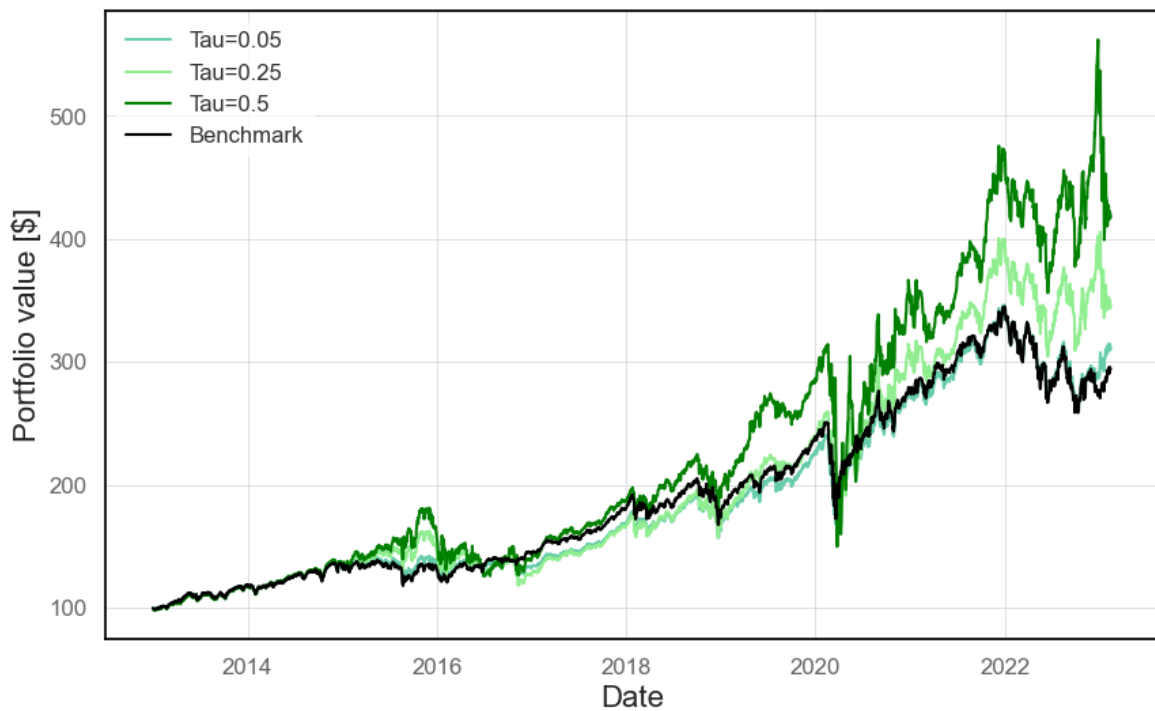
3.2 Strategy 2: Adapting weights based on a consensus score

As part of Strategy 2, I first constructed a non-constrained Black-Litterman portfolio utilizing sell-side analysts' consensus scores. Like with Strategy 1, I normalized the data and simulated an initial investment of \$100 with both the Black-Litterman portfolio and the benchmark.

Figure 22 displays the simulation's visual results of the non-constrained portfolio with daily rebalancings for multiple values of parameter τ . The results indicate that the Black-Litterman portfolio formed with consensus scores outperformed the benchmark regarding outright return when being rebalanced daily for $\tau = 0.25$ and $\tau = 0.5$. Setting τ to 0.05, the portfolio does not tend to deviate considerably from the market equilibrium portfolio; thus, it closely tracks the benchmark with occasional minor deviations. Compared to Strategy 1, where the outperformance is most vivid after the onset of the Covid-19 pandemic in early 2020, this strategy tends to outperform the benchmark much earlier for $\tau = 0.5$.

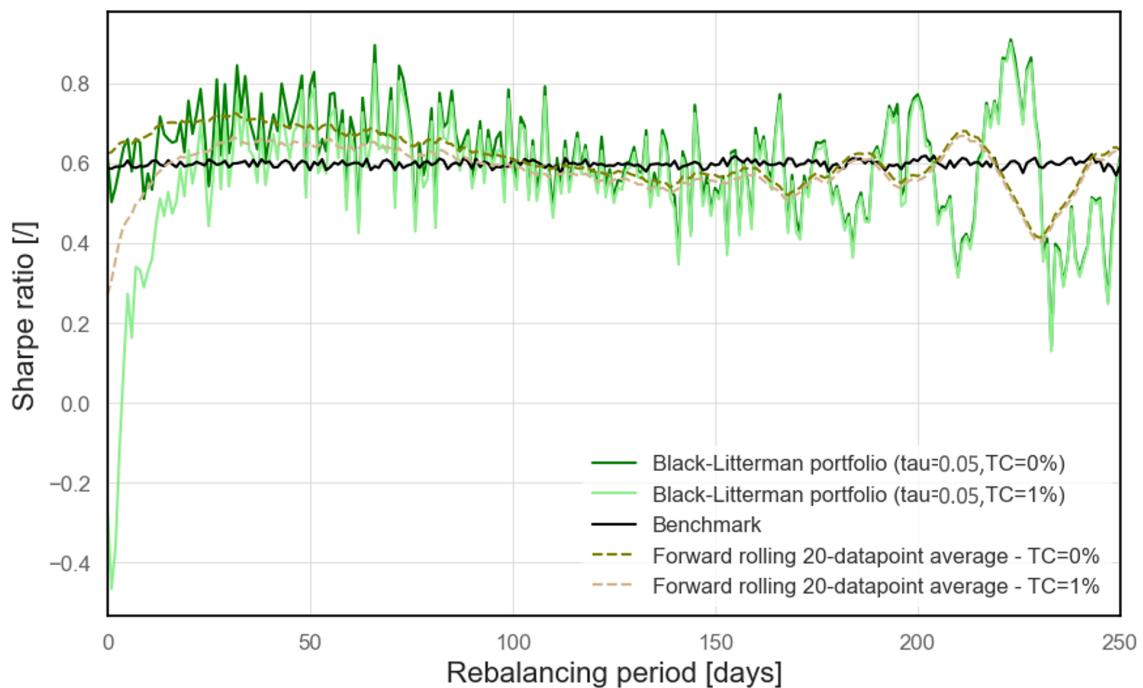
The performance in terms of risk-adjusted returns and the effect of the rebalancing period are shown in Figure 23 ($\tau = 0.05$). Another important finding suggests that the overall Sharpe ratio increases with the frequency of rebalancing, which aligns with findings from Strategy 1. However, in this scenario, the ratio is maximized in the approximate interval of 30 to 45 trading days. Extremely frequent rebalancing, for example, daily, underperforms in terms of outright return and Sharpe ratio relative to rebalancing in the before-mentioned interval. I analyzed whether the outperformance in terms of a Sharpe ratio would still be present after transaction costs, the results of which are also presented in Figure 23. It can be observed that while accounting for a transaction cost of 1%, the portfolio, on average, still outperforms the benchmark when the rebalancing period is appropriate. The approximate rebalancing period interval lies between rebalancing every 23 trading days and rebalancing every 77 trading days.

Figure 22: The sensitivity of an unconstrained portfolio's performance to the parameter τ



Source: Own work.

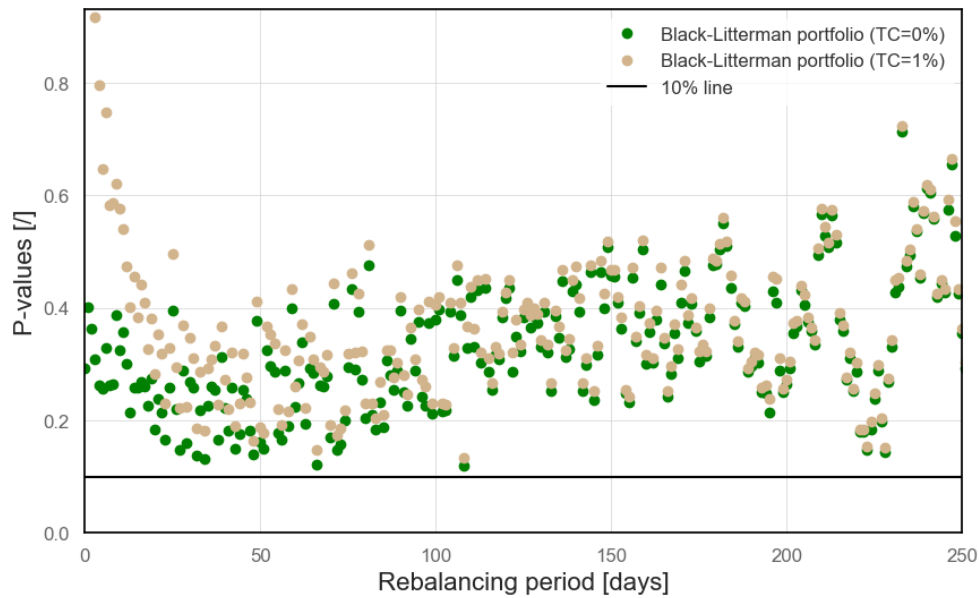
Figure 23: Sharpe ratio relative to the rebalancing period before and after transaction costs ($\tau = 0.05$)



Source: Own work.

Figure 24 presents the p-values obtained after performing a one-sided two-sample t-test where I tested if the average daily returns generated by the portfolio are larger than the benchmark. For a value of 0.5 for τ , the average daily returns are not statistically significantly larger than the benchmark, although they are close to the 10% level. This figure confirms that frequent rebalancing (1 to 23 trading days) is not optimal for this strategy as the p-values enlarged more, the larger was the rebalancing frequency when including transaction costs.

Figure 24: P-values of a one-sided two-sample t-test of an unconstrained portfolio's average daily returns vs. benchmark

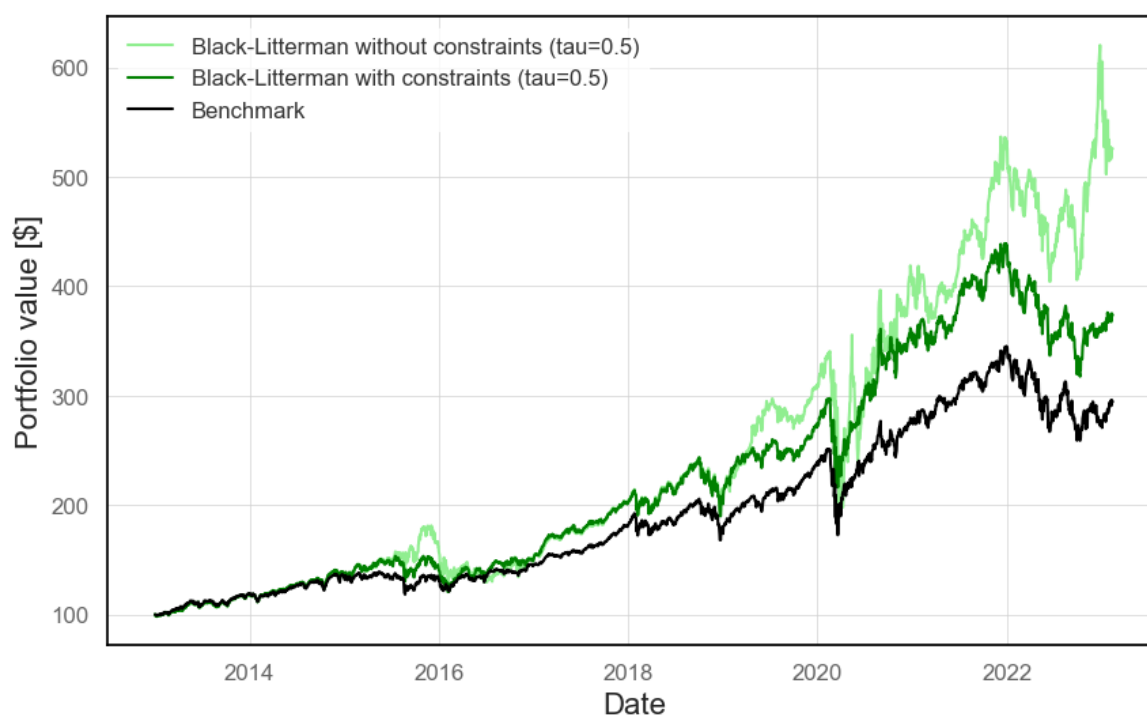


Source: Own work.

3.2.1 The effect of constraints

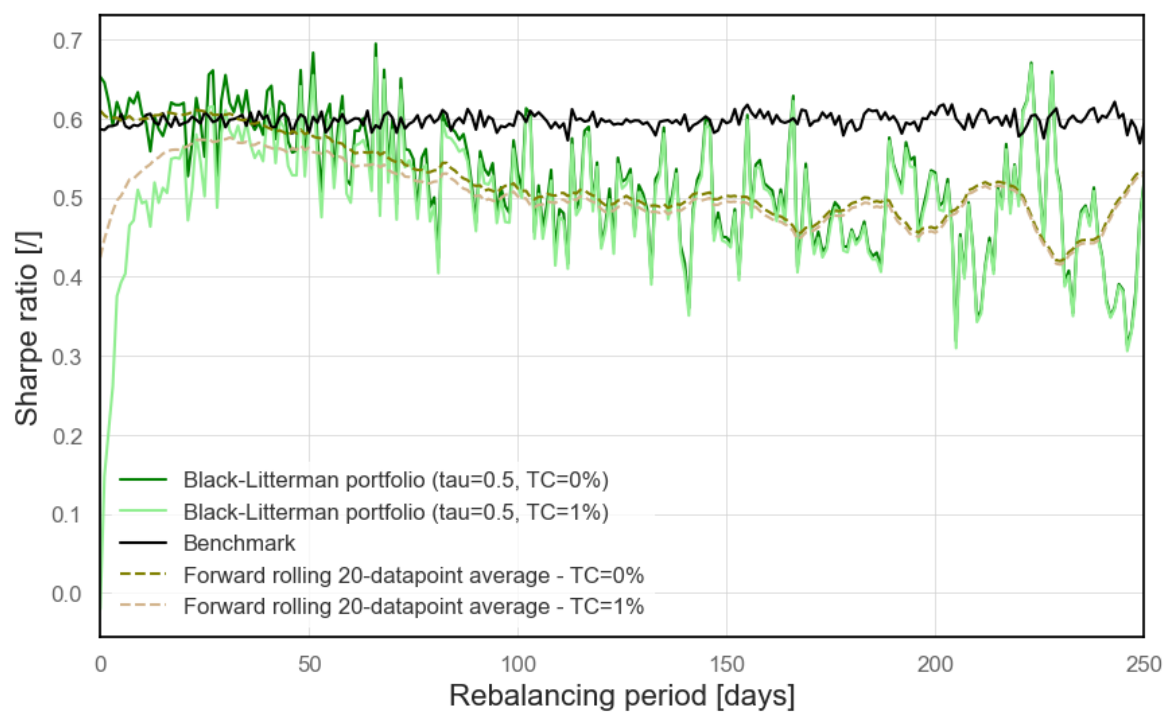
To replicate an average investor, who is prohibited from short-selling stocks and does not want to concentrate the majority of the portfolio value on only a few stocks, I performed a simulation under the same conditions as before ($\tau = 0.5$ and daily rebalancings) with the same two constraints as with Strategy 1: short selling was prohibited and the highest weight an individual stock could have on any given trading day was 40%. The results with daily rebalancing are shown in Figure 25. In line with the expectations for the former and the results of Strategy 1, the constrained portfolio underperformed the unconstrained one in terms of outright return while still outperforming the benchmark. In this case, the constrained portfolio was also much less volatile than the unconstrained one, which resembled a higher Sharpe ratio. Figure 26 presents the Sharpe ratios relative to the rebalancing frequency. The constrained portfolio's Sharpe ratio merely outperforms the benchmark up to the rebalancing period of approximately 25 trading days, judging by the forward-rolling 20-datapoint average. After accounting for transaction costs of 1%, this strategy underperforms the benchmark no matter the rebalancing frequency.

Figure 25: The effect of a no short-selling constraint on a portfolio's performance ($\tau=0.5$)



Source: Own work.

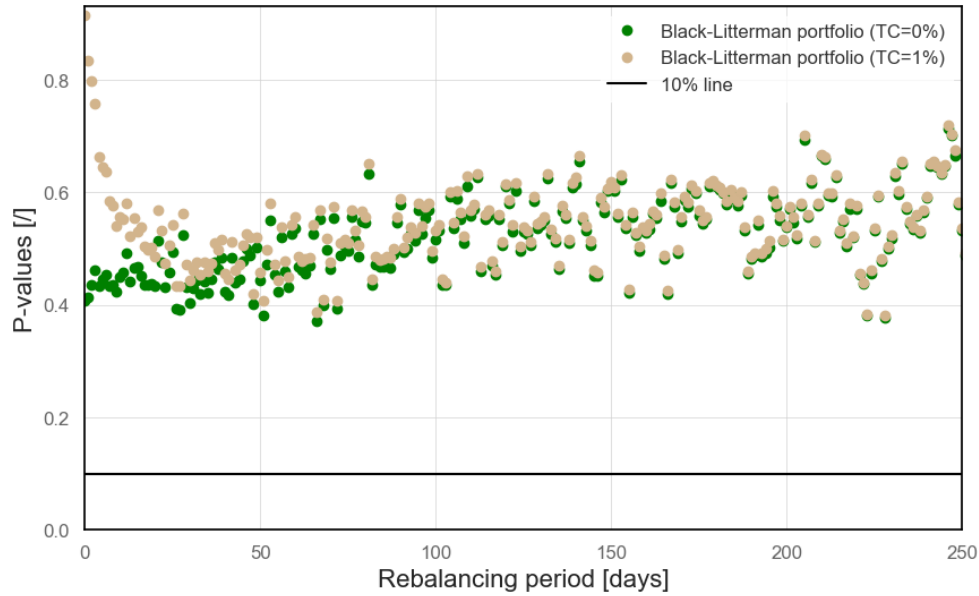
Figure 26: Sharpe ratio of a constrained portfolio relative to the rebalancing period after transaction costs ($\tau=0.5$)



Source: Own work.

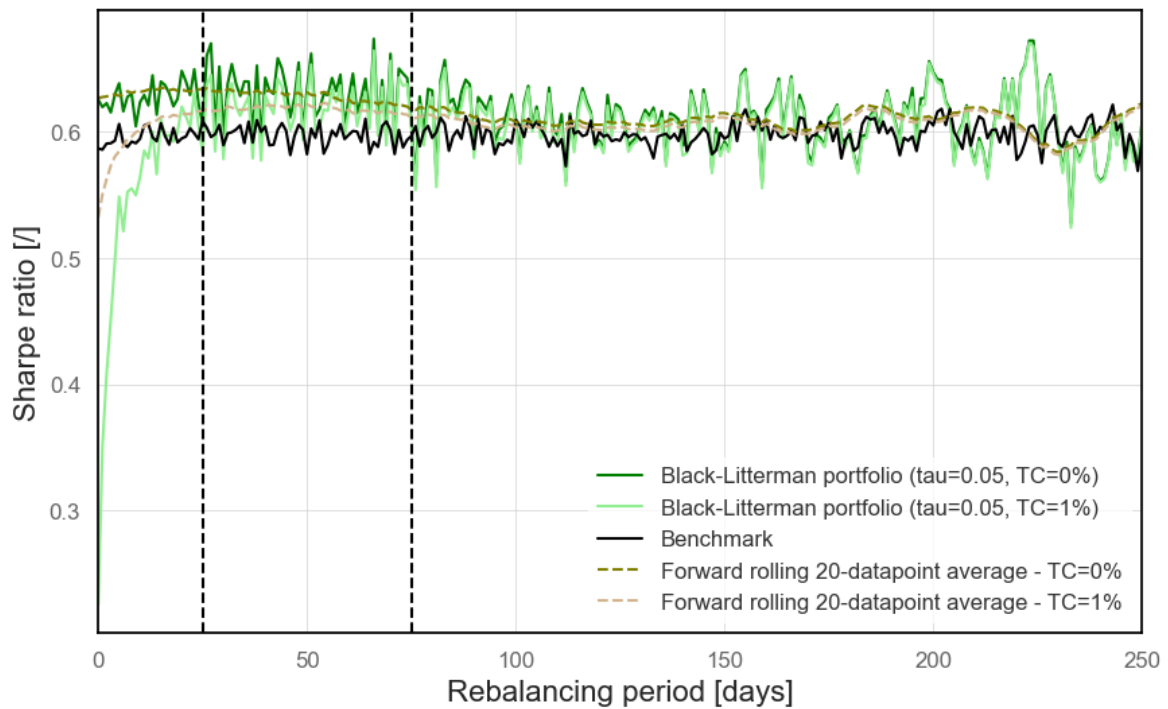
Figure 27 presents the p-values of a one-sided two-sample t-test of average daily returns vs. benchmark. For a value of 0.5 for τ , the average daily returns are not statistically significantly larger than the benchmark's returns. P-values are generally higher than without constraints.

Figure 27: P-values of a one-sided t-test of a constrained portfolio's average returns



Source: Own work.

Figure 28: Sharpe ratio of a constrained portfolio with monthly rebalanced ($\tau = 0.05$)

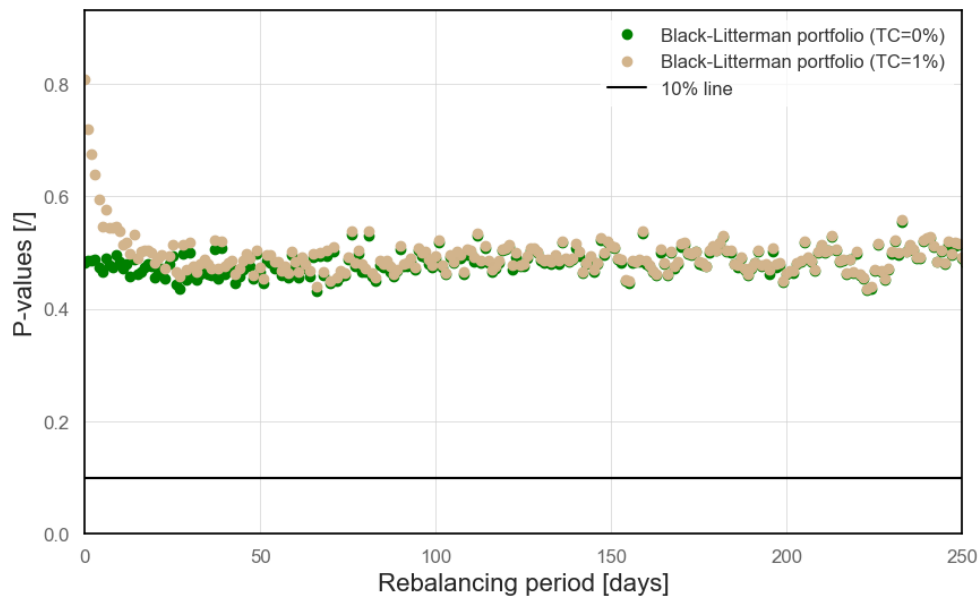


Source: Own work.

As evident from Table 2, after transaction costs, the constrained portfolio with $\tau = 0.05$ performs the best in terms of the Sharpe ratio when monthly rebalanced. Figure 28 shows that rebalancing between 25 and 75 trading days produces, on average, larger Sharpe ratios than the benchmark. After performing a one-tailed two-sample t-test for this interval against the benchmark's Sharpe ratios, the mean is statistically significantly larger than the benchmark's mean. T-statistic was 6.603, and the p-value was 1.044e-9. Figure 29 displays the p-values of a one-tailed two-sample t-test for the before-mentioned portfolio. No significant outperformance is achieved before and after adjusting for the transaction costs.

The sensitivity of the portfolio's performance to the parameter τ , with daily rebalancing, is presented in Figure 30. It can be observed that, like the unconstrained portfolio, setting τ to 0.05 only slightly outperforms the benchmark in terms of outright return. Setting τ higher than 0.25 does not achieve any significant additional contribution, as constraints do not allow the Black-Litterman model to capitalize on additional profits.

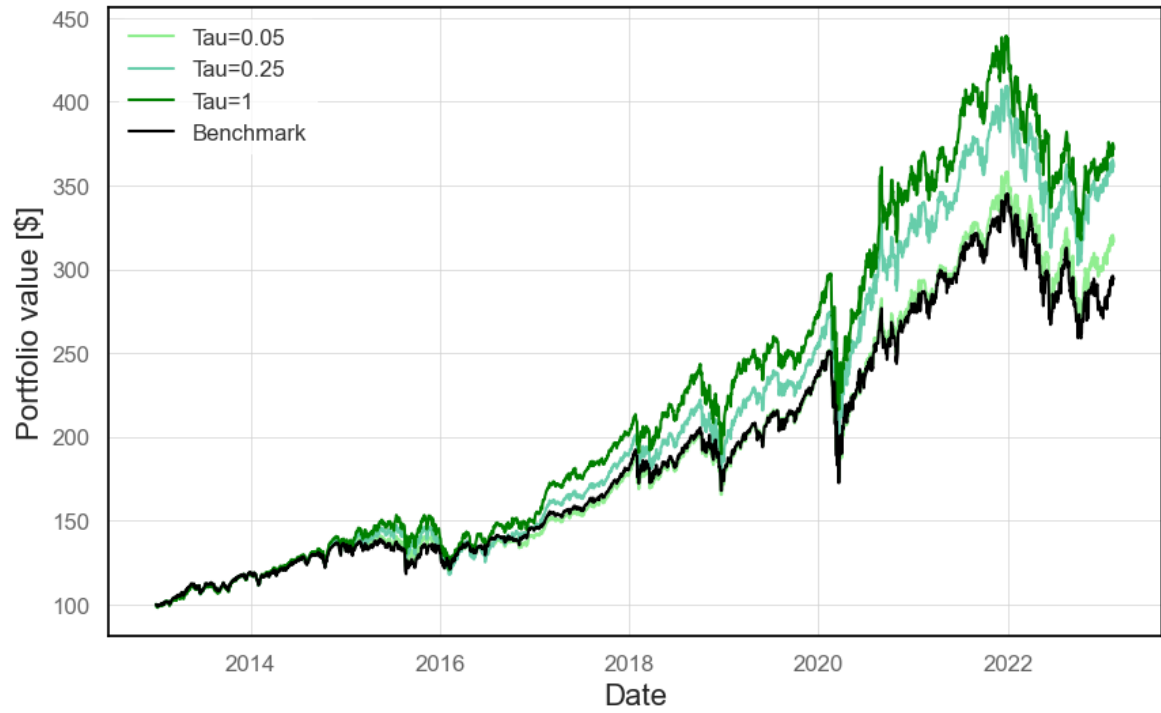
Figure 29: P-values of a one-sided two-sample t-test of the before-mentioned constrained portfolio's average daily returns vs. benchmark



Source: Own work.

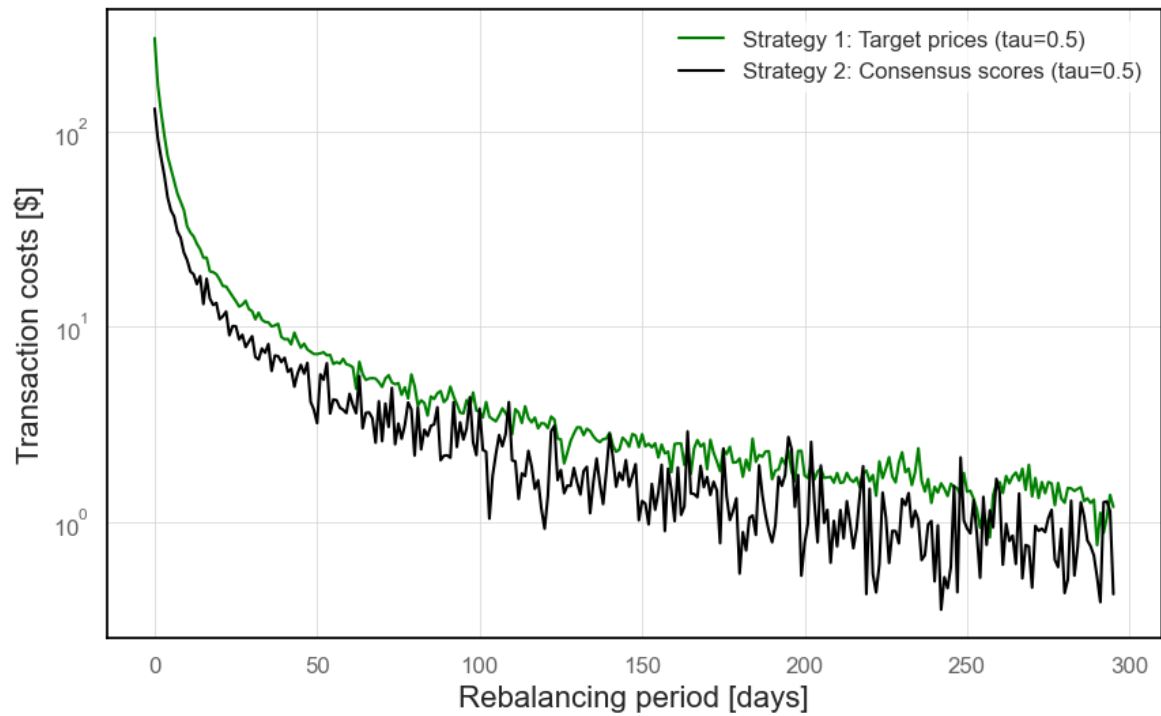
Another finding is that trading Strategy 2 has a much lower turnover than Strategy 1; hence, there is a lower amount of transaction costs incurred, which can be observed in Figure 31 (for portfolios with $\tau = 0.5$). On average, for a given rebalancing period, the trading costs of Strategy 1 are 73.6% higher than Strategy 2 at the end of the simulated period. Additional benefits of lower transaction costs incurred is a higher compounding effect, all else equal. On average, incurred transaction costs by Strategy 2 are more volatile, despite being displayed in a log scale in Figure 31. Another interesting observation is the exponential growth of the incurred transaction costs with increased rebalancing frequency.

Figure 30: The sensitivity of a constrained portfolio's performance to the parameter τ



Source: Own work.

Figure 31: Comparison of transaction costs incurred for both trading strategies as a function of the rebalancing period (shown in log scale)



Source: Own work.

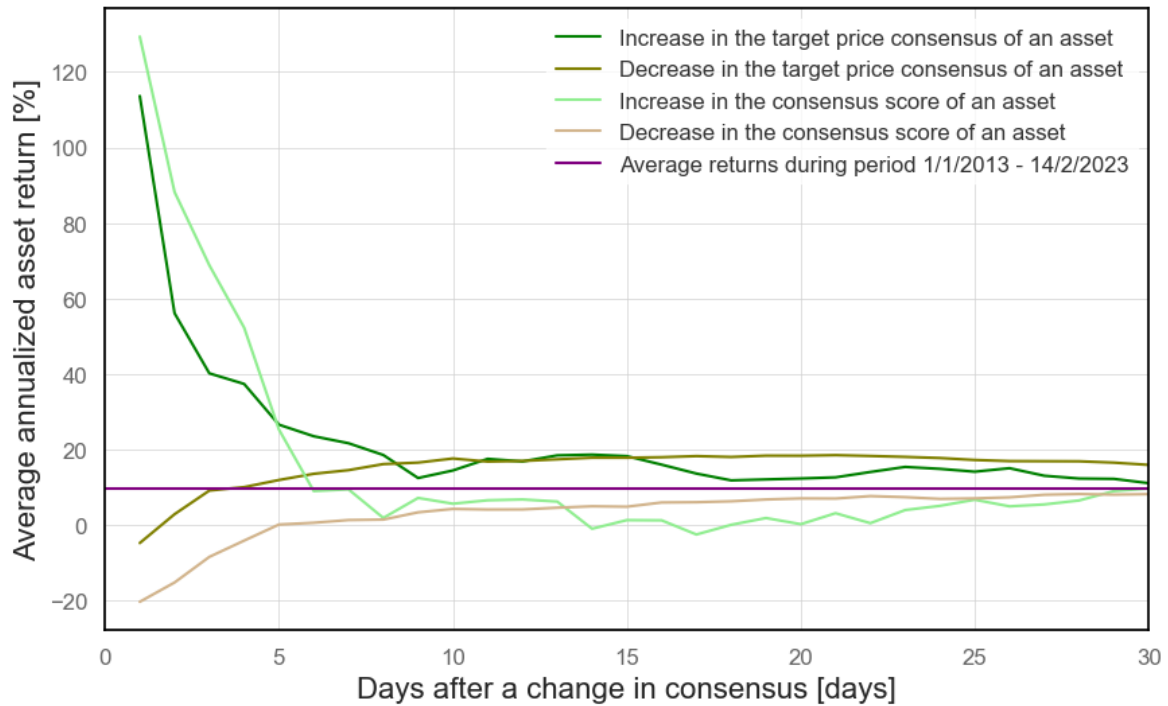
3.3 Analyzing the causes of frequently rebalanced portfolios' outperformance

One possible explanation I generated for the portfolios' outperformance with frequent rebalancing is the following. I analyzed the causal relationship between two datasets: analysts' consensus scores (and target prices) and stock returns. By observing changes in consensus scores (consensus target prices) for each stock throughout the whole period, I tracked the subsequent daily returns of individual stocks and grouped them separately. This allowed me to assess the annualized average returns resulting from shifts in consensus, thus, providing valuable insights into the influence of sell-side analysts' recommendations on stock performance. The analysis revealed statistically significant differences in stock returns following changes in analysts' consensus scores and target prices. This finding supports the notion that analysts' recommendations can act as self-fulfilling prophecies and, at least short-term, drive stock prices in the direction they predict. These results are consistent with the research of Okada and Azuma (2013), who also concluded that sell-side analysts' recommendations hold substantial sway over market behavior. It underscores the importance of considering the potential impact of analysts' recommendations when evaluating investment strategies. A similar effect of analysts' recommendations on stock price was also found by Aitken, Muthuswamy & Wong (2000) and Murg, Pachler & Zeitlberger (2014).

Average annualized returns after a change in an individual asset's consensus are presented in Figure 32. It can be observed that after a positive change in the specific asset's consensus score and consensus target price, the stock price, on average, increases drastically. This phenomenon is most apparent in the first 2 to 3 days after a change in consensus. The change in consensus score is, on average, more influential on the price increase than the change in the asset's target price consensus. The individual stock's price, on average, increased the next trading day by 129.38% (annualized data) following an increase in the consensus score as opposed to 113.65% following an increase in target price. The same holds for a decrease in the consensus. On average, the individual stock's price decreased the next trading day by 20.26% (annualized data) following a decline in the consensus score. In comparison, it decreased by 4.71% following a decrease in the target price. The effects of the recommendations are, on average, most apparent up until the fourth trading day after a consensus change. Performing the one-tailed t-tests showed that judging by the p-values, the calculated average returns are also statistically significant for numerous days after a consensus change compared to the mean of all individual returns during the observed period. Table 3 shows that, on average, the first four days after an increase and the first two days after a decrease in the individual stock's target price consensus, as well as the first four days after an increase and the first fifteen days after a reduction of the specific stock's consensus score are statistically significantly different than the mean of all the returns. An important finding is that the effect of the change in the target price consensus as well as the increase in the consensus score is, on average, priced in by the market in just a few days, while a decrease in the consensus score, which means that analysts tilt more to the »sell« recommendation, drags the stock price lower for several consequent days. This could imply

that, as opposed to the short-term effect on the price by the change in the target price consensus, a negative recommendation affects the overall market sentiment for numerous following trading days. The same was concluded by Murg, Pachler & Zeitlberger (2014).

Figure 32: Average annualized return after a change in individual asset's consensus



Source: Own work.

Table 3: *p*-values of one-tailed two-sample-t-tests when testing for a difference in means of returns after a change in consensus vs. all recorded returns during the observed period

Number of days after the change in consensus	Target price		Consensus score	
	Increase	Decrease	Increase	Decrease
Day 1	p < 0.001	p = 0.0039	p < 0.001	p < 0.001
Day 2	p = 0.0029	p = 0.0490	p < 0.001	p < 0.001
Day 3	p = 0.0133	p = 0.4567	p < 0.001	p < 0.001
Day 4	p = 0.0101	p = 0.5644	p = 0.0034	p < 0.001
Day 5	p = 0.0566	p = 0.7843	p = 0.1334	p = 0.0082
Day 6	p = 0.0777	p = 0.9175	p = 0.5157	p = 0.0069
Day 7	p = 0.0927	p = 0.9632	p = 0.5020	p = 0.0080
Day 8	p = 0.1466	p = 0.9925	p = 0.7496	p = 0.0064

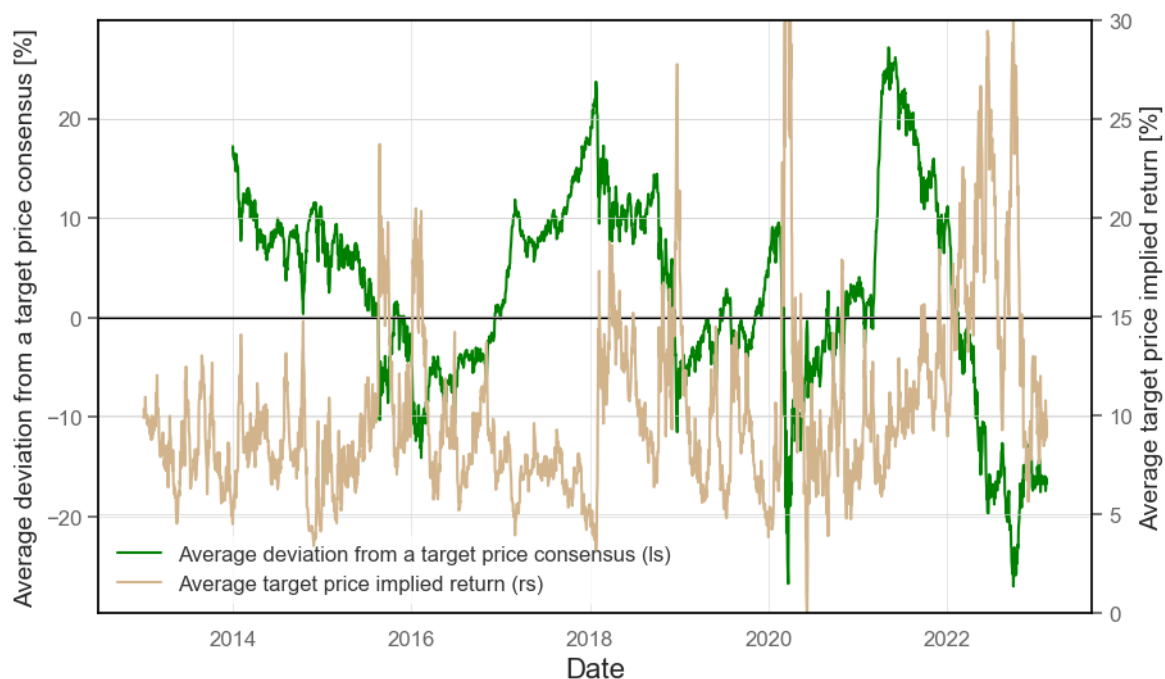
Source: Own work.

Judging by these results, the portfolios with frequent rebalancing, especially daily, tend to marginally capture the effects of new analysts' recommendations on the stock's price. Suppose a particular value of a stock's weight in the portfolio at the end of the trading day

before the new analyst recommendation (take a target price as an example). After a new recommendation is published before the start of a new trading day that increases the consensus target price, the latest analyst target price implied return will increase relative to the one calculated the day earlier, as the stock's actual price will remain the same. The Black-Litterman model will, therefore, overweight that stock relative to the day earlier. As stocks' prices, on average, increase after the better-than-consensus recommendation is published, there is a good chance the portfolio value will marginally increase more because of that than it would have if we would not rebalance the portfolio, given the target price implied return at rebalancing is higher than it was at the previous rebalancing. After the stock's price increases, and if the consensus target price remains the same, the analyst target price implied return will again decrease. The same stock might momentarily be less overweight, or it can even become underweight. The more frequent the portfolio rebalancing is, the more such situations contribute to the overall portfolio value. The same goes for when an analyst publishes a target price lower than the consensus target price, hence, dragging the consensus target price lower, which causes the Black-Litterman model to momentarily underweight that stock relative to the trading day or rebalancing before. This occurrence is more relevant for trading Strategy 1, which forms the view vector indirectly through target price consensus.

I also analyzed the accuracy of the analysts' consensus target prices for the whole observed period (except the year 2013, as target prices are set 12 months in advance). I compared the actual prices to the target price consensus 12 months prior for all 30 stocks (Figure 33).

Figure 33: Average deviation of a real price from a target price consensus made 12 months prior (left) and average implied returns during the observed period (right)

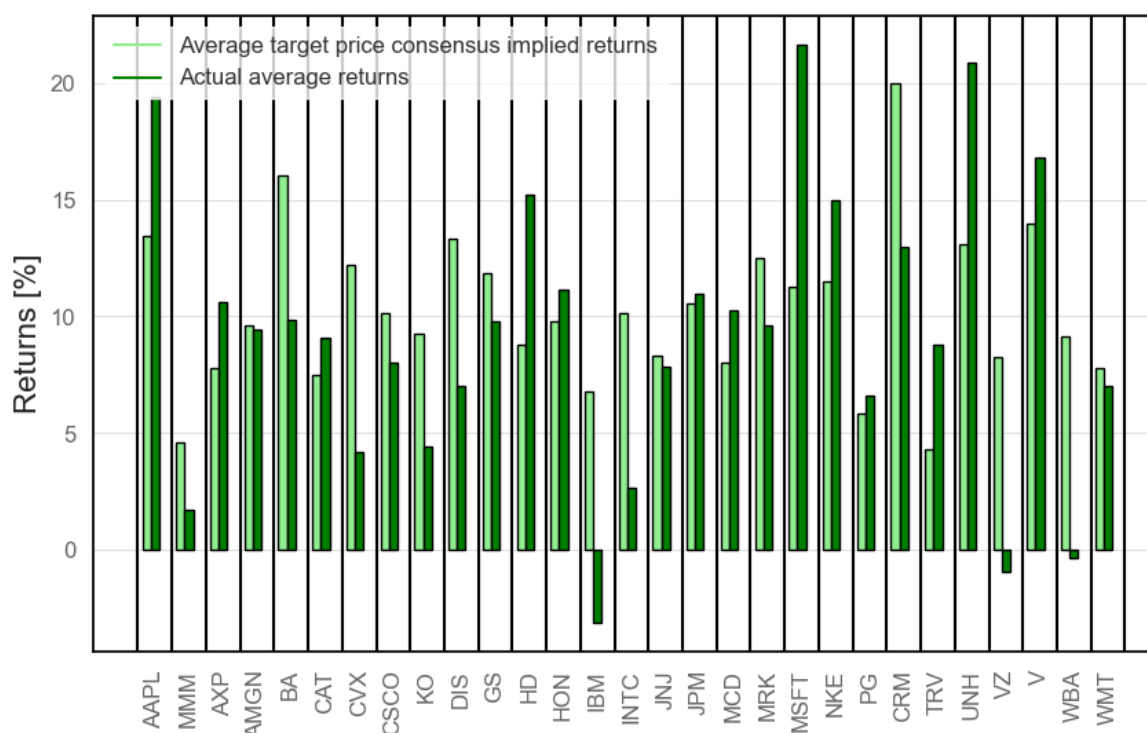


Source: Own work.

The figure shows the average target price implied returns and the average deviations from the target price consensus, representing the percentual difference of a stock's actual price from the 12-month prior target price consensus. On average, only 52.153% of the time, the actual price equaled or exceeded the target price consensus set 12 months earlier, which might act as one of the factors of why, on average, Strategy 1's portfolios did neither outperform nor underperform the benchmark in terms of Sharpe ratio when rebalanced infrequently, for instance, once every year. I believe being accurate in setting a target price is only one aspect of the forecast performance. Other factors that are as much important are accurately forecasting the direction of the price movement and the timing as target price consensus implied returns could increase when a particular stock is supposed to have a bullish trend which would translate into some economic value of analysts' recommendations if it does even though the actual price would not reach the target price, and vice versa.

By comparing the average target price consensus implied returns with the actual average returns of the stocks included in the portfolios (Figure 34), it can be concluded that, on average, analysts' consensus seemed to have underestimated all six best-performing stocks, which were Apple (AAPL), Microsoft (MSFT), Home Depot (HD), Visa (V), UnitedHealth Group (UNH), and Nike (NKE), while significantly overestimated various low-performers, such as 3M (MMM), Boeing (BA), Chevron (CVX), Coca-cola (KO), Disney (DIS), IBM (IBM), Intel (INTC), Salesforce (CRM), Verizon (VZ), and Walgreens Boots Alliance (WBA).

Figure 34: Average target price consensus implied returns vs. actual average returns



Source: Own work.

3.3.1 Limitations and opportunities for further research

Despite gaining valuable insights in writing this thesis, some limitations must be addressed. First and foremost, the conclusions formed are based on a limited and selected scope of stocks and are relevant only for the analyzed period. The results can vary when using a different investment universe and time frame. I recommend further research with equivalent trading strategies on other datasets, which should include a more significant number of stocks and be simulated over a more extended period. In my case, I was limited to 10 years as Bloomberg Terminal's database of analyst recommendations did not span for periods earlier than 1/1/2013.

Another factor that might have influenced the results is that two of the stocks with the largest market capitalization in the investment universe simultaneously exhibited above-average growth throughout the examined period (Apple and Microsoft). As the Black-Litterman model starts with market capitalization weights and then deviates from them, the analysts' recommendations on these two specific stocks might have affected the overall portfolio performance too much, slightly distorting the conclusions on the overall economic value of sell-side analysts' recommendations. Another aspect that should be accounted for in future research is the dividends various companies pay in the investment universe. There could be some marginal discrepancies due to holding different individual weights relative to the benchmark when these companies pay out dividends. Furthermore, the effects of taxes on the profitability of the two trading strategies I used should be analyzed.

I analyzed the consensus target prices and consensus scores for individual stocks, which are formed by Bloomberg's selection of analysts. It would be insightful to obtain individual analysts' historical recommendations and filter out the worst-performing ones, thus, increasing the prediction accuracy and potentially the portfolios' performance. According to findings by Li (2005), the research should be promising as they found that analysts who exhibit above-median risk-adjusted returns also outperform the below-median-performing analysts in the succeeding periods. Another property of using the consensus recommendations is the following. If, for a particular stock, an individual analyst increases their target price or consensus score from their previous forecast, but the target price or score falls short of the overall consensus, the consensus is lowered, and the Black-Litterman strategy accounts for it as a negative recommendation. However, the sentiment of this specific analyst might be positive, hence, the upgraded forecasts. Further research should analyze the changes in individual analysts' forecasts, as this resembles their sentiment for a covered stock.

Another suggestion for further research is the econometric analysis of the economic value of the analysts' 12-month EPS forecasts, where one could, through multiple regression, statistically determine the effect of the EPS on the P/E ratio, thus, incorporating the Black-Litterman's view vector with this type of analysts' forecasts.

4 CONCLUSION

In this master's thesis, I have comprehensively analyzed the value of sell-side analysts' recommendations in active portfolio management. Firstly, despite numerous research papers on the subject, a clear consensus on the profitability of these recommendations remains elusive due to conflicting results. On the other hand, the literature on forming a trading strategy based on the sell-side analysts' consensus is very scarce, and only a few provided a robust empirical analysis through historical simulations. No study has simultaneously examined the incorporation of two forms of analysts' recommendations: the target price consensus and the consensus scores. To closely emulate a real-life investor, I employed the Black-Litterman asset allocation model as the primary methodology. This approach allowed me to initiate the analysis with market equilibrium weights and deviate from them by overweighting stocks with positive analysts' recommendations and underweighting those with negative ones while implementing both before-mentioned types of analysts' recommendations. Many research papers were written on implementing the Black-Litterman model, addressing its advantages and disadvantages. One of the most controversial aspects of the model is its scaling parameter τ , the value of which academics cannot agree. Consequently, I assessed the practical importance of its value for real-life investors and identified the relationship between this parameter, portfolio mean returns, the Sharpe ratio, and its effect on the volatility of the Sharpe ratio as a function of the rebalancing period. To supplement the existing literature on the Black-Litterman model, I also performed the simultaneous simulations using the Treynor-Black model, utilizing the same analyst-implied returns as input and compared the outcomes. The analysis was conducted on the 30 constituent companies of the widely established index Dow Jones Industrial Average over a period spanning from January 1st, 2013, to February 14th, 2023. The historical recommendations consensus was obtained from the Bloomberg terminal.

The first part of the main findings suggests that the sell-side analysts' recommendations provide an economic value to the average investor to some extent. It is especially apparent with high-frequency trading before adjusting for the incurred transaction costs when both trading strategies utilizing the Black-Litterman model outperformed the respective benchmark regarding mean returns and the risk-adjusted returns expressed by Sharpe and Sortino ratios. Interestingly, with decreasing the rebalancing frequency, the outperformance of the portfolios, on average, also decreased. After accounting for the transaction costs, the economic value provided by the analysts mostly diminishes when rebalancing frequently, especially in constrained portfolios. This is, to some extent, in accordance with He, Grant & Fabre (2012) and Gertzell (2013). The former found that the Black-Litterman portfolio with analysts' recommendations tends to significantly outperform the respective benchmark with daily rebalancing; however, no positive statistical significance is achieved after transaction costs. The latter found that the Black-Litterman portfolio tends to outperform the benchmark; however, no statistically significant differences in mean returns were found. I identified five variations of monthly rebalanced trading strategies using target prices as an input that

outperformed the benchmark in terms of mean returns. Two of them, which were non-constrained, also outperformed the benchmark in terms of Sharpe and Sortino ratios. When utilizing consensus scores as an input, I identified four variations of the trading strategy that outperformed the benchmark in terms of mean returns, while one of them outperformed it also in both risk-adjusted return measures. The latter was a constrained portfolio with prohibited short-selling and an upper weight limit of 40% of the portfolio value. This important finding is relevant for an everyday investor as unconstrained portfolios that produce extreme short and long positions are impractical in real life. Performing a monthly rebalancing and assuming transaction costs equal to 1% of the purchased/sold amount, I simulated that an average investor could entirely rely on the analyst consensus scores and outperform the benchmark using the Black-Litterman model with τ set to 0.05, resulting in an annualized mean benchmark-adjusted return of +0.416% and a slightly larger Sharpe and Sortino ratio. This adds to the findings of He, Grant & Fabre (2012), who found that reducing the rebalancing frequency decreases the turnover and the transaction costs incurred, while the portfolio performance is reduced too, which results in underperformance against the benchmark. One aspect that suggests the sell-side analysts provide some economic value is the increased risk-adjusted performance before transaction costs and with daily rebalancing when increasing τ (except the non-constrained Strategy 2 portfolios). This is in accordance with Chen, Da & Schaumburg (2015), who came to the same conclusion in their study based on their results. Overall, it is essential to note that no statistically significant differences in mean returns between the portfolios and the benchmark were found after accounting for the transaction costs.

The second part of the main findings suggests that the sell-side analysts' recommendations can be self-fulfilling prophecies, which is in accordance with the findings of Aitken, Muthuswamy & Wong (2000), Azuma and Okada (2013), and Murg, Pachler & Zeitlberger (2014). The analysis of the average returns on the subsequent days after the change in an individual stock's consensus implied that, on average, new recommendations that are better than consensus cause a positive spike in the stock's price over the next several days, while the recommendations that are worse than consensus drag the stock's price lower. These findings stand for target prices and consensus scores and are statistically significant for multiple subsequent days. A change in consensus score has, on average, a more profound effect on the amplitude of the stock's price drift, and a negative change in a consensus score has the most prolonged effect on the price drift. Similar findings were obtained by Murg, Pachler & Zeitlberger (2014). The following findings act as one of the factors that could explain the outperformance of the portfolios based on the target price consensus strategy with high-frequency trading. I assessed that, on average, analyst target price consensus alone does not provide an investor with much value, as only 52.153% of target prices were achieved or surpassed by an actual price over the 12 months. The self-fulfilling effect of new recommendations can be marginally captured more frequently the more frequent the rebalancing is, as the trading strategy momentarily overweights the specific stock a day

before the positive recommendation's price spike, given the target price implied return at rebalancing is higher than it was at the previous rebalancing, and vice versa.

Another finding that suggests the value of sell-side analysts exists is, on average, higher two-year trailing returns during the observed period of stocks with higher consensus scores. This is in accordance with findings by Jegadeesh, Kim, Krische & Lee (2004) and He, Grant & Fabre (2012). The trading strategy that utilized the consensus scores was formed in a way that it overweighted stocks with higher consensus scores while underweighting stocks with lower ones, which helps explain the outperformance in terms of mean returns relative to the benchmark.

Furthermore, the analysis of the Sharpe ratio relative to the rebalancing period suggests that the controversial parameter τ profoundly affects its volatility. The larger the scaling parameter is, the more significant the overall volatility of the Sharpe ratio is; hence, the lower the predictability of the risk-adjusted performance becomes when varying the rebalancing period. This finding adds to the existing literature and deepens the understanding of the practical effect of τ while using analyst recommendations as an input. It should be in an investor's interest, among other reasons, to apply a low value of τ to reduce the volatility of the portfolio's risk-adjusted performance.

Lastly, I compared the performance of the Black-Litterman trading strategy to another well-established asset allocation model, the Treynor-Black. For an average investor, the Black-Litterman model is much more suitable for utilization of the analysts' target prices as input. The unconstrained Treynor-Black portfolio produced absurd weight amplitudes, while the constrained one underperformed the benchmark even before transaction costs.

All in all, the findings imply that a market is a semi-strong form inefficient as inefficiencies exist; however, they are difficult for traders to take advantage of due to transaction costs, allowing them to continue to exist (Barber, Lehavy, McNichols & Trueman, 2001). The findings of this master's thesis not only offer valuable insights into the economic value of sell-side analysts' recommendations in active portfolio management but also contribute to the ongoing debate regarding the Black-Litterman model and its practical application for real-life investors while also comparing its performance to another popular active asset allocation model, the Treynor-Black. The practical aspect of the controversial parameter τ was assessed. The general relationship between the Sharpe ratio, rebalancing period, transaction costs, and the scaling parameter was determined for two different investment strategies utilizing two of the most well-regarded forms of analysts' recommendations: target prices and scores. This thesis provided a direct comparison of the two, addressing the value of each when it comes to active portfolio management. It should help an average investor choose which recommendation to rely on while making investment decisions. Another contribution to the existing literature was assessing the effect of τ on the volatility of the Sharpe ratio when varying the rebalancing period for constrained and unconstrained portfolios. In this case, Bloomberg's consensus was chosen to represent analysts'

recommendations, which also smooths out low and high-performing individual analysts. Therefore, further work should analyze multiple individual analysts and filter the best and worst-performing ones. Furthermore, the conclusions I formed are based on a limited and selected scope of stocks and are relevant only for the analyzed period. I recommend further research with equivalent trading strategies on other datasets over an extended period while considering the dividends and personal taxes.

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APPENDIX

Appendix 1: Summary in Slovene

Povprečen upravitelj premoženja sledi investicijskim priporočilom nakupnih analitikov (»buy-side«). Raziskave so pokazale, da so priporočila prodajnih analitikov (»sell-side«) donosnejša od priporočil nakupnih (Busse, Green & Jegadeesh, 2012), hkrati pa skladi pri več kot 90% ameriških upraviteljev ne presežejo donosnosti referenčnega indeksa oz. premoženja (Otuteye & Siddiquee, 2019). Namen magistrskega dela je določiti ekonomsko vrednost priporočil prodajnih analitikov pri aktivnem upravljanju premoženja z vključitvijo le-teh v Black-Littermanov model alokacije sredstev. S tem želim posnemati obnašanje upravitelja portfelja, ki po eni strani glede donosnosti sledi referenčnemu indeksu, po drugi pa upošteva priporočila analitikov glede posameznih naložb, zaradi česar relativne uteži teh naložb odstopajo od uteži pri sestavi referenčnega indeksa. Ciljev magistrskega dela je več. Prvi cilj je bil izvesti zgodovinske simulacije donosnosti in tveganja premoženja, katerega sestava temelji na priporočilih analitikov in primerjati tveganju prilagojeno donosnost z referenčnim indeksom. Hkrati sem želel oceniti uspešnost dveh različnih metod vključitve priporočil, to je uporabe 12-mesečnih ciljnih cen in razvrstitve delnic v štiri skupine glede na povprečno oceno analitikov. Poleg tega sem želel analizirati vpliv omejitev, ko gre za uteži posameznih naložb, pogostosti ponovnega uravnoteženja portfelja in transakcijskih stroškov na tveganju prilagojeno donosnost. Drugi cilj, s katerim sem želel prispevati k oceni praktičnosti modela, je bil učinek parametra τ na volatilnost Sharpovega razmerja ob spreminjanju pogostosti uravnoteženja premoženja. Nazadnje sem želel primerjati uspešnost ekvivalentne trgovalne strategije z modelom Treynor-Black. Oblikovana so bila naslednja raziskovalna vprašanja:

- Kakšna je ekonomska vrednost uporabe priporočil prodajnih analitikov v modelu Black-Litterman?
- Katera metoda vključitve priporočil analitikov da portfelj z najvišjo tveganju prilagojeno donosnostjo pred in po transakcijskih stroških?
- Kateri vhodni parametri najbolj vplivajo na tveganju prilagojeno donosnost portfelja?
- Kakšna je tveganju prilagojena donosnost portfelja, oblikovanega na podlagi Black-Littermanovega modela v primerjavi s portfeljem, oblikovanim na podlagi Treynor-Blackovega modela?

V prvem poglavju tega dela je predstavljeno teoretično ozadje obeh modelov in pregled predhodnih raziskav o vrednosti priporočil prodajnih analitikov. Naslednje poglavje opisuje metodologijo. Izvedel sem simulacije v desetletnem časovnem okviru od 1. januarja 2013 do 14. februarja 2023. Portfelje je sestavljalo 30 delnic, ki so trenutno vključene v indeks Dow Jones Industrial Average. Priporočila analitikov so bila v Black-Littermanov model vključena na dva načina. Prvi pristop je temeljil na 12-mesečnem konsenzu analitikov glede ciljne cene. Pričakovana donosnost je bila izračunana in vključena v vektor pogledov glede na trenutno ceno delnice. Drugi pristop je temeljil na razdelitvi naložb v štiri skupine glede na povprečno oceno analitikov. Pričakovana donosnost delnice je bila določena kot drseča

dvoletna zgodovinska povprečna donosnost pripadajoče skupine. Rezultati empiričnega dela so skupaj z diskusijo predstavljeni v tretjem poglavju.

Prvi del ugotovitev nakazuje, da priporočila prodajnih analitikov za povprečnega upravitelja premoženja do neke mere predstavljajo ekonomsko vrednost. To je še zlasti opazno pri pogostem preuravnateževanju brez upoštevanja transakcijskih stroškov, ko sta obe trgovalni strategiji presegli donosnost referenčnega indeksa tako glede povprečnih letnih donosov kot tudi donosov prilagojenih na tveganje. Z znižanjem pogostosti ponovnega uravnateženja se je v povprečju znižala tudi presežna donosnost portfeljev. Po upoštevanju transakcijskih stroškov se ekonomska vrednost, ki jo nudijo priporočila prodajnih analitikov, pri pogostem preuravnateženju večinoma izniči, zlasti v omejenih portfeljih. To je v skladu z ugotovitvami, ki so jih dobili He, Grant & Fabre (2012) ter Gertzell (2013). Identificiral sem več različic trgovalnih strategij z mesečnim preuravnateževanjem, ki po transakcijskih stroških še vedno presegajo referenčni indeks v smislu povprečne in tveganju prilagojene donosnosti. Za povprečnega upravitelja je zanimiv portfelj z omejitvami na podlagi trgovalne strategije z analitskimi ocenami. Scenarij, kjer je imel τ vrednost 0.05, je privedel do 0.416 odstotnih točk višje povprečne letne donosnosti glede na referenčni indeks in rahlo višjih razmerij Sharpe ter Sortino. Po upoštevanju transakcijskih stroškov sicer niso bile dosežene statistično značilne razlike v povprečnih donosnostih med portfelji in referenčnim indeksom. Eden izmed vidikov, ki nakazujejo, da priporočila prodajnih analitikov v neki meri nudijo ekonomsko vrednost, je povečana tveganju prilagojena donosnost pred transakcijskimi stroški in ob dnevnem preuravnateževanju, ko povečujemo τ (razen pri neomejenih portfeljih strategije z analitskimi ocenami). To je v skladu z Chen, Da & Schaumburg (2015).

Analiza Sharpovega razmerja glede na pogostost preuravnateževanja nakazuje, da sporni parameter τ močno vpliva na volatilitnost, ki se viša z vrednostjo parametra, kar pa zmanjšuje predvidljivost tveganju prilagojene donosnosti. Ta ugotovitev dopolnjuje obstoječo literaturo in pogloblja razumevanje praktičnega učinka τ . Primerjava modelov Black-Litterman in Treynor-Black pa je pokazala, da je za povprečnega upravitelja prvi pristop veliko bolj primeren za snovanje trgovalne strategije, kjer so ciljne cene analitikov vhodni podatek.

Drugi del ugotovitev nakazuje, da so lahko priporočila prodajnih analitikov samouresničujejoče se prerokbe, kar so ugotovili tudi Aitken, Muthuswamy & Wong (2000), Azuma in Okada (2013) in Murg, Pachler & Zeitlberger (2014). Analiza povprečnih donosnosti v dneh po spremembi analitskega konsenza nakazuje, da izboljšanje konsenza priporočil v povprečju povzroči rast cene v naslednjih dneh in obratno. Te ugotovitve veljajo za ciljne cene in analitske ocene ter so statistično značilne za več dni, ki sledijo spremembi konsenza. Sprememba konsenza analitske ocene ima v povprečju večji vpliv na ceno, negativna sprememba le-tega pa ima tudi najdalgotrajnejši vpliv. Do podobnih ugotovitev so prišli tudi Murg, Pachler & Zeitlberger (2014). Dodatna ugotovitev, ki tudi nakazuje, da priporočila prodajnih analitikov nudijo ekonomsko vrednost, je v povprečju višja donosnost

delnic z višjimi konsenzualnimi ocenami v opazovanem obdobju, kar je v skladu z ugotovitvami, ki so jih dobili Jegadeesh, Kim, Krische & Lee (2004) ter He, Grant & Fabre (2012).

Ugotovitve tega dela nakazujejo, da za ameriški delniški trg velja srednje močna oblika informacijske učinkovitosti, saj neučinkovitosti obstajajo, vendar jih vlagatelji zaradi transakcijskih stroškov težko izkoristijo (Barber, Lehavy, McNichols & Trueman, 2001). Ugotovitve ne ponujajo le vpogleda v ekonomsko vrednost priporočil prodajnih analitikov pri aktivnem upravljanju portfelja, temveč tudi prispevajo k razpravi o modelu Black-Litterman in njegovi praktični uporabi. Prvi prispevek dela k obstoječi literaturi je neposredna primerjava uspešnosti in praktičnosti vključitve dveh vrst priporočil prodajnih analitikov v model Black-Litterman: ciljne cene in povprečne ocene. Drugi prispevek dela k obstoječi literaturi je ocena učinka τ na volatilitnost Sharpovega razmerja ob spreminjanju pogostosti uravnoteženja tako za omejene kot tudi neomejene portfelje. Sklepi dela temeljijo na analizi na podlagi omejenega števila delnic in se nanašajo le na analizirano obdobje, zato za posplošitev sklepov priporočam enakovredno analizo na večjem številu delnic v daljšem časovnem obdobju.