UNIVERSITY OF LJUBLJANA
FACULTY OF ECONOMICS

MASTER`S THESIS

THE LOW VOLATILITY EFFECT ON STOCK MARKET RETURNS- A MARKET ANOMALY OR A RISK FACTOR?

Ljubljana, December 2016

LAURA GODLER ČOH

AUTHORSHIP STATEMENT

The undersigned Laura Godler Čoh, a student at the University of Ljubljana, Faculty of Economics, (hereafter: FELU), author of this written final work of studies with the title The low volatility effect on stock market returns - a market anomaly or a risk factor? prepared under supervision of izr. prof. dr. Igor Lončarski,

DECLARE

1. this written final work of studies to be based on the results of my own research;
2. the printed form of this written final work of studies to be identical to its electronic form;
3. the text of this written final work of studies to be language-edited and technically in adherence with the FELU's Technical Guidelines for Written Works, which means that I cited and / or quoted works and opinions of other authors in this written final work of studies in accordance with the FELU's Technical Guidelines for Written Works;
4. to be aware of the fact that plagiarism (in written or graphical form) is a criminal offence and can be prosecuted in accordance with the Criminal Code of the Republic of Slovenia;
5. to be aware of the consequences a proven plagiarism charge based on the this written final work could have for my status at the FELU in accordance with the relevant FELU Rules;
6. to have obtained all the necessary permits to use the data and works of other authors which are (in written or graphical form) referred to in this written final work of studies and to have clearly marked them;
7. to have acted in accordance with ethical principles during the preparation of this written final work of studies and to have, where necessary, obtained permission of the Ethics Committee;
8. my consent to use the electronic form of this written final work of studies for the detection of content similarity with other written works, using similarity detection software that is connected with the FELU Study Information System;
9. to transfer to the University of Ljubljana free of charge, non-exclusively, geographically and time-wise unlimited the right of saving this written final work of studies in the electronic form, the right of its reproduction, as well as the right of making this written final work of studies available to the public on the World Wide Web via the Repository of the University of Ljubljana;
10. my consent to publication of my personal data that are included in this written final work of studies and in this declaration, when this written final work of studies is published.

Ljubljana, December 22nd 2016 Author's signature: Laura Godler Čoh

TABLE OF CONTENTS

INTRODUCTION **1**

1 THEORETICAL BACKGROUND **3**

1.1 Portfolio selection and the Capital Asset Pricing Model (CAPM) **3**

1.1.1 Portfolio selection theory 3

1.1.2 CAPM 3

1.1.3 Testing the CAPM 6

1.1.3.1 Cross-sectional tests 6

1.1.3.2 Two pass procedure 7

1.2 Factor models **8**

1.2.1 Size and value factors 10

1.2.2 Liquidity factor 11

1.3 Low volatility and low beta effect **13**

1.3.1 Volatility, beta and Sharpe ratio 13

1.3.2 Possible explanations of low volatility and low beta effect 14

1.3.2.1 Low volatility effect as an anomaly 14

1.3.2.2 Behavioral explanations 15

1.3.2.3 Limits to arbitrage 17

1.3.2.4 Fundamental factors 17

1.3.2.5 A liquidity explanation 17

2 PREVIOUS EMPIRICAL RESEARCH **18**

2.1 The initial tests of CAPM **18**

2.2 More recent tests **20**

3 DATA AND METHODOLOGY **23**

3.1 Data **23**

3.1.1 The S&P 600 index 23

3.1.2 The data on the constituents of S&P 600 25

3.1.2.1 Stock returns 26

3.1.2.2 Ask and bid prices (in U.S. dollars) 27

3.1.3 The data on Fama-French factors 28

3.1.3.1 Size (SMB) and value (HML) 28

3.1.3.2 Market return in excess of the risk-free rate of return (R_m-R_f) 29

3.2 Methodology **29**

3.2.1 Testing for the low volatility effect 29

3.2.2 Creating low volatility portfolios 30

3.2.2.1 Sorts on volatility - details 33

3.2.3 Double sorted portfolios – Independent sorts 35

3.2.4 Double sorted portfolios - Dependent sorts on liquidity and volatility 36

3.2.5 Double sorted portfolios - Dependent sorts on volatility and liquidity 37

3.3 Assessment of performance of the portfolios **37**

4 RESULTS **38**
4.1 Overview of the data **38**
4.2 Volatility portfolios **39**
4.3 Double sorted portfolios – independent sorts **47**
4.4 Double sorted portfolios - volatility within the liqudity groups **56**
4.5 Double sorted portfolios - liqudity within the volatility groups **62**
4.6 Robustness tests **66**
4.6.1 Sorting stocks into five volatility portfolios 66
4.6.2 Using a different proxy for the market 67
4.6.3 The impact of the financial crisis on the results 69
4.6.3.1 Volatility sorted portfolios 69
4.6.3.2 Independent sorts 70
4.6.3.3 Dependent sorts on liquidity and volatility 72
4.6.3.4 Dependent sorts on volatility and liquidity 74
CONCLUSION **76**
REFERENCE LIST **79**

LIST OF TABLES

Table 1. Independent sorts on liquidity and volatility 35
Table 2. Dependent sorts on liquidity and volatility 36
Table 3. Dependent sorts on volatility and liquidity 37
Table 4. Properties of the low volatility portfolios calculated in the period 6 Jan 2006 – 29 Jan 2016 (annualized log returns) 40
Table 5. End-of-period (total) return, risk and Sharpe ratio of volatility sorted portfolios, S&P 600 index, the market and equal-weighted portfolio of S&P 600 constituents (6 Jan 2006 – 29 Jan 2016) 43
Table 6. Annualized Fama-French adjusted alphas and R-squared values of the volatility-sorted portfolios 44
Table 7. Annual (end-of-year) excess returns, ex-post risk and Sharpe ratios of volatility sorted portfolios (in %) 45
Table 8. Annual (end-of-year) excess returns, ex-post risk and Sharpe ratis of minimum volatility portfolio, S&P 600 and the market (in %) 46
Table 9. Annualized average excess returns of the double sorted portfolios - independent sorts 48
Table 10. Annualized standard deviations of the double-sorted portfolios – independent sorts 48
Table 11. Sharpe ratios of the double-sorted portfolios – independent sorts 48
Table 12. CAPM alphas of the double-sorted portfolios – independent sorts (the alphas are annualized) 49

Table 13. R-squared values of the double-sorted portfolios – independent sorts 49
Table 14. Average number of stocks in the double-sorted portfolios – independent sorts . 49
Table 15. The differences between annualized average excess returns of minimum and maximum volatility portfolios at similar levels of liquidity 49
Table 16. Annualized Fama-French adjusted alphas of the double-sorted portfolios – independent sorts 50
Table 17. Year-by-year excess returns, risk and Sharpe ratios of maximum liquidity portfolios and the market (in %) 54
Table 18. Annualized average excess returns of the double sorted portfolios - dependent sorts on liquidity and volatility 56
Table 19. Annualized standard deviations of the double-sorted portfolios – dependent sorts on liquidity and volatility 56
Table 20. Sharpe ratios of the double-sorted portfolios – dependent sorts on liquidity and volatility 56
Table 21. Annualized CAPM alphas of the double-sorted portfolios – dependent sorts on liquidity and volatility 57
Table 22. R-squared values from the CAPM regressions of the double-sorted portfolios – dependent sorts on liquidity and volatility 57
Table 23. Annualized Fama-French adjusted alphas of the double-sorted portfolios – dependent sorts on liquidity and volatility 58
Table 24. Year-by-year excess returns (in %), standard deviations (in %) and Sharpe ratios of maximum liquidity portfolios and the market 61
Table 25. Annualized average excess returns of the double sorted portfolios - dependent sorts on volatility and liquidity 63
Table 26. Annualized standard deviations of the double-sorted portfolios – dependent sorts on volatility and liquidity 63
Table 27. Sharpe ratios of the double-sorted portfolios – dependent sorts on volatility and liquidity 63
Table 28. Annualized CAPM alphas of the double-sorted portfolios – dependent sorts on volatility and liquidity 63
Table 29. R-squared values from the CAPM regressions of the double-sorted portfolios – dependent sorts on volatility and liquidity 64
Table 30. Annualized Fama-French adjusted alphas of the double-sorted portfolios – dependent sorts on volatility and liquidity 64
Table 31. Properties of 5 volatility sorted portfolios (annualized log returns)................... 67
Table 32. Annualized average excess returns, standard deviations and Sharpe ratios of the volatility-sorted portfolios (observations from 30 May 2008 – 31 July 2009 excluded) 70
Table 33. Annualized Fama-French adjusted alphas of the volatility-sorted portfolios (observations from 30 May 2008 – 31 July 2009 excluded from the regression) 70

Table 34. Annualized average excess returns of the double-sorted portfolios – independent sorts (observations from 30 May 2008 – 31 July 2009 excluded)...................... 71
Table 35. Annualized standard deviations of the double-sorted portfolios – independent sorts (observations from 30 May 2008 – 31 July 2009 excluded)...................... 71
Table 36. Sharpe ratios of the double-sorted portfolios – independent sorts (observations from 30 May 2008 – 31 July 2009 excluded).. 72
Table 37. Annualized Fama-French adjusted alphas of the double-sorted portfolios – independent sorts (observations from 30 May 2008 – 31 July 2009 excluded from the regression)... 72
Table 38. Annualized average excess returns of the double-sorted portfolios –dependent sorts on liquidity and volatility (observations from 30 May 2008 – 31 July 2009 excluded).. 73
Table 39. Annualized standard deviations of the double-sorted portfolios –dependent sorts on liquidity and volatility (observations from 30 May 2008 – 31 July 2009 excluded).. 73
Table 40. Sharpe ratios of the double-sorted portfolios –dependent sorts on liquidity and volatility (observations from 30 May 2008 – 31 July 2009 excluded)............... 73
Table 41. Annualized Fama-French adjusted alphas of the double-sorted portfolios – dependent sorts on liquidity and volatility (observations from 30 May 2008 – 31 July 2009 excluded from the regression).. 74
Table 42. Annualized average excess returns of the double-sorted portfolios – dependent sorts on volatility and liquidity (observations from 30 May 2008 – 31 July 2009 excluded).. 75
Table 43. Annualized standard deviations of the double-sorted portfolios – dependent sorts on volatility and liquidity (observations from 30 May 2008 – 31 July 2009 excluded).. 75
Table 44. Sharpe ratios of the double-sorted portfolios – dependent sorts on volatility and liquidity (observations from 30 May 2008 – 31 July 2009 excluded)................ 75
Table 45. Annualized Fama-French adjusted alphas of the double-sorted portfolios – dependent sorts on volatility and liquidity (observations from 30 May 2008 – 31 July 2009 excluded from the regression).. 76

LIST OF FIGURES

Figure 1. Sector breakdown of S&P 600 (in %) 25
Figure 2. Number of stocks for which the data for the calculation of (1-year) standard deviation is available 39
Figure 3. Cumulative excess returns of volatility sorted portfolios 42
Figure 4. The values of Fama-French adjusted alphas and betas of volatility sorted and independently double sorted portfolios 51
Figure 5. Cumulative excess returns of minimum liquidity portfolios and the market 52

Figure 6. Cumulative excess returns of medium liquidity portfolios and the market 53
Figure 7. Cumulative excess returns of maximum liquidity portfolios and the market 54
Figure 8. Cumulative excess returns of all three minimum liquidity portfolios and the market 59
Figure 9. Cumulative excess returns of all three medium liquidity portfolios and the market 60
Figure 10. Cumulative excess returns of all three maximum liquidity portfolios and the market 60
Figure 11. Cumulative excess returns of all three minimum volatility portfolios and the market 65
Figure 12. Cumulative excess returns of all three medium volatility portfolios and the market 65
Figure 13. Cumulative excess returns of all three maximum volatility portfolios and the market 66
Figure 14. Cumulative excess returns of S&P 600 constituents, the market and S&P 600 index 68

INTRODUCTION

One of the basic predictions in finance is the positive relation between risk and expected return. Investors who are willing to take on more risk, should be rewarded with higher returns and those who are more risk-averse should be paid lower returns. However, several empirical studies of the risk-return relation such as Clarke, De Silva and Thorley (2006), Blitz and van Vliet (2007) and Blitz, Pang and van Vliet (2012) have shown the exact opposite: low risk stocks yield higher returns on average than do high risk stocks. This effect has in the existing literature become known as the low volatility effect (see for example Yamada (2013)).

The risk-return relationship is commonly expressed by the Capital Asset Pricing Model (hereinafter: the CAPM) which was built in the early 1960`s on the grounds of Markowitz`s (1952) portfolio selection theory. The CAPM states that the expected return of a stock is positively related to market risk premium and beta, the former being the return on the market portfolio in excess of the risk-free rate and the latter being the measure of a stock`s systematic risk. Since beta is by definition positively related to return volatility of an asset, the low volatility effect is as such a contradiction of the CAPM.

The outperformance of low volatility stocks suggests that high volatility stocks are overpriced compared to their low volatility counterparts, which leads to the former having lower returns on average than the latter. What is the reason for such mispricing? Is it an anomaly or perhaps an incompletion of the single factor model (CAPM)? The existing literature offers some potential reasons for what appears to be an anomalous risk-return relation, for example, leverage restrictions and behavioral biases of the investors. I have focused on the one that perhaps has been given less attention, but may just as important: the possibility that this mispricing is in fact a realization of a liquidity risk premium. The intuition behind it is that less liquid stocks are often not traded as frequently as the liquid ones, and thus have lower return volatility due to less frequent price movements. It follows that such low volatility stocks are in fact more risky than they appear to be and thus yield higher returns because of the liquidity risk premium.

Since small cap stocks are often the ones that are less liquid and have a lower trading volume (are less frequently traded), I have chosen a sample of small cap stocks for my empirical research. I have gathered the data on the constituents of the Standard & Poor`s SmallCap 600 (hereinafter: S&P 600) index in the period from 2006 – 2016. With the aim of testing the existence of a low volatility effect in the United States (hereinafter: U.S.) small cap equity segment and its connection with liquidity, I have formed three sets of research questions that I will try to answer based on the tests I have performed on my sample of data:

The first set of research questions refers to finding evidence of a low volatility effect in the U.S. small cap equity segment in the period from 2006 – 2016:

- Is the alpha of the portfolio with the lowest volatility positive and statistically significant?
- Does the value of alpha of low volatility portfolio exceed the value of medium and maximum volatility portfolios?
- Does the inclusion of the Fama-French factors significantly change the results? So, does the alpha of any portfolio decrease in value and statistical significance when these two variables are added into the model?

The second set of research questions refers to the relation of the low volatility effect to stocks` liquidity:

- When the portfolios are independently sorted by both, liquidity and volatility, what happens to the values and the significance of the intercepts (alphas)?
- Is there evidence of a low volatility effect when the portfolios are compared at similar levels of liquidity?

The third set of research questions refers to how the use of dependent sorting (volatility within liquidity sorts and vice-versa) changes the results:

- What happens to the values and the significance of the intercepts (alphas) when the stocks are sorted by liquidity and by volatility within the liquidity groups?
- Do the results change significantly when the stocks are first sorted by volatility and then by liquidity?

In order to answer the research questions, I have sorted stocks into portfolios by their return volatility and assessed the results of these volatility-sorted portfolios. I have then used a double sorting procedure where stocks were ranked by both, their average bid-ask spread-price ratio (liquidity) and by volatility. Three types of sorting were used. First the stocks were sorted independently by both variables where groups with different combinations of liquidity and volatility were formed. Next, the stocks were sorted dependently, thus they were sorted by liquidity and subsequently by volatility within each liquidity group. The procedure was then the reversed, so that the stocks were sorted on volatility first and subsequently by liquidity within each volatility group. Then the performance of all the double sorted portfolios was assessed. For all the volatility sorted and the double sorted portfolios simple CAPM regressions and Fama-French regressions were run - the latter included size and value factors to control for possible exposures to these effects. The performance of all the portfolios was compared to each other, the market portfolio and the S&P 600 index itself.

The thesis is organized as follows. The next section briefly describes the theoretical background of low volatility and low beta effect. In Section 2 some of the important work that has been done so far on this subject is presented. Section 3 describes the data and methodology of my empirical research. Section 4 presents the results and the last section concludes.

1 THEORETICAL BACKGROUND

Since the early 1960`s, several models have been developed to explain the cross section of expected stock returns. The ones that are most relevant for my research are briefly described in this section.

1.1 Portfolio selection and the Capital Asset Pricing Model (CAPM)

1.1.1 Portfolio selection theory

The theory of asset pricing as we know today was to a great extent built on the foundations of Harry Markowitz`s Nobel prize winning paper on portfolio selection which he wrote in 1952. Markowitz`s main contribution was in recognizing the importance of the diversification effect that arises when an investor combines individual assets into portfolios. In his mean-variance analysis a risk-averse investor focuses on the expected return and risk, but also on the comovement of returns of individual assets that are included in the portfolio. Having this in mind, Markowitz solved the problem of constructing an efficient portfolio (Pennacchi, 2007).

Markowitz`s portfolio selection rules can be interpreted as both, normative and positive. On the one hand, they provide investors with a "recipe" on how to invest, but on the other hand they can also be seen as a description of investors` behavior. Looking from the latter perspective, a logical extension is to consider an equilibrium of all the investors behaving rationally, and then think of the asset pricing consequences of investors` individual actions. This way, portfolio choice theory has provided a foundation for an asset pricing model – the CAPM (Pennacchi, 2007).

1.1.2 CAPM

The CAPM was developed independently by four individuals: Jack Treynor (1961), William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) (Pennacchi, 2007). Before its development, there was no formal model that would describe and quantify the trade-off between risk and return (Ackert & Deaves, 2010). The model assumes that investors behave rationally in the sense of Markowitz, i.e. they only care about expected return and risk and they want to achieve the highest return at a given amount of risk (or, equivalently, the lowest risk at a given return).

Two types of risk exist according to CAPM: **diversifiable** (also known as idiosyncratic or non-systematic), which can be eliminated by investing in assets whose returns move in the opposite directions and **non-diversifiable** (systematic or market risk) which reflects general conditions in the market and thus cannot be diversified away. The idea behind it is that idiosyncratic risk is the individual risk that belongs to each asset and has to do with events that are specific to each firm. For example, launching a new product or changing the company`s management will only affect this particular company, and consequently the price of its stocks. Systematic risk, on the other hand, is common to all companies and leads to broader market volatility. So, it is natural to assume that the idiosyncratic risk can be eliminated by holding a well diversified portfolio. Thus, in the CAPM world, the expected return on any asset is related solely on the part of the total risk that is generated by the common risk factor. And if rational and risk-averse investors hold diversified portfolios with no asset-specific risk, the unsystematic component should not be priced (Kurach, 2013).

The CAPM is based on the following assumptions (Alexander, 2008):

- There exists a risk-free asset and there is unlimited lending and borrowing at the risk-free rate of return. The risk-free rate is known in advance and is therefore not a random variable.
- All assets are fully described by their expected return, standard deviation of returns and correlations of returns with other assets.
- There are no limitations with buying and selling assets, i.e. they can be bought and sold in any quantity.
- All investors share the same information.
- All investors are risk-averse which means they prefer the portfolio with the lowest risk for a given level of return.

Under these assumptions it can be shown that all investors will hold portfolios that are a combination of the market portfolio and the risk-free asset. The CAPM introduces the concept of **market beta** of an asset which is a measure of an asset`s systematic risk. The model states that an asset with no systematic risk should earn the risk free rate and any additional return in excess of the risk-free rate should be proportional to the systematic risk. The market beta is defined as the ratio between the covariance of the asset return and the market and the variance of the market return and as such represents the share of an asset`s risk in the market portfolio. Assuming all investors hold the market portfolio, they wish to know how much compensation in terms of excess return must there be in order to add this risky asset into their portfolio (Alexander, 2008).

The linear relation between excess return on the market and beta can be expressed by the following equation, called the **security market line**[1]:

$$R_i - R_f = \beta_i(R_m - R_f) \quad (1)$$

where $R_i - R_f$ and $R_m - R_f$ are the excess return of the i-th asset and excess return on the market portfolio, respectively and β_i is the CAPM beta. The beta measures an asset`s sensitivity to the market and is defined as the ratio of the covariance of an asset`s returns ($cov\ (R_i, R_m)$) with the market returns and the variance of market returns ($var\ (R_m)$):

$$\beta_i = \frac{cov\ (R_i, R_m)}{var\ (R_m)} \quad (2)$$

Assuming R_i and R_m are random variables equation (1) can be written in terms of expectations:

$$E(R_i - R_f) = \beta_i E(R_m - R_f) \quad (3)$$

So, expected returns depend linearly on the expected return on the market through beta. Since R_f is non-stohastic ($E(R_f) = R_f$) it could be taken out of the expectations.

The existence of a positive linear relation between expected returns of securities and their market betas is the primary implication of CAPM. Variables other beta should not capture the cross-sectional variation in expected returns (Bali & Cakici, 2008). Thus, if CAPM is the correct pricing model, the cross section of stock returns should be entirely explained by their betas. As described in the later sections of this thesis, this is often not the case in practice.

The model described above is the Sharpe-Lintner-Mossin version of CAPM. Another more general version of the model was developed by Black (1972) in response to the lack of empirical power of the original model. The main difference between the two models is that Black`s version, often referred to also as zero-beta CAPM does not assume the existence of the risk-free asset. The latter is replaced by the unobservable return on a zero-beta portfolio which is uncorrelated with the market, so: $cov(R_0, R_m) = 0$ (Greene, 2002).

Black`s version of the model assumes a positive relation between market beta and expected return, but the relation is weaker than originally suggested by the Sharpe-Lintner`s version, so that it allows a weak form of the low beta effect (Yamada, 2013). The main difference

---

[1] When volatility is used instead of beta, the equation is called capital market line.

in testing the Sharpe-Lintner-Mossin and Black`s version is that in the latter real returns are used instead of excess returns.

1.1.3 Testing the CAPM

1.1.3.1 Cross-sectional tests

According to the CAPM, differences in average returns in a cross sections of stocks depend linearly and only on CAPM betas. Under the assumption of homogenous investors` expectations and a known non-stohastic risk-free rate, the most straightforward test of CAPM is $\alpha_i = 0$ in the following excess return regression (Cuthbertson & Nitzsche, 2004):

$$ER_{i,t} - r_t = \alpha_i + \beta_i(ER_m - r)_t \tag{4}$$

where $ER_{i,t} - r_t$ and $(ER_m - r)_t$ are the expected excess return on asset i and the expected excess return on the market, respectively, and α_i and β_i are the constant and coefficient to be estimated. As regards the return distributions, it can only be assumed they are temporally independently and identically distributed (iid), because returns can be contemporaneously correlated across assets: $E(\varepsilon_{it}\varepsilon_{jt}) \neq 0$.

Since expectations are not observable, the parameters in equation (4) must be estimated using historical data.

There are two serious problems with the estimation of equation (4). The first one has to do with individual stock`s volatility which is too large compared to the number of time periods (T)[2] and it is causing problems with statistical inference, i.e. one cannot reject the hypothesis that average returns across different stocks are the same. One way to solve this problem is to sort the stocks into portfolios in such a way that the differences in average returns will be maximized. Thus, the grouping needs to be based on a variable that has a significant effect on the cross section of stock returns, otherwise the differences in average returns between groups will be negligible and we will not be able to test the CAPM. For example, grouping on size and book-to-market ratio is very common[3] (Cuthbertson & Nitzsche, 2004).

---

[2] The estimated standard errors are calculated using the following formula: $= \frac{\sigma}{\sqrt{T}}$, which means even if T is large, we might not be able to reject the null hypothesis of average returns being the same across stocks.

[3] The size and value effect are described in section 1.2.1.

The second problem is the measurement error in betas that can arise in the two-stage procedure that is usually used for the estimation of cross-section tests (described in the next section). The procedure includes the estimation of time-series regressions for each asset in the sample (so, regressing the excess return of an asset on the excess return of the market) and then using the estimates of beta from the first step in the cross-section regression of assets` average stock return on the betas. The issue is that the betas from the first step are measured with error. This is known as the "errors-in-variables" problem[4]. The stocks whose beta is high will tend to have a positive measurement error, and the opposite holds for the low-beta stocks, these will tend to have a negative measurement error. So, when the estimates of betas form the first step are used in the second step as regressors in the cross section regression, the systematic pattern in the measurement error will cause the coefficient to be an underestimate of its true value, undervaluing the importance of beta in the regression. This is because a low beta stock, whose estimated beta is an underestimation, will have an average return that is too high compared to the (undervalued) beta estimate and a high beta stock will have an average return that is too low. This will be reflected in the lower value of the regression coefficient. The "errors-in-variables" problem can be solved by assigning individual stocks into a small number of portfolio betas which are estimated through a time series regression of portfolio returns on the market returns. Such a procedure should minimize the error in estimating betas (Cuthbertson & Nitzsche, 2004).

With the use of portfolios instead of individual stocks the effect is at least partially removed (cancelled out). In addition, using portfolios also takes into account the diversification effect so that the systematic component of risk is measured more precisely. This is because a part of asset-specific risk is shifted away (Kurach, 2013).

1.1.3.2 Two pass procedure

As mentioned, the cross section tests of CAPM usually take form of a two pass procedure where in the first pass returns on individual stocks are regressed on a market index (which serves as a proxy for the return on the market) in order to obtain betas for individual stocks. For each stock the following time series regression is run:

$$R_{it} - r_t = \alpha_i + \beta_i(ER_m - r)_t + \varepsilon_{it} \tag{5}$$

[4] The errors-in-variables problem was first pointed out by Blume (1970) and later used by Friend and Blume (1970), Black, Jensen and Scholes (1972) and Fama and MacBeth (1973).

where α_i and β_i are the coefficients to be estimated. The estimates of β_i can then be used in the second pass of the procedure where the sample average monthly returns $\bar{R}_\iota$ are regressed on these estimates in the following cross-section regression (on all k-securities):

$$\bar{R}_\iota = \lambda_0 + \lambda_1 \hat{\beta}_\iota + v_i \quad (6)$$

where the coefficients λ_0 and λ_1 are constant across assets. If we compare equation (6) with the standard CAPM equation:

$$R_i = r + \beta_i (R_m - r) + \varepsilon_i \quad (7)$$

we expect $\lambda_0 = \bar{r}$ and $\lambda_1 = \bar{R}_m - \bar{r} > 0$ in order for the CAPM to hold. The bar here indicates sample mean value of returns of each individual stocks. Additionally, one could include one or more other variables in the second pass regression in order to test whether beta ($\beta_i, i = 1, \ldots k$) is the only factor that affects $\bar{R}_\iota$. This would be an even stronger test of the validity of the model (Cuthbertson & Nitzsche, 2004). In practice, the two pass procedure is widely used for the estimation of (multi)factor models which were developed in response of the lack of theoretical and empirical power of the CAPM as a single factor model.

1.2 Factor models

Te evidence against the CAPM has led to the development of factor models which are generally based on the **arbitrage pricing theory (hereinafter: the APT)**. The APT, which was introduced by Ross (1976), assumes there are several market-wide variables (factors) affecting asset returns. So, unlike the CAPM which assumes there is only one factor that is priced, i.e. the return on the market portfolio, the APT assumes the expected returns are a function of the sensitivity of stock returns to multiple factors which are assumed to reflect systematic risk (Cavenaile, Dubois, & Hlavka, 2009).

The sensitivities of the return on asset i to each of the factors are known as **factor betas**. The APT leads to a regression model:

$$R_{it} = a_i + \sum_{j=1}^{k} b_{ij} F_{jt} + \varepsilon_{it} \quad (8)$$

where $F_{j,t}$ is the j-th factor, $b_{i,j}$ is the beta of the j-th factor and $\varepsilon_{i,t}$ is the error term. With the use of the condition on the absence of arbitrage, it can be proven that equation (8) gives an expression for the equilibrium return ER_i on any risky asset:

$$ER_i = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \cdots + \lambda_k b_{ik} \quad (9)$$

where λ_k is the risk premium (i.e. the price of beta risk) of the j-th factor. Equation (9) shows that the expected excess return on any asset i depends linearly on a set of factor betas (Alexander & Sheedy, 2004). Thus, instead of including only the return on the market into the regression, one can add several macroeconomic and firm specific factors. For example Chen, Roll and Ross (1986) include macroeconomic factors such as growth in the industrial production and change in the expected inflation, and Fama and French (1993) propose firm specific factors such as size and book-to-market ratio. The APT is in this sense more general than the CAPM because it allows variation in multiple factors to influence the average returns (Alexander & Sheedy, 2004).

Although the CAPM and the APT use similar estimation techniques, the theoretical background is different: while the CAPM is an equilibrium model derived from an agent`s utility maximization, the APT is driven by the absence of arbitrage opportunities in an efficient market (Cavenaile et al., 2009).

The APT is based on two fundamental principles: (i) **a linear return generating k-factor model** (see equation 8) and (ii) **the absence of arbitrage**. The idea of the no-arbitrage condition is straightforward: if no arbitrage opportunities exist, perfect substitutes in financial markets should have the same price. In other words, two assets with the same expected cash flows and the same amount of risk should have the same price[5]. This is known as the **law of one price** and it is an immediate implication of the absence of arbitrage without being equivalent (Koch, 1996).

Even though CAPM and APT are different by conception, it is possible to view CAPM as a special case of the APT with a single factor. The CAPM has also been extended to multi-beta setting and approaching the APT from the other side (Koch, 1996).

The intercept in the multifactor models is called Jensen`s alpha and is often used to measure abnormal performance after the effects of all the factors in the model have been taken into account (Alexander & Sheedy, 2004). Put differently, Jensen`s alpha is the part of the return that is left unexplained by the model (in addition to the error term). In the empirical part of my research I have focused on the values of the intercepts from the CAPM and Fama-French regressions as the most important indicator of the portfolios` performance.

---

[5] If this is not the case, arbitrage opportunities exist and profits can be made without having to bear any risk.

1.2.1 Size and value factors

Some of the early empirical tests of CAPM have raised doubt in the model`s ability to explain stock returns leading the researches to wonder whether sensitivity to market risk is in fact the only factor that affects stock returns. The two great anomalies discovered around 1980 were the size and value anomaly. Small cap stocks were earning higher returns compared to large cap stocks and value stocks, so stocks with a low price-to-earnings ratio and high book-to-market ratio, were outperforming growth stocks (Falkenstein, 2010).

The intuition behind the **size effect** is the following: smaller stocks are riskier because small firms are usually more distressed, less stable and more likely to fail than larger stable firms. Therefore higher return should be paid on small stocks, in other words, they are sold at a price discount. The issue with many empirical studies was the fact that CAPM beta was in fact capturing the size effect. Once the size effect was taken into account, the relationship between beta and return became flat (see for example, Fama and French, 1992). This is because beta and size are strongly correlated and it is often hard to distinguish between the two effects (Falkenstein, 2010).

Another great financial anomaly was discovered in the late 1970`s known as the **value effect** and has since then been reported in several papers. The value effect refers to the positive relation between asset returns and the ratio of accounting based measures of cash flow or value to the market price of an asset such as earnings per share and book value of equity per share. Basu (1977) was the first to include value-related variables into the original model (CAPM) and found a significant positive relation between earnings-to-price ratio (E/P) and average returns of U.S. stocks that could not be explained by the CAPM. Rosenberg, Reid and Lanstein (1985) and DeBondt and Thaler (1987) have found book-to-price ratio (B/P) to positively affect stock returns. Other value ratios which include cash flow in the numerator instead of earnings were also found to significantly affect stock returns (Keim, 2006).

The work on size and value effect was further complemented by Fama and French (1992) who used data on size (proxied by market equity), book equity divided by market equity (BE/ME), E/P and leverage of all non-financial firms[6] of NYSE, AMEX and NASDAQ in the period 1962 – 1989 and showed that the combination of size and book-to-market absorbs the roles of all other variables in the model (leverage and E/P).

---

[6] Non-financial firms are used because these firms are often highly leveraged which is considered normal in this industry, while in other industries it more often means distress. The results could have been biased if all the firms were used.

Fama and French (1993) continued their work by developing their famous three-factor model where expected returns are linearly related to the risk premium on the market, the premium related to size of a firm and the premium related to its book-to-market ratio (Cavenaile et al., 2009). The expected return on a stock (or portfolio) then equals:

$$E(R_i) - R_f = b_i\left[E(R_m) - R_f\right] + s_i E(SMB) + h_i E(HML), \qquad (10)$$

where $E(R_m) - R_f$ is the expected excess return on a broad market portfolio, SMB (small-minus-big) is the difference between the return on a portfolio of small stocks and a portfolio of large stocks and HML (high-minus-low) is the difference between the return on a portfolio of value stocks (high book-to-market ratio) and a portfolio of growth stocks (low book-to-market ratio). The former can be thought of as a size premium and the latter as a value premium. Since equation (10) is written in terms of expectations, $E(R_m) - R_f$, $E(SMB)$ and $E(HML)$ represent expected premiums and b_i, s_i and h_i are factor sensitivities which can be estimated from running a time series regression of (10).

Fama and French (2015) continued their work by adding new factors to the three factor model described above. Their five factor model which includes profitability and investment factors in addition to size and value, was found to perform better than the original three factor model. In addition, Fama and French found the inclusion of profitability and investment factors made the value factor redundant for describing average returns.

In my analysis I have used both factors, SMB and HML in order to adjust the alpha for the possible exposure to these factors. This is because it is very possible that my choice of a small cap universe of stocks captures the size and value bias (as well as liquidity bias as mentioned before). Small stocks often tend to be value stocks and tend to be less liquid, at least the ones which are less frequently traded.

1.2.2 Liquidity factor

One of the CAPM assumptions is that markets are perfect and in that sense all stocks are liquid. In the perfect CAPM world assets are freely bought and sold without any limitations. However, in reality not all assets are liquid or at least do not have the same level of liquidity. For example, currency is the most liquid asset in the world, while coins, real estate and stamps on the other hand are considered some of the least liquid assets. As argued by Amihud and Mendelson (1986) the latter three have yielded substantial returns for its holders due to their low liquidity, as have art work and some privately placed security issues.

Liquidity is defined as the ease of trading in security, in other words, it means how quickly an asset can be transformed into cash. It depends on liquidity how much price discount is needed to liquidate the security in a short period of time (Multi-factor Models and Liquidity, 2013). It is rather intuitive that less liquid stocks should yield higher returns on average because they are in that way riskier. An investor should thus earn a liquidity risk premium in order to compensate him for bearing the risk of not being able to sell the stock as soon as he would want to and at a price that he would want to sell it. The price of such stock should thus include a discount.

Given that investors require compensation for liquidity costs such as search costs, transaction costs, etc., this affects security prices. Another thing to note is that liquidity varies in time and risk averse investors will also require compensation for liquidity risk (Multi-factor Models and Liquidity, 2013).

Since liquidity is not directly observable, some proxies need to be established for its inclusion into any model that we want to empirically test. There are a few proxies one could use, such as bid-ask spread or trading volume. In my analysis I have used bid-ask spread as a share of price in order to make the variable comparable across stocks.

Amihud and Mendelson (1986) were one of the first that examined the role of liquidity in the capital assets pricing. They focused on the relation between stock returns and their bid-ask spreads taking into account also the planning horizons of investors, that is, how long they plan on holding the assets. They tested the hypothesis that expected return of an asset is an increasing and concave function of its bid-ask spread. The evidence on NYSE stocks in the period of 1961 – 1980 showed a highly significant positive effect on stock returns and that there was indeed a concave relationship between liquidity and returns since the biggest differences in returns per unit change in bid-ask spread were at low values of bid-ask spread (high liquidity). As illiquidity increased, the compensation (additional return) per unit of illiquidity decreased.

Amihud and Mendelson (1986) point to another characteristic of liquidity risk worth mentioning at this point. While investors can reduce security risk by holding well diversified portfolios, there is little they can do to avoid the costs of illiquidity. This is because non-systematic risk can be almost entirely eliminated by forming a zero-beta portfolio, which is not the case for illiquidity costs since these are always additive.

Pastor and Stambaugh (2003) also included liquidity into their asset pricing model, more precisely they investigated whether return sensitivity to aggregate liquidity is priced. They tested whether cross sectional differences in expected stock returns are related to returns sensitivity to fluctuations in aggregate liquidity. The intuition behind their model was the following: when a negative shock hits the market, investors which are subject to certain constraints may be forced to liquidate assets. If an asset`s liquidity covaries with the market liquidity (aggregate liquidity), investors will be less willing to hold it, i.e. they will require compensation for holding such an asset. If liquidation is more likely where aggregate liquidity is low (assuming liquidity is time varying), investors will prefer assets whose returns are less sensitive to aggregate liquidity. Therefore, there should be a liquidity premium for assets whose returns positively covary with aggregate market liquidity. The findings supported the proposed liquidity adjustment. Assets with higher sensitivity to aggregate liquidity were found to produce higher expected returns.

Acharya and Pedersen (2005) proposed a liquidity-adjusted CAPM in which the expected return on an asset depends not only on its market beta and illiquidity, but also on three other betas that reflect its liquidity. They tested the predictions of the model on a sample of stocks from the NYSE and AMEX in the period 1963 – 1999. Their results showed that stock returns are better explained by the liquidity-adjusted CAPM in terms of R^2 and p-values of specification tests. Their finding of "flight to liquidity" suggests that the stocks that are less liquid in absolute terms also have higher liquidity risk (they have higher values of all three liquidity betas). The overall conclusion is that investors should care about an asset`s performance and tradability not only when the market is down, but also when the aggregate liquidity is down.

An interesting question is whether low volatility is itself a factor? As has been done for size, value and liquidity, a factor that represents the risk premium on low volatility stocks could be created. The creation of such a factor is beyond the scope of my thesis, but could be an interesting task for future research.

1.3 Low volatility and low beta effect

1.3.1 Volatility, beta and Sharpe ratio

The flat or negative relation between return volatility, i.e. variance or standard deviation of returns, and average returns has in the existing literature become known as the **low volatility effect**. This contradicts one of the basic assumptions in finance: the positive risk-return relation. Taking on extra risk should be rewarded with a return premium, and conversely, assets that are less risky should pay lower returns.

When CAPM beta is used instead of volatility, the relation is called **low beta effect**. As described in previous sections, the CAPM predicts that higher beta assets have higher risk premiums, thus low beta effect is as such a contradiction of CAPM.

In the existing literature the two effects are often not distinguished from one another, but these are in fact two separate phenomena. The results should, however, point in the same direction regardless of whether beta or volatility of returns is used as a risk measure. This is because beta and volatility are by definition related to each other. It follows from equation (2):

$$\beta_i = \rho_i \frac{\sigma_i}{\sigma_m} \tag{11}$$

where ρ_i is the correlation between asset i and the market portfolio (Yamada, 2013). It is apparent from equation (11) that assets with higher return volatility should have a higher beta as well. In my thesis I have focused on the low volatility effect, therefore standard deviation of returns was used as a risk measure.

The risk-adjusted performance of assets and portfolios is usually measured by Sharpe ratio which is defined as the ratio of excess return and standard deviation:

$$SR_i = \frac{R_i - r_f}{\sigma_i} \tag{12}$$

Sharpe ratio is thought of as a better measure of performance than simple average excess return because it enables the comparison of assets and portfolios by taking into account the risk associated with the excess return. For that reason, I have used this measure in the performance evaluation of portfolios in the empirical part of my research.

Since CAPM was built on several assumptions, which are often unrealistic and violated in practice, some authors suggest that the empirical failures are attributable to one or more such violations (see for example Blitz, Falkenstein and van Vliet (2014)). The most important ones (the ones that are most often mentioned in the existing literature) are briefly described in the next section.

1.3.2 Possible explanations of low volatility and low beta effect

This section describes the most commonly offered explanations for low volatility and low beta effect. In general, the issue can be viewed from two separate angles: if the proposed theoretical model is not supported by the data, we can either call the result an anomaly or adjust the model appropriately (assume assets are priced correctly and any deviations from the theory are driven by the inappropriateness of the theory itself).

1.3.2.1 Low volatility effect as an anomaly

Low volatility effect is often categorized as an anomaly. According to Keim (2006) financial market anomalies are patterns in security returns that are not predicted by the generally accepted (mainstream) theory. The term "anomaly" was introduced by Kuhn (1970). Anomalies are usually discovered through empirical tests of the theoretic predictions about informational efficiency and specific return behavior, for example CAPM, which assumes the differences in asset returns can be attributed to the differences in systematic risk. However, the rejection of generally accepted models such as the CAPM does not necessarily suggest market inefficiency - it can mean the equilibrium model that is being used is incorrect or incomplete. Some believe that anomalies, once discovered, will tend to disappear since investors will make use of them or because the discovery was a consequence of data mining or simply a sample specific characteristic. Though this has happened for some findings, for example the weekend effect[7], these are more an exception than a rule since most of the discovered anomalies tend to persist for longer time periods. The fact that so many of them have persisted for decades suggests that perhaps the reason for their existence is not to be searched for in market inefficiency. Instead, the benchmark models that are used may indeed be incorrect or at least an incomplete description of actual price formation (Keim, 2006).

Three popular explanations for financial market anomalies are (Engelberg, McLean, & Pontiff, 2015):

- Risk-based or fundamental-based explanations such as size, value and momentum. Low volatility effect can also be categorized as a fundamental anomaly.
- Behavioral-based explanations such as overconfidence and representativeness bias, preference for lotteries, etc. Human bias combined with market frictions (limits to arbitrage) causes the anomalies to persist.
- Data mining such as survivorship bias and data selection bias. Finding connections between variables does not always mean there exists an actual relation between them.

The volatility effect has often been categorized as an anomaly suggesting low volatility stocks are underpriced compared to high volatility stocks which is reflected in the higher future returns of the former and lower future returns of the latter. The next sections briefly describe some of the often used explanations for low volatility effect.

1.3.2.2 Behavioral explanations

Some principles of behavioral finance can be applied to the low volatility and low beta anomalies. According to some behavioral models the most important source that drives the anomalies is irrationality of investors (Baker, Bradley, & Wurgler, 2010).

[7] The weekend effect is the tendency of stocks to exhibit higher returns on Fridays compared to those on Mondays (Weekend effect, 2016).

Investors` rationality is one of the most important assumptions of asset pricing models such as CAPM, APT and others. In truth, the entire modern finance has its foundations in assuming rational decision-making of economic agents.

All of the traditional finance models are built on economics` foundations, especially neoclassical economics which has been the dominant paradigm. In such neoclassical frame, all individuals and firms are self-interested agents who attempt to optimize their future well-being (future utility, future profits) given the constraints on resources. The prices of assets are determined in a market as a consequence of the rational decisions made by economic agents (Ackert & Deaves, 2009). In other words, prices are formed in accordance with the law of demand and supply. The three fundamental assumptions that neoclassical economics makes about individuals are (i) people have rational preferences across possible states of nature, (ii) people maximize utility, while firms maximize profits and (iii) people make independent decisions based on all relevant information.

One of the commonly mentioned biases that relates to the low volatility effect is **loss aversion.** In their seminal paper Kaqhneman and Tversky (1979) found that losses play a more important role in investors` decision making than originally believed. They found the decisions of individuals are driven by loss aversion and the prospect of ending up with a lower amount of wealth than they currently have and not only by risk aversion which includes positive and negative deviations from the expected wealth. Prospect theory which is based on these findings assumes investors put additional weight on losses, so they either overestimate the magnitude or the probability of losses (Maringer, 2007).

Loss aversion, also known as preference for lotteries due to high gains with low probability and minimum losses, could be the reason behind high demand for risky assets which pushes their prices above their actual value and this leads to lower future returns. The opposite happens for low risk stocks whose prices are underestimated due to lower demand which in turn leads to higher future returns. This effect can be taken into account by including the third moment of the return distribution into account, i.e. skewness, by assuming investors do not only care about the average returns and standard deviations, but also whether the distribution is (positively) skewed.

There are two other biases that are also often mentioned in the existing literature: the **representativeness bias** and the **overconfidence bias**. The former means that investors associate high volatility stocks with high returns and vice versa regardless of the actual historical data and previous research. The latter has to do with the observation that more aggressive investors are usually the ones who are overconfident about their abilities and knowledge and also more risk-taking. Consequently, they invest more in risky stocks, pushing their prices up and future returns down. The opposite then happens for the low risk stocks.

1.3.2.3 Limits to arbitrage

Another source that can drive the anomalies is limits to arbitrage. An intuitive question is why don`t sophisticated institutional investors make use of the low volatility anomaly. One of the reasons is benchmarking. Institutional investors normally use fixed benchmarks which are usually cap-weighted indices that discourage investments in low risk stocks. Instead of reducing it, institutional investors might actually magnify the volatility effect. A typical institutional contract for delegated portfolio management can in fact increase the demand for higher-beta investments (Baker et al., 2011).

Active managers are more interested in volatile stock because of the higher potential gains, so portfolios often include more stocks that are very volatile (Clermont Alpha, 2016). In addition, they are often more interested in outperforming during bull markets than underperforming during bear markets which is why they increase their demand for higher-beta assets (Baker et al., 2011).

1.3.2.4 Fundamental factors

Fundamental factors refer to firm characteristics which, as has been shown in several papers so far, can also affect expected stock returns. These are for example size, value and momentum effect. The size and value effects were described in section 1.2.1.

1.3.2.5 A liquidity explanation

It is known that small cap less liquid stocks as are not as frequently traded as larger liquid stocks. This causes all the risk measures such as beta, volatility and correlation to be underestimated for the small cap stocks. The underestimation effect is also present in other illiquid asset classes such as real estate and private equity. So, in general it holds that the risk measures are well estimated for (large) liquid stocks and underestimated for (small) illiquid stocks. If we then rank the stocks based on these biased estimates, the estimates we get in later steps (for portfolios) will be underestimated even more (Clermont Alpha, 2016).

Some interesting characteristics of low volatility stocks have been documented: They tend to be large cap stocks that have a lower average sales growth and are less liquid. In addition, they tend to have lower prices because of their low price-to-earnings (P/E) ratios, have higher average profit margins and higher average dividend yield. While the finding of a large cap bias is somewhat surprising, the value and illiquidity bias are more understandable since it has been shown that value stocks and less liquid stocks yield higher returns on the long run. Therefore, the low volatility anomaly might be a realization of a value and liquidity risk premium (Clermont Alpha, 2016).

To sum up, several theories have been suggested so far to explain what appears to be anomalous risk-return relation. Only the most important ones were mentioned in this section. But is low volatility effect in fact an anomaly or is the pricing model missing some important variable(s)? I have focused on the latter view and I will try to find out whether the factor that we are missing is liquidity. If this is the case, then low volatility effect is in fact low liquidity effect which is also closely related to the size effect. This is why I have chosen a small cap universe of U.S. stocks to test whether evidence of low volatility effect can be found in the last 10 years. If the true reason behind the outperformance of low volatility stocks is in fact the liquidity effect, then this should be most apparent in the small cap segment since small cap stocks are the ones that often suffer from market "thinness" which reduces their liquidity.

2 PREVIOUS EMPIRICAL RESEARCH

The low volatility/low beta effect is not a new finding in finance. In fact, some very early empirical tests of CAPM have already found the risk-return relation to be flat or even negative. This section presents the most important work that has been done so far in the field of low volatility and low beta effect, briefly describes the methodologies used and some of the issues with estimation, and summarizes the main findings of the empirical tests.

2.1 The initial tests of CAPM

The initial tests of CAPM focused mainly on the ability of the market to price the risk of securities. The main question was whether the model can be used to describe actual behavior of security returns (Defining risk, 2016).

One of the earliest empirical tests was done by Douglas (1969) who applied a very straightforward test of the CAPM by regressing stocks` returns on beta and residual variance (both from separate time-series regressions for individual assets). His results have already shown a deviation from the proposed theory since residual variance was significant indicating that systematic risk is not the only risk that is priced. The intercept from Douglas`s the regression was also significant and statistically different form the risk-free rate (real returns were used, hence the intercept should converge to the risk-free rate), indicating a consistent mispricing and thus market inefficiency.

There were some problems with Douglas`s methodology, so the results did not necessarily show the real picture. The main problem was the use of individual securities in the risk-return analysis instead of groups of stocks (portfolios). By using estimates of beta from a different regression he might have introduced the "errors-in-variables" problem since the estimates, though they could have been unbiased, were measured with error. Had he used portfolios of stocks instead of individual securities, the effect of significant residual terms could have been controlled for. Using portfolios of stocks also makes sure that a part of asset-specific risk is shifted away, as there are many stocks belonging to the same portfolio (diversification).

Miller and Scholes (1972) continued the work of Douglas, correcting his results for the measurements errors in beta and some other biases. However, even their corrections did not entirely eliminate Douglas`s findings[8] (Falkenstein, 2010). Their results showed that the alphas of individual assets depended systematically on their betas: the stocks with high beta values had a negative alpha, and the stocks with low value of beta had a negative alpha (Black, Jensen, & Scholes, 1972).

Black, Jensen and Scholes (1972) found very similar results when they tested the model on a sample of securities listed on the NYSE in the period 1926 – 1966. They used a two stage procedure, as described in the previous section and also found a systematic pattern between betas and alphas: high beta securities had significantly negative intercepts and vice versa. This finding itself contradicts the predictions of the traditional form of the model. Additionally, they found this effect to be getting even stronger over time. They concluded that the traditional form of the model is not consistent with the data.

Soon after Black, Jensen and Scholes (1972), Fama and MacBeth (1973) wrote an important paper that has in many ways contributed to the existing literature. They tested the model on a sample of NYSE common stocks in the period from 1926 – 1968 and found the relation between beta and expected returns to be flatter than predicted by the model[9]. They came to similar conclusions as Friend and Blume (1970) and Black, Jensen and Scholes (1972) who, at least in the post-World War II period, found the average value of the constant term to exceed the risk-free rate significantly.

A few years later Haugen and Heins (1975) took a sample of stocks listed on the NYSE in 1926, formed portfolios and regressed average portfolio returns on standard deviations and betas. They found a clear evidence of a low volatility effect in the period from 1926 – 1971 and concluded that, based on their sample, risk does not generate a special reward, regardless of which risk measure is used - standard deviation or beta.

---

[8] Falkenstein (2010) also argues that adding so many new variables into the model in order to control for possible biases will diminish any effect, because the effect is then spread among several variables as opposed to one.

[9] However, they could not reject the hypotheses about market efficiency.

2.2 More recent tests

When researchers started testing the validity of the proposed theoretical risk-return relation, their tests were affected by the limited computer ability and computational costs, so the results of the early tests perhaps need to be looked at by having this is mind. For example, Falkenstein (2010) notes that most of the size effect discovered in the 1980`s was in fact measurement error. Now that the state of the art technology is available to almost every individual, testing various risk-return hypotheses with the use of different econometric techniques no longer poses a problem. The low volatility/low beta anomaly has therefore been tested over and over again by several researchers, but to conserve space I shall only mention some of the most important work that has been done recently. The studies that are relevant for my work are described in more detail.

The first thing to note here is that there are (at least) two most commonly used approaches for creating minimum volatility portfolios. The first one consists of the estimation of the covariance matrix between returns of individual stocks and then minimization of the ex-ante risk for any given expected return. This is the approach that Clarke, de Silva and Thorley (2006) used in their paper. They continued/improved the work by Haugen and Baker (1991) by extending the period under examination from 1972 – 1989 to 1968 – 2005 and by using some more advanced techniques for the estimation of the covariance matrix. They used data on 1000 U.S. stocks with the largest market cap and estimated the covariance matrix by using (i) principal components and (ii) Bayesian shrinkage method in order to find the weights of the minimum volatility portfolio. The description of these methods is beyond the scope of my thesis, especially since I do not use any of them in my empirical work.

The results of Clarke, de Silva and Thorley (2006) showed that the minimum variance portfolio created with the use of Bayesian shrinkage yielded 6.5% return (annualized excess return) with an ex-post standard deviation of 11.7 % (Sharpe ratio 0.55), which is much more than the market (annualized excess return of 5.6 % with 15.4 % risk, hence the Sharpe ratio was 0.36). Not only did the minimum variance portfolio experience higher excess return, this return was also achieved at a lower level of risk. These results contradict the conventional equilibrium portfolio theory. The results were similar when the principal components method was used. The regression of the minimum variance portfolio returns on the market return over the entire period (1968 - 2005) produced an alpha of 2.8% and beta of 0.65. Controlling for Fama-French factors and exposures to these factors (they found that minimum variance portfolios have a size and value bias), led to an increase of beta and a decrease of alpha, but the latter was not entirely eliminated. Overall, the findings of Haugen and Baker (1991) which contradict the theoretical risk-return relation were confirmed.

Clarke, de Silva and Thorley used the first method of creating minimum volatility portfolios as was proposed by Markowitz (1952). I have used the second approach which involves sorting stocks into groups by a historical risk measure. The approach Blitz and van Vliet (2007) used in their study bears a resemblance to the one I have used in the empirical part of my research. Blitz and van Vliet used a sample of data on all the constituents of FTSE World Developed index (December 1985 – January 2006). The stocks were ranked by their historical (3-years) return volatility every month and assigned to 10 portfolios based on this estimate. The stocks were also ranked by size (free float market value), value (book-to-market ratio) and momentum (past 12 minus 1 month total return). Next, they calculated excess return on each of the portfolios over the month that followed portfolio formation and for the resulting time series of portfolio returns they calculated the average return, standard deviation and Sharpe ratio.

Blitz and van Vliet (2007) then used regressions and double sorting of stocks in order to separate the volatility effect from other effects. They constructed the Fama-French factors themselves by sorting the stocks on size and value and then defining SMB and HML as the return difference between the top 30 % and bottom 30 % small and big stocks and value and growth stocks, respectively. In my analysis I used Fama-French factors from Kenneth French`s website which is a simplification, but the results should not suffer because of this. Blitz and van Vliet then used the SMB and HML factors in the portfolio time series regressions. They also applied a double sorting procedure where the stocks were first sorted by size and book-to-market and then by volatility within the size/value portfolios. The difference between their methodology and the one I used is that they used dependent sorting only while I used both, dependent and independent sorts in order to find out how the sorting procedure affects the results.

The results of Blitz and van Vliet (2007) showed that low risk portfolios outperformed their high risk counterparts, especially on a risk-adjusted basis. The Sharpe ratio of global minimum volatility portfolio achieved a value of 0.72, while Sharpe ratio of the market was 0.4 and that of the high volatility portfolio only 0.05 (the latter was to a great extent driven by the high value of standard deviation). The values of Sharpe ratios declined as the prior volatility increased which itself is a clear low volatility effect. The regression results spoke in favor of this, since the low risk portfolio had a low beta (0.56) and a positive significant alpha (4.0 %) as opposed to high risk portfolio with beta of 1.58 and a negative alpha of -0.08 %. The alpha spread of low minus high risk portfolio was 12 %. Further analysis showed that the observed negative risk-return relation does not weaken over time, in fact it was even larger in the more recent sub-period (1996 – 2005).

In addition to the global results, Blitz and van Vliet (2007) analyzed the risk-return relation in U.S., European and Japanese markets in isolation. The results confirmed the low volatility effect - the regional results were similar to the global results. The Sharpe ratio improvement compared to the market was the biggest in Europe, followed by Japan and then the U.S. The alpha spreads were again similar to the global results, implying regional effects are not the main driver of this "anomaly".

As regards the size, value and momentum sorts of stocks, Blitz and van Vliet (2007) concluded that the volatility effect is a stronger effect and more importantly, a separate effect. In addition, the inclusion of SMB and HML factors into the regressions reduced, but did not eliminate the volatility effect and it turned out to be robust also to the ex ante sorts on size, value and momentum (double sorting).

Another interesting finding of this study is that volatility effect turned out to be a stronger effect compared to the beta effect. And even further, the sorts on beta and subsequently on volatility showed that some of the outperformance (measured by alpha) still remains even when one controls for beta: within portfolios with similar beta, low volatility portfolios still capture additional alpha. The authors argue that "this finding suggests that both the idiosyncratic part and the systematic part of volatility are mispriced".

A similar research to that of Blitz and van Vliet (2007) focused on the emerging markets only was carried out by Blitz, Pang and van Vliet (2013). They tested the model on a sample of all constituents of the S&P/IFC Investable Emerging Markets Index from the period of its inception (December 1988) until December 2010. Their findings showed a flat or even negative relation that persists even after controlling for large caps only, longer holding periods and exposures to size, value and momentum factors. They also note that the empirical risk-return relation appears to be getting even stronger over time. In addition, the mispricing seems to be occurring independently in different markets.

Yamada (2013) also expanded the horizon of the tests on low volatility and low beta effect by testing the long term performance of low volatility stocks in the U.S., Japan and other developed countries and found evidence of the two effects in almost all the markets. He tested (i) a global minimum variance portfolio defined as the lowest volatility portfolio on the efficient frontier and (ii) a reciprocal of a variance weighted portfolio which holds assets in proportions that are inverse to their historical variance ($w_i = \frac{1}{Var_i}$). This way the least volatile stocks were given higher weights. Capitalization weighted and equal weighted portfolios were used as benchmarks. Low volatility portfolios showed higher risk-adjusted return compared to the market-value-weighted indices in almost all markets.

Some of important empirical confirmations of low volatility effect that are also worth mentioning are Bali and Cakici (2008) who found a significant relationship between idiosyncratic volatility and expected returns, Baker, Bradley and Wurgler (2011) who identified benchmarking of institutional investors combined with behavioral biases as main drivers of low volatility and low beta effect, Frazzini and Pedersen (2014) who considered a model which takes into account investors` different constraints and found that the funding constraints flatten the slope of the security market line. Haugen and Baker (2012) complemented the existing literature by attributing the low volatility effect they found in almost all equity markets to the agency issues between investment managers within an organization and between the managers and their clients.

3 DATA AND METHODOLOGY

3.1 Data

In order to verify the existence of low volatility effect in the U.S. small cap equity market, I have chosen the S&P 600 index which consists of 600 U.S. stocks with the smallest market capitalization. My universe of stocks covers all of the S&P 600 constituents and the sample covers the periods from 31 December 2004 through 29 January 2016. All the data have weekly frequency. Transaction costs and dividends are ignored throughout my analysis.

3.1.1 The S&P 600 index

The S&P 600 was introduced in 1994 and it consists of 600 U.S. stocks with the smallest value of market capitalization (market cap), the latter is calculated as a product of a stock`s current price and the number of stocks outstanding. The index is formed in a way that includes only liquid and financially viable firms of the small cap U.S. equity segment. More precisely, for a stock to be included in the S&P 600 it must meet the following eligibility (inclusion) criteria (S&P U.S. Indices Methodology, 2016):

- Unadjusted market capitalization of $400 million to $1.8 billion.
- Adequate liquidity and reasonable price: the ratio of the annual dollar value traded and float-adjusted market capitalization should be at least 1.00 and the stock should trade a minimum of 250,000 shares in each of the 6 months prior to the evaluation date.
- It has to be an U.S. company stock: Criteria based on the company`s headquarters, employees, etc.
- Public float of minimum 50 % of the stock.
- Sector classification: Contribution to the sector balance maintenance.
- Financial viability: The sum of the four most recent consecutive quarters` as-reported earnings and the as-reported earnings of the most recent quarter by itself should be positive. As-reported earnings equal net income minus discontinued operations and extraordinary items.
- Treatment of initial public offerings (IPOs): IPOs should be seasoned for 6 – 12 months before the stock is considered for addition to the index.

- Eligible securities.[10]

Companies that are involved in mergers and acquisitions or restructured in a way that they do not longer meet the inclusion criteria, and those who substantially violate one or more of the addition criteria are deleted from S&P 600. In general, turnover in index membership is avoided and since addition criteria is much stricter than the criteria for continued membership, a stock that violates addition criteria is not deleted unless “ongoing conditions warrant an index change” (S&P U.S. Indices Methodology, 2016).

For reweighting purposes the S&P 600 is rebalanced quarterly in March, June, September and December. However, constituent changes to the index due to corporate actions or market developments can still be made at any time unless the change is considered not significant, i.e. less than 5 % of total shares outstanding. In that case the change is accumulated and implemented with the quarterly rebalancing (S&P U.S. Indices Methodology, 2016).[11]

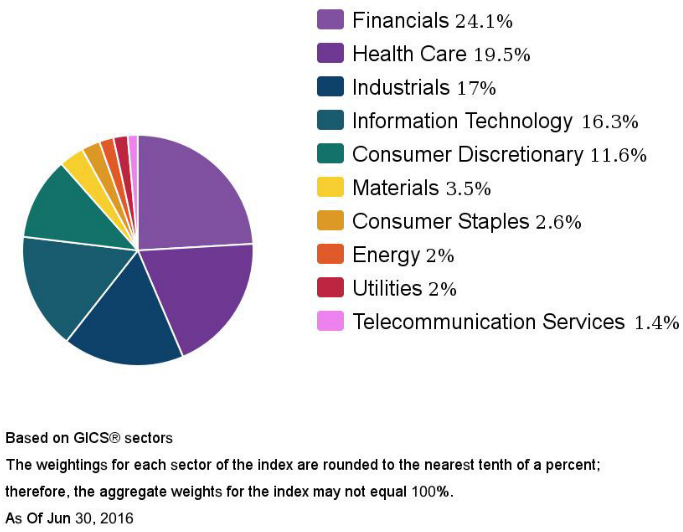
Figure 1 shows the sector breakdown of S&P 600. Almost one fourth (24.1 %) of the firms included in the index are financial firms, followed by health care (19.5 %) and industrial firms (17 %). Information technology firms (16.3 %) and consumer discretionary (11.6 %) also take a rather significant part. Other industries take a rather small, perhaps negligible part in the index.

---

[10] »Eligible securities include all U.S. common equities listed on NYSE, NYSE Arca, NYSE MKT, NASDAQ Global Select Market, NASDAQ Select Market, NASDAQ Capital Market, Bats BZX, Bats BYX, Bats EDGA, and Bats EDGX exchanges.«

[11] A more detailed description on the methodology of the S&P 600 index can be found in the S&P U.S. Indices Methodology.

Figure 1. Sector breakdown of S&P 600 (in %)



Source: S&P Dow Jones Indices, *S&P U.S. Indices Methodology*, 2016.

3.1.2 The data on the constituents of S&P 600

The data on closing prices, bid prices and ask prices on all S&P 600 constituents that were included in the index on 2 March 2016 were downloaded. All the data have weekly frequency, but do not represent weekly averages. So, every week on a particular day the prices are recorded. The dollar prices on stocks were then used to calculate logarithmic returns (log returns) and relative bid-ask spreads were calculated from bid and ask prices.

The data were downloaded on 2 March 2016 which means I have only gathered data on the stocks that were included in the index on that particular day. So, if for example, a firm was established in 2010 and died (went bankrupt) in 2013, the data on that firm is not included. This can bias the results upwards since only the surviving firms are taken into account. The firms that died would probably exhibit very low or even negative returns in the period of distress which would lower the average returns. The issue described is known as the "survivorship bias[12]" or "survivor bias" and according to Investorwords.com this is "a tendency for failed companies to be excluded from performance studies due to the fact that they no longer exist. This causes the results of some studies to skew higher because only the companies that were successful enough to survive over the entire period of examination are included" (Survivorship bias, 2016).

[12] Survivorship bias was first introduced by Haugen and Heins (1975). They measured its impact on the studies of Jensen (1969), Soldfosky and Miller (1969) and Sharpe (1964) (Haugen & Baker, 2012).

3.1.2.1 Stock returns

The log returns were calculated in the following way:

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \tag{13}$$

Where $P_{i,t}$ denotes the price of stock i on week t, and $P_{i,t-1}$ denotes the price of the same stock on week t-1. As mentioned, dividend payments and stock splits were ignored in the return calculations. Log returns were used in the entire analysis.

There are many benefits of using log returns instead of simple returns. Firstly, taking logarithms of variables is a normalization which means all variables can be measured in a comparable metric: they can all be measured on the logarithmic scale. Secondly, it is often assumed that stocks prices follow a log normal distribution (prices cannot take negative values) and if that is the case, then the log returns follow a normal distribution:

$$ln(1 + R_i) \sim N(E(R_i), \sigma_i) \tag{14}$$

This is very convenient for statistical inference. Thirdly, when returns are small enough, log returns and raw returns are approximately equal:

$$ln(1 + R) \approx R, \qquad R < 1 \tag{15}$$

And most importantly, log returns are additive because logarithms convert products to sums. So the compounding return of the n-th period becomes:

$$ln(1 + R_{total}) = \sum_{i=1}^{n} ln(1 + R_i) \tag{16}$$

This is especially convenient since the product of normally distributed random variables is not itself normally distributed, but the sum of normally distributed variables does follow a normal distribution (assuming the variables are independent). Therefore, compounding returns are normally distributed and can be calculated by simply adding the log returns:

$$\begin{aligned}\sum_{i} ln(1 + R_i) &= ln(1 + R_1) + ln(1 + R_2) + \cdots + ln(1 + R_n) \\ &= ln(P_n) - ln(P_0)\end{aligned} \tag{17}$$

Or, even simpler, subtracting the log of initial price from the log of final price (Why log returns, 2016).

The average returns of portfolios were calculated as simple averages (equal weighting). The time series averages of portfolios were also calculated as simple (arithmetic) average because the compounding effect has already been taken into account due to the use of log returns:

$$\bar{R}_p = \sum_{t=1}^{T} R_{p,t} \tag{18}$$

where $\bar{R}_p$is the arithmetic mean for portfolio p and $R_{p,t}$ is the average return of portfolio p on week t. The average portfolio returns were also annualized to make the values more comparable.

3.1.2.2 Ask and bid prices (in U.S. dollars)

From ask and bid prices bid-ask spreads were calculated by simply subtracting the latter from the former. The bid-ask spread is defined as the difference between the highest price a buyer is willing to pay for an asset and the lowest price a seller is willing to sell the asset. The size of the spread differs mostly due to the difference in liquidity of assets (Investopedia, 2016). A low value of bid-ask spread suggests high liquidity of an asset, for example cash has one of the lowest values of bid-ask spread. On the other hand, small-cap stocks usually have higher values of bid-ask spreads and are considered less liquid. Since prices of stocks vary substantially between individual stocks, the bid-ask spread cannot be a comparable measure of their liquidity. For that reason, I have divided the bid-ask spreads ($AP_{i,t} - BP_{i,t}$) by their respective price ($P_{i,t}$):

$$Relative\ BA\ spread_{i,t} = \frac{AP_{i,t} - BP_{i,t}}{P_{i,t}} \tag{19}$$

The relative bid-ask spreads were then used for sorting stocks by their liquidity where stocks with a low value of the spread were considered the most liquid.

Amihud and Mendelson (1986) provide another definition of the bid-ask spread: It can be thought of as the price the dealer demands for providing liquidity services and immediate execution. The ask price or offer price includes a premium for immediate buying and the bid price reflects a price concession for immediate sale.

Studies have shown bid-ask spread, defined as a percentage of the stock price is strongly negatively correlated with other stock characteristics that reflect liquidity, for example trading volume, number of shareholders, number of dealers forming a market in the stock and the degree of price continuity. The bid-ask spread as a sum of buying premium and the selling concession is thus a natural measure of the illiquidity cost (Amihud & Mendelson, 1986).

There are also some drawbacks with the use of bid-ask spreads to proxy for (il)liquidity. It is based on market microstructure data which is not available for long time series. Also, it measures well the cost of selling a small number of shares, which is not necessarily the case for a large number of stocks (Acharya & Pedersen, 2005).

3.1.3 The data on Fama-French factors

Fama-French factors were also downloaded and they served as the explanatory variables in the CAPM and Fama-French regressions.

3.1.3.1 Size (SMB) and value (HML)

As mentioned in section 1.2.1, SMB and HML are return premiums on small and value stocks compared to large and growth stocks. The following methodology is used for the construction of SMB and HML: At the end of each June the stocks are sorted into two size groups (small and big) and three B/M groups (value, neutral and growth). Small and big stocks are the ones belonging to bottom 10 % and top 90 % of June market cap, respectively and the breakpoints for B/M portfolios are the 30th and 70th percentiles of B/M for the big stocks of the region (in my case U.S.). These independent sorts produce six portfolios which are combinations of size and B/M groups of stocks. Size factor (Small minus Big - SMB) is then the difference between the average returns on three small and three big portfolios:

$$SMB = \frac{1}{3}(Small\,Value + Small\,Neutral + Small\,Growth) - \frac{1}{3}(Big\,Value + Big\,Neutral + Big\,Growth) \tag{20}$$

Value factor (High minus Low B/M - HML) is the difference between average returns on two portfolios with the lowest and two portfolios with the highest book-to-market ratio (Fama-French Data Library, 2016):

$$HML = \frac{1}{2}(Small\,Value + Big\,Value) - \frac{1}{2}(Small\,Growth + Big\,Growth) \tag{21}$$

3.1.3.2 Market return in excess of the risk-free rate of return (R_m-R_f)

The excess return on the market (Rm-Rf) is the value-weighted return of all Center for Research in Security Prices (CRSP) firms incorporated in the U.S. and listed on NYSE, AMEX or NASDAQ[13] in excess of the one-month Treasury bill rate (Fama-French Data Library, 2016).

Since the true market portfolio is unobservable and finding the most suitable proxy for it is a rather difficult task, the results should be interpreted having this in mind. Market return from Kenneth French`s website is merely one of the possible proxies that could have been used and perhaps using a different one would notably change the results.

The values of the one-month Treasury bill rate were also downloaded separately and used for the calculations of excess returns on portfolios.

The data on S&P 600 constituents were downloaded from Thomson Reuters Datastream and the Fama-French factors, including excess return on the market and the riskless rate, were downloaded from Kenneth French website. I used STATA for the empirical research and Excel for some minor calculations.

3.2 Methodology

This section describes the approach I used for the empirical part of my research. The testing implications and methodologies of CAPM and its Fama-French adjusted version were already discussed. The main thing to note is that there are some differences between the approach that has been used in some of the early empirical work and the one I used. My approach is very similar to the approach of Blitz and van Vliet (2007) described in section 2.2, but with some adjustments and simplifications.

3.2.1 Testing for the low volatility effect

Though there are some differences between testing for low volatility effect and for low beta effect, the basic intuition behind it is the same. The main simplification of using stocks` volatility of returns as a risk measure is in that there is no need for the estimation of beta in the first step which means no need for running a time series regression for each of the stocks. Since standard deviation of returns does not enter the regressions in the second step, there is also no fear of errors-in-variables problem described in previous sections.

[13] The stocks must have a CRSP share code of 10 or 11 (ordinary common shares) at the beginning of month t, good shares and price data at the beginning of t, and good return data for t. More details can be found on the following website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

Another important difference is that in the second step of the procedure time series regression is used for portfolios and not individual stocks like for example in Black, Jensen and Scholes (1972). Portfolio returns are regressed on the market returns for each of the portfolios. This way it is controlled for systematic risk, i.e. the sensitivity of each portfolio`s returns to the market. The intercepts from these regressions are then the main indicator of a portfolio`s out- or underperformance. This is the part of stocks` returns that is left unexplained by the model. And since it is very possible that minimum volatility stocks are small and value stocks, the intercepts are corrected by including SMB and HML factors into the regressions. Most importantly, the sample of small stocks suggests these stocks are also less liquid because it is possible they are less frequently traded compared to large stocks. Therefore, liquidity is included into the model with the use of double sorting the stocks on liquidity and volatility which produces nine liquidity-volatility portfolios.

3.2.2 Creating low volatility portfolios

There are (at least) two basic approaches for creating low volatility portfolios. The first one is the mean-variance approach which was proposed by Markowitz (1952) and has been a mainstay of financial economics and portfolio management until the development of CAPM. It includes estimating expected returns and a covariance matrix for individual stocks and then minimizing the portfolio`s ex-ante risk for any given expected return by adjusting stocks` weights in the portfolio (Clarke, de Silva, & Thorley, 2006).

Under Markowitz`s framework, the optimal portfolio depends on the information that is available to an investor. When no prior information is available, the optimal portfolio is equally weighted portfolio (1/N), when only volatilities of assets` returns are estimated, the reciprocal of a variance weighted portfolio is the optimal one (1/Var) and when assets` correlations are available in addition to variances, the minimum variance portfolio is optimal (Yamada, 2013). This is the portfolio on the efficient frontier with the lowest volatility described in section 1.1. Thus in order to calculate the weights of the minimum variance portfolio one needs to know both the variances and covariances of assets` returns.

Minimum variance technique then makes full use of covariances of returns in order to get the portfolio with the lowest volatility. In addition, it a very flexible approach as it allows the use of constraints which enable greater customization or taking into account some weaknesses of the model (Clermont Alpha, 2016).

The second approach is ranking-based and it includes sorting stocks into portfolios based on a historical measure of risk. I have used this approach for my empirical research mostly due to the simplicity of calculations. In addition, previous empirical research has shown that the results do not suffer because of this simplification.

The ranking-based approach is much simpler to apply compared to the mean-variance approach and it does not require advanced expertise and technology. However, there are some limitations of using this approach, for instance, it does not take into account the covariances between assets` returns in building a low volatility portfolio and therefore also neglects the benefits of diversification in lowering risk at the portfolio level (Clermont Alpha, 2016).

Once the stocks are ranked (screened) the portfolio returns need to be calculated by weighting the stocks` individual returns. Various weighting schemes are used with the most popular being equal-weighting and inverse of volatility weighting[14]. Both approaches will tend to bias the portfolio toward small stocks. Weighting of stocks by their inverse volatility will also tend to overweight the stocks whose volatility is underestimated which are usually the ones with low liquidity caused by infrequent trading (Clermont Alpha, 2016).

I have sorted the stocks into equally-weighted portfolios, so each stock was given a weight of 1/N, N being the number of stocks in a portfolio. The stocks were sorted based on their historical volatility. Other risk measures could be used instead, such as beta or idiosyncratic risk (the residuals from simple CAPM regressions, for example).

Both, beta and volatility can be calculated over different periods of time. Looking from a statistical point of view it is better to include more periods into the estimation in order to make the estimate more precise. However, taking very long periods might make the estimate less relevant if the company has significantly changed during that time (Clermont Alpha, 2016). I have calculated past 1-year volatility and it is hoped that 1-year period is long enough for the estimates to be precise, yet short enough to account for any possible changes in the companies (that regard their return volatility). Also, this way only 1 year of data (31 December 2004 – 30 December 2005) was lost for further analysis (used only for volatility estimation).

Double sorting procedure was also used in order to separate the liquidity effect (if there is some) from the volatility effect. Stocks were sorted on their average 1-year bid-ask spread/price ratio where a low value of this measure indicates high liquidity, and on their past 1-year volatility of returns where low value is preferred as well. This resulted in nine (3x3) liquidity-volatility portfolios.

The stocks were first sorted independently which means their past volatilities and average relative bid-ask spreads were calculated separately and then portfolios were formed on the

---

[14]A reciprocal of a variance (or volatility) weighted portfolio holds individual stocks in inverse proportion to their historical variance (volatility) assigning higher weights to less volatile stocks (Yamada, 2013).

basis of these estimates. This type of procedure may result in portfolios of different size (different number of stocks in each of the nine portfolios). Dependent sorts on liquidity and volatility were also made where the stocks were first sorted by liquidity and then each of the three portfolios was further divided into three equally sized volatility portfolios. This means that, in a given period (year) each of the nine portfolios contained the same number of stocks.[15] This enables the volatility-average returns relationship to be observed independently of liquidity. Then the opposite procedure was used and the stocks were sorted first by volatility and subsequently by liquidity within the three volatility groups. The latter procedure enables the relationship between liquidity and average returns to be observed independently of volatility (inside each volatility bucket).

While it is irrelevant for the use of independent sorting by which variable the sorts are made first, it makes a difference for the dependent sorts. So, it makes a difference whether the stocks are first sorted by liquidity and then by volatility within the liquidity sorts or whether it is the other way around. It depends on which variable we want to "control for" or hold constant and which effect we want to observe independently. By sorting stocks on liquidity first, I was able to observe whether there exists a low volatility effect within each liquidity group and by sorting stocks on volatility first, I was able to observe whether there exists a liquidity effect within each volatility group. So, by performing both grouping procedures, I was able to examine whether there is a low volatility effect in my sample of stocks and whether it is in any way related to liquidity.

This approach bears a resemblance to the addition of variables into a regression model: by including the variable(s) we want to control for into the regression model, we can observe the relationship between the other variable(s) and the dependent variable separately. However, sorting and regression analysis are still two different techniques. Blitz and van Vliet (2007) and Blitz, Pang and van Vliet (2013) point out the main advantages of using a (dependent) sorting procedure compared to the use of regression. By sorting the stocks on size and value and subsequently on volatility within size and value portfolios they control for the two effects ex ante, so that it can be observed what happens with average returns when volatility changes independently of size and value. Dependent sorting is a robust, non-parametric technique and it systematically neutralizes other effects ex ante as opposed to adjusting the estimated alphas (the intercepts in the regressions) ex post.

As regards the size and value effects my approach is again a simplification since Fama-French factors – SMB and HML – from Kenneth French`s website were used in the portfolio regressions in order to control for size and value effects, respectively. The more

---

[15] The number of stocks in portfolios is not the same every period which is due to the missing values in the sample.

precise way to do it would mean the construction of the two factors from the size and value sorts.

3.2.2.1 Sorts on volatility - details

For each stock that was included in the S&P 600 on 2 March 2016, I first calculated past 1-year volatility of weekly returns, starting with 7 January 2005. The first calculation of volatility thus covered the period from 7 January 2005 through 30 December 2005. Each stock was then assigned to one of the three portfolios: minimum volatility, medium volatility and high volatility based on this estimate. I next calculated the returns on the three portfolios for the year following the portfolio formation, starting with 6 January 2006 and ending with 29 December 2006 for the first period. So, there was no overlap between periods used for calculations of volatility. The first period starts with 7 January 2005 and ends with 30 December 2005, the next period then starts with 6 January 2006 and ends with 29 December 2006, and so on. This is a simplification and the results would be more precise if there was some overlap between the periods. The portfolios could then be rebalanced more often, for example once a month which would lead to more accurate data on portfolio returns. If volatility is assumed to vary a lot in time, firms would probably move from one portfolio to another more often than once a year.

As already mentioned, the weekly portfolio returns were calculated as an arithmetic average of the returns on individual stocks that were included in each portfolio. All the stocks had an equal weight in the portfolios and thus equal relevance. The outcome for the first period then consisted of three time series of weekly portfolio returns starting with 6 January 2006 and ending on the 29 December 2006. The whole procedure was then repeated exactly one year later, thus the portfolios were rebalanced on a yearly basis. So, volatility was calculated (6 January 2006 – 29 December 2006), stocks were assigned to the three portfolios and average returns on portfolios were calculated for the year that followed portfolio formation (5 January 2007– 28 December 2007). This procedure was then repeated until the end of time series was reached.

Each time the volatility of returns was calculated the portfolios were rebalanced which means there were different stocks belonging to the three portfolios each period. For example, low volatility portfolio contained the stocks that had the lowest value of past 1-year standard deviation of returns, but these were different stocks each year. Since volatility is time varying sorting stocks only once would produce less precise results. In general it holds: the more often the rebalancing, the "better" or more precise the results will be. It is a kind of a trade-off between having precise results and computational simplicity.

One of the important features of such a procedure is also the fact that one period is used for the volatility calculations, and then the subsequent time period is used for the portfolio return calculations. This minimizes the errors that could arise if the same period was used.

The whole procedure resulted in a time series of weekly portfolio returns for each of the three volatility sorted portfolios for 10 years, more precisely from 6 January 2006 – 29 January 2016 which is 526 weeks. For each of the portfolios average excess return, ex-post standard deviation and Sharpe ratio, which is simply the ratio of the former and the latter, were then calculated. The average excess returns and standard deviations were annualized to make the numbers more comparable with each other. Next, I ran a CAPM regression for each of the portfolios. Thus, the following time series equation was estimated:

$$R_{i,t} - R_f = \alpha_i + \beta_i(R_m - R_f) + \mathrm{e}_{\mathrm{i,t}} \tag{22}$$

for each of the three portfolios, where $R_{i,t} - R_f$ is the log excess return on the i-th portfolio in time t, α_i and β_i are the intercept and regression coefficient, respectively, $R_m - R_f$ is the excess return on the market portfolio[16] and $e_{i,t}$ is the error term. The beta in equation (22) is the CAPM beta that measures portfolios` exposures to the market, i.e. systematic risk. Additionally, SMB and HML factors were added to the regressions to control for size and value effects, respectively. Thus, the following time series regression was run for each portfolio:

$$R_{i,t} - R_f = \alpha_{FF} + \beta_{FF}(R_m - R_f) + s_i SMB_t + h_i HML_t + v_{i,t} \tag{23}$$

where α_{FF} is the Fama-French adjusted alpha, β_{FF} is CAPM beta adjusted for size and value and s_i and h_i are the factor exposures on SMB_t and HML_t factors, respectively. The error term is denoted by $v_{i,t}$. The basic point of estimating equations (22) and (23) is to capture any additional effects that influence average returns as opposed to calculating the unconditional means. The part that is left unexplained by these factors is captured in the intercept (and also in the error term).

Standard errors of the regression coefficients were estimated using a robust estimator. This is because the normality of the error term cannot be assumed and consequently, the normality of the OLS coefficients cannot be assumed either. If this is the case, the t-values do not follow a t-distribution which makes any inference based on t-distribution meaningless (the P-values are not correct).

---

[16] The return on the market was not log-transformed. It is assumed the values are small enough for the simple and log returns not do differ substantially. For the sake of simplicity the riskless rate was not log-transformed either before it was subtracted from the return on portfolios.

The robust estimation does not cause the values of the estimated coefficients to change, however estimates of standard errors and consequently the t-values are different (Gujarati, 2004). The standard errors become unbiased, but can be less efficient (they are no longer the lowest possible).

STATA uses the Huber –White estimator of the variance-covariance matrix which relaxes the normality assumption. It also relaxes the condition of identical distribution of errors (the errors still need to be independently distributed), so the estimated matrix is robust to heteroskedasticity as well (Stata Manuals: Robust and clustered standard errors, 2016).

Another issue should be noted at this point. The standard deviations and later average bid-ask spread/price ratios were calculated every 52 weeks which is accurate for all the years except for 2010 which has 53 weeks. For the sake of simplicity the volatility was calculated for 52 weeks as well, so the values for the last week of 2010 were already a part of the calculation for the year that followed. It is hoped that the results will not suffer because of this simplification. The total number of observations for each of the volatility sorted portfolios was 526 (10 years times 52 weeks plus 1 extra week in 2010 and 5 weeks in 2016).

3.2.3 Double sorted portfolios – Independent sorts

In order to test whether the low volatility effect is a separate effect or merely a realization of a liquidity risk premium, I used a double sorting procedure where stocks were sorted on both, liquidity and volatility. The stocks were first sorted independently which means that they were split into three groups based on the estimates of each variable taking into account the estimates from the whole sample. For example, when the stocks in minimum liquidity group were further divided on the basis of their volatility, the volatility of stocks of the whole sample was taken into account. Table 1 shows all the combinations of liquidity and volatility. There may be different number of stocks belonging to each of the nine portfolios, so the equal-sized cells in Table 1 do not suggest all the portfolios are of the same size.

Table 1. Independent sorts on liquidity and volatility

Min liq./Min vol.	Med liq./Min vol.	Max liq./Min vol.
Min liq./Med vol.	Med liq./Med vol.	Max liq./Med vol.
Min liq./Max vol.	Med liq./Max vol.	Max liq./Max vol.

The procedure that followed was then the same as described before, so the stocks were assigned to nine liquidity-volatility portfolios based on the average 1-year bid-ask spread – price ratio (6 January 2006 – 29 December 2006) and past 1-year volatility of returns (6 January 2006 – 29 December 2006). The reason for postponing the calculations for one year, is the lack of data on bid and ask prices before 2006. The returns on all portfolios were calculated for the next year (5 January 2007 – 28 December 2007) and the procedure was repeated exactly one year later. The results were times series of weekly portfolio returns for the nine portfolios. I then calculated average excess returns, standard deviation - both of which were annualized - and Sharpe ratios for each of the portfolios. Once again, a simple CAPM (equation 22) and Fama-French regression (equation 23) was run for each of the portfolios.

3.2.4 Double sorted portfolios - Dependent sorts on liquidity and volatility

Dependent sorting was also used in order to find out whether the use of this different approach affects the results significantly. The stocks were first sorted into three portfolios based on their relative bid-ask spreads (yearly averages) and then within these three liquidity sorts on volatility, thus past 1-year standard deviation of returns. The procedure resulted in nine portfolios as well, only this time each of the three portfolios belonging to the same level (group) of liquidity had different levels of volatility. But their liquidity was in the same 1/3 of total liquidity. For example, the standard deviations for further division for the stocks in the minimum liquidity group were calculated and stocks were divided into three volatility groups only on the basis of the estimates for minimum liquidity group. Thus the standard deviations of stocks belonging to medium and maximum liquidity groups were not taken into account when dividing the stocks in the minimum liquidity group. All the possible combinations of liquidity and volatility were formed. Table 2 shows all these combinations.

Table 2. Dependent sorts on liquidity and volatility

Min liq.	Med liq.	Max liq.
Min vol.	Min vol.	Min vol.
Med vol.	Med vol.	Med vol.
Max vol.	Max vol.	Max vol.

Once again, the performance of portfolios double sorted portfolios was assessed and CAPM and Fama-French regressions (equations 22 and 23) were run.

Dependent sorting on liquidity and subsequently on volatility enables the relationship between volatility and average returns to be observed independently of liquidity since liquidity is the same in each of the liquidity buckets. This may shed new light on the low volatility effect because the two effects can be observed separately.

3.2.5 Double sorted portfolios - Dependent sorts on volatility and liquidity

The procedure described in the previous section (3.2.4) was then repeated, the only difference being the sorting order: this time the stocks were first sorted by their past volatility and then by their average bid-ask spread/price ratio within the three volatility groups. All other details of the sorting have remained the same. The performance of all nine portfolios was assessed and the alphas were calculated using CAPM and Fama-French regressions (equations 22 and 23). Table 3 shows all the possible combinations of volatility and liquidity where the stocks are first sorted by volatility.

Table 3. Dependent sorts on volatility and liquidity

Min vol.	Med vol.	Max vol.
Min liq.	Min liq.	Min liq.
Med liq.	Med liq.	Med liq.
Max liq.	Max liq.	Max liq.

3.3 Assessment of performance of the portfolios

The performance of volatility sorted and of double sorted portfolios was assessed by calculating:

- **Average excess returns** to see whether portfolios with ex-ante lower risk produce higher ex-post returns on average and to see whether the least liquid portfolios produce higher average returns than portfolios with higher liquidity. The averages were annualized in order to make them more comparable.
- **Standard deviations** of portfolio returns to see whether higher ex-ante risk is producing higher ex-post risk as well. Standard deviations were annualized as well.
- **Sharpe ratios** to compare the performance of the portfolios on a risk-adjusted basis. They were calculated with the use of equation (12).
- **Alphas of simple CAPM regression and Fama-French adjusted alphas** to see whether portfolios are producing any added value compared to the market. This is the average excess return that is left unexplained by the model. Positive value of alpha of a portfolio indicates that the portfolio has earned more than predicted by the model, while a negative value suggests underperformance compared to the market. The term Jensen`s alpha is also often used in the literature.

Since all three sets of research questions are based on the values of the alphas, these were thought of as the most important measure of the portfolios` performance and the most important indicator of a low volatility (and a low liquidity) effect.

The performance of volatility sorted portfolios was compared to the performance of double sorted portfolios in order to find out whether the results have changed after the inclusion of stocks` liquidity into the model. The performance of all portfolios was also compared to the performance of the market, S&P 600 index and the equal-weighted average of all S&P 600 constituents.

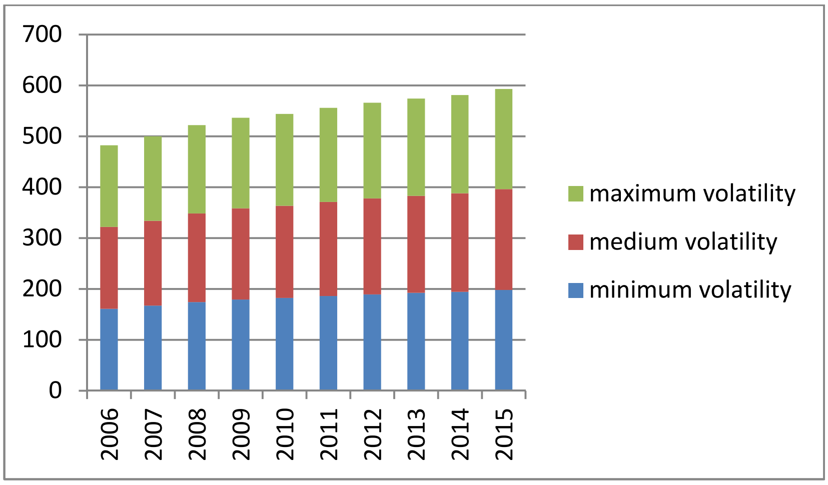
Given there is evidence of low volatility effect in my sample of stocks it can either be a consequence of illiquidity of the less volatile stocks or not. Therefore, if the results show that liquidity has a significant effect on the performance of the portfolios, this will suggest low volatility effect (if there is evidence of its existence in my sample of data) is not an anomaly. Rather, it reflects another effect which is liquidity related. The opposite results might indicate anomalous risk–return relation, although many other factors could be taken into account here, so it is hard to draw some firm conclusions without additional tests.

4 RESULTS

4.1 Overview of the data

The data consist of all the S&P 600 constituents, which includes 600 U.S. stocks with the smallest market capitalization, and covers the period from 31 December 2004 through 29 January 2016. There are some missing values in the dataset, especially at the beginning when some firms have not been established yet. As mentioned, if a firm has died, its data is no longer in the Thomson Reuters Datastream database. Figure 1 shows the number of stocks for which the data is available for each year (year 2005 is omitted because it used for the calculation of standard deviations only). It is obvious from Figure 1 that the number of data available has increased in time. Figure 1 also shows how the stocks are assigned to the three volatility portfolios. Each year the number of stocks for which the data is available is divided into three equal groups based on the estimated standard deviation from the previous year. If the total number is not even, the number of stocks in the portfolio with maximum volatility is properly adjusted. For example, 482 stocks in year 2006 were assigned to minimum (161), medium (161) and maximum (160) volatility portfolios.

Figure 2. Number of stocks for which the data for the calculation of (1-year) standard deviation is available



4.2 Volatility portfolios

To conserve some space, the results of volatility sorting are described in a bit more detail than are the double sorting results.

The stocks were first sorted by their past 1-year volatility of returns. The 1-year standard deviations of weekly returns were calculated and, based on this estimate stocks were assigned to one of the three portfolios: minimum, medium and maximum volatility. Portfolios were rebalanced once a year and each time returns were calculated for the year that followed portfolio formation, which resulted in a time series of weekly portfolio returns. The values of mean excess returns and standard deviation were annualized using the following formulas:

$$Return_{ann} = Return_{weekly} * 52 \quad (24)$$

$$SD_{ann} = SD_{weekly} * \sqrt{52} \quad (25)$$

The values of alphas were also annualized using the same formula as for excess returns because these values are in fact weekly average excess returns after the market factor has been controlled for. This makes their comparison easier. Table 4 shows the properties of volatility-sorted portfolios. The statistical significance of the alphas is denoted with *, ** or *** depending on the level of significance[17].

[17] If the P-value is smaller or equal to 0.1, the estimate is given *, if the P-value is smaller or equal 0.01, the intercept is given ** and if the P-value is smaller or equal to 0.001, it is given ***. The following marks of statistical significance were used throughout the thesis.

The results seem to confirm the theory (CAPM) to a certain extent at least as regards the average excess returns. The annualized average excess return of the portfolio with minimum volatility (0.72 %) is lower than the excess return of the most volatile portfolio (1.33 %). The portfolio with medium volatility produces an excess return of 0.71 % per year on average, thus the least volatile portfolio slightly outperforms its medium volatility counterpart. The average difference between the excess return of maximum and minimum volatility portfolios is -0.61 %. Standard deviation of this difference equals 13.8 %. So, according to Table 4, higher risk is producing higher excess return on average.

Higher ex-ante risk seems to be producing higher ex-post risk as well which means past volatility is a good predictor of future volatility. The least volatile portfolio has the lowest and the most volatile portfolio has the highest value of standard deviation (20.48 % and 30.5 %, respectively). Medium volatility portfolio is somewhere in the middle (25.27 %). The results also confirm the standard deviation-beta relationship since the values of beta (not shown) increase with the values of standard deviations.

Table 4. Properties of the low volatility portfolios calculated in the period 6 Jan 2006 – 29 Jan 2016 (annualized log returns)[18]

	Min. volatility	**Med. volatility**	**Max. volatility**	**Spread (min-max)**	**Market (Rm-Rf)**	**S&P 600 constit.**	**S&P 600 index**
Mean	0.72 %	0.71 %	1.33 %	-0.61 %	7.15 %	0.93 %	4.28 %
St. dev.	20.48 %	25.27 %	30.5 %	13.8 %	18.91 %	24.95 %	22.91 %
Sharpe	0.04	0.03	0.04	/	0.38	0.04	0.19
Alpha	-6.38 %*	-8.23 %**	-9.4 %**	3.02 %	/	-7.95 %**	-3.63 %
R^2	0.84	0.88	0.87	/	1	0.89	0.9

Although the least volatile portfolio does not outperform the other two on the basis of excess returns, its risk-adjusted performance is the same as of the most volatile portfolio. The values of Sharpe ratios are 0.04 for both portfolios. The middle volatility portfolio is not very far behind with a Sharpe ratio of 0.03. Such a result does not indicate a low volatility effect, but it does not support the theory either. According to Table 4 the risk – risk-adjusted excess return relationship is flat instead of positive. The least volatile was able to produce the same excess return per unit of risk as the most volatile portfolio.

[18] The values of S&P 600 constituents (the second column from the right) were calculated as simple averages of weekly returns of all the constituents of S&P 600.

The intercept is negative for all the portfolios, indicating underperformance compared to the market portfolio. However, the annualized value of alpha is least negative for the portfolio with the lowest volatility (-6.38 %), more negative for the medium volatility portfolio (-8.23 %) and the most negative for the most volatile portfolio (-9.4 %). The results thus show an evidence of a volatility effect since the portfolio with minimum variance is underperforming the least. All the intercepts are statistically significant – the ones of middle and high volatility portfolio are significant at 1 % level and the one of low volatility portfolio at 5 % level.

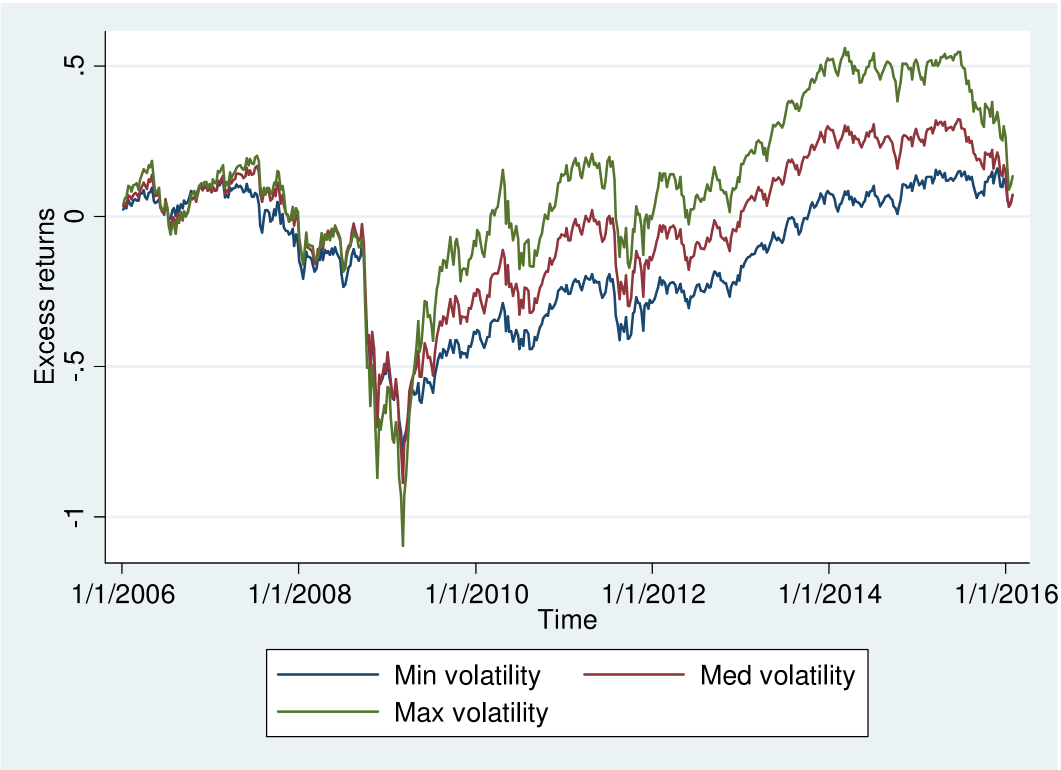
As regards the overall fitness of the three regression models, little (less than 20 %) variability is left unexplained by the market factor: the values of R-squared are 0.84, 0.88 and 0.87 for the least, medium and the most volatile portfolios, respectively. The underperformance of all three portfolios is apparent also from the comparison of their characteristics with the market. In the entire period under observation all the portfolios have underperformed compared to the market. The latter has produced 7.15 % excess return per year on average with 18.91 % of risk which yields a Sharpe ratio of 0.38. Thus, the risk-adjusted performance of the market exceeds the performance of all three volatility sorted portfolios substantially.

In order to get a bit more insight, I compared the results of the portfolios with the performance of S&P 600 index and an equal-weighted average of S&P 600 constituents as well. The results show outperformance of S&P 600 compared to all three volatility-sorted portfolios. The S&P 600 has yielded 4.28 % of excess return per year on average with 22.91 % risk, which gives the Sharpe ratio of 0.19. However, compared to the market, S&P 600 has underperformed in terms of excess returns and on a risk-adjusted basis as well. The equal-weighted average of all S&P 600 constituents is shown in the second column from the right. The numbers are somewhere between the medium and maximum volatility portfolios with an average excess return of 0.93 % per year and risk of 24.95 % per year. Sharpe ratio thus equals 0.04 which is the same as of minimum and maximum volatility portfolios. The value of alpha (-7.95 %) is somewhere between the values of minimum and medium volatility portfolios. Apparently, the equal-weighted average of S&P 600 has underperformed the market in the years 2006 – 2016. This is also apparent from Figure 14 in the Section 4.6.1.

Next, I wanted to see the development of portfolio returns in time. Figure 2 shows the cumulative excess returns of all three volatility portfolios for the whole period under examination, so from 6 January 2006 – 29 January 2016. The returns follow the same pattern throughout almost the whole time period: the most profitable is the highest volatility portfolio, followed by middle volatility and low volatility portfolio, although it is rather hard to tell the difference between them in the beginning of the series. Up until mid-2008 the three return series have stayed closely together, but after the 2008 crisis the difference between them has become very apparent. So, with a few smaller exceptions the most volatile portfolio has outperformed the other two in the whole period under observation. In other words, higher risk has produced higher excess returns over the entire 10-years period.

The three time series have reached their low in the years 2008 and 2009 and their peak around the years 2014 and 2015. It was very recently that the returns began to fall again and yielded much lower returns in the end of series (see Table 5). These are thus the 10-period returns in excess of the one month Treasury bill return. This means if, for example, the least volatile was held (all stocks in equal proportions) for the past 10 years, the return on this investment would be 7.3 %.

Figure 3. Cumulative excess returns of volatility sorted portfolios



Next, I wanted to compare the 10-year period returns and risk of the three portfolios with the market, S&P 600 index and the equal-weighted average of S&P 600 constituents. Table 5 shows the numbers. Since the returns are logarithmic, these are simply the sums over the entire period. According to the results, the positive risk - return relation still holds. The 10-year return on minimum volatility portfolio over the riskless rate equals 7.3 % which is far less than the return on the maximum volatility portfolio which yielded 13.5 %. However, the performance of medium volatility portfolio (7.15 %) is worse than the performance of the least volatile portfolio, which shows a deviation from the theoretical prediction.

It is important to note that the end-of-period returns were much higher about a year before the end of series, for example if portfolios were held from the beginning of 2006 until the end of June 2015 the total returns would be around 15 %, 32 % and 55 % for minimum, medium and maximum volatility portfolio, respectively. This can also be seen from Figure 2.

Meanwhile, the S&P 600 index and the market[19] yielded a return of 43.22 % and 54 % in the whole period, respectively, which exceeds all the volatility portfolios substantially. The reason for the large difference between the performance of the portfolios and the index lies in the construction methodology behind the index. S&P 600 is a cap weighted index where large cap stocks are given higher weights and judging by the results these stock have outperformed in almost entire period under observation pushing the value of the index up. The difference is also apparent from the comparison of S&P 600 and the equal-weighted portfolio of all S&P 600 constituents. The end-of-period return of the latter is shown in the last column of Table 5 and is somewhere between the medium and maximum volatility portfolios (9.5 %), but is way lower than the return on the actual S&P 600 index (43.22 %).

Table 5. End-of-period (total) return, risk and Sharpe ratio of volatility sorted portfolios, S&P 600 index, the market and equal-weighted portfolio of S&P 600 constituents (6 Jan 2006 – 29 Jan 2016)

	Min volatility	**Med volatility**	**Max volatility**	**S&P 600**	**Market**	**S&P 600 constit.**
Excess return	7.3 %	7.15 %	13.5 %	43.22 %	54 %	9.5 %
St. deviation	20.48 %	25.24 %	30.5 %	22.93 %	19.11 %	24.95 %
Sharpe	0.35	0.28	0.44	1.88	2.82	0.38

The standard deviations in Table 5 were annualized so that the annual values of the Sharpe ratios could be calculated. As before, higher ex-post risk is associated with higher ex-ante risk: the values of standard deviations equal 20.48 %, 25.24 % and 30.5 % for minimum, medium and maximum volatility portfolio, respectively. The S&P 600 and especially the market exhibit a rather low value of risk (22.93 % and 19.11 %, respectively) compared to the total return they produce, and the risk of equal-weighted return of all S&P 600 constituents is somewhere in between (24.95 %). Of all the volatility portfolios, the end-of-period Sharpe ratio was the highest for the most volatile portfolio (0.44), followed by the least volatile (0.35) and middle volatility portfolio (0.28).

Table 6 shows the results of Fama-French regressions of all three excess portfolio returns on the excess return on the market and two additional factors, SMB and HML. As before, the annualized alphas are negative and decrease in value with increased volatility. The

[19] For the calculation of cumulative returns the market returns were log-transformed.

inclusion of size and value factors into the regression does not seem to have an important effect on the value of alpha, although the yearly values have increased (become less negative) to -5.63 % for the least volatile, -7.30 % for medium and -8.15 % for the most volatile portfolio. As before, the values are less negative for portfolios with less ex-ante risk (the spread is positive and equals 2.52 % per annum) which speaks in favor of the volatility effect, despite the underperformance of all three portfolios compared to the market.

Table 6. Annualized Fama-French adjusted alphas and R-squared values of the volatility-sorted portfolios

	Minimum volatility	**Medium volatility**	**Maximum volatility**	**Spread (min-max)**
FF-adjusted alpha	-5.63 %**	-7.30 %***	-8.15 %***	2.52 %
R^2	0.93	0.97	0.96	/

The value of beta for each of the three portfolios (not shown) is lowered as well which is expected because of the correlation between beta and both factors that is now reflected in the SMB and HML coefficients. The SMB and HML coefficients (also not shown) are highly significant and positive which is in line with the expectations. With the additional factors the model is now able to explain more than 90 % of excess return variability of all three portfolios. Medium volatility returns are almost perfectly explained by the three factors (R-squared 0.97). This suggests that there are size and value biases in my sample of small U.S. stocks which are now controlled for. The two coefficients had the highest values for maximum volatility portfolio which may suggest that the size and value biases are of larger magnitude for high volatility stocks (within the sample of small stocks).

Since all the portfolios are very volatile (see for example Table 4 and Figure 2), average excess returns and end-of-period returns are perhaps not the best indicator of what was going on with the returns over the entire period. Table 7 thus shows the end-of-year excess returns, risk and Sharpe ratios of all volatility sorted portfolios separately for each year. The last column of Table 7 shows the difference between the excess return of minimum and maximum volatility portfolios.

Table 7. Annual (end-of-year) excess returns, ex-post risk and Sharpe ratios of volatility sorted portfolios (in %)

	Min. volatility			**Med. volatility**			**Max. volatility**			**Spread**
Year	**Mean**	**Std.dev.**	**Sharpe**	**Mean**	**Std.dev.**	**Sharpe**	**Mean**	**Std.dev.**	**Sharpe**	
2006	9.78	12.94	0.76	9.60	16.63	0.58	10.82	19.14	0.57	-1.04
2007	-18.97	18.79	-1.01	-10.80	18.29	-0.59	-9.46	20.25	-0.47	-9.51
2008	-43.41	38.23	-1.14	-50.65	44.62	-1.14	-66.88	49.28	-1.36	23.46
2009	14.21	26.17	0.54	26.30	36.11	0.73	57.38	48.20	1.19	-43.16
2010	14.04	17.94	0.78	19.36	24.27	0.80	21.74	31.18	0.70	-7.70
2011	-4.44	23.22	-0.19	-8.32	30.53	-0.27	-14.59	35.28	-0.41	10.15
2012	7.46	13.43	0.56	11.16	16.63	0.67	12.44	20.44	0.61	-4.97
2013	29.89	11.54	2.59	33.47	13.25	2.53	41.15	14.45	2.85	-11.26
2014	5.02	12.71	0.39	-0.94	14.71	-0.06	-0.27	17.80	-0.02	5.28
2015	-1.01	13.62	-0.07	-12.18	16.02	-0.76	-22.28	21.21	-1.05	21.27

It is apparent from Table 7 that the returns of all three portfolios have varied substantially from year to year as has their volatility. It is hard to say which of the portfolios has performed better judging from the annual numbers, though minimum volatility portfolio is less sensitive to the changes in general conditions in the market compared to the other two portfolios (which is in accordance with the values of CAPM betas). For example, when the market was down in 2008, the yearly returns fell by 43.41 %, 50.65 % and 66.88 % for the least, the middle and the most volatile portfolio, respectively. And when the market was up, for example in 2013 the returns increased the most for the most volatile portfolio (41.15 %) and the least for minimum volatility portfolio (29.89 %). Middle volatility portfolio was somewhere in between (33.47 %). The difference between the returns of the least and the most volatile portfolios is shown in the last column. The values are negative for 6 out of 10 years which is a bit of an ambiguous result. As already noted, the low volatility stocks seem to do better (annual returns are less negative) than high volatility stocks when the market is down (when the yearly returns are negative), but the returns are also less positive when the market is up. The difference seems to be positive when the market is down and both, minimum and maximum volatility, are negative. But when the returns are positive, high volatility portfolio usually outperforms (negative difference).

As regards the ex-post risk on a year-by-year basis, it seems that the most volatile portfolio has produced more risk than the other two portfolios, but the differences are not very large (see also Figure 3).

The results from Table 7 are quite intriguing, since minimum volatility has outperformed in almost half of the years. To complement and compare the results of Table 7 annual return and risk were calculated for the S&P 600 index and the market as well. Table 8 shows the yearly numbers. The numbers of minimum volatility portfolio are added and the last two columns show the spreads of minimum volatility portfolio returns minus the S&P 600 and the market, respectively. Standard deviations are annualized with the use of formula (25).

A quick glance at Table 8 indicates underperformance of the least volatile portfolio compared to both benchmarks. The differences with the S&P 600 are negative 6 out of 10 times and 7 out of 10 compared to the market (the spreads are not shown to conserve space). The volatility of both benchmarks does not seem to exceed the one of minimum volatility portfolio substantially. The Sharpe ratios of minimum volatility portfolio, S&P 600 and the market also vary substantially, but in general the values for minimum volatility portfolio do not exceed those of S&P 600 and the market.

To sum up, on the basis of the CAPM and Fama-French alphas, there is an evidence of a low volatility effect when the stocks are sorted by their past volatility. Even though the effect has not been found according the average excess returns, the values of Sharpe ratios also suggest the risk-excess return relationship is flat rather than positive and thus speak in favor of a low volatility effect.

Since the numbers on excess returns and alphas are lower than expected, I performed some additional robustness checks which are summarized in the last section. An interesting test is the formation of five volatility sorted portfolios where maximum volatility portfolio still outperforms, but the second lowest volatility portfolio is a close second (see section 4.6.2).

Table 8. Annual (end-of-year) excess returns, ex-post risk and Sharpe ratis of minimum volatility portfolio, S&P 600 and the market (in %)

Year	**Min. vol. Average**	**Min. vol. Std.dev.**	**Min. vol. Sharpe**	**S&P 600 Average**	**S&P 600 Std. dev.**	**S&P 600 Sharpe**	**Market Average**	**Market Std. dev**	**Market Sharpe**
2006	9.78	12.94	0.76	4.91	15.32	0.32	9.26	10.49	0.88
2007	-18.97	18.79	-1.01	-5.45	17.90	-0.30	1.30	13.91	0.09
2008	-43.41	38.23	-1.14	-45.72	39.40	-1.16	-51.63	34.60	-1.49
2009	14.21	26.17	0.54	27.39	32.34	0.85	29.29	26.15	1.12
2010	14.04	17.94	0.78	20.74	22.80	0.91	14.94	17.92	0.83
2011	-4.44	23.22	-0.19	-19.89	27.59	-0.72	-0.44	22.47	-0.02
2012	7.46	13.43	0.56	11.80	15.36	0.77	13.33	12.19	1.09
2013	29.89	11.54	2.59	35.24	12.15	2.90	31.41	10.26	3.06
2014	5.02	12.71	0.39	5.08	13.86	0.37	12.75	11.60	1.10
2015	-1.01	13.62	-0.07	-2.43	14.63	-0.17	-0.23	13.78	-0.02

4.3 Double sorted portfolios – independent sorts

The stocks were sorted on both, volatility and liquidity. The yearly average relative bid-ask spreads were calculated in addition to yearly standard deviations from before. Stocks were then assigned to three groups based on the bid-ask spread calculation and to three groups based on standard deviation, and combined together which resulted in nine groups of stocks. Excess returns for all portfolios were calculated for the first year (2007) and then the procedure was repeated exactly one year later. The first year for which the data for calculations of average relative bid-ask spreads (bid and ask prices) was available is 2006, hence the first year of data for double sorted portfolios is 2007. The properties of the nine time series of weekly portfolio returns are shown in Tables 9 – 14. When interpreting the results the portfolios with similar levels of liquidity were compared with each other in order to see how their performance is affected by volatility. In addition, the effect of stocks` liquidity can also be observed from these results.

It is apparent from Table 9 that excess returns and standard deviations follow different patterns depending on the liquidity of stocks. Comparing stocks with low liquidity shows that the average excess return is the highest for middle volatility portfolio, while comparing stocks with medium liquidity shows that the number is the highest for the most volatile portfolio. Within the most liquid group minimum volatility portfolio outperforms (produces the least negative result). Ex-post standard deviations (Table 10) on the other hand, increase with ex-ante risk for all the liquidity groups.

The values of average excess returns vary from very negative to positive and it looks like liquidity has a lot do with the numbers. For example, all the portfolios with maximum liquidity exhibit negative excess returns while all the least liquid ones have positive values. In the middle liquidity group the most volatile portfolio produces (bearly) positive excess returns, the other two are negative. The “winner” portfolio is a combination of the least liquid and middle volatility stocks. With an average excess return of 6.52 % and 25.71 % of risk it yields a Sharpe ratio of 0.25 (see Table 11). The “looser” portfolio contains the most liquid and most volatile stocks and has a negative value of average excess return and therefore Sharpe ratio (-0.28).

The comparison of volatility portfolios with different levels of liquidity again shows that the performance of portfolios worsens when liquidity increases. For example, the least volatile portfolios produce the following Sharpe ratios (from least liquid to most liquid): 0.18, -0.03 and -0.13. The same pattern holds for medium and maximum volatility portfolios.

The values of the CAPM alphas in Table 12 are, as for the volatility sorted portfolios, negative for all the portfolios and are insignificant for all the minimum liquidity portfolios. As we move to the right and liquidity increases, the alphas become more and more significant (so their significance increases with volatility too), but their values are still negative which suggests underperformance compared to the market. The alpha is most negative for the maximum liquidity / maximum volatility combination of stocks (-18.20 %) which on one hand contradicts the theoretical positive risk - return relation and on the other hand indicates a strong role of liquidity in the portfolio performance. This is in line with the expectations since this is evidence of low volatility and low liquidity effect at the same time.

Since it is one of the three most liquid portfolios that is underperforming[20] the most (in terms of alpha and Sharpe ratio as well), it seems lower liquidity stocks do offer some compensation for being less liquid and harder to trade.

The betas (not shown) increase in accordance with the theory as do the standard deviations: higher ex-ante risk produces higher betas (which measure ex-post systematic risk) and it produces higher standard deviations (which measure the overall ex-post risk) as well.

Table 9. Annualized average excess returns of the double sorted portfolios - independent sorts

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	3.78 %	6.52 %	3.70 %
Med. liquidity	-0.68 %	-1.66 %	0.21 %
Max. liquidity	-2.71 %	-4.75 %	-8.41 %

Table 10. Annualized standard deviations of the double-sorted portfolios – independent sorts

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	20.86 %	25.71 %	32.25 %
Med. liquidity	22.81 %	27.49 %	32.68 %
Max. liquidity	21.07 %	25.95 %	30.55 %

Table 11. Sharpe ratios of the double-sorted portfolios – independent sorts

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	0.18	0.25	0.12
Med. liquidity	-0.03	-0.06	0.01
Max. liquidity	-0.13	-0.18	-0.28

[20] Here under- and outperformance is thought of as under- and outperformance compared to the other portfolios. Obviously, all the portfolios underperform compared to the market.

Table 12. CAPM alphas of the double-sorted portfolios – independent sorts (the alphas are annualized)

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	-2.66 %	-1.59 %	-6.54 %
Med. liquidity	-7.85 %**	-10.53 %**	-10.38 %**
Max. liquidity	-9.49 %***	-13.22 %***	-18.20 %***

Table 13. R-squared values of the double-sorted portfolios – independent sorts

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	0.78	0.82	0.83
Med. liquidity	0.81	0.85	0.86
Max. liquidity	0.85	0.87	0.84

Table 14. Average number of stocks in the double-sorted portfolios – independent sorts

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	48	57	79
Med. liquidity	60	66	59
Max. liquidity	60	66	59

Table 14 shows the average number of stocks in each of the portfolios (the actual number changes every year). Since these are combinations of stocks with different levels of liquidity and volatility, it is not surprising the portfolios are not equally sized. Note the number of stocks in each liquidity group is the same and equals 185. The exception is the first group where the number is 184, because the whole number (554) is not dividable by three. The differences might also be caused by the missing values in the sample.

Table 15 shows the differences between average returns of minimum and maximum volatility portfolios for stocks with the same level of liquidity. The difference is the largest for the most liquid group of stocks (5.7 %), however, both averages are negative values and the average of minimum volatility portfolio is less negative than the one of maximum volatility. In the low liquidity group, minimum volatility portfolio outperforms, but not by much (0.08 %), and in the middle liquidity group low volatility stocks underperform their high volatility counterparts. It will also be very informative comparing these results to the results of both versions of dependent sorting (see next sections).

Table 15. The differences between annualized average excess returns of minimum and maximum volatility portfolios at similar levels of liquidity

	Min liq. **Min vol. – max vol.**	**Med liq.** **Min vol. – max vol.**	**Max liq.** **Min vol. – max vol.**
Mean	0.08 %	-0.89 %	5.7 %
St. dev.	16.79 %	14.86 %	16.80 %

Table 16 shows the Fama-French adjusted alphas for double sorted portfolios. The values of alphas are negative, but are less negative than before when only the market return was included in the model. As before, the alphas of least liquid portfolios are insignificant and their significance increases as we move towards higher liquidity and volatility. Interestingly, of all three medium liquidity groups the one with middle volatility underperforms the most – its alphas are most negative (-9.05 %). Among the most liquid stocks the portfolio with the highest volatility has the lowest value of the intercept (-16.93 %). The "winner" or the group of stocks with the least negative alpha is the minimum liquidity/medium volatility (-0.17 %). However, the value is insignificant.

Compared to the CAPM alphas the values are higher (less negative) for all the portfolios, but follow the same (or similar) pattern: their values get more negative as their liquidity and volatility increase. The differences are even bigger when liquidity groups are compared for the same level of volatility, for example the least volatile group of stocks has the following alphas moving from low liquidity to high: -1.61 %, -6.49 % and -8.77 %. So, with some small exceptions, these results again show evidence of a low volatility and a low liquidity effect.

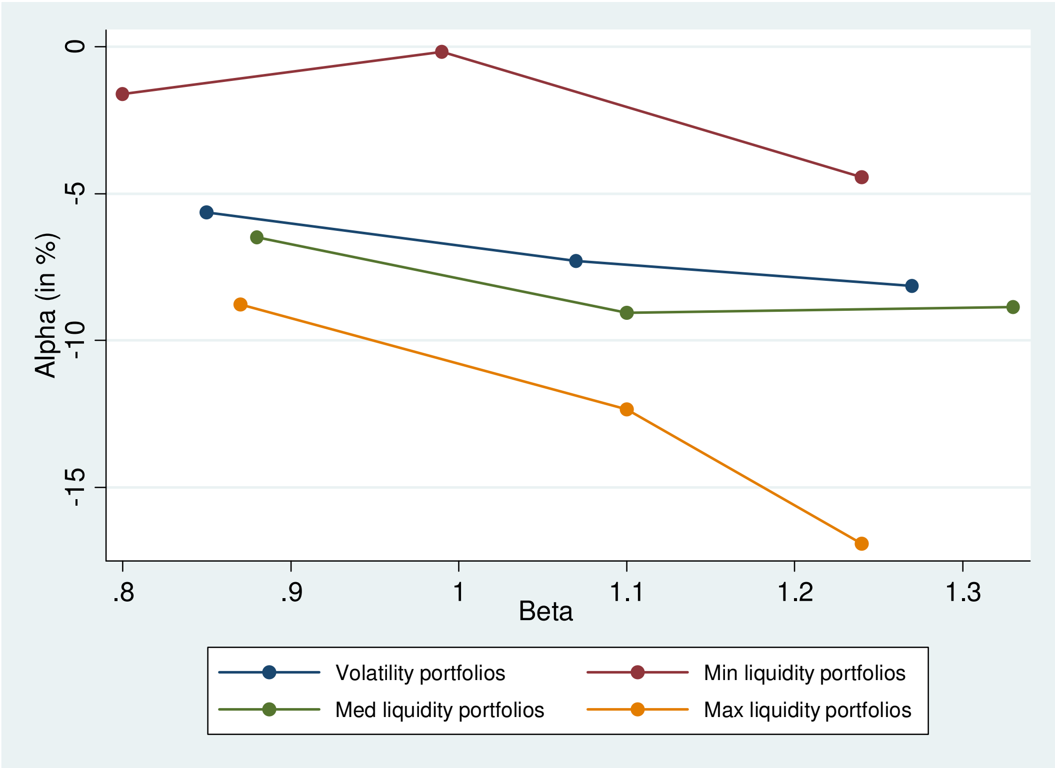
Table 16. Annualized Fama-French adjusted alphas of the double-sorted portfolios – independent sorts

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	-1.61 %	-0.17 %	-4.43 %
Med. liquidity	-6.49 %**	-9.05 %***	-8.87 %***
Max. liquidity	-8.77 %***	-12.34 %***	-16.93 %***

Figure 4 plots the numbers of Table 6 (Fama-French adjusted alphas of the volatility-sorted portfolios) and Table 16. The high beta-negative alpha pattern that has been documented in the existing literature (for example Miller and Scholes (1972), Black, Jensen and Scholes (1972) and Blitz and van Vliet (2007)) is apparent from the graph. The relation is most negative for the most liquid portfolios (the slope is the steepest) and the least for portfolios sorted by volatility only, although the least liquid portfolios stand out with middle volatility portfolio having the least negative intercept. The opposite holds for the middle liquidity group where alpha is the lowest for the middle volatility portfolio.

An interesting thing to note from Figure 4 is also the positioning of the lines. The three least liquid portfolios are positioned the highest, followed by middle and most liquid groups of stocks. As said, liquidity seems to affect excess returns to a large extent.

Figure 4. The values of Fama-French adjusted alphas and betas of volatility sorted and independently double sorted portfolios[21]

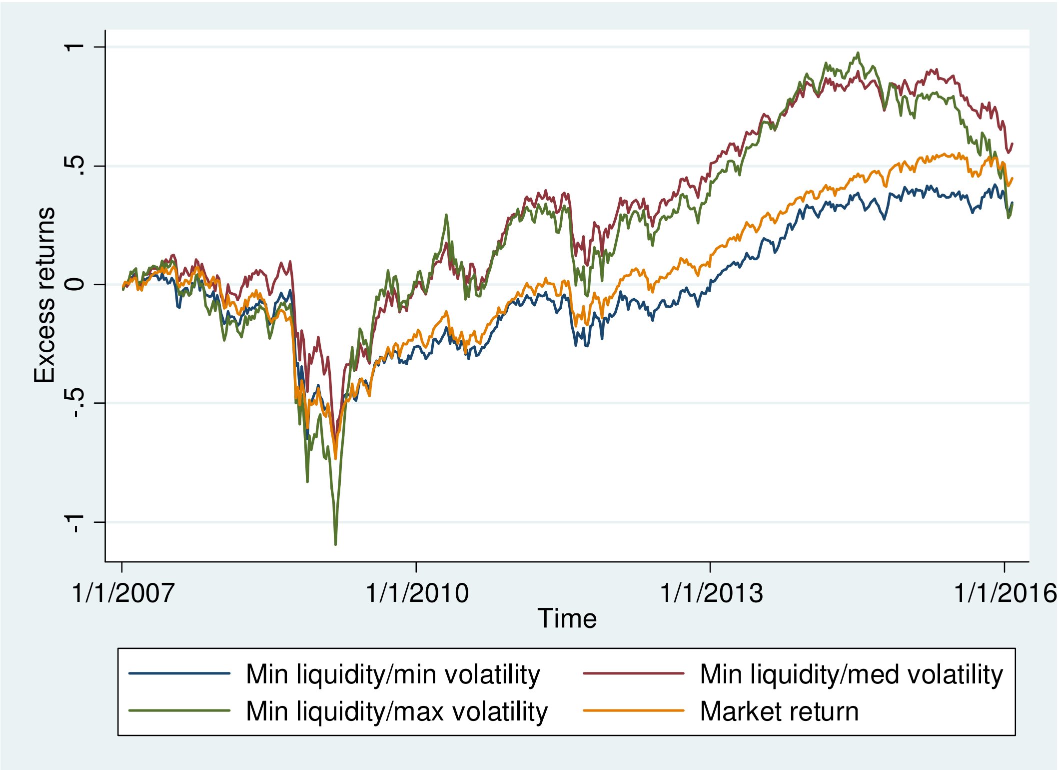


Next, I plotted the cumulative excess returns in time. Figures 5 - 7 show the cumulative excess returns on minimum, medium and maximum liquidity portfolios, respectively. Each figure includes the cumulative return on the market as well for comparison. By plotting all portfolios with different levels of volatility but the same level of liquidity, the relation between volatility and returns can be observed more thoroughly.

According to Figure 5 the results seem to confirm the theoretical risk-return relation, although the middle and high volatility portfolios move very closely together making it hard to distinguish between the two time series. Both portfolios (middle and maximum volatility) outperform the market for almost the entire time and the minimum volatility group is outperformed by the market, but not by much though. All the time series move very closely together in the begging and until the crisis in 2008, but after that the differences become very apparent, at least between the least volatile portfolio and the other two. It seems like the latter have bounced back from the crisis rather quickly and in greater magnitude compared to the minimum volatility and the market as well.

[21] The values of beta are not reported in order to conserve space.

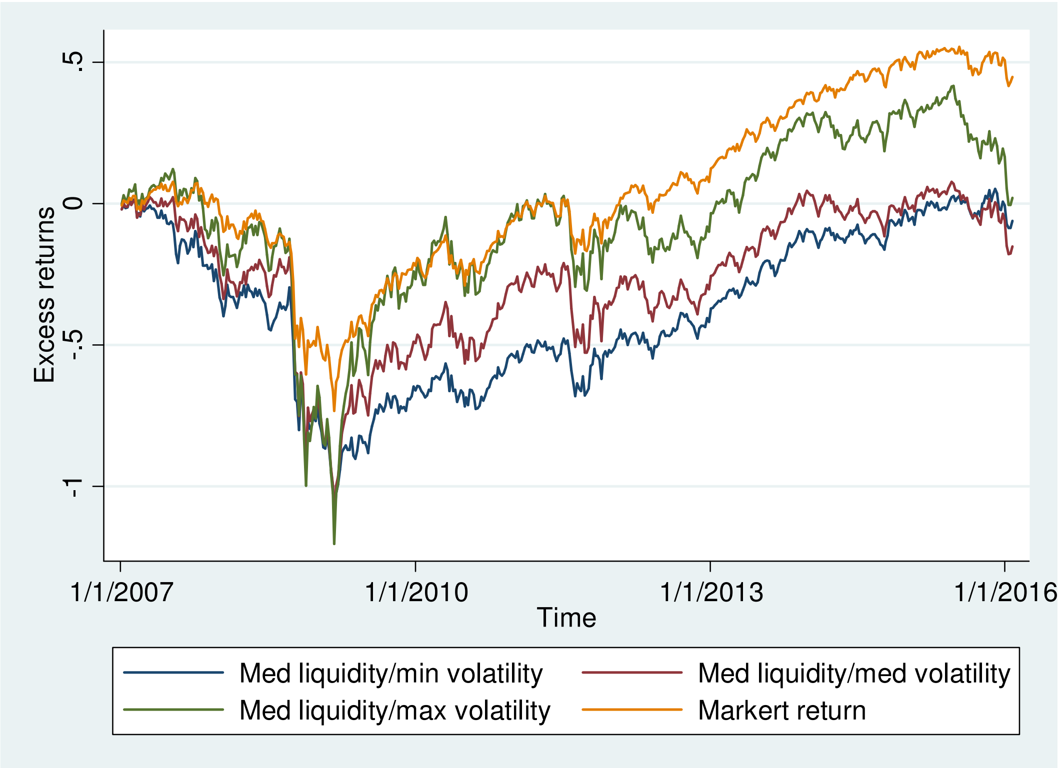
Figure 5. Cumulative excess returns of minimum liquidity portfolios and the market



The situation is similar for the medium liquidity portfolios: the risk – return relation seems to hold for (almost) the entire period. However, the overall performance of all three medium liquidity portfolios is lower compared to the market. The market outperforms all three portfolios almost over the entire period under observation. There are periods though (especially after the 2008 crisis), where maximum volatility outperforms the market, but not by much. The two series move very closely together. In the end of series, all three portfolios have taken a downturn while the market kept moving up. So, on the basis of cumulative excess returns, the positive relation between risk and excess return seems to hold in the middle liquidity group.

Interestingly, Figure 7 shows another result that is quite different from the previous two. It seems that in the most liquid group of stocks the ones who perform better are the low and middle volatility portfolios. The former and the latter move very closely together for almost the entire time period, so it is hard to tell them apart. The “winner” portfolio from Table 11 is middle volatility portfolio with a Sharpe ratio of 0.25, but judging by the cumulative returns it might as well be the minimum volatility. And it seems that in the end of series the difference between them has become larger (in favor of the minimum volatility portfolio). As regards the comparison with the market, all the high liquidity portfolios have shown an underperformance over almost the entire time span. Thus, Figure 7 once again confirms the strong role of liquidity in stocks` performance: higher liquidity seems to reduce the average excess returns.

Figure 6. Cumulative excess returns of medium liquidity portfolios and the market



So, according to cumulative excess returns, among the most liquid stocks, the ones who are the most volatile (ex-ante) underperform the most. This is again an evidence of a low volatility effect.

Since “anomalous” behavior of returns is found for the most liquid stocks, the year-by-year numbers were calculated only for the maximum liquidity portfolios. Table 17 shows the year-by-year excess returns, standard deviations and Sharpe ratios for all three portfolios with maximum liquidity.

Figure 7. Cumulative excess returns of maximum liquidity portfolios and the market

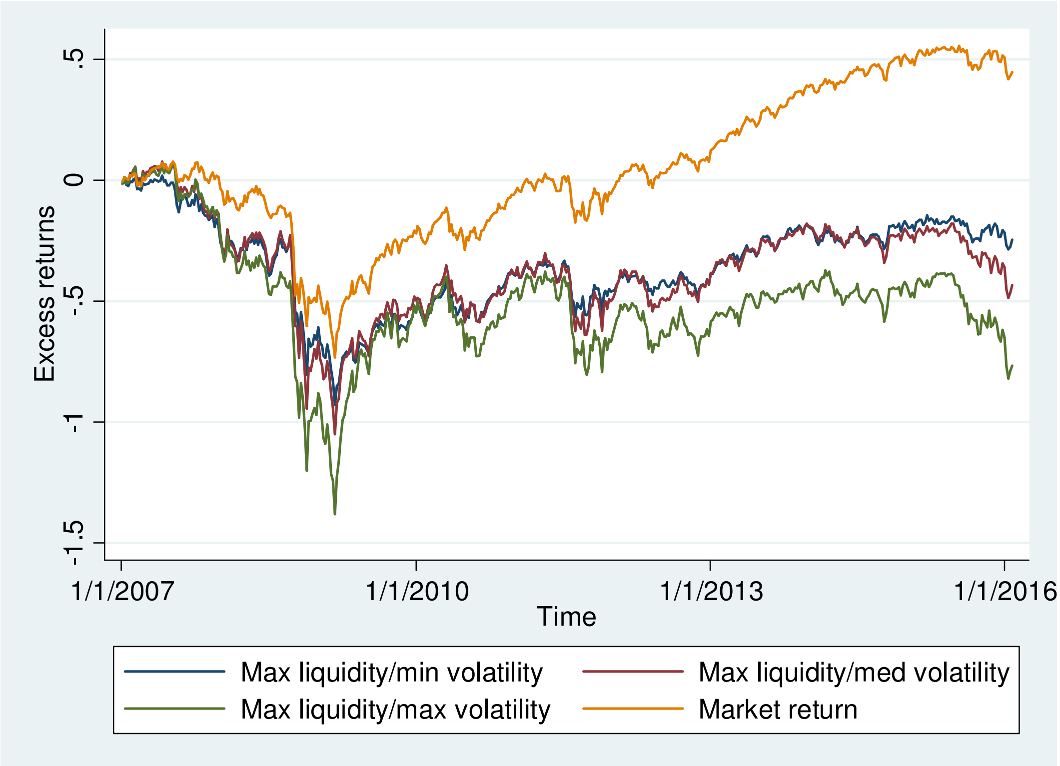


Table 17. Year-by-year excess returns, risk and Sharpe ratios of maximum liquidity portfolios and the market (in %)[22]

Year	Max. liq./Min. vol.			Max. liq./Med. vol.			Max. liq./Max. vol.			Spread
	Average	St. dev.	Sharpe	Average	St. dev.	Sharpe	Average	St. dev.	Sharpe	
2007	-19.13	16.66	-1.15	-14.87	18.82	-0.79	-13.75	19.90	-0.69	-5.38
2008	-46.84	38.94	-1.20	-58.83	46.37	-1.27	-83.20	49.11	-1.69	36.36
2009	13.03	25.38	0.51	25.66	34.25	0.75	45.11	43.63	1.03	-32.08
2010	14.07	18.39	0.77	9.79	22.50	0.44	5.15	27.19	0.19	8.93
2011	-7.07	23.72	-0.30	-11.80	29.71	-0.40	-20.34	35.41	-0.57	13.27
2012	2.68	12.55	0.21	6.81	17.09	0.40	2.86	22.28	0.13	-0.18
2013	24.97	11.68	2.14	25.17	13.12	1.92	20.80	15.36	1.35	4.17
2014	1.36	13.63	0.10	-2.14	14.71	-0.15	1.24	17.96	0.07	0.12
2015	-3.71	13.56	-0.27	-14.19	16.59	-0.86	-19.89	22.14	-0.90	16.18

[22] The last column of Table 17 shows the difference in average excess returns of Max. liq./Min. vol. and Max. liq./Max. vol.

Interestingly, the difference between the least and the most volatile portfolio is positive six out of nine times. Of the most liquid group, the stocks with minimum ex-ante volatility have outperformed their maximum volatility counterparts almost all the years. As before when stocks were sorted by volatility only (see Table 7), when the returns go down for all the portfolios (the general conditions in the market go down) the returns go less negative for least volatile portfolios. Nevertheless, this is still a desirable characteristic of a portfolio. In addition, the minimum volatility stocks were able to produce these returns at lower levels of risk compared to the most volatile stocks which had a positive effect on the Sharpe ratios.

To sum up, the results of independent double sorts shed some new light on the issue, showing signs of both, liquidity and volatility effect. Liquidity without doubt affects the returns of stocks, which is apparent from the tables and graphs of this section: the higher the liquidity, the lower the excess returns. This is not in line with the fundamental theory of the CAPM which states no such effect exists since only systematic risk should be priced. In the CAPM framework all other risks, i.e. asset specific risks, are shifted away as investors increase the number of assets in their portfolios. Liquidity is in this sense overlooked by the CAPM since liquidity costs actually cannot be diversified away, but are not included in the model either (see for example Amihud and Mendelson, 1986). Of course the results are in line with several previous studies and are intuitive: less liquid stocks earn a liquidity risk premium to serve as compensation for their lower liquidity.

The effect of volatility on the excess returns and Sharpe ratios is not as straightforward since it seems to affect returns differently at different levels of liquidity. With the use of independent sorting the nine portfolios were simply combinations of stocks with different levels of liquidity and volatility - not to confuse it with dependent sorting where liquidity (or volatility) is controlled for in the sense that volatility-returns relationship (liquidity-returns relationship) is observed completely independently of liquidity (next sections). For portfolios with low levels of liquidity, the relationship looks concave: the middle volatility portfolio outperforms the other two. But the low volatility portfolio outperforms (although only for 0.08 %) the most volatile portfolio and on a risk-adjusted basis as well (Sharpe ratio 0.18 compared to 0.12). So, the relationship between ex-ante volatility and ex-post risk adjusted returns is not found to be positive for the least liquid stocks. Middle liquidity portfolios follow a different pattern: the relationship is slightly convex with middle volatility portfolio underperforming the most. Out of middle liquidity portfolios the most volatile is the only one that actually produces positive excess return, so no evidence of low volatility effect is found here. The most liquid portfolios all produce negative excess return, but the return is more negative as volatility increases. Sharpe ratios are thus all negative and also go more negative with higher volatility. So, the theory is again not supported by these results.

The alphas of the double sorted portfolios, which are the most important indicator of a low volatility effect, are least negative for less volatile portfolios and decrease with higher volatility which speaks in favor of a low volatility effect. So, the independent sorts of stocks showed a low volatility effect combined with a low liquidity effect, that is, at least to some degree. The next thing to do is sort the stocks on liquidity and subsequently on volatility within the liquidity groups.

4.4 Double sorted portfolios - volatility within the liqudity groups

The stocks were sorted into three equally-sized groups by their 1-year average relative bid-ask spreads and within this sorts by 1-year volatility of returns. The returns were then calculated for the subsequent year and the procedure was repeated until the end of series was reached. Tables 18 - 22 show the characteristics of the nine portfolios. Table 18 includes the average number of stocks that were included in each of the portfolios each year (61). The actual number of stocks in each portfolio was changing from year to year. The overall number has increased each year which is probably due to the fact that there are more missing values at the beginning of the sampling period.

Table 18. Annualized average excess returns of the double sorted portfolios - dependent sorts on liquidity and volatility

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	7.36 %	8.16 %	0.8 %
Med. liquidity	5.53 %	0.29 %	-8.63 %
Max. liquidity	3.36 %	0.07 %	-18.22 %
Average no. of stocks	61		

Table 19. Annualized standard deviations of the double-sorted portfolios – dependent sorts on liquidity and volatility

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	18.25 %	26.73 %	38.14 %
Med. liquidity	19.02 %	26.78 %	37.47 %
Max. liquidity	17.89 %	24.08 %	33.22 %

Table 20. Sharpe ratios of the double-sorted portfolios – dependent sorts on liquidity and volatility

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	0.4	0.31	0.02
Med. liquidity	0.29	0.01	-0.23
Max. liquidity	0.19	0.003	-0.55

Table 21. Annualized CAPM alphas of the double-sorted portfolios – dependent sorts on liquidity and volatility

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	1.7 %	-0.34 %	-11.15 %*
Med. liquidity	-0.37 %	-8.38 %*	-20.71 %***
Max. liquidity	-2.34 %	-7.78 %**	-28.92 %***

Table 22. R-squared values from the CAPM regressions of the double-sorted portfolios – dependent sorts on liquidity and volatility

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	0.79	0.83	0.8
Med. liquidity	0.79	0.86	0.85
Max. liquidity	0.93	0.87	0.85

As can be seen from Table 18, there is an even stronger evidence of a low volatility effect when the stocks are sorted dependently: average excess returns decrease with the increased ex-ante risk within each liquidity group, the only exception being the outperformance of the minimum liquidity / medium volatility compared to its minimum volatility counterpart (average excess return of 8.16 % compared to 7.36 %). Nevertheless, the average excess return of the least volatile portfolio within the least liquid group is almost 10 times the value of the most volatile portfolio of the same group of stocks (0.8 %). The excess returns seem to decrease with higher liquidity as well.

The standard deviations increase monotonically within each liquidity bucket (see Table 19) confirming the positive ex-ante risk and ex-post risk relation.

The negative risk-return relation becomes even more apparent when the performance is evaluated on a risk-adjusted basis. The values of Sharpe ratio decrease as we move from low to high volatility (see Table 20). The "winner" portfolio is the one with the lowest volatility within the low liquidity group and produces a Sharpe ratio of 0.4 and the "looser" is the portfolio with maximum volatility within the maximum liquidity group (Sharpe ratio of -0.55).

The CAPM alphas are all negative except for the first (minimum liquidity / minimum volatility) portfolio and a lot of them are insignificant (see Table 21). Their significance increases with higher liquidity and even more so with higher volatility. Once again, the alpha is most negative for the highest volatility portfolio within highest liquidity group and equals -28.92 % per annum. The highest alpha is produced by winning minimum liquidity / minimum volatility portfolio (1.7 % per annum, but it is insignificant). This speaks in favor of the low volatility effect.

I next ran Fama-French regressions for all the portfolios. The results are shown in Table 23.

Table 23. Annualized Fama-French adjusted alphas of the double-sorted portfolios – dependent sorts on liquidity and volatility

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	2.53 %	1.19 %	-8.19 %*
Med. liquidity	0.41 %	-7.2 %***	-18.49 %***
Max. liquidity	-2.02 %	-7.16 %***	-27.3 %***

The Fama-French adjusted alphas decrease as liquidity and volatility get higher. They are mostly negative except for the least and medium volatility portfolios within the least liquid group of stocks and the least volatile portfolio within the medium liquidity group of stocks. However, these are all insignificant as are about half of the alphas. Interestingly, a comparison of the results with the results from simple CAPM regressions shows an increase in all the values of the intercept. So, the alphas that were previously negative which was almost all of them, become less negative and the only alpha that was positive before (minimum liquidity / minimum volatility portfo1io; 1.7 %) is now higher and equals 2.53 % per year. The alpha of medium liquidity / minimum volatility portfolio that was previously negative (-0.37 %) is now positive and equals 0.41 % per year. The same holds for the alpha of minimum liquidity / medium volatility portfolio (-0.34 % compared to 1.19%). As do the CAPM alphas, the Fama-French adjusted alphas show evidence of a low volatility effect: within each liquidity group their values get lower as volatility increases.

The values of beta (not shown to conserve space) follow the same pattern as before: they monotonically increase with volatility. Their values are all lowered, which again makes sense since two significant variables were added in the regression, but they are all still highly significant.

Roughly speaking, the model is able to explain about 10 % more variability in excess returns when SMB and HML are added as regressors in addition to the market factor. The coefficients of SMB and HML are also highly significant and positive which is in line with the theory (the R-squared values and the coefficients estimates are not shown to conserve space).

Next I wanted to see the evolution of the portfolio returns in time. Figures 10 - 12 show the cumulative excess returns of the dependently double sorted portfolios and the market. The situation is now reversed compared to single sorts on volatility: minimum volatility portfolio is outperforming the other two portfolios and the market for almost the entire period under observation. There are some minor exceptions where medium volatility outperforms and there is a short period of time when maximum and medium volatility portfolios outperform the least volatile portfolio (within minimum liquidity group - see

Figure 10), but otherwise the results show a clear low volatility effect for all the liquidity groups. So, when liquidity is controlled for and the risk-return relation is observed independently, the least volatile group of stocks outperform the other two and the market. Note also that the effect is even more severe as liquidity increases. It can be seen from Figure 12 that the most volatile group of stocks has taken an entirely different "path" compared to the other portfolios and has experienced much lower returns.

As was done for volatility sorts and independent sorts on liquidity and volatility, I have once again calculated the year-by-year values of excess returns, standard deviations and Sharpe ratios. I have chosen the trinity of the most liquid portfolios since there seems to be more evidence of a low volatility effect in this group of stocks (see Table 18 and Figure 12). Table 24 shows the year-by-year numbers of the portfolios. The last column shows the difference between the yearly returns of the minimum and maximum volatility portfolios within the maximum liquidity group.

Figure 8. Cumulative excess returns of all three minimum liquidity portfolios and the market

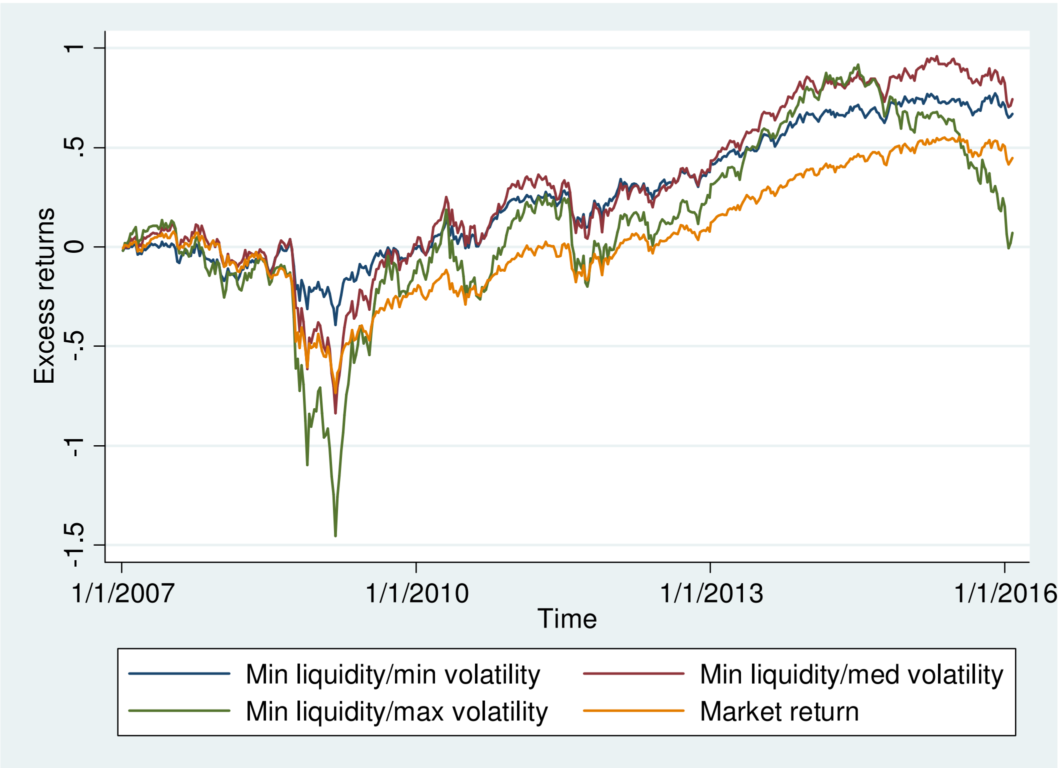


Figure 9. Cumulative excess returns of all three medium liquidity portfolios and the market

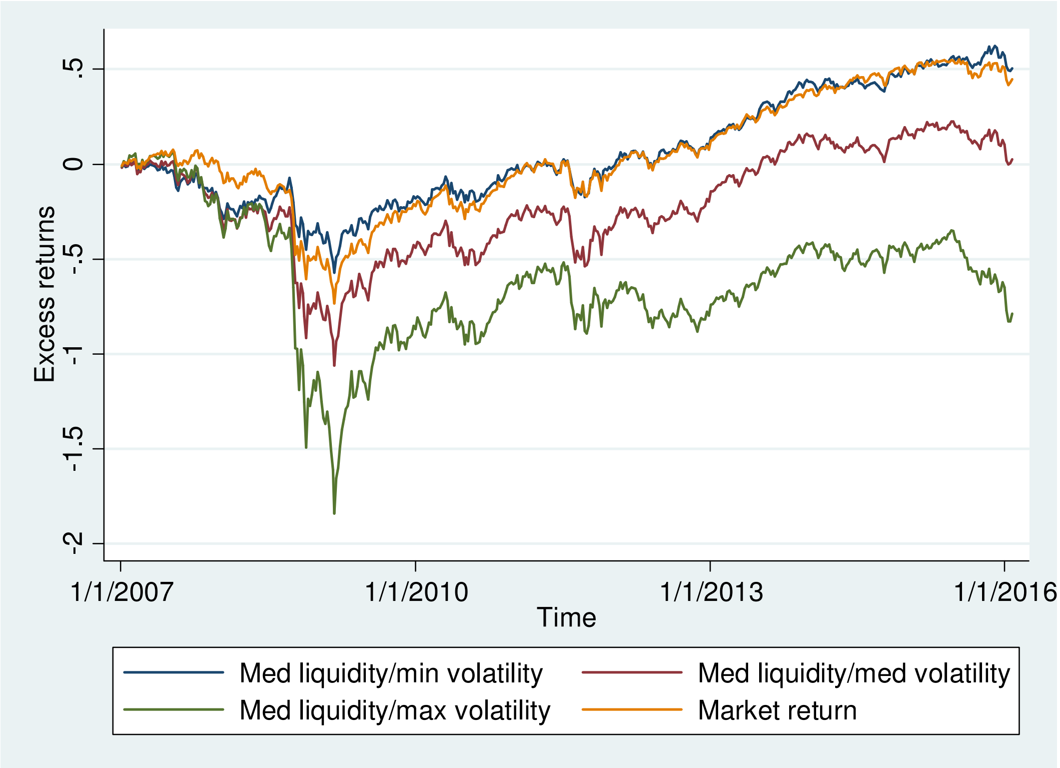
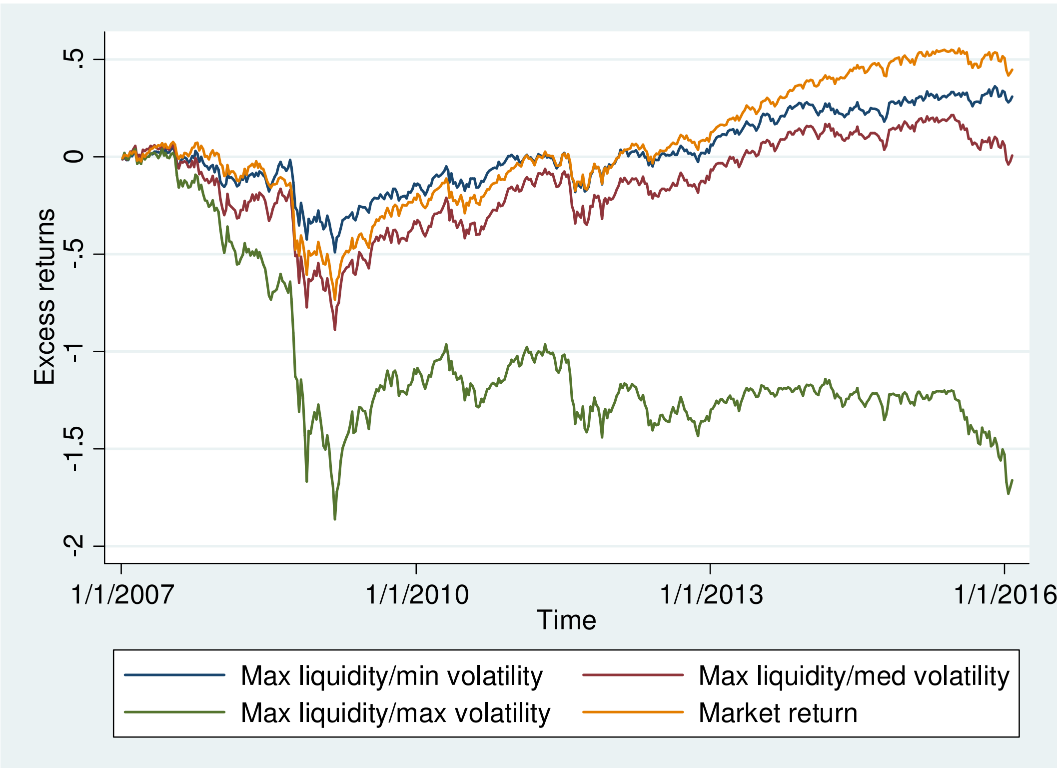


Figure 10. Cumulative excess returns of all three maximum liquidity portfolios and the market



The spread of minimum minus maximum volatility excess return is positive every year except for 2009 where the most volatile portfolio outperforms, but it rises back from a very low value in 2008. Table 24 thus confirms the results of Tables 18 – 21 and Table 23 and Figures 10 - 12 which all show evidence of a low volatility effect. A quick glance at the annualized values of standard deviations shows that higher values of excess returns were achieved at lower levels of risk which further magnifies the low volatility effect and has a favorable effect on the Sharpe ratios.

To sum up, the results of this section have shown that there is strong evidence of a low volatility effect when the stocks are first sorted on liquidity and then on volatility within the liquidity groups. The results of average excess returns, standard deviations and Sharpe ratios all speak in favor of volatility effect within each liquidity bucket. The alphas are mostly negative but decrease with the increased volatility. The cumulative excess returns also exhibit a specific pattern: low volatility stocks (mostly) outperform the other two groups and the market within each liquidity group. The results also confirm the liquidity effect: higher liquidity reduces the performance of the portfolios.

Table 24. Year-by-year excess returns (in %), standard deviations (in %) and Sharpe ratios of maximum liquidity portfolios and the market[23]

Year	Max. liq./Min. vol.			Max. liq./Med. Vol.			Max. liq./Max. vol.			Spread
	Average	**St. dev.**	**Sharpe**	**Average**	**St. dev.**	**Sharpe**	**Average**	**St. dev.**	**Sharpe**	
2007	-6.69	15.0	-0.45	-13.41	18.19	-0.74	-13.41	21.57	-1.31	21.5
2008	-26.6	30.3	-0.88	-47.89	42.16	-1.14	-47.89	61.21	-1.74	80.16
2009	19.4	21.24	0.91	27.9	31.74	0.88	27.9	44.45	0.58	-6.37
2010	13.79	15.97	0.86	16.45	21.44	0.77	16.45	25.32	0.16	9.76
2011	-4.53	21.41	-0.21	-5.23	27.59	-0.19	-5.27	36.54	-0.70	21.21
2012	6.57	12.33	0.53	10.32	15.52	0.67	10.32	21.37	-0.22	11.23
2013	26.09	10.98	2.38	27.69	12.73	2.17	27.69	15.07	1.22	7.71
2014	2.58	12.72	0.20	2.04	14.41	0.14	2.04	18.55	-0.21	6.45
2015	3.04	12.79	0.24	-10.05	15.82	-0.63	-10.05	23.02	-1.26	32.12

[23] The last column of Table 24 shows the difference between average excess returns of Max. liq./Min. vol. and Max. liq./Max. vol.

4.5 Double sorted portfolios - liqudity within the volatility groups

In addition to independent sorts and dependent sorts where stocks were sorted by volatility within the liquidity groups, the sorts of stocks by liquidity within the volatility groups were also produced. Thus the stocks were first sorted by volatility (past 1-year standard deviation of returns) and subsequently by liquidity (past 1-year average bid-ask spread/price ratio). Other details of the sorting procedure have remained the same. The procedure has produced three liquidity groups within each volatility group which enables the liquidity-excess return relation to be observed independently of volatility. Tables 25 – 29 show the properties of the nine volatility-liquidity portfolios. The average number of stocks in each portfolio is 61 and is shown in Table 25.

As regards the liquidity-excess returns relation, the results are as expected and in line with the results of previous sorting procedures. The excess returns are lower at higher levels of liquidity and this holds in each volatility bucket. All the values in the three most liquid groups are negative and two out of three are negative in the middle liquidity group. The volatility-excess returns relation, on the other hand, is not as clear. If we compare stocks with similar levels of liquidity (note that these are not independent sorts, nor they are dependent sorts by liquidity and subsequently by volatility), the averages follow different patterns. Among the groups of stocks with minimum liquidity, medium volatility portfolio is the one with the highest excess return (6.19 %), among the groups with medium liquidity, maximum volatility portfolio has the highest (although barely positive; 0.44 %) return and among the groups with the highest liquidity, minimum volatility portfolio has the highest (least negative; -3.62 %) average excess returns. On the basis of excess returns the "winner" is the medium volatility / minimum liquidity portfolio (average excess return of 6.19 %) and the "looser" is the maximum volatility / maximum liquidity portfolio (average excess return of -5.39 %).

The values of standard deviations (see Table 26) are in line with the theoretical predictions since they increase with higher ex-ante risk. In addition, the values are almost the same in each volatility bucket.

The Sharpe ratios (see Table 27) increase with lower liquidity and higher volatility. The "winner" is the portfolio made of the least liquid stocks within the middle volatility group (it yields 0.24 of risk-adjusted return) and the "looser" is the portfolio with the most liquid group within the same volatility group (it yields -0.18 of risk-adjusted return).

Table 25. Annualized average excess returns of the double sorted portfolios - dependent sorts on volatility and liquidity

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	2.98 %	6.19 %	5.83 %
Med. liquidity	-0.7 %	-2.27 %	0.44 %
Max. liquidity	-3.62 %	-4.74 %	-5.39 %
Average no. of stocks	61		

Table 26. Annualized standard deviations of the double-sorted portfolios – dependent sorts on volatility and liquidity

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	21.35 %	25.97 %	31.66 %
Med. liquidity	21.93 %	27.77 %	33.43 %
Max. liquidity	21.57 %	25.72 %	31.29 %

Table 27. Sharpe ratios of the double-sorted portfolios – dependent sorts on volatility and liquidity

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	0.14	0.24	0.18
Med. liquidity	-0.03	-0.08	0.01
Max. liquidity	-0.17	-0.18	-0.17

Table 28. Annualized CAPM alphas of the double-sorted portfolios – dependent sorts on volatility and liquidity

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	-3.71 %	-2.05 %	-4.08 %
Med. liquidity	-7.59 %*	-11.25 %**	-10.33 %*
Max. liquidity	-10.55 %***	-13.11 %***	-15.51 %***

Table 28 shows the CAPM alphas of the nine portfolios. All the alphas are negative and follow similar patterns within each volatility group as the average excess returns (see Table 25). The portfolios with more liquid stocks have produced more negative alphas suggesting greater underperformance compared to the market. In addition, the statistical significance increases with higher liquidity. As for the comparison of alphas of the portfolios with similar levels of liquidity (in order to observe the volatility-abnormal returns relationship), higher volatility seems to produce lower (more negative) abnormal returns with some small exceptions. According to the values of the intercepts, the "winner", i.e. the portfolio with the highest value of the CAPM alpha, is the medium volatility / minimum liquidity portfolio (-2.05 %, but note that the value is insignificant) and the "looser" is the maximum volatility / maximum liquidity portfolio (-15.51 %).

Table 29. R-squared values from the CAPM regressions of the double-sorted portfolios – dependent sorts on volatility and liquidity

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	0.81	0.83	0.8
Med. liquidity	0.81	0.86	0.85
Max. liquidity	0.85	0.87	0.86

The Fama-French adjusted alphas were calculated as well and their values are shown in Table 30. The values of the alphas follow the same pattern as they did without the Fama-French adjustment: within each volatility group, the alphas decrease with higher liquidity. The comparison of the intercepts at same levels of liquidity (so, between minimum, medium and maximum volatility) produces the desired results (low volatility effect) at least to some extent since almost all of the least volatile portfolios have outperformed their high risk counterparts. As before, without the Fama-French adjustment, the portfolio with the highest, i.e. less negative value of alpha, is the medium volatility / minimum liquidity portfolio (-0.61 % per annum, but the value is not significant) and the one with the lowest value of alpha is the maximum volatility / maximum liquidity (-14.12 % per annum). The comparison of CAPM and Fama-French adjusted alphas shows that the alphas have increased in value (they have become less negative) after the size and value adjustment.

Table 30. Annualized Fama-French adjusted alphas of the double-sorted portfolios – dependent sorts on volatility and liquidity

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	-2.54 %	-0.61 %	-2.18 %
Med. liquidity	-6.45 %**	-9.77 %***	-8.44 %**
Max. liquidity	-9.81 %***	-12.26 %***	-14.12 %***

Next, I plotted the cumulative excess returns of all nine portfolios and the market in time. The results are shown in Figures 11 – 13. The results are quite interesting and in line with the results I got so far. Within each volatility bucket, there seems to be a rather strong liquidity effect: the portfolios with the lowest levels of liquidity outperform the other counterparts of the same volatility groups. The effect is the strongest in the middle volatility group of stocks (see Figure 12), although the most volatile group is a close second. According to the Figures 11 – 13, liquidity seems to have an important effect on the performance of stocks.

Figure 11. Cumulative excess returns of all three minimum volatility portfolios and the market

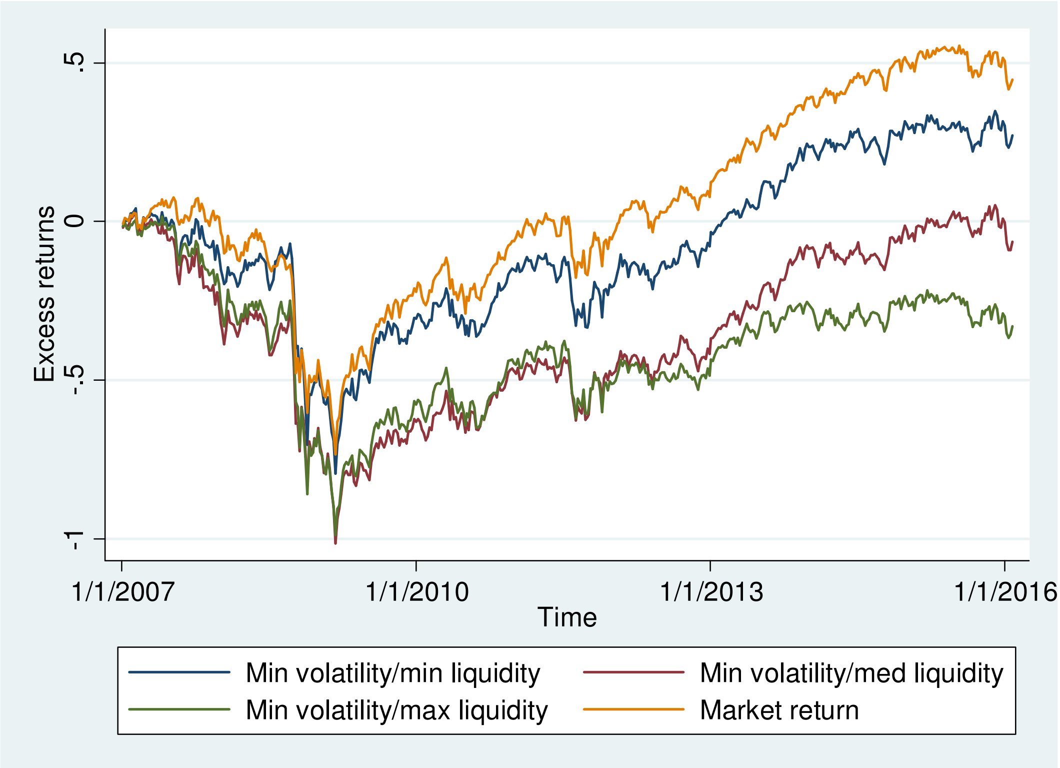


Figure 12. Cumulative excess returns of all three medium volatility portfolios and the market

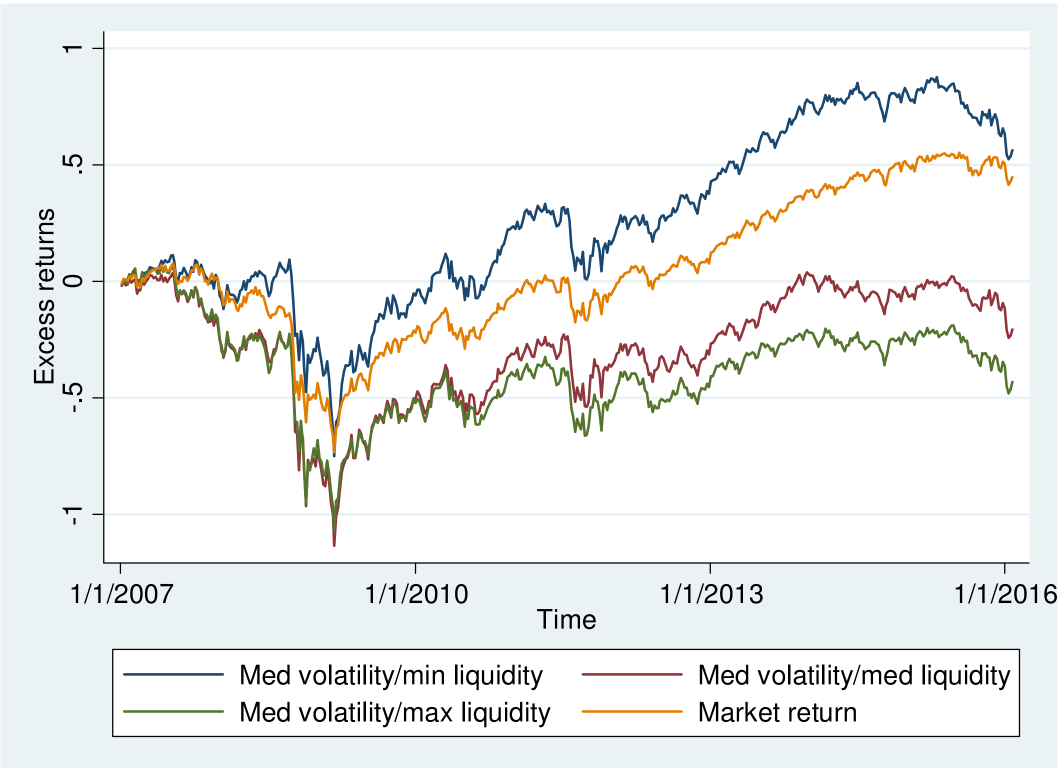
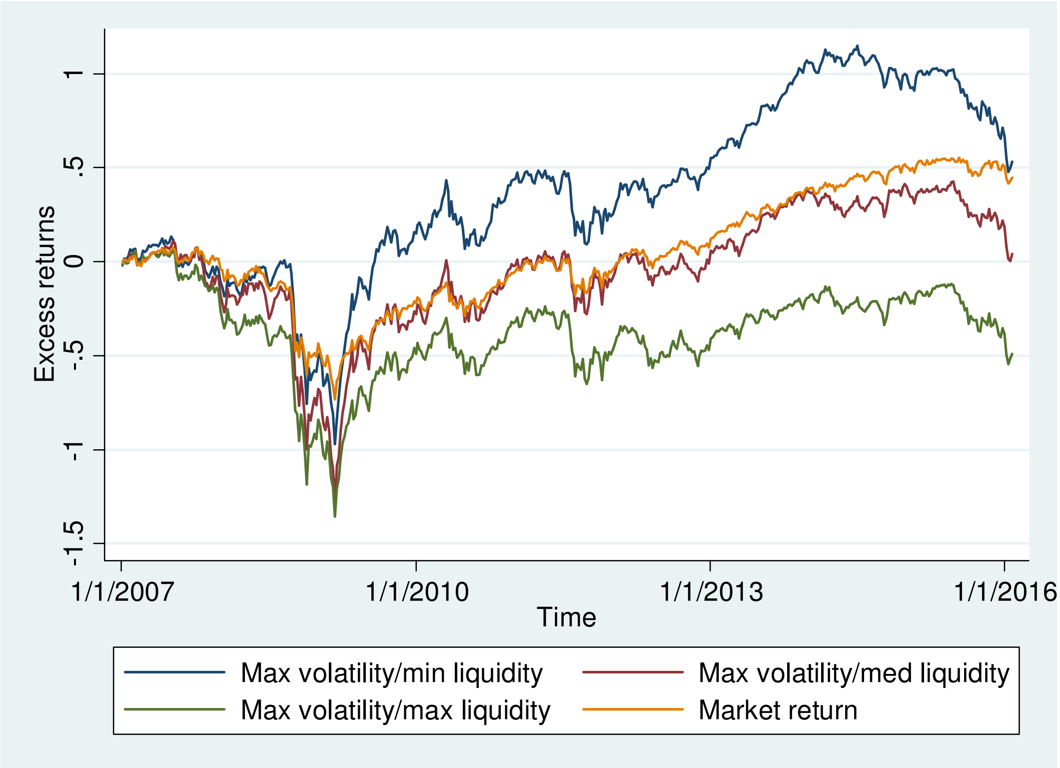


Figure 13. Cumulative excess returns of all three maximum volatility portfolios and the market



To sum up, the results of dependent double sorting on volatility and subsequently on liquidity have produced very unambiguous results regarding the effect of liquidity on the performance of portfolios: higher liquidity produces lower excess returns on average which is intuitive and in line with the previously reported results. As regards the effect of volatility on the performance of the portfolios, the results tend to show evidence of a low volatility effect as well (with some minor exceptions). Though there seems to be a slightly different pattern of stocks` performance at different levels of liquidity, the alphas are higher for low volatility stocks and lower for high volatility stocks confirming the "anomalous" risk-return relation.

4.6 Robustness tests

4.6.1 Sorting stocks into five volatility portfolios

Since it is a bit difficult to draw some firm conclusions about the performance of the portfolios when the stocks are sorted into only three groups, I additionally sorted the stocks into five groups based on their 1-year historical volatility. As before, I have calculated the characteristics of the resulting time series for all the portfolios. The results are summarized in Table 31. The portfolios are denoted with Q1 – Q5, where Q1 indicates the lowest and Q5 the highest volatility portfolio, respectively.

As regards the values of average excess returns and Sharpe ratios, the results support the theoretical risk-return relation almost completely, the only exception being the outperformance of the second quintile (with the excess return of 1.2 % per year it outperforms all but maximum volatility portfolio). The "winner" in terms of risk-adjusted returns is the portfolio with the highest volatility and the "looser" is the low volatility portfolio.

Table 31. Properties of 5 volatility sorted portfolios (annualized log returns)[24]

	Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Spread (Q1-Q5)**
Mean	0.3 %	1.2 %	0.6 %	0.7 %	1.8 %	-1.6 %
St. dev.	19.2 %	23.7 %	24.9 %	28.0 %	32.3 %	18 %
Sharpe	0.01	0.05	0.02	0.03	0.06	/
FF-Alpha	-5.64 %**	-6.1 %***	-7.28 %***	-8.11 %***	-8.01 %**	2.37 %
R^2	0.89	0.94	0.97	0.96	0.93	/

However, the Fama-French adjusted alphas follow the same pattern that was observed throughout the entire analysis: all values are negative (but significant) and tend to decrease with higher volatility. So, according to the abnormal returns, there seems to be evidence of a low volatility effect even when the stocks are sorted into more than three groups.

To sum up, additional sorting did not add much to the existing results. It seems that increasing the number of portfolios has improved the results (average excess returns and Sharpe ratios) in the sense of supporting the theory. However, the Fama-French adjusted alphas decrease with higher volatility, which once again shows evidence of a low volatility effect.

4.6.2 Using a different proxy for the market

So far, my analysis has shown some interesting results: there seems to be an evidence of a low volatility effect in the U.S. small cap equity segment and the effect seems to be highly influenced by liquidity. However, the values of CAPM and Fama-French adjusted alphas were mostly negative suggesting underperformance compared to the market. There can be many reasons for such results, for example, the choice of a proxy for the market excess

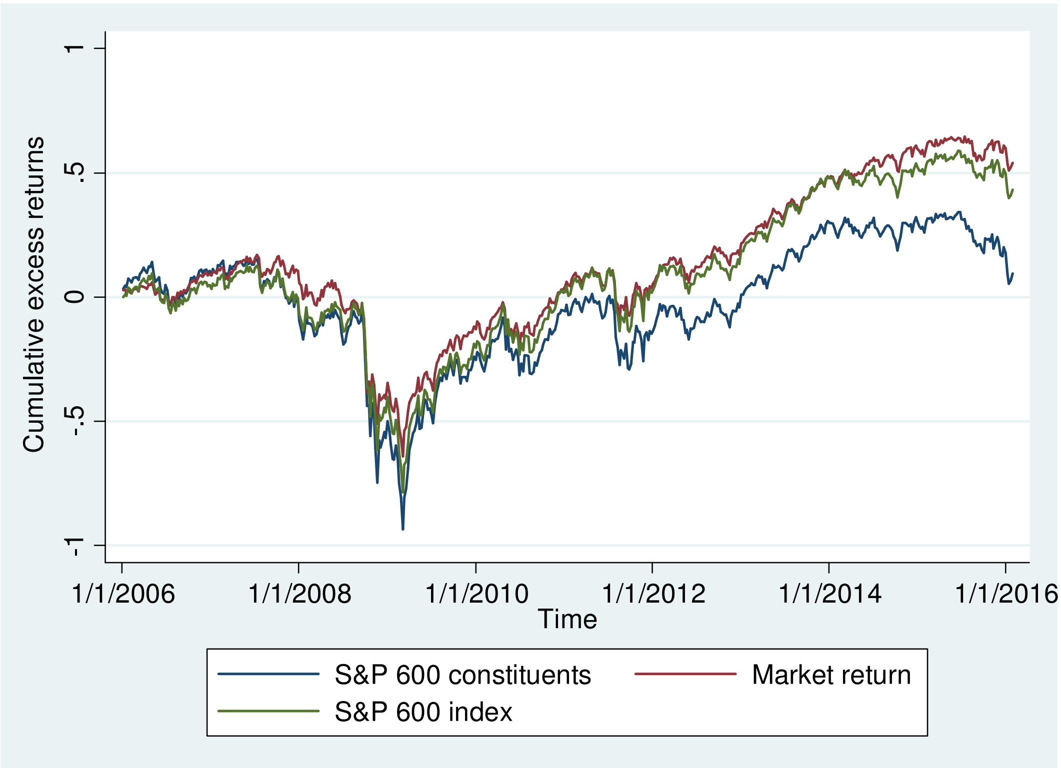
[24] The last column in Table 31 show the difference between the average excess returns of minimum and maximum volatility portfolio, so Q1 – Q5.

return is of great importance since it enters all the regressions. And since the "true" market portfolio is unobservable, there is always a need for a proxy. So, perhaps the use of some other proxy would be more appropriate in this case.

I tried replacing the proxy for the market return in the CAPM and Fama-French regressions first with the S&P 600 index and with the MSCI USA Small Cap index. The results, which are not shown here to conserve space, were very similar than before which suggests the problem was probably not in the choice of the proxy. Most of the intercepts had a small and negative value. It seems S&P 600 constituents have in fact produced lower returns in the period under examination compared to the market. This is also apparent form Figure 14.

Figure 14 plots the cumulative excess returns of S&P 600 constituents, the market and S&P 600 index. While the market and S&P 600 index remain very close together for the entire period under observation, the equally-weighted average of S&P 600 constituents underperforms, although the three series move very closely together from the beginning and up to somewhere around 2009. After the crisis, and even more so recently, the equal-weighted average somehow "chooses" a path of its own by producing much lower returns compared to the benchmarks.

Figure 14. Cumulative excess returns of S&P 600 constituents, the market and S&P 600 index



To sum up, the negative values of the CAPM and Fama-French alphas are probably not caused by the choice of the proxy for the market portfolio. Perhaps some of the low performance of the volatility sorted and double sorted portfolios can be attributed to the financial crisis in the years 2008 and 2009 (see next section).

4.6.3 The impact of the financial crisis on the results

As can be seen from several figures and tables throughout the thesis, the performance of S&P 600 and of all the portfolios that were formed from its constituents has worsened substantially in the mid-2008 due to the financial crisis. Consequently, the overall performance of the portfolios was lower. For this reason, I have analyzed the data again, only this time without taking the data from mid-2008 to mid-2009 into account. Exactly 62 weeks of observations were not taken into account when doing the calculations. The number of observations was thus 464 for volatility sorts and 412 for all the double sorts (since the data on bid-ask spreads is not available from the beginning of the time series). Average excess returns, standard deviations and Sharpe ratios were calculated and Fama-French regressions[25] were run for all the volatility sorted portfolios and for all the double sorts.

4.6.3.1 Volatility sorted portfolios

As can be seen from Table 32, the average excess returns of volatility sorted portfolios are much higher when the returns from mid 2008 to mid 2009 are excluded from the analysis. Namely, minimum volatility portfolio now yields 4.12 % which is almost six times as before when all the data were taken into account (0.72 %; see Table 4). Similar holds for medium volatility portfolio which yields 3.72 % of excess return on average compared to 0.71 % when all the data were taken into account. The most volatile portfolio also performs better, since it yields 2.31 % on average compared to 1.33 %. Interestingly, there is evidence of a low volatility effect: the excess return on minimum volatility portfolio is almost double of that of the maximum volatility portfolio. The same holds for the risk-adjusted performance of the portfolios. With the data in the period of the crisis excluded from the analysis, the minimum volatility portfolio was able to yield a Sharpe ratio 0.25 which is much more than the maximum volatility portfolio. It is also much more than before when all the observations were taken into account (see Table 4). The improvement in the Sharpe ratios can also be partially contributed to the lower values of standard deviations.

[25] To conserve space, the CAPM regressions were not run here. It is assumed that the CAPM alphas should follow similar patterns than the Fama-French adjusted values. Also, the Fama-French adjustment has been proven to be justified in several papers which makes these alphas even more precise compared to the CAPM alphas.

Table 32. Annualized average excess returns, standard deviations and Sharpe ratios of the volatility-sorted portfolios (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	**Med. volatility**	**Max. volatility**
Mean	4.12 %	3.72 %	2.31 %
St. deviation	16.67 %	20.55 %	24.94 %
Sharpe	0.25	0.18	0.09

Table 33 shows the values of the Fama-French adjusted alphas from the regressions that do not include the data on the crisis. All the values are higher (less negative) and the spread between minimum and maximum volatility portfolio is larger than before (5.63%; not shown compared to 2.52 %; see Table 6).

Table 33. Annualized Fama-French adjusted alphas of the volatility-sorted portfolios (observations from 30 May 2008 – 31 July 2009 excluded from the regression)

Min. volatility	**Med. volatility**	**Max. volatility**
-3.04 %*	-5.52 %***	-8.67 %***

It appears the financial crisis has had a significant effect on the performance of volatility sorted portfolios. Since there is a stronger evidence of a low volatility effect when the crisis is not taken into account, this might suggest minimum volatility portfolio was perhaps even more negatively affected by the crisis than were the other portfolios. In any case, the crisis seems to have distorted the results, suggesting this robustness test is justified. Clearly, if the crisis had not occurred, a low volatility effect would have been immediately present in the data (not only according to the values of the intercepts, but on the basis of the average excess returns and Sharpe ratios as well).The next sections summarize the results of the double sorting procedures when the data on the crisis is not taken into account.

4.6.3.2 Independent sorts

Tables 34 - 37 show the performance of the independently sorted portfolios when the data on the financial crisis is excluded from the calculations. As can be seen from the tables, the average excess returns are much higher for all the portfolios (for comparison see Table 9), the standard deviations are lower (for comparison see Table 10) and consequently, the Sharpe ratios show an improvement compared to the original calculations (for comparison see Table 11).

Only two out of five values of average excess returns that were previously negative have remained negative and all of the previously positive values have increased (for example, minimum liquidity / medium volatility portfolio has been able to produce 10.38 % of excess return on average compared to 6.52 %). Interestingly, with the crisis not taken into account, there is evidence of a low volatility effect at similar levels of liquidity, the exception being the least liquid group where middle volatility portfolio outperforms. But the minimum volatility / minimum liquidity portfolio still produces much higher excess returns than maximum volatility / minimum liquidity portfolio (7.52 % compared to 4.21 %, respectively).

The standard deviations (Table 35) have decreased substantially compared to the original values (Table 10) suggesting the higher returns were produced at lower levels of risk. This makes sense since the returns are usually more volatile in times of crisis. For example, the portfolios with minimum liquidity have produced standard deviations of 16.41 %, 20.97 % and 25.94 % for minimum, medium and maximum volatility portfolio, respectively, which is much less than 20.86 %, 25.71 % and 32.25 % (all the data taken into account).

The patterns of the Sharpe ratios are the same as for average excess returns which speaks in favor of a low volatility effect. All the values have improved substantially. The "winner" remains to be the minimum liquidity / medium volatility group of stocks which now produces 0.5 of risk-adjusted excess return. Its value has doubled (it was 0.25 when all the data were taken into account). The "looser" has also remained the same: the most liquid and most volatile portfolio now yields a Sharpe ratio of -0.23 which is less negative than before (-0.28; see Table 11). So, at similar levels of liquidity minimum volatility portfolios outperform their maximum volatility counterparts. It seems eliminating the data on the crisis has made the low volatility effect even more apparent.

Table 34. Annualized average excess returns of the double-sorted portfolios – independent sorts (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	7.52 %	10.38 %	4.21 %
Med. liquidity	4.75 %	2.56 %	4.02 %
Max. liquidity	1.17 %	-1.09 %	-5.82 %

Table 35. Annualized standard deviations of the double-sorted portfolios – independent sorts (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	16.41 %	20.97 %	25.94 %
Med. liquidity	17.81 %	21.43 %	25.21 %
Max. liquidity	16.73 %	20.38 %	24.86 %

Table 36. Sharpe ratios of the double-sorted portfolios – independent sorts (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	0.46	0.50	0.16
Med. liquidity	0.27	0.12	0.16
Max. liquidity	0.07	-0.05	-0.23

Almost all of the Fama-French adjusted alphas have improved as well (Table 37 compared to Table 16). Two of the values that were previously negative have become positive: the minimum liquidity / minimum volatility and minimum liquidity / medium volatility, although both values are still insignificant. Two of the maximum volatility portfolios have experienced a lower (more negative) value of alpha (maximum liquidity / maximum volatility and minimum liquidity / minimum volatility), but other than that, it is safe to assume that the crisis has negatively affected the performance of all the independently double sorted portfolios. The alphas once again exhibit a low volatility effect when comparing the portfolios at similar levels of liquidity, the only exception being the minimum liquidity / medium volatility portfolio. And even in the low liquidity group, the minimum volatility has produced a much higher alpha than its maximum volatility counterpart.

Table 37. Annualized Fama-French adjusted alphas of the double-sorted portfolios – independent sorts (observations from 30 May 2008 – 31 July 2009 excluded from the regression)

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	0.61 %	1.72 %	-6.76 %*
Med. liquidity	-2.4 %	-6.68 %***	-7.6 %***
Max. liquidity	-6.83 %***	-11.21 %***	-17.62 %***

4.6.3.3 Dependent sorts on liquidity and volatility

Tables 38 – 41 show the results of the dependent sorts on liquidity and subsequently on volatility when the data on the financial crisis is excluded from the calculations.

The comparison of the results of Table 38 with the original results of Table 18 shows a substantial increase in average excess returns of all the nine portfolios. The excess returns remain to decrease with higher volatility within each liquidity group, with the exception of minimum liquidity / medium volatility portfolio which outperforms its lower volatility counterpart. So, there is evidence of a low volatility effect within each liquidity bucket. The highest increase in excess returns compared to the results of Table 18, was produced by medium liquidity / maximum volatility: the average return has increased by 8.61 percentage points (from -8.63 % to -0.02 %).

Table 38. Annualized average excess returns of the double-sorted portfolios –dependent sorts on liquidity and volatility (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	8.25 %	10.94 %	3.49 %
Med. liquidity	6.97 %	4.10 %	-0.02 %
Max. liquidity	5.31 %	2.80 %	-12.03 %

The higher excess returns were again produced at lower levels of risk. Table 39 shows the values of standard deviations when the data on the crisis is excluded from the calculations. The comparison of the results with the original results of Table 19 shows a decrease in value of standard deviations in all nine liquidity-volatility groups.

Table 39. Annualized standard deviations of the double-sorted portfolios –dependent sorts on liquidity and volatility (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	15.64 %	21.47 %	29.61 %
Med. liquidity	16.42 %	21.41 %	26.79 %
Max. liquidity	14.99 %	19.46 %	25.24 %

As a consequence of higher excess returns combined with lower levels of risk, the values of the Sharpe ratios have increased (see Table 40). Compared to the original results shown in Table 20, the values of the risk-adjusted return of all nine portfolios are higher, but the biggest increase was again (as was for the average excess returns) produced by the medium liquidity / maximum volatility portfolio. A clear low volatility effect can be seen from Table 40: the Sharpe ratios are the highest for low volatility portfolios and decrease with higher volatility (within each liquidity bucket).

Table 40. Sharpe ratios of the double-sorted portfolios –dependent sorts on liquidity and volatility (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	0.53	0.51	0.12
Med. liquidity	0.42	0.19	-0.001
Max. liquidity	0.35	0.14	-0.48

Interestingly, not all values of the intercept have increased with the crisis being excluded from the regression (see Table 41 and Table 23). Namely, the minimum liquidity / minimum volatility and medium liquidity / minimum volatility have previously produced a slightly larger alpha (2.53 % compared to 1.83 % and 0.41 % compared to 0.21 %, respectively) as has the maximum liquidity / maximum volatility portfolio (-8.19 % compared to -8.63 %). All the other values of alpha have improved compared to the original values. So, it is safe to say that the portfolios would have performed better (compared to the market) if there was no crisis in 2008. The patterns have mainly remained the same: the values of the alphas within each liquidity group decrease with higher volatility, thus showing evidence of a low volatility effect, the only exception being the least liquid group of stocks where medium volatility portfolio outperforms. And even in the least liquid group, the least liquid stocks still outperform the most liquid ones.

Table 41. Annualized Fama-French adjusted alphas of the double-sorted portfolios – dependent sorts on liquidity and volatility (observations from 30 May 2008 – 31 July 2009 excluded from the regression)

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	1.83 %	2.01 %	-8.63 %*
Med. liquidity	0.21 %	-5.48 %**	-11.93 %***
Max. liquidity	-1.96 %	-6.82 %***	-24.18 %***

To sum up, the crisis seemed to have a substantial effect on the performance on the liquidity-volatility portfolios as well. When the data on the crisis (30 May 2008 – 31 July 2009) is excluded from the analysis, the average excess returns are higher, standard deviations lower and Sharpe ratios are appropriately higher. The values of the alphas are also mostly higher as well. Compared to the original analysis, there is stronger evidence of a low volatility effect when the crisis is excluded from the analysis.

4.6.3.4 Dependent sorts on volatility and liquidity

Tables 42 – 45 show the results of the dependent double sorts on volatility and subsequently on liquidity when the data on the financial crisis is excluded from the calculations.

The comparison of the results in Table 42 with the original results (Table 25) shows the average excess returns have increased for all of the nine portfolios. Only two of the previously five negative excess returns have remained negative. The biggest increase in value (5 percentage points) was produced by the minimum volatility / medium liquidity portfolio (4.31 % compared to -0.7 %). The values of excess returns still tend to decrease with higher liquidity within each volatility bucket suggesting there is a liquidity effect within each of the groups. The "winner" remains to be the medium volatility / minimum liquidity portfolio (10.14 %) and the "looser" remains to be maximum volatility / maximum liquidity portfolio (-2.37 %). Comparing the values at similar levels of liquidity shows a low volatility effect as well: at each level of liquidity the least volatile group of stocks was able to outperform the most volatile group.

Table 42. Annualized average excess returns of the double-sorted portfolios – dependent sorts on volatility and liquidity (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	6.91 %	10.14 %	5.96 %
Med. liquidity	4.31 %	1.98 %	3.33 %
Max. liquidity	0.52 %	-0.97 %	-2.37 %

It is not surprising the values of standard deviations (Table 43) have decreased compared to the original results (shown in Table 26) with the data on the crisis being excluded from the calculations. All the values have decreased, but the pattern remains the same: higher ex-ante risk produces higher ex-post risk. In addition, there are little differences between the liquidity groups within the same volatility group.

Table 43. Annualized standard deviations of the double-sorted portfolios – dependent sorts on volatility and liquidity (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	16.65 %	21.13 %	25.91 %
Med. liquidity	17.48 %	21.39 %	25.52 %
Max. liquidity	16.97 %	20.34 %	25.23 %

As a consequence of higher values of excess returns and lower values of standard deviations, the values of Sharpe ratios have increased (see Table 44). Compared to the original results shown in Table 27 all the Sharpe ratios have improved in value, but the pattern of decreasing with higher liquidity within each volatility group has remained the same. According to the risk-adjusted return the "winner" remains to be the medium volatility / minimum liquidity portfolio (0.48) and the "looser" has become the maximum volatility / maximum liquidity portfolio (-0.09). A comparison of the Sharpe ratios at similar levels of liquidity shows that the values are much higher at lower levels of volatility (the exception is again the medium volatility / minimum liquidity portfolio) speaking in favor of a low volatility effect.

Table 44. Sharpe ratios of the double-sorted portfolios – dependent sorts on volatility and liquidity (observations from 30 May 2008 – 31 July 2009 excluded)

	Min. volatility	Med. volatility	Max. volatility
Min. liquidity	0.41	0.48	0.23
Med. liquidity	0.25	0.09	0.13
Max. liquidity	0.03	-0.05	-0.09

The values of the Fama-French adjusted alphas (see Table 45) have also improved compared to the original values shown in Table 30, the only two exceptions being the least and the most liquid group of stocks within the most volatile group (-4.9 % compared to -

2.18 % and -14.18 % compared to -14.12 %, respectively) and even these have not decreased by much. The pattern of the alphas decreasing with higher liquidity within each volatility group has remained the same. So, the crisis has had a negative impact on the abnormal performance of the volatility-liquidity portfolios. Note that the improvement of the alphas is much smaller compared to the improvement in excess returns, standard deviations and Sharpe ratios which may suggest underperformance of most of the portfolios compared to the market regardless of the crisis.

Comparing the alphas at similar levels of liquidity once again shows a low volatility effect: at each level of liquidity, the value of alpha of the minimum volatility portfolio exceeds the value of alpha of the maximum volatility portfolio.

Table 45. Annualized Fama-French adjusted alphas of the double-sorted portfolios – dependent sorts on volatility and liquidity (observations from 30 May 2008 – 31 July 2009 excluded from the regression)

	Min. volatility	**Med. volatility**	**Max. volatility**
Min. liquidity	-0.13 %	1.29 %	-4.9 %
Med. liquidity	-2.8 %	-7.26 %***	-8.22 %***
Max. liquidity	-7.68 %***	-11.07 %***	-14.18 %***

To sum up, the crisis seemed to have a negative impact on the performance of the volatility-liquidity portfolios. When the data from mid-2008 to mid-2009 are not taken into account, the average excess returns increase, standard deviations decrease and the Sharpe ratios show improvement. Most of the values of the Fama-French adjusted alphas also increase and become less negative suggesting a bit higher abnormal return compared to the market (although most of them remain negative). The results show a much clearer low volatility effect than before. In addition, a liquidity effect was documented as well.

CONCLUSION

The low volatility effect contradicts the very core of finance: that bearing extra risk produces higher returns on average. Low risk stocks have been found to generate high risk-adjusted returns and to produce higher abnormal returns (measured by alphas) compared to riskier stocks. The "anomaly" has so far been confirmed in several papers, some of them date back to the earliest tests of the CAPM.

I have analyzed the data on all the S&P 600 constituents which are small cap U.S. stocks in order to find evidence of low volatility effect. It was expected that this effect should be present in my sample of data due to the fact that these are small stocks which are often not as frequently traded as are the large liquid stocks, so the reason behind their low volatility of returns can be less frequent price changes.

In order to find out whether the risk-excess return relation is in fact flat or negative as reported in many previous studies, I have sorted the stocks by their 1-year past volatility of returns and assigned them to three portfolios based on this estimate. Returns on portfolios were calculated for the next year and the procedure was repeated until the end of series was reached. I have then calculated some of the characteristics of portfolios and ran some regressions. Since liquidity has in the existing literature been found to be an important determinant of stocks` returns I have included liquidity into the model by the use of independent and dependent double sorting. Even though some different performance measures were calculated for each of the portfolios, the alphas from the regressions (especially Fama-French regressions where the size and value are both controlled for) were seen as the most important indicator of a low volatility effect, and of a low liquidity effect as well.

Even though the results varied depending on the sorting procedures, some evidence of a low volatility effect was found regardless of the type of sorting that was used. When the stocks were sorted by volatility only, the risk-return relation was found to be flat according to the value of Sharpe ratios (0.04 for minimum and maximum volatility portfolio). On the basis of the estimated alphas from the CAPM and Fama-French regressions, there was a clear evidence of a low volatility effect (even though the values were negative).The year-by-year returns showed some ambiguous results: low volatility outperforming in some years and underperforming in other years compared to the high volatility. The cumulative excess returns of the portfolios seemed to confirm the theory with some small exceptions.

When the stocks were independently sorted by liquidity and volatility, excess returns and standard deviations followed different patterns at different levels of liquidity. The portfolio with the highest risk-adjusted return was the one combining low liquidity and medium volatility (0.25). The portfolio that underperformed the most was the maximum liquidity / maximum volatility portfolio (-0.28). The CAPM and Fama-French adjusted alphas were again negative for all portfolios and tended to decrease with higher volatility and higher liquidity suggesting that both, a low volatility and a low liquidity effect are present in the data. The plots of cumulative returns seemed to support the theory to a certain extent, although for the most liquid group of stocks maximum volatility portfolio underperformed all other portfolios including the market.

When the stocks were sorted on liquidity and on volatility within the liquidity groups, the results showed a clear low volatility effect in each liquidity group. The largest Sharpe ratio was produced by the minimum liquidity / minimum volatility portfolio (0.4) and the same portfolio has also produced the highest (and the only positive) value of Fama-French adjusted alpha (1.7 % per year), although the value of the latter was insignificant. The CAPM and Fama-French alphas decreased with higher volatility and with higher liquidity. Plotting the cumulative excess returns in time again showed a low volatility effect within each liquidity group. The effect was most apparent in the medium and maximum liquidity group.

When the stocks were sorted on volatility and on liquidity within the volatility groups, the results showed a liquidity effect within each volatility group: the Sharpe ratios were decreasing with higher liquidity. The highest value of Sharpe ratio was produced by the medium volatility / minimum liquidity portfolio (0.24) and the Fama-French adjusted alpha was the largest for minimum volatility / minimum liquidity portfolio (-2.54 % per year), although the value of the latter was insignificant. The results also showed tendency towards a low volatility effect. The plots of cumulative returns showed a liquidity effect within each volatility group. The effect was the largest in the medium and maximum volatility group where the least liquid stocks have outperformed the market as well.

Overall, there seems to an evidence of a low volatility effect in the U.S. small cap equity segment in the past 10 years. In addition, it also looks like low volatility effect has a lot do with stocks` liquidity. Judging by the values of alphas, the low volatility and low liquidity effect are, with some small exceptions, confirmed regardless of the sorting procedure that was used.

One of the robustness tests in the last section shows that the underperformance of all the portfolios compared to the market (negative alphas) is most likely not a consequence of a poor proxy for the market portfolio. Instead, it was shown that the U.S. small equity segment, measured by the constituents of the S&P 600, has in fact underperformed the market in the past few years, especially in the years that followed the crisis.

The performance of all the volatility sorted and double sorted portfolios has worsened substantially during the financial crisis as can be seen from the plots of cumulative returns and calculations of end-of-year returns. In order to eliminate the negative effect of the financial crisis on the results, I have excluded the data from mid-2008 to mid-2009 and repeated the whole sorting procedure. The results show an improvement in the values of average excess returns, Sharpe ratios and Fama-French adjusted alphas compared to the original results and this holds for almost all the volatility sorted and double sorted portfolios. In addition, the results show a much clearer low volatility effect combined with a liquidity effect.

To conclude, there seems to be an evidence of a low volatility effect in the U.S. small equity segment in the last 10 years and it seems a large part of it can be explained by a low liquidity effect. A logical extension of my study would be the creation of a low volatility effect as a factor and its inclusion into the regression models. This way the effect would be directly reflected in the regression coefficient. However, the creation of a low volatility factor is beyond the scope of my thesis and it shall be left for future research.

REFERENCE LIST

1. Acharya, V. V., & Pedersen, L. H. (2005). Asset pricing with liquidity risk. *Journal of Financial Economics, 77*(2), 375–410.
2. Ackert, A., & Deaves, R. (2010). *Behavioral Finance: Psyhology, Decision-Making, and Markets.* USA: South-Western, Cengage Learning.
3. Alexander, C. (2008). *Practical Financial Econometrics, Vol. II.* West Sussex: John Wiley & Sons, Ltd.
4. Alexander, C., & Sheedy, E. (2004). *The Professional Risk Management`s Handbook, A Comprehensive Guide to Current Theory and Best Practices*. USA: The Professional Risk Managers` International Association.
5. Amihud, Y., & Mendelson, H. (1986). Liquidity and Stock Returns. *Financial Analysts Journal, 42*(3), 43-48.
6. Bali, T. G., & Cakici, N. (2008). Idiosyncratic Volatility and the Cross Section of Expected Returns. *Journal of Financial and Quantitative Analysis, 43*(1), 29-58.
7. Baker, M., Bradley, B., & Wurgler, J. (2009). A Behavioral Finance Explanation for the Success of Low Volatility Portfolio. *Research Gate.* Retrieved October 10, 2016, from file:///C:/Users/Laura/Downloads/A_Behavioral_Finance_Explanation_for_the_Success_o%20(1).pdf
8. Baker, M., Bradley, B., & Wurgler, J. (2011). Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly. *Financial Analysts Journal, 67*(1), 1-15.
9. Banz, R. W. (1981). The Relationship between Return and Market Value of Common Stocks. *The Journal of Financial Economics, 9*(1), 3-18.
10. Basu, S. (1983). The Relationship between Earnings` Yield, Market Value and Returns for NYSE Common Stocks, Further Evidence. *The Journal of Financial Economics, 12*(1), 129-156.
11. *Bid-Ask Spread*. (n.d.) In *Investopedia*. Retrieved July 20, 2016, from http://www.investopedia.com/terms/b/bid-askspread.asp
12. Black, B., Jensen, M. C., & Scholes, M. (1972). The Capital Asset Pricing Model: Some Empirical Tests. *Studies in the Theory of Capital Markets*. Retrieved August 22, 2016, from http://papers.ssrn.com/abstract=908569
13. Blitz, D. C., & van Vliet, P. (2007). The Volatility Effect: Lower Risk Without Lower Return. *The Journal of Portfolio Management, 34*(1), 102-113.
14. Blitz, D., & van Vliet, P. (2011). Benchmarking Low-volatility Strategies. *Journal of Index Investing, 2*(1), 44-49.
15. Blitz, D., Pang, J., & van Vliet, P. (2013). The Volatility Effect in Emerging Markets. *Emerging Markets Review, 16*(1), 31-45.
16. Blitz, D., Falkenstein, E. G., & van Vliet, P. (2014). Explanations for the Volatility Effect: An Overview Based on the CAPM Assumptions. *Journal of Portfolio Management, 44*(1), 61-76.
17. Cavenaile, L., Dubois, D., & Hlavka, J. (2009). Cross-Sectional Tests of Fama and French Three-Factor Model and Fama-MacBeth Method. Retrieved July 10, 2016,

from
http://www.hec.ulg.ac.be/sites/default/files/workingpapers/WP_HECULg_20091101_Cavenaile_Dubois_Hlawka.pdf

18. Clarke, R., de Silva, H., & Thorley, S. (2006). Minimum-Variance Portfolios in the US Equity Market, *Journal of Portfolio Management*, *33*(1), 10-24.
19. Cuthbertson, K., & Nitzsche, D. (2004). *Quantitative Financial Economics, Stocks, Bonds and Foreign Exchange.* West Sussex: John Wiley & Sons, Ltd.
20. Davis, L. J. (2003, July 2). Explaining Stock Returns: A Literature Survey. Retrieved June 2016, from https://www.ifa.com/pdfs/explainingstockreturns.pdf
21. Dempsey, M. (2013). The Capital Asset Pricing Model (CAPM): The History of a Failed Revolutionary Idea in Finance? *ABACUS, A Journal of Accounting, Finance and Business Studies, 49*(S1), 7-23.
22. *Defining risk*. Retrieved July 22, 2016, from http://www.d42.com/equilibrium/evidence/initial-tests-of-the-standard-capm
23. Engelberg, R. McLean, J. D., & Pontiff, R. (2015). Anomalies and News. Retrieved July 22, 2016, from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2631228
24. Falkenstein, E. (2010). Risk and Return in General: Theory and Evidence. Retrieved May 2, 2016, from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1420356
25. Fama, E. F. (1991). Efficient Capital Markets: II. *The Journal of Finance, 46*(5). 1575-1617.
26. Fama, E. F., & French, K. R. (1992). The Cross Section of Expected Stock Returns. *The Journal of Finance, vol. 47*(2), 427-465.
27. Fama, E. F., & French, K. R. (1993). Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics 33*(1), 3-56.
28. Fama, E. F., & French, K. R. (2015). A Five Factor Asset Pricing Model. *Journal of Financial Economics 116*(1), 1-22.
29. Fama, E. F., & MacBeth, J. D. (1973). Risk, Return and Equilibrium: Empirical Tests, *Journal of Political Economy, 81*(3), 607-636.
30. Frazzini, A., & Pedersen, L. H. (2014). Betting Against Beta. *Journal of Financial Economics 111*(1), 1-25.
31. Fama-French Data Library. Retrieved May 6, 2016, from http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
32. Greene, W. H. (2002). *Econometric Analysis*. New Jersey: Prentice Hall.
33. Gujarati, D. N. (2003). *Basic Econometrics*. New York: McGraw-Hill.
34. Haugen, R. A, Heins, A. J. (1975). Risk and the Rate of Return on Financial Assets: Some Old Wine in New Bottles. *Journal of Financial and Quantitative Analysis, 10*(5), 775-784.
35. Haugen, R. A., & Baker, N. L. (1991). The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios. *The Journal of Portfolio Management, 17*(3), 35-40.
36. Haugen, R. A., & Baker, N. R. (2012). Low Risk Stocks Outperform within All Observable Markets of the World. Retrieved July 20, 2016, from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2055431

37. Keim, D. B. (2006, May 30). Financial Market Anomalies. Retrieved August 1, 2016, from http://finance.wharton.upenn.edu/~keim/research/NewPalgraveAnomalies(May302006).pdf
38. Koch, C. (1996). *The Arbitrage Pricing Theory as an Approach to Capital Asset Valuation.* Diploma Thesis, European Business School, Munich, Germany.
39. Kurach, R. (2013). Does Beta Explain Global Equity Market Volatility – Some Empirical Evidence. *Vizija Press&IT, 7*(2), 55-66.
40. Lončarski, I. (2013). Multi-factor Models and Liquidity. *Empirical Finance slides.* (Lecture 4). Ljubljana: Faculty of Economics.
41. Maringer, D. (2007). Risk Preferences and Loss Aversion in Portfolio Optimization. In E.J. Kontoghiorges, B. Rustem & P. Winker (Eds.), *Computational Methods in Finance Engineering* (27-45). Berlin, Germany: Springer-Verlag Berlin Heidelberg.
42. Markowitz, H. (1952). Portfolio Selection, *Journal of Finance, 7*(1), 77-91.
43. *Minimum Volatility Portfolio*. Retrieved April 30, 2016, from http://www.clermontalpha.com/minvolatility.htm
44. Pastor, L., & Stambaugh, R. F. (2003). Liquidity Risk and Expected Stock Returns. *Journal of Political Economy, 111*(3), 642–685.
45. Pennachi, G. (2007). *Theory of Asset Pricing*. New Jersey: Prentice Hall.
46. Prohaska, Z. (2004). *Finančni trgi* [Financial markets]. Ljubljana: Faculty of Economics.
47. Stata Manuals. (n.d.). *Robust and clustered standard errors*. Retrieved August 22, 2016, from http://www.stata.com/manuals14/semintro8.pdf
48. *Survivorship bias*. (n.d.) In *Investorwords*. Retrieved August 1, 2016, from http://www.investorwords.com/5814/survivorship_bias.html#ixzz4Fig222PB
49. *S&P SmallCap 600*. Retrieved April 2, 2016, from http://us.spindices.com/indices/equity/sp-600
50. S&P Dow Jones Indices. (2016). *S&P U.S. Indices Methodology*. Retrieved August 10, 2016, from https://us.spindices.com/documents/methodologies/methodology-sp-us-indices.pdf
51. *Weekend Effect*. Retrieved August 22, 2016, from http://calendar-effects.behaviouralfinance.net/weekend-effect/
52. *Why Log Returns*. Retrieved July 20, 2016, from https://quantivity.wordpress.com/2011/02/21/why-log-returns/
53. Yamada, T. (2013). Long-term Verification of Low Volatility Stock Investment. *Policy Research Institute, Ministry of Finance, Japan, Public Policy Review, 9*(3), 553-574.