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MASTER'S THESIS

AN EFFICIENCY ANALYSIS OF EQUITY MUTUAL FUNDS: THE CASE OF SLOVENIA

Ljubljana, September 2015 ALEXANDR GUTIREA

AUTHORSHIP STATEMENT

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INTRODUCTION

Mutual funds are one of the pillars of the modern financial system. Millions of investors worldwide decide to pursue investment goals using mutual funds. Among these investors are individuals and households as well as institutional investors, both financial and non-financial. One of the reasons mutual funds are so popular is that they act as transparent investment vehicles that invest in identifiable financial instruments that are regularly marked-to-market (Khorana, Servaes, & Tufano, 2005) and could thus be perceived as an almost perfect link between savers and borrowers.

When investors, either individual or institutional, decide to invest in mutual funds, they express their faith that experienced and educated mutual fund employees who are directly responsible for investment decisions will be vigilant advocates of their interests (Boggle, 2010). However, delegation of investment decisions could lead to agency conflicts because fund managers and investors do not always share common objectives. As Lückoff (2011) points out, investment skills of fund managers are unknown and their true effort cannot be observed. Ceteris paribus investors possess incomplete information about how well their assets are being managed and have only two choices – 1) to do nothing or 2) to make certain efforts to control mutual fund performance. Since mutual funds are obliged to regularly publish specific information regarding their business operations, investors or other interested parties could use these data to determine how efficient and/or lucky are fund managers. It stands to mention, however, that the performance appraisal process could become quite demanding due to specific knowledge required and necessity to choose the most suitable performance indicator from a plethora of developed measures.

The objective of this paper is to estimate the efficiency of actively managed equity mutual funds in Slovenia and to investigate what factors determine superior performance. Efficiency refers to how successful mutual funds are in transforming available resources into results, while taking into consideration the technology available and applied. Only equity mutual funds are chosen due to the fact that in Slovenia mutual funds with such investment policies dominate others in terms of number and cumulative value of assets under management.

The efficiency measurement in the context of the mutual fund industry becomes more and more frequent, also due to the fact that estimated efficiencies could be used instead of more traditional performance measures. The main disadvantage of the traditional performance measures such as the Sharpe ratio, the Treynor ratio, the Sortino ratio, the Information ratio, Jensen's alpha, Fama-French's alpha, Carhart's alpha and M^2 measure is that they do not include certain important characteristics of the mutual fund management process (for example fees and management structure) and are thus not flexible enough to account for important differences that exist between mutual funds. Additionally, at least some of the traditional performance measures are exposed to the following problem: due to the calculation mechanics it is extremely hard to interpret results in the case of mutual funds that are underperforming their benchmarks.

The results of the investigation on Slovenian equity mutual fund efficiency could be of interest to both individual and institutional investors as well as mutual fund companies and legal persons marketing mutual funds. Investors could improve efficiency of their financial portfolios exploiting data on efficiencies of individual mutual funds and investigating what characteristics of mutual funds could potentially influence fulfillment of financial goals. Mutual fund companies could improve performance and the level of investors' satisfaction by determining critical factors of efficiency and by channeling resources to problematic areas of operations defined by the analysis. Companies and individuals working in the area of financial consulting could increase satisfaction of their clients discovering and offering the most efficient mutual funds.

In order to conduct the research, a non-parametric approach that allows researchers to include in analysis multiple outputs and inputs called data envelopment analysis (DEA) is employed, while some other measures of mutual fund performance are only briefly presented. DEA-based models have been quite extensively used in the last 15 years for the purpose of mutual fund performance evaluation. However, this approach has not been yet employed in any research on the Slovenian mutual fund industry. For this reason, it is important to note that proven possibility of collecting the required data, constructing the model and actually performing DEA in the context of the Slovenian mutual fund industry could be perceived as a result worth special attention.

This paper combines theoretical and empirical approaches to the analysis of the mutual fund efficiency. The theoretical part includes a brief review of methods of the estimation of mutual fund performance, an overview of findings on persistence in mutual funds' results, a presentation of the DEA method of the efficiency determination and conclusions of previously performed research in the area of the DEA-based mutual fund efficiency analysis. The empirical part consists of efficiency analysis which was performed by employing data collected on equity mutual funds and managed by Slovenian mutual fund companies, a test of performance persistence existence and investigation on the relationships between mutual fund efficiency and certain uncontrollable factors.

The research hypotheses evaluated in this research paper are as follows:

1. There is no performance persistence among Slovenian equity mutual funds.
2. Equity mutual funds with higher amount of assets under management are less efficient.
3. Equity mutual funds from mutual fund families, i.e., mutual fund companies with higher cumulative amount of assets under management are more efficient.
4. Younger, i.e., more recently established equity mutual funds are more efficient.

It should be noted that all research hypotheses are formulated on the basis of already existing findings; in other words, each research hypothesis has theoretical and empirical grounds.

This master's thesis consists of eight parts. In introduction a brief explanation of the research is provided, together with the rationale for efficiency analysis of Slovenian equity mutual funds.

The first chapter describes the mutual fund industry in Slovenia, with an emphasis on equity mutual funds. In this part of the research certain characteristics of the industry are presented. These are assets under management of individual mutual fund companies, cumulative net assets of different types of mutual funds (according to the classification of the Securities Market Agency) as well as monthly net inflows.

The second chapter sheds light on the literature on mutual fund performance persistence phenomenon and performance measures. This part is important due to the fact that if persistence does not exist, potential and existing investors should take into account criteria other than past performance when choosing the most appropriate mutual funds. A brief presentation of the existing performance measures is required to show that there are multiple approaches to performance evaluation, with DEA potentially becoming a useful complement.

Among the performance measurement indicators described in this chapter are the Sharpe ratio, the Treynor ratio, the Sortino ratio, the Information ratio, Jensen's alpha, Fama-French's alpha, Carhart's alpha, Characteristic-based models, Holdings-based models, Trade-based models as well as M² measure.

The third chapter is devoted to efficiency measurement and the DEA approach in particular. In this part basic efficiency concepts are presented, such as productivity, the production frontier, technical efficiency, etc. Additionally, this chapter describes the DEA approach and provides a brief discussion on strengths and weaknesses of this method. Different approaches to dealing with uncontrollable factors are discussed as well (a separation approach, a one-stage DEA, a two-stage DEA, a three-stage DEA, a four-stage DEA). Additionally, this part of the paper provides a brief review on the literature on mutual fund analysis employing a DEA approach.

In the fourth chapter certain determinants of mutual fund performance are presented with the emphasis on the literature review. This chapter is important because the inclusion of relevant factors as well as the omission of irrelevant variables is the key to a successful performance analysis.

The fifth chapter provides a discussion on risk and its relationship with performance. The risk return interaction is a paramount element of the modern financial system and deserves to be presented in a separate chapter. It should be noted that in many cases return is assumed to be a function of risk; greater risk is rewarded with higher performance. However, empirical evidence shows that the risk-return relationship is not always significant, which leads to a conclusion that until there are no models that unequivocally reveal the influence of risk on performance, it would be better to assume that portfolio managers are pursuing two objectives, more specifically, return maximization and risk minimization.

In the sixth chapter the analysis of mutual fund performance is executed. Firstly, variables employed in DEA are described and the actual data are presented, with the primary source of information regarding mutual fund operations being mutual fund companies and the Securities Market Agency. Secondly, an efficiency analysis employing the DEA method is executed. Thirdly, an additional analysis of obtained efficiency results is performed. In this chapter possible suggestions are also brought to light, providing details on how to increase the quality and the informational value of the DEA investigation of the mutual fund efficiency.

In the conclusion findings of the investigation of efficiency of Slovenian equity mutual funds are summarized.

1 SLOVENIAN MUTUAL FUND INDUSTRY

Mutual funds are defined as companies "that pool money from a group of people with common investment goals to buy securities such as stocks, bonds, money market instruments, a combination of these investments, or even other funds" (Mobius, 2007) or, slightly different, as "collective investment vehicles that pool money from individual investors to buy the most attractive securities in order to achieve the maximum benefit in terms of risk-adjusted return" (Babalos, Caporale, & Philippas, 2009). Mutual funds act as systemically important financial intermediaries which reduce negative effects of market frictions on direct contacts between surplus units and deficit units. Lückoff (2011) includes among such market distortions: 1) local divergence that arises due to the fact that deficit units and surplus units are located at different places, 2) divergent lot sizes that arise due to the fact that monetary needs of deficit units are larger than one surplus unit can provide, 3) divergent risks that arise due to the fact that deficit units and surplus units do not share the same risk characteristics, 4) divergent maturities that arise due to the fact that deficit units and surplus units do not share the same maturity characteristics, 5) asymmetric information that arises due to the fact that deficit units have more information about their business operations than surplus units do. Mutual funds can diminish negative impacts of these market frictions, as they channel assets of investors, which are sometimes distinctly different in terms of investment goals and capital available, to financial professionals who are in theory able to analyze information more efficiently than the owners of mutual fund shares and are capable of constructing and managing diversified and liquid portfolios of financial instruments.

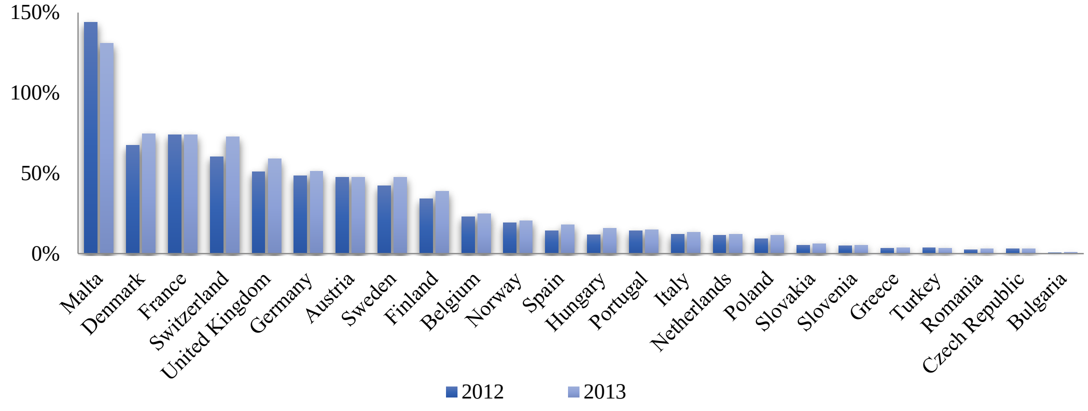
Data provided by the Investment Company Institute (2014) help understand how important and massive the mutual fund industry on the global scale is: at the end of the year 2013, total net assets of all 73,243 mutual funds in the world were equal to approximately 30 trillion US dollars. Taking into consideration the degree of sophistication of the US financial market and the level of the economic development of this country, it is not surprising that the US mutual fund industry is by far the largest in the world, being a domicile for approximately 10.1% of all mutual funds in the world and accounting for approximately 50% of all assets invested in mutual funds (Investment Company Institute, 2014).

Although the position of the mutual fund industry in Slovenia is hardly comparable with the situation in the US and other developed markets since it is relatively young and underdeveloped, if total assets invested in the mutual fund industry measured as the share of the GDP are taken into account (European Fund and Asset Management Association, 2014; Eurostat, 2014), it still offers existing and potential investors a plethora of mutual funds. At the end of the year 2013, 10 mutual fund companies existed in Slovenia, offering 117 mutual funds with EUR 1,855 million of assets under management invested in a wide range of financial instruments (Securities Market Agency, 2014a). It should also be noted that in Slovenia are available mutual funds that are managed by foreign mutual fund companies – at the end of the year 2013 mutual fund companies which were registered in the EU marketed 113 mutual funds in Slovenia (Securities Market Agency, 2014g), of which EUR 121 million were provided by Slovenian residents (Securities Market Agency, 2014h).

As of the end of the year 2013 cumulative assets of all Slovenian mutual funds were equal to 5.24% of the GDP, with European median being equal to 15.85%[1], which ranks Slovenia at the bottom of the list of the European countries. The relative size of the Slovenian mutual fund industry is higher if compared with Greece, Turkey, Romania, the Czech Republic and Bulgaria, but at the same time it is lower if compared with all "old" members of the European Union, except Greece, and even certain "new" members of the European Union, more specifically, Slovakia, Poland and Hungary. The fact that Slovenia is characterized by a below European average size of the mutual fund industry is primarily the result of underdevelopment of the industry in terms of how long it has existed, which is a cause of a certain immaturity in terms of investment and marketing processes.

[1] Calculated taking into account all EU countries except Croatia, Cyprus, Estonia, Ireland, Latvia, Lithuania, Luxembourg and including such non-EU countries as Norway, Switzerland and Turkey.

Figure 1. The size of mutual fund industries of certain European countries as a percentage of GDP

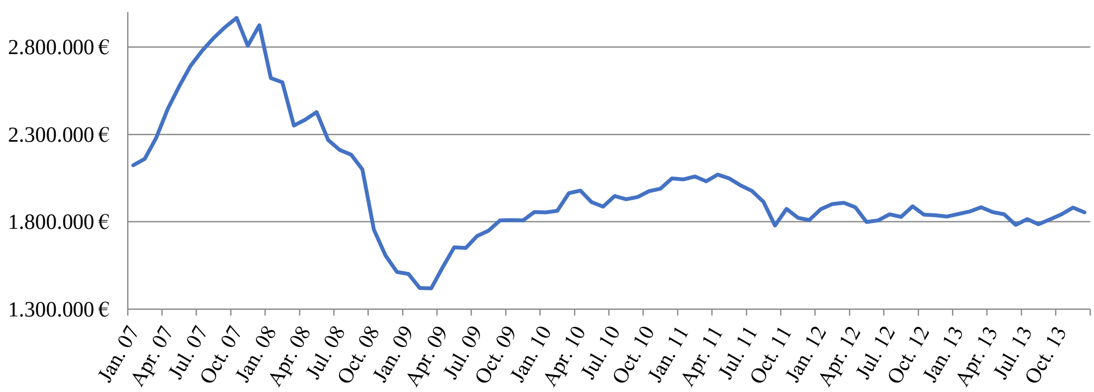


Notes: a) Luxembourg and Ireland are excluded as these countries act as financial hubs, b) both UCITS and non-UCITS mutual funds are included.

Source: European Fund and Asset Management Association, *Quarterly statistical reports*, 2014; Eurostat, *GDP and main components - Current prices*, 2014.

Monthly data on cumulative net assets of the Slovenian mutual fund industry show that the negative effect of the Global Financial crisis on Slovenian mutual funds companies has been extremely high, with cumulative net assets falling by massive 52.14% in the period from October 2007 to March 2009. From April 2009 to April 2011 the trend was positive. However, after April 2011 another downturn in the Slovenian mutual fund industry started, which resulted in a 14.08% drop in the cumulative net asset value in a 5-month period. In the period from October 2011 to December 2012 Slovenian mutual fund companies experienced a 2.87% increase in assets under management, while in 2013 cumulative net assets of Slovenian mutual funds increased by 1.35%.

Figure 2. Slovenian mutual fund industry net assets

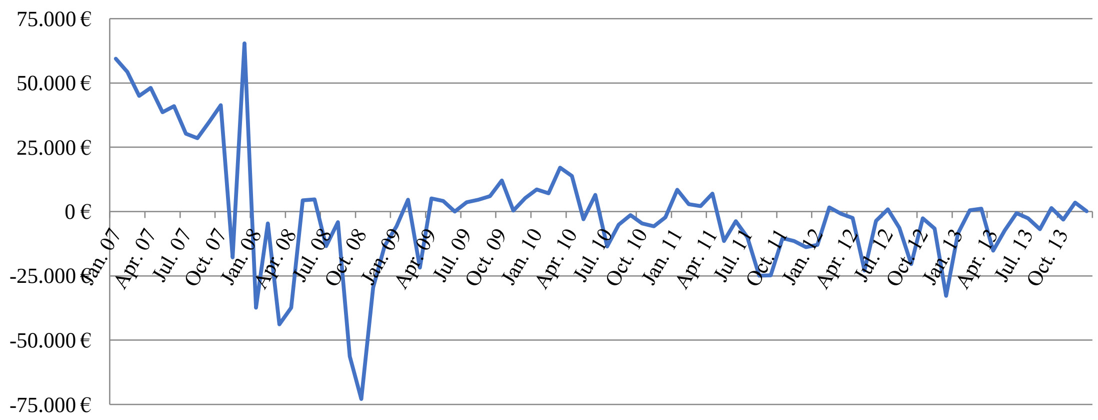


Source: Securities Market Agency, *Composition of the assets of mutual funds (ALL), net contribution and No. of subscribers*, 2014a.

The fluctuations of Slovenian mutual fund net assets reflect to a certain degree a situation on the capital markets, both globally and locally. However, it has certainly been influenced by investors' sentiment and their financial needs as well. Both of these factors could be proxied by the information on monthly net inflows in mutual funds, which reveals that investors in

Slovenia still do not have the possibility and/or willigness to use mutual funds for financial saving purposes.

Figure 3. Monthly net inflows in Slovenian mutual funds

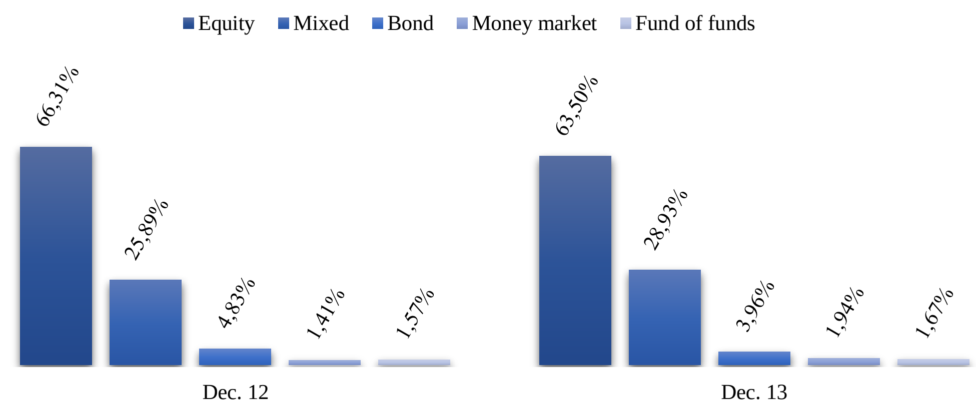


Source: Securities Market Agency, *Composition of the assets of mutual funds (ALL), net contribution and No. of subscribers*, 2014a.

The dynamics of monthly net inflows disclose that in the last three years it has been extremely difficult to attract new customers and retain the existing ones. As a result of this, in the mid-term perspective, inorganic growth through acquisitions and growth that is based on new services would most probably become the primary ways to significantly increase the amount of assets under management.

An important characteristic of the Slovenian mutual fund industry is its structure or, in other words, the relative importance of different types of mutual funds measured as weights of their net assets in cumulative net assets.

Figure 4. Slovenian mutual fund industry structure as of December 31, 2012 and December 31, 2013

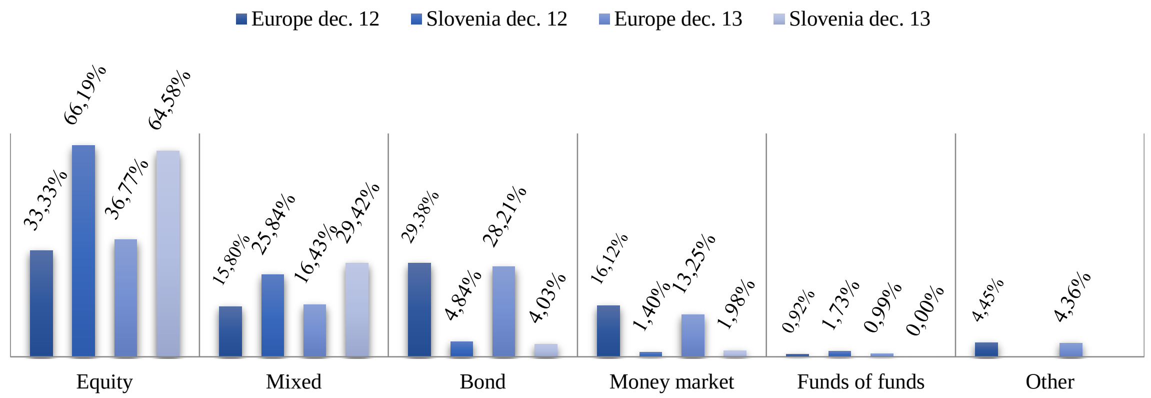


Source: Securities Market Agency, *Composition of the assets of equity mutual funds, net contribution and No. of subscribers*, 2014b; Securities Market Agency, *Composition of the assets of mixed mutual funds, net contribution and No. of subscribers*, 2014c; Securities Market Agency, *Composition of the assets of money-market mutual funds, net contribution and No. of subscribers*, 2014d; Securities Market Agency, *Composition of the assets of mutual funds-funds of funds, net contribution and No. of subscribers*, 2014e; Securities Market Agency, *Composition of the assets of bond mutual funds, net contribution and No. of subscribers*, 2014f.

Figure 4 shows that the Slovenian mutual fund industry is dominated by equity mutual funds, which represent almost two thirds of net assets of all mutual funds managed by local companies. The second largest group of mutual funds are mixed mutual funds, which have attracted a third of all assets invested in Slovenian mutual funds. As a reflection of a certain shift in the risk aversion of Slovenian investors it could be noticed that in 2013 the share of equity mutual funds decreased by approximately 3 percentage points, while the share of mixed mutual funds increased by approximately 3 percentage points. Other types of mutual funds represent less than 10% of cumulative net assets.

A simple comparison with the situation in other European countries discloses that the proportion of assets invested in equity and mixed mutual funds is significantly higher than the European average, while cumulative net assets of bond and money market mutual funds are significantly lower. A conclusion could be drawn that the Slovenian mutual fund industry is skewed towards more risky equity and mixed mutual funds, which will most probably change as the industry matures and mutual funds become saving vehicles to a greater extent and to a lesser extent mechanisms for speculative investments.

Figure 5. Slovenian mutual fund industry structure in comparison with Europe as of December 31, 2012 and December 31, 2013



Notes: 1) in the case of Europe only UCITS mutual funds are taken into account, 2) as of beginning of the year 2013 Securities Market Agency's classification does not include funds of funds

Source: Securities Market Agency, *Composition of the assets of equity mutual funds, net contribution and No. of subscribers*, 2014b; Securities Market Agency, *Composition of the assets of mixed mutual funds, net contribution and No. of subscribers*, 2014c; Securities Market Agency, *Composition of the assets of money-market mutual funds, net contribution and No. of subscribers*, 2014d; Securities Market Agency, *Composition of the assets of mutual funds-funds of funds, net contribution and No. of subscribers*, 2014e; Securities Market Agency, *Composition of the assets of bond mutual funds, net contribution and No. of subscribers*, 2014f; European Fund and Asset Management Association, *Quarterly statistical reports*, 2014.

The Slovenian mutual fund industry is relatively small in terms of the number of mutual fund companies and it is not problematic to make a brief review of the situation in the industry on the level of individual mutual fund providers. As it has already been alluded to, as of the end of the year 2013, 10 mutual fund companies existed in Slovenia, managing 117 mutual funds, 79 of which being equity funds, 24 balanced funds, 10 bond funds and 4 money market funds. The largest mutual fund company in terms of managed assets is Triglav Skladi d.o.o., which controls more than a quarter of the market. Among mutual fund providers with significant market shares are also KD Skladi d.o.o, NLB Skladi d.o.o. and KBM Infond d.o.o.

2 MUTUAL FUND PERFORMANCE

2.1 Performance persistence

One of the primary goals of mutual funds is to satisfy financial expectations of investors, by generating risk-adjusted returns that are higher compared to peers and the benchmark. However, even if specific mutual funds are able to outperform, the question arises as to whether investors are capable of identifying winning mutual funds and what factors are critical for investors' choice. Taking into account the information available to investors, it is not surprising that data on past performance play an important role in the decision making process (Ippolito, 1992). It should be noted though that mutual fund companies and regulators explicitly inform investors that past performance does not guarantee future results. Nevertheless, it does not change the fact that investors frequently contradict the advice of mutual fund companies and base their decisions on mutual fund past performance.

Another dimension of the mutual fund performance persistence phenomenon is connected with portfolio managers' evaluation and supervision. If superiority and inferiority dissapear with results experiencing mean reversion, the estimation of mutual fund managers' qualification through a simple peer or benchmark comparison may be biased and inappropriate. If the punishment-remuneration system is based on indicators that cannot be used to distinguish between manager-driven part of the performance and the performance that is the result of other factors, the process of performance appraisal could be useless or could even lead to lower future results by promoting employees ranked higher solely on the basis of luck and penalizing financial managers who had a mischance.

Since both of the above-mentioned problems are extremely important for individuals and institutions connected with the mutual fund industry, the question how strong the connection between past and future results is, should be addressed at least briefly.

A lot of research on performance persistence in the mutual fund industry has been performed in the last decades. Contradicting a semi-strong form of the efficient market hypothesis, some of this research has found evidence of performance persistence, which implies that it is possible to achieve abnormal returns, following the "hot hands" investment strategy, i.e., buying recent outperformers. The possibility of predicting future mutual fund results using past performance is, for example, documented in works published by Grinblatt and Titman (1992), who reveal a positive persistence in mutual fund results and stress that past performance could assist investors in the investment process, as well as in works by Elton, Gruber and Blake (1996), who discover persistence in one-year and three-year risk-adjusted returns. These findings are consistent with conclusions made by Gruber (1996), who suggests that past performance has certain predictive power and divides investors into two groups: a sophisticated clientele, which invests in recent outperformers, and a disadvantaged clientele, which consists of unsophisticated investors, who are influenced by other factors apart from performance, institutionally disadvantaged investors, who are restricted by investment plans, and, finally, tax disadvantaged investors, who held shares of recent underperformers in order to avoid capital gain tax.

The results of the research on the predictability of mutual fund results are not without contradiction. There is an empirical evidence that the performance persistence phenomenon can be found only in the short-run, while in the long-run fund managers cannot constantly achieve superior results. The evidence of the short-term performance persistence is found in the research conducted by Hendricks, Patel and Zeckhauser (1993), who find signs of both under- and outperformance persistence in the near-term evaluation horizon and stress that these results are not due to survivorship bias or known anomalies. Bollen and Busse (2005) show that superior performance is observable only when mutual funds are analyzed several times a year, whereas Huij and Verbeek (2007) state that short-term performance persistence do exist and manifest itself more evidently in the case of young, small cap mutual funds.

Among academics who find weak signs of managerial skills or no relationship between past and future results are:

- Jensen (1968), who analyzes managerial skills of mutual fund managers and concludes that mutual funds cannot outperform buy-and-hold investment strategy, stressing that there are only weak signs of managers' forecasting abilities.
- Carhart (1997), who shows that expenses, beta, market capitalization, one-year return momentum, portfolio type (value or growth) almost completely explain short-term performance persistence and finds only evidence of underperformance persistence, concluding that the existence of skilled managers or informed mutual funds is not supported by empirical results.
- Detzel and Weigand (1998), who develop a model that takes into account properties of mutual fund holdings and prove that persistence could be explained by the size of the stocks held by mutual funds and fund manager's investment styles.
- Porter and Trifts (1998), who analyze results of experienced mutual fund managers and discover that superior performance in the past is not predictive of superior performance in the future, also revealing that results of inferior managers are not experiencing complete mean reversion.
- Berk and Green (2004), who develop a flow-perfomance relationship model, according to which rational investors provide funds based on past results until performance is eroded due to decreasing returns for managers in exploiting their skills.
- Poti and Duffy (2007), who assess performance persistence of Irish mutual funds and find that excess returns achieved by mutual funds can be replicated, applying three strategies – high-versus-low beta stocks, value-versus-growth stocks and size.
- Bessler, Blake, Lückoff and Tonks (2010), who argue that the mean reversion in mutual fund returns can be explained by asset inflows and portfolio manager changes.

The literature review on performance persistence reveals that studies that document the existence of the relationship between past and future results are relatively rare. Pätäri (2009) argues that the results of the research on performance persistence phenomenon cannot be generalized since their direction is often dependent on the choice of methodology, performance indicators exploited, the length of the analyzed time periods and stresses that "the persistence literature seems to be quite unanimous that if performance persistence exists it is rather short-term phenomenon ranging from one month … and in addition, that it can be to large extent explained by persistence in inferior performance" (Pätäri, 2009). Possible reasons for the lack of persistence include (Lückoff, 2011): 1) the absence of superior investment skills; 2) the inability of applied statistical methods to detect connection between past and future results; 3) systematic factors that hinder best fund managers from continually beating the market.

2.2 Performance measures

Even though academic studies reveal mixed evidence of mutual fund performance persistence, the measurement and comparison of performance of mutual funds remains to be an important issue for fund managers, investors, regulators and providers of financial information. Since information on simple mutual fund share price appreciation does not reflect such an important performance dimension as the level of undertaken risk, the majority of measures are risk-adjusted. Lückoff (2011) divides existing performance measures into three broad groups: 1) measures that are based on ratios of excess returns and different risk indicators, 2) "alpha"- measures that reflect the systematic risk estimated by factor models, and 3) measures that are based on endogenous benchmarks, determined by using portfolio information.

Measures from the first group specify the return per unit of risk and are in most cases simple to compute and interpret. Ratio-based performance measures include, for example:

- The Sharpe ratio (Sharpe, 1966), defined as the amount of mutual fund excess return, relative to the return of the risk-free asset, divided by the standard deviation of mutual fund returns.

$$\textit{Sharpe ratio} = \frac{r_{mf} - r_{rf}}{\sigma_{r_{mf}}} \quad (1)$$

  where

  r_{mf} denotes the return of the mutual fund,
  r_{rf} denotes the return of the risk-free asset,
  $\sigma_{r_{mf}}$ denotes the standard deviation of mutual fund returns.

- The Treynor ratio (Treynor, 1965), calculated as the amount of mutual fund's excess return, relative to the return of the risk-free asset, divided by the mutual fund beta.

$$\textit{Treynor ratio} = \frac{r_{mf} - r_{rf}}{\beta} \quad (2)$$

  where

  r_{mf} denotes the return of the mutual fund,
  r_{rf} denotes the return of the risk-free asset,
  β denotes the mutual fund beta.

- The Sortino ratio (Sortino & van der Meer, 1991), determined as the amount of mutual fund's excess return, relative to the minimum acceptable return (often abbreviated as MAR), which is also called the hurdle rate, divided by the downside deviation of mutual fund returns (which means that when calculating standard deviation only mutual fund returns lower than MAR are taken into account).

$$\textit{Sortino ratio} = \frac{r_{mf} - MAR}{\sigma_{down_{r_{mf}}}} \quad (3)$$

  where

  r_{mf} denotes the return of the mutual fund,
  MAR denotes the minimum acceptable return,
  $\sigma_{down_{r_{mf}}}$ denotes the downside deviation of mutual fund returns.

- The information ratio (Sharpe, 1994), calculated as the amount of mutual fund excess return, relative to the benchmark return, divided by the standard deviation of this difference, i.e., the tracking error.

$$Information\ ratio = \frac{r_{mf} - r_b}{\sigma_{r_{mf} - r_b}} \tag{4}$$

where

r_{mf} denotes the return of the mutual fund,
r_b denotes the return of the benchmark,
$\sigma_{r_{mf} - r_b}$ denotes the tracking error.

"Alpha'' – measures represent the spread between mutual fund return and the return of the hypothetical benchmark, determined by the mutual fund systematic risk exposure. Common measures from this group of performance estimators are:

- Jensen's alpha (Alpha_J) (Jensen, 1968), calculated as the amount of mutual fund excess return, relative to the sum of the return of the risk free asset and the return explained by the market risk.

$$Alpha_J = r_{mf} - (r_{rf} + \beta * (r_m - r_{rf})) \tag{5}$$

where

r_{mf} denotes the return of the mutual fund,
r_{rf} denotes the return of the risk-free asset,
r_m denotes the return of the market,
β denotes the mutual fund beta.

- Fama-French's alpha (Alpha_FF) (Fama & French, 1992; Fama & French, 1993), determined as the amount of mutual fund excess return, relative to the sum of the return of the risk free asset, the return explained by the market risk, the return explained by the size effect and the return explained by the value effect.

$$Alpha_{FF} = r_{mf} - (r_{rf} + \beta * (r_m - r_{rf}) + \beta_{SMB} * SMB + \beta_{HML} * HML) \tag{6}$$

where

r_{mf} denotes the return of the mutual fund,
r_{rf} denotes the return of the risk-free asset,
r_m denotes the return of the market,
β denotes the mutual fund beta,
β_{SMB} denotes the mutual fund size beta,
β_{HML} denotes the mutual fund value beta,
SMB denotes the difference between returns of a small-capitalization portfolio and a large-capitalization portfolio,
HML denotes the difference between returns of a portfolio with a high book-to-market ratio and a portfolio with a low book-to-market ratio.

- Carhart's alpha ($Alpha_C$) (Carhart, 1997), determined as the amount of mutual fund excess return, relative to the sum of the return of the risk free asset, the return explained by the market risk, the return explained by the size effect, the return explained by the value effect and the return explained by the momentum effect.

  $$Alpha_C = r_{mf} - (r_{rf} + \beta * (r_m - r_{rf}) + \beta_{SMB} * SMB + \beta_{HML} * HML + \beta_{MOM} * MOM) \quad (7)$$

  where

  r_{mf} denotes the return of the mutual fund,
  r_{rf} denotes the return of the risk-free asset,
  r_m denotes the return of the market,
  β denotes the mutual fund beta,
  β_{SMB} denotes the mutual fund size beta,
  β_{HML} denotes the mutual fund value beta,
  β_{MOM} denotes the mutual fund momentum beta,
  SMB denotes the difference between returns of a small-capitalization portfolio and a large-capitalization portfolio,
  HML denotes the difference between returns of a portfolio with a high book-to-market ratio and a portfolio with a low book-to-market ratio,
  MOM denotes the difference between the average of the highest returns and the average of the lowest returns achieved in the previous year.

Portfolio-information-based models use information on mutual fund holdings or trades to construct hypothetical benchmarks. Calculated performance measures can be interpreted as the covariance between excess returns and mutual fund holdings or trades. Lückoff (2011) emphasizes that since more observations are available on portfolio specific information than on periodical mutual fund returns, even relatively young mutual funds can be evaluated by employing portfolio-information-based models.

- Characteristic-based models use benchmarks which are constructed by employing the information on the characteristics of stocks held by evaluated mutual funds. Utilizing the information on the market capitalization, the book-to-market ratio and prior-year return, Daniel, Grinblatt, Titman and Wermers (1997) build 125 passive portfolios and break down the expected return into three components: 1) return achieved due to selection skills (CS measure); 2) return achieved due to timing skills (CT measure), 3) return achieved due to tendency to hold stocks with certain characteristics (AS measure).
- Holdings-based models interpret the performance of fund managers as a positive correlation between returns of single stocks and portfolio weights. Grinblatt and Titman (1993) develop a portfolio change measure (PCM) that brings to light differences between informed and uninformed fund managers emphasizing different portfolio weights of different assets in various time periods.
- Trade-based models assume that information on holdings reflect passive management and use information on trades to catch active management performance. Cohen, Coval and Pástor (2005) develop a performance measure that estimates skills of fund managers, comparing their trades with the trades of fund managers with outstanding track records.

Three categories of performance measures presented above do not include all approaches used to estimate fund managers' investment skills. One of such "unclassified" methods is a M² measure, developed by Modigliani and Modigliani (1997). This performance measure is calculated as the product of market risk premium and the ratio between standard deviations of the market and mutual fund returns plus the return of the risk free asset. M² measure could also be presented as the product of the Sharpe ratio of the mutual fund and standard deviation of the market, plus the return of the risk free asset.

$$M^2 = \frac{\sigma_{r_m}}{\sigma_{r_{mf}}} * \left(r_{mf} - r_{rf}\right) + r_{rf} \tag{8}$$

where

r_{mf} denotes the return of the mutual fund,
r_{rf} denotes the return of the risk-free asset,
σ_{r_m} denotes the standard deviation of the returns of the market,
$\sigma_{r_{mf}}$ denotes the standard deviation of mutual fund returns.

The question of what performance measures to employ remains open. Among factors influencing the choice of which technique to use in order to estimate the investment skill are mutual fund characteristics, availability of data on mutual fund returns, holdings and trades, the problem of benchmark identification and quality, ease of implementation, accuracy of results, frequency of performance evaluation and chronological focus (ex post or ex ante) as well as method(s) used by fund managers and regulators to evaluate risk exposure.

In practice, mutual fund managers often estimate performance, comparing mutual funds with benchmarks. However, the results of benchmark-based performance analysis could be biased if certain rules are not followed. Amenc and Le Sourd (2003) stress that the chosen benchmark should have a similar asset structure and investment strategy and use similar calculation methodology. The first criterion relates to the fact that mutual fund and benchmark should ideally have the same investment universe. The second rule stems from the fact that in most cases benchmark returns do not take into account dividends, while mutual fund performance is calculated net of management fees and other expenses. Bailey (1992) provides a list of characteristics of a good benchmark:

- High coverage, i.e., high proportion of the mutual fund assets in the benchmark.
- Low turnover, i.e., low proportion of the benchmark market value allocated to transactions.
- Positive active positions, i.e., securities that are attractive from the point of view of the mutual fund manager, should have higher weight if compared to the benchmark; the unattractive ones, however, should have lower weight.
- Investable position sizes, i.e., weights of securities in the benchmark scaled to the size of the mutual fund assets should be lower than a certain threshold level.
- Reduced observed active risk, i.e., low variability of the mutual fund active return, measured as the difference between returns of the mutual fund and the benchmark.
- Significantly positive extra-market return correlations between the mutual fund and the benchmark, i.e., high proportion of the mutual fund return in excess of the market should be explained by the benchmark.
- Insignificant extra-market return correlation between the benchmark and the managed portfolio versus the benchmark, i.e., the ability of mutual fund managers to add value to

the benchmark should not be influenced by the fact whether the investment style is in or out of favor.

- Similarity of managed portfolio and benchmark style exposures, i.e., profiles of mutual fund and benchmark in relation to market capitalization and value/growth companies should be similar.

3 EFFICIENCY ANALYSIS

3.1 Efficiency basics

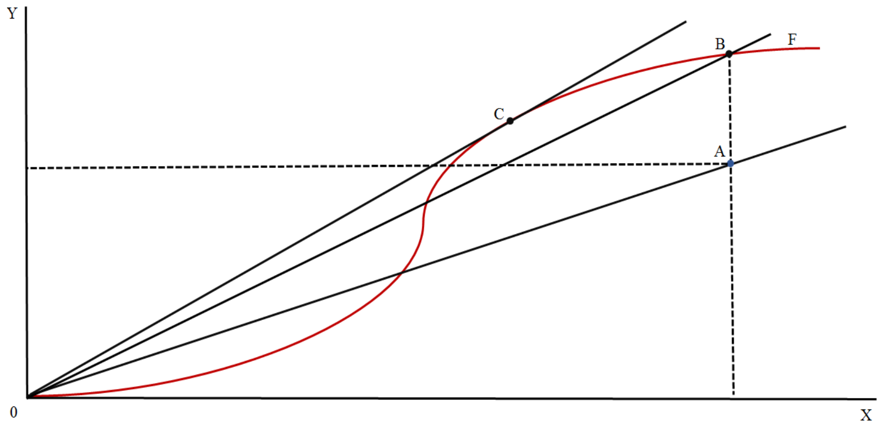
Managing mutual funds involves transformation of certain inputs, i.e., resources to certain outputs, i.e., results, with the primary goal being the creation of services and products that clients require, employing available resources. Without a doubt a vast majority of mutual fund companies struggle to maximize desirable outputs, while minimizing inputs that are not related to mutual fund management fees. The analysis of success of these activities helps to find sources of weak results, increase productivity and efficiency and is thus one of the pillars of successful operations. In other words, performance measurement is critical for mutual fund companies. It is also important not to forget that in most cases, performance is a relative concept, which means that, for example, in the case of performance analysis of a certain mutual fund, the final result could be relative to the performance of the analyzed mutual fund in the past, relative to the performance of another mutual fund or relative to the benchmark performance. This characteristic of success indicators allows mutual fund companies to add other dimensions to analysis and determine the factors that influence final results more accurately.

The basic performance measure is a productivity ratio, measured as the ratio of outputs to inputs, where larger values of this ratio are associated with better performance (Coelli, Rao, O'Donnell, & Battese, 2005). The same definition is also provided by Lovell (1993), who defines productivity of the production unit as the ratio of its output to its input. It is worth noting that when there are only one input and one output, the calculation of the productivity ratio is a trivial task; however, when there are several outputs and (or) inputs, the aggregation of outputs and (or) inputs is required. Lovell (1993) and Coelli et al. (2005) also make an important remark and stress that there is a difference in concepts of productivity and efficiency. The fundamental difference between the two concepts is easier to describe using an example of two productive units that both use a single input to produce a single output.

Assume that a decision making unit (often abbreviated as DMU) A produces X_A units of output X using Y_A units of input Y. DMU B, on the other hand, uses Y_B units of input Y to obtain X_B units of output X. The productivity of the productive unit A is defined as $P_A = \frac{Y_A}{X_A}$, while the productivity of the productive unit B is equal to $P_B = \frac{Y_B}{X_B}$. If P_A is higher than P_B, then DMU A is more productive than DMU B. However, if information regarding technology is not known, P_A and P_B remain to be merely indicators of productivity, and do not allow to evaluate performance. If DMU A and DMU B operate in a multiple input/multiple output universe, P_A and P_B are referred to as partial productivity measures, and if all factors of production are taken into account when calculating a productivity measure, the final result is defined as the total factor productivity.

Suppose that the technology is described by the production function $y^* = f(x)$. In that case $y_A^* = f(x_A)$ is equal to maximum output that can be produced from input X_A and $y_B^* = f(x_B)$ is equal to maximum output that can be produced from input X_B. The comparison of actual output with maximum producible output allows to measure technical efficiency. An example of the production function is shown in figure 6.

Figure 6. Productivity and technical efficiency



Source: Coelli et al., *An introduction to efficiency and productivity analysis (2nd ed.)*, 2005.

Three rays starting at the origin reveal productivities at different data points. For example, productivity at point A is lower than productivity at point B; therefore, the higher the slope of the ray, the higher the productivity. Curve 0F represents a production frontier, the maximum feasible output produced given each quantity of input. If the productive unit is operating on the production frontier, it is considered to be technically efficient. Therefore, the DMUs at points B and C in figure 6 are technically efficient, while the DMU at point A is technically inefficient.

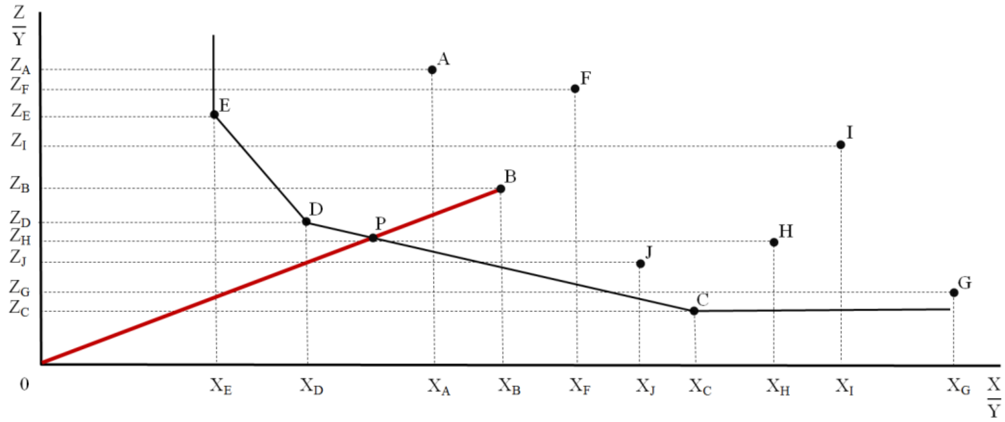
According to Koopmans (1951), "producer is technically efficient if an increase in any output requires a reduction in at least one other output or an increase in at least one input, and if a reduction in any input requires an increase in at least one other input or a reduction in at least one output". In other words, a technically inefficient productive unit could produce a given amount of output employing less inputs or produce more outputs using a given amount of inputs. Using an example presented in figure 6, in order to become technically efficient the DMU at point A could increase the amount of output and thus move to point B. On the other hand, the DMU located at point A could also become efficient by decreasing the amount of input used. In this case the location of the DMU would shift to the left and would be located on the projection of point A on the production frontier 0F. At this point, it is worth noting that as Lovell (1993) stresses, the analysis of technical efficiency could be oriented towards either output augmentation or input conservation.

It should be stressed that despite the fact that both points B and C are points of technical efficiency, only point C is the point where the ray from 0 is at a tangent to the curve 0F, revealing the optimal scale. In other words, the DMU operating at point C is achieving the maximum possible productivity.

DMUs operating in the single input/single output universe are rare. An understanding of approaches to the efficiency evaluation of multiple input/multiple output DMUs is a prerequisite of a successful analysis of mutual fund efficiency. For example, assume that 10 DMUs employ two inputs X and Z to produce one unit of output Y. Therefore, DMU A uses X_A units of input X and Z_A units of input Z to produce one unit of output Y, DMU B uses X_B units of input X and Z_B units of input Z to produce one unit of output Y, and so on. Described

DMUs can be plotted taking the ratio between input X and output Y ($\frac{X}{Y}$) as axis y and the ratio between input Z and output Y ($\frac{Z}{Y}$) as axis x. An example of the described situation is shown in figure 7.

Figure 7. Two inputs and one output case

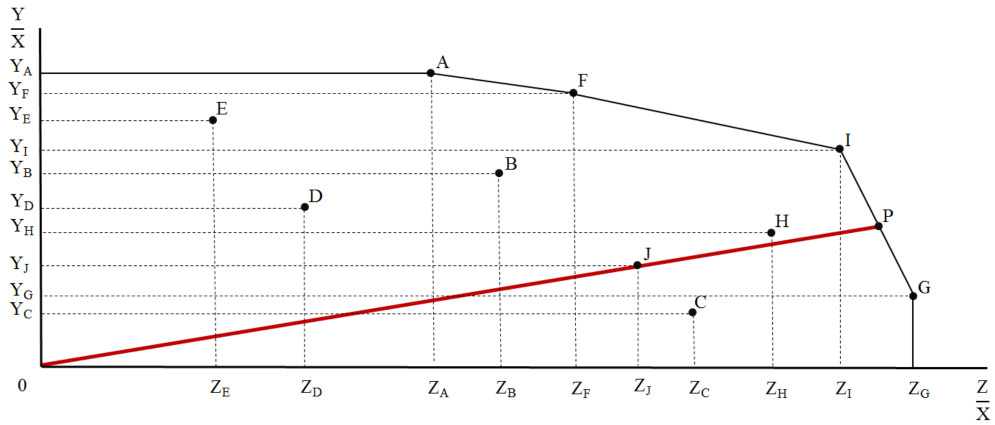


Source: Cooper, Seiford, & Tone, *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software (2nd ed.)*, 2007.

Since DMUs that use less input to get one unit of output are more efficient, it is possible to identify the line connecting E, D and C as the efficient frontier. The region including all data points (DMUs) enveloped by the efficient frontier is called the production possibility set. In order to evaluate efficiency of the inefficient DMU, for example B, it is possible to calculate a ratio $\frac{0P}{0B}$, where 0B is the distance between zero and point B, while 0P is the distance between zero and the point in which 0B crosses the efficiency frontier (point P). The efficiency measure calculated as described is always between zero and one. Because the point P lies on the line DC, which connects DMUs D and C, the efficiency of B is estimated using DMUs D and C as the reference set. It is clear that reference sets could be different for different DMUs; for example, the reference set for DMU A consists of DMUs E and D. The efficiency of DMU B could be improved by changing the amount of inputs used and/or output produced until productive unit moves from point B to point P or any other point that lies on the efficient frontier.

If in the previous example DMUs are operating in a two input/one output environment, for the purpose of presenting a situation in which DMUs are a part of a different universe, assume that 10 DMUs employ one unit of input X to produce two outputs Y and Z. In this case DMU A uses one unit of input X to produce Y_A units of output Y and Z_A units of output Z, DMU B uses one unit of input X to produce Y_B units of output Y and Z_B units of output Z, and so on. Described DMUs can be plotted taking the ratio between output Y and input X ($\frac{Y}{X}$) as axis y and the ratio between output Z and input X ($\frac{Z}{X}$) as axis x. An example of the described situation is shown in figure 8.

Figure 8. One input and two outputs case



Source: Cooper et al., *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software (2nd ed.)*, 2007.

Since all DMUs employ exactly one unit of input X, it is understandable that DMUs that produce higher amounts of outputs Y and Z are more efficient. The line connecting A, F, I and G represents the efficient frontier and the production possibility set lies in the region bounded by the efficient frontier and the axes. In order to evaluate the efficiency of the DMU, let it be the DMU J, it is possible to calculate a ratio $\frac{0J}{0P}$, where 0J is the distance between zero and point J, and 0P is the distance between zero and the point in which extension of line 0J crosses the efficiency frontier (point P). The efficiency measure calculated as described is always between zero and one. Because the point P lies on the line IG, which connects DMUs I and G, the efficiency of J is estimated using DMUs I and G as the reference set. It is clear that reference sets could be different for different DMUs; for example, the reference set for DMU D consists of DMUs A and F. The efficiency of DMU J could be improved by changing the amount of input used and/or outputs produced until productive unit moves from point J to point P or any other point that lies on the efficient frontier.

When performing efficiency analysis, it is important to keep in mind that since the production function is dependent on technology, the productivity level could be increased by introducing new inventions and innovations to a production process. The employment of a more sophisticated and advanced technology could result in the shift of the production functions, allowing DMUs to increase the output to input ratio.

3.2 DEA approach

Examples presented in the previous chapter are relatively easy to interpret. However, in most cases efficiency analysis includes DMUs with higher number of inputs and outputs and it becomes impossible to use graphical analysis and arbitrary assumptions. One of the principal models that allow researchers to estimate efficiency in the efficient frontier context is data envelopment analysis (DEA).

Ramanathan (2003) defines DEA as “a linear programming-based technique for measuring the performance efficiency of organizational units”. Frontier analysis techniques were introduced by Farrel (1957); however, a mathematical approach to frontier analysis was developed only two decades later by Charnes, Cooper and Rhodes (1978).

Mathematical formulation of DEA looks as follows (Ramanathan, 2003):

Suppose x represents inputs with $i = 1, 2, \ldots, n$ defining particular inputs (for example x_1, x_2, etc.), while y represents outputs with $j = 1, 2, \ldots, m$ defining particular outputs (for example y_1, y_2, etc). Assume that I and J represent the total number of inputs and outputs respectively, with both I and J being more than zero. The DEA approach linearly aggregates a) multiple inputs using weights and creating a virtual input $\sum_{i=1}^{I} u_i x_i$; b) multiple outputs using weights and creating a virtual output $\sum_{j=1}^{J} v_j y_j$. When virtual outputs and inputs are calculated, it is possible to calculate efficiency in the following manner:

$$Efficiency = \frac{\sum_{j=1}^{J} v_j y_j}{\sum_{i=1}^{I} u_i x_i} \tag{9}$$

where

y_j denotes the j-th output of the DMU,
x_i denotes the i-th input of the DMU,
v_j denotes the weight of output y_j $(v_j\ ^{3}\ 0)$,
u_i denotes the weight of input x_i $(u_i\ ^{3}\ 0)$.

From the previous discussion it is clear that the most problematic part of DEA is to assess weights, which according to Ramanathan (2003), “should be flexible and reflect the requirement (performance) of individual DMUs”. DEA solves this problem by employing mathematical programming and determining a unique set of weights for each DMU in such a way that the efficiency of this specific DMU is maximized subject to the following condition: if the obtained set of weights is attributed to any other DMU efficiency of this DMU is between zero and one. An important characteristic of the DEA approach is that weights are not fixed and known in advance, but derived from data.

The basic DEA model is the CCR model, introduced by Charnes et al. (1978). The basis of the CCR model is the following fractional programming problem:

$$max\, E_m = \frac{\sum_{j=1}^{J} v_{jm} y_{jm}}{\sum_{i=1}^{I} u_{im} x_{im}} \tag{10}$$

subject to

$$0 \leq \frac{\sum_{j=1}^{J} v_{jm} y_{jn}}{\sum_{i=1}^{I} u_{im} x_{in}} \leq 1;\ n = 1, \ldots, N$$
$$v_{jm} \geq 0;\ j = 1, \ldots, J$$
$$u_{im} \geq 0;\ i = 1, \ldots, I$$

where

y_{jm} denotes the j-th output of the DMU m,
x_{im} denotes the i-th input of the DMU m,
y_{jn} denotes the j-th output of the DMU n,
x_{in} denotes the i-th input of the DMU n,
v_{jm} denotes the weight of the output y_{jm},
u_{im} denotes the weight of the input x_{im},

Since fractional programming problems are generally complex to solve, it is logical to convert the formulation presented above to a linear programming format by normalizing either the numerator or the denominator. In the context of the output-oriented CCR model, which attempts to maximize outputs, the denominator is normalized and, as presented by Ramanathan (2003), the linear programming problem is:

$$max\, z = \sum_{j=1}^{J} v_{jm} y_{jm} \quad (11)$$

subject to

$$\sum_{i=1}^{I} u_{im} x_{im} = 1$$
$$\sum_{j=1}^{J} v_{jm} y_{jn} - \sum_{i=1}^{I} u_{im} x_{in} \leq 0;\ n = 1,\ \ldots,\ N$$
$$v_{jm} \geq \varepsilon;\ j = 1,\ \ldots,\ J$$
$$u_{im} \geq \varepsilon;\ i = 1,\ \ldots,\ I$$

where

y_{jm} denotes the j-th output of the DMU m,
x_{im} denotes the i-th input of the DMU m,
y_{jn} denotes the j-th output of the DMU n,
x_{in} denotes the i-th input of the DMU n,
v_{jm} denotes the weight of the output y_{jm},
u_{im} denotes the weight of the input x_{im},
ε denotes an infinitesimal or non-Archimedian constant.

In the context of the input-oriented CCR model, which attempts to minimize inputs, the numerator is normalized and, as presented by Ramanathan (2003), the linear programming problem is:

$$min\, z' = \sum_{i=1}^{I} u'_{im} x_{im} \quad (12)$$

subject to

$$\sum_{j=1}^{J} v'_{jm} y_{jm} = 1$$
$$\sum_{j=1}^{J} v'_{jm} y_{jn} - \sum_{i=1}^{I} u'_{im} x_{in} \leq 0;\ n = 1,\ \ldots,\ N$$
$$v'_{jm} \geq \varepsilon;\ j = 1,\ \ldots,\ J$$
$$u'_{im} \geq \varepsilon;\ i = 1,\ \ldots,\ I$$

where

y_{jm} denotes the j-th output of the DMU m,
x_{im} denotes the i-th input of the DMU m,
y_{jn} denotes the j-th output of the DMU n,
x_{in} denotes the i-th input of the DMU n,

v_{jm} denotes the weight of the output y_{jm},
u_{im} denotes the weight of the input x_{im},
ε denotes an infinitesimal or non-Archimedian constant.

In the case of the input-oriented model the results are equal or less than one, while in the case of the output-oriented model the results are more or equal to one. Additionally, in the context of the CCR DEA model, the input-oriented optimal solution is equal to the reciprocal of the output-oriented optimal solution and vice versa. If the input orientation approach is employed, the solution reveals by how much it is possible to proportionally decrease inputs in order to leave outputs constant. If the output-oriented model is used, the solution shows by how much it is possible to proportionally increase outputs in order to leave inputs unchanged. The DMU could be considered efficient according to one model only if it is also efficient according to the other model. In other words, the DMU is efficient if both the input-oriented optimal solution and the output-oriented optimal solution are equal to one.

It is known that every optimization problem could be approached from two directions; in other words, every linear programming problem (primal) has an associated linear programming problem (dual). The DEA approach employing linear programming problems presented earlier in this chapter is called the multiplier DEA, while the method which uses duals of the linear programming problems described above, is referred to as the envelopment DEA.

It should be noted that the CCR model operates under the assumption of constant returns to scale (often abbreviated as CRS) and this approach is also called the CRS CCR model. However, it is understandable that operating units often operate under conditions of the variable returns to scale (often abbreviated as VRS). This limitation has been reduced by Banker et al. (1984) and the developed model is termed the BCC model. In the case of the envelopment approaches the difference between the CCR and the BCC model is that the model that assumes variable returns to scale has the convexity constraint $\sum_{n=1}^{N} \lambda_n = 1$ (Cooper et al., 2007). If the multiplier form is being utilized, the BCC model differs from the CCR model in that the former one employs a variable ϑ_0, which can take any value and is smaller than zero in the case of decreasing returns to scale, equals zero in the case of constant returns to scale and is larger than zero in the case of increasing returns to scale.

As Coelli et al. (2005) note, by employing results obtained by the CCR and BCC models, it is possible to decompose technical efficiency scores calculated under the constant returns to scale assumption into “pure” technical efficiency and efficiency that stems from the economies of scale, i.e., scale efficiency.

It is not out of place to note here that apart from the models presented earlier in this chapter, multiple other DEA models exist. Among them are the additive, the slack-based, the hybrid as well as the free disposable hull.

Another important remark is that when analyzing efficiency employing the DEA approach, concepts of input and output slacks are often mentioned. It is also important to note that slacks could be radial and non-radial. Radial input/output slacks reflect by how much the analyzed DMU should decrease/increase inputs/outputs in order to lie on the efficient frontier. Non-radial input/output slacks, on the other hand, are not equal to zero if it is possible to decrease/increase inputs/outputs even when the analyzed DMU lies on the efficient frontier. The sum of radial and non-radial slacks is equal to the total slack.

Additionally, it is not out of place to stress a distinction between weakly and strongly efficient DMUs. The DMU is considered to be weakly efficient if it lies on the efficient frontier, but has non-radial slacks. If, on the other hand, the DMU, lying on the efficient frontier, has no non-radial slacks, this productive unit is considered to be strongly efficient.

3.3 Advantages and limitations of the DEA method

Researchers analyzing efficiency with the DEA method should understand strengths and weaknesses of this approach. Some of the advantages of the DEA method are (Cooper et al., 2007; Murthi, Choi, & Desai, 1997; Ramanathan, 2003):

- DEA helps to determine sources and the amount of inefficiency in inputs, outputs and DMUs, which could help mutual fund companies to exploit existing resources better and use the best practices in the industry.
- DEA allows identifying best performing DMUs that act as benchmarks for less efficient productive units, which is particularly important for researchers who estimate mutual fund performance, where benchmark identification could sometimes become a problem.
- DEA does not require a subjective opinion of researchers and examines efficiency using numerical data, which lowers the negative effect of various biases and mistakes.
- DEA can evaluate efficiency of DMUs operating in the multiple input/multiple output universe without restrictions on units in which inputs and outputs are measured. This characteristic makes DEA a good tool for analyses on mutual fund performance that include costs and expenses in the evaluation.
- Since DEA is a non-parametric approach, it does not need a prespecification of the functional relationship between inputs and outputs.
- In contrast with parametric approaches that calculate statistical averages, DEA results in a set of efficiency indices, where each DMU has its own score.

Without a doubt DEA is not a panacea and has certain weaknesses. Coelli et al. (2005) and Ramanathan (2003) provide a list of limitations that could lower the usefulness of the DEA method:

- The shape and position of the efficiency frontier could be influenced by the measurement error and other noise.
- Efficiency scores could be influenced by outliers.
- Results could be biased if important input or output is not included in the analysis.
- Obtained efficiency scores are relative to the best performing DMUs in the sample and the inclusion of new DMUs could reduce efficiency scores.
- The comparison of mean efficiency scores of two samples could be meaningless.
- Technical efficiency scores of the DMUs in the sample could not be increased by adding an extra DMU in the analysis.
- Technical efficiency scores of the DMUs in the sample could not be decreased by adding extra input or output in the analysis.
- In the case of few observations and many inputs and/or outputs, multiple DMUs could be located on the efficiency frontier.
- Results could be biased if heterogeneous inputs and/or outputs are considered to be homogenous.
- Conclusions regarding managerial efficiency could be incorrect if environmental differences are not taken into account.

- Multi-period optimization and the risk in management decision making process are not taken into account by standard DEA.
- The efficiency analysis of samples including many DMUs could be computationally intensive.
- Employing DEA to analyze efficiency makes it difficult to test statistical hypotheses.
- It could be difficult to explain methodology of DEA.
- DEA requires at least one input and output.
- DEA does not allow researchers to directly influence weights of inputs and outputs.
- Results obtained by the DEA approach are sometimes unexpected and counterintuitive.
- DMUs could manipulate DEA results by concentrating on improving a limited number of inputs or outputs.

Although DEA limitations outweigh advantages of this approach quantitatively, certain weaknesses presented earlier are either cautionary notes, which warn researchers of potentially biased results of improperly performed analyses, or shortcomings that are common to other methods of the mutual fund performance evaluation. The knowledge of advantages and disadvantages of DEA should allow researchers to correctly assess whether this approach is appropriate for achieving their goals and should help them increase the quality of the obtained results.

3.4 Efficiency analysis using DEA approach in practice

The DEA approach presented earlier is a very useful method for measuring efficiency. However, in practice the results obtained in the process of the DEA efficiency evaluation are not enough to analyze all processes and variables influencing efficiency. As Fried, Lovell, Schmidt and Yaisawarng (2002) note, "producer performance is influenced by three very different phenomena: the efficiency with which management organizes production activities, the characteristics of the environment in which production activities are carried out, and the impact of good and bad luck, omitted variables, and related phenomena." For purposes of increasing the added value of DEA several procedures have been developed, which are commonly used together or instead of the basic DEA. A separation model as well as two-, three- and four-stage DEA methods are briefly described below.

Researchers that employ the separation model stratify the sample and employ as criteria categorical variables that reflect certain characteristics of the operating environment. In the next part of the analysis, efficiency frontiers are constructed and efficiency scores calculated, whereas each earlier determined stratum is analyzed separately, i.e., there are as many efficiency frontiers as there are subpopulations. It is understandable that results of this approach are dependent on the quality of the stratification process; however, the inability to directly compare efficiency scores of DMUs from different subsamples could also be seen as a limitation.

The procedure of the two-stage efficiency analysis could be described as follows: calculate efficiency scores and then regress them against explanatory variables. Although the analysis algorithm seems to be rather trivial Lovell (1993) provides three cautionary notes:

- Since optimal efficiency solutions are bounded either by zero and one or below by one, they must be either transformed or an appropriate limited dependent variable regression technique must be employed.
- It is important to define what inputs and outputs should be used in the first stage of the analysis and what dependent variables should be considered in the second phase. Lovell

(1993) advises to employ in DEA variables that are under control of the decision maker while leaving variables that cannot be controlled for regression analysis.

- Explanatory variables used in regression analysis influence the efficiency of the output generation. However, they do not have an impact on the transformation process.

Additional notes should be provided about the first point. Simar and Wilson (2011), who, interestingly, do not recommend using efficiency scores in regression analysis, evaluate statistical models employed in the second part of the two-stage DEA in their work. They examine two approaches developed by Simar and Wilson (2007) (truncated regression) and Banker and Natarajan (2008) (ordinary least squares, often abbreviated as OLS), concluding that restrictions of the statistical methods chosen for the second stage should be identified, carefully considered and tested. Hoff (2007) compares different approaches to the execution of a two-stage DEA and concludes that tobit regression is suitable in most cases, with OLS being equally sufficient in many situations. McDonald (2009) argues that tobit regression is not an appropriate statistical method, since efficiency scores are fractional data, and suggests employing the OLS regression. Bogetoft and Otto (2011) name as alternatives bootstrapping methods and stochastic frontier analysis (often abbreviated as SFA).

An example of extension to the two-stage DEA is the three-stage DEA, developed by Fried et al. (2002). In the first stage the classic DEA method is applied to determine efficiency scores, without taking into account environmental variables or statistical noise. The second stage includes stochastic frontier analysis (SFA) used to evaluate impact of managerial inefficiency, environmental variables and statistical noise on the DMU's performance. Fried et al. (2002) assume that total slacks revealed in the first stage of the analysis are composed of managerial efficiency, environmental variables and statistical noise. In the last part DEA is performed using inputs and outputs, one set of which is adjusted to account for the environmental effects and statistical noise discovered in the second stage (what set to transform depends on the orientation of DEA in the first stage). The advantage of the three-stage DEA is that this approach reveals what impacts controllable and uncontrollable factors have on total slacks. Additionally, this method allows the inclusion of multiple uncontrollable variables where there is no need of prespecification of the direction and magnitude of their influence on the efficiency of DMUs.

Due to the fact that the three-stage DEA approach employs stochastic frontier analysis, which is one of the most widely used tools for measuring efficiency and is in a certain sense a competitor to DEA, it is not out of place to describe stochastic frontier analysis in more details.

Stochastic frontier analysis and data envelopment analysis are the two dominant methods of benchmarking. According to Bogetoft and Otto (2011), an important difference between both techniques is that DEA is a non-parametric approach, while SFA is a parametric approach, which means that SFA requires multiple a priori assumptions. However, due to the parametric nature of SFA it is possible to consider a stochastic relationship between inputs and outputs.

The stochastic frontier production function model has been proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977) and can be presented in the following form:

$$ln\, q_i = x_i'\beta + v_i - u_i \tag{13}$$

where

q_i denotes the output of the *i*-th firm,
x_i denotes a vector with logarithms of inputs,
β denotes a vector of parameters to be estimated,
v_i denotes a symmetric random error,
u_i denotes a non-negative variable reflecting technical inefficiency.

An important characteristic of the SFA method is that it takes into account the statistical noise, which could arise due to the unintentional omission of relevant variables or measurement and approximation errors due to the choice of functional form (Coelli et al., 2005). Coelli et al. (2005) illustrate this feature of SFA using an example of two DMUs (A and B) that operate in a one input/one output universe and present a Cobb-Douglas stochastic frontier in the following form:

$$ln\, q_i = \beta_o + \beta_1 \, ln\, x_i + v_i - u_i \tag{14}$$

or

$$q_i = exp(\beta_o + \beta_1 \, ln\, x_i + v_i - u_i) \tag{15}$$

or

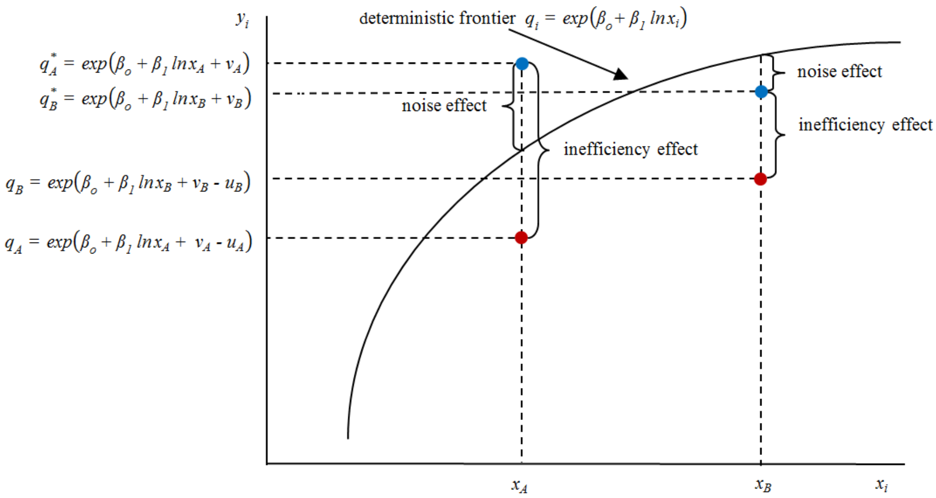
$$q_i = exp(\beta_o + \beta_1 \, ln\, x_i) * exp(v_i) * exp(-u_i) \tag{16}$$

where

q_i denotes the output of the *i*-th firm,
x_i denotes a vector with logarithms of inputs,
β_o and β_1 denote parameters to be estimated,
v_i denotes a symmetric random error,
u_i denotes a non-negative variable reflecting the technical inefficiency.

The stochastic production frontier is presented in figure 9.

Figure 9. The stochastic production frontier



Source: Coelli et al., *An introduction to efficiency and productivity analysis* (2nd ed.), 2005.

If both DMUs operate efficiently, i.e., u_A and u_B are equal to zero, then A would produce $q_A^* = exp\ (\beta_0 + \beta_1 \ln x_A + v_A)$ outputs and B would produce $q_B^* = exp\ (\beta_0 + \beta_1 \ln x_B + v_B)$ outputs. q_A^* lies above the deterministic frontier due to the positive statistical noise effect (v_A is larger than zero), while q_B^* lies below the deterministic frontier due to the negative statistical noise effect (v_B is lower than zero). It should also be noted that the observed output could only be above the deterministic frontier if the positive effect of the statistical noise is larger than the inefficiency effect.

In the context of the SFA approach, the technical efficiency could be calculated as the ratio between actual achieved output and stochastic frontier output (Coelli et al., 2005):

$$TE_i = \frac{q_i}{exp(x_i'\beta + v_i)} = \frac{exp(x_i'\beta + v_i - u_i)}{exp(x_i'\beta + v_i)} = exp(-u_i) \tag{17}$$

where

q_i denotes the output of the *i*-th firm,
x_i denotes a vector with logarithms of inputs,
β denotes a vector of parameters to be estimated,
v_i denotes a symmetric random error,
u_i denotes a non-negative variable reflecting the technical inefficiency.

In order to obtain the technical efficiency, parameters of the stochastic frontier production function should be estimated. The commonly used approach is to make an assumption regarding distribution characteristics of the statistical noise v_i and inefficiency u_i, and to employ the maximum likelihood method. Aigner et al. (1977) use the following distributional assumptions in their paper: $v_i \sim N\ (0,\sigma_v^2)$ and $u_i \sim N_+\ (0,\sigma_v^2)$. Additionally, it is not out of place to note that Coelli et al. (2005) bring to notice that it is possible to replace the half-normality assumption by other assumptions, for example, that the distribution of u_i is truncated normal, exponential or gamma.

Battese and Coelli (1988) describe the approach to estimating the efficiency of specific DMUs by providing the following technical efficiency predictor:

$$E\left[exp\left(-u\right)|e_i\right] = \frac{1 - \Phi\left(\sigma_A + \frac{\gamma e_i}{\sigma_A}\right)}{1 - \Phi\left(\frac{\gamma e_i}{\sigma_A}\right)} exp(\gamma e_i + \frac{\sigma_A^2}{2}) \tag{18}$$

where

$\sigma_A = \sqrt{\gamma\left(1 - \gamma\right)(\sigma_u^2 + \sigma_v^2)}$,
$\gamma = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2}$,
$\Phi(.)$ denotes the cumulative density function of a standard normal random variable,
σ_u^2 denotes the variance of u_is,
σ_v^2 denotes the variance of v_is,
$e_i = u_i + v_i$.

The last model presented in this chapter is referred to as the four-stage DEA. This model was introduced by Fried, Schmidt and Yaisawarng (1999). The first part of this method includes a standard DEA, performed by using unadjusted inputs and outputs. Afterwards the impact of uncontrollable variables on total slacks is analyzed using tobit regression. In the third stage the original dataset is adjusted using parameters calculated in the second stage. The final step of the analysis is a DEA evaluation of efficiency using adjusted dataset.

Researchers who decide to employ either three- or four-stage DEA models should also anticipate potentially incorrect results due to choosing the inappropriate input adjustment algorithm. As Tone and Tsutsui (2009) argue, the input and output transformation approach chosen by Fried et al. (2002) leads to incorrect efficiency scores. Trying to satisfy the non-negativity constraint, demanded by most DEA models, Fried et al. (2002) account for environmental factors and statistical noise by summing original variable values with calculated values, which are fixed for all DMUs. This adjustment approach results in efficiency scores which tend toward unity as fixed adjustment values tend toward infinity and Tone and Tsutsui (2009) present an alternative transformation algorithm. Different adjustment procedures are presented below:

- Input adjustment technique developed by Fried et al. (2002)

$$x_{ij}^{A} = x_{ij} + \left[\max_{j}\left\{z_j^i\hat{\beta}^i\right\} - z_j^i\hat{\beta}^i\right] + \left[\max_{j}\left\{\hat{v}_{ij}\right\} - \hat{v}_{ij}\right] \tag{19}$$

where

x_{ij} denotes the initial *i*-th input of the *j*-th DMU,
x_{ij}^{A} denotes the adjusted *i*-th input of the *j*-th DMU,
$z_j^i\hat{\beta}^i$ denotes the *i*-th total input slack of the *j*-th DMU, attributable to operating evironment,
$\hat{v}_{ij}$ denotes the *i*-th total input slack of the *j*-th DMU, attributable to statistical noise.

- Output adjustment technique proposed by Avkiran and Rowlands (2008)

$$y_{rj}^{A} = y_{rj} + \left[z_j^r\hat{\beta}^r - \min_{j}\left\{z_j^r\hat{\beta}^r\right\}\right] + \left[\hat{v}_{rj} - \min_{j}\left\{\hat{v}_{rj}\right\}\right] \tag{20}$$

where

y_{rj} denotes the initial *r*-th output of the *j*-th DMU,
y_{rj}^{A} denotes the adjusted output *r*-th output of the *j*-th DMU,
$z_j^r\hat{\beta}^r$ denotes the *i*-th total output slack of the *j*-th DMU attributable to operating evironment,
$\hat{v}_{rj}$ denotes the *i*-th total output slack of the *j*-th DMU attributable to statistical noise.

- Two-step input and output adjustment procedure developed by Tone in Tsutsui (2009)

    - In the case of inputs

$$x_{ij}^{A} = x_{ij} - z_j^i\hat{\beta}^i - \hat{v}_{ij} \tag{21}$$

$$x_{ij}^{AA} = \frac{x_{imax} - x_{imin}}{x_{imax}^{A} - x_{imin}^{A}}\left(x_{ij}^{A} - x_{imin}^{A}\right) + x_{imin} \tag{22}$$

where

x_{ij} denotes the initial input,
x_{ij}^{A} denotes the adjusted input,
x_{ij}^{AA} denotes the re-adjusted input,
$z_j^i\hat{\beta}^i$ denotes the *i*-th total output slack of the *j*-th DMU attributable to operating evironment,
$\hat{v}_{ij}$ denotes the *i*-th total input slack of the *j*-th DMU attributable to statistical noise.

- In the case of outputs

$$y_{rj}^{A} = y_{rj} + z_j^r\hat{\beta}^r + \hat{v}_{rj} \tag{23}$$

$$y_{rj}^{AA} = \frac{y_{rmax} - y_{rmin}}{y_{rmax}^{A} - y_{rmin}^{A}}\left(y_{rj}^{A} - y_{rmin}^{A}\right) + y_{rmin} \tag{24}$$

where

y_{rj} denotes the initial output,
y_{rj}^{A} denotes the adjusted output,
y_{rj}^{AA} denotes the re-adjusted output,
$z_j^r\hat{\beta}^r$ denotes the *i*-th total output slack of the *j*-th DMU attributable to operating evironment,
$\hat{v}_{rj}$ denotes the *i*-th total output slack of the *j*-th DMU attributable to statistical noise.

The last two presented models are similar in the sense that both three- and four-stage approaches attempt to increase the objectivity of efficiency scores by lowering the impact of uncontrollable factors. The main difference between these two models is that the three-stage DEA employs the SFA method in the second stage, while the four-stage DEA usually uses OLS or tobit regression.

Which DEA model to choose in order to estimate efficiency scores while taking into account uncontrollable factors and, if possible, statistical noise, depends on the goals of the research, available data, knowledge of the researchers, resources available, etc. In theory, the three-stage DEA reveals the most dimensions of the efficiency. However, this approach is also among the most complex DEA methods and is used less often than the two-stage DEA.

3.5 Efficiency analysis and the mutual fund industry

Taking into consideration the flexibility of the DEA approach and the importance of the mutual fund industry, it is not surprising that there is so much research conducted to investigate the efficiency of mutual funds employing the DEA method. Murthi et. al (1997) are among pioneers who propose using data envelopment analysis to evaluate performance of mutual funds. A developed model includes the expense ratio, turnover, standard deviation of mutual fund returns and total loads as inputs, while actual annual return, which is calculated as the value of an investment of one US dollar in a mutual fund after a oneyear period, is used as a single output. An important finding of this research is a positive correlation between efficiency scores and traditional performance measures (Jensen's alpha and the Sharpe ratio).

Basso and Funari (2001) construct several DEA-based performance indexes. The first one (I_{DEA-1}) uses one or more risk measures (standard deviation of mutual fund returns, beta and half-variance of mutual fund returns) and subscription and/or redemption costs as inputs and expected return or expected excess return as an output. The second index (I_{DEA-2}) is a modification of I_{DEA-1}; the difference between these measures is that additional output is employed in I_{DEA-2}, more specifically, a stochastic dominance indicator. The analysis of the relative efficiency of 47 Italian mutual funds with the utilization of both indexes stresses the importance of subscription and redemption costs in determining efficiency scores. Researchers also find a positive correlation between DEA-based and traditional performance measures, which is consistent with the results obtained by Murthi et al. (1997). Basso and Funari (2001) find that the correlations between DEA-based measures and Jensen's alpha are higher if compared with findings obtained by Murthi et al. (1997), which could be explained by the employment of beta.

DEA is involved in performance analysis of 257 Australian mutual funds, conducted by Galagedera and Silvapulle (2002). Based on investor survey results, historical evidence, data availability and subjective judgment, researchers develop 11 DEA models including different inputs and outputs – among inputs are standard deviations of the 1-, 2-, 3- and 5-year gross returns, sales charges including subscription and redemption fees, operating expenses (the management expense ratio, often abbreviated as MER) as well as a minimum initial investment; among outputs, however, are gross returns in 1-, 2-, 3- and 5-year long periods. An interesting result is that mutual funds which are efficient in the short-term tend to be efficient in the long-run. Galagedera and Silvapulle (2002) also perform logistic regression analysis and find that positive net flows have negative effect on efficiency.

The usefulness of the DEA approach in measuring performance that accounts for other characteristics of mutual funds, apart from risk and return, is shown in the research published by Basso and Funari (2003). In this research 50 randomly generated ethical or socially responsible mutual funds are analyzed using several DEA models. Subscription costs per 5,000, 25,000 and 50,000 US dollars, redemption costs after a 1-, 2-, and 3-year long holding period as well as standard deviation of mutual fund returns and beta are employed as inputs, while the two outputs taken into account are the expected return and ethical indicator.

Haslem and Scheraga (2003; 2006) investigate efficiency of small-cap and large-cap mutual funds in the Morningstar's 500 and determine the inputs and outputs following Hancock (1986), who analyzes user costs to classify individual variables as inputs or outputs. The input variables selected for analysis of large-cap mutual funds are the share of cash in total assets, the expense ratio, the share of stocks in total assets, the P/E ratio, the P/B ratio and mutual funds total assets, while the Sharpe ratio is the only output employed in the research. In the case of small-cap funds the share of cash in total assets, the expense ratio, the share of stocks in total assets, the P/E ratio, the P/B ratio, the number of securities held as well as portfolio turnover are utilized as inputs, while total assets are selected as the only output. Results of the analysis of large-cap mutual funds show that efficient mutual funds tend to have the highest Sharpe ratio, the highest Jensen's alpha, the lowest beta, the lowest standard deviation of mutual fund returns, the lowest portfolio turnover, the lowest share of stocks, the highest share of bonds and other financial instruments, the lowest P/E ratio, the lowest P/B ratio and the lowest three-year earnings growth. Researchers find out that inefficient small-cap mutual funds tend to have the highest beta, the highest P/E, P/B, P/CF ratios, the highest three-year earnings growth as well as the highest median market capitalization. On the other hand, inefficient small-cap mutual funds tend to have the lowest share of bonds and other financial instruments as well as the lowest share of foreign securities and total assets.

Margaritis, Otten and Tourani-Rad (2007) focus their attention on the New Zealand mutual funds. The expense ratio, load and volatility are used as inputs, while a 5-year return serves as the only output. While performing censored tobit regression analysis, researchers discover that the size of the mutual fund has a positive effect on the efficiency score.

In another paper, Gregoriou (2007) employs three DEA models (CCR, cross-efficiency and super-efficiency) to appraise 25 US mutual funds. The average monthly standard deviation of mutual fund returns, monthly downside deviation and maximum drawdown are used as inputs, whereas the share of positive months and annualized monthly compounded return are utilized as outputs.

Lin and Chen (2008) argue that risk measures used in DEA models should reflect fat tails and asymmetry in return distributions. They propose several DEA indices that employ as inputs value-at-risk (often abbreviated as VaR), which is a measure that "summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence" (Jorion, 2007), and conditional value-at-risk (often abbreviated as CVaR), which is defined as the conditional expectations of losses exceeding VaR (Rockafellar & Uryasev, 2000). Additional inputs used are standard deviation of mutual fund returns, half-variance of mutual fund returns, beta, the turnover ratio, the expense ratio, the redemption fee and loads. Outputs, on the other hand, include expected return and Jensen's alpha. It should be noted that in their analysis researchers consider 24 combinations of inputs and outputs. Academics conclude that the utilization of traditional performance indices may not be useful since certain DEA indices can be seen as the generalization of Treynor, Sharpe and reward-to-half-variance indices. Beta and costs, on the other hand, have a great effect on the performance appraisal. Results also show that VaR or CVaR should be used together with traditional risk measures. An interesting novelty of the research is the way the efficiency performance is analyzed – several time periods are investigated and each mutual fund is treated as a different mutual fund in these periods.

Hu and Chang (2008) decompose mutual fund underperformance using the three-stage DEA method, which has been already discussed in this paper, and reveal a positive relationship between performance and size, previous performance, mutual fund manager's tenure and education as well as a negative relationship between the number of mutual funds under management and performance.

Hu, Yu and Wang (2012) employ a four-stage DEA approach to analyze performance of 60 mutual funds from Taiwan in the period 2006-2010 and discover that balanced mutual funds, mutual funds managed by female financial experts and larger mutual funds achieve higher performance, while persistence, mutual fund manager's tenure, replacement as well as the number of funds under management negatively affect results.

A brief review of the literature on the DEA efficiency analysis of mutual funds reveals that although academics are pursuing a common objective, there is no generally accepted methodology in terms of input-output definition and DEA model employment. From one point of view it could be seen as a limitation, especially for researchers who are not proficient in non-parametric analysis. However, the absence of strict rules of analysis is an advantage for those efficiency investigators who need flexibility and search for a highly adaptable method.

4 DETERMINANTS OF MUTUAL FUND PERFORMANCE

The areas of scientific research on mutual fund performance measurement and managerial skills are interconnected with studies on determinants of superior results. A plethora of papers is dedicated to analysis of how size, holdings characteristics, fund flows, characteristics of mutual fund managers, trading activities, expenses and other factors influence mutual fund performance. Findings of these investigations are of interest to investors who screen for mutual funds best suited for fulfilling their investment objectives and mutual fund companies that are constantly trying to enhance clients' satisfaction.

Lückoff (2011) divides performance determinants into five groups:

- Investment style, which includes portfolio turnover, active share, portfolio concentration and style consistency.
- Information access, which includes financial centres, regional proximity, political proximity and information networks.
- Manager characteristics, which include education, experience, gender and management structure.
- Cost-related determinants, which include fees, transaction costs and taxes.
- Fund-related determinants, which include mutual fund size, mutual fund family size, mutual fund age and regulatory environment.

This chapter provides a literature review primarily on the effects that fees, mutual fund size, mutual fund family size, and mutual fund age have on performance. Fees are an endogenous factor, while the last three performance determinants are exogenous. In this paper the endogenous factor is used as an input in DEA, while exogenous factors are employed to estimate the relationships between mutual fund efficiency and the forces that are out of mutual fund managers' control. It should be noted that the selection of the uncontrollable factors covered in this chapter is at least partially a reflection of limited data availability.

4.1 Fees

The relationships between mutual funds and investors imply the existence of two fundamental principles. First, mutual funds provide investment services managing collected assets in clients' best interest. Second, owners of mutual fund shares compensate mutual fund companies for their efforts by paying different fees, some of which are known ex ante, while others are not. Taking into account that the subject of choosing the best performance measure is still open for discussion, and in view of the fact that, according to the empirical evidence, there seems to be no performance persistence, questions regarding the appropriate level of charged fees and their correlation with mutual fund results are extremely important.

In principle, the correlation between charged fees and results should be positive. Mutual funds with a higher level of fees should offer an appropriately higher level of customers' satisfaction; otherwise investors would punish such mutual funds by switching to mutual funds with lower fees or higher returns. This process should continue until the fair price in the form of units of investors' satisfaction for every unit of fees is determined. However, if the exact impact of fees on performance is not known to investors, mutual funds could try to charge high fees, justifying their action by more active and successful portfolio management. In a situation like this, the subjective perception regarding fees becomes a factor in the investment decision making process, at least for those mutual fund investors that evaluate mutual funds from the fee-based perspective. Needless to say, if higher fees are perceived by investors as a sign of better and more sophisticated services, the relationship between expenses and mutual fund inflows could be positive even if in reality there are no superior returns.

Empirical evidence does not seem to support the existence of a positive relationship between fees and performance. Carhart (1997) in his seminal work finds that expense ratios and load fees are negatively correlated with performance. The performance is reduced by expense ratios slightly greater than one-to-one, while in the case of load fees, there is an 80 basis-point difference in returns achieved by average load and average no-load mutual funds, wherein returns are controlled for the correlation between load fees and expense ratios as well as the effect of 20% of the worst-performing mutual funds. Similar results are obtained by Pollet and Wilson (2008), who analyze the relationship between size and performance and find that expenses and load fees are negatively associated with returns. Chen, Hong, Huang, and Kubik (2004) perform regression analysis of mutual fund returns, employing as independent variables different mutual fund characteristics and find no correlation neither between fees and returns nor between load fees and returns. Murthi et al. (1997) analyze the efficiency of mutual funds employing the DEA method and find that expense ratios and load fees are not related to efficiency; the conducted research indicates, however, that mutual funds are inefficient in transforming load fees in results. Gil-Bazo and Ruiz-Verdu (2009) document a negative relationship between mutual fund results before fees and charged fees. In order to understand the reasons for this phenomenon, authors provide two possible explanations: first, there are factors that are not included in regression analysis and are negatively correlated with fees and positively correlated with return; second, mutual funds strategically determine fees on the basis of achieved or expected results. Such strategic behavior could arise because of one of the following reasons: first, mutual funds with inferior past returns have investors that are less sensitive to results, which allows them to raise fees; second, mutual funds with lower expected returns decide not to compete for sophisticated investors and instead focus on investors with inelastic demand in order to increase fees; third, mutual funds with different expected returns pursue different marketing strategies, which influence distribution costs. In recent research Ferreira, Keswani, Miguel and Ramos (2012) analyze determinants of mutual fund performance and document a negative relationship between performance after fees and the expense ratio, noting that the relationship is statistically significant only in the case of some specifications of non-US mutual funds. In the same paper authors indicate that there is no statistically significant relationship between subscription fees and performance as well as between redemption fees and performance. Murcia (2011) divides fees into two groups: implicit costs, which include management and custody provisions, and explicit fees, which include subscription and redemption expenses. The researcher finds that implicit fees have no effect on net results, while evidencing a positive relationship between explicit fees and performance.

The decision making process of investors takes place in conditions of high uncertainty and risk. Even in the case of mutual funds with very defensive portfolio structures a possibility of tail risks exists. Taking into account that future returns of mutual funds are unknown, investors could employ information on fees in order to make the screening and performance evaluation process as objective as possible. It is important to note, though, that analysis of the empirical evidence reveals zero or negative correlation between management fees and performance. Similar results for the relationship between load fees and performance are obtained in most, but not all papers.

4.2 Size

One of the main characteristics of the mutual fund is size, which is usually measured with such proxies as assets under management and net asset value. The current size of a specific mutual fund and its past fluctuations provide information on performance of this mutual fund and, at the same time, reveal investors' perception regarding future results of the mutual fund under review. This information is particularly important for existing and prospective investors; however, it could be also used for peer comparison and benchmarking by mutual fund companies as well as mutual fund managers.

Apart from being a source of information, which increases the level of transparency, asset base could also influence operational results of mutual funds. Mutual fund companies, which should always act on behalf of investors increasing their satisfaction, could achieve lower expenses, better transaction terms and a higher level of fund managers' attention by deciding to manage a lower number of large funds instead of offering dozens of funds with low assets under management. It should be noted though that previously mentioned benefits could be achieved even in the case of small average size of mutual funds, given that cumulative assets of mutual fund companies under management are large enough to negotiate favorable transaction terms and commissions and mutual fund managers are not overloaded with duties. The required level of mutual fund managers' attention and affiliation could be achieved, for example, by hiring additional employees or by rationalizing investment decision making processes.

Discussing scale effects in the mutual fund industry, it is not out of place to acknowledge that in certain cases mutual fund managers could personally benefit from managing large funds, receiving higher management fees, gaining access to additional perks or achieving ego-based goals. While admitting that performance-based motivation of fund managers and their psychological needs could negatively affect owners of mutual fund shares by skewing the risk profile of managed portfolios, potential benefits of mutual fund employees trying to achieve better results should not be underestimated.

Keeping in mind positive effects of economies of scale, a question of potential size-related diseconomies should be addressed. The most obvious source of problems is connected with a shift in characteristics of portfolio holdings and management style. Managers of large mutual funds are facing a dilemma of whether to exclude small cap companies from available investment universe, sacrificing potential higher return or to invest in such companies, making a choice between two options: to invest until controlling interest is attained or to keep non-controlling number of shares, while increasing the number of stocks in portfolio beyond reasonable boundaries. Another problem that arises when a specific mutual fund grows large is that it becomes increasingly hard to open and liquidate positions without attracting attention of market participants and influencing the prices. As Ciccotello and Grant (1996) point out, managers of large mutual funds are struggling to maneuver a battleship in a bathtub, which, understandably, implies that managing a large mutual fund is a risky task that requires special care and attention.

Analyzing scale effects, Chen et al. (2004) also conjecture that hierarchy costs may be particularly important for large mutual funds. This scale diseconomy stems from the fact that decisions of employees in large hierarchically structured organizations are ex ante affected by the competitive process of idea implementation. This problem becomes more profound in the case of organizations in which large amounts of soft information are analyzed as it is more difficult to justify and implement decisions in such conditions.

A lot of research has been executed to investigate the relationship between mutual fund size and performance.

- Ciccotello and Grant (1996) analyze mutual funds classified as Aggressive Growth, Long-Term Growth or Growth & Income and find evidence of small fund superiority only in the case of more aggressive funds, stressing that additional inflows cause more problems than opportunities for managers of mutual funds with aggressive investment policies.
- Chen et al. (2004) examine mutual fund data from 1962 to 1999 and make several important conclusions: first, performance declines as fund size increases, second, liquidity plays an important role in eroding performance, third, performance improves as fund family size grows and fourth, due to hierarchy costs size and liquidity have an inverse relationship with results.
- Berk and Green (2004) investigate the relationship between mutual fund flows and performance and argue that higher inflows lead to lower performance because mutual fund managers increase expense ratios, or due to the fact that as mutual fund grows in size, diseconomies of scales, such as the need to add lower quality holdings, organizational inefficiencies or higher transaction cost, diminish superior performance.
- Pollet and Wilson (2008) examine how size affects the behavior of mutual fund managers and discover that in the case of typical mutual funds the level of portfolio diversification grows at a slower rate than the mutual fund size, i.e., when mutual fund experiences growth in assets under management, mutual fund managers prefer to increase monetary value of existing holdings instead of generating new investment ideas. Researchers find evidence that diminishing returns to scale may appear due to the inability of mutual fund managers to appropriately change investment strategies in response to increased asset base. They also document that when controlling for the size of the mutual fund and mutual fund family, there is a positive correlation between diversification and performance, with the relationship being stronger in the case of small-cap funds.
- Liquidity constraints as the reason for large US mutual funds underperforming the smaller ones is evidenced in the research conducted by Ferreira et al. (2012). An important finding of this research is related to the fact that researchers document scale economies in the case of non-US mutual funds and US mutual funds that invest abroad. The explanation for this phenomenon stems from larger and more liquid investment universes available for mutual funds, located outside the US, and US mutual funds with investment policies that allow holding overseas assets.
- Murcia (2011) investigates determinants of performance of Spanish mutual funds and finds no evidence of scale efficiencies, stressing that Spanish mutual funds are on average smaller than European and US mutual funds and may not be large enough to experience positive scale effects.
- Research on efficiency of European equity mutual funds, conducted by Annaert, van den Broeck and Vander Vennet (2003), employing a Bayesian stochastic frontier approach, finds a positive scale effect.

Since this research employs the DEA method of efficiency evaluation, it is not out of place to bring to notice that there has also been some research conducted that analyzes mutual funds employing DEA and investigates scale effects in the industry at the same time. Babalos et al. (2009) employ DEA analysis to evaluate the Greek mutual fund industry and demonstrate that larger size could negatively affect performance. This phenomenon is attributed to the specific structure of the Greek equity market, which is illiquid and has small market capitalization. If DEA-based research, cited earlier, evidences an existence of scale diseconomies, Galagedera and Silvapulle (2002), on the other hand, find no relationship between size and efficiency. Murthi et al. (1997) estimate efficiency of 731 mutual funds and document a correlation between size and efficiency that is not significantly different from zero. Researchers note, though, that there are certain groups of mutual funds with a positive relationship between the net asset value and efficiency scores, and provide lower transaction costs as a possible explanation of this phenomenon. Fernandez-Sanchez and Luna (2007) examine scale effects in the Spanish mutual fund industry and find a significant and positive correlation between size and performance. Researchers note, though, that the positive relationship disappears when mutual fund size approaches a certain point. Similar results are documented by Haslem and Scheraga (2006), who discover that mutual funds with a lower asset base are managed less efficiently than larger mutual funds. Researchers stress, however, that mutual funds included in the analysis may not be large enough to experience management inefficiency, which is consistent with the findings of Latzko (1999). Margaritis et al. (2007) analyze the New Zealand mutual funds and in the process of a censored tobit regression analysis discover that mutual fund size has a positive effect on the efficiency score.

Summarizing the results of the existing papers on scale effects in the mutual fund industry, several important conclusions could be made. Firstly, the relationship between size and performance in the mutual fund industry still puzzles practitioners and academics, with a theoretical approach supporting the existence of both negative and positive scale effects in mutual fund operations. Secondly, when it comes to recent empirical evidence, the relationship between size and performance seems to be negative, though this result is not completely unanimous. Thirdly, there are several explanations of scale diseconomies in the mutual fund industry. Empirical evidence reveals that diseconomies of scale seem to appear due to organizational inefficiencies, among which could be included the inability of mutual fund managers to appropriately customize investment strategy when the asset base changes, and liquidity constraints.

4.3 Family size

The phenomenon of scale efficiency on the level of individual funds is often analyzed together with the relationship between mutual fund family size, i.e., the size of the mutual fund company and performance. From the theoretical point of view, the increase in the cumulative asset base should lead to a higher amount of collected management fees and should allow mutual fund companies to decrease fixed costs per unit of managed monetary value. Relieved monetary resources could thus be employed more efficiently, for example, mutual fund managers could get access to additional sources of information in form of different types of research or news and information aggregating systems. Larger mutual fund companies could also negotiate more favorable terms of cooperation with brokers and investment banks, by lowering the burden of expenses that are covered by mutual fund investors and by increasing net of fees return.

Since increased value of cumulative assets under management does not necessarily lead to asset base growth of all individual mutual funds, problems related to a shift in characteristics of portfolio holdings and management style could be avoided. Organizational diseconomies, i.e., the negative impact of hierarchy costs, on the other hand, should be insignificant if individual mutual funds are managed independently and do not compete for resources.

Among research studies that reveal a positive effect of mutual fund family size on performance of individual mutual funds is the research conducted by Chen et al. (2004), who investigate scale effects in the mutual fund industry and find that a two-standard deviation shift in the size of the mutual fund family which excludes the mutual fund under review leads to a 4 to 6 basis point change in the monthly performance of the analyzed mutual fund. Results that confirm the existence of a positive relationship between the mutual fund family size and performance are likewise obtained in the analysis conducted by Ferreira et al. (2012), who point out that members of smaller mutual fund families suffer from higher lending fees and trading comissions.

Interesting findings are documented in the research conducted by Bhojraj, Jun Cho and Yehuda (2012), who examine the effect of regulatory changes on the relationship between mutual fund family size and performance. Researchers find that the positive effect of larger cumulative asset base on performance of individual mutual funds dissapears after the adoption of new regulatory rules. Since the research also reveals that, after controlling for mutual fund size, managers of mutual funds from larger families have better stock-picking ability prior to the regulatory changes that limit selective information disclosure, the information advantage explanation of the analyzed phenomenon seems to be the most appropriate.

From the above discussion the following conclusion could be made: according to the empirical evidence, there seems to be a positive relationship between the mutual fund family size and performance.

4.4 Age

Mutual fund age which is measured as a time period from the mutual fund inception date could potentially be a source of important information for investors and researchers who are active in the area of mutual fund performance. However, findings on the relationship between mutual fund longevity and performance could have informational value only if the age-performance linkage reflects development of the ability and willingness of mutual fund managers to increase performance or evolution of other mutual fund characteristics that are significant only for older mutual funds. Investors should be careful when making conclusions about mutual funds based on their age because longevity could be a product of successful and out of the ordinary marketing efforts of the managing company or investors' unwillingness to penalize underperformance. If at least one of the latter two explanations holds true, then results obtained in the analysis could be misleading for those investors who believe that only efficiently managed mutual funds could survive for many decades. It should also be noted that the direction of the age effect could be potentially skewed by other determinants of mutual fund performance which are in the relationship with age.

As Ferreira et al. (2012) stress, age could theoretically be a source of both, outperformance and underperformance. A negative relationship between longevity and performance could arise due to higher agility and activities aimed at achieving superior results which are needed to survive in the initial phases of operation. If the mutual fund company routinely launches mutual funds, it could decide to appoint younger and less experienced investment professionals as portfolio managers, which could skew the risk and return profile of the investment policy if these mutual fund managers try to secure their position in the company by achieving higher performance. Arguments that age has a positive effect on mutual fund performance include higher costs and lack of experience during the first months of mutual fund existence or, in other words, the experience curve effect.

Among researchers who analyze the relationship between age and performance and find it negative or non-existent are:

- Ferreira et al. (2012), who find no correlation between longevity and performance in the case of the US mutual funds, evidencing at the same time a negative association between age and results in the case of mutual funds outside the US.
- Otten and Bams (2002), who investigate performance of mutual funds in France, Germany, Italy, the UK and the Netherlands and show that younger mutual funds achieve higher results than older mutual funds, stressing that while coefficients are negative in the case of all countries, only mutual funds in Germany and the UK have a significantly negative relationship between longevity and performance. It is important to note, however, that due to the lack of information on individual mutual fund characteristics in Italy, authors do not report results for this country.
- Karoui and Meier (2009), who study the performance of newly launched US equity mutual funds and document higher excess and abnormal returns as well as higher risk-adjusted performance if compared with older mutual funds. Interestingly, researchers also find that younger mutual funds exhibit higher unsystematic and total risk, are less diversified and invest in smaller and less liquid stocks.
- Yong and Jusoh (2012), who analyze Islamic and conventional mutual funds in Malaysia and reveal that younger mutual funds outperform older mutual funds. An interesting finding of this research is that the negative age effect is more pronounced in the case of Islamic mutual funds, which could be explained by different regulatory frameworks.
- Murcia (2011), who analyzes the performance determinants in the Spanish mutual fund industry, and does not evidence any significant relationship between age and returns. At the same time, the researcher detects a negative relationship between longevity and performance when fixed income and balanced mutual funds are analyzed.
- Annaert et al. (2003), who employ a Bayesian stochastic frontier approach to determine factors influencing performance of European equity mutual funds and reveal no link between age and efficiency.

Signs of the positive age-performance relationship are detected by Eid Jr. and Rochman (2009), who analyze the Brazilian mutual fund industry and investigate whether active management adds or destroys value. Researchers conclude that age is a significant variable and that there is a positive relationship between age and performance which is measured using alpha. Positive, but insignificant, is the age variable in the research on the Pakistani mutual fund performance, conducted by Afza and Rauf (2009), who conclude that older mutual funds achieve the same or slightly higher results than the younger ones.

To sum up, a brief review of the literature on the relationship between longevity and performance in the mutual fund industry reveals that researchers are not completely unanimous regarding the age effect. However, there seems to be more evidence that younger mutual funds achieve higher performance than the older ones.

5 DISCUSSION ON RISK

Risk could be interpreted as the likelihood that the achieved return is different from expected, which means that risk includes not only lower than expected returns (downside risk), but also higher than expected returns (upside risk). An important characteristic of risk is that it is a subjective and relative concept; in other words, what is considered to be an extremely risky investment by one investor could be perceived as relatively safe by another. In theory and practice, risk is often considered to be the price that investors pay for return, which leads to the assumption that higher risk should result in higher return and vice versa. If this assumption is correct, then risk could be considered as an input variable in the context of efficiency analysis; however, in the case of non-positive relationship between risk and return, the risk-minimization objective could be justifiably included in the DEA procedure. It should also be noted that the choice of inputs and outputs employed in the efficiency study should be based on the characteristics of the production process. In other words, if the mutual fund is following a strategy of risk-minimization or includes this element in the investment process, then risk could be perceived as the result of the mutual fund management procedures. In other cases it may be more appropriate to treat the mutual fund risk exposure as an input.

In most cases the risk is measured ex post and therefore a question arises of whether the information on the historical variability of returns could be useful for the mutual fund management process. Bacon (2012) provides three reasons for calculating ex post risk:

- Performance appraisal should include both, return and risk, which leads to the conclusion that historical risk is a critical component of investment process analysis.
- Mutual fund managers should follow certain investment policies ex ante determined by investors and/or regulators and/or senior managers; therefore, constant analysis of the risk deviations is required.
- Forecasted level of risk should be compared with actual historical risk, which is a prerequisite of an efficient investment process.

For an obvious reason, the risk-reward relationship remains to be one of the principal elements of the financial system. In fact, as it has already been mentioned, multiple measures of risk adjusted performance have been developed in the last decades and their usage has become a de facto standard in the financial industry. However, definitions and methods of measuring risk are still a subject of discussion.

The decision of what measure of risk to employ remains to be an essential factor in the risk-return analysis. The basic and one of the most widely used risk measures is the standard deviation, which reveals the dispersion of a set of periodic returns around the average value of these returns. However, when analyzing risk, it is important to understand that the total risk, which could be measured by the standard deviation, consists of two components: the systematic risk, also referred to as non-diversifiable or market risk, and the unsystematic risk, often called diversifiable or firm-specific risk. A portfolio manager could erase the unsystematic risk through diversification; however, it is not possible to nullify the systematic risk. In theory, since it is assumed that the marginal investor holds a diversified portfolio, only non-diversifiable risk should be priced and rewarded. From this perspective standard deviation could not be accepted as the universal risk measure which is appropriate for all situations.

There are several models developed for the purpose of measuring market risk. The most widely used is the CAPM proposed by Sharpe (1964) and Lintner (1965). The mutual fund risk measure in the context of the CAPM is beta measured as follows (Fama & French, 2004):

$$\beta = \frac{cov_{im}}{\sigma_m^2} \tag{25}$$

where

cov_{im} denotes the covariance of mutual fund returns and market returns,
σ_m^2 denotes the variance of market returns.

Since covariance of mutual fund returns and market returns could be presented as a product of standard deviations of mutual fund returns and market returns as well as a correlation between mutual fund returns and market returns, it is clear that beta is dependent on relative volatility, equal to mutual fund volatility divided by market volatility, and the correlation between mutual fund returns and market returns.

The relationship between return and risk, according to the CAPM, could be defined in the following form (Fama & French, 2004):

$$E(R_i) = R_f + [E(R_m) - R_f] * \beta \tag{26}$$

where

$E(R_i)$ denotes the expected return,
R_f denotes the risk-free interest rate,
$E(R_m)$ denotes the expected return of the market portfolio,
β denotes the beta.

From the formula presented above it is clear that in the context of the CAPM there is a positive relationship between market risk measured by beta and expected return if the expected market return exceeds the risk-free rate.

Due to the fact that the CAPM requires fulfilment of certain assumptions, among which are the absence of transaction costs, unlimited tradability and divisibility of financial instruments, the absence of under- or overinformed investors, certain alternative market risk measuring models were introduced such as the arbitrage pricing model (often abbreviated as APM), proposed by Ross (1976). One of the differences between the two models is that the APM includes multiple factors of market risk with multiple factor betas reflecting their impact on the analyzed investment. Another approach involves multi-factor models, which differ from the APM in that they include in calculations certain predetermined macroeconomic factors that influence the market risk. There are also risk and return models that are built on the assumption that there is a positive relationship between risk and return, and therefore certain characteristics of financial instruments, such as price multiples, could be used as proxies for risk.

When it comes to the empirical evidence, the critique of the CAPM becomes an important and sometimes even controversial part of conducted research – Black, Fama and French (2004) even warn that “despite its seductive simplicity, the CAPM’s empirical problems probably invalidate its use in applications”. Basu (1977) questions the validity of the price-ratio hypothesis and finds that returns predicted by the CAPM are lower than the actual returns of stocks with high P/E ratios. Banz (1981) investigates the relationship between market capitalization and returns and concludes that the CAPM is misspecified since smaller firms tend to have higher risk-adjusted return than the large ones. Interestingly, Banz (1981) also states that the P/E effect identified by Basu (1977) is actually a proxy for the size effect. Lakonishok and Shapiro (1986) investigate the impact of beta, standard deviation (or total variance) and the size on stock market returns, concluding that “… neither the traditional measure of risk (beta) nor the alternative measures (variance or residual standard deviation), can explain – again, at standard levels of significance – the cross-sectional variation in returns; only size appears to matter”. Fama and French (1992) employ the cross-section regression method and conclude that such factors as the size and book-to-market ratio increase the explanatory power of the market beta. In recent research Baker, Bradley and

Wugler (2011) apply principles of behavioral finance to investigate the historical outperformance of low-volatility and low-beta stocks and conclude that this anomaly could be partly the result of institutional investors' benchmarking that interrupts arbitrage trading in low-alpha, high-beta stocks and high-alpha, low-beta stocks. On the basis of the brief literature review, it is possible to conclude that results obtained by multiple academics challenge the theoretical groundings of the market risk models or question the validity of empirical testing approaches.

In addition to standard deviation and beta, the risk measure that is particularly often employed is the tracking error (often abbreviated as TE), sometimes referred to as the active risk or the relative risk. The tracking error is calculated as the standard deviation of active returns, i.e., differences between mutual fund returns and the market returns. Thus the formula for calculating ex post active risk is the following (Alexander, 2008):

$$TE = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(R_t - \overline{R})^2} \tag{27}$$

where

T denotes the number of observations in the sample,
R_t denotes the active return at time t,
$\overline{R}$ denotes the average active return.

This risk measure is usually employed for assessing risk of the passively managed mutual funds or sub-portfolios; however, sometimes it is used by investment companies that utilize active strategies. At this point, it is crucial to note that even though at first glance tracking error seems to be an appropriate risk measure for mutual funds with active investment policies, Alexander (2008) argues that "there is a real problem with ex post tracking if risk managers try to apply this metric to active funds, or indeed any fund that has a non-zero mean active return." Due to the calculation mechanics a constantly underperforming or outperforming mutual fund could have a very low tracking error as long as the dispersion of active returns around average active return is not too wide. In fact, tracking error is useful for determining how closely passive mutual fund is tracking its benchmark; however, it does not reveal the risk of active mutual fund achieving lower return than the reference index.

The literature review discloses that there is evidence that existing models and approaches do not unequivocally reveal the relationship between risk and return, thus the reliance on risk measures as inputs in DEA calculations may prove to be misleading, especially in cases when the mutual fund management process implicitly treats risk-minimization as one of the objectives. If the final result of the successful investment process is a combination of return maximization and simultaneous risk minimization, then risk could be applied as a DEA output. Mutual fund managers pursuing objectives of maximizing return and minimizing risk are actually struggling to maximize risk-adjusted return, which means that implementation of the risk as the DEA output is not so controversial as it may seem and could be appropriate at least in some cases.

It is important to stress though that the negative relationship between risk and efficiency, i.e., the risk minimization objective, results in impossibility to use conventional DEA models, which assume that efficiency increases (decreases) when inputs decrease (increase) or (and) outputs increase (decrease). One way to deal with this issue is to use the reciprocal of the risk, which in this paper is measured as the ratio between mutual fund daily total return volatility and volatility of daily gross total returns of the benchmark. In this case the risk management objective could be stated as “maximize benchmark risk relative to mutual fund risk” as opposed to “minimize mutual fund risk relative to benchmark risk”; however, these goals are essentially equivalent. At this point, it is important to note that in this paper the reciprocal of the risk is referred to as the relative risk.

It should also be noted that Seiford and Zhu (2002) introduce a model that allows that outputs which are negatively related with efficiency, and are referred to as undesirable or anti-isotonic, are incorporated in the VRS DEA model. Researchers employ DEA classification invariance, i.e., classifications of efficiencies and inefficiencies do not change with the data transformation. The developed model transforms undesirable outputs by multiplying them by “-1” and adding to the obtained numbers such value that turns negative anti-isotonic outputs into positive ones.

6 EFFICIENCY ANALYSIS

6.1 Data

The data on mutual funds used in this analysis are compiled from several sources. The information on net asset values and net asset values per share is provided by the Securities Market Agency, while the information on age and total expense ratios (often abbreviated as TER) is available on webpages of Slovenian mutual fund companies. At this point, it should be stressed that unfortunately, it turned out to be a rather complicated task to receive any additional information from mutual fund companies. For this reason, information on manager characteristics and investment style, which characterizes investment culture and decision making processes that exist in different mutual fund companies, is not included in the research.

The total expense ratio is the only input employed in DEA, whereas relative return and relative risk are the two outputs included in the model. The total expense ratio is a ratio of certain mutual fund expenses and the average net asset value of the mutual fund. These are not all fees and expenses that mutual fund investors are exposed to; for example, the total expense ratio does not include provisions paid to brokers, entrance and exit fees. Relative return is measured as the end-of-period gross value of one monetary unit invested in the mutual fund divided by the end-of-period gross value of one monetary unit invested in the benchmark. Relative risk is calculated as the ratio between benchmark volatility and mutual fund volatility, whereby volatility is measured as the standard deviation of daily gross returns. It should be noted that most Slovenian mutual fund companies do not reveal the information on benchmarks and for this reason this paper employs internally generated indexes that reflect investment policies of mutual funds under review. Additionally, it is important to stress that daily gross total returns are used for the calculation of return and risk in the case of benchmarks, while in the case of mutual funds daily net asset values per share adjusted for the expenses are employed.

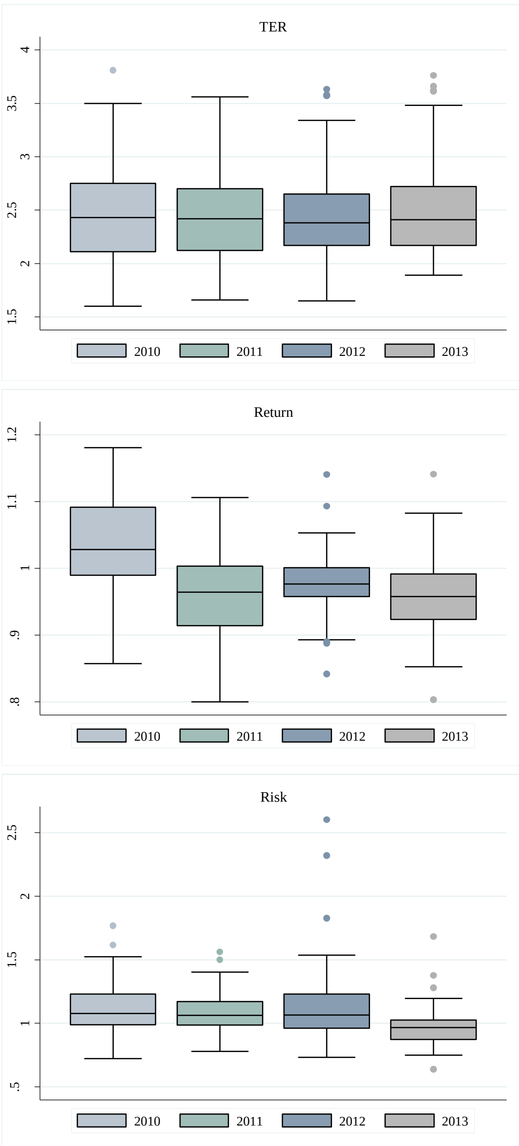
Descriptive statistics are presented in table 1, which reveals among other information that there are 62 mutual funds under review. Further explanation regarding this element of the data selection is required – in the analysis are considered only actively managed equity mutual funds that did not experience significant changes in the investment policy in the period 2010-2013.

Table 1. Descriptive statistics – input and outputs employed in DEA

Variable	Year	Number of observations	Average	Median	Standard deviation	Minimum	Maximum
TER (%)	2010	62	2.4453	2.4300	0.4229	1.6000	3.8100
	2011	62	2.4558	2.4200	0.4327	1.6600	3.5600
	2012	62	2.4687	2.3800	0.4320	1.6500	3.6300
	2013	62	2.5169	2.4100	0.4432	1.8900	3.7600
Relative return	2010	62	1.0312	1.0281	0.0721	0.8575	1.1808
	2011	62	0.9637	0.9644	0.0641	0.8003	1.1061
	2012	62	0.9769	0.9765	0.0480	0.8420	1.1404
	2013	62	0.9589	0.9578	0.0608	0.8032	1.1409
Relative risk	2010	62	1.1143	1.0776	0.1966	0.7213	1.7675
	2011	62	1.0745	1.0636	0.1522	0.7786	1.5596
	2012	62	1.1317	1.0657	0.3149	0.7331	2.6007
	2013	62	0.9731	0.9664	0.1548	0.6383	1.6802

Additional information on variables is presented in figure 10, where boxplots are graphed.

Figure 10. Boxplots – input and outputs employed in DEA



6.2 DEA efficiency

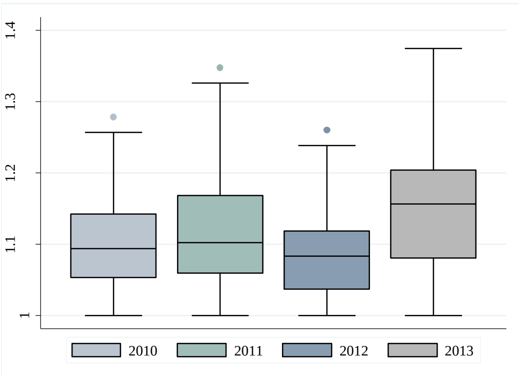
In this research the efficiency analysis of the Slovenian equity mutual fund sector is performed, employing a VRS output-oriented DEA model. The VRS formulation is chosen because it allows the usage of ratios as inputs and outputs (Hollingsworth & Smith, 2003), while the output orientation is employed due to the fact that in the context of the mutual fund management process it is more important to evaluate how successful the DMUs are in terms of maximizing outputs and simultaneously leaving inputs intact rather than assume that the primary objective of portfolio managers is to minimize inputs. As it has already been noted earlier, the total expense ratio is used as the only input, while relative risk and relative return are employed as the two outputs.

Descriptive statistics on obtained optimal solutions are presented in table 2, while boxplots are displayed in figure 11. When looking at the obtained results, it is important to keep in mind that these optimal solutions do not take into account effects of factors that cannot be controlled by mutual fund managers. Therefore, these results can accurately reveal efficiency levels relevant to mutual fund investors; however, at the same time they are potentially biased if used to rank mutual fund managers. It is understandable that if further analysis reveals no relationship between DEA optimal solutions and uncontrollable factors than the efforts of mutual fund managers could be judged on the basis of DEA findings.

Table 2. Descriptive statistics – output-oriented VRS model optimal solutions

Year	Number of observations	Average	Median	Standard deviation	Minimum	Maximum
2010	62	1.0975	1.0937	0.0648	1.0000	1.2782
2011	62	1.1167	1.1020	0.0802	1.0000	1.3476
2012	62	1.0863	1.0831	0.0615	1.0000	1.2598
2013	62	1.1457	1.1562	0.0871	1.0000	1.3745

Figure 11. Boxplots – output-oriented VRS model optimal solutions



The obtained results are not transformed to be less than 1; in other words, they do not directly show the levels of efficiency, but reveal by how much could the outputs be proportionally increased without changing the inputs. For example, in 2010 average equity mutual fund produced on average 8.88% (calculated as 1 minus the reciprocal of the optimal solution 1.0975) proportionally fewer outputs than possible or, in other words, it produced only 91.12% (calculated as the reciprocal of the optimal solution 1.0975) of efficient amount of outputs or, looking at the results from different perspective, it was operating in such a manner that in order to be efficient it was required to proportionally increase outputs by on average 9.75% (calculated as the optimal solution 1.0975 minus 1). Table 2 also shows that in 2010 the least efficient mutual fund produced only 78.23% (calculated as the reciprocal of the optimal solution 1.2782) of achievable outputs and could have proportionally increased them by 27.82% (calculated as the optimal solution 1.2782 minus 1). It is understandable that results in 2011, 2012 and 2013 are interpreted in a similar way. At this point, it is also necessary to note that the obtained results are radial efficiency measures and do not take into account possible non-radial slacks.

In order to analyze the differences between optimal solutions in different years, non-parametric tests are employed (Wilcoxon-Mann-Whitney and Kolmogorov-Smirnov). It should be noted that the decision of what statistical tests to utilize is influenced by the findings of Banker, Zheng and Natarajan (2010), who investigate the issue of choosing the appropriate statistical technique for the purpose of comparing the efficiencies of two groups of DMUs.

Non-parametric tests, the results of which are presented in table 3, reveal that the differences between optimal solutions in different years are significant ($\alpha = 0.01$) only when optimal solutions in 2013 are compared with optimal solutions in 2010 and 2012. Taking into account the information presented in table 2 and figure 11, it is possible to conclude that the average optimal solution in 2013 was higher than the average optimal solutions in 2010 and 2012.

Table 3. Optimal solutions - results of the non-parametric tests

		Wilcoxon-Mann-Whitney test			Kolmogorov-Smirnov test		
		2011	2012	2013	2011	2012	2013
2010	test statistic	-1.232	0.913	-3.320	0.1290	0.1774	0.3065
	p / exact p	*0.2178*	*0.3614*	*0.0009*	*0.683*	*0.258*	*0.006*
2011	test statistic		2.052	-1.975		0.2581	0.2419
	p / exact p		*0.0401*	*0.0483*		*0.022*	*0.036*
2012	test statistic			-3.950			0.4355
	p / exact p			*0.0001*			*0.000*

Note: information on exact p is provided in the case of Kolmogorov-Smirnov test

Since analyzed optimal solutions are obtained using four different datasets (one dataset for every analyzed year), there are four different frontiers, i.e., a mutual fund considered to be efficient in one year could be inefficient in another year. In other words, it is incorrect to directly compare mutual funds in different years and make conclusions regarding efficiency levels. It is understandable that the same applies to the measures of central tendency. The attained results could be treated as follows: in order to become efficient, i.e., to lie on the efficient frontier, the average mutual fund in 2013 should have proportionally expanded its outputs by a higher factor than in the previous year and in 2010.

In order to reveal relationships between optimal solutions in different years, several approaches (Spearman’s rank correlation, Kendall’s tau, pairwise correlation, OLS regression, tobit regression and SFA regression) are employed. At this point, it is not out of place to note that multiple methods are utilized because there is no uniformly accepted method of testing the relationship between results obtained in the DEA process and other variables. The issue of choosing the appropriate regression procedure has already been brought to light in this paper in chapter 3.4; this paper therefore considers that it is preferable to use multiple approaches than to choose only one.

The complete results of performed analyses are presented in the appendix of this paper (six tables from C to H), while the summary of the findings is revealed in table 4.

Table 4. Results of the analyses of existence of relationships between optimal solutions in different years

	2011	2012	2013
2010	A positive but insignificant (α = 0.01) relationship	A positive but insignificant (α = 0.01) relationship	A negative but insignificant (α = 0.01) relationship
2011		A positive and significant (α = 0.01) relationship	A positive and significant (α = 0.01) relationship (except for the Spearman's rank correlation test and Kendall's tau test)
2012			A positive and significant (α = 0.01) relationship

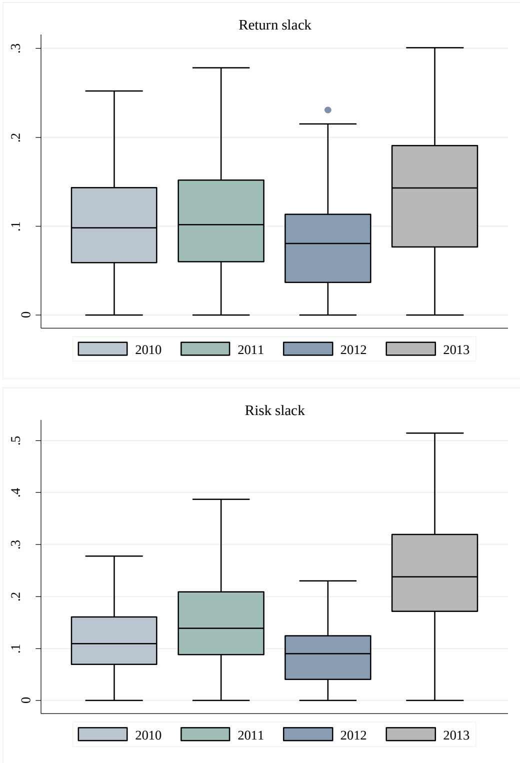
Although all findings of the analyses presented above are undoubtedly important, it is more useful to focus on the following question: "If the mutual fund is (in)efficient this year, is it rational to assume that this mutual fund will be (in)efficient next year?" Statistical tests, the main findings of which are displayed above, reveal significant (α = 0.01) positive relationships between optimal solutions in the periods 2011-2012 and 2012-2013. However, the analysis of the relationship between optimal solutions in 2010 and 2011 reveals a positive, but insignificant (α = 0.01) relationship. Taking into account the findings of the correlation and regression analyses, it could be concluded that in certain time frames performance persistence does exist in the Slovenian mutual fund industry; however, this positive relationship is not particularly strong and in some cases disappears.

Another characteristic of the efficiency of mutual funds in Slovenia is revealed in table 5 and figure 12, where the information on total output slacks is displayed. Total output slacks show the difference between strongly efficient output values and the actual output values. For example, it is clear that in order to become strongly efficient in 2010, the average mutual fund should have increased the relative return by on average 0.0983 and the relative risk by 0.1128. At this point, it is not out of place to note that since the relative risk in this DEA investigation is equal to benchmark risk divided by mutual fund risk, maximization of this output is equivalent to minimization of mutual fund risk relative to benchmark risk.

Table 5. Descriptive statistics – total output-oriented VRS model slacks

Variable	Year	Number of observations	Average	Median	Standard deviation	Minimum	Maximum
Relative return	2010	62	0.0983	0.0982	0.0611	0.0000	0.2523
	2011	62	0.1083	0.1018	0.0685	0.0000	0.2782
	2012	62	0.0827	0.0804	0.0560	0.0000	0.2306
	2013	62	0.1358	0.1431	0.0758	0.0000	0.3008
Relative risk	2010	62	0.1128	0.1096	0.0694	0.0000	0.2779
	2011	62	0.1484	0.1388	0.0935	0.0000	0.3869
	2012	62	0.0912	0.0900	0.0602	0.0000	0.2301
	2013	62	0.2331	0.2381	0.1147	0.0000	0.5142

Figure 12. Boxplots – total output-oriented VRS model slacks



The results of the performed non-parametric tests (Wilcoxon-Mann-Whitney and Kolmogorov-Smirnov) presented in tables 6 and 7 reveal that the differences between total return slacks in different years are significant (α = 0.01) when total return slacks in 2013 are compared with total return slacks in 2010 and 2012. On the other hand, the differences between total risk slacks in different years are significant (α = 0.01) when total risk slacks in 2013 are compared with total risk slacks in 2010, 2011 and 2012 and when total risk slacks in 2012 are compared with total risk slacks in 2011. Taking into account the information presented in table 5 and figure 12, it is possible to conclude that firstly, the average total return slack in 2013 was higher than the average total return slacks in 2010 and 2012, secondly, the average total risk slack in 2013 was higher than the average total risk slacks in 2010, 2011 and 2012, and thirdly, the average total risk slack in 2012 was lower than the average total risk slack in 2011.

Table 6. Total return slacks - results of the non-parametric tests

		Wilcoxon-Mann-Whitney test			Kolmogorov-Smirnov test		
		2011	2012	2013	2011	2012	2013
2010	test statistic	-0.697	1.418	-2.965	0.1129	0.1935	0.2903
	p / exact p	*0.4855*	*0.1563*	*0.0030*	*0.829*	*0.186*	*0.008*
2011	test statistic		2.122	-2.205		0.2419	0.2419
	p / exact p		*0.0338*	*0.0275*		*0.036*	*0.036*
2012	test statistic			-4.045			0.4355
	p / exact p			*0.0001*			*0.000*

Note: information on exact p is provided in the case of Kolmogorov-Smirnov test

Table 7. Total risk slacks - results of the non-parametric tests

		Wilcoxon-Mann-Whitney test			Kolmogorov-Smirnov test		
		2011	2012	2013	2011	2012	2013
2010	test statistic	-1.977	1.728	-6.195	0.2097	0.1935	0.6129
	p / exact p	*0.0480*	*0.0840*	*0.0000*	*0.085*	*0.186*	*0.000*
2011	test statistic		3.482	-4.329		0.3548	0.4516
	p / exact p		*0.0005*	*0.0000*		*0.001*	*0.000*
2012	test statistic			-6.945			0.7097
	p / exact p			*0.0000*			*0.000*

Note: information on exact p is provided in the case of Kolmogorov-Smirnov test

Several statistical tests (Spearman's rank correlation, Kendall's tau, pairwise correlation, OLS regression, tobit regression and SFA regressions) are employed in order to reveal the relationships between total slacks in different years. The complete results of these analyses are presented in the appendix of this paper (six tables from I to N), while the summary of the findings could be found in tables 8 and 9.

Table 8. Results of the analyses of the existence of relationships between total return slacks in different years

	2011	2012	2013
2010	A positive but insignificant (α = 0.01) relationship	A positive but insignificant (α = 0.01) relationship	A negative but insignificant (α = 0.01) relationship
2011		A positive and significant (α = 0.01) relationship	A positive and significant (α = 0.01) relationship
2012			A positive and significant (α = 0.01) relationship

Table 9. Results of the analyses of the existence of relationships between total risk slacks in different years

	2011	2012	2013
2010	A positive but insignificant (α = 0.01) relationship	A positive but insignificant (α = 0.01) relationship	A positive but insignificant (α = 0.01) relationship
2011		A positive but insignificant (α = 0.01) relationship	A positive and significant (α = 0.01) relationship (except Spearman's rank correlation test and Kendall's tau test)
2012			A positive but insignificant (α = 0.01) relationship

As in the case of optimal solutions, it is rational to focus on results in consequent years, one of the reasons being that significant positive relationships between total slacks in consequent years could be a sign of efficiency persistence. Statistical tests reveal no significant (α = 0.01) relationships between total risk slacks in the periods 2010-2011, 2011-2012 and 2012-2013, while in the periods 2011-2012 and 2012-2013 significant (α = 0.01) positive relationships between total return slacks could be revealed. Therefore, the results of the total slack analysis confirm to some extent the main findings of the earlier executed persistence investigation, which shows that performance persistence does exist in the Slovenian mutual fund industry.

However, this positive relationship is not particularly strong and it disappears in certain time frames.

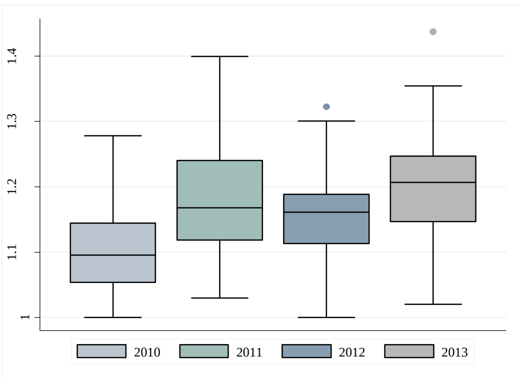
Optimal solutions and total slacks presented earlier in this chapter are results of a cross-sectional analysis. Assuming that in the period 2010-2013 the Slovenian mutual fund industry did not experience technology changes, it is possible to perform a panel DEA. The difference between the cross-sectional and panel DEA is that the former one estimates four efficiency frontiers (one for each examined year), while the latter one constructs only one efficiency frontier, pooling all data in one sample. It should be stressed that since there were no major innovations in regard to mutual fund management techniques in the period 2010-2013, this paper considers utilization of the panel DEA to be justifiable.

Descriptive statistics on obtained optimal solutions are presented in table 10, while boxplots are displayed in figure 13. As in the case of the cross-sectional DEA, average optimal solutions show by how much on average should DMUs proportionally increase outputs, leaving inputs unchanged at the same time, in order to become efficient. In this particular case, however, there is only one set of efficient DMUs and not four as in the case of cross-sectional analysis performed earlier in this chapter. The obtained results could be treated in the following manner: in 2010 the average mutual fund should have proportionally increased its outputs by on average 10.06% (calculated as optimal solution 1.1006 minus 1) in order to become efficient in the four-year period starting in 2010. It is understandable that the results in 2011, 2012 and 2013 are interpreted in a similar way.

Table 10. Descriptive statistics – panel output-oriented VRS model optimal solutions

Year	Number of observations	Average	Median	Standard deviation	Minimum	Maximum
2010	62	1,1006	1,0955	0,0647	1,0000	1,2782
2011	62	1,1814	1,1678	0,0843	1,0297	1,3995
2012	62	1,1533	1,1612	0,0691	1,0000	1,3224
2013	62	1,2029	1,2068	0,0788	1,0203	1,4372

Figure 13. Boxplots – panel output-oriented VRS model optimal solutions



Similarly to the case of the cross-sectional approach, an additional analysis is performed employing non-parametric tests (Wilcoxon-Mann-Whitney and Kolmogorov-Smirnov). According to the attained results, revealed in table 11, optimal solutions in 2010 are significantly ($\alpha = 0.01$) different from optimal solutions in 2011, 2012 and 2013, while optimal solutions in 2012 are significantly ($\alpha = 0.01$) different from optimal solutions in 2013. Taking into account the information presented in table 10 and figure 13, it is possible to draw a conclusion that in 2010 Slovenian mutual funds were operating more efficiently than in other years, and that in 2012 Slovenian mutual funds were operating more efficiently than in 2013.

Table 11. Panel output-oriented VRS model optimal solutions - results of the non-parametric tests

		Wilcoxon-Mann-Whitney test			Kolmogorov-Smirnov test		
		2011	2012	2013	2011	2012	2013
2010	test statistic	-5.228	-4.487	-6.592	0.4516	0.4297	0.5645
	p / exact p	*0.0000*	*0.0000*	*0.0000*	*0.000*	*0.000*	*0.000*
2011	test statistic		1.514	-1.559		0.2419	0.1935
	p / exact p		*0.1300*	*0.1190*		*0.053*	*0.186*
2012	test statistic			-3.543			0.3710
	p / exact p			*0.0004*			*0.000*

Note: information on exact p is provided in the case of Kolmogorov-Smirnov test

At this point, the difference between cross-sectional and panel DEA results should be stressed. In the former case results show by how much on average mutual funds should increase the outputs, leaving the inputs unchanged at the same time, in order to become efficient in a particular analyzed year. In other words, it is not possible to conclude that a mutual fund X in the year t is less efficient than a mutual fund Y in the year other than t, simply because the optimal solution of X is higher than the optimal solution of Y. In the case of the panel DEA it is possible to compare different mutual funds in different years and, what is probably even more important, to make conclusions regarding industry efficiencies in different time periods. Therefore, employing the previous example with mutual funds X and Y, if the optimal solution of X is higher than the optimal solution of Y, then the first DMU is less efficient than the second one. If the objective is to estimate the industry efficiency, it is possible to conclude that the industry of managing equity mutual funds is more efficient in the year other than t if compared to the year t in case the average optimal solution in the year t is higher than the average optimal solution in the year other than t.

As opposed to the results of the cross-sectional DEA, findings of the panel DEA are not analyzed from the total slack perspective. Additionally, efficiency persistence analysis is not executed. The rationale for this decision is that in the case of panel data, this paper considers information revealed by the total slack and relationship analysis to be less important and useful if compared to additional analysis of the cross-sectional DEA total slacks and optimal solutions.

Since this chapter is one of the most important parts of the paper, a short summary of the obtained results should be made. First, the obtained optimal solutions are the result of the cross-sectional and panel DEA, and do not take into account possible positive or negative impacts of uncontrollable factors. Second, cross-sectional optimal solutions show that in terms of efficiency the difference between the average mutual fund and the efficient mutual fund in 2013 was higher if compared to the situation in 2010 and 2012. Third, in the periods 2011-2012 and 2012-2013 there were positive, but not particularly strong relationships between last year’s optimal solutions and this year’s optimal solutions. However, there was no relationship between last year’s optimal solutions and this year’s optimal solutions in the period 2010-2011. These findings could be perceived as a sign of not very strong and time-varying efficiency persistence. Fourth, analysis of the cross-sectional DEA total slacks shows that the average total return slack in 2013 was higher if compared to the situation in the previous year and in 2010, while the average total risk slack in 2013 was higher than in the previous years and the average total risk slack in 2012 was lower than in 2011. Fifth, correlation and regression analyses of the cross-sectional DEA total slacks reveal that there were no relationships between total risk slacks in the periods 2010-2011, 2011-2012 and 2012-2013. Moreover, there were positive, but not particularly strong, relationships between total return slacks in the periods 2011-2012 and 2012-2013, but there was no relationship between total return slacks in the period 2010-2011. These findings could be seen as evidence of not very strong and time-varying efficiency persistence. Sixth, panel DEA shows that when different mutual funds in different years are directly compared to each other, it is possible to conclude that in 2010 the Slovenian mutual fund industry was more efficient than in other years and that in 2012 it was more efficient than in 2013.

6.3 Uncontrollable factors

As it has already been mentioned in one of the previous chapters, there are several ways to increase the informational value of DEA and to take into account the factors that are not under the control of mutual fund managers. The most complex and sophisticated approach is a three-stage DEA, where total slacks are analyzed in the process of SFA (total slacks are employed as dependent variables and uncontrollable factors are utilized as independent variables) and afterwards inputs or outputs used in DEA are adjusted to put all DMUs in the same environment, also in such regard as luck. On the other hand, it should be stressed that regression analysis of the efficiency scores obtained as a result of DEA is still the method that is most frequently applied in order to estimate the relationships between efficiency and uncontrollable factors.

In order to increase the possibility of obtaining informationally valuable results, this research employs the following algorithm to analyze effects of the uncontrollable factors:

1. Calculate optimal solutions and total slacks employing DEA.
2. Analyze total slacks using SFA.
3. If results obtained in the previous stage allow it, adjust outputs to put all mutual funds in the same environment and execute DEA again.
4. If SFA results are insignificant, investigate the relationships between efficiency and uncontrollable factors using different regression methods.

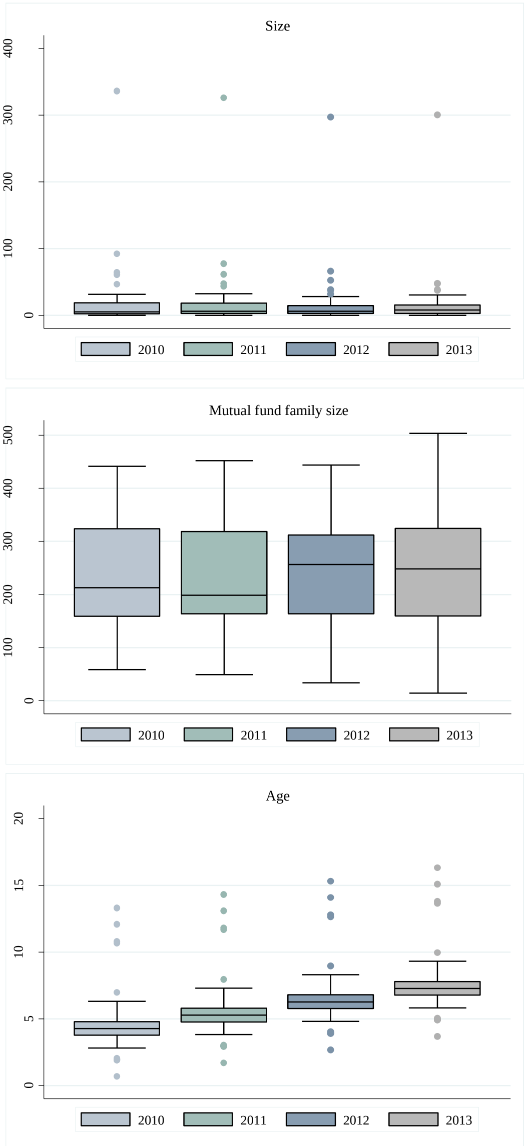
Among uncontrollable variables included in this research are mutual fund size and age as well as mutual fund family size. The mutual fund size and the mutual fund family size are measured as an average of monthly results, while the age is measured as an average of the age at the end of the year and the age at the beginning of the year. These variables have already been discussed earlier in this paper; however, it is not out of place to note that according to the literature review on the performance determinants in the mutual fund industry, there seems to be a negative relationship between size and performance as well as age and performance, while family size seems to be positively correlated with performance.

Descriptive statistics and boxplots are displayed in table 12 and figure 14.

Table 12. Descriptive statistics – uncontrollable factors

Variable	Year	Number of observations	Average	Median	Standard deviation	Minimum	Maximum
Size (million EUR)	2010	62	17.5526	5.2891	44.2124	0.2325	336.1341
	2011	62	17.1364	6.1401	42.3084	0.1664	325.9181
	2012	62	15.5021	6.3700	38.2661	0.1833	297.0844
	2013	62	16.2378	8.2073	38.2276	0.1949	300.4422
Mutual fund family size (million EUR)	2010	62	251.8415	212.6173	110.2769	58.5585	441.3440
	2011	62	244.0490	198.8477	117.2827	49.1085	451.6754
	2012	62	243.7097	256.5046	117.4679	33.8285	443.9469
	2013	62	251.7505	248.2918	132.1893	14.2728	503.2846
Age (years)	2010	62	4.5918	4.2836	2.2242	0.6918	13.3137
	2011	62	5.5918	5.2836	2.2242	1.6918	14.3137
	2012	62	6.5918	6.2836	2.2242	2.6918	15.3137
	2013	62	7.5918	7.2836	2.2242	3.6918	16.3137

Figure 14. Boxplots – uncontrollable factors



Note: Size and mutual fund family size are measured in million EUR, while age is measured in years

As it has already been described earlier in chapter 3.4, the relationships between total slacks and uncontrollable factors should be determined with the help of SFA in order to perform a three-stage DEA investigation. The key results of the SFA regressions are provided in table 13 while additional information could be found in the appendix (four tables from O to R).

Table 13. SFA regression results

			Total return slack	Total risk slack
2010	Size	coefficient	-0.0002564	-0.0001556
		p	*0.196*	*0.506*
	Family size	coefficient	0.0001752	0.0000998
		p	*0.011*	*0.222*
	Age	coefficient	0.0027311	0.0028275
		p	*0.476*	*0.532*
2011	Size	coefficient	0.0000199	0.000058
		p	*0.934*	*0.858*
	Family size	coefficient	-0.0001287	-0.0002035
		p	*0.089*	*0.047*
	Age	coefficient	-0.0006382	0.0030455
		p	*0.884*	*0.607*
2012	Size	coefficient	0.0000747	0.0000998
		p	*0.725*	*0.660*
	Family size	coefficient	-0.0001347	-0.0001557
		p	*0.027*	*0.017*
	Age	coefficient	0.0006903	0.0010551
		p	*0.844*	*0.778*
2013	Size	coefficient	0.000463	0.0004115
		p	*0.120*	*0.348*
	Family size	coefficient	-0.0001662	-0.0001064
		p	*0.028*	*0.360*
	Age	coefficient	-0.0055981	-0.0142588
		p	*0.246*	*0.049*

The obtained results reveal that the relationships between total slacks and such uncontrollable factors as mutual fund size and age as well as mutual fund family size were insignificant ($\alpha = 0.01$) in all years under review. To conclude, it is not possible to claim that in 2010, 2011, 2012 and 2013 the uncontrollable factors included in the analysis were associated with total slacks; therefore, it is not reasonable to utilize a three-stage DEA. Following the algorithm presented earlier, the next step is to execute regression analysis of optimal solutions.

The problem of choosing the appropriate regression model for the two-stage DEA has already been discussed earlier in this paper. This research employs three regression approaches; more specifically, OLS, tobit and SFA. The results of these analyses are displayed in table 14.

Executed regression analyses reveal that in 2010, 2011, 2012 and 2013 none of the included uncontrollable factors had a significant ($\alpha = 0.01$) relationship with the optimal solutions obtained in the process of DEA. Additionally, special attention should be paid to the fact that the results of the total slack analysis are similar to the findings of the regression analysis which employs optimal solutions as dependent variable. This is not surprising if taking into account that total slacks and optimal solutions are both efficiency indicators.

Table 14. OLS, tobit and SFA regression results

			OLS	Tobit	SFA
2010	Size	coefficient	-0.0002456	-0.0002567	-0.0002456
		p	*0.265*	*0.280*	*0.244*
	Family size	coefficient	0.0001815	0.000203	0.0001815
		p	*0.020*	*0.017*	*0.014*
	Age	coefficient	0.0026861	0.0025012	0.0026861
		p	*0.526*	*0.589*	*0.510*
2011	Size	coefficient	0.0000424	0.0000238	0.0000424
		p	*0.884*	*0.937*	*0.880*
	Family size	coefficient	-0.0001586	-0.0001539	-0.0001586
		p	*0.088*	*0.113*	*0.073*
	Age	coefficient	-0.0005208	0.0002277	-0.0005208
		p	*0.922*	*0.967*	*0.919*
2012	Size	coefficient	0.000095	0.0000922	0.000095
		p	*0.694*	*0.722*	*0.683*
	Family size	coefficient	-0.000154	-0.000154	-0.000154
		p	*0.030*	*0.043*	*0.021*
	Age	coefficient	0.0007384	0.0011132	0.0007384
		p	*0.853*	*0.795*	*0.848*
2013	Size	coefficient	0.000546	0.0006213	0.000546
		p	*0.125*	*0.102*	*0.108*
	Family size	coefficient	-0.0001904	-0.0002072	-0.0001904
		p	*0.037*	*0.032*	*0.027*
	Age	coefficient	-0.0060765	-0.0084561	-0.0060765
		p	*0.288*	*0.181*	*0.268*

Summarizing the results of the analysis of the relationships between mutual fund efficiency and certain uncontrollable factors, the conclusion could be made that the mutual fund size and age as well as the mutual fund family size seem to be irrelevant for the efficiency.

6.4 Discussion on results of the efficiency analysis

This chapter discusses the efficiency analysis executed earlier. The objective of this part of the research is to structure the achieved results and to formally answer the question whether or not the following research hypotheses could be supported:

1. There is no performance persistence among Slovenian equity mutual funds.
2. Equity mutual funds with higher amount of assets under management are less efficient.
3. Equity mutual funds from mutual fund families, i.e., mutual fund companies with higher amount of cumulative assets under management are more efficient.
4. Younger, i.e,. more recently established equity mutual funds are more efficient.

Although it has already been stressed, it is not out of place to note once again that all formulated research hypotheses are a result of thorough literature review, findings of which are revealed in chapters on mutual fund performance persistence and determinants of mutual fund success. The first research hypothesis deals with the performance persistence phenomenon, the existence or absence of which significantly influences the quality of the investment decisions. If past results are irrelevant for the purpose of ranking mutual funds, then DEA optimal solutions and slacks should not be employed to construct an optimal portfolio. The remaining research hypotheses consider the relationships between efficiency and factors, such as mutual fund size, mutual fund age and mutual fund family size. The reason why it is so important to understand whether these research hypotheses are supported is that when analyzing mutual fund efficiency it is rational to account for environmental effects.

Taking into account that this research is the first DEA investigation on mutual fund efficiency in the context of Slovenia, it is important to stress that the fact that such analysis has actually been executed in practice is worth the attention of investors and researchers. The existence of this master's thesis proves that it is possible to collect data, construct a model and define efficiencies of Slovenian mutual funds. It is true, however, that DEA utilization could become significantly less time-consuming and less complex to adopt. The main constraint is the availability of data, more specifically, the data on benchmark characteristics, investment style and manager characteristics. If mutual fund companies decided to become more transparent or if the regulator obliged mutual fund companies to publish more information, efficiency analysis would become substantially less tedious. If DEA was implemented in a less complex way, mutual fund managers, investors and academics might be persuaded to consider implementing this performance evaluation approach as an alternative or at least as an addition to traditional measures. It is also understandable that higher quality and amount of accessible information could potentially result in higher relevance of constructed DEA models, i.e., the inclusion of certain other inputs and outputs apart from total expense ratio, relative risk and relative return.

Existing and potential investors as well as academics interested in evaluating mutual fund efficiency should be aware of the fact that if the data is available for more than one time period, it is possible to execute both cross-sectional and panel DEA. An important criterion that should be met, however, is the condition of constant technology in the case of the panel efficiency estimation. If technology shifts are present, the obtained results could be biased and could lead to incorrect conclusions. The assumption related to constant technology is not particularly severe in the case of this efficiency investigation, taking into account that this research includes in the panel DEA a relatively short time frame in which the Slovenian mutual fund industry did not experience any significant changes in the portfolio management procedures, such as the implementation of computers two decades ago or start of the widespread usage of Bloomberg terminals more than a decade ago.

The cross-sectional DEA investigation reveals that the difference between the average mutual fund and the efficient fund was higher in 2013 if compared to the situation in 2010 and 2012. Looking at the obtained optimal solutions, it could also be concluded that the average mutual fund is constantly producing outputs which are materially lower if compared to the efficient peers. This also means that if the results achieved by best mutual funds are not the result of luck, owners of the average mutual fund shares have an objective reason to demand 1) better results in terms of relative risk and relative return and/or 2) lower TER.

An additional perspective on efficiency could be obtained with the help of total slack analysis, which shows that the average total return slack in 2013 was higher than the average total return slacks in 2010 and 2012, while the average total risk slack in 2013 was higher than the average total risk slacks in the previous years and the average total risk slack in 2012 was lower than the average total risk slack in 2011.

As it has already been mentioned earlier, researchers who investigate efficiency by employing both, optimal solutions and total slacks, should be careful when making conclusions since optimal solutions are given in an enlargement factor form and total slacks are absolute numbers. However, total slacks are useful in the sense that they actually show by how much exactly the analyzed DMUs should increase outputs in specific years in order to become strongly efficient.

Looking at the results of panel DEA, the findings of this analysis could be perceived as an answer to the question regarding the industry-level efficiency. Employing the data on Slovenian mutual funds in the period 2010-2013, it is possible to draw a conclusion that in 2010 Slovenian mutual funds were operating more efficiently than in other years and that in 2012 Slovenian mutual funds were operating more efficiently than in 2013. The results of panel DEA could become even more useful if further analyses are executed. One example of such analysis could be the investigation on the relationship between efficiencies and mutual fund inflows in different years. Of a certain interest could also be identification of the connection between perceived and achieved efficiency levels.

Correlation and regression analyses of optimal solutions and total slacks obtained in the process of the cross-sectional DEA investigation on the efficiency in the Slovenian mutual fund industry reveal that there are, in fact, signs of efficiency persistence, which is not particularly strong, however, and disappears in certain time periods. In the case of optimal solutions and total return slacks, there was a positive relationship in the periods 2011-2012 and 2012-2013, while in the case of total risk slacks, there was no relationship between total risk slacks in the periods 2010-2011, 2011-2012 and 2012-2013. However, due to the fact that there is no evidence that efficiency in the year t is connected with efficiency in the year t-1 in the period 2010-2011, the results of the performance persistence analysis are rather inconclusive. It is not possible to claim that the first formulated research hypothesis is supported; however, it is important to keep in mind that in certain time periods there could be no relationship between past and future performance. Investors, mutual fund managers and academics interested in mutual fund analysis should therefore be very careful when forecasting future efficiency on the basis of historical performance. They could also decide to completely exclude past results from the decision making process.

Since traditional DEA does not account for environmental variables, this research tries to employ a three-stage DEA approach and when SFA reveals insignificant coefficients, a simple two-stage DEA method is utilized. Three regression models used are OLS, tobit and SFA, and all of them show that mutual fund efficiency is not correlated with the size, age or family size of mutual funds. These findings prove that research hypotheses number two, three and four are not supported, which demonstrates that at least in Slovenia mutual fund efficiency is not associated with such factors as the size, age and family size of mutual funds. The main limitation of the two-stage DEA employed in this research stems from a relatively low level of transparency in the Slovenian mutual fund industry. It is quite possible that other uncontrollable variables are connected with the efficiency of Slovenian mutual funds. Therefore, the most logical way to increase the informational value of DEA is to require that mutual fund companies and the regulator publish information on certain other characteristics of the mutual fund management process, for example manager characteristics. This could notably increase the utility of DEA and help gain deeper knowledge on the processes that make some mutual funds efficient and others not.

Main conclusions drawn in this chapter are therefore as follows:

- It is possible to execute a DEA study on efficiency in the context of Slovenia.
- The DEA investigation of mutual fund efficiency could become deeper in the informational sense if mutual fund companies and the regulator decide to publish more information on benchmarks, investment style and manager characteristics.
- There are signs of time-varying and not very strong efficiency persistence in the Slovenian mutual fund industry. However, the results of performance persistence analysis are rather inconclusive.
- The analysis of the relationships between the mutual fund size, the mutual fund age, the mutual fund family size and mutual fund efficiency reveals that these relationships are not significant.

CONCLUSION

Mutual funds are one of the most convenient ways to fulfil financial goals of individual and institutional investors. Therefore, it is not surprising that mutual funds possess massive assets under management, more specifically, approximately 30 trillion US dollars as of the end of the year 2013 (Investment Company Institute, 2014).

Looking at the share of net assets in the GDP, the Slovenian mutual fund industry is among the least developed in the European Union. Nevertheless, the number of mutual funds managed by Slovenian mutual fund providers is relatively high; therefore, prospective as well as existing mutual fund investors are exposed to a dilemma of how to choose the most appropriate mutual funds.

This master's thesis investigates performance of Slovenian equity mutual funds. The topic of mutual fund performance is important for academics who analyze the situation in the industry, investors who are interested in constructing the optimal portfolio as well as portfolio managers and their supervisors who struggle to increase competitiveness of their products and services. Taking into account the importance of the issue, it is not strange that multiple approaches to measuring portfolio management performance have been developed. However, currently used traditional performance measures do not include certain characteristics of the mutual fund management process and are thus not flexible enough to be suitable for all situations, i.e., researchers' objectives. Additionally, at least some of the performance indicators are exposed to the problem of the following nature: it is exceptionally difficult to construe the results when mutual fund return is lower than the benchmark return.

Due to the previously mentioned disadvantages of classical approaches to ranking mutual funds, this paper proposes to employ an alternative method, called data envelopment analysis, also abbreviated as DEA, which allows researchers to estimate how successful mutual funds are in achieving objectives by taking into consideration the amount of resources utilized. The DEA method has multiple advantages, and if used appropriately, it could become a powerful tool for measuring mutual fund performance. The DEA investigation on Slovenian mutual fund efficiency has not been executed yet, and the fact that this master thesis exists could serve as a proof that it is possible to collect data, develop a model and perform DEA calculations in the context of Slovenia. Undoubtedly, further analyses are required; however, it is possible to assume that results described in this paper would be useful for future researches.

The investigation on Slovenian equity mutual fund efficiency executed in this research has several parts. At the beginning, the cross-sectional and panel DEA are executed by employing the total expense ratio as the only input and relative return and relative risk as the two outputs. Cross-sectional DEA results in four sets of optimal solutions, one for every year under investigation, while the panel DEA is utilized for the purpose of analyzing industry efficiency in different years and results in just one set of optimal solutions, in which every mutual fund appears four times.

Cross-sectional efficiency analysis reveals that the difference between the average mutual fund and the efficient fund was higher in 2013 if compared to the situation in 2010 and 2012. The obtained optimal solutions also show that the average equity mutual fund operates notably worse than possible. If the results achieved by best mutual funds are not a result of luck, investors of average Slovenian equity mutual fund have objective reasons to require 1) higher relative return and lower relative risk and/or 2) lower TER.

Total slack analysis, which focuses on the differences between strongly efficient and achieved amounts of outputs, sheds additional light on mutual fund performance; more specifically, it reveals that the average total return slack in 2013 was higher than the average total return slacks in 2010 and 2012, while the average total risk slack in 2013 was higher than the average total risk slacks in the previous years. Moreover, the average total risk slack in 2012 was lower than the average total risk slack in 2011.

The cross-sectional DEA and additional analyses also shed light on one of the cornerstone questions puzzling mutual fund investors and academics. The results of this research reveal that in the Slovenian mutual fund industry performance persistence does exist; however, it is not particularly strong and time-varying. In other words, in certain time periods past results could be exploited for the purpose of choosing mutual funds, while in other time periods past performance could turn out to be an insignificant indicator of future results. The findings are therefore inconclusive and although it is not possible to support the research hypothesis that there is no performance persistence in the Slovenian mutual fund industry, the information on past performance should be considered very carefully in the decision making process.

Returning back to the elements of this efficiency research, it is worth noting that panel DEA shows that in the period 2010-2013 the highest efficiency in the Slovenian mutual fund industry was achieved in 2010. Additionally, panel DEA reveals that in 2012 Slovenian mutual funds were operating more efficiently than in 2013. The usefulness of the obtained results of the panel DEA depend on the condition whether the assumption of absence of technology shifts in the period 2010-2013 holds true. However, due to the fact that there were no game-changing innovations in the analyzed period, this research considers the results of panel DEA trustworthy.

Without additional research it is not possible to decide whether achieved results are the result of the activities of mutual fund managers or whether they are connected with environmental variables and luck. For this reason, this paper investigates the relationships between efficiency and certain uncontrollable factors; more specifically, the size, age and company size of mutual funds. Due to the fact that the effects of these uncontrollable factors on total slacks are found to be insignificant, it turns out to be impossible to execute the three-stage DEA procedure, which employs SFA and allows the calculation of efficiencies which account for environmental effects and luck. Further analysis utilizing different regression methods reveals that the size, age and family size of mutual funds are not associated with DEA optimal solutions. Taking into account that optimal solutions and total slacks are both efficiency indicators, it is not surprising that the SFA investigation on slacks and regression analysis of optimal solutions yield similar results.

Findings of the DEA presented in this master's thesis are important; however, further investigation is undoubtedly needed. Future research could shed more light on the relationships between efficiency and environmental factors not included in the analysis presented in this paper. Special attention could be devoted to the inclusion of certain other variables in DEA models. Among information which could increase the relevance of the DEA investigation is information on benchmarks, investment style and manager characteristics.

It is unlikely that DEA would completely substitute traditional performance measures; however, it could definitely become one of the most powerful and widely used tools for assessing the efficiency of mutual fund management.

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APPENDICES

TABLE OF APPENDICES

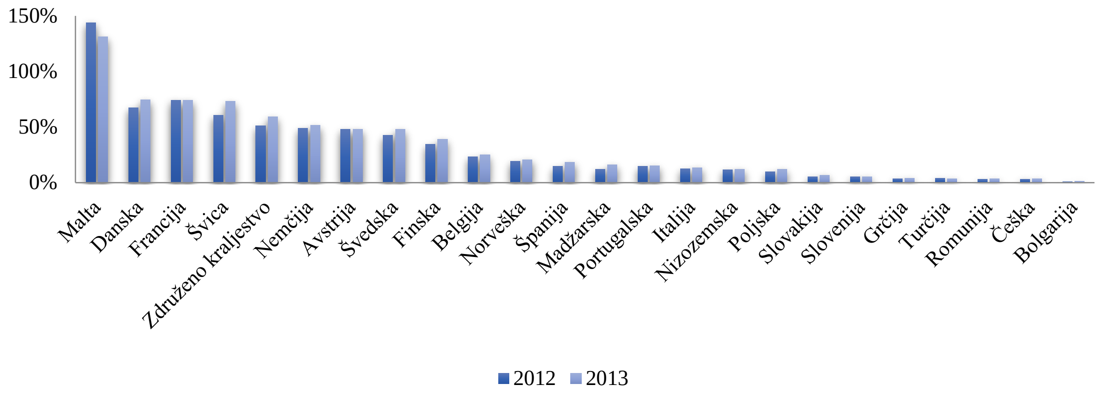
Appendix A. Summary in Slovene language (povzetek v slovenskem jeziku)

Vzajemni skladi so eden ključnih elementov globalnega finančnega sistema. Njihova pomembnost izhaja iz tega, da zmanjšujejo negativni vpliv tržnih neučinkovitosti na odnose med suficitarnimi in deficitarnimi enotami, pri čemer se kot vmesni člen pojavljajo družbe za upravljanje oziroma drugi upravljavci finančnih sredstev.

Ni torej presenetljivo, da se številni vlagatelji, tako individualni kot tudi institucionalni, za doseganje svojih finančnih ciljev odločajo za investiranje v vzajemne sklade. Podatki Investment Company Institute (2014) kažejo, da je ob koncu leta 2013 na svetu obstajalo več kot 73 tisoč vzajemnih skladov, skupna vrednost sredstev v upravljanju pa je znašala približno 30 bilijonov ameriških dolarjev.

Čeprav je industrija upravljanja vzajemnih skladov v Sloveniji manj razvita v primerjavi z razvitimi državami, kar na primer potrjujejo podatki o velikosti industrije upravljanja vzajemnih skladov glede na BDP v evropskih državah, imajo slovenski vlagatelji na izbiro približno dvesto vzajemnih skladov v upravljanju domačih in tujih družb za upravljanje.

Slika 1. Velikosti industrije upravljanja vzajemnih skladov glede na BDP v evropskih državah

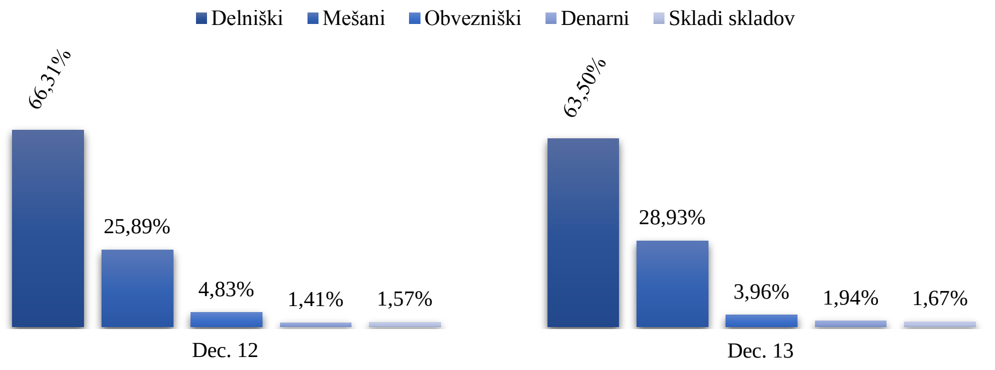


Vir: European fund and asset management association, *Quarterly statistical reports*, 2014; Eurostat, *GDP and main components - Current prices*, 2014.

Cilj magistrske naloge je oceniti učinkovitost delniških vzajemnih skladov v Sloveniji v upravljanju domačih družb za upravljanje, in preučiti, kateri dejavniki so povezani z učinkovitostjo. Učinkovitost v primeru vzajemnih skladov se nanaša na raven uspešnosti pri doseganju rezultatov ob upoštevanju dostopnih virov in uporabljene tehnologije. V analizo so vključeni samo delniški vzajemni skladi, predvsem zaradi tega, ker je ta vrsta vzajemnih skladov v Sloveniji daleč najpomembnejša, kar potrjujejo tudi podatki predstavljeni na sliki 2.

Rezultati analize učinkovitosti vzajemnih skladov so lahko pomembni za vlagatelje, družbe za upravljanje, ter podjetja, ki ponujajo storitve finančnega svetovanja. Ocenjene stopnje uspešnosti so lahko alternativa klasičnim kazalnikom uspešnosti, med katerimi so na primer kazalnik Sharpe, kazalnik Treynor, kazalnik Sortino. Ti včasih niso dovolj fleksibilni, da bi omogočali raziskovalcem doseganje zastavljenih ciljev, ko vzajemni sklad dosega slabše rezultate kot kriterijski indeks pa so nekateri celo neuporabni.

Slika 2. Struktura industrije upravljanja vzajemnih skladov v Sloveniji na dan 31. december 2012 in na dan 31. december 2013



Vir : Securities market agency, *Composition of the assets of equity mutual funds, net contribution and No. of subscribers*, 2014b; Securities market agency, *Composition of the assets of mixed mutual funds, net contribution and No. of subscribers*, 2014c; Securities market agency, *Composition of the assets of money-market mutual funds, net contribution and No. of subscribers*, 2014d; Securities market agency, *Composition of the assets of mutual funds-funds of funds, net contribution and No. of subscribers*, 2014e; Securities market agency, *Composition of the assets of bond mutual funds, net contribution and No. of subscribers*, 2014f.

Za namen analize učinkovitosti vzajemnih skladov se v raziskavi uporablja neparametrična metoda, ki omogoča upoštevanje večjega števila vložkov (angl. *input*) in izložkov (angl. *output*), in ki se imenuje metoda ovojnice podatkov (angl. *data envelopment analysis*) (v nadaljevanju DEA).

V začetni fazi analize učinkovitosti so predstavljeni nekateri kazalniki uspešnosti vzajemnih skladov in metodologija merjenja učinkovitosti s pomočjo DEA pristopa. Sledi pregled literature s področja vztrajnosti uspešnosti oziroma povezav med preteklimi in prihodnjimi rezultati poslovanja vzajemnih skladov, dejavnikov, ki vplivajo na rezultate poslovanja vzajemnih skladov, in uporabe neparametrične metode DEA za namen ocenjevanja učinkovitosti vzajemnih skladov.

Rezultat pregleda relevantne literature so štiri raziskovalne hipoteze:

1. Vztrajnost uspešnosti ni značilna za slovenske delniške vzajemne sklade, kar pomeni, da pretekla donosnost ni pozitivno korelirana s prihodnjo.
2. Večji vzajemni skladi oziroma skladi, ki imajo več sredstev v upravljanju, so manj učinkoviti kot manjši vzajemni skladi oziroma skladi, ki imajo manjši obseg sredstev v upravljanju.
3. Vzajemni skladi večjih družb za upravljanje so bolj učinkoviti kot vzajemni skladi iz manjših družb za upravljanje.
4. Mlajši vzajemni skladi so bolj učinkoviti kot starejši vzajemni skladi.

Ker se raziskava osredotoča na analizo učinkovitosti s pomočjo DEA metode, sta v nadaljevanju definirana pojma učinkovitost in DEA pristop. Koopmans (1951) piše, da tehnična učinkovitost predpostavlja, da je za doseganje višje vrednosti kateregakoli izložka potrebno zmanjšanje vrednosti kateregakoli drugega izložka, ali pa zvišanje vrednosti vsaj enega vložka. Podobno je za znižanje vrednosti kateregakoli vložka potrebno zvišanje vrednosti kateregakoli drugega vložka, ali pa znižanje vrednosti vsaj enega izložka.

Analiza učinkovitosti je lahko orientirana k zviševanju izložkov ali k zniževanju vložkov (Lovell, 1993). DEA metoda je eden izmed načinov merjenja učinkovitosti, ki temelji na rezultatih raziskav Farrella (1957) ter Charnesa et al. (1978). Ramanathan (2003) definira DEA metodo kot pristop za merjenje učinkovitosti organizacijskih enot, imenovanih tudi DMU (angl. *decision making unit*), ki temelji na linearnem programiranju.

Matematična predstavitev logike DEA je naslednja (Ramanathan, 2003):

Predpostavimo, da x predstavlja izložke, kjer $i = 1, 2, ..., n$ definira posamezne vložke (x_1, x_2 itd) in y predstavlja izložke, kjer $j = 1, 2, ..., m$ definira posamezne izložke (y_1, y_2 itd). Naj I in J predstavljata celotno število vložkov oziroma izložkov, pri čemer sta oba večja od 0. DEA metoda linearno agregira 1) večje število vložkov in generira virtualni vložek $\sum_{i=1}^{I} u_i x_i$, kjer je u_i utež vložka x_i in $u_i \geq 0$, 2) večje število izložkov in generira virtualni izložek $\sum_{j=1}^{J} v_j y_j$, kjer je v_j utež izložka y_j in $v_j \geq 0$. Učinkovitost se izračunava s pomočjo formule $\frac{\sum_{j=1}^{J} v_j y_j}{\sum_{i=1}^{I} u_i x_i}$.

Razumljivo je, da je eden izmed ključnih elementov DEA definiranje uteži posameznih vložkov in izložkov. DEA za ta namen uporablja linearno programiranje in določa uteži za vsako analizirano proizvodno enoto tako, da če so enake uteži uporabljene v primeru katerekoli druge proizvodne enote, je izračunana učinkovitost le-te med 0 in 1.

Obstaja več različnih modelov DEA, ključna pa sta CCR, ki so ga predstavili Charnes et al. (1978), in BCC, ki so ga razvili Banker et al. (1984). CCR DEA predpostavlja konstantne donose obsega, medtem ko BCC DEA omogoča ocenjevanje učinkovitosti tudi v primeru variabilnih donosov obsega. Smiselno je tudi ločevati med modelom, ki je orientiran k vložkom, in modelom, ki je orientiran k izložkom. Prvi model se uporablja v primerih, ko ima DMU vpliv na izložke in lahko generira različne obsege le-teh v odvisnosti od stopnje učinkovitosti, medtem ko je drugi model koristen takrat, ko lahko DMU dosega višjo ali nižjo učinkovitost s tem, da uporablja različne obsege vložkov. Izbira enega ali drugega modela je v veliki meri odvisna od značilnosti proizvodnega procesa, ki se uporablja v DMU, in od ciljev raziskave.

Ključne prednosti DEA so (Cooper et al., 2007; Murthi et al., 1997; Ramanathan, 2003):

- DEA pomaga definirati vire in obseg neučinkovitosti na ravni vložkov, izložkov in celotnih DMU.
- DEA omogoča definiranje najbolj uspešnih DMU, ki so lahko uporabljeni za namen primerjave.
- DEA ne temelji na subjektivnih ocenah raziskovalcev in uporablja numerične podatke.
- DEA omogoča analizo učinkovitosti DMU, ki operirajo v okolju z več vložki in izložki, pri čemer metoda ne zahteva, da so vsi vložki in izložki v istih enotah.
- DEA ne zahteva predhodne določitve povezave med vložki in izložki.
- Rezultat DEA so ocene učinkovitosti, kjer ima DMU svojo oceno.

Potrebno se je zavedati, da ima DEA tudi določene pomanjkljivosti (Coelli et al., 2005; Ramanathan, 2003), ki pa so bodisi značilne tudi za druge načine merjenja uspešnosti bodisi nastopajo v vlogi priporočil, ob upoštevanju katerih se lahko raziskovalci izognejo netočnim in pristranskim rezultatom.

- Oblika in položaj meje proizvodnih možnosti je lahko pod vplivom napak merjenja.
- Na rezultate DEA lahko vplivajo izstopajoči DMU.
- Rezultati DEA so lahko netočni, če v modelu ni pomembnega vložka ali izložka.
- Vključitev novega DMU v model lahko povzroči padec ocen učinkovitosti.
- Medsebojno primerjanje povprečnih ocen učinkovitosti dveh vzorcev ni mogoče.
- Vključitev novega DMU v model ne more povzročiti rast ocen učinkovitosti.
- Vključitev novega vložka ali izložka v model ne more povzročiti padec ocen učinkovitosti.
- V primeru manjšega števila DMU in večjega števila vložkov in/ali izložkov je lahko več DMU definiranih kot učinkovitih.
- Rezultati DEA so lahko netočni, če so heterogeni vložki in/ali izložki vključeni v model kot homogeni.
- Neupoštevanje razlik med okolji, v katerih operirajo DMU, lahko povzroči napačne sklepe.
- DEA ne upošteva učinke dolgoročnega optimiziranja in tveganje.
- Ocenjevanje učinkovitosti v primeru večjega števila DMU je lahko prezahtevno z vidika obsega potrebnih izračunov.
- Testiranje statističnih hipotez je lahko oteženo.
- Uporaba DEA zahteva vključitev vsaj enega vložka in vsaj enega izložka.
- DEA ne dovoli, da raziskovalci direktno vplivajo na uteži posameznih vložkov in izložkov.
- Rezultati DEA so včasih nepričakovani oziroma celo nasprotujejo pričakovanjem.
- DMU lahko manipulirajo z ocenami uspešnosti tako, da se osredotočajo na izboljšanje omejenega števila vložkov ali izložkov.

Pregled rezultatov preteklih raziskav na področju praktične uporabe DEA kaže, da obstaja več različnih pristopov k analiziranju povezav med učinkovitostjo in spremenljivkami, na katere DMU nima vpliva. Prvi način je uporaba separacijskega modela, pri katerem je prvotni vzorec stratificiran na podlagi določenih kriterijev in šele nato je izvedeno več DEA ocenjevanj. Drugi pristop je dvostopenjska analiza učinkovitosti, pri kateri se najprej uporablja DEA, nato pa se izvaja regresijska analiza pridobljenih ocen učinkovitosti. Tretji način je tristopenjska analiza, pri kateri se v prvi fazi izvaja DEA, v drugi fazi se s pomočjo stohastične analize mejne funkcije (angl. *stochastic frontier analysis*) (v nadaljevanju SFA) ocenjuje vpliv neučinkovitosti, zunanjih spremenljivk in statističnega šuma, v zadnji fazi pa se ponavlja DEA, vendar tokrat s podatki, popravljenimi za vpliv zunanjih spremenljivk in statističnega šuma. Četrti način se imenuje štiristopenjska DEA metoda, pri kateri se najprej izvaja DEA ocenjevanje, nato se s pomočjo tobit regresije ocenjuje vpliv zunanjih dejavnikov, v tretji fazi se začetni vložki/izložki popravljajo za vpliv zunanjih spremenljivk, v zadnji fazi pa se izvaja DEA s popravljenimi podatki.

Analiza učinkovitosti delniških vzajemnih skladov v Sloveniji je sestavljena iz naslednjih elementov:

- Presečno DEA ocenjevanje učinkovitosti 62 delniških vzajemnih skladov v Sloveniji v letih 2010-2013 in dodatne analize pridobljenih rezultatov.

- Panelno DEA ocenjevanje učinkovitosti 62 delniških vzajemnih skladov v Sloveniji v letih 2010-2013 in dodatne analize pridobljenih rezultatov.
- Analiza povezanosti med učinkovitostjo in dejavniki, na katere vzajemni skladi nimajo vpliva.

DEA model, uporabljen za namen ocenjevanja učinkovitosti delniških vzajemnih skladov, vključuje en vložek in dva izložka. Edini vložek je kazalnik celotnih stroškov poslovanja vzajemnega sklada (kazalnik CSP), dva izložka pa sta relativni letni donos in relativno tveganje, kjer je relativni donos merjen kot vrednost denarne enote investirane v vzajemni sklad na koncu leta deljene z vrednostjo denarne enote investirane v kriterijski indeks na koncu leta. Relativno tveganje pa predstavlja količnik med standardnim odklonom dnevnih donosnosti kriterijskega indeksa in standardnim odklonom dnevnih donosnosti vzajemnega sklada. Pomembno je izpostaviti, da je v primeru vzajemnih skladov donosnost popravljena za vpliv celotnih stroškov poslovanja, medtem ko je v primeru kriterijskih indeksov uporabljen donos ob predpostavki reinvestiranja prejetih dividend. Uporabljen model predpostavlja, da vzajemni sklad za doseganje dveh ciljev - maksimiranje relativnega donosa vzajemnega sklada glede na donos kriterijskega indeksa in minimiziranje relativnega tveganja vzajemnega sklada glede na tveganje kriterijskega indeksa - uporablja določene vire, merjene s pomočjo CSP. Ker CCR DEA in BCC DEA predpostavljata, da večja vrednost vložka pomeni višjo učinkovitost, se relativno tveganje uporablja v prej predstavljeni obliki. Maksimiranje vrednosti tega kazalnika pomeni minimiziranje relativnega tveganja vzajemnega sklada glede na tveganje kriterijskega indeksa.

Uporabljen DEA model predpostavlja variabilne donose obsega (v veliki meri je ta model uporabljen zato, ker omogoča uporabo vložkov in izložkov, ki so v obliki razmerij (Hollingsworth & Smith, 2003)), ter je orientiran k izložkom (ta orientacija je izbrana zato, ker sta primarna cilja poslovanja vzajemnega sklada maksimiranje donosa in minimiziranje tveganja.

Rezultati presečnega DEA ocenjevanja so predstavljeni v tabeli 1.

Tabela 1. Rezultati analize učinkovitosti s pomočjo presečne DEA metode

Leto	Število opazovanj	Povprečje	Mediana	Standardni odklon	Minimum	Maksimum
2010	62	1.0975	1.0937	0.0648	1.0000	1.2782
2011	62	1.1167	1.1020	0.0802	1.0000	1.3476
2012	62	1.0863	1.0831	0.0615	1.0000	1.2598
2013	62	1.1457	1.1562	0.0871	1.0000	1.3745

Pridobljeni rezultati niso dodatno transformirani z namenom, da se dobi vrednost od 0 do 1, ki je hkrati ocena učinkovitosti. Podatke predstavljene v tabeli 1 je mogoče razumeti na naslednji način: v letu 2010 je povprečni delniški vzajemni sklad proizvedel 8,88% (1 - 1/1,0975) proporcionalno manj izložkov kot bi lahko oziroma 91,12% učinkovitega obsega izložkov. Učinkovit bi postal, če bi proizvedel za 9,75% (1,0975 - 1) proporcionalno več izložkov.

S pomočjo neparametričnih statističnih testov (Wilcoxon-Mann-Whitney in Kolmogorov-Smirnov) je ugotovljeno, da je bil povprečni rezultat DEA analize podatkov v letu 2013 statistično značilno ($\alpha = 0.01$) višji kot v letih 2010 in 2012. Na podlagi teh rezultatov je mogoče sklepati, da bi moral v letu 2013 povprečni delniški vzajemni sklad z namenom, da postane učinkovit, proporcionalno zvišati obseg učinkov za več odstotkov kot v letih 2010 in 2012. Pomembno pa je izpostaviti, da rezultati presečnega DEA ocenjevanja ne dajejo podlage za sklepanja, da so bili vzajemni skladi v enem ali drugem letu bolj učinkoviti, saj vzajemni skladi v posameznih letih niso direktno primerjani med seboj.

Korelacijske in regresijske analize rezultatov presečnega DEA kažejo, da je v obdobjih 2011-2012 in 2012-2013 obstajala statistično značilna ($\alpha = 0.01$) in pozitivna povezava med učinkovitostjo v letu t in učinkovitostjo v letu t-1, medtem ko v obdobju 2010-2011 statistično značilne ($\alpha = 0.01$) povezave med učinkovitostjo v letu t in učinkovitostjo v letu t-1 ni bilo.

Eden izmed rezultatov DEA so tudi podatki o tako imenovanih celotnih mrtvih izložkih, ki kažejo razliko med močno učinkovitim obsegom izložkov in doseženim obsegom izložkov. Rezultati dodatnih analiz celotnih mrtvih izložkov (neparametrična pristopa Wilcoxon-Mann-Whitney and Kolmogorov-Smirnov) kažejo, da do statistično značilnih ($\alpha = 0.01$) razlik prihaja v naslednjih primerih: 1) povprečni celotni mrtvi izložek na področju relativnega donosa je bil v letu 2013 višji kot v letih 2010 in 2012, 2) povprečni celotni mrtvi izložek na področju relativnega tveganja je bil v letu 2013 višji kot v letu 2012, 3) povprečni celotni mrtvi izložek na področju relativnega tveganja je bil v letu 2011 višji kot v letu 2012.

Korelacijske in regresijske analize celotnih mrtvih izložkov kažejo, da 1) v primeru celotnih mrtvih izložkov na področju relativnega tveganja statistično značilnih ($\alpha = 0.01$) povezav med rezultati v letu t in rezultati v letu t-1 v obdobjih 2010-2011, 2011-2012 in 2012-2013 ni bilo, 2) v primeru celotnih mrtvih izložkov na področju relativnega donosa je obstajala statistično značilna ($\alpha = 0.01$) pozitivna povezava med rezultati v letu t in rezultati v letu t-1 v obdobjih 2011-2012 in 2012-2013, medtem ko v obdobju 2010-2011 statistično značilne ($\alpha = 0.01$) povezave med rezultati v letu t in rezultati v letu t-1 ni bilo.

Če podatke o CSP, relativnem donosu in relativnem tveganju v posameznih letih združimo v en vzorec, lahko izvedemo panelno DEA ocenjevanje učinkovitosti. Treba pa je izpostaviti, da so rezultati podobne analize korektni le v primeru, če v analiziranem obdobju ni večjih sprememb v tehnologiji transformacije vložkov v izložke. Rezultati panelne analize so predstavljeni v tabeli 2.

Tabela 2: Rezultati analize učinkovitosti s pomočjo panelne DEA metode

Leto	Število opazovanj	Povprečje	Mediana	Standardni odklon	Minimum	Maksimum
2010	62	1,1006	1,0955	0,0647	1,0000	1,2782
2011	62	1,1814	1,1678	0,0843	1,0297	1,3995
2012	62	1,1533	1,1612	0,0691	1,0000	1,3224
2013	62	1,2029	1,2068	0,0788	1,0203	1,4372

Kot je razvidno iz tabele 2, bi moral v letu 2010 povprečni delniški vzajemni sklad v Sloveniji proporcionalno zvišati izložke v povprečju za 10,06% (1,1006 - 1), da bi bil učinkovit v obdobju 2010-2013.

Neparametrična analiza rezultatov panelnega DEA ocenjevanja razkriva, da so v letu 2010 delniški vzajemni skladi v Sloveniji poslovali statistično značilno ($\alpha = 0.01$) bolj učinkovito kot v ostalih analiziranih letih in da je bila povprečna učinkovitost v letu 2012 statistično značilno ($\alpha = 0.01$) višja kot v letu 2013.

V zadnji fazi raziskave so analizirane povezave med učinkovitostjo in različnimi dejavniki, ki niso pod vplivom finančnih strokovnjakov, ki upravljajo s sredstvi analiziranih delniških vzajemnih skladov. Pri izbiri teh dejavnikov so bili upoštevani rezultati preteklih raziskav na področju definiranja dejavnikov uspešnosti poslovanja vzajemnih skladov, ključen vpliv pa je imelo tudi dejstvo, da je obseg dostopnih podatkov o poslovanju delniških vzajemnih skladov v Sloveniji in okolju, v katerem le-ti poslujejo, relativno omejen. Izbrane spremenljivke so tako velikost sklada, velikost družbe za upravljanje in starost sklada.

Pri analizi je upoštevan naslednji algoritem:

1. Izvedba DEA ocenjevanja učinkovitosti delniških vzajemnih skladov.
2. Analiza celotnih mrtvih izložkov s pomočjo SFA metode.
3. Če rezultati SFA to omogočajo, izvedba prilagoditve prvotnih izložkov tako, da se upoštevajo razlike med poslovnimi okolji, in ponovna izvedba DEA ocenjevanja učinkovitosti delniških vzajemnih skladov.
4. Če so rezultati SFA neznačilni, izvedba analize povezav med učinkovitostjo in zunanjimi spremenljivkami s pomočjo različnih regresijskih pristopov.

Ker so v analiziranih letih povezave med celotnimi mrtvimi izložki in dejavniki, kot so velikost vzajemnih skladov, velikost družb za upravljanje in starost vzajemnih skladov, ocenjene s pomočjo SFA pristopa, statistično neznačilne ($\alpha = 0.01$), je dejanska analiza povezav med učinkovitostjo in zunanjimi dejavniki izvedena s pomočjo regresijskih modelov, kjer kot odvisna spremenljivka nastopa rezultat DEA. Trije regresijski pristopi (OLS, tobit in SFA) kažejo, da so bile v letih 2010, 2011, 2012 in 2013 povezave med učinkovitostjo in velikostjo vzajemnih skladov, velikostjo družb za upravljanje ter starostjo vzajemnih skladov statistično neznačilne ($\alpha = 0.01$).

Na podlagi izvedenih analiz je mogoče sklepati, da:

- V Sloveniji obstajajo znaki vztrajnosti uspešnosti, ki pa ni pretirano močna, in v določenih obdobjih izginja. Čeprav ni mogoče trditi, da je prva raziskovalna hipoteza podprta, se je potrebno zavedati, da v določenih primerih povezava med preteklo in bodočo uspešnostjo ne obstaja.
- Povezave med učinkovitostjo in zunanjimi dejavniki, kot so velikost vzajemnega sklada, velikost družbe za upravljanje in starost vzajemnega sklada, so statistično neznačilne, kar hkrati pomeni, da druga, tretja in četrta raziskovalne hipoteze niso podprte.

Glede na to, da DEA metodologija doslej ni bila uporabljena v nobeni analizi uspešnosti vzajemnih skladov v Sloveniji, si že sama izvedba DEA ocenjevanja učinkovitosti delniških vzajemnih skladov v Sloveniji zasluži pozornost. Gre za zelo fleksibilno metodo merjenja uspešnosti, ki se lahko uporablja namesto ali skupaj s tradicionalnimi kazalniki uspešnost. Treba pa je izpostaviti, da DEA ocenjevanje lahko postane opazno bolj uporabno v primeru višje stopnje transparentnosti industrije vzajemnih skladov v Sloveniji, kar se nanaša predvsem na poročanje oziroma objavljanje podatkov o sestavi kriterijskih indeksov, značilnostih stila investiranja in karakteristikah upravljavcev.

Appendix B. List of commonly used abbreviations

APM	Arbitrage pricing model
AS	Average style
b	Benchmark
BCC	Banker, Charnes, Cooper
CAPM	Capital asset pricing model
CCR	Charnes, Cooper, Rhodes
CRS	Constant returns to scale
CS	Characteristic selectivity
CT	Characteristic timing
CVaR	Conditional value-at-risk
DMU	Decision making unit
EU	European Union
EUR	Euro
GDP	Gross domestic product
HML	High-minus-low
im	Asset i - Market
m	Market
MAR	Minimjm acceptable return
max	Maximum
MER	Management expense ratio
mf	Mutual fund
min	Minimum
MOM	Momentum
OLS	Ordinary least squares
P/B	Price to book
P/CF	Price to cash flow
P/E	Price to earnings
PCM	Portfolio change measure
rf	Risk-free
SMB	Small-minus-big
TER	Total expense ratio
TE	Tracking error
UK	United Kingdom
US	United States of America
VaR	Value-at-risk
VRS	Variable returns to scale

Appendix C. Optimal solutions - results of the Spearman's rank correlation tests

Table 1. Optimal solutions - results of the Spearman's rank correlation tests

		2011	2012	2013
2010	ρ	0.1335	0.1037	-0.0201
	p	*0.3009*	*0.4224*	*0.8768*
2011	ρ		0.3771	0.3098
	p		*0.0025*	*0.0143*
2012	ρ			0.3969
	p			*0.0014*

Appendix D. Optimal solutions - results of the Kendall's tau tests

Table 2. Optimal solutions - results of the Kendall's tau tests

		2011	2012	2013
2010	tau b	0.1040	0.0652	-0.0272
	p	*0.2381*	*0.4616*	*0.761*
2011	tau b		0.2650	0.2103
	p		*0.0026*	*0.0166*
2012	tau b			0.2894
	p			*0.010*

Appendix E. Optimal solutions - results of the pairwise correlations

Table 3. Optimal solutions - results of the pairwise correlations

		2011	2012	2013
2010	coefficient	0.1223	0.0873	-0.0321
	p	*0.3436*	*0.4996*	*0.8044*
2011	coefficient		0.3743	0.446
	p		*0.0027*	*0.0003*
2012	coefficient			0.3552
	p			*0.0046*

Appendix F. Optimal solutions - results of the OLS regressions

Table 4. Optimal solutions - results of the OLS regressions

		2011	2012	2013
2010	coefficient	0.1514	0.0829	-0.0431
	p	*0.344*	*0.500*	*0.804*
2011	coefficient		0.4880	0.4108
	p		*0.003*	*0.000*
2012	coefficient			0.2509
	p			*0.005*

Appendix G. Optimal solutions - results of the tobit regressions

Table 5. Optimal solutions - results of the tobit regressions

		2011	2012	2013
2010	coefficient	0.1889	0.0993	-0.3212
	p	*0.267*	*0.462*	*0.864*
2011	coefficient		0.3170	0.5381
	p		*0.003*	*0.000*
2012	coefficient			0.5793
	p			*0.003*

Appendix H. Optimal solutions - results of the SFA regressions

Table 6. Optimal solutions - results of the SFA regressions

		2011	2012	2013
2010	coefficient	0.1514	0.0829	-0.0431
	p	*0.332*	*0.490*	*0.800*
2011	coefficient		0.2871	0.4843
	p		*0.001*	*0.000*
2012	coefficient			0.5029
	p			*0.003*

Appendix I. Total slacks - results of the Spearman's rank correlation tests

Table 7. Total slacks - results of the Spearman's rank correlation tests

		Risk slack			Return slack		
		2011	2012	2013	2011	2012	2013
2010	ρ	0.1536	0.0239	-0.1912	0.1745	0.1107	-0.011
	p	*0.2332*	*0.8538*	*0.1365*	*0.1749*	*0.3916*	*0.9326*
2011	ρ		0.1565	0.3095		0.4049	0.3293
	p		*0.2246*	*0.0144*		*0.0011*	*0.009*
2012	ρ			0.0507			0.4049
	p			*0.6956*			*0.0011*

Appendix J. Total slacks - results of the Kendall's tau tests

Table 8. Total slacks - results of the Kendall's tau tests

		Risk slack			Return slack		
		2011	2012	2013	2011	2012	2013
2010	tau b	0.1296	0.0235	-0.1393	0.1328	0.0738	-0.0166
	p	*0.1411*	*0.7936*	*0.1138*	*0.1315*	*0.4046*	*0.8552*
2011	tau b		0.1125	0.2156		0.2853	0.2305
	p		*0.2016*	*0.0141*		*0.0012*	*0.0086*
2012	tau b			0.0331			0.2936
	p			*0.7106*			*0.0008*

Appendix K. Total slacks - results of the pairwise correlations

Table 9. Total slacks - results of the pairwise correlations

		Risk slack			Return slack		
		2011	2012	2013	2011	2012	2013
2010	coefficient	0.1345	0.0712	-0.1299	0.1605	0.1179	-0.0102
	p	*0.2974*	*0.5826*	*0.3141*	*0.2127*	*0.3616*	*0.937*
2011	coefficient		0.1312	0.3352		0.4017	0.4346
	p		*0.3095*	*0.0077*		*0.0012*	*0.0004*
2012	coefficient			0.09			0.3902
	p			*0.4868*			*0.0017*

Appendix L. Total slacks - results of the OLS regressions

Table 10. Total slacks - results of the OLS regressions

		Risk slack			Return slack		
		2011	2012	2013	2011	2012	2013
2010	coefficient	0.1812	0.0617	-0.2147	0.1797	0.1079	-0.0127
	p	*0.2970*	*0.5830*	*0.3140*	*0.2130*	*0.3620*	*0.9370*
2011	coefficient		0.0844	0.4110		0.3185	0.4816
	p		*0.3100*	*0.0080*		*0.0010*	*0.0000*
2012	coefficient			0.1714			0.5288
	p			*0.4870*			*0.0000*

Appendix M. Total slacks - results of the tobit regressions

Table 11. Total slacks - results of the tobit regressions

		Risk slack			Return slack		
		2011	2012	2013	2011	2012	2013
2010	coefficient	0.2215	0.0656	-0.2077	0.2146	0.1250	-0.0043
	p	*0.2320*	*0.5960*	*0.3680*	*0.1630*	*0.3380*	*0.9800*
2011	coefficient		0.0768	0.4438		0.3624	0.5412
	p		*0.4060*	*0.0080*		*0.0010*	*0.0000*
2012	coefficient			0.2642			0.6021
	p			*0.3260*			*0.0000*

Appendix N. Total slacks - results of the SFA regressions

Table 12. Total slacks - results of the SFA regressions

		Risk slack			Return slack		
		2011	2012	2013	2011	2012	2013
2010	coefficient	0.1812	0.0617	-0.2378	0.1797	0.1079	-0.0344
	p	*0.2850*	*0.5600*	*0.2390*	*0.2000*	*0.3500*	*0.8340*
2011	coefficient		0.0844	0.4229		0.3285	0.4285
	p		*0.2970*	*0.0030*		*0.0010*	*0.0000*
2012	coefficient			0.0571			0.5288
	p			*0.8630*			*0.0010*

Appendix O. SFA regressions results in the year 2010

Table 13. SFA regressions results in the year 2010

		Return slack		Risk slack	
		Coefficient	Standard error	Coefficient	Standard error
Size	coefficient	-0.0002564	0.0001983	-0.0001556	0.000234
	p	*0.196*		*0.506*	
Family size	coefficient	0.0001752	0.0000693	0.0000998	0.0000817
	p	*0.011*		*0.222*	
Age	coefficient	0.0027311	0.0038305	0.0028275	0.0045199
	p	*0.476*		*0.532*	
Ln(δ_v^2)	coefficient	-5.694652	0.1810932	-5.363686	0.1816257
	p	*0.000*		*0.000*	
Ln(δ_u^2)	coefficient	-12.98906	92.73886	-12.60286	103.0653
	p	*0.889*		*0.903*	
δ_v		0.0579992	0.0052516	0.0684369	0.006215
δ_u		0.0015117	0.0700961	0.0018337	0.0944943
δ^2		0.0033662	0.0006193	0.004687	0.0008701
λ		0.0260639	0.070949	0.0267937	0.0956061
Likelihood-ratio test of $\delta_u = 0$	coefficient	0.00		0.00	
	p	1.000		1.000	

Appendix P. SFA regressions results in the year 2011

Table 14. SFA regressions results in the year 2011

		Return slack		Risk slack	
		Coefficient	Standard error	Coefficient	Standard error
Size	coefficient	0.0000199	0.0002392	0.000058	0.0003232
	p	*0.934*		*0.858*	
Family size	coefficient	-0.0001287	0.0000757	-0.0002035	0.0001023
	p	*0.089*		*0.047*	
Age	coefficient	-0.0006382	0.0043868	0.0030455	0.0059258
	p	*0.884*		*0.607*	
Ln(δ_v^2)	coefficient	-5.412308	0.000	-4.810832	0.1827767
	p	*0.000*		*0.000*	
Ln(δ_u^2)	coefficient	-12.69074	93.63743	-12.15238	143.5685
	p	*0.892*		*0.933*	
δ_v		0.0667932	0.0060505	0.0902279	0.0082458
δ_u		0.0017549	0.0821599	0.0022969	0.1648822
δ^2		0.0044644	0.0008223	0.0081464	0.0015405
λ		0.0262729	0.083158	0.0254568	0.1665961
Likelihood-ratio test of $\delta_{\mathrm{u}} = 0$	coefficient	0.00		0.00	
	p	1.000		1.000	

Appendix Q. SFA regressions results in the year 2012

Table 15. SFA regressions results in the year 2012

		Return slack		Risk slack	
		Coefficient	Standard error	Coefficient	Standard error
Size	coefficient	0.0000747	0.0002125	0.0000998	0.0002269
	p	*0.725*		*0.660*	
Family size	coefficient	-0.0001347	0.0000611	-0.0001557	0.0000652
	p	*0.027*		*0.017*	
Age	coefficient	0.0006903	0.0035072	0.0010551	0.0037447
	p	*0.844*		*0.778*	
$\mathrm{Ln}(\delta_v^2)$	coefficient	-5.845	0.1815166	-5.71463	0.1972083
	p	*0.000*		*0.000*	
$\mathrm{Ln}(\delta_u^2)$	coefficient	-12.9925	91.19282	-11.67593	86.85061
	p	*0.887*		*0.893*	
δ_v		0.053799	0.0048827	0.0574227	0.0056621
δ_u		0.0015091	0.068809	0.0029148	0.1265744
δ^2		0.0028966	0.0005367	0.0033059	0.0007568
λ		0.0280505	0.0696738	0.507597	0.1290053
Likelihood-ratio test of $\delta_u = 0$	coefficient	0.00		0.00	
	p	1.000		1.000	

Appendix R: SFA regressions results in the year 2013

Table 16. SFA regressions results in the year 2013

		Return slack		Risk slack	
		Coefficient	Standard error	Coefficient	Standard error
Size	coefficient	0.000463	0.0002978	0.0004115	0.0004387
	p	*0.120*		*0.348*	
Family size	coefficient	-0.0001662	0.0000754	-0.0001064	0.0001162
	p	*0.028*		*0.360*	
Age	coefficient	-0.0055981	0.0048222	-0.0142588	0.0072436
	p	*0.246*		*0.049*	
$\mathrm{Ln}(\delta_v^2)$	coefficient	-5.284257	0.8390585	-4.822704	0.5869856
	p	*0.000*		*0.000*	
$\mathrm{Ln}(\delta_u^2)$	coefficient	-7.585949	22.5841	-4.459722	1.166498
	p	*0.737*		*0.000*	
δ_v		0.0712095	0.0298745	0.089694	0.0263245
δ_u		0.0225285	0.2543929	0.1075434	0.0627246
δ^2		0.0055783	0.0073742	0.0196106	0.0095302
λ		0.3163691	0.2836003	1.199004	0.0869981
Likelihood-ratio test of $\delta_u = 0$	coefficient	0.0013		0.33	
	p	0.486		0.284	