UNIVERSITY OF LJUBLJANA
SCHOOL OF ECONOMICS AND BUSINESS

MASTER'S THESIS

THE DISCOUNT FOR LACK OF LIQUIDITY FOR CONTROLLING OWNERSHIP INTEREST: UNDERSTANDING AND APPLICATION IN THE BUSINESS VALUATION PROCESS ON THE SLOVENIAN MARKET

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LIST OF ABBREVIATIONS

ASA – American society of appraisers
CAEs – Court appraisers and experts
CAIA – Chartered alternative investment analyst
CBAs – Certified business appraisers
CCBs – CFA, CAIA and stock broker
CFA – Chartered financial analyst
DCF – Discounted cash flow
DLOC – Discount for lack of control
DLOL – Discount for lack of liquidity
DLOM – Discount for lack of marketability
EBIT – Earnings before interest and tax
EBITDA – Earnings before interest, tax, depreciation and amortisation
EV – Enterprise value of a company
FCFE – Free cash flow to the equity
FCFF – Free cash flow to the firm
IPO – Initial public offering
IVS – International valuation standards
LTM – Last twelve months
M&A – Merger and acquisition
MVIC – Market value of invested capital
NAICS – North America industry classification system
P/E – Price to earnings
PCD – Private company discount
POPM – Put option pricing model
SEC – Security and exchange commission
SIA – Slovenian institute of auditors
SIR – Slovenski inštitut za revizijo // *Slovenian institute of auditors*

1 INTRODUCTION

When individuals, companies or society are making a decision about an investment into a particular asset, a question of its value arises. Knowing what the value of an asset is and what determines it is important when the decision to buy or to sell an asset is taking place (Damodaran, 2006). In case where the subject of valuation is not the 100% ownership interest of a company, but only a part of it, the size of the valuated part is crucial. This is important from the perspective of corporate governance, business policy, financial structure determination, as well as profit distribution and other business-related factors, which all have an impact on the value of the valuated ownership interest.

The subject of the valuation can be a non-controlling or controlling ownership interest and to both discounts for lack of marketability (hereinafter "DLOM") and/or liquidity (hereinafter "DLOL") can be applied, but the reasons to do so are different. Measurement of a DLOM is more related to a non-control ownership interest and therefore marketability denotes the right or possibility to sell an asset on the ready market, while DLOL is more related to the controlling ownership interest and refers to the ability to readily convert an asset into cash without significant loss in value or within a reasonable timeframe (Dodel, 2014).

In practice, one can face the problem of lack of studies considering DLOL for controlling ownership interest, most probably because non-controlling ownership interest valuations are more frequent. The estimation of DLOL for controlling ownership interest is not as clear as estimation of DLOM for non-controlling ownership interests. Even though the owners of the controlling ownership interest in a private company typically have no legal restrictions to sell their interest, they cannot convert it into cash quickly. It may take time and costs to sell it. Due to that it is more appropriate that in the cases of controlling ownership interest valuation, DLOL studies are examined more closely, whereby one does not use shortcuts with a usage of DLOM studies.

1.1 Goals and purpose of the master's thesis

By writing this master's thesis I would like to contribute to a better understanding of DLOL through the summarisation of the results and findings of the existing studies and analysis of conditions on the Slovenian market. I hope that my contribution to this field of science would make valuation processes easier and estimations more accurate.

The key objective that I would like to reach at the end of the writing of this master's thesis relates to analysis of conditions on the Slovenian market. I would like to find out what is a practice of Slovenian market participants regarding the estimation and application of DLOL for controlling ownership interest.

I believe that it is possible to provide a good market overview that could be used as a guideline in future determinations of DLOL in the valuation processes of controlling ownership interest. It means that the main purpose of the thesis is the summarization of existing studies and the analysis of conditions on Slovenian market.

1.2 Methodology

The master's thesis consists of two content parts, a literature review and empirical research of conditions on the Slovenian market, which is further divided into survey among Slovenian market participants and analysis of actually applied discounts by Certified Business Appraisers (hereinafter "CBAs").

The first part of the master's thesis is a review of the existing studies and literature about the estimation of the DLOL. It contains the presentation of the existing studies, the presentation of the methodological approaches used by authors, as well as the presentation of the obtained results.

The second part of the master's thesis is an empirical research. The first part of the empirical research is a survey among participants on Slovenian market about their practice and usage of the DLOL in the process of controlling ownership interest valuation. It is carried out on the primary data.

The second part of the empirical research analyses the actually applied discounts of CBAs authorized by the Slovenian Institute of Auditors (hereinafter "SIA").

1.3 The set hypotheses

To examine how Slovenian market participants determine and use DLOL for controlling ownership interest I have set the following hypotheses:

Hypothesis 1.1: Most of the Slovenian market participants use adjusted selected mean value of different studies as a primary approach to determine the appropriate DLOL level.

Hypothesis 1.2: According to Slovenian market participants, DLOL mainly reflects the time value of money and with that the adjustment for time uncertainty risk.

Hypothesis 1.3: Deduction of DLOL from estimated equity value is the most commonly used approach of DLOL application among Slovenian market participants.

Hypothesis 1.4: Among the Slovenian market participants, determining the appropriate level of DLOL does not take more than 5% of total time in the business valuation process.

Hypothesis 1.5: Slovenian market participants acknowledge that the size of the valuated interest has an impact on applied DLOL and that they are inversely related.

The second set of hypotheses is set up to test actually applied values of DLOM and DLOL by CBAs authorized by SIA. I have set the following hypotheses:

Hypothesis 2.1: With an increase of valuated ownership interest the size of applied discount decreases.

Hypothesis 2.2: Applied value of a discount to a certain size of valuated ownership interest does not vary over time.

1.4 Structure of the master's thesis

The first part of the master's thesis is built gradually. In the second chapter, I present concepts of marketability and liquidity and discounts associated with them. The purpose of this chapter is to draw a clear line between DLOM and DLOL, their determination and application. The third chapter of the master's thesis is about DLOM for non-controlling ownership interest, since it presents a base for the estimation of DLOL for controlling ownership interest. It means that studies of DLOM for non-controlling ownership interest are briefly presented and summarized here. The fourth chapter of the thesis contains an examination of papers and studies related to the main research. It obtains studies examining determination of DLOL for controlling ownership interest. This part is divided into several subsections, depending on the content of studies and their approaches to determine DLOL.

The second part of the master's thesis is an analysis of the conditions on Slovenian market. The fifth chapter is divided into two parts. In the first part, using a survey questionnaire, I analyse a practice of determination and usage of DLOL for controlling ownership interest among Slovenian market participants, while in the second part I analyse actually applied discounts by CBAs authorized by SIA.

The final chapter of my master's thesis is a summarization of key findings, limitations of the thesis and suggestions for further research.

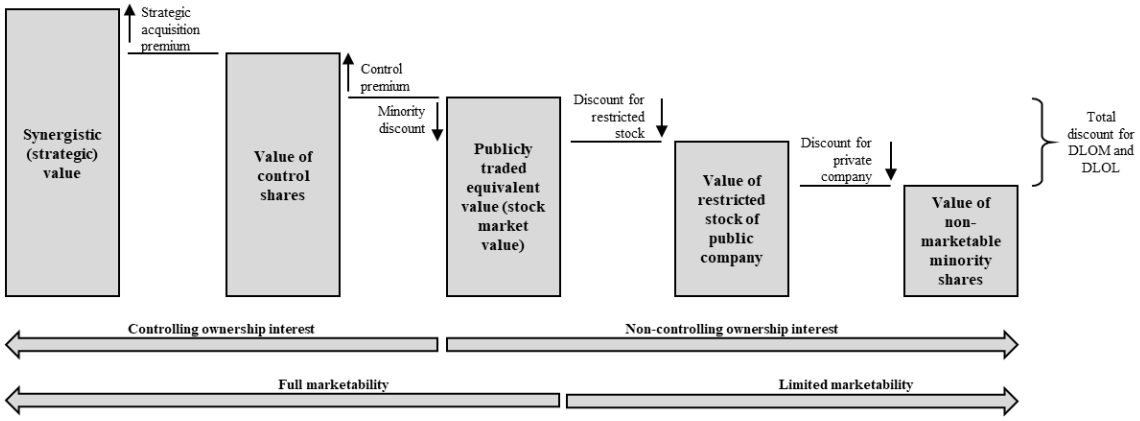
2 MARKETABILITY, LIQUIDITY AND RELATED DISCOUNTS

Since there is no ready market where ownership interests of a private companies could be sold or bought immediately and without additional costs, the risk of inability to execute transaction of those interests increase substantially compared to the freely traded interests of listed companies (Praznik, 2004). It could be said that DLOL and DLOM represent a bridge between values of private and public companies or that they are a difference between the prices of liquid and illiquid companies, considering the same other characteristics. In

practice, the determination of appropriate level of a discount is a time-consuming process including many assumptions, causing many to simply subtract a certain percentage from the estimated base value of an asset (Chipalkatti et al., 2013).

DLOM and DLOL as well as discount for lack of control (hereinafter "DLOC") are discounts that reflect characteristics of the ownership in the company. There are also discounts that refer to the entire company and are unrelated to the ownership features of the company. Such discount is a discount for key personnel. Overview of different levels of company value, regarding DLOM, DLOL and other discounts, is shown in the Figure 1 (Dodel, 2014).

Figure 1: Overview of the value levels



Source: Dodel (2014).

Pratt (2009) said that if an asset has a ready market, the marketability refers to the right to sell it and liquidity refers to the time it takes to convert asset into cash. Assets like real estate, machinery and controlling ownership interest in private and public companies belong to the group of assets that are marketable illiquid. This means that an individual has a right to sell them but there is no ready market to sell the asset and no received cash. Real estate is marketable, but it takes some time to convert it into cash on the bank account. On the other hand, assets like non-controlling ownership interest in a private company are considered non-marketable and assets like actively traded public stocks are considered as liquid (Hitchner, 2017).

Investors are risk averse and prefer marketability, because they will always choose and pay more for the asset that is readily marketable against the identical asset that is not, other things being equal. The commonly used valuation approaches and methods do not take into account degree of the marketability of valuated ownership interest. Instead, they usually consider valuated ownership interest as it would be fully marketable (Bajaj et al., 2001). Due to that, they applied DLOM and DLOL at the end of the valuation process. They are applied to the base value right after the application of a DLOC or application of controlling premium. This is because controlling or non-controlling ownership interest can be affected by the lack of marketability or liquidity. Sometimes discount for lack of liquidity enter into valuation

process through the adjustment of the discount rate in the discounted cash flow method or through firm-specific risk adjustment as a percentage of the derived multiple (Pratt, 2009).

In the presence of the additional costs and the restrictions, investors require discount on the estimated base value of a company to neutralize them. Furthermore, they require a discount in case of inability to sell a company for a period of time as it represents a compensation for a time that they do not have money at their account and cannot allocate it elsewhere and achieve a higher return (Bajaj et al., 2001).

2.1 Definition of marketability and DLOM

Pratt and Niculita (2008) explain marketability as an ability of the ownership interest holder to convert his investment into cash quickly, with minimum predictable costs and with high degree of realization certainty. They say that measurement base for DLOM is a transfer of non-controlling ownership interest such as small block of shares listed on stock exchange. The DLOM is the result of inherent liquidity differences between non-controlling ownership interest in the valuated company and comparable publicly traded companies' ownership interests.

Pratt and Niculita (2008) define marketability as: "The relative ease and promptness with which a security or commodity may be sold when desired, at a representative current price, without material concession in price merely because of the necessity of sale."

Dodel (2014) relates marketability and discount for lack of it to the non-controlling event and defines it as the right to sell company's ownership interest or some other asset in established and efficient public or private market with minimum effort, low transaction costs and within reasonable time.

According to the IVS (International Valuation Standards Council, 2017), DLOM should be applied when the comparable companies or transactions have superior marketability compared to the valuated company. It means that in the comparison of otherwise identical companies, one that is readily marketable has higher value compared to the one with restrictions to sell or long marketing period.

Above given descriptions of the marketability and DLOM are well summarized in the definition given by American Society of Appraisers (hereinafter "ASA") (2009) where marketability is defined as the ability of the holder of the ownership interest to quickly convert his or her interest into cash with lowest possible costs, while DLOM reflects the relative absence of marketability and it is an amount or a percentage deduction from the value of the ownership interest. Abbott (2009) said that marketability represents the capability and ease of transfer or saleability of the ownership interest.

To calculate DLOM any reasonable method can be used. IVS (International Valuation Standards Council, 2017) highlights two methods typically used in the valuation process, which are option pricing models where the value of publicly traded shares and restricted shares in the same company are compared, and studies where the value of shares in a company before and after an initial public offering (hereinafter "IPO") are compared.

2.2 DLOM for controlling ownership interest

Many individuals working in the field of financial services makes adjustment of the estimated value of controlling ownership interest for the time needed to sell the interest, sale-related costs and risk to complete the sale. Adjustment of the risk to complete the sale is the most significant one (Pratt & Niculita, 2008).

Some authors and individuals working in the field of financial services believe that DLOM for the controlling ownership interest is not justified and as such unnecessary in valuation process, while on the other hand, many believe that it is a necessary and an important part of it. Doubts about justification of DLOM for controlling ownership interest will be always present since there are no clear guidelines to determine it. Based on Pratt et al. (2000), even the U.S. Tax Court has been inconsistent in determining the size of DLOM. In some cases, it allowed discount in the amount of 30.0%, while in some other case it denied the discount entirely.

I believe that ignoring DLOM of the controlling ownership interest is an anomaly in the valuation process. Doing so one ignores market conditions and individual characteristics of companies. Additionally, Rotkowski (2011) explains that it is important to understand companies' individual characteristics that affect the size of DLOM and not always apply the same discount. Not applying DLOM to the controlling ownership interest is just as inappropriate as applying 35% DLOM to every non-controlling ownership interest. It means that appropriate size of DLOM needed to be applied to every company.

2.3 Definition of liquidity and DLOL

Pratt and Niculita (2008) explain liquidity as an ability of the holder of the controlling ownership interest to convert investment into cash quickly with predictable low cost. The measurement base for DLOL is a transfer of the controlling ownership interest or entire company. The DLOL is the result of inherent liquidity differences between controlling ownership interest in the valuated company and the benchmark of comparable companies.

Dodel (2014) relates liquidity and discount for lack of it to the controlling event and defines it as the ability to convert ownership interest or some other asset into cash without significant loss of principal.

In economics, liquidity is used as a synonym for the desire to possess an asset in the form of cash. Pratt and Niculita (2008) on the other hand define liquidity as the time required to convert an asset into cash or to pay a liability.

Damodaran (2005) said that any asset can be sold, no matter how illiquid it is. It is just a question of how willing one is to accept lower price when selling the asset. Because of that it can be said that all assets are illiquid, but the magnitude of illiquidity variates between assets.

Above given description of the liquidity and DLOL are well summarized in the definition given by ASA (2009) where liquidity is defined as the ability to readily convert an ownership interest into cash without significant loss of principal, while DLOL reflects the relative inability to quickly convert ownership interest into cash and it is an amount or a percentage deduction from the value of the ownership interest.

According to the above given definitions and understanding of a controlling and non-controlling ownership interest and their influence on the decision making in a company and business performance of a company, it can be concluded that in general DLOM is used for the non-controlling ownership interest and represents a right to sell it on a market, while DLOL is used for the controlling ownership interest and represents an ability to convert it into cash without significant loss of principal.

2.4 Transactional uncertainties affecting controlling ownership interest in the private company

Process of selling the controlling ownership interest in the private company is not as simple as selling the ownership interest of the publicly traded company, where holder call security broker, sell ownership interest with normal costs and receive cash from sale in three days, which is a benchmark for that kind of transactions (Pratt & Niculita, 2008).

Holder of the controlling ownership interest in the private company that wants to sell ownership interest has to face the following transactional uncertainties (Pratt & Niculita, 2008):

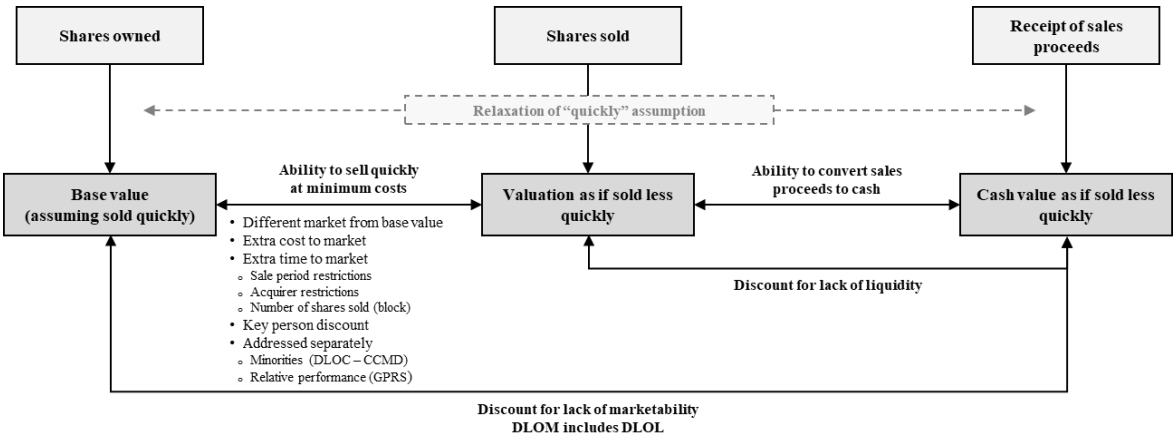
- Uncertain time horizon, because it can take months to complete offering or sale of the private company. Time factor for the seller of the controlling ownership interest can be offset to some extent by cash flows of the company in the waiting period if they are equal or higher then costs on the stand-alone basis.
- Costs of preparation and execution of the offering or sale refers to an auditing and accounting fees to prepare financial information for potential buyers, legal costs to prepare all necessary documents, and administrative costs to connect and hold together all the included parties.

- Risk of difference in actual and estimated sale price, because valuations are just estimates and appearance of other factors that could affect value of a company during the selling process. Holder also faces the risk of actual realisation of a transaction.
- Form of the proceeds refers to the possibility that all proceeds may not be in cash, but the holder or seller can receive part of the transaction price in the form of note or contingent compensation, or in some other form.
- Inability to hypothecate, because even controlling ownership interest in the private company sometimes does not represent good enough collateral for banks.

2.5 A comprehensive demonstration of the interaction between marketability and liquidity

Many definitions of marketability, liquidity and related discounts had been given. In the Figure 2, the interaction between marketability (DLOM) and liquidity (DLOL) is presented. In the figure, the base anchor value is referring to the application of an income, market or asset-based valuation approach. This is the value of the asset that is highly possible to be obtained in the market under the assumption that the asset can be converted into cash quickly. Quickly conversion into cash refers to the net cash value from hypothetical deliberative sales process discounted to the present value at the valuation date rather than from sale in three days, which one can take as a benchmark, since this could anticipate a fire sale and reduce the value of the asset (Shaw, 2016). The chapter will be concluded with the presentations of the factors that according to Pratt affects both DLOM and DLOL.

Figure 2: Interaction between marketability and liquidity



Source: Shaw (2016).

2.6 Factors that affect DLOM and DLOL

It cannot be simply said that one company's ownership interest is marketable and liquid and another's is not. Because of that the adjustment for non-marketability and illiquidity is not fixed value or the percentage deducted from the base value. Companies operate in different industries, have different financial positions, different management, and because of that they

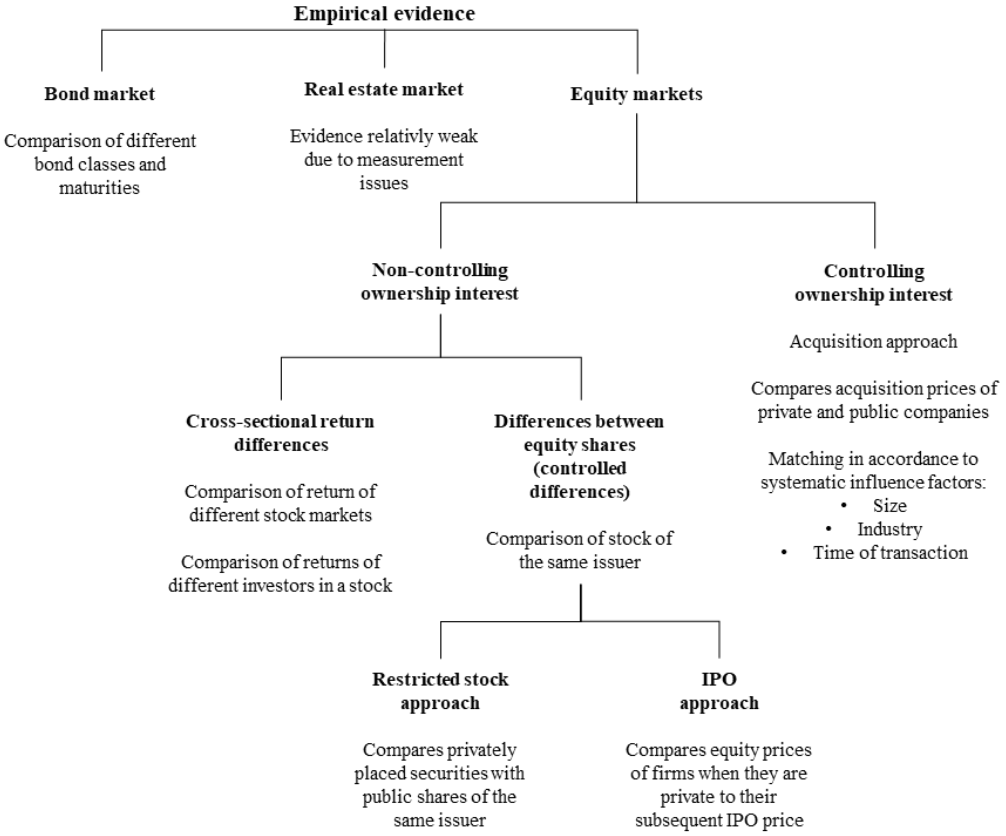
have different characteristics. This results in different degrees of marketability and liquidity and because of that also in different levels of DLOM and DLOL. There are different factors that influence the level of DLOM and DLOL and give a guide about the range of discounts. Pratt and Niculita (2008) recognise the following factors:

- Contractual right of the ownership interest holder to sell it, which acts as holder's "put" right and represents the market.
- Ownership interest that provides little or no dividend payments is more susceptible for the lack of marketability and liquidity compared to the ownership interest that provide high dividend payments. This is because the return of the ownership interest holder with low or no future dividend payments depended strictly on the ability to sell interest to the next investor. In case of high returns of the ownership interest, holder generates return regardless of his or her ability to sell the interest.
- Number of potential buyers: To lower the degree of discounts it is enough that there is one strong potential buyer.
- Interest size: Empirical evidence shows that larger block of shares or bigger ownership interests are subjected to a higher degree of DLOM and DLOL compared to the smaller ones, since there may be lower number of potential buyers because of the difficulty to finance the transaction. Some argue that larger block of shares should be subject to the lower DLOM and DLOL because of the controlling component, but this is a factor of controlling discount or premium.
- Prospect of public offering may offset the degree of the DLOM and DLOL. Because these events are uncertain the offset is also uncertain. On the other hand, if company is committed to stay private, in the future this would increase DLOM and DLOL.
- Availability and reliability of information affect the degree of DLOM and DLOL.
- Sometimes ownership interests are subject to provisions that restrict the right of the holder to sell it. Restrictive provisions can either be applied as a fixed amount or act as a ceiling that caps the discount, but regardless of the type of restriction it influences the degree of DLOM and DLOL.
- Company's characteristics, such as historical bed performance, high stock price volatility, unstable earnings and other increased degrees of discounts.

3 DLOM FOR NON-CONTROLLING OWNERSHIP INTEREST

Over the years, different empirical studies of DLOM and DLOL appeared. In the Figure 3 below are shown classifications of empirical studies dealing with discounts for lack of marketability and liquidity for controlling and non-controlling ownership interest.

Figure 3: Overview of empirical studies for lack of marketability and liquidity



Source: Dodel (2014).

This chapter mainly refers to the presentation of restricted stock and prior IPO studies to estimate appropriate degree of DLOM for non-controlling ownership interest. Those two are just one of possible approaches and methods to determine the DLOM for non-controlling ownership interest but they often appear in the studies and it can be said that they are most commonly used by the analysts. Next chapter examines studies of determination of DLOL for controlling ownership interest.

In practice, valuation reports comprise fifty or more pages to determine and explain the estimated value of privately held company or some other asset, but examination and explanation of factors to determine appropriate level of discount can only take few pages. Literature provides various ways how to divide or classify methods to determine appropriate degree of DLOM. In the Table 1, one of the possible classifications of DLOM estimation approaches and methods is shown.

Table 1: Classification of the DLOM estimation methods

Approach	Method / Study
Benchmark approach	Restricted stock studies
	Pre-initial public offering (pre-IPO) studies
	Restricted stock equivalent studies
	Cost of flotation
	Mandelbaum Factors
Security-based approach	Long-Term Equity Anticipation Securities (LEAPS)
	The Longstaff Study
	The Chaffee Study
	Bid-Ask Spread Methods
Analytical approach	Karen Hopper Wruck
	Hertzel and Smith
	Bajaj, Denis, Ferris and Sarin
	Ashok B. Abbott
Other approaches	Quantitative Marketability Discount Model (QMDM)
	Non-marketable Investment Company Evaluation (NICE)
	National Economic Research Associates (NERA)
	Partnership Profiles
	Public vs. Private P/E Multiples in Acquisitions

Source: Internal Revenue Service (2009).

Among all approaches, the most frequently used and examined is the benchmark approach and within that restricted stock and pre-IPO studies. In the following pages, these two studies are going to be closely examined. This will give a good base for the examination of DLOL for controlling ownership interest in the following chapters.

The security-based approaches to estimating DLOM are based on theoretical option pricing models and observing illiquidity demonstrated by traded stock prices and option prices (Internal Revenue Service, 2009).

The analytical approaches examine the sales where the issuer of the stock is generally the seller and some institutional entity the buyer. The sale price of the publicly traded stocks sold through private placements are compared to the sale prices of the stocks as traded in the primary market where they were listed. Obtained data is statistically analysed and examined with regression models to determine the degree of the discount and to further break it down to the various causal factors (Internal Revenue Service, 2009).

3.1 Benchmark approaches

Benchmark approaches are divided into two main forms. First ones estimate appropriate DLOM based on IPO pricing compared to the price of earlier privately traded transactions and the second ones estimate DLOM based on restricted stocks. Out of these approaches some derivative approaches have developed over time (Internal Revenue Service, 2009).

These two main forms can also be classified as the empirical models. These models rely on actual transaction data to provide the appropriate level of DLOM (Novak, 2016).

3.1.1 Pre-initial public offering (pre-IPO) studies

Pre-IPO studies compare stock prices of companies following IPO, when they are public, to transaction prices of those same stock prior to the IPO, when they are private. These studies estimate discount by comparing the price of the asset during the period when it is more marketable to its price during the period in which it is less marketable (Bajaj et al., 2001).

One of the most important researchers in the field of estimation of DLOM by comparing stock prices before IPO and at the day of IPO is John D. Emory who has done several studies. His estimates of DLOM can be seen in the Table 2. In the same table the result of study made in the 1997-2000 period including only dot.com companies can be seen. Study was carried out during the boom period of those companies which resulted in a higher degree of discount (Block, 2007).

Table 2: Results of John D. Emory studies and study with dot.com companies

Study	Number of IPO prospectuses reviewed	Number of qualifying transactions	Discounts	
			Average	Median
dot.com companies				
1997-2000	92	53	54%	54%
Emory studies				
1995-1997	732	91	43%	42%
1994-1995	318	46	45%	45%
1991-1993	443	54	45%	44%
1990-1992	266	35	42%	40%
1989-1990	157	23	45%	40%
1987-1989	98	27	45%	45%
1985-1986	130	21	43%	43%
1980-1981	97	13	60%	66%

Source: Block (2007).

Time length between the pre-IPO chosen date and date of execution of the IPO affects the degree of DLOM. The results shown above do not show this effect. Pearson (in Novak, 2016) in his Valuation advisors' lack of marketability discount study shows the importance of the period length and that the larger it is the higher is the value of the discount. Results of the study can be seen in the Table 3.

Table 3: Results of Valuation advisors' lack of marketability discount study

IPO year	Period before IPO in which transaction occurred					Number of transactions
	0-3 months	4-6 months	7-9 months	10-12 months	1-2 years	
1995	37.8%	28.6%	60.4%	50.3%	60.6%	34
1996	30.8%	53.0%	56.4%	69.4%	71.8%	270
1997	34.2%	50.0%	67.1%	76.0%	80.0%	212
1998	23.4%	46.7%	68.9%	71.4%	71.9%	212
1999	30.8%	53.9%	75.0%	76.9%	82.0%	694
2000	28.7%	45.1%	61.5%	68.9%	76.6%	653
2001	14.7%	33.2%	33.4%	52.1%	51.6%	115
2002	6.2%	17.3%	21.9%	39.5%	55.0%	81
2003	28.8%	22.3%	38.4%	39.7%	61.4%	123
2004	16.7%	22.7%	40.0%	56.3%	57.9%	334
2005	14.8%	26.1%	41.7%	46.1%	45.5%	296
2006	23.5%	20.7%	40.2%	46.5%	56.3%	264
2007	12.7%	32.6%	43.7%	56.0%	54.2%	459
2008	20.0%	24.2%	45.9%	52.2%	41.2%	41
2009	6.2%	31.9%	26.8%	41.0%	34.9%	108
2010	15.8%	29.9%	44.4%	47.5%	51.9%	358
2011	23.3%	34.6%	43.3%	50.8%	62.1%	281
2012	18.9%	24.1%	28.9%	35.5%	44.8%	292
Average 1995 - 2012	**21.5%**	**33.2%**	**46.5%**	**54.2%**	**58.9%**	
Average 2008 - 2012	**16.8%**	**28.9%**	**37.9%**	**45.4%**	**47.0%**	

Source: Novak (2016).

Pre-IPO studies are not treated as the most suitable or the most precise to estimate the degree of DLOM. They have some shortcomings by which results of the studies are affected. One of these shortcomings, and by the opinion of some authors the most important one, is the selection bias, since only companies that have successful IPO are being taken into account, while other companies are being ignored. Buyers of the pre-IPO companies' stocks do not know if the IPO will succeed or not. This represents a certain level of risk that disappears when companies go public. Once companies are public, prices of their stocks are higher simply as a result of the removing this risk, even if everything else remains the same. That is why pre-IPO studies are subject to the upward bias in the estimation of the DLOM (Block, 2007).

Second shortcoming is a different type of transactions prior IPO and at the time of IPO. In the case of prior transactions, buyers are most commonly insiders providing services to the company, like venture capitalists that help the management of the company with advice. Because of that discount can reflect a part of compensation for those services rather than a compensation for lack of marketability. Buyers at the time of the IPO are less likely to be insiders and because of that the price of the transaction is less likely to provide compensation for services performed (Bajaj et al., 2001).

There are also some other shortcomings like high estimated value of discount that can result in a too high rate of return and no adjustment of time value of money. Because of

shortcomings that pre-IPO studies are facing many individuals working in the financial sector also use the restricted stock studies to determine appropriate level of DLOM for non-controlling ownership interest.

3.1.2 Restricted stock studies

Restricted stock studies at the same time compare prices of two claims on the same underlying asset, where one is marketable and the other is not (Bajaj et al., 2001), or they compare prices of restricted, unregistered shares of publicly traded companies with transaction prices in a public market (Dodel, 2014). Both calculations compare variables from the same day. The Table 4 presents fourteen restricted stock studies. When using restricted stock studies to determine appropriate level of DLOM for non-controlling ownership interest one has to consider U.S. Security and Exchange Commission (hereinafter "SEC") restriction via Rule 144, which forbids to trade unregistered shares for a certain period of time. Those shares are less liquid in comparison to the unrestricted ones, because of which the difference in their values represents a discount (Bajaj et al., 2001).

Table 4: Summary of restricted stock studies

Study	Number of transactions	Reporting dates	Discounts	
			Average	Median
SEC Institutional Investor	398	1966-1969	24%	26%
Gelman	89	1968-1970	33%	33%
Moroney	148	1968-1972	34%	35%
Maher	33	1969-1973	33%	35%
Trout	60	1968-1972	n/a	34%
Stryker/Pittock (Standard Research Consultants)	28	1978-1982	45%	n/a
Willamette Management Association	33	1981-1984	31%	n/a
Silber	69	1981-1988	n/a	34%
Hall/Polacek Study (FMV Opinions, Inc.)	100+	1979-1992	n/a	23%
Management Planning, Inc.	49	1980-1990	29%	28%
Johnson	72	1991-1995	n/a	20%
Achwald (Columbia Financial Advisors (pre-1997))	23	1996-1997	14%	21%
Achwald (Columbia Financial Advisors (post-1997))	15	1997-1998	9%	13%
FMV Opinions Database	475	1980-2005	19%	22%
LiquiStat Database	41	2005-2006	32%	31%

Source: Dodel (2014).

All studies have their own characteristics and used assumptions. Wruck (in Bajaj et al., 2001) in her 1989 study compares restricted stocks to the registered ones and finds out that the difference between average discounts was 17.6%, while the difference of the obtained medians was 10.4%. Since only restricted stocks have marketability impairments this difference implies the DLOM. She does not control for the systematic differences in the characteristics of the companies' stocks included in the study and because of that differences

between average discounts and discounts' median could be caused by some other factors rather than marketability.

Hertzel and Smith (in Bajaj et al., 2001) in their 1993 study consider Wruck's shortcomings and find out that discounts obtained by comparing values of restricted and registered stocks are not only affected by the lack of marketability but also by factors such as limited tangible assets, financial distress and engagements in speculative developments of new products. After controlling for those non-marketable factors, they find out that the average discount for restricted stocks was 13.5% and show that earlier studies give the results with too high discounts.

Changes in SEC Rule 144 have an impact on a decline in estimated levels of DLOM. First change in 1990 allowed investors to trade with unregistered securities among themselves which increased the liquidity of restricted stocks. Additional increase in liquidity of the restricted stocks appeared in year 1997 when obligated holding period was reduced from two to one year (Dodel, 2014).

Compared to the IPO studies, the restricted stock studies are improved and more suitable. Although, at the beginning of the restricted stock studies the estimated values of discount were higher due to the impact of other factors, not related to the lack of marketability. DLOM estimates of the first studies moved around 35.0%. After taking into account all the necessary factors and some changes in the regulations, the estimated degree of DLOM more than halved.

3.2 The use of DLOM studies for non-controlling ownership interest in the valuation process of controlling ownership interest

Dodel (2014) explains why DLOM studies for non-controlling ownership interest should not be used in the process of controlling ownership interest valuations by giving the following four explanations:

- Holder of the controlling ownership interest in the company can influence its business process and change the strategy of the company. By doing so holder can optimize company's business performance, and ceteris paribus increase the value of his ownership interest compared to non-controlling ownership interest with no influence on the company's business performance. Additionally, holder of the controlling ownership interest can launch an IPO which makes his interest more liquid compared to the non-controlling ownership interest.
- Costs and uncertainty of potential sale for the controlling ownership interest are different when compared to the non-controlling ownership interest.
- Methodology to provide empirical evidence for the discount for controlling ownership interest is significantly different. While there are databases of restricted stock and pre-IPO studies for non-controlling ownership interest from which individuals can draw a

guidance, there is no empirical transaction database for controlling ownership interest. Also, high variance of the control premium makes it even more difficult to value controlling ownership interest, compared to non-controlling ownership interest.

- Existing studies for DLOL for controlling ownership interest show that it is more difficult to determine the part of the discount that is relating to the lack of liquidity as in the case of the studies for DLOM for non-controlling ownership interest.

Although studies of DLOL for controlling ownership interest exist some authors still question eligibility of their usage and some even believe that their usage is not needed. Because of that an important question that arises is whether the application of DLOL for controlling ownership interest is even reasonable choice. According to Dodel (2014) there are three factors that affect the liquidity and the value of the controlling ownership interest and have to be taken into account in the process of its valuation. These factors are:

- uncertainty in the transaction process,
- costs associated with the selling process,
- the inability to hypothecate.

In practice, one can face the problem of the lack of studies considering the DLOM or DLOL for controlling ownership interest. Since the DLOM represents the right to sell the asset and because the owner of the controlling ownership interest has no legal limitations to do so, it is more appropriate that in case of controlling ownership interest valuation DLOL studies are examined more closely. That is done in the following chapter.

4 DLOL FOR THE CONTROLLING OWNERSHIP INTEREST

Studies presented above manly refer to the non-controlling ownership interest and do not distinguish between different levels of ownership. As already mentioned, holding the controlling ownership interest or holding the non-controlling ownership interest is different from many perspectives. Two of those are the possibility to influence the business process of the company and the other is an impact and cause of the lack of liquidity. Among the other reasons, those two are enough to show that pre-IPO and restricted stock studies are not suitable in the case of controlling ownership interest valuation.

Some authors are trying to solve the problem of how to determine DLOM or DLOL correctly through theoretical concepts while others are trying to do so through empirical examinations. Empirical examinations are often based on the acquisition approach and pricing multiple approach where researchers compare different multiples between public and private companies. Some researchers try to deal with the problem of determining an appropriate level of DLOL through modelling regression models including different factors, such as macroeconomic and industry specific factors, that could affect the value of ownership interest.

Studies show that private companies are typically sold with a price discount compared to public companies. This may be because private companies are illiquid and therefore sold at transaction pricing multiples that reflect this illiquidity.

The case for estimating DLOL for controlling ownership interest is not as clear as estimating a discount for minority interests. Even though the owners of the controlling ownership interest in a private company typically have no legal restrictions to sell their interest, they cannot merely call a stockbroker, execute a transaction in seconds, and have cash in hand within three business days. It may take time to prepare a controlling interest for sale, with significant legal, accounting, investment banking, marketing costs, and management time incurred in the process. Also, compared to public companies, most closely held companies have less ready access to the capital markets to raise additional capital and provide less accounting information (Pratt & Grabowski, 2014). In terms of the above, it can be said that controlling ownership interests in private companies are marketable, since owners have no legal restrictions to sell ownership interest, but are illiquid since it takes time and costs to convert them into cash.

4.1 Theoretical solutions

For many years, appraisers and analysts applied discounts that were based on the empirical researches and actual data, but in the last three decades they have been provided with additional type of analysis to estimate DLOL of already marketable assets. They were provided with the theoretical models or put option pricing models (hereinafter "POPM"). These models are not based on the empirical observations and are used to explain allegedly irrational pricing that can be seen in the market under certain conditions. At least theoretically, DLOL resulting from an option pricing model is a starting point for calculating a discount that is inherent in interests in closely held companies (Pratt & Grabowski, 2014).

POPM assume that by buying a put option on the underlying asset, such as ownership interest, the holder obtains a right to sell the interest at a predetermined price in the future. It could be said that with that the holder buys a liquidity. Because of that the price paid for the option can be understood as the price of liquidity (Pratt & Grabowski, 2014).

In the following three models that relate cost of the option or cost of the liquidity to the volatility of the underlying asset and the expiration time are presented. These models are:

- The Chaffe model.
- The Longstaff model.
- The Finnerty model.

Although the models are giving theoretical solutions to determine DLOM they can be replicated into determination of DLOL for controlling ownership interest and because of that they are included into analysis.

4.1.1 The Chaffe model

In 1993 Chaffe was the first one to introduce the concept of a put option model to estimate DLOM. He pointed out that with a purchase of an option to sell the non-marketable security at the free market price, one effectively purchases the marketability for security (Chaffe, 1993). In that case price of the option represents a price of liquidity and by dividing it by the price of the security at the end of the holding period the value of DLOM is obtained (Rotkowski & Harter, 2013).

Chaffe model is based on the European put option and Black-Scholes model. The equation (1) on the next page shows how to estimate DLOM. The model is based on the following assumptions (Elmore, 2017):

- The option is trade in the liquid and organized market that provides effective trading,
- contract terms are standardized.
- In the holding period interest rate and volatility are constant.
- Stock prices are continuous and without probability of sudden extreme jumps.
- Investor have some market timing ability.

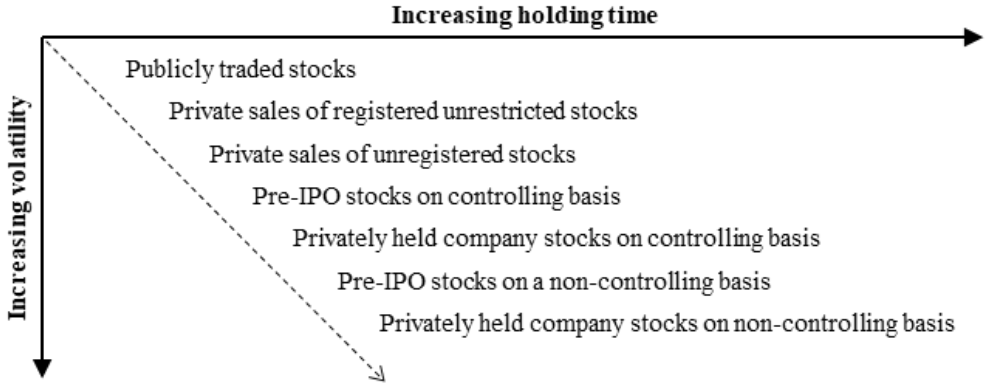
$$\text{DLOM} = \frac{Xe^{-rT}N(-d_2) - Se^{-qT}N(-Nd_1) - S + Xe^{-rT}}{S} \tag{1}$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + T\left(r - q + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T},$$

where *S* denotes current price of the security,
X denotes exercise price of the security,
N(•) denotes cumulative probability function for standard normal distribution,
ln(•) denotes natural logarithm function,
e denotes the base of the natural logarithm,
T denotes time to expiration (in years),
r denotes risk-free interest rate,
q denotes dividend yield,
σ denote annualized volatility of security price.

Time to expiration is the same as the restriction period. This means that it is the same as the time period in which security stays non-marketable after the decision to sell was made. At the expiration date the stock and the strike price of the security are the same as the marketable value of the privately held company's ownership interest. Volatility of security is usually estimated by reference to guideline publicly traded stocks (Elmore, 2017). Volatility along with time component presents one of the most important factors in the process of DLOM estimation with the put options, therefore in the Figure 4 relationship between these two factors and the corresponding levels of securities is shown.

Figure 4: Levels of securities corresponding to the level of volatility and holding time



Source: Elmore (2017).

Since Chaffe's model is based on the European put option which can only be exercised at the end of the holding period, the value of the option is lower compared to the American put option that can be exercised earlier. This means that Chaffe's model is downward biased and as such represents a minimum applicable discount (Elmore, 2017). Estimated values of DLOM obtained by the Chaffe's model are shown in the Table 5.

Table 5: DLOM obtained from Chaffe's Black-Scholes put option model

Volatility (σ)	**Holding period (T) - years**						
	0.25	**0.50**	**0.75**	**1**	**2**	**3**	**4**
25%	4.7%	6.5%	7.8%	8.9%	11.8%	13.8%	15.2%
50%	9.7%	13.5%	16.3%	18.6%	25.2%	29.6%	33.0%
75%	14.6%	20.3%	24.5%	28.0%	37.7%	44.1%	48.8%
100%	19.4%	27.0%	32.5%	36.9%	49.1%	56.7%	61.8%
125%	24.2%	33.5%	40.1%	45.4%	59.2%	67.1%	72.0%
150%	28.9%	39.7%	47.3%	53.1%	67.8%	75.4%	79.5%
200%	37.9%	51.3%	60.2%	66.6%	80.7%	86.1%	87.9%
250%	46.4%	61.5%	70.8%	77.1%	88.5%	91.2%	91.1%
300%	54.3%	70.3%	79.3%	84.8%	92.8%	93.3%	92.1%

Source: Elmore (2017).

It can be seen that the longer holding period and the higher volatility of the underlying asset result in greater size of the discount and vice versa.

4.1.2 The Longstaff model

Longstaff published his study in 1995. He shows an analytical upper bound in the value of marketability. Study is based on the American put option and the intuition behind the model is that hypothetical investor has the ability of perfect market timing, but also has a restriction from selling his asset until a certain time. If there were no restriction investor would be able to sell his asset at the time the asset reaches its maximum value and his incremental cash flow would be essentially the same as if he would have swapped the value of the asset at the end of the restriction period for the asset's maximum value during the restriction period. The

liquidity swap's present value represents hypothetical investor's value of the marketability and upper bound for the actual investor without perfect market timing ability (Longstaff, 1995).

Longstaff (1995) shows that DLOM can be large even in the case of short-term marketability restriction and that upper bound presents good benchmark for estimation of valuation effects of marketability restrictions.

The framework can be seen in the equation (2) below. Its advantage is that it is not necessary to make all the assumptions about information asymmetry, investor preferences or others that would be necessarily in the equilibrium model, but on the other hand that results in obtained bounds rather than explicit model of the level of marketability (Longstaff, 1995).

$$dV = \mu V dt + \sigma V dZ, \qquad (2)$$

where V denotes current or time zero value of the asset,
μ denotes average value of the asset,
σ denotes standard deviation of the value of the asset,
Z denotes standard Wiener process.
It is considered that riskless interest rate r is constant.

Value of the asset for investor that cannot sell it until time T which represents the end of restriction period is equal to the present value of the cash flow of the asset at time T, denoted by VT. If the investor does not have time restriction and has perfect market timing ability, he or she is able to sell the asset at its highest value and reinvest it in the riskless asset at time τ. In this case, investor would obtain MT which denotes present value of the cash flow of the asset at time τ. While investor is limited with restrictions, he or she cannot benefit from the perfect market timing ability. Difference in the value of the MT and VT imposes important opportunity costs and because of that it can be said that the value of the marketability for the investor with the ability of perfect market timing is the incremental cash flow (MT – VT) that investor would receive if marketability restrictions were omitted. For the investor without the perfect market timing ability the value of the marketability would be lower, since he or she cannot determine the highest value of the asset (Longstaff, 1995).

Because of all of the above, present value of the incremental cash flow represents an upper bound on the value of the marketability and it is captured in the following equation (3) (Longstaff, 1995).

$$F(V,T) = e^{-rT}E[M_T] - e^{-rT}E[V_T], \qquad (3)$$

where $F(V, T)$ denotes present value of the $M_T - V_T$,
V denotes the risk-neutral dynamics for V,

e denotes mathematical constant – the base of the natural logarithm.

Further development of the equation (3) leads to the Longstaff's lookback put option model that is as follows in the equation (4) (Longstaff, 1995):

$$F(V,T) = V\left(2 + \frac{\sigma^2 T}{2}\right) N\left(\frac{\sqrt{\sigma^2 T}}{2}\right) + V\sqrt{\frac{\sigma^2 T}{2\pi}}\, e\left(-\frac{\sigma^2 T}{8}\right) - V, \qquad (4)$$

where $N(\bullet)$ denotes cumulative probability function for a normal distribution,
Π denotes mathematical constant.

As it can be seen in the Table 6, upper bound increases with the increase of the length of marketability restriction (T) and with the increase of the volatility (σ) of the return since higher return volatility means higher opportunity cost of inability to trade. Upper bound represents the highest value of DLOM (Longstaff, 1995).

Table 6: DLOM obtained from Longstaff's lookback put option model

Volatility (σ)	**Holding period (T) - days**						
	1	**5**	**10**	**20**	**30**	**40**	**90**
1.25%	5.3%	11.8%	16.6%	23.5%	28.8%	40.8%	50.0%
2.5%	10.5%	23.5%	33.3%	47.1%	57.7%	81.7%	100.1%
5%	21.0%	47.1%	66.7%	94.4%	115.7%	163.9%	201.0%
	Holding period (T) - years						
	0.25	**0.50**	**0.75**	**1**	**2**	**3**	**4**
20%	8.2%	11.8%	14.6%	17.0%	24.6%	30.8%	36.1%
50%	21.6%	31.5%	39.5%	46.6%	70.1%	90.0%	108.1%
75%	33.6%	49.8%	63.3%	75.3%	116.7%	153.0%	186.9%
100%	46.6%	70.1%	90.0%	108.1%	172.0%	229.9%	284.9%
125%	60.4%	92.3%	119.8%	145.2%	236.9%	321.9%	404.0%
150%	75.3%	116.7%	153.0%	186.9%	311.9%	430.0%	545.4%
200%	108.1%	172.0%	229.9%	284.9%	494.3%	697.5%	898.9%
250%	145.2%	236.9%	321.9%	404.0%	722.8%	1036.8%	1349.8%
300%	186.9%	311.9%	430.0%	545.4%	999.2%	1449.8%	1900.0%

Source: Elmore (2017); Longstaff (1995).

Values of DLOM obtained with the Longstaff's model are substantially higher compared to the ones obtained with the restricted studies and also exceed the 100% discount at reasonable levels of returns' volatility. Because of that Abbot transformed and adjusted the Longstaff's model (Elmore, 2017).

4.1.3 Transformed Longstaff model

Abbott (in Elmore, 2017) believes that value of the put option premium estimating the cost of liquidity is incorrectly presented as DLOL and neglecting conversion of option premium

into applicable discount creates an illusion of discounts being greater of 100%. He adjusted Longstaff's model so that discount is calculated as follows in the equation (5):

$$DLOM = \frac{Put\ value}{1 + Put\ value} \tag{5}$$

Obtained results can be seen in the Table 7. Obtained values still represent an upper bound but they do not exceed 100%.

Table 7: DLOM obtained from transformed Longstaff's lookback put option model

Volatility (σ)	**Holding period (T) - years**						
	0.25	**0.50**	**0.75**	**1**	**2**	**3**	**4**
20%	7.6%	10.6%	12.7%	14.5%	19.8%	23.5%	26.5%
50%	17.7%	23.9%	28.3%	31.8%	41.2%	47.4%	51.9%
75%	25.2%	33.3%	38.8%	43.0%	53.8%	60.5%	65.1%
100%	31.8%	41.2%	47.4%	51.9%	63.2%	69.7%	74.0%
125%	37.7%	48.0%	54.5%	59.2%	70.3%	76.3%	80.2%
150%	43.0%	53.8%	60.5%	65.1%	75.7%	81.1%	84.5%
200%	51.9%	63.2%	69.7%	74.0%	83.2%	87.5%	90.0%
250%	59.2%	70.3%	76.3%	80.2%	87.8%	91.2%	93.1%
300%	65.1%	75.7%	81.1%	84.5%	90.9%	93.5%	95.0%

Source: Elmore (2017).

Like in the case of the Chaffe model it can be seen that the longer holding period and the higher volatility of the underlying asset result in the greater size of the discount and vice versa. The Longstaff's lookback put option model and Finnerty's average-strike put option model that will be presented below avoid Black-Scholes model assumption that in the restricted period the strike price is fixed by the adjustment of the strike price in the way that during the period of non-marketability it reflects the changing price of the unrestricted asset (Finnerty, 2016).

4.1.4 The Finnerty model

Black-Scholes model represents the beginning of the DLOM estimates by POPM. Their model estimates DLOM with respect to the holder's loss of flexibility to sell the stock at current market price when holder is restricted to sell asset for a certain period of time. Assumption that in the restriction period the strike price is fixed limits the model's applicability to DLOM since the future assets' prices are not fixed. Holder's loss of timing flexibility should be measured relative to the opportunity to sell the asset under no restrictions at any time at any market price (Finnerty, 2016).

While Longstaff's assumption of investors perfect market timing is inconsistent with empirical evidence, Finnerty designed his model to estimate DLOM as the value of the average-strike put option where investor does not have this ability, but he assumed that the

probability of investor to sell his or her interest share is equal for any time in the restriction period (Finnerty, 2016). Because of that it can be said that Finnerty's model is an upgrade of the Longstaff's model.

Finnerty's model is based on the Asian put option and the average-strike put model is derived by the lognormal approximation that results in an effective cap on the volatility and the discount, but it can become problematic in cases where restriction period for the high-volatile stocks exceed the period of four years. The model gives the best results when restriction period is no longer than two years. The shorter the restriction period is the more accurate the model is (Finnerty, 2016). Finnerty's formula to calculate one-year DLOM is shown in the equation (6) below (Finnerty, 2016).

$$D(T) = V_0 e^{-qT}\left[N\left(\frac{v\sqrt{T}}{2}\right) - N\left(-\frac{v\sqrt{T}}{2}\right)\right] \tag{6}$$

$$v\sqrt{T} = \left[\sigma^2 T + ln\left\lfloor 2\left\{e^{\sigma^2 T} - \sigma^2 T - 1\right\}\right\rfloor - 2ln\left\lfloor e^{\sigma^2 T} - 1\right\rfloor\right]^{1/2},$$

where *N(•)* denotes cumulative standard normal distribution function,
D(T) denotes proportion to the current share price – DLOM,
V_0 denotes value of the otherwise identical unrestricted interest,
σ denotes annualized volatility of the underlying security,
T denotes holding period – time of put option expiration,
r denotes risk-free interest rate,
e denotes mathematical constant – the base of the natural logarithm,
q denotes annualized volatility of the underlying security,
v denotes volatility of the ratio between the average forward contract price and the price of the underlying unrestricted share.

It can be noticed that DLOM is directly related to the stock's price volatility σ, directly related to the length of the transfer restriction period *T* and inversely related to the stock's dividend yield *q* (Finnerty, 2016). The results of the Finnerty's average-strike put option model are shown in the Table 8.

Table 8: DLOM obtained from Finnerty's average-strike put option model

Volatility (σ)	Holding period (T) - years						
	0.25	**0.50**	**0.75**	**1**	**2**	**3**	**4**
20%	2.3%	3.3%	4.0%	4.6%	6.5%	7.9%	9.1%
50%	5.7%	8.0%	.9.8%	11.2%	15.5%	18.5%	20.9%
75%	8.5%	11.9%	14.4%	16.3%	21.8%	25.3%	27.5%
100%	11.2%	15.5%	18.5%	20.9%	26.6%	29.5%	30.9%
125%	13.9%	18.8%	22.2%	24.6%	29.7%	31.5%	32.1%
150%	16.3%	21.8%	25.3%	27.5%	31.4%	32.1%	32.3%
200%	20.9%	26.6%	29.5%	30.9%	32.2%	32.3%	32.3%
250%	24.6%	29.7%	31.5%	32.1%	32.3%	32.3%	32.3%
300%	27.5%	31.4%	32.1%	32.3%	32.3%	32.3%	32.3%

Source: Elmore (2017).

This model may not generate reliable DLOM for the private companies' securities and for the valuing restricted stocks when liquidity event may not occur for time period of seven years or more (Finnerty, 2016). Due to this, limitation model was generalized for a longer restriction period.

At this point presentation of three theoretical solutions to estimate DLOM is completed. It follows a summarization of obtained results and cognitions as well as a presentation of key findings of some other theoretical solutions to obtain additional information.

4.2 Private company discount

Privately held company valuation is difficult and often subjective process. This is because private companies do not have observable market values, since prices of company shares that represents objective measure of market value are not freely traded on the market. Also, it is not possible to use other private companies to compare valuated company to them, since they also do not have observable market values (Koeplin et al., 2000).

The approach one can use to estimate the value of private company is to find a set of comparable transactions, that is acquisitions of similar companies in the similar industry at around the same time horizon, and calculate valuation multiples of different financial parameters and then apply these multiples to the values of valuated company parameters. These multiples do not provide exact value of company being valuated but do provide evidence on the prices paid on a comparable transaction (Koeplin et al., 2000).

Restricted stock and pre-IPO studies are based on the data of public companies and are mainly used in the valuation process of non-controlling interests. Some authors believe that in the valuation process of ownership interest more direct methods should be used. One of these methods is private company discount (hereinafter "PCD") method. This is a method that is not so commonly used since it represents a newer guidance (Zanni, 2013). In practice,

there is a lack of modern approaches that estimate appropriate level of DLOL. Newer approaches are able to fix some shortcomings of older approaches.

DLOL can be estimated as a difference between valuation multiples in transactions of private and public companies where controlling ownerships interests were acquired. Researchers try to account for differences between private and public companies but cannot block out all of them. Therefore, they acknowledge that differences in valuation multiples of private and public companies are explained by liquidity and other factors included in the PCD (Dodel, 2014). Due to the PCD, method can be used as a DLOL.

In theory, two types of PCD studies can be noticed. The first type are studies based on an acquisition approach, that is a comparison of public and private companies' transactions, while the second type of studies are pricing multiples studies that are based on publicly traded companies' stock prices and the private companies' enterprise values (hereinafter "EV") derived from transaction (Zanni, 2013).

In the following pages four acquisition multiple studies and one pricing multiple study are presented. These studies are:

- Koeplin, Sarin and Shapiro study (2000).
- Kooli, Kortas and L'Her study (2003).
- Officer study (2007).
- Block study (2007).
- Paglia and Harjoto study (2010) – pricing multiple study.

4.2.1 The Koeplin, Sarin and Shapiro study

The Koeplin, Sarin and Shapiro study was published in 2000. In the continuation of the master's thesis, it will be referred to as Koeplin study. The study is one of the most recognizable and most cited studies in the field of PCDs using the acquisition approach. The main goal of the study is to find a set of companies comparable with the one being valuated by industry and the time of acquisition, and calculate multiples of different financial parameters. Obtained multiples are then compared between transactions of private and public companies to obtain a discount (Koeplin et al., 2000).

Authors believe that quality and reliability of this approach depend on the availability of similar companies' transactions that poses similar value characteristics as the valuated company. Those characteristics are risk, capital structure, growth rate, size, and timing of cash flows and liquidity. The latter represents the biggest difference between private and public companies (Koeplin et al., 2000).

Koeplin study takes into observation time period between 1984 and 1998. In the study only companies for which necessary data was available were included but all financial and

regulated utilities companies were excluded. From suited private companies they choose only companies where the subject of transaction was controlling ownership interest. Finally, they matched chosen private companies with the best fitting comparable public companies. Match up was made for each private company individually. Best fit was estimated by the comparison of operating industry, acquisition time and the size of the company. At the end, considering all the selection criteria, the sample contains 84 transactions in the USA and 108 transactions in non-USA countries (Koeplin et al., 2000).

Authors chose four different financial parameters in the process. EBIT and EBITDA were selected rather than after-tax earnings because of their independency from the capital structure and they both measure company's cash flow availability to pay off the debt and pay dividends. Other two selected financial parameters are sales and book value. All these four financial parameters are compared to the EV of an individual company. Obtained multiples are commonly used in the merger and acquisition (hereinafter "M&A") analysis to see whether the price paid or offered is appropriate (Koeplin et al., 2000). All used parameters and associated multiples are described in the following Table 9.

Table 9: Used valuation multiples in the Koeplin study

Variable	Description
Enterprise value	EV is calculated by multiplying number of outstanding shares with offering price adding the book value of short-term debt, straight debt, convertible debt and preferred stock reduced by the value of marketable securities based on the financial information closest to the announcement of the transaction.
EBIT multiple	EV to EBIT ratio. EBIT is defined as earnings before interest income and expenses, non-operating income, taxes and minority interest for last twelve months (hereinafter "LTM") closest to the announcement of the transaction.
EBITDA multiple	EV to EBITDA ratio. EBITDA is defined as earnings before interest, taxes, depreciation and amortisation for LTM closest to the announcement of the transaction.
Sales multiple	EV to sales ratio. Sales remain primary source of revenues after taking into account returned goods and allowances for price reductions. LTM closest to the announcement of the transaction are taken into the calculation.
Book multiple	EV to book value ratio. Into book value of the company long- and short-term debt and equity closest to the announcement of the transaction are included.

Source: Koeplin et al. (2000).

Results obtained in the study show that the value of public companies' multiples are higher compared to the values of private companies' multiples. This implies that private companies are sold at a discount compared to the public companies which is in line with the theory that investors prefer more liquid assets that are quickly convertible into cash compared to the less liquid assets (Koeplin et al., 2000). Public companies are such assets compared to the private companies and the difference in values of their multiples is captured in DLOL. Discount for every estimated multiple was calculated by the following equitation (7).

$$Private\ Company\ Discount = 1 - \left(\frac{Private\ Company\ Multiple}{Public\ Company\ Multiple}\right) \quad (7)$$

Four different discounts were calculated, based on every financial parameter, for USA and non-USA markets. Results are shown in the Table 10 and it can be noticed that in all cases private companies' multiples are smaller than corresponding multiples of public companies.

Table 10: Private Company Discounts – Koeplin study results

	Private companies		**Public companies**		**Discount**	
	Average	**Median**	**Average**	**Median**	**Average**	**Median**
USA transactions						
EV/EBIT	11,8x	8.6x	16.4x	12.4x	28.3%***	30.6%***
EV/EBITDA	8,1x	7.0x	10.2x	8.5x	20.4%***	18.1%***
EV/Book value	2,4x	1.9x	2.9x	1.7x	17.8%***	-7.0%
EV/Sales	1,4x	1.1x	1.3x	1.1x	-2.3%	0.8%
Non-USA transactions						
EV/EBIT	16,3x	11.4x	29.0x	12.1x	43.9%***	6.0%**
EV/EBITDA	12,0x	7.1x	25.9x	9.3x	53.9%**	23.5%*
EV/Book value	2,4x	1.4x	3.7x	1.7x	34.9%	19.6%
EV/Sales	2,6x	1.4x	4.6x	1.6x	42.7%	17.2%

*Note: Statistically significant at *** - 0.01; ** - 0.05; * - 0.1.*
Source: Koeplin et al. (2000).

Obtained results show that discounts for USA companies are lower compared to non-USA companies. Average value of discounts for USA companies moves between 17.8% and 28.3%, while for non-USA companies it moves between 34.9% and 53.9%. It has to be pointed out that different values of discounts were obtained at different significance levels which can affect one's decision in regard to the chosen discount.

Problem with the study that authors expose is that acquired private companies are smaller and so have different growth rates compared to the matched public companies. That could be the reason for differences in estimated values of discounts. To control for this shortcoming, they estimated regression model for sample transactions relating to size, growth, multiple and dummy variable. While results for non-USA companies were statistically significant only in the case of discount calculated with EBIT multiple, results for the USA companies indicate that results are insignificant for sales and book valuation multiples, which leaves some doubt on the study's results.

4.2.2 The Kooli, Kortas and L'Her study

One of their first highlights in the Kooli, Kortas and L'Her study published in 2003 is that standard techniques for estimating private company risk are in general ignored because of

the absence of a ready market. Furthermore, they highlighted the problem of hard to sell asset. It means that investor is going to demand discount on the price of the asset, since he or she is faced not only with the downside in terms of cash flows, but also with missed opportunities (Kooli et al., 2003). In the continuation of the master's thesis it will be referred to as Kooli study.

Their selection of suitable companies for research is quite straightforward. They identified all the suitable M&A transactions of public and private companies in the period from 1995 until 2002 in the DoneDeals and SDC databases. At the end, the sample contains 331 companies' transactions of controlling ownership interests, of which majority are USA companies (Kooli et al., 2003).

Used multiples are price to sales ratio, price to earnings ratio, and price to cash flow ratio. Unlike Koeplin study where they used technique of matching public companies Kooli study used reference portfolio over the matching company. They believe that public transactions included into a benchmark multiple are more suitable, more rationally priced and less influenced by arbitrary factors (Kooli et al., 2003).

For every private company transaction included in the study they create a benchmark portfolio of public companies' transactions. Private companies were ranked based on the year, industry and size. They form a size-quartile portfolios and for each quartile breakpoint was calculated. Companies were ranked based on the same parameters and with same procedure. This means that for each private company transaction control public portfolio including all transaction of public companies with the same parameters used for ranking private companies was created (Kooli et al., 2003). To calculate discount, the following equation (8) was used.

$$Discount_{k,s,t,i} = 1 - \left(\frac{Private\ company\ multiple_{k,s,t,i}}{Median\ public\ company\ multiple_{k,s,t,i}} \right), \qquad (8)$$

where k, s, t and i denote valuation multiple, transaction size quartiles, time period from 1995 till 2002, and industry, respectively. Classification of companies by industry is shown in the Table 11.

Table 11: Industrial Classification of 331 Private Companies included into the Kooli study

Industry	SIC Code	Number	Frequency
Agriculture and mining	01-14	15	4.5%
Construction	15-17	3	0.9%
Manufacturing	20-39	94	28.4%
Transportation and communication	40-49	22	6.6%
Wholesale and retail trade	50-59	43	13.0%
Finance, insurance and real estate	60-67	71	21.5%
Services	70-88	83	25.1%
Total		**331**	**100.0%**

Source: Kooli et a. (2003).

As it can be seen, the majority of companies included in the study operate in manufacturing industry (28.4%), services (25.1%) and finance, insurance and real estate industry (21.5%). In following two tables, results of estimated discounts by the year from 1995 until 2002 and by the industries are shown.

Table 12 shows the time profile of the medians of three discounts in the time period from 1995 until 2002. Movements in the value of the discount can be explained by the M&A activities in the market. This means that in the years with higher number of transactions the discount is lower and vice versa. For example, discounts in years 1999 and 2000 are lower compared to the previous years, due to the increased investors' confidence in the time of the internet bubble (Kooli et al., 2003).

Table 12: Discounts median movement between 1995-2002

Year	Price to Sales	Price to Earnings	Price to Cash flow
1995	20.2%	19.7%	/
1996	-111.7%*	29.2%**	-52.7%
1997	36.4%**	50.7%*	-5.6%
1998	46.7%**	43.4%**	36.8%**
1999	6.1%	30.2%*	-21.2%
2000	46.8%**	34.1%**	24.8%
2001	-8.8%	40.9%	25.2%
2002	3.3%	9.3%	27.4%

*Note: Statistically significant at * - 0.01; ** - 0.05; *** - 0.1.*

Source: Kooli et al. (2003).

In the Table 13 discount medians across seven industries are shown. The highest value of discount, based on the Price to Sales multiple, was obtained for construction and wholesale and retail trade industries where discounts can go up to 70.2% and 60.1%, respectively.

Table 13: Discounts movement between industries

Industry	Price to Sales	Price to Earnings	Price to Cash flow
Agriculture and mining	-58.6%	49.0%	31.5%
Construction	70.2%	59.0%	19.1%
Manufacturing	36.7%*	30.5%**	21.6%
Transportation and communication	-30.3%	18.1%	21.6%
Wholesale and retail trade	60.1%*	55.7%*	-10.4%
Finance, insurance and real estate	-35.3%**	29.2%*	3.8%
Services	15.4%	33.6%**	34.1%

*Note: Statistically significant at * - 0.01; ** - 0.05; *** - 0.1.*

Source: Kooli et al. (2003).

Overall results of the study show that private company discounts calculated with Price to Sales, Price to Earnings and Price to Cash flows multiples amounted to 17%, 34% and 20%, respectively, but there is a variance in their values depending on the individual company and industry in which company operates (Kooli et al., 2003).

4.2.3 The Officer study

Officer in his study, published in 2007, highlights that sales of private companies and their subsidiaries became an important source of liquidity and restructuring for corporations. Additionally, the number of private companies' transactions was increasing and because of that M&A market of private companies was becoming as important as M&A market of public companies (Officer, 2007).

Main focus of the study is determination of multiples paid to acquire private company, followed by the analysis of impact of target's pre-sale financial conditions on a level of a discount, as well as an impact of equity and debt market characteristics at the time of the sale. Private companies cannot be traded easily but can solve this problem by going public, while parent companies can spin-off their subsidiaries into public market. Both processes are included in the study and can be accompanied by substantial transaction costs. This can result in a lower demand in a sale process for private companies compared to the demand volume in the sale process of public companies. Result of this is a sale of private company at the discount compared to the public company. In the case of the subsidiaries' sales by parent companies, this action can be understood as the parent companies' desire for liquidity and it should be reflected in the paid price (Officer, 2007).

Officer takes into account the time period from 1979 until 2003. The beginning sample contained 12,716 successful and unsuccessful bids. In the final sample he included only bids for at least 50.0% ownership interest with deal value greater than USD 50 million. Bids included in the study were all-cash, all-stock bids or their combination. The total sample size was reduced to roughly 3,000 companies (Officer, 2007).

To choose companies into the study comparable industry transaction technique was used. It was implemented in three stages. First, for every private company public target in the same two-digit SIC code was found. Second, deal value of public company excluding assumed liabilities within 20,0% of the private company target deal value. And third, public company's acquisition has to be announced within the three-year window compared to the announcement of the private company's acquisition (Officer, 2007).

In the study, four different acquisition multiples are examined. These are price to book value of equity, price to earnings per share, deal value to EBITDA and deal value to sales. This means that all multiples are calculated as the ratio between the value offered by bidder divided with the value of the company's financial parameter one year before the acquisition attempt (Officer, 2007). The results of the study can be seen in the Table 14.

Table 14: Private Company Discounts – Officer study results

	Stand-alone			**Subsidiary**		
	Average	**Median**	**N**	**Average**	**Median**	**N**
Excess price to book value of equity	-15.6%	-15.2%	106	27.5%	35.2%	145
Excess price to earnings per share	22.9%	27.8%	148	28.9%	38.0%	136
Excess deal value to EBITDA	17.2%	20.1%	111	26.9%	35.1%	107
Excess deal value to sales	18.2%	18.7%	308	30.0%	40.9%	590
Average acquisition discount	17.3%	19.5%	364	28.3%	36.0%	643

Source: Officer (2007).

Average value of the discount moves between 17.2% and 30.0% for private companies and unlisted subsidiaries, compared to a public company. The highest average value of discount was obtained when it was calculated from price to earnings per share multiple for private companies (22.9%) and deal value to EBITDA multiple for unlisted subsidiary companies (30.0%). From the study it can be concluded that parent companies are willing to sell assets with a price discount in the case of liquidity need and that greater that need is the higher is the price discount.

4.2.4 The Block study

Block followed the approach used in the Koeplin study but with some modifications. His study was published in 2007. He compared privately held companies to publicly traded companies from the same industry and traded at the same time period. Study included transactions in the years between 1999 and 2006. In the study were included only transactions of the controlling ownership interest. 91 private companies and 91 matched public companies were included in the study (Block, 2007).

In the Block study the same concepts are used as in the Koeplin study, but time period moved from period 1984–1998 to period 1999–2006. Included companies were divided into eight

industry groups to determine if there is any difference in the liquidity discount between them (Block, 2007). Groups deviation can be seen in the Table 15.

Table 15: Industrial Classification of 91 Private Companies included into Block study

Finance (9)	**Manufacturing (19)**	**Technology (12)**	**Retail (28)**
Commercial Banks	Automobile	Aerospace	Household Products
Insurance Companies	Chemicals	Computers	Beverages
Saving institutes	Electronics	Computer Software	Food Products
Brokerage Firms	Metals	Engineering	Drugstores
		Semiconductors	Sporting Goods
Energy (5)	**Healthcare (12)**	**Transportation (5)**	**Utilities (1)**
Oil and Gas	Hospitals	Airlines	Electric Utilities
Energy Services	Pharmaceuticals	Railroads	
Petroleum Refining	Medical Products		
Pipelines			

Source: Block (2007).

Block used the same valuation multiples as they were used in the Koeplin study. These multiples are EV to EBIT, EV to EBITDA, EV to Book value and EV to Sales. Block included the fifth multiple, that is Price/Earnings per share. Results of the study can be seen in the Table 16.

Table 16: Private Company Discount by different multiple – Block study results.

	Private companies		**Public companies**		**Discount**	
	Average	**Median**	**Average**	**Median**	**Average**	**Median**
Price/Earnings per share	14.0x	10.3x	18.7x	13.4x	25.2%	23.3%
EV/EBIT	12.2x	9.1x	16.7x	12.0x	27.1%	24.3%
EV/EBITDA	9.1x	7.2x	12.1x	9.3x	24.6%	22.5%
EV/Book value	2.4x	2.0x	2.9x	2.4x	16.3%	14.5%
EV/Sales	2.1x	1.2x	2.0x	1.5x	26.4%	24.5%

Source: Block (2007).

As in the Koeplin study, the lowest average value of discount is obtained when it is calculated with EV to Book multiple. Its average amounted to 16.3%, calculated on the entire sample. The highest average value of the discount is obtained when it is calculated with EV to EBIT multiple. Its average value is 27.1%. Overall, discounts cluster around 25.0%.

Block believe that the nature of the assets can affect companies' liquidity and liquidity discount. There is an important difference between assets of the commercial bank or other

financial institutions that are highly liquid and assets of some manufacture company that has very specific equipment that has low liquidity. To assess impact of the industrial classification, companies were divided into groups and within every group average value and median of private and public companies were compared and discount calculated (Block, 2007). In Table 17 results for each industry are shown.

Table 17: Liquidity Discount Associated with Industry Classification

	Price/Earnings per share		**EV/EBIT**		**EV/EBITDA**	
	Average	**Median**	**Average**	**Median**	**Average**	**Median**
Energy	28.3%	25.2%	30.1%	26.2%	29.3%	27.0%
Manufacturing	37.2%	32.2%	40.1%	35.1%	36.1%	30.2%
Finance	10.0%	11.2%	12.1%	9.1%	13.0%	10.2%
Utilities	21.0%	21.0%	23.2%	23.2%	25.1%	25.1%
Technology	26.2%	19.0%	24.7%	21.2%	28.1%	24.1%
Retail	24.2%	21.6%	28.7%	22.1%	26.3%	21.6%
Healthcare	30.2%	26.1%	32.2%	28.4%	27.1%	22.9%
Transportation	26.3%	24.2%	27.1%	25.0%	26.3%	25.2%

	EV/Book value		**EV/Sales**	
	Average	**Median**	**Average**	**Median**
Energy	20.1%	16.0%	23.9%	19.7%
Manufacturing	25.2%	22.7%	31.1%	29.1%
Finance	7.6%	2.1%	8.4%	6.6%
Utilities	15.4%	15.4%	26.4%	26.4%
Technology	14.9%	12.8%	25.2%	24.1%
Retail	17.9%	14.6%	23.1%	22.2%
Healthcare	19.0%	17.3%	26.1%	24.3%
Transportation	18.3%	16.6%	27.2%	26.4%

Source: Block (2007).

Two industries stand out. First one is the financial industry where average discount deviates from 7.6% to 13.0% among five estimated multiples and is by far the lowest. The other industry is manufacturing where four out of five multiples give average discount of over 30.0%. Other companies reach discount values between these two extremes. It can be seen that discounts vary among industries and because of that it is important that in the valuation process the industry in which company operates is taken into account and not some generalized consensus is used.

4.2.5 The Paglia and Harjoto study

Study was published in 2010 and it is known as Paglia study. Authors believe that four acquisition multiples studies presented above have one major downfall and that is a problem of finding suitable matches for private companies among public companies. A significant problem for above studies is also the sample size, since some of them have less than 100 pairs of companies' transactions taken into research (Zanni, 2013).

In the study they used data from Pratt's Stats database for private companies and Compustat database for publicly traded companies. In research, they are focusing on the time period from 1994 till 2008 and have excluded utilities and other services companies. Also, they included only private companies with annual net sales of at least USD 50 million, since there is a higher probability that those companies became public. To match the private companies with public companies they used criteria based on six digits of North America Industry Classification System (hereinafter "NAICS"), year of transaction and annual net sales. They put additional focus to use one-year lag public companies' data, since private companies' financial data is usually lagged one year as of the sale data. They also focused on that used public companies stayed listed for at least three years from a matching date, because of the duration of selling process of a private company that can take on average from six up to nine months (Paglia & Harjoto, 2010).

In the Paglia study discount was calculated based on two multiples; the market value of invested capital (hereinafter "MVIC") to annual sales multiple, and MVIC to EBITDA multiple. MVIC for private companies was taken from Pratt's stats database, while MVIC for public companies was calculated as total number of outstanding common shares multiplied by fiscal year stock price plus book values of long-term debt and preferred stock. Discounts were calculated with equations (9) and (10) shown below (Paglia & Harjoto, 2010):

$$DLOMSALE\ (\%) = \left(1 - \frac{\frac{MVIC}{Sale}\,for\ private\ company}{\frac{MIVC}{Sale}\,for\ public\ company}\right) * 100 \qquad (9)$$

$$DLOMEBITDA\ (\%) = \left(1 - \frac{\frac{MVIC}{EBITDA}\,for\ private\ company}{\frac{MIVC}{EBITDA}\,for\ public\ company}\right) * 100 \qquad (10)$$

Discount was calculated across different industries based on the two digits of NAICS and in the final research they included only pairs of private and public companies that provide discount between 0.0% and 100.0%. Final sample included 431 companies for discount calculated from MVIC to annual sales ratio and 283 companies for discount calculated from

MVIC to EBITDA ratio (Paglia & Harjoto, 2010). Results of the study can be seen in the Table 18.

Table 18: Private Company Discounts – Paglia study results

Sector	2 digits NAICS	MVIC/annual sales		MVIC/EBITDA	
		No. of firms	Average	No. of firms	Average
Mining	21	20	55.2%	14	48.0%
Construction	23	14	65.5%	12	53.1%
Manufacturing	31	163	55.4%	91	49.6%
Wholesale trade	42	22	60.9%	20	42.4%
Retail trade	44	17	70.8%	16	44.5%
Transportation	48	9	57.5%	9	29.0%
Information	51	63	57.2%	47	60.2%
Finance and Insurance	52	22	70.0%	13	48.1%
Real Estate	53	5	65.5%	6	53.1%
Professional services	54	63	66.3%	36	67.7%
Staff support &Waste management	56	13	66.9%	6	34.2%
Healthcare	62	9	40.7%	6	29.3%
Art and Entertainment	71	2	60.3%	2	17.9%
Accommodation and Food services	72	9	58.9%	5	68.5%
Number of companies		**431**		**283**	

Source: Paglia & Harjoto (2010).

Measuring discount with the MVIC to annual sales ratio shows that the highest average value of discount is obtained in the retail trade industry (70.8%), while the lowest average value is obtained in the healthcare industry (40.7%). Measuring discount with the MVIC to EBITDA shows that the highest average value of discount is obtained in accommodation and food services (68.5%), while the lowest average value is obtained in the art and entertainment industry (17.9%). It can be seen that larger and more profitable companies, companies with positive net income and companies bought by publicly traded buyers have lower values of discount.

In the Appendix 2 the summarization of the theoretical solutions and PCD studies is given.

4.3 Studies on how large illiquidity discount experts apply

In practice, it can be noticed that market participants do not take enough time and care to apply the appropriate level of DLOL. It seems they just use an average value of existing studies or rely too heavily on the researches of DLOL for minority interest. Reasons to apply DLOL are specific for every company. Many times, characteristics of an individual company are not taken into account. This means that market participants do not examine company, its

market position and macroeconomic environment well enough. Taking all this into account, two studies that take more qualitative approach are presented below. These studies are:

- Petersen, Plenborg and Schøler study (2006).
- Vydržel and Soukupova study (2012).

Both studies used surveys and interviews to obtain information about market participants deals with critical components in the valuation of private companies. First one was made in the Danish market, while the other study was performed on the Czech market.

4.3.1 The Petersen, Plenborg and Schøler study

The main focus of the Petersen, Plenborg and Schøler study is presenting how financial advisors and private equity funds managers use present value approaches to determine the value of private companies. Their study was published in 2006. In the continuation, the study will be referred to as the Petersen study.

In the study, they are focusing on a Danish market and as authors said they are the first ones to examine the issue of how concerned market participants deal with the problem of using present value approaches to determine the value of private company (Petersen et al., 2006)

In the sample they included investors and financial advisors. Final sample included 42 individuals and their division into groups can be seen in Table 19. Because of the more qualitative approach used in the study the obtained information about corporate behaviour is unique and cannot be obtained with large sample studies because of the inability to ask qualitative questions (Petersen et al., 2006).

Table 19: Petersen study – Participants and response rate

	Number of target participants	**Number of participants included in study**	**Response rate**	**Addition of a present value approach**
Private equity funds	15	15	100.0%	14
Dependent financial advisors	12	11	91.7%	11
Independent financial advisors	17	16	94.1%	14
Total	**44**	**42**	**95.5%**	**39**

Source: Petersen et al. (2006).

Out of 44 target participants only two were not willing to participate in the survey, thus the basic sample included 42 participants. As additional criteria participants had to apply present value approach as their way to determine the value of private company, since authors are also focusing on this method. 14 private equity funds and 25 corporate financial advisors applied present value approach to estimate the value of private company, thus final sample included 39 participants. Selected participants were asked to select which present value approach they use. Obtained results can be seen in Table 20. Participants could choose more

than one model or method and because of that number of answers does not match with the number of participants, it exceeds it.

Table 20: Petersen study – Used model and methods

	Dependent financial advisors	Independent financial advisors	Private equity funds	Total
DDM	5 45.5%	2 14.3%	0 0.0%	7 17.9%
DCF	11 100.0%	12 85.7%	11 78.6%	34 87.2%
EVA	5 45.5%	1 7.1%	2 14.3%	8 20.5%
APV	1 9.1%	0 0.0%	0 0.0%	1 2.6%
Other (LBO)	1 9.1%	3 21.4%	8 57.1%	12 30.8%
Number of Participants	**11**	**14**	**14**	**39**

Note: DDM – Dividend Discount Model; DCF – Discounted Cash Flow Model, EVA – Economic Value Added; APV – Adjusted Present Value; LBO – Leverage Buyout model.
Source: Petersen et al. (2006).

To select the used valuation model and methods participants could choose more than one option and because of that number of answers exceeds the number of participants. It can be noticed that DCF model is significantly more used. 87.2% of participants use DCF model, while only 20.5% use EVA model. In most cases, EVA model is used to check the results obtained with DCF model. The DDM model is primarily used in valuing banks and utility services companies. LBO model is most frequently used by private equity funds as they use it to calculate internal rate of return (Petersen et al., 2006).

Petersen study shows that participants apply DLOL as a discount to adjust multiple or final value or as a risk premium incorporated into discount rate. Results are shown in Table 21.

Table 21: Petersen study – Estimated discount and risk premium

	Average	Median	Number of Participants
Discount			
Dependent financial advisors	25.0%	22.5%	6
Independent financial advisors	33.8%	30.0%	8
Private equity funds	35.0%	30.0%	5
Total	**31.3%**	**30.0%**	**19**
Risk premium			
Dependent financial advisors	2.6%	2.0%	10
Independent financial advisors	2.0%	2.0%	10
Private equity funds	3.3%	4.0%	3
Total	**2.8%**	**2.0%**	**23**

Source: Petersen et al. (2006).

19 out of all 42 participants adjust the estimated value of a company at the end of the valuation process with the application of DLOL as a discount. Estimated average value of discount is 31.3%. There is no significant difference between the average values of discount between three groups of participants, although dependent financial advisors applied slightly lower value compared to the average value. 23 participants incorporated DLOL into a discount rate. The average value of added risk premium is 2.8%. Again, there are no significant differences between applied values of risk premiums, but dependent financial advisors applied slightly lower value compared to the average value.

4.3.2 The Vydržel and Soukupova study

Vydržel and Soukupova published their study in 2012. As they presented in the study, companies' valuations become more interesting and drew a lot of attention in the last years due to the increased level of M&A activity in the Czech Republic (Vydržel & Soukupova, 2012). From here on, the study will be referred to as the Vydržel study.

Main goal of the study was to examine the valuation methodologies used by Czech private equity professionals and financial advisors. Authors examined the preferences among valuation methods and related issues in the process of valuation of private companies. Study is divided into four main areas. Those are (Vydržel & Soukupova, 2012):

- Cash flow forecasting.
- Cost of capital estimation practice.
- Estimation of other individual parameters in DCF model.
- Application and size estimation of risk premiums.

Into the research they included private equity fund managers and financial advisors. Authors decided to do so because first ones usually appear as a buyer in the transaction process, while others look onto the transaction form different point of view and can work for both parts, for a buyer and for a seller. Furthermore, financial advisors are divided into a group of independent financial advisors and a group of dependent financial advisors, because of their different composition of customer portfolios. In the research they included all the market participants because of the limitations of the Czech market in the context of limited number of financial corporations, and the classification into the sample would only decrease the number of included participants. Out of 73 target participants 45 fully filled questionnaires and interviews (Vydržel & Soukupova, 2012). In the Table 22, the number of participants by three professions and the associated response rate can be seen.

Table 22: Vydržel study – Participants and response rate

Participants	Number of target participants	Number of participants included in study	Response rate
Private equity fund managers	37	18	48.6%
Dependent financial advisors	11	8	72.7%
Independent financial advisors	25	19	76.0%
Total	**73**	**45**	**61.6%**

Source: Vydržel & Soukupova (2012).

In the selection of used valuation methods participants could choose more than one valuation method and because of that number of answers exceeds the number of participants. Results of the study for preferred valuation methods are slightly different from what one would expect. As it can be seen in the Table 23, the most popular method of valuation is the transaction multiple method, followed by the DCF method and the market multiples method. Methods are used by 91.1%, 88.9%, and 73.3% of the participants, respectively (Vydržel & Soukupova, 2012).

Table 23: Vydržel study – Used valuation methods

Valuation Method	Private equity fund manager	Dependent financial advisors	Independent financial advisors	Total
DCF Methodology	14	8	18	40
	77.8%	*100.0%*	*94.7%*	*88.9%*
Market Multiples	12	6	15	33
	66.7%	*75.0%*	*78.9%*	*73.3%*
Transaction Multiples	18	8	15	41
	100.0%	*100.0%*	*78.9%*	*91.1%*
Substantial Multiples	3	1	10	14
	16.7%	*12.5%*	*52.6%*	*31.1%*
Option Based Models	0	0	2	2
	0.0%	*0.0%*	*10.5%*	*4.4%*
Other Valuation Models	2	0	1	3
	11.1%	*0.0%*	*5.3%*	*6.7%*
Number of Participants	**18**	**8**	**19**	**45**

Source: Vydržel & Soukupova (2012).

Taking into account market multiples, 94% of participants use EV to EBITDA multiple and as such it is most commonly used market multiple. It is followed by EV to Sales multiple with 55% and EV to EBIT and Price to Earnings multiples with 42%, each. In the case of transaction multiples ranking stays the same (Vydržel & Soukupova, 2012).

Closer look at the DCF and multiple methods reveals that the most frequently used DCF methods are FCFF (87.5%) and FCFE (50.0%). In the case of multiple approach 69.8% of participants chose conventional approach; this means that participants apply specific multiple to a specific industry in which the company operates. Authors also highlighted that

participants due to the time intensity rarely choose the best fitting multiple based on R^2 (Vydržel & Soukupova, 2012).

Research shows that 45% of participants do not apply DLOM, while 33% apply discount as a percentage of the final estimated value and 26% apply discount as a risk premium in the cost of equity. Size of the discount depends on the industry company operates in, but in general discount moves between 15% and 30% (Vydržel & Soukupova, 2012). As it has been pointed out earlier in the thesis this is one of the main problems in determining the discount. Inadequate research of the company and its environment gives incorrect values of discount and can significantly affect the estimated value of the companies' ownership interests.

4.4 Summarization of DLOL for controlling ownership interest calculation methods and approaches

Studies presented and analysed show that the level of the discount, DLOM and DLOL, can vary across different studies and approaches. As presented on the previous pages, DLOM can be calculated by using pre-IPO or restricted stock approaches. Under the pre-IPO approach, the average value of DLOM estimated in the eight Emory's studies carried out in the period of 1980–1997 ranged from 43% to 60% (Block, 2007), while the adjusted study in 2002 gives the average value of the discount in the amount of 46% (Emory Sr. et al., 2002). Additionally, average value of the discount for the dot.com companies in the period of 1997–2000 amounted to 54% (Block, 2007).

Under the restricted stock approach, the Wruck study carried out in 1989 shows that the average value of the discount amounted to 17.6% (Bajaj et al., 2001), while the Hertzel and Smith (1993) concluded that the average discount for the restricted stock studies amounted to 13.5%.

DLOL can be determined through theoretical concepts (solutions) or empirical examinations (acquisition approaches). Three theoretical solutions presented are based on the POMP. They provide the following discount ranges for 20% (25%) - 150% volatility range and the time period of six months. Chaffe model estimated the discount range of 6.5% - 39.7%, transformed Longstaff model estimated range of 10.6%-53.8% and Finnerty model estimated the discount range from 3.3% to 21.8% (Elmore, 2017).

Under the acquisition approach, the Koeplin study obtained the average value of the discount in the range of 17.8% - 28.3% for USA companies and 34.9% - 53,9% for other companies (Koeplin et al., 2000), Kooli study obtained the average value of the discount in the range of 17% - 34% (Kooli et al., 2003), Officer study obtained the average value of the discount in the range of 17.2% - 22.9% for stand-alone companies and 26.9% - 30.0% for subsidiaries (Officer, 2007), Block study obtained the average value of the discount in the range of 16.3%

- 27.1% (Block, 2007), while Paglia study obtained range of 17.9% - 68,5% (Paglia & Harjoto, 2010).

As it can be noticed, the spread among estimated discounts varies across approaches and studies, but it is clear that all of the studies apply a significant discount to the values of private companies.

5 EMPIRICAL RESEARCH OF THE CONDITIONS ON THE SLOVENIAN MARKET

Empirical part of this master's thesis covers the analysis of the understanding, usage and applied size of DLOL for controlling ownership interest on the Slovenian market. It consists of two parts, namely:

- Survey among Slovenian market participants to obtain information about their understanding and usage of DLOL in the practice.
- Analysis of the applied DLOL and DLOM by Slovenian CBAs, with a main focus on the DLOL for controlling ownership interest.

With the help of the obtained results from both parts of the Slovenian market research, I will check the set hypotheses and summarize the key findings.

5.1 Survey among Slovenian market participants

In the first part of the Slovenian market analysis the survey among market participants was carried out. In the continuation, I present the set hypotheses, an implementation of the survey questionnaire, the sample description and an analysis of the obtained results.

The purpose of this part of empirical research is to obtain the information about understanding and usage of DLOL for controlling ownership interest of the Slovenian market participants. The main focus is to show how different groups of experts perceived and understand DLOL in the process of the valuation of the controlling ownership interest and if their understanding is in line with the theory. According to that the following hypotheses have been set:

Hypothesis 1.1: Most of the Slovenian market participants use adjusted selected mean value of different studies as a primary approach to determine the appropriate DLOL level.

Hypothesis 1.2: According to Slovenian market participants, DLOL mainly reflects the time value of money and with that the adjustment for time uncertainty risk.

Hypothesis 1.3: Deduction of DLOL from estimated equity value is the most commonly used approach of DLOL application among Slovenian market participants.

Hypothesis 1.4: Among the Slovenian market participants, determining the appropriate level of DLOL does not take more than 5% of total time in the business valuation process.

Hypothesis 1.5: Slovenian market participants acknowledge that the size of the valuated interest has an impact on applied DLOL and that they are inversely related.

5.1.1 Methodology

In this part of the Slovenian market research, I used an analytical approach and inductive reasoning. I used the survey method to collect the necessary data and information. I designed a survey questionnaire and published it on the EnKlikAnketa (1KA) website. Thus, I obtained data from various market participants in Slovenia. In the survey I used closed and open questions.

With the help of the obtained data, I analysed how different market participants understand and use in practice DLOL for controlling ownership interest. Based on the information obtained, I will test previously set up hypotheses and synthesized key findings.

5.1.2 Collection of data

In the survey 208 individuals were included that in the course of their professional career must have been faced with the usage of DLOL for controlling ownership interest. Individuals included into the survey work as financial consultants, court appraisers and experts (in the field of stock exchange, finance, valuing companies or business finance), or work in a bank or fund.

The survey was conducted in the period between the 11th of November 2019 and the 11th of February 2020 with the help of an online questionnaire, where the link to the questionnaire was provided to all participants. 141 respondents started to answer the questionnaire which represents 67.8% of all respondents, though the survey has been concluded by 33 respondents, which represents 15.9% of all respondents.

The questionnaire consisted of two parts. The first part referred to demographic variables and the second part to the discounts for lack of liquidity and marketability. The questionnaire was designed in accordance with the previously studied literature and a consultation with individuals working in the field of business valuations. The questionnaire consisted of 6 demographic and individual specific questions, 4 questions related to DLOM, 13 related to DLOL and additional 4 related to DLOC. The subject of the analysis are only questions related to understanding the DLOL. The questionnaire can be seen in the Appendix 4.

5.1.3 Sample description

Out of 208 individuals to whom the questionnaire was send, 33 of them completed the questionnaire. In what follows, the basic demographic and individual-specific characteristics of those 33 respondents are presented. From here on the term "all respondents" is referring to the 33 respondents that have fully concluded the survey.

Most respondents fall into the age group between 30 and 39 years, that is 14 respondents, which represents 42.4% of all respondents, while the lowest number of respondents was in the age group between 20 and 29 years, namely 6.1% of all respondents. The age structure of all respondents can be seen in the Table 24.

Table 24: Age structure of respondents

	Number	**Share**
From 20 to 29 years	2	6.1%
From 30 to 39 years	14	42.4%
From 40 to 49 years	7	21.2%
From 50 to 59 years	4	12.1%
60 years or older	6	18.2%

Source: Own work.

More than half of all respondents have more than 10 years of working experience in the field of finance or similar industry, namely 36.4% have 11 to 20 years of working experience, while 24.2% have 21 or more years of experience. As can be seen from Table 25, the analysis of respondents' work experience showed that the least of respondents (12.1%) have up to 5 years of experience.

Table 25: Work experience of respondents

	Number	**Share**
Up to 5 years	4	12,1%
From 6 to 10 years	9	27,3%
From 11 to 20 years	12	36,4%
21 years or more	8	24,2%

Source: Own work.

More than half of all respondents (51,5%) have master's degree, followed by bachelor's degree (42,4%). Among all respondents there is one with a high school graduate and one with a PhD. The educational structure of all respondents can be seen in the Table 26.

Table 26: Educational structure of respondents

	Number	**Share**
High school graduate	1	3.0%
Bachelor's degree	14	42.4%
Master's degree	17	51.5%
PhD	1	3.0%

Source: Own work.

In practice, individuals can obtain multiple titles. Due to that the sum of shares in Table 27 exceeds 100%. The most frequently obtained title among all respondents is certified business appraiser. 63.6% of all respondents have obtained this title. CFA and CAIA titles had been obtained by 4 and 1 respondents, respectively. 3 respondents are stockbrokers, 6 of them are court experts and 4 are court appraisers. 6 of all respondents have obtained other additional titles.

Table 27: Obtained titles of respondents

	Number	**Share**
Certified business appraisers (SIR)	21	63.6%
CFA	4	12.1%
CAIA	1	3.0%
Stock broker	3	9.1%
Court expert	6	18.2%
Court appraiser	4	12.1%
Other	6	18.2%

Source: Own work.

In practice, individuals can work on different positions. Due to that the sum of the shares in Table 28 exceeds 100%. The most respondents work as a business appraiser (66.7%), financial consultant (45.5%) and/or as analyst (39.4%). This is expected, since those work positions are most frequently faced with a usage of discounts in the process of business valuation. The structure of work positions of all respondents can be seen in the table below.

Table 28: Work positions of respondents

	Number	**Share**
Business Appraiser	22	66.7%
Financial consultant	15	45.5%
Investment Manager	6	18.2%
Analyst	13	39.4%
Court appraiser	3	9.1%
Court expert	4	12.1%
Other	4	12.1%

Source: Own work.

In practice, individuals can perform different activities in work. Due to that the sum of the shares in Table 29 exceeds 100%. In compliance with a working position also work activities have the same pattern. 81.8% of all respondents are engaged in the business valuation, followed by data analysis with 57.6% and M&A consulting and business valuation as certified appraiser with 51.5% each. All of the work activities can be seen in the table below.

Table 29: Work activities of respondents

	Number	Share
M&A consulting	17	51.5%
Business valuation	27	81.8%
Business valuation as a certified appraiser	17	51.5%
Obtaining financial resources	10	30.3%
Refinancing	7	21.2%
Expert opinions	12	36.4%
Investment management	7	21.2%
Data analysis	19	57.6%

Source: Own work.

5.1.4 Analysis of the obtained results

In this subchapter, the obtained answers in the survey questionnaire are analysed. Each question is being analysed separately with two main focuses. The first one is the analysis in terms of all the respondents, while the other focus is given to the analysis of answers of a specific group of respondents. In the Table 30 it can be seen how I divided respondents into several groups that have been used in a further analysis.

Table 30: Groups of respondents used in a further analysis

Title	Number of respondents	Group name	Number of respondents in a group
Certified business appraisers (SIR)	21	CBAs	21
CFA	4	CCBs	8
CAIA	1		
Stock broker	3		
Court expert	6	CAEs	10
Court appraiser	4		
Others	6	Others	6

Source: Own work.

It can be noticed that the total value of obtained titles exceeds the number of all respondents. This is due to the fact that some individuals obtained more than one title in their careers. Those individuals are included in two groups. Three out of six respondents with other title have stated that they are obtaining CFA certificate, work as a certified tax adviser and work as operator or manager. Other three respondents did not obtain any title.

5.1.4.1 Time needed to convert controlling ownership interest into money for an interest to be marked as liquid

According to the SEC it should not take more than three business days for the sale of interest to become official and the funds made available. Current rule is referred to as T+3 settlement, meaning trade day plus three more business days. This defines the length of time within which a stock seller has to deliver the stock to the buyer's brokerage firm and the length of time the buyer can take to pay for the shares (SEC, 2004).

Applying DLOM to the controlling ownership interest is controversial for some individuals. Some believe there should never be a DLOM applied in the process of controlling ownership interest valuation, while others believe that the appropriateness of the applications depends on the how the control value was estimated (Pratt, 2009).

Although, according to the Pratt (2009), even those that do not believe in the appropriateness of application of DLOM to the controlling ownership interest agree that owner of the interest cannot simply call the broker, sell the interest and receive money in three business days. This is called the problem of lack of liquidity rather than the problem of lack of marketability.

Ability to call the broker, sell interest in the company at known price and receive money within three days has been known as the benchmark for marketability. As mentioned above, that is not case with the controlling ownership interest. That is why Pratt (2009) does not believe there is a thing such as "control, marketable", regardless of the company's status, private or public. He believes that only for the minority ownership interests there exists the distinction between marketable and non-marketable. Pratt also delivers the results of the survey where the average time to sell an ownership interest is around six months, without considering time to prepare the interest to sell.

Among all respondents, 8 of them or 24.2% believe that the upper boundary for controlling ownership interest to be marked as liquid is an ability to be converted into money in three days. This is in line with the T+3 rule by SEC. The most respondents, namely 9 or 27.3% of all respondents, answered that the upper boundary for controlling ownership interest to be marked as liquid is an ability to be converted into money in six months, which is in line with Pratt's findings. All the answers can be seen in the Table 31.

Among CBAs, the answers have the similar magnitude as the answers of all respondents and 23.8% of them answered that the appropriate time to convert controlling ownership interest into money for the interest to be marked as liquid is within three days, while 28.6% of the CBAs believe that receiving money in six months is an appropriate time. Answers of the CCBs are divided between all possible answers, except one-year period, while none of CAEs answered that the time period shorter than one month is appropriate.

Table 31: Time needed to convert controlling ownership interest into money for an interest to be marked as liquid

	CBAs		CCBs		CAEs		Others		Total	
Answer	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***
In 24 hours	1	*4.8%*	1	*12.5%*	0	*0.0%*	1	*16.7%*	**3**	***9.1%***
In three days	5	*23.8%*	2	*25.0%*	0	*0.0%*	2	*33.3%*	**8**	***24.2%***
In one week	3	*14.3%*	2	*25.0%*	0	*0.0%*	0	*0.0%*	**4**	***12.1%***
in one month	4	*19.0%*	2	*25.0%*	1	*10.0%*	0	*0.0%*	**5**	***15.2%***
In six months	6	*28.6%*	1	*12.5%*	5	*50.0%*	2	*33.3%*	**9**	***27.3%***
In one year	2	*9.5%*	0	*0.0%*	4	*40.0%*	1	*16.7%*	**4**	***12.1%***

Source: Own work.

5.1.4.2 Determination of a discount for lack of liquidity in the process of the controlling ownership interest valuation

To determine the appropriate level of DLOL 27.3% respondents use a selected mean of several studies and then adjusted it for the characteristics of the valuated company, while 18.2% of them use the same approach as in the case of valuation of non-controlling ownership interest and setting DLOM. More detailed breakdown of the answers is shown in the Table 32.

Three of the respondents did not choose any of the listed methods. Two of them said that they use the same method for the determination of DLOM and DLOL, since DLOL is already included in or it is a part of the DLOM, while the third one said that he or she use the value of DLOL in accordance with the court cases.

Among CBAs prevails determination of the DLOL by using selected mean value of a discount of several studies with adjustments according to the characteristics of the valuated company, while the second most often used method to determine the appropriate level of DLOL is the usage of the same approach as in the case of valuation of non-controlling ownership interest and setting the DLOM.

CCBs have divided their answers among different choices. Two respondents each chose the usage of discount determined in one study with adjustments according to the characteristics of the valuated company, trusting their own feelings and use subjective evaluation, and not using the discount. CAEs' answers are also divided among several choices, but the most frequently chosen one is a determination of DLOL based on their feeling and experiences.

Table 32: Determination of a discount for lack of liquidity in the process of the controlling ownership interest valuation

	CBAs		CCBs		CAEs		Others		Total	
Answer	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***
I use the same approach as in the case of valuation of non-controlling ownership interest and setting the discount for lack of marketability	4	*19.0%*	1	*12.5%*	2	*20.0%*	2	*33.3%*	**6**	***18.2%***
Use of a discount as determined in one study	0	*0.0%*	0	*0.0%*	0	*0.0%*	0	*0.0%*	**0**	***0.0%***
Use of a discount as determined in one study with adjustments based on the characteristics of the valuated company	2	*9.5%*	2	*25.0%*	1	*10.0%*	1	*16.7%*	**5**	***15.2%***
Use of the selected mean of discount rates of multiple studies	1	*4.8%*	0	*0.0%*	0	*0.0%*	2	*33.3%*	**3**	***9.1%***
Use of the selected mean of discount of several studies with adjustments according to the characteristics of the valuated company	7	*33.3%*	1	*12.5%*	2	*20.0%*	1	*16.7%*	**9**	***27.3%***
I trust my own feeling and experience (subjective evaluation)	2	*9.5%*	2	*25.0%*	3	*30.0%*	0	*0.0%*	**4**	***12.1%***
I do not use a discount in valuation of controlling ownership interest	2	*9.5%*	2	*25.0%*	0	*0.0%*	0	*0.0%*	**3**	***9.1%***
Other	3	*14.3%*	0	*0.0%*	2	*20.0%*	0	*0.0%*	**3**	***9.1%***

Source: Own work.

5.1.4.3 Most frequently used studies as the basis for determination of DLOL

Among all respondents, the most frequently used study that represents a basis to determine appropriate level of DLOL are other PCD studies used by 22.4% of all respondents, followed by Koeplin study used by 20.4% of all respondents and theoretical solutions used by 16.3% of all respondents. Due to multiple-choice nature of the question, the total number of responses exceeds the number of respondents.

Among CBAs, the most frequently used is Koeplin study, namely by 24.2%, followed by theoretical solutions and other PCD studies, both used by 15.2% of CBAs. Among CCBs, the most frequently used studies to determine appropriate level of DLOL were not listed, followed by Koeplin study and other PCD studies, while CAEs most frequently use something else.

None of the respondents use Kooli study or Paglia and Harjoto study as the basis for their determination of the appropriate magnitude of DLOL. Among other answers respondents stated usage of Pratt (discount should range among 3% and 33%), Damodaran, Officer, De Franco, Gavius, Jin and Richardson study or simply apply the value of DLOL in accordance with the court cases.

Table 33: Most frequently used studies as the basis for determination of DLOL

	CBAs		CCBs		CAEs		Others		Total	
Answer	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***
Theoretical solutions (Chaffe model, Longstaff model, Finnerty model and other theoretical models)	5	*15.2%*	2	*15.4%*	2	*16.7%*	1	*14.3%*	**8**	***16.3%***
Koeplin, Sarin and Shapiro Study	8	*24.2%*	3	*23.1%*	1	*8.3%*	1	*14.3%*	**10**	***20.4%***
Kooli, Kortas and L'Her Study	0	*0.0%*	0	*0.0%*	0	*0.0%*	0	*0.0%*	**0**	***0.0%***
Officer Study	3	*9.1%*	0	*0.0%*	0	*0.0%*	0	*0.0%*	**3**	***6.1%***
Block Study	1	*3.0%*	0	*0.0%*	0	*0.0%*	0	*0.0%*	**1**	***2.0%***
Paglia and Harjoto study	0	*0.0%*	0	*0.0%*	0	*0.0%*	0	*0.0%*	**0**	***0.0%***
Other Private Company Discount Studies	5	*15.2%*	3	*23.1%*	2	*16.7%*	4	*57.1%*	**11**	***22.4%***
Nothing listed	7	*21.2%*	5	*38.5%*	2	*16.7%*	1	*14.3%*	**11**	***22.4%***
Other	4	*12.1%*	0	*0.0%*	5	*41.7%*	0	*0.0%*	**5**	***10.2%***

Source: Own work.

5.1.4.4 General understanding of DLOL

In this question, respondents had to choose how strongly they agree or disagree with the listed arguments. The levels of agreeing were determined according to the Liker 5-level scale. Respondents were able to choose between the following levels of agreeing:

- 1 – Entirely disagree.
- 2 – Disagree.
- 3 – Indifferent.
- 4 – Agree.
- 5 – Entirely agree.

As can be seen above, to each level of agreeing a certain value (from one to five) has been given. This represents the basis for calculation of the average value of agreeing with the certain argument and possible comparison among different groups of respondents. The average value of agreement of each respondent group was determined by first calculating the average value for each title, followed by the computation of the group's overall average (titles breakdown into respondent groups is shown in Table 30). Average values of 1.49 or below are classified as entirely disagree, values from 1.50 to 2.49 are classified as disagree, average values from 2.50 to 3.49 are classified as indifferent, values from 3.50 to 4.49 are classified as agree, and average values of 4.50 or above are classified as entirely agree. The results are presented in the Table 34.

Table 34: Arguments about DLOL

	CBAs	CCBs	CAEs	Others	Total
Answer	**Average**	**Average**	**Average**	**Average**	**Average**
The use of DLOL in the valuation of controlling ownership interest is required due to the adjustment to market conditions	3.62	3.53	3.96	3.83	**3.70**
The use of DLOL in the valuation of controlling ownership interest is required due to the presence of transaction costs	3.43	3.33	3.42	3.50	**3.42**
The use of DLOL in the valuation of controlling ownership interest is required due to time uncertainty of the transaction	3.76	3.72	4.21	3.67	**3.76**
The use of DLOL in the valuation of controlling ownership interest is required to adjust for the risk of change in the value of the holding	3.05	3.22	3.29	3.17	**3.12**
The use of DLOL in the valuation of controlling ownership interest is not sufficiently theoretically supported and therefore unjustified	2.38	2.97	1.96	2.17	**2.39**
The use of DLOL in the valuation of controlling ownership interest is not sufficiently supported by empirical studies and therefore unjustified	2.57	2.78	2.17	2.17	**2.52**
Any ownership interest can be sold, no matter how illiquid it is, it is just a subject of acceptance of lower price	3.00	4.19	2.04	3.00	**3.15**

Source: Own work.

All respondents on average agree with the argument that the use of DLOL in the valuation of controlling ownership interest is required due to the adjustment to market conditions and external factors, such as macroeconomic conditions, industry's current situation and expected future developments, state laws, and other factors. Average value of all respondents amounted to 3.70. The highest average value of the variable was reached by CAEs, namely 3.96, while CBAs and CCBs reached 3.62 and 3.53, respectively.

On average, all respondent groups are indifferent to the argument that the use of DLOL in the valuation of controlling ownership interest is required due to the presence of transaction costs, while other groups of respondents on average agree with the given argument. They received average value of 3.50 which is a limit value between being indifferent and agreeing with the argument. The average value of the variable of all respondents is 3.42 and they are on average indifferent to the argument. The lowest average value was obtained by CCBs.

All groups of respondents agree with the argument that the use of DLOL in the valuation of controlling ownership interest is required due to time uncertainty of the transaction. The average value of the variable amounted to 3.76. CAEs most strongly agreed with a given argument (4.21 average value of the variable), while other respondents agreed least strongly with the argument (3.67 average value of the variable).

It can be noticed in the theoretical overview that the selling process of the company or controlling ownership interest is associated with costs, such as administrative and transaction costs, and with a time uncertainty. I believe it is necessary to take into account these two characteristics when the appropriate level of DLOL is being calculated. With time uncertainty not only time value of money is related, but also the risk of change in value of company and controlling ownership interest; the higher is the time uncertainty, the higher is the risk of change in value of interest.

On average, none of respondent groups agree with the argument that the use of DLOL in the valuation of controlling ownership interest is required to adjust for the risk of change in the value of the holding. All groups of respondents are on average indifferent to the argument. The lowest average value of the variable was obtained by CBAs, namely 3.05, followed by other respondents with 3.17 average value. The highest value was obtained by CAEs and amounted to 3.29.

On average, only CCBs are indifferent to the argument that the use of DLOL in the valuation of controlling ownership interest is not sufficiently theoretically supported and therefore unjustified, resulting in the highest average value of the variable, which is 2.97. Other groups of respondents on average disagree with the argument. CBAs' average value of the variable amounts to 2.38, other respondents' average value amounts to 2.17, while CAEs gave the lowest average value, namely 1.96. Average value of the variable of all respondents amounted to 2.39, meaning that all respondents on average disagree with the argument.

On average, all respondents are indifferent to the argument that the use of DLOL in the valuation of controlling ownership interest is not sufficiently supported by empirical studies and is therefore unjustified. CBAs and CCBs are indifferent to the argument (achieving average value of the variable in the amount of 2.57 and 2.78, respectively), while CAEs and other respondents disagree with the argument (achieving 2.17 average value of the variable).

According to the answers to the previous two arguments, the usage of DLOL in the valuation process of controlling ownership interest is sufficiently supported with theoretical and empirical studies, but the agreeing is not as strong as I have expected.

The argument that any ownership interest can be sold, no matter how illiquid it is, it is just a subject of accepting the lower price, is in line with Damodaran's theory that has been presented in previous chapters. All respondents are on average indifferent. Average value of

the variable of all respondents amounted to 3.15. CBAs and other respondents are on average indifferent to the argument, while CCBs on average agree with it and CAEs disagree.

The results of the CBAs were surprising since the average value of the answer on four out of seven given arguments showed indifference to the argument. This was unexpected since the business valuation is one of their main expertise areas and it would be expected that they have a strong opinion on subjects related to the valuation process, which DLOL is.

Additionally, taking into account all respondents, the picture stays the same, since average value of the answer indicates indifference of respondents to the given argument in four out of seven arguments. This can be an indicator that the determination of DLOL in the process of the controlling ownership interest valuation does not have appropriate attention among Slovenian market participants, although it can have substantial impact on the estimated value.

5.1.4.5 Approach to application DLOL in the business valuation process

Majority of the respondents (81.8%) applicate DLOL in the valuation process of the controlling ownership interest by deducting discount from the estimated equity value. Among all respondents, 12.1% of them value the equity of the company in accordance with valuations of similarly liquid equity of other companies, while the rest, 6.1% of them, consider DLOL as a premium in percentage points in the discount rate or capitalization rate.

Among CBAs 19 deduct the discount from the estimated equity value, while other two answers were chosen each by one respondent. CBAs have the highest percentage of choosing deduction of DLOL from the estimated equity value as their application approach in the process of the business valuation. Other groups of respondents used this approach less frequently, namely CCBs in 75.0% of the answers, CAEs in 70.0% of the answers, and other respondents in 66.7% of the answers. The answers to the used approach to applicate DLOL in the valuation process can be seen in the Table 35.

Table 35: Approach to application of DLOL in the business valuation process

	CBAs		**CCBs**		**CAEs**		**Others**		**Total**	
Answer	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***
I deduct the discount from the estimated equity value	19	*90.5%*	6	*75.0%*	7	*70.0%*	4	*66.7%*	**27**	***81.8%***
I consider DLOL in the discount rate as a premium in percentage points in the discount rate or capitalization rate	1	*4.8%*	0	*0.0%*	3	*30.0%*	0	*0.0%*	**2**	***6.1%***
I value the equity of the company in accordance with valuations of similarly liquid equity of other companies	1	*4.8%*	2	*25.0%*	0	*0.0%*	2	*33.3%*	**4**	***12.1%***

Source: Own work.

5.1.4.6 The impact of different factors on the size of DLOL

With this question, the respondents had to determine how strong impact on the size of DLOL different factors have. The magnitudes of impact were determined according to the Likert 5-level scale. Respondents were able to choose between the following magnitudes of impact:

- 1 – Do not have impact.
- 2 – Small impact.
- 3 – Medium impact.
- 4 – Strong impact.
- 5 – Very strong impact.

As it can be seen above, to each level of impact a certain value (from one to five) has been given. This represents the basis for a calculation of the average impact of individual factor and a possible comparison among different groups of respondents. The average value of impact of each respondent group was determined by first calculating the average value for each title, followed by the computation of the group's overall average (titles breakdown into respondent groups is shown in Table 30). Average values of 1.49 or below are classified as no impact, values from 1.50 to 2.49 are classified as small impact, average values from 2.50 to 3.49 are classified as medium impact, values from 3.50 to 4.49 are classified as strong impact, and average values of 4.50 or above are classified as very strong impact. The results are presented in the Table 36.

Table 36: The impact of different factors on the size of DLOL

	CBAs	CCBs	CAEs	Others	Total
Answer	**Average**	**Average**	**Average**	**Average**	**Average**
The right to sell (put option)	3.90	3.31	3.88	4.33	**3.94**
Dividend payments	3.52	2.56	3.13	3.50	**3.27**
Potential buyers	4.10	3.56	4.13	3.83	**3.94**
The size of the share block to be sold	3.90	3.83	4.08	3.83	**3.88**
Expected public sale of shares	3.62	3.50	3.42	3.33	**3.48**
Access to and reliability of information	3.19	2.92	3.50	3.83	**3.33**
Restrictive provisions in the Statute/Partnership agreement	3.71	4.08	3.50	4.00	**3.88**
Uncertain timeline of transaction execution	3.71	3.11	3.71	3.50	**3.61**
Transaction preparation cost	3.38	2.50	2.96	3.33	**3.24**
Risks associated with the selling price	3.19	2.94	2.63	3.83	**3.30**
Different forms of purchase	2.90	2.72	2.00	2.83	**2.82**
The difficulty of selling a privately held company	3.81	353	4.04	4.50	**3.94**

Source: Own work.

All respondents on average answered that the right to sell the ownership interest has a strong impact on the size of the DLOL. Average value of the answer of all respondents amounted to 3.94. Comparing results of the individual groups of respondents shows that other

respondents reach the highest average value of the coefficient, namely 4.33. Second largest average value was obtained by CBAs with 3.90. CCBs are the only group of respondents that on average answered that the right to sell the ownership interest has a medium impact.

CBAs and other respondents on average answered that the dividend payments have strong impact on the size of DLOL. Average values of the variables amounted to 3.52 and 3.50, respectively, which means that the values are at the boundary between strong and medium impact. CCBs and CAEs answered on average that the dividend payments have medium impact on the size of DLOL, while brokers on average answered that it has small impact. All respondents reached average value of 3.27, meaning that dividend payments have medium impact on the size of DLOL.

All groups of respondents on average answered that the existence of potential buyer has strong impact on the size of DLOL, whereby CAEs and CBAs reached the highest average values of variable, namely 4.13 and 4,10, respectively. Average value of the variable of all respondents amounted to 3.94.

According to the average value of variable of all respondents the size of the share block to be sold has strong impact on the size of DLOL. The highest average value vas obtained by CAEs (4.08) and the lowest by CCBs and other respondents (3.83).

Expected public sale of shares has according to the CBAs and CCBs strong effect on the size of DLOL. Their average value of the variable amounted to 3.62 and 3.50, respectively. Other groups of respondents answered on average that the expected public sale of shares has medium impact on the size of DLOL. All respondents on average answered that expected public sale of share has medium impact on the size of DLOL, with 3.48 average value of the variable, which is at the boundary between strong and medium impact.

Other respondents (3.83) and CAEs (3.50) answered that the access and reliability of information have strong impact on the size of DLOL, while other two groups of respondents on average answered that it has a medium impact. Average value of the answer of all respondents amounted to 3.33 and it is above the average value of CBAs (3.19) and CCBs (2.92).

All groups of the respondents answered that on average the restriction provisions in the statute or partnership agreement have a strong impact on the size of DLOL. Average value of the answer of all respondents amounted to 3.88, while the highest average value was obtained by CCBs (4.08) and the lowest by the CAEs (3.50).

CBAs, CAEs and other respondents on average answered that the uncertain timeline of transaction execution has a strong impact on the size of DLOL. The highest average value was obtained by CBAs and CAEs, namely 3.71, while the lowest average value was obtained

by CCBs (3.11). All respondents on average answered that the uncertain timeline of transaction has strong impact on the size of DLOL. Obtained average value amounted to 3.61.

According to the average value of all respondents the transaction preparation cost has a medium impact on the size of DLOL. The average value of all respondents amounted to 3.24. The highest average value of the variable was obtained by CBAs, namely 3.38, while the lowest one was obtained by CCBs, namely 2.50.

All groups of the respondents, except other respondents, on average answered that the risks associated with the selling price have a medium impact on the size of DLOL. Average value of the variable of all respondents amounted to 3.30, while the highest average value was obtained by other respondent (3.83), the only group of respondents that on average answered that risks associated with the selling price have a strong impact on the size of DLOL. Among other groups of respondents, the highest average value of the variable was obtained by CBAs, namely 3.19, and the lowest by the CAE, namely, 2.63.

On average, all respondents answered that different forms of purchase have a medium impact on the size of DLOL. Obtained average value of all respondents amounted to 2.82. Only CAEs answered that different forms of purchase have on average a small impact on the size of DLOL. Their average value amounted to 2.00. The highest average value of the variable was obtained by CBAs, namely 2.90.

The difficulty of selling a privately held company has a strong impact on the size of DLOL, since average value of the variable of all respondents amounted to 3.94. Other respondents reached the highest average value of variable, namely 4.50, which is a boundary between strong and very strong impact on the size of DLOL. Other groups of respondents on average answered that the difficulty of selling a privately held company has a strong impact on the size of DLOL.

5.1.4.7 Share of time in the business valuation process dedicated to determine the size of DLOL

In this question, respondents had to reveal the percentage of the total time in the business valuation process intended to determine the appropriate size of DLOL. Among all respondents 45.5% of them spend up to 5% of the time in the business valuation process to determine the appropriate size of DLOL, while 36.4% of them spend from 6% to 10% of the time to determine the appropriate size of DLOL. The rest of respondents spend from 11% to 15% of the time to determine the appropriate size of DLOL. None of the respondents spend 16% or more of time in the business valuation process to determine the appropriate size of DLOL. Answers to the question are shown in the Table 37.

Among CBAs, 47.6% of them spend up to 5% of the time in the business valuation process to determine the appropriate size of DLOL, while 28.6% and 23.8% of them spend from 6% to 10% or from 11% to 15% of the time in the process of the business valuation process to determine the appropriate level of DLOL, respectively.

Six out of eight CCBs answered that they spend up to 5% of the time in the business valuation process to determine the appropriate size of DLOL, while other two respondents spend from 6% to 10% of the time. Among CAEs 40% of them answered that they spend up to 5% of the time to determine the appropriate size of DLOL.

Table 37: Share of time in the business valuation process intended to determine the size of DLOL

	CBAs		CCBs		CAEs		Others		Total	
Answer	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***
Up to 5% of the time	10	*47.6%*	6	*75.0%*	4	*40.0%*	2	*33.3%*	**15**	***45.5%***
6% - 10% of the time	6	*28.6%*	2	*25.0%*	3	*30.0%*	4	*66.7%*	**12**	***36.4%***
11% - 15% of the time	5	*23.8%*	0	*0.0%*	3	*30.0%*	0	*0.0%*	**6**	***18.2%***
16% - 20% of the time	0	*0.0%*	0	*0.0%*	0	*0.0%*	0	*0.0%*	**0**	***0.0%***
More than 20% of the time	0	*0.0%*	0	*0.0%*	0	*0.0%*	0	*0.0%*	**0**	***0.0%***

Source: Own work.

5.1.4.8 Impact of the size of the valuated interest on the size of the applied DLOL

Among all respondents, only one (broker) answered that the size of the valuated interest does not affect the size of DLOL. This means that 97.0% of all respondents answered that the size of the valuated interest does affect the size of DLOL. Breakdown by groups of respondents can be seen in the Table 38.

The size of the valuated interest refers to the share of voting rights and not to the absolute size of the interest in the ownership structure.

Table 38: Does the size of the valuated interest impact the size of the DLOL?

	CBAs		CCBs		CAEs		Others		Total	
Answer	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***
Yes	21	*100%*	7	*87.5%*	10	*100%*	6	*100%*	**32**	***97.0%***
No	0	*0.0%*	1	*12.5%*	0	*0.0%*	0	*0.0%*	**1**	***3.0%***

Source: Own work.

5.1.4.9 The average value of the applied DLOL to a certain value of valuated interest

Respondents had to state the average value of the applied DLOL to a certain size or range of valuated interest (reflecting the share of the voting rights). The valuated interest was divided into seven units. Out of those seven units, CCBs stated the lowest average value of the

applied DLOL in two units and the other respondents in the remaining five units. CAEs on average stated the highest average value of the applied DLOL in all seven units.

The average value of applied discount of each respondent group was determined by first calculating the average value for each title, followed by the computation of the group's overall average (titles breakdown into respondent groups is shown in Table 30).

As can be noticed from the Table 39, CBAs are the only group of respondents where the average value of the stated applied DLOL to a certain size or range of valuated interest decreased with an increase of the valuated interest range. Average values of stated applied discount for other groups increased with an increased valuated ownership interest at least in one successive range of the ownership interest.

The average value of the stated DLOL to a certain size or range of valuated interest by CBAs does not stand out on the stand-alone basis, but on the other hand exceeds an average value of the applied DLOL by all participants.

All respondents stated an average value of DLOL in the amount of 17.1% to a valuated ownership interest up to 24.99%. For this range of valuated interest CBAs on average stated an average value of DLOL of 20.5%, while the CCBs stated an average value of 4.3%. A big gap can be noticed between an average stated value of applied DLOL by individual groups of respondents. CAEs stated a 22.1% average value of applied DLOL.

To the 25.00% - 49.99% range of valuated ownership interest all respondents stated a 16.1% average value of applied DLOL. CBAs stated an average value of DLOL in the amount of 18.6%. CCBs stated an average value of applied DLOL in the amount of 6.8%, while other respondents stated an 8.3% average value of discount to applied interest. A big gap can be noticed between average values of applied DLOL by individual groups of respondents. CAEs stated a 22.7% average value of the applied DLOL to the range of valuated interest.

To the 50.00% valuated ownership interest, all respondents stated an average value DLOL of 14.5%, while the CBAs on average stated DLOL in the amount of 16.2%. The CCBs on average stated 10.5% DLOL, CAEs 24.0% and other respondents 9.9%.

To the 50.01% - 74.99% range of valuated interest all respondents on average stated 12.5% value of the applied DLOL, while CBAs on average stated 14.0% applied DLOL. CCBs stated an average value of the applied DLOL in the amount of 9.4%, CAEs 22.1% and other respondents 8.7%. A big gap can be noticed between the average stated values of the applied DLOL by individual groups of respondents.

To the 75.00% valuated interest all respondents on average stated 10.9% DLOL, while the CBAs on average stated 12.0%. The CCBs and other respondents stated the average values

of the DLOL in the amount of 10.8% and 8.2%, respectively. CAEs stated the average value of applied DLOL to 75.00% valuated ownership interest in the amount of 21.7%.

To the 75.01% - 99.99% range of the valuated interest all respondents on average stated 8.8% DLOL, while the CBAs stated an average value of the applied DLOL of 9.7%. CCBs stated an average value of the applied DLOL in the amount of 8.4%. A big gap can be noticed between the average stated values of the applied DLOL by individual groups of respondents. CAEs stated an average value of the applied DLOL to the valuated interest range in the amount of 21.0%, while other respondents stated 7.0%.

To the 100% valuated interest all respondents on average stated 8.4% DLOL, while the CBAs stated an average value of the DLOL in the amount of 9.7%. CCBs stated the average values of the DLOL in the amount of 9.0%, CAEs in the amount of 24.3% and the other respondents 5.4%.

Table 39: The average value of the applied DLOL to a certain value of valuated interest

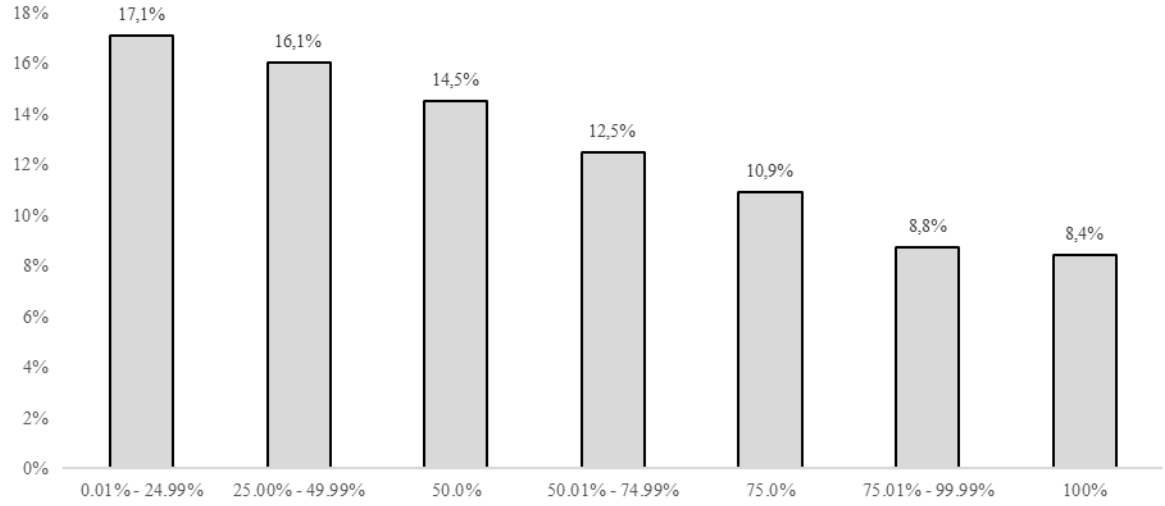
	CBAs	CCBs	CAEs	Others	Total
Answer	**Average**	**Average**	**Average**	**Average**	**Average**
0.01% - 24.99%	20.5%	4.3%	22.1%	12.1%	**17.1%**
25.00% - 49.99%	18.6%	6.8%	22.7%	8.3%	**16.1%**
50%	16.2%	10.5%	24.0%	9.9%	**14.5%**
50.01% - 74.99%	14.0%	9.4%	22.1%	8.7%	**12.5%**
75%	12.0%	10.8%	21.7%	8.2%	**10.9%**
75.01% - 99.99%	9.7%	8.4%	21.0%	7.0%	**8.8%**
100%	9.7%	9.0%	24.3%	5.4%	**8.4%**

Source: Own work.

As it can be noticed from the table above there are substantial differences in the average values of stated applied DLOL to the certain size or range of valuated interest between groups of respondents. This can be an indication that CBAs are the most qualified and have the most knowledge or better understanding of the business valuation process. This also resulted in DLOL determination, since the CBAs respondents are the only group of respondents that have inversely proportional movement of DLOL and the size of valuated interest.

Based on the stated values of applied discount of all respondents it can be noticed that the average value of DLOL decreases with an increase of the valuated ownership interest. Stated can be seen in the Chart 1, where the average applied DLOL decreased from 17.1% for ownership interest up to 24.99% to 8.4% for a 100% ownership interest.

Figure 5: Average stated values of applied DLOL to a certain size or range of valuated interest



Source: Own work.

5.1.4.10 Usage of discounts in the process of ownership interest valuation for the purpose of the squeeze out of minority shareholders (in accordance with ZGD-1)

To the question which discounts respondents use in the process of share valuation for the purpose of the squeeze out of minority shareholders (that is in accordance with the Companies act, Zakon o gospodarskih družbah (ZGD-1), Ur. l. RS, št. 65/09 – uradno prečiščeno besedilo), 37.8% of all respondents chose the answer that they do not use any discount. The usage of discounts in the process of squeezing out of minority shareholders is shown in the Table 40. Due to multiple-choice nature of the question, the total number of responses exceeded the number of respondents.

CBAs obtained the highest percentage of not using any discount in the process of minority interest valuation for the purpose of squeeze out among all groups of respondents, namely 48.1%, followed by DLOL (25.9%), DLOM (22.2%) and DLOC (3.7%).

45.5% of CCBs use DLOM, 27.3% use DLOL and 27.3% do not use any discount, while none of CCBs use DLOC. Answers of CAEs were spread through all four answers. Usage of DLOC was chosen only by one respondent, while the other answers were chosen by five respondents. Other respondents mostly use DLOC and DLOL (37.5% each), while 25% of them do not use any discount.

Table 40: Usage of discounts in the process of interest valuation for the purpose of the squeeze out of minority shareholders

	CBAs		CCBs		CAEs		Others		Total	
Answer	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***	**Val.**	***%***
No discounts	13	*48.1%*	3	*27.3%*	5	*31.3%*	2	*25.0%*	**17**	***37.8%***
Discount for lack of control	1	*3.7%*	0	*0.0%*	1	*6.3%*	3	*37.5%*	**5**	***11.1%***
Discount for lack of marketability	6	*22.2%*	5	*45.5%*	5	*31.3%*	3	*37.5%*	**13**	***28.9%***
Discount for lack of liquidity	7	*25.9%*	3	*27.3%*	5	*31.3%*	0	*0.0%*	**10**	***22.2%***

Source: Own work.

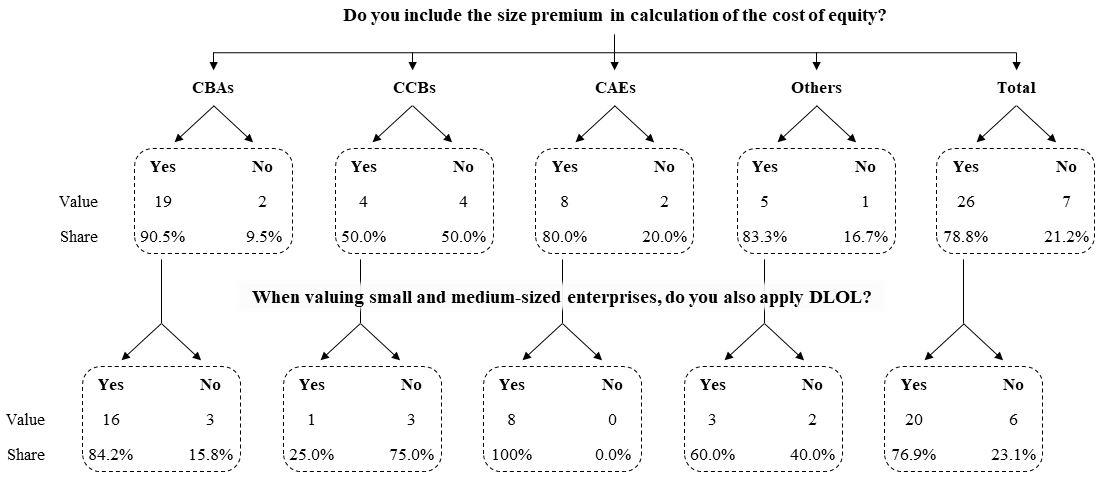
5.1.4.11 Usage of size premium and DLOL in the valuation process of small and medium sized companies

The last question of the questionnaire was a two-part question. It was based on the research article from Torchio and Surana (2014) where they analysed the effect of liquidity on size premium. They came to the conclusion that most frequently used method to evaluate business is a discounted cash flow method, which includes a calculation of a discount rate to convert future cash flows into a present value. Part of the discount rate is cost of equity and within it a size premium. They show that major part of the size premium reflects a lack of liquidity, which affects smaller companies more than bigger ones. This results in an underestimation of smaller companies; the smaller the company, the greater the underestimation.

The Torchio and Surana (2014) research showed that the substantial part of the size premium is reflecting a lack of liquidity, which means that in case where a discount rate includes a size premium through the cost of equity, and appraisers also apply DLOL to the enterprise value to obtain an equity value of a company, the problem of double counting occurs. Double counting occurs since the lack of liquidity is a significant part of the size premium that is included in the calculation of discount rate through a cost of equity, while the second time the lack of liquidity is taken into account is in a calculation of equity value, where DLOL is applied to an enterprise value of a company or valuated ownership interest.

Among all the respondents, 26 of them use size premium in the calculation of a discount rate. Out of those respondents, 20 of them additionally use a DLOL in the process of valuation of small and medium sized companies, which results in the problem of potential double counting for lack of liquidity. The answers given to those two questions are presented in the Figure 5.

Figure 6: Usage of size premium and DLOL in the valuation process of small and medium sized companies



Source: Own work.

Further analysis of the obtained data with a regression model was not made due to the low response to the survey and not sufficient number of the input data. This can be a subject of a further development of this master's thesis, with a higher response, which would give a better market representation.

5.1.5 Summarization and hypotheses testing

From the survey questionnaire analysis, it is clear that the Slovenian market participants are not consistent with each other and that answers of some market participants deviate from the practice presented in the theory overview. Some drawbacks could result from the underdevelopment of the market, compared to the more advanced EU or the USA economies. However, viewed as a whole, the market analysis shows that the development of the Slovenian market is at a high level.

Among the Slovenian market participants, 27.3% of them use the selected mean of several studies for discounts with adjustments according to the characteristics of the valuated company. Based on that the Hypothesis 1.1 cannot be rejected. The most commonly used is the Koeplin study and other PCD studies (excluding Kooli, Officer and Block studies). On the other hand, 18.2% of the market participants use the same approach as in the case of non-controlling ownership interest. As can be seen from the literature review, the latter may be reflected in an inconsistent assessment of DLOL level and consequently in overestimated or underestimated value of controlling ownership interest.

Liquidity can be defined as an ability to quickly convert asset, namely controlling ownership interest into cash without substantial loss of principal, or put it differently, without substantial transaction costs. Results of the survey show that the most common argument for

using DLOL in the business valuation is the time uncertainty of transaction. This means that the Hypothesis 1.2 cannot be rejected.

In practice, there are several ways to include DLOL in the business valuation process. According to the results of a survey, Slovenian market participants most commonly used deduction from an equity value as an approach to apply DLOL into business valuation, which is in line with the Hypothesis 1.3 and it means that it cannot be rejected.

Determination of an appropriate level of DLOL is one of the key steps in the business valuation process, since it can have significant impact on the estimated equity value. I have expected that the Slovenian market participants do not devote significant share of time in the valuation process to determine an appropriate level of DLOL and in the light of this the Hypothesis 1.4 has been set. The hypothesis can be rejected, since 18 out of 33 participants spend more than 6% of time in the business valuation process to determine the appropriate level of DLOL, which I believe to be a sufficient amount of time, considering the necessary analyses when preparing a business valuation.

Based on the presented literature, the value of the applied DLOL in the business valuation process should decline with an increased valuated ownership interest. The hypothesis 1.5 was set up to test this and based on the answers provided from the Slovenian market participants the hypothesis cannot be rejected, since the average value of applied DLOL based on the provided answers decreased from 17.1% for valuation of ownership interest up to 24.99% to 8.4% for valuation of 100% ownership interest.

5.2 Statistical analysis of applied DLOL and DLOM on Slovenian market

In the second part of the Slovenian market analysis, the statistical analysis of the actual applied discounts by CBAs is carried out. This part of the analysis includes the descriptive statistics and further statistical analysis of the applied values of DLOM and DLOL in the business valuation process. This will give an additional comparison to the stated values of applied discount obtained in the survey among the Slovenian market participants.

Analysis of the applied values of DLOM and DLOL by CBAs will give the answer if the value of applied DLOL declines with an increased valuated ownership interest and if the size of the applied discount varies over time. According to that, the following hypotheses have been set:

Hypothesis 2.1: With an increase of valuated ownership interest the size of applied discount decreases.

Hypothesis 2.2: Applied value of a discount to a certain size of valuated ownership interest does not vary over time.

5.2.1 Methodology

In this part of the Slovenian market research, I carried out a statistical analysis of applied DLOM and DLOL by CBAs in the selected time period and for different valuated ownership interest sizes. The analysis is based on the realized DLOM and DLOL in the business valuation process. The data was obtained by SIA.

With the help of the obtained data, I analysed what is the correlation between the valuated interest size and the applied discount and if applied discounts vary over time, namely if it increases or decreases. Based on the information obtained, I will check the previously set hypotheses and summarized key findings.

5.2.2 Data collection and sample description

Data for this part of the research was received from the SIA. The obtained information refers to the applied DLOM and DLOL in the business valuation process in the 2004–2018 period.

The initial data base consists out of 1,474 valuations. Data base was adjusted for the valuations without all the needed information, such as valuated interest size, date of valuation and the size of the applied discount. After the adjustment, the total number of valuations included into analysis amounted to 1,108. The number of transactions by year can be seen in the Table 41.

Table 41: Number of transactions with all required information

Year	Total sample		Adjusted sample		Adjusted / Total
	Number	Share	Number	Share	Share
n/a	23	1.6%	0	0.0%	0.0%
2004	1	0.1%	1	0.1%	100.0%
2005	16	1.1%	16	1.4%	100.0%
2006	16	1.1%	14	1.3%	87.5%
2007	2	0.1%	1	0.1%	50.0%
2008	5	0.3%	5	0.5%	100.0%
2009	4	0.3%	3	0.3%	75.0%
2010	0	0.0%	0	0.0%	0.0%
2011	3	0.2%	3	0.3%	100.0%
2012	48	3.3%	37	3.3%	77.1%
2013	**261**	**17.7%**	**200**	**18.1%**	**76.6%**
2014	**313**	**21.2%**	**248**	**22.4%**	**79.2%**
2015	**305**	**20.7%**	**235**	**21.2%**	**77.0%**
2016	**289**	**19.6%**	**205**	**18.5%**	**70.9%**
2017	**182**	**12.3%**	**137**	**12.4%**	**75.3%**
2018	6	0.4%	3	0.3%	50.0%
Total	**1,474**	**100%**	**1,108**	**100%**	**75.2%**

Source: Own work based on SIA (2018).

Since 2014, SIA has been collecting certain financial parameters related to business valuation in order to monitor the development of the valuation process. Since information is

given for the previous year the starting point of the further analysis will be year 2013 and because the data was obtained in the middle of 2018 and due to that last full year is 2017, this will be my analysed period. The final sample that I am going to use in the further analysis can be seen in the Table 42 and consists out of 1,025 transactions, which represent 75.9% of all valuations dated in the period 2013–2017.

Table 42: Final sample of valuations used in an analysis

Year	Total sample		Adjusted sample		Adjusted / Total
	Number	Share	Number	Share	Share
2013	261	19.3%	200	19.5%	76.6%
2014	313	23.2%	248	24.2%	79.2%
2015	305	22.6%	235	22.9%	77.0%
2016	289	21.4%	205	20.0%	70.9%
2017	182	13.5%	137	13.4%	75.3%
Total	**1,350**	**100%**	**1,025**	**100%**	**75.9%**

Source: Own work based on SIA (2018).

5.2.3 Analysis of the obtained results

Analysis of 1,025 business valuations focused mainly on an analysis of a relationship between the size of valuated ownership interest and the applied level of discount. The analysis is summarized in the Table 43, where average values of applied discount are shown for every ownership interest decile for every year in the period 2013–2017, and the average value of an ownership interest decile for the entire analysed period.

Table 43: Average value of applied discount for every ownership interest decile and year in 2013-2017 period

Year	Size of the valuated interest										
	Up to 10%	11% - 20%	21% - 30%	31% - 40%	41% - 50%	51% - 60%	61% - 70%	71% - 80%	81% - 90%	91% - 99%	100%
2013	4.06%	12.67%	5.73%	0.00%	2.00%	0.13%	0.00%	0.00%	4.08%	1.22%	2.49%
2014	11.00%	2.85%	6.43%	0.00%	0.43%	0.00%	3.29%	0.00%	n/a	2.17%	2.61%
2015	11.25%	4.00%	0.00%	0.00%	3.21%	0.00%	0.00%	0.00%	0.00%	1.07%	1.84%
2016	8.43%	3.58%	7.67%	7.00%	7.77%	0.00%	3.33%	5.25%	0.00%	0.00%	3.61%
2017	6.38%	8.80%	5.00%	17.00%	9.20%	5.40%	n/a	18.50%	0.00%	1.50%	2.80%
Average	**8.43%**	**5.54%**	**5.86%**	**3.65%**	**4.39%**	**0.97%**	**2.06%**	**2.79%**	**2.88%**	**1.09%**	**2.63%**

Source: Own work based on SIA (2018).

As it can be noticed, there is a negative correlation between the size of the valuated ownership interest and applied discount, although the pattern is not as clear as one obtained with the analysis of questionnaire among Slovenian market participants.

5.2.4 Correlation coefficients analysis

In practice, the correlation is the most commonly used approach to measures association between two variables. It summarizes the relationship between two variables in a so-called correlation coefficient. Pearson correlation coefficient ranges from +1 to -1. These are two extremes that represent a perfect correlation between the two variables. This correlation can be positive or negative. If a value of the correlation coefficient is +1 variables move in the same direction (if one increases the other increases) with the same magnitude, while the correlation coefficient with the value of -1 means that the variables move in different directions (if one increases the other decreases). With the lowering of the value of the correlation coefficient the magnitude of the correlation between two variables decreases. When the value of the coefficient is equal to 0 there is no correlation between two variables, meaning that the increase or decrease of one variable does not affect the value of the other variable (Daly et al., 1995). Magnitude of the Pearson correlation coefficient expressed in absolute values can be divided into the following levels (BenSTAT, 2022):

- 0.00; no correlation.
- 0.01 – 0.19; negligible correlation.
- 0.20 – 0.39; low correlation.
- 0.40 – 0.69; medium correlation.
- 0.70 – 0.89; high correlation.
- 0.90 – 0.99; very high correlation.
- 1.0; perfect correlation.

Analysis of Person correlation coefficients for value of discount as dependent variable indicates that it is negatively and statistically significantly correlated with a size of the valuated ownership interest, while year of valuation is positively and statistically significantly correlated with the discount value. Values of correlation coefficients among dependent and independent variables are shown in the Table 44.

Correlation between size of the valuated ownership interest and the year of valuation is positive and not high enough that there would be a risk of multicollinearity. The correlation is also statistically insignificant. Multicollinearity is a situation where two independent variables are highly correlated and change in the value of one variable has impact on the value of the other variable. The latter has negative impact on the quality of the model.

Table 44: Correlation coefficient analysis

	Discount	Valuated ownership interest size	Year of valuation
Discount	**1.000** -		
Valuated ownership interest size	**-0.191** ***0.000***	**1.000** -	
Year of valuation	**0.066** ***0.018***	**0.038** ***0.113***	**1.000** -

Source: Own work based on SIA (2018).

5.2.5 Summarization and hypotheses testing

Statistical analysis of the applied discount on the Slovenian market showed that, with some deviations, Slovenian CBAs take into account the characteristics of liquidity and how to consider a lack of it when determining the appropriate level of discount. This can be noticed in the negative and statistically significant impact of the size of valuated ownership interest on the value of applied discount (negative correlation coefficient), but some deviations exist, as in some cases, the average value of the applied discount increases, compared to the previous decile of valuated ownership interest. However, in general, the value of the applied discount decreases with an increase of the valuated ownership interest, which means that the Hypothesis 4.1 cannot be rejected.

From the correlation coefficient between the year of valuation and the value of applied discount can also be noticed that the year of valuation has a positive and statistically significant impact on the value of applied DLOL, but the magnitude of the impact is negligible, meaning that the Hypothesis 4.2 cannot be rejected.

6 CONCLUSION

Determination and application of DLOL is a very complex task and has an important role in understanding how market behaves. Different factors affect the value of DLOL, authors have different opinions about it, views how to use or estimate it and market participants understand and use it in different ways. In what follows, I present the key findings of the master's thesis, followed by analysis of shortcomings and suggestions for further research.

6.1 Summary and key findings

The main purpose of this master's thesis was to analyse understanding and application of DLOL on the Slovenian market. It was carried out by two separate researches. First one was a survey among relevant market participants, while the second one was an analysis of actual, realized discounts applied to an ownership interests of different sizes. Results of a survey indicated that market participants are not always consistent with each other, which may be due to the wide range of participants included into survey.

Overall conclusion can be that Slovenian market participants have an appropriate level of DLOL understanding. Based on the answers provided in the survey, the average value of applied DLOL in the business valuation decreases from 17.1% % for valuation of ownership interest up to 24.99% to 8.4% for valuation of 100% ownership interest.

Above stated can be further proven by the movement of the actual applied values of discount carried out in the second research on the Slovenian market. The results of the analysis showed that in general the value of the applied discount decreases with an increase of the valuated ownership interest. Average applied DLOL decreased from 8.4% for up to 10.0% ownership interest to 2.6% for 100% ownership interest. Although some deviations occurred, meaning that the average value of the applied DLOL in a certain ownership interest decile increased compared to the applied DLOL in previous decile.

6.2 Limitations, shortcomings and weaknesses

A key limitation of an analysis of a survey among Slovenian market participants is a poor response, as only 15.9% of participants finished the survey. In case of a higher response, the results could give a substantially better market overview.

Analysis of an actually applied values of DLOL obtained from SIA faced a shortcoming in terms of short time period taken into analysis. On the one hand, SIA only started collecting needed data in 2014, while on the other hand, the cut-off date of the analysis was in the middle of 2018, meaning that because the data was collected for a previous year, the analysis was based on a data only for the 2013–2017 period.

6.3 Suggestions for further research

The key objective of this master's thesis was an analysis of usage of DLOL on the Slovenian market, its understanding and application in the business valuation process. Given that the survey was conducted in 2020, it would be interesting to conduct it again and compare the results and the development of understanding of DLOL. The market research could be further deepened by conducting interviews with target groups and analysing case studies.

The analysis of applied discounts could be further extended over a longer period, as the data originally collected is from 2018. Longer time period would result in a better market overview and in a more holistic understanding. The research could be further deepened by including additional variables, such as the purpose of the sale and the basis of value or any other factors that affect the size of the applied discount.

Further research could be carried out on a wider range, including additional markets or geographical regions. This would give additional insight into the understanding and use of

DLOL in the wider European area. The limitation of an analysis on a wider European area could be a fragmentation of the market, as in many countries the data of private companies' transactions are not publicly available.

Furthermore, additional analysis of potential double counting of DLOL in case of included size premium in the cost of equity calculation and application of DLOL to estimated equity value at the same time could be carried out.

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APPENDICES

Appendix 1: Summary in Slovene language (Povzetek v slovenskem jeziku)

Vrednotenje podjetij oziroma lastniškega deleža v le-teh je kompleksen proces, ki vključuje širok nabor predpostavk in spremenljivk. Slednje imajo močan vpliv na končno vrednost ocenjevanega lastniškega deleža, zaradi česar je potrebno predpostavke in spremenljivke v procesu vrednotenja analizirati in določiti njihovo vrednost z vso skrbnostjo.

Pri izračunu vrednosti za lastniški kapital ocenjevalec vrednosti aplicira diskont za pomankanje kontrole ter diskont za pomankanje tržljivosti in/ali likvidnosti, slednje pa je odvisno od velikosti ocenjevanega lastniškega deleža.

Pregled obstoječe literature pokaže, da lahko metode za izračun diskonta za pomankanje tržljivosti in/ali likvidnosti delimo v dve skupini, in sicer na metode, primernejše za uporabo v primerih vrednotenja manjšinskega lastniškega deleža, ter na metode, primernejše za uporabo v primerih vrednotenja obvladujočega lastniškega deleža. Zaradi pomankanja literature, ki bi primarno naslavljala določitev diskonta v primeru vrednotenja obvladujočega lastniškega deleža, v praksi prihaja do uporabe metod, namenjenih vrednotenju neobvladujočega lastniškega deleža tudi v vrednotenjih obvladujočega lastniškega deleža. To se lahko odrazi v nepravilno določeni ravni diskonta za pomankanje likvidnosti obvladujočega lastniškega deleža, saj je pri izračunu diskonta potrebno izhajati iz razlik med neoblvadujočim in obvladujočim lastniškim deležem ter pravicami, ki jih slednja posedujeta.

Tako ima imetnik večinskega lastniškega deleža vpliv na poslovni proces in strategijo podjetja, s čimer lahko optimizira denarne tokove podjetja, zaradi česar je, ob vsem ostalem nespremenjenem, njegov lastniški delež vreden več v primerjavi z neobvladujočim lastniškim deležem. Prav tako so stroški in negotovost morebitne prodaje obvladujočega lastniškega deleža drugačni v primerjavi z neobvladujočim lastniškim deležem, zaradi česar je slednji manj likviden. Metodologija, ki prikazuje empirične dokaze o diskontu pri vrednotenju obvladujočega lastniškega kapitala je bistveno drugačna od metodologije za določitev diskonta za neobvladujoč lastniški delež. Nadaljnje, visoka varianca premije za obvladovanje otežuje določitev primerne ravni diskonta za pomanjkanje likvidnosti obvladujočega lastniškega deleža iz perspektive neobvladujočega lastniškega deleža.

Metode, ki so primernejše za določitev diskonta za pomankanje tržljivosti pri vrednotenju neobvladujočega lastniškega deleža so pre-IPO študije ter študije delnic z omejitvami (*restricted stock studies*). Te metode ne razlikujejo med različnimi stopnjami lastništva in so kot take primernejše za vrednotenje manjšinskega lastniškega deleža. V okviru pre-IPO pristopa je povprečna vrednost diskontov osmih Emoryjevih študij, izvedenih v obdobju 1980–1997, znašala od 42% do 60%, medtem ko prilagojena študija iz leta 2002 daje povprečno vrednost diskonta v višini 46%. Poleg tega je povprečna vrednost diskonta za podjetja dot.com v obdobju 1997–2000 znašala 54%. V okviru študij delnic z omejitvami je

Wruck v svoji študiji iz leta 1989 pokazala, da povprečna vrednost diskonta znaša 17,6%, medtem ko sta Hertzel in Smith ugotovila, da povprečni diskont znaša 13,5%.

Izračun diskonta za pomankanje likvidnosti za obvladujoč lastniški delež ni tako jasen kot ocena diskonta za neobvladujoč lastniški delež. Čeprav lastniki obvladujočega lastniškega deleža v zasebnih družbah običajno nimajo zakonskih omejitev za prodajo svojega deleža, pa transakcij svojega deleža ne morejo izvršiti hipno in prejeti gotovine v treh delovnih dneh, kar nekateri avtorji postavljajo kot mejo likvidnostnega sredstva. Priprava obvladujočega lastniškega deleža za prodajo lahko traja nekaj časa, pri tem pa nastanejo stroški. Tako lahko rečemo, da so obvladujoči lastniški deleži v zasebnih podjetjih tržni, saj lastniki nimajo zakonskih omejitev za prodajo lastniškega deleža, vendar so nelikvidni, ker je za njihovo preoblikovanje v denar potreben čas in povzroča stroške.

Nekateri avtorji poskušajo s teoretičnimi koncepti rešiti problem pravilne določitve velikosti diskonta, medtem ko se drugi tega problema lotevajo z empiričnimi raziskavami. Prvi naslavljajo problem pravilne določitve diskonta z modeliranjem regresijskih modelov, vključno z različnimi dejavniki, kot so makroekonomski in dejavniki, specifični za posamezno industrijo, ki bi lahko vplivali na vrednost lastniškega deleža, medtem ko empirične raziskave pogosto temeljijo na prevzemih podjetij in mnogokratnikih vrednotenja, kjer se primerjajo mnogokratniki javnih in zasebnih podjetij.

V magistrski nalogi predstavljeni teoretični koncepti so Chaffe model, Longstaff model ter Finnerty model, ki vsi nakazujejo na obstoj diskonta. Modeli temeljijo na različnih prodajnih opcijah in zajemajo različna časovna obdobja držanja teh opcij ter različne volatilnosti opcij. Ob upoštevanju šestmesečnega držanja opcije in razponu volatilnosti med 20% in 150% je razpon diskonta navedenih modelov sledeč: razpon Chaffe modela (*European fixed strike put option*) znaša med 6,5% in 39,7%, razpon preoblikovanega Longstaff modela (*American fixed strike lookback put option*) znaša med 10,6% in 53,8%, razpon Finnerty modela (*Asian-style average strike put option*) pa med 3,3% in 21,8%.

V okviru empiričnih raziskav, temelječih na prevzemih podjetij in mnogokratnih vrednotenja, v okviru te magistrske naloge analiziram pet študij. Študije temeljijo na primerjavi različnih mnogokratnikov v različnih časovnih obdobjih, avtorji pa imajo različne poglede glede upoštevanja vhodnih podatkov in določitve parov zasebnih in javnih podjetij. Študije kažejo, da se zasebna podjetja običajno prodajajo z diskontom v primerjavi z javnimi podjetji, kar je lahko posledica manjše likvidnosti, ki se odraža v nižjih mnogokratniki vrednotenja. Koeplin študija poda razpon povprečne vrednosti diskonta med 17,8% in 28,3% za podjetja s sedežem v ZDA ter med 34,9% in 53,9% za druga podjetja. Kooli študija poda razpon povprečne vrednosti diskonta v razponu med 17% in 34%, Officer v razponu med 17,2% in 30,0%, Block študija pa v razponu med 16,3% in 27,1%. Paglia in Harjoto študija, z nekoliko alternativnim pogledom na izračun diskonta za pomankanje likvidnosti

obvladujočega lastniškega deleža, poda razpon vrednosti diskonta med 17,9% (umetniška in zabavna industrija) in 68,5% (nočitvene in prehranske storitve).

V empiričnem delu magistrske naloge sem analiziral stanje na slovenskem trgu, analiza pa je razdeljena na dva dela. Iz prvega dela analize, to je anketnega vprašalnika med 33 deležniki na trgu, je razvidno, da slednji med seboj niso usklajeni ter da v določenih primerih njihova praksa odstopa od prakse, predstavljene v teoretičnem delu naloge. Iz analize anketnega vprašalnika gre izpostaviti, da največ deležnikov na slovenskem trgu uporablja izbrano srednjo vrednost več študij, s prilagoditvijo za značilnosti vrednotenega podjetja, kot pristop k določitvi primerne ravni diskonta za pomanjkanje likvidnosti, da sta časovna komponenta ter prilagoditev tržnim razmeram glavna dejavnika aplikacije diskonta ter da v večini deležniki odbijejo vrednost diskonta od vrednosti za celoten kapital.

Drugi del analize stanja na slovenskem trgu je zajemal analizo dejansko apliciranih vrednosti diskonta. Cilj tega dela je bil ugotoviti, ali dejanski podatki kažejo na obratno povezanost med velikostjo ocenjevanega lastniškega deleža in velikostjo apliciranega diskonta, na kar nakazujejo tako študije predstavljene v teoretičnem delu naloge kot rezultati anketnega vprašalnika, ter ugotoviti vpliv časovne komponente na velikost apliciranega diskonta. Slednje sem preverjal s pomočjo korelacijskih koeficientov. Vrednosti korelacijskih koeficientov med velikostjo diskonta ter velikostjo ocenjevanega deleža in letom vrednotenja sta statistično značilni, pri čemer ima velikost ocenjevanega deleža negativen vpliv, leto transakcije pa pozitiven, a zanemarljiv vpliv.

Določitev vrednosti diskonta za pomankanje likvidnosti obvladujočega lastniškega deleža je kompleksna naloga, ki pa ima pomemben vpliv na vrednost ocenjevanega lastniškega deleža. S povzetkom raziskav drugih avtorjev ter izvedbo lastne analize stanja na slovenskem trgu menim, da lahko ta magistrska naloga pripomore k boljšemu razumevanju ter pravilnejši določitvi in uporabi diskonta za pomankanje likvidnosti pri ocenjevanju vrednosti obvladujočega lastniškega deleža.

Appendix 2: Summarization of the DLOL determination methods

Theoretical solutions	Year	Characteristics	Comment	Discount range (T: 6 months; σ: 20%-150%)
The Chaffe model	1993	European fixed strike put option	• Based on BSM option theory. • Ignores risk of price increases. • Indicated DLOM is excessively high at higher volatilities.	6.5% - 39.7%
The Longstaff model – transformed	1995	American fixed strike lookback put option	• Considers risk of price increases during holding period. • Assumes perfect market timing ability. • May overstate the DLOM at higher volatilities.	10.6% - 53.8%
The Finnerty model	2003	Asian-style average strike put option	• Considers risk of price increases during holding period without perfect market timing ability. • Upper DLOM bound at 32.3% regardless of higher volatilities or longer time period. • Tends to produce a minimum DLOM.	3.3% - 21.8%

PCD studies	Year	Period	Area	No. of observations	Discount range
Koeplin, Sarin and Shapiro	2000	1984-1998	USA	84	17.8% - 28.3%
			non-USA	108	34.9% - 53.9%
Kooli, Kortas, and L'Her	2003	1995-2002	USA	331	17% - 34%
Officer	2007	1979-2003	Global - Stand-alone	364	17.2% - 22.9%
			Global - Subsidiaries	643	26.9% - 30.0%
Block	2007	1999-2006	USA	91	16.3% - 27.1%
Paglia & Harjoto	2010	1993-2008	Global	431	17.9% - 68.5%

Source: Block (2007); Elmore (2017); Koeplin et al. (2000); Kooli et al. (2003); Longstaff (1995); Officer (2007) and Paglia & Harjoto (2010).

Appendix 3: Analysis of applied discounts by CBAs in the period 2013-2017

Descriptive statistics	Up to 10%						11% - 20%						21% - 30%					
	FY13	FY14	FY15	FY16	FY17	**Total**	FY13	FY14	FY15	FY16	FY17	**Total**	FY13	FY14	FY15	FY16	FY17	**Total**
Count	17	22	12	17	8	**76**	6	13	11	12	10	**52**	11	7	2	6	2	**28**
Min	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**
Max	40.0%	46.0%	33.0%	32.0%	29.0%	**46.0%**	46.0%	24.0%	28.0%	27.0%	31.0%	**46.0%**	32.0%	35.0%	0.0%	23.0%	10.0%	**35.0%**
Average	4.06%	11.00%	11.25%	8.43%	6.38%	**8.43%**	12.67%	2.85%	4.00%	3.58%	8.80%	**5.54%**	5.73%	6.43%	0.00%	7.67%	5.00%	**5.86%**
Median	0.0%	0.0%	7.0%	0.0%	0.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%	0.5%	**0.0%**	0.0%	0.0%	0.0%	0.0%	5.0%	**0.0%**
StDev	10.6%	17.4%	12.5%	12.4%	10.7%	**13.5%**	20.3%	7.3%	9.3%	8.7%	12.1%	**11.1%**	12.7%	13.1%	0.0%	11.9%	7.1%	**11.4%**
CV*	2.60	1.58	1.11	1.47	1.69	1.60	1.60	2.57	2.32	2.43	1.37	2.00	2.23	2.04	n/a	1.55	1.41	1.95

Descriptive statistics	31% - 40%						41% - 50%						51% - 60%					
	FY13	FY14	FY15	FY16	FY17	**Total**	FY13	FY14	FY15	FY16	FY17	**Total**	FY13	FY14	FY15	FY16	FY17	**Total**
Count	5	2	4	4	2	**17**	10	7	14	13	5	**49**	8	9	5	2	5	**29**
Min	0.0%	0.0%	0.0%	0.0%	16.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**
Max	0.0%	0.0%	0.0%	28.0%	18.0%	**28.0%**	20.0%	3.0%	25.0%	33.0%	20.0%	**33.0%**	1.0%	0.0%	0.0%	0.0%	16.0%	**16.0%**
Average	0.00%	0.00%	0.00%	7.00%	17.00%	**3.65%**	2.00%	0.43%	3.21%	7.77%	9.20%	**4.39%**	0.13%	0.00%	0.00%	0.00%	5.40%	**0.97%**
Median	0.0%	0.0%	0.0%	0.0%	17.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%	12.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**
StDev	0.0%	0.0%	0.0%	14.0%	1.4%	**8.4%**	6.3%	1.1%	8.2%	11.8%	8.9%	**8.7%**	0.4%	0.0%	0.0%	0.0%	7.6%	**3.5%**
CV*	n/a	n/a	n/a	2.00	0.08	2.31	3.16	2.65	2.56	1.51	0.97	1.99	2.83	n/a	n/a	n/a	1.41	3.67

Descriptive statistics	61% - 70%						71% - 80%						81% - 90%					
	FY13	FY14	FY15	FY16	FY17	**Total**	FY13	FY14	FY15	FY16	FY17	**Total**	FY13	FY14	FY15	FY16	FY17	**Total**
Count	2	7	4	3	0	**16**	7	12	7	4	4	**34**	12	0	1	3	1	**17**
Min	0.0%	0.0%	0.0%	0.0%	n/a	**0.0%**	0.0%	0.0%	0.0%	0.0%	7.0%	**0.0%**	0.0%	n/a	0.0%	0.0%	0.0%	**0.0%**
Max	0.0%	23.0%	0.0%	10.0%	n/a	**23.0%**	0.0%	0.0%	0.0%	16.0%	27.0%	**27.0%**	10.0%	n/a	0.0%	0.0%	0.0%	**10.0%**
Average	0.00%	3.29%	0.00%	3.33%	n/a	**2.06%**	0.00%	0.00%	0.00%	5.25%	18.50%	**2.79%**	4.08%	n/a	0.00%	0.00%	0.00%	**2.88%**
Median	0.0%	0.0%	0.0%	0.0%	n/a	**0.0%**	0.0%	0.0%	0.0%	2.5%	20.0%	**0.0%**	0.0%	n/a	0.0%	0.0%	0.0%	**0.0%**
StDev	0.0%	8.7%	0.0%	5.8%	n/a	**6.1%**	0.0%	0.0%	0.0%	7.5%	8.3%	**6.9%**	5.1%	n/a	n/a	0.0%	n/a	**4.6%**
CV*	n/a	2.65	n/a	1.73	n/a	2.96	n/a	n/a	n/a	1.44	0.45	2.49	1.24	n/a	n/a	n/a	n/a	1.60

Descriptive statistics	91% - 99%						100%					
	FY13	FY14	FY15	FY16	FY17	**Total**	FY13	FY14	FY15	FY16	FY17	**Total**
Count	9	6	14	9	6	**44**	113	163	161	132	94	**663**
Min	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**
Max	10.0%	13.0%	10.0%	0.0%	7.0%	**13.0%**	34.0%	32.0%	39.0%	30.0%	24.0%	**39.0%**
Average	1.22%	2.17%	1.07%	0.00%	1.50%	**1.09%**	2.49%	2.61%	1.84%	3.61%	2.80%	**2.63%**
Median	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**	0.0%	0.0%	0.0%	0.0%	0.0%	**0.0%**
StDev	3.3%	5.3%	2.9%	0.0%	2.8%	**3.0%**	6.6%	7.1%	6.5%	6.8%	4.8%	**6.5%**
CV*	2.71	2.45	2.70	n/a	1.87	2.78	2.64	2.70	3.53	1.88	1.73	2.48

Note: CV - Coefficient of variation
Source: Own work based on SIA (2018).

Appendix 4: Survey questionnaire among Slovenian market participants

UNIVERSITY OF LJUBLJANA
SCHOOL OF ECONOMICS AND BUSINESS

SURVEY QUESTIONNAIRE FOR MASTER'S THESIS

DISCOUNT FOR LACK OF LIQUIDITY FOR CONTROLLING OWNERSHIP INTEREST: UNDERSTANDING AND APPLICATION IN THE BUSINESS VALUATION PROCESS ON THE SLOVENIAN MARKET

Ljubljana, November 2019 Author: Nejc Jakomini
Supervisor: asst. prof. Matjaž Črnigoj, PhD

Dear Sir or Madam,

My name is Nejc Jakomini and I am a student at the School of Economics and Business, University of Ljubljana. I am preparing a master's thesis with the title **Discount for lack of liquidity for controlling ownership interest: understanding and application in the business valuation process on the Slovenian market**, within which I also analyse business valuations in Slovenia. The supervisor of the master's thesis is **asst. prof. Matjaž Črnigoj, PhD**.

The purpose of the survey questionnaire is to determine what practice prevails in Slovenia in the field of business valuation and more specifically in determination of appropriate level and application of discount for lack of liquidity. Since the chosen topic affects different groups of experts, that have different views on the business valuations and application of the mentioned discount, I would like to find out what are their views and whether they differ between the expert groups. **Your participation is crucial for the research**, because only with your answers will I be able to get a realistic picture of what is happening on the Slovenian market.

It will take you about 5-10 minutes to complete the survey. The collected data will be treated strictly confidentially and only at a consolidated level and used for the preparation of this master's thesis, possible further research or presentations.

I sincerely thank you for your participation.

Nejc Jakomini

For any question, I am available at:
+386-XX-XXX-XXX
XXXX.XXXXXXXX@gmail.com

1. General section

The general section includes questions about demographic variables and descriptive questions related to your field of work.

1.1 Select your age group.

a) 20-29
b) 30-39
c) 40-49
d) 50-59
e) 60-69

1.2 Select your years of experience group.

a) Up to 5 years
b) From 6 to 10 years
c) From 11 to 20 years
d) 21 years or more

1.3 Select your formal education level.

a) High school graduate
b) Bachelor's degree
c) Master's degree
d) PhD

1.4 Select title or titles you have acquired. (multiple answers are possible)

a) Certified Business Appraisers (SIR)
b) Certified Business Appraisers (ASA)
c) CFA
d) CAIA
e) FRM
f) Stockbroker
g) Court expert
h) Court appraiser
i) Other: _____
j) Nothing listed

1.5 Select the functions you perform in your workplace. (multiple answers possible)

a) Business appraiser
b) Financial consultant
c) Investment manager
d) Analyst
e) Court appraiser
f) Court expert
g) Other

1.6 Select the services you perform in your workplace. (multiple answers possible)

h) M&A consulting
i) Business valuation
j) Business valuation as a certified appraiser
k) Obtaining financial resources
l) Refinancing
m) Expert opinions
n) Investment management
o) Data analysis

2. Discount for lack of marketability

In this part, the questions refer to determination and usage of discount for lack of marketability. When referring to marketability I used the following definitions:

"Marketability and discount for lack of it relate to the non-controlling ownership interest and it is defined as the right to sell company's ownership interest in established and efficient market with minimum effort, low transaction costs and within reasonable time."

2.1 What do you think discount for lack of marketability represents in the ownership interest valuation?

_____

2.2 How do you determine the value of the discount for lack of marketability when valuing a non-controlling ownership interest?

a) Use of a discount as determined in one study
b) Use of a discount as determined in one study with adjustments based on the characteristics of the valuated company
c) Use of the selected mean of discount rates of multiple studies
d) Use of the selected mean of discount of several studies with adjustments according to the characteristics of the valuated company
e) I trust my own feeling and experience (subjective evaluation)
f) Other:

_____

2.3 Which of the following studies represent your basis for determining the discount for lack of marketability? (multiple answers possible)

a) Restricted stock studies
b) Pre-IPO studies
c) Theoretical solutions (discounted cash flows, option models)
d) I trust my own feelings and experiences (subjective assessment)
e) Other:

_____

2.4 In your opinion, how strong is the impact of each stated factor on the size of the discount for lack of marketability? (select one answer)

Answer	Do not have impact	Small impact	Medium impact	Strong impact	Very strong impact
The right to sell (put option)					
Dividend payments					
Potential buyers					
The size of the share block to be sold					
Expected public sale of shares					
Access to and reliability of information					
Restrictive provisions in the Statute/ Partnership agreement					

3. Discount for lack of liquidity

In this part, the questions refer to determination and usage of discount for lack of liquidity. When referring to liquidity I used the following definitions:

"Liquidity and discount for lack of it relate to the controlling ownership interest and it is defined as the ability to convert ownership interest into cash without significant loss of principal."

3.1 What do you think discount for lack of liquidity represents in the ownership interest valuation?

_____

3.2 In your opinion, how many days to convert the controlling ownership interest into cash represent the liquid asset limit?

a) In 24 hours
b) In three days
c) In one week
d) In one month
e) In six months
f) In one year

3.3 How do you determine the value of the discount for lack of liquidity when valuing a controlling ownership interest?

a) I use the same approach as in the case of valuation of non-controlling ownership interest and setting the discount for lack of marketability
b) Use of a discount as determine in one study
c) Use of a discount as determined in one study with adjustments based on the characteristics of the valuated company
d) Use of the selected mean of discount rates of multiple studies
e) Use of the selected mean of discount of several studies with adjustments according to the characteristics of the valuated company
f) I trust my own feeling and experience (subjective evaluation)
g) I do not use a discount in valuation of controlling ownership interest
h) Other: _____

3.4 Which of the following studies represent your basis for determining the discount for lack of marketability? (multiple answers possible)

a) Theoretical solutions (Chaffe model, Longstaff model, Finnerty model and other theoretical models)
b) Koeplin, Sarin and Shapiro study
c) Kooli, Kortas and L'Her study
d) Officer study
e) Block study
f) Paglia and Harjoto study
g) Other Private Company Discount studies
h) Nothing listed
i) Other: _____

3.5 Do you agree with the following statements? (select one answer)

Answer	Entirely disagree	Disagree	Indifferent	Agree	Entirely agree
The use of DLOL in the valuation of controlling ownership interest is required due to the adjustment to market conditions					
The use of DLOL in the valuation of controlling ownership interest is required due to the presence of transaction costs					
The use of DLOL in the valuation of controlling ownership interest is required due to time uncertainty of the transaction					
The use of DLOL in the valuation of controlling ownership interest is required to adjust for the risk of change in the value of the holding					
The use of DLOL in the valuation of controlling ownership interest is not sufficiently theoretically supported and therefore unjustified					
The use of DLOL in the valuation of controlling ownership interest is not sufficiently supported by empirical studies and therefore unjustified					
Any ownership interest can be sold, no matter how illiquid it is, it is just a subject of acceptance of lower price					

3.6 Which approach of DLOL application do you use most frequently

a) I deduct the discount from the estimated equity value
b) I consider DLOL in the discount rate as a premium in percentage points in the discount rate or capitalization rate
c) I value the equity of the company in accordance with valuations of similarly liquid equity of other companies

3.7 In your opinion, how strong is the impact of each stated factor on the size of the discount for lack of liquidity? (select one answer)

Answer	Do not have impact	Small impact	Medium impact	Strong impact	Very strong impact
The right to sell (put option)					
Dividend payments					
Potential buyers					
The size of the share block to be sold					
Expected public sale of shares					
Access to and reliability of information					
Restrictive provisions in the Statute/ Partnership agreement					
Uncertain timeline of transaction execution					
Transaction preparation cost					
Risks associated with the selling price					
Different forms of purchase					
The difficulty of selling a privately held company					

3.8 How much time in the business valuation process do you dedicate to determine the size of discount for lack of liquidity?

a) Up to 5% of the time
b) 6% - 10% of the time
c) 11% - 15% of the time
d) 16% - 20% of the time
e) More than 20% of the time

3.9 Is the size of the discount for lack of marketability and liquidity affected by the size of the valuated ownership interest?

a) Yes
b) No

3.10 For each range of the valuated ownership interest, write down the average value of applied discount for lack of marketability or liquidity.

- 0.01% - 24.99% _____
- 25.00% - 49.99% _____
- 50% _____
- 50.01% - 74.99% _____
- 75% _____
- 75.01% - 99.99% _____
- 100% _____

3.11 Which discounts do you use when valuating ownership interest for the purpose of squeezing out minority shareholders (according to ZGD-1)?

a) No discounts
b) Discount for lack of control
c) Discount for lack of marketability
d) Discount for lack of liquidity

3.12 When calculating the cost of capital, do you take into account size premium of the company being valuated (Cost of capital = Risk-free rate of return + (Beta * Premium for capital risk) + Premium for the size of the company + Other risks)?

a) Yes
b) No

3.13 If yes, when valuing small and medium-sized companies, do you also consider a discount for lack of liquidity when calculating the final, enterprise value?

a) Yes
b) No

4. Discount for lack of control

In this part, the questions are related to the determination and usage of the discount for the lack of control, and I used the following definition:

"A discount for lack of control refers to an amount or percentage that is deducted from the equity value due to a lack of control over the corporate governance."

4.1 Where do you consider discount for lack of control more often, in your cash flow projections or in calculation of enterprise value?

a) In cash flow projections
b) In enterprise value calculation

4.2 How do you determine the value of the discount for lack of control?

a) I use the same approach as in the case of valuation of non-controlling ownership interest and setting the discount for lack of marketability
b) Use of a discount as determine in one study
c) Use of a discount as determined in one study with adjustments based on the characteristics of the valuated company
d) Use of the selected mean of discount rates of multiple studies
e) Use of the selected mean of discount of several studies with adjustments according to the characteristics of the valuated company
f) I trust my own feeling and experience (subjective evaluation)
g) I do not use a discount in valuation of controlling ownership interest
h) Other:

_____

4.3 How do you determine the value of the discount for lack of control?

a) Appointment of management board
b) Appointment of supervisory board
c) Establishment of bonuses for members of management and supervisory board
d) Establishment of business policies and strategies
e) Purchase, sale and rent of business assets
f) Selection of external partners
g) Acquisition of another company
h) Liquidation, sale or capitalization of the company
i) Obtainment or sale of own shares
j) Issue of new shares
k) Issue of debt securities
l) Definition and pay-out of dividends and other payments
m) Changes to statute
n) Prevention of the decision stated above

4.4 For each range of the valuated ownership interest, write down the average value of applied discount for lack of control.

- 0.01% - 24.99% _____
- 25.00% - 49.99% _____
- 50% _____
- 50.01% - 74.99% _____
- 75% _____
- 75.01% - 99.99% _____