UNIVERSITY OF LJUBLJANA
SCHOOL OF ECONOMICS AND BUSINESS

MASTER'S THESIS

THE USEFULNESS OF THE REAL OPTIONS VALUATION APPROACH IN VALUING START-UP COMPANIES

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LIST OF ABBREVIATIONS

CAPEX – Capital expenditures
CAPM – Capital asset pricing model
DCF – Discounted cash flow
FCF – Free cash flow
FCFF – Free cash flow to firm
FY – Financial year
IPO – Initial public offering
MAD – Marketed asset disclaimer
NPV – Net present value
PY – Project year
ROA – Real options analyses
R&D – Research and development
WACC – Weighted average cost of capital

INTRODUCTION

Startups have become an important part of today's economy. Since late 1970s, 43% of all companies that went public were backed by venture capitalists prior to their initial public offering (IPO) (Gornall & Strebulov, 2015). Startups that succeed in their early stages can become corporate giants or "unicorns" – a company backed by venture capital with a reported valuation above 1 billion USD (Gornall & Strebulov, 2017). As of early 2017 there were more than 200 unicorns in the World (Gornall & Strebulov, 2017). These two facts show that startup companies are becoming more and more important and account for a large part of the economy.

I will illustrate that startup companies normally have a payoff scheme that corresponds to the payoff diagram of a financial call option. Black and Scholes (1973) define an option as a security that gives a right to buy or sell an asset, subject to certain conditions, within a specified time period. The aim of the thesis is to explore whether an option valuation approach is applicable to and can be useful in valuing startup companies. I examine the possible application of real options analyses (ROA) as the most optimal valuation method for startups, both from a theoretical perspective, as well as through practical examples.

Direct beneficiaries of an improved startup valuation model are the two main stakeholder groups associated with startups – entrepreneurs (or founders) and investors. Investors will gain more insight into their investments, whereas founders will be able to make a better case for their business idea and raise funds. With an improved model, key value drivers would become easier to determine and value. This should make startup valuations more accurate and better business ideas easier to screen, which would improve technology development and affect everyone that is to some extent connected to startups, their products and services or their funding.

Startups use external financing sources, but due to their risky nature, financing differs from a typical established enterprise. Startups usually use equity financing as they are too risky for debt financing vehicles. They normally turn to friends and family, angel investors and, at a later stage, to venture capital investors. Allen, Brealey & Myers (2011) separate the phases of startup development into the following investment groups: seed financing (family and friends), early investment rounds (business angels), later investment rounds (venture capital firms) and IPO of the firm. All the above-mentioned groups of investors are potential users and beneficiaries of ROA applications to startup valuations.

The thesis has several goals. I wish to contribute to the theoretical discussions about real options analysis, its place in the valuation theory and its application to actual cases. Furthermore, I compare ROA to various valuation models and argue against their advantages and disadvantages. Comparisons are planned to be done from a theoretical as well as practical perspective. I aim to examine the models not just as valuation techniques, but also as tools to determine the key value drivers of the underlying company.

The main research topic is whether startup companies can be valued using real options analysis approach and if there is any value added by using this method compared to the traditional ones.

To answer this question, I investigate other supporting research questions to thoroughly investigate the theoretical feasibility of real options, the key assumptions and to compare it with other established valuation models on real-life startup cases.
I focus on the advantages and disadvantages of the standard valuation approaches such as the discounted cash flow method (DCF) and their effectiveness in startup valuation.
A case is made to bridge the disagreements between the advocates and critics of ROA. I argue that ROA does not conflict with the DCF approach but can instead be used as a supplementary model, where DCF does not work on a stand-alone basis.
I determine the key value drivers by ROA and DCF approaches and perform sensitivity analyses of the value based on individual factors in each model to show where the variability of value is originating from. I analyse the model dependency on individual factors and the consequences of it on valuation accuracy for various valuation approaches.

The thesis is structured as follows. After this introduction, I devote the second section to theoretical background of various valuation techniques. Its focus is mainly on the most popular DCF and ROA. It provides theoretical foundations and arguments for and against the use of ROA. The third part builds on the theoretical discussions and transforms the theory to startup world. The key objective here is to examine how the theoretical assumptions function in this highly risky environment. The fourth section serves practical purposes. It presents the startup companies used in the research and performs simple valuations with the selected valuation methods. The fifth section concludes.

1 THEORETICAL BACKGROUND

This thesis is dealing primarily with real options and how they can be used to value startup companies. Even though startup companies are a very special asset group and ROA a specific valuation tool, the basis for applying ROA to startups (or any other asset for that matter) comes from the same fundamentals that are also used with other key valuation models. This chapter breaks down the theory behind the two main valuation methods of the thesis – ROA and DCF. I build up from the basic essence of value and valuation and then present each of the two methods separately.

1.1 Valuation

When looking at an asset and analysing its value it is not only important to understand what the value is, but also to understand where it originates from and what are its key value drivers. Valuation is often said to be a mixture of science and art and as such it is not prone to biases. Hence, an important aspect of valuation is to understand who is performing the valuation and what it is being used for as there can be various purposes for it. Damodaran (2012) separates three main roles of valuation: Portfolio management, M&A activities and corporate finance. The importance of valuation in the first two is straight-forward but it is also very much present in corporate finance. The goal of portfolio management is to make money on investments, meaning you should in general part invest in the assets you believe are undervalued and sell the ones you believe are overvalued. There are various types of portfolio investors and the importance of valuation depends on their beliefs of the market and the role they play as investors – for passive investors such as market timers or efficient marketers (the first group believes it makes more sense to value markets as a whole rather than individual stocks, while the second one believes that market price provides the best estimate of actual value) valuation is not as often used as with more active investors such as fundamental analysts or franchise buyers (the two groups of investors that focus most on finding the assets that are not valued accurately and exploit this inefficiency of the markets) (Damodaran, 2012). Valuation should also be a major factor when dealing with M&A decisions as you do not want to overpay or undersell your company. Startups can often be associated with M&A as it is often the goal of the founders or investors to make an exit (sell the company) after the company develops. Valuation should also play an important role in corporate finance as most corporate financial theory is built on the premise that companies' goal is the maximisation of firm value (Damodaran, 2012). Following this goal, each management decision should be done in a way that enhances the company's value.

Valuation is obviously a crucial piece in the financial world and it is important to know what are the valuation methods that we can use, which one should be chosen in a given scenario and how to apply them to get to the best estimate of value. Value is defined as the single time-value discounted number where all future net profitability is taken under account (Mun, 2002). To capture the value of an entire company in a single number sounds like a difficult task and it is, especially if a company is subject to high future uncertainty. It may happen that stock prices on the market do not equal the value of the underlying company, which can be a sign of market inefficiency. An asset can have its value based on its physical or non-physical, intrinsic or intangible aspects and the idea of the financial market is to capture the true value of tradeable assets and assign a price to it (Mun, 2002).

Since the source of an asset's value can be of various nature, multiple valuation approaches have been established. In broad terms, we can separate valuation models based on three main approaches: market, income and cost valuation approach (Mun, 2002). The market approach assumes the market tends to go towards equilibrium and price assets at a fair market value. By accepting such an assumption, one can find comparable assets in the financial markets with similar characteristics of the company in question. The income approach considers the future free cash flows that the company will bring and discounts them to a present value.

The cost approach looks at an asset and values it based on what costs a company would incur if it were to reproduce the future free cash flows of this asset. In the thesis I mainly focus on the second approach as both DCF and ROA belong there since they are trying to determine the intrinsic value of an asset. I also investigate the fit for relative valuation in the startup world, but there are not many public companies that would resemble startups enough (at least early and mid-stage ones). The comparable transactions might often be a useful valuation technique but are again more applicable for late-stage startups.

DCF based on the net present value approach is the valuation method I have found the most when researching the literature on valuation and even when researching valuation in uncertain environments. Even though other valuation tools such as ROA have shown great promise in valuation of certain new projects or capital budgeting of large corporations, it has been DCF analyses that has shown as the most popular method (Copeland & Antikarov, 2001; Brandao, Dyer & Hahn, 2005). Survey evidence implies that ROA has not been as popular among corporate managers as it has been within the academics' circles. Graham and Harvey (2001) show results of ROA ranking eighth out of 12 capital budgeting techniques, whereas a similar survey of Ryan and Ryan (2001) indicates ROA ranked as the least popular tool with 11,4% usage rate. Among the respondents of the research in the mid-2000s that included 279 out of Fortune 1000 largest companies, only 14.3% of respondents declared to use ROA (Block, 2007). In a research of the top Canadian companies, Baker, Dutta and Saadi (2015) observe similar trends with 16,8% of 214 respondents declaring to use ROA.

An interesting research of startup valuation techniques in Brasil (Oliveira & Zotes, 2018) shows that option-based-valuation-models have been the least popular with only 8% while DCF and comparable valuation techniques have been confidently holding the top spots.

1.2 Discounted cash flow method

DCF is the method that is ultimately used for comparisons with any other alternative approaches. In the thesis I do the same as I compare the process, methodology, findings and results of doing valuation using ROA with the ones being produced by applying DCF valuation. To do that I first need to present the basic model of the DCF, its foundations and intuition. Scenarios, where DCF should be used as the main valuation tool and scenarios, where DCF can get exposed by its pitfalls are described. As I will show, valuation of startup companies fits under the latter. I go more into details of that in later chapters, where I try to apply DCF valuation to startup cases, whereas in this subchapter, I focus on presenting the general characteristics of the model along with its advantages and disadvantages.

I present the model mainly following the terminology of Damodaran (2012), but similar interpretations can be found in just about any corporate finance or valuation textbook. In short, DCF valuation determines the value by forecasting future free cash flows of an asset and discounting them all to the present time, thus getting the present value of an asset discounted by the asset's risk factor, called the discount rate. Discounting future cash flows to the present state is also called the present value rule. In general, the formula for calculating the net present value (NPV) of an asset's cash flows discounted by a certain discount rate is the one below.

$$DCF = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_n}{(1+r)^n} \tag{1}$$

$$DCF = \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t} \tag{2}$$

where,
CF_t = the cash flow of an asset in time t
r = the discount rate reflecting the risk of the asset's future cash flows
N = the life span of the asset

Looking at the basic DCF formula, it makes sense that whenever we can clearly define the parameters mentioned in it, DCF would provide us with an accurate valuation. Hence, the key to DCF valuation is estimating future cash flows and the risk associated with them. This can work perfectly if estimating the value of a government bond, but when valuing a complex enterprise in a risky business environment, determining the future cash flows and their uncertainty becomes more difficult. Making similar arguments, Damodaran (2012) lists the areas, where DCF cannot work optimally in its original form. Some of these are:
- firms in trouble (negative cash flows, possibly even expected to go bankrupt);
- cyclical firms (cash flows very dependent on the state of the economy);
- firms with unutilised assets (assets that are not (yet) producing cash flows cannot be estimated with DCF);
- firms with patents or product options (patents or licenses that are not expected to produce cash flows in the future, but are still of some value).

Startup companies often belong to one of the above categories or have characteristics very similar to them. Negative cash flows, possibly expected to fail, unutilised assets, product options are all typical associations of a pre-revenue startup company or a venture developing new technologies.

Despite its drawbacks, DCF is reported to be the most used method of doing intrinsic company valuations, whereas the most used method is the relative valuation (Damodaran, 2012). Damodaran (2012) states that DCF is the basic method on which all other valuation approaches are built. Koller, Goedhart & Wessels (2010) write that DCF is the most accurate and flexible method for valuing projects, divisions and companies. I do not go into all the details of DCF valuation of a company as this is not the purpose of the thesis, but I do break down the basic DCF formula to its main components and show the framework of DCF that is usually used in practice.

1.2.1 Discount rate

Damodaran (2012) presents three ways of calculating the DCF: valuing cash flows to firm, cash flows to equity and breaking down the company to pieces and valuing them separately. All three approaches lead to the same result and it depends on the appraiser to use the one that is the most relevant for him. Cash flows to equity value only the equity stakes or the cash flows that equity holders can expect to get, whereas cash flows to firm value also the cash flows that debt holders can expect to get. Based on which approach you choose, you need to select the appropriate discount rate, so that it will reflect either just the risk associated with equity or also the risk associated with debt. When using cash flows to equity, we need to discount them by the cost of equity, which is the rate of return the equity holders in the firm are requiring. When using cash flows to firm, we must apply the cost of capital which is most commonly classified as the weighted average cost of capital (WACC) and combines the required rate of return of equity holders (cost of equity) as well as debt holders (cost of debt). I will use mainly the cash flow to firm model (FCFF) in the future mentions. The DCF equation can be restated to the following.

$$DCF = \sum_{t=1}^{N} \frac{CF_t}{(1+WACC)^t} \tag{3}$$

and

$$WACC = \frac{D}{E+D} K_d + \frac{E}{E+D} K_e \tag{4}$$

where,
$WACC$ = the weighted average cost of capital
K_d = the cost of debt
K_e = the cost of equity
D = the market value of company’s debt
E = the market value of company’s equity

Risk-free rate

There are multiple ways of deriving the cost of equity and the cost of debt. I will present only the main ones and the intuition behind them. For discussions regarding cost of debt or cost of capital we must first define the risk-free rate, which serves as the main building block of majority of corporate finance theory. The main job of DCF valuation is predicting future cash flows of the company and estimating the risk associated with these predictions. This risk can also be interpreted as the standard error of these expectations as the actual cash flows will likely defer from the predictions. When it comes to a risk-free asset, we are looking for an asset, where the actual returns match the predicted returns. Damodaran (2012) defines the risk-free rate as the rate of such a security that the future returns can be predicted with certainty and that carries no default risk and no reinvestment risk. To meet these criteria, we usually turn to the returns of a solid government's bond, where default rate is minimal. The bond needs to be long-term to meet the second criteria – since we are looking for a long-term rate, we could not use short-term ones and assume we would just be able to invest there again after maturity as the bond rate may change at that time. Normally, we should go with as long-term government bond as we can find (Damodaran, 2012), but we need to be careful that it matches the other data that we are using from a currency and longevity perspective.

Cost of debt

The cost of debt is what it takes the company today to borrow funds to finance its projects. It is driven by the risk-free rate, the default spread of the company and the tax rate. The first two items need to be added together, whereas the last one refers to the tax advantage of interest payments as they are tax deductible. Hence, if the company has sufficient income, taxes will be lower due to the interest payments, which simply means that after we determine the pre-tax cost of debt we multiply with (1 – tax rate) to get to the after-tax cost of debt. If the company has long-term bonds issued that are widely traded, the simplest way is to take the characteristics of that bond (market price, coupons, maturity) and derive a yield out of it which essentially means the company's cost of debt. If there are no such bonds, we can try to find a rating's agency rating of the company and get the cost of debt out based on the assumptions for a company of such rating. However, a lot of companies have neither long-term traded bonds nor rating scores, especially if they are not publicly traded, and we need to come up with a rate of our own. We can see if the company has had any recent long-term loans given to it and take the interest rate from there or we need to create our own synthetic rating. This can be done based on the financial statements. A common way is to calculate the interest coverage ratio, which divides earnings (EBIT) by interest paid and tells how many times more does a company earn compared to the interests it pays. Based on the ratio, we can then estimate a rating score and default spread.
(Damodaran, 2012)

To sum up, to get to the cost of debt we usually add together the risk-free rate and the default spread. To find the appropriate default spread or the total cost of debt directly, there are multiple ways depending on what data you have available:

- take the yield of a significantly traded long-term corporate bond as the cost of debt;
- get a rating score by a rating's agency to determine the default spread;
- look at recent long-term borrowings of the company;
- create your own synthetic rating out of the financial data available and estimate the default spread.

Cost of equity

There are several models that can compute the cost of equity, but I present only the capital asset pricing model (CAPM) as it has been by far the most popular model for calculating it since its introduction (Graham & Harvey, 2001). Popular alternatives, among others, are arbitrage pricing model, multi factor model and proxy model (Damodaran 2012).

Using CAPM we calculate the cost of equity or the required rate of return by equity holders through this equation:

$$E(r_i) = r_f + \beta_i[E[r_M] - r_f] \tag{5}$$

where,
$E(r_i)$ = the required rate of return
β_i = the measure of risk that refers to the company's exposure to market movements
$E[r_M] - r_f$ = the market risk premium (Expected market return – risk-free rate)

It is important to understand how risk is treated within the CAPM model as accounting for risk will be key throughout the discussion regarding DCF and ROA. In most cases in corporate finance risk is viewed at as from the eyes of a marginal investor; i.e. a fully-diversified investor. Such an investor does not care about the idiosyncratic risk associated directly with the company at hand as this is the risk that can be decreased to zero through diversification. Marginal investor only cares about the systematic risk or market risk as it cannot be diversified. The company's exposure to such risk is what should be considered when discounting expectations of future cash flows and this is the only risk that a marginal investor can get rewarded for taking (Damodaran, 2012). In CAPM, the parameter that captures this risk is beta. There are many ways of computing beta, the two most common ones being: regression beta (only possible with traded companies with enough data) and bottom-up beta (Damodaran, 2012). The first option requires us to regress the stock returns of the company to the returns of the market and the slope of the regression is the beta. With the second process, we need to find comparable traded companies, get their regression betas, unlever them of the effective debt to equity structure of each company, take an average of them and relever with the market values of debt to equity of the valued company.

The last missing bit required to calculate the cost of equity is the expected market return from which we need to subtract the risk-free rate to get to the market risk premium. The expected market return is usually calculated simply by taking the historical market returns over a certain (long) horizon, whereas there are also other, forward looking ways that may be preferable (Damodaran, 2012).

1.2.2 Predicting free cash flows

I have now gone through the key determinants of discount rate – the risk measure under the DCF approach. The other part of the DCF are the expected future cash flows and this subchapter focuses on estimating them. In DCF, the future cash flows or more precisely the future free cash flows represent the value of the company and their estimations are thus one of the most important bits of the process. First, let us define what the free cash flows are. Usually, the following approach is taken when calculating FCFF:

Table 1: Free cash flow to firm

Free cash flow to firm
+ EBIT*(1 - tax rate)
+ Depreciation and amortisation
- Change in working capital
- Capital expenditures
= Free cash flows to firm

Adapted from Damodaran (2012).

Intuitively, free cash flows represent the amount of cash the company earns that is free of any liabilities or reinvestment costs. This is what the company can earn for its shareholders and debtholders. In order to get to the future free cash flows we thus need to estimate future EBIT (sales and operating expenses) as well as the dynamics of future working capital and estimates of capital expenditures along with depreciation and amortization. We prepare such forecasts for every year in the foreseeable future and get to yearly future free cash flow predictions. These numbers are then discounted back to the present, but as the premise when doing company valuation is usually going concern, we need to predict the cash flows into perpetuity. Since it is extremely difficult to make accurate predictions even a few years in advance, making them for ten, fifteen or more years into the future is often not very realistic. That is why future cash flow predictions are usually divided into two parts, the yearly predictions and the terminal value. The terminal value can be computed in three ways (Damodaran, 2012): by liquidation value, applying a multiple or using a perpetuity growth factor of the cash flows and discounting them back like for the years before. All of these methods apply to the expectations of certain values (liquidation value, item value that a multiple is being applied to or yearly cash flow) at a specified point in time – the year after which we choose to no longer make individual yearly predictions. It should be a point in time when the company has reached stable growth. The DCF formula can be rewritten as follows:

$$DCF = \frac{CF_1}{(1+WACC)^1} + \frac{CF_2}{(1+WACC)^2} + \cdots + \frac{CF_n}{(1+WACC)^n} + \frac{TV_n}{(1+WACC)^n} \quad (6)$$

$$DCF = \sum_{t=1}^{t=n} \frac{CF_t}{(1+WACC)^t} + \frac{TV_n}{(1+WACC)^n} \quad (7)$$

where,

TV = the terminal value
n = the point in time in the future after which we need to calculate the terminal value

In most cases we assume the going concern feature of the company, which leaves us to two options for calculating the terminal value. We could apply certain multiples retrieved from replicable traded companies such as the price to earnings ratio (P/E ratio) or market to book ratio. But most commonly in practice we wish to continue with our intrinsic valuation without including the relatives and choose to use the growth factor. However, it is important to mention that using this method, we can get very different values of the terminal value by making only smaller changes to a few key assumptions (Brealey, Myers & Allen, 2011). Either method we choose, we need to first get the value of the terminal value at time t=n and then we can discount it back to the present time. Let us imagine what this means in formulated terms of the growth in perpetuity method:

$$TV_n = \frac{CF_{n+1}}{(1+WACC)^1} + \frac{CF_{n+2}}{(1+WACC)^2} + \cdots + \frac{CF_{n+\infty}}{(1+WACC)^\infty} \qquad (8)$$

For all the cash flows after t = n we need to assume certain long-term growth (that is usually labeled as g) since without growth a company would eventually cease to exist. There can also be a negative growth rate, when we expect the company's cash flows to continue decreasing. An example would be a company in an industry that is becoming obsolete and it is expected that the company would slowly fade away (Damodaran, 2012). How to define g is a difficult but important decision as it is usually the most influential factor on the final value (Damodaran, 2012). As I will move forward to the chapters that are more case-oriented, I will show that variations in g in fact can significantly affect the value. As terminal value should include only cash flows after the company has already reached its steady growth, we could in most cases assume that the company has already fully developed its potential and grown in size. If we are dealing with a startup company, we should then start the terminal value after the company's growth phases when the operations are already in steady motions. If we are dealing with such cash flows, factor g should not be as high as the company's current growth rate is. There are some guidelines and limitations regarding choosing it. The most important one is that g should not be higher than the growth rate of the economy in which the company is producing its cash flows as no company can be expected to exceed the growth of the economy in perpetuity (Damodaran, 2012). The optimal way of choosing the long-term steady growth factor is in most cases probably taking the expected long-term growth rate of the consumption for industry's products in which the company functions and add inflation (Koller, Goedhart & Wessels, 2010). One should also take under consideration other factors when deciding about the longevity of rapid growth (Damodaran, 2012): the size of the company as smaller companies tend to grow faster, capital expenditures and other amount of money it spends on growth, sustainability of competitive advantages, existing growth rate and excess returns.

By defining the perpetuity growth factor, we can estimate all cash flows based on it. We come up with an infinite geometric series that can be very neatly reorganized into a single short equation.

$$TV_n = \frac{CF_n(1+g)}{(1+WACC)^1} + \frac{CF_n(1+g)^2}{(1+WACC)^2} + \frac{CF_n(1+g)^3}{(1+WACC)^3} + \cdots + \frac{CF_n(1+g)^\infty}{(1+WACC)^\infty} \quad (9)$$

$$TV_n = \frac{CF_n(1+g)}{WACC - g} \quad (10)$$

Where,
g = the perpetuity growth factor of the company

By using the above formulas, we should be able to apply DCF to the valuation of a company. But when it comes to startup companies, a lot of the parameters that I have described so far and can normally be estimated well for a steady, profitable, established company, can become very tricky and almost impossible to estimate for a company that is showing negative cash flows, is growing at a pace of 100% or more, is disrupting an industry or introducing a complete novelty or has not even entered the market yet. In such cases it is crucial to keep the intuition behind these parameters and the rationale behind each process of the valuation. The main objective of the DCF in general is to predict the future free cash flows and to estimate a risk measure to discount them back to their present value. Risk should account for the required rate of return of equity and debt holders. Looking from an equity holders' perspective and assuming the investors are fully diversified, the only risk that companies should get rewarded for, is the systematic or market risk.

1.3 Real options approach

To define real options, we need to understand both terms, "real" and "options". Those two words were joined together by Myers (1977), who pointed out to the existence of many corporate assets, especially opportunities for growth, that can be viewed as call options and identified the value in future flexibility for a company holding an option on such asset. With that he laid grounds for future researchers that have been working on using theory of pricing financial options for valuing real assets with option-like characteristics. In this chapter I develop the option pricing theory based on financial assets and then show how it can be applied on real assets.

Financial theory lists various industries and areas where real options are applicable, including the valuation of startups. Mun (2002) provided examples, ranging from oil and gas exploration and production, to pharmaceutical research and development, e-commerce valuation, IT infrastructure investment justification, prioritisation of venture capital investments, mergers and acquisitions, research and development (R&D), internet startup valuation, structuring of venture capital contracts, timing of investments, parallel portfolio development, profitability profiling and other. In addition to enterprise valuation, real options have been widely used in other areas. Their introduction has opened doors to new possibilities in various areas of strategic management, such as market entry timing, modes of entry, joint ventures analysis, foreign direct investment and MNC performance, cooperation versus competition trade-offs, etc. (Trigeorgis & Reuer, 2017). Boer (2003) shows that real options are the preferred type of valuation tools for valuing R&D plans. Similar conclusion can be drawn from Brach & Paxson (2001), as they find that the primary value of R&D lies in the physical options it creates. Any similar project such as an early stage startup dependent on growth capital should also be valued using real options.

Not only startup companies, but all ventures should commit to finding the real options embedded in their business and assess whether and when to explore them. Failure to perform pre-investments or to investigate follow-up opportunities or in other ways neglect real options, may result in companies not being able to engage in the same investment opportunities as other competitors (Trigeorgis & Reuer, 2017). Such behaviour is value-decreasing and can lead a company to fall behind in the market.

1.3.1 Financial and real options

The definition of an option seems obvious and intuitive and so it is; yet it has somewhat developed over the years to truly capture the full scope of its meaning in the world of finance. Black and Scholes (1973, p. 1) defined an option as "a security giving the right to buy or sell an asset, subject to certain conditions, within a specified period of time". They were obviously referring to financial options and back then there were not so many complex options in the market as there are now and there was yet no talk of real options. A definition that I prefer and, in my opinion, better covers an entire spectrum of options, is the one by Trigeorgis and Reuer (2017, p. 2): "An option is a right, but not an obligation, to take some future specified action at a specified cost". In finance, the two basic and most common options are the "buy" or "call" option and the "sell" or "put" option. We can insert these two words into any of the two definitions above (or a similar one) and get the definitions specifically for these two types:

- a call option gives you a right to buy a particular asset for a specific price at a specific time in the future;
- a put option gives you a right to sell a particular asset for a specific price at a specific time in the future.

To illustrate, let me show the payoffs on a call option of an imaginary company ABC that has a stock currently trading at 70 EUR. Assume you have an option to buy such an asset at 80 EUR one year from now. If the price remains at the same level, you would not exercise the option as you would be buying the stock 10 EUR above the market price. Hence, you let it expire and you do not lose nor gain any money. On the other hand, if the price goes up to 90 EUR, you would certainly exercise it as you can buy the stock at 80 EUR and if you choose to, immediately sell it at a market price of 90 EUR, consequently making a positive payoff of 10 EUR. We can write such a payoff down:

$$Payoff\ of\ a\ call\ option = \max\left(S_{t=1} - E, 0\right) \qquad (11)$$

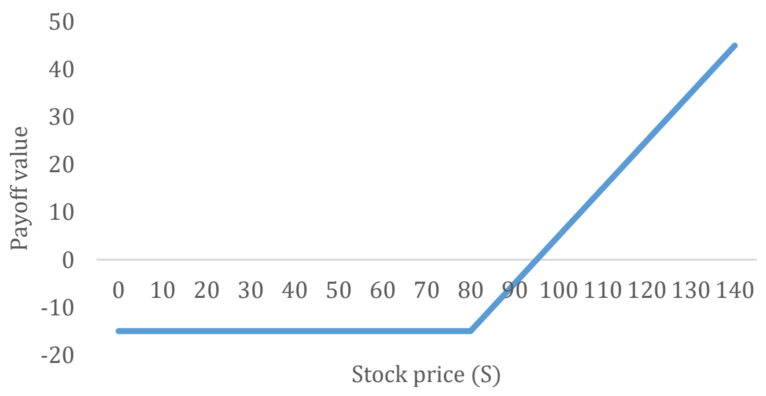
where,
$S_{t=1}$ = the price of the underlying asset at expiration date
E = the exercise or strike price (the price at which the underlying can be bought at expiration)

The process of a put option is just a reversed one. Assume the same example, only that this time we have a put option on the ABC stock expiring in a year with strike price 60 EUR. If the price remains at 70, you would not exercise as you would be selling at 10 EUR below the market price. A put option should be exercised when the price is below the strike price. If at time of expiry, the price quotes at 50, you would exercise and effectively have a positive payoff of 10 EUR. To help us decide when to exercise, we thus use this payoff:

$$Payoff\ of\ a\ put\ option = \max\left(E - S_{t=1}, 0\right) \qquad (12)$$

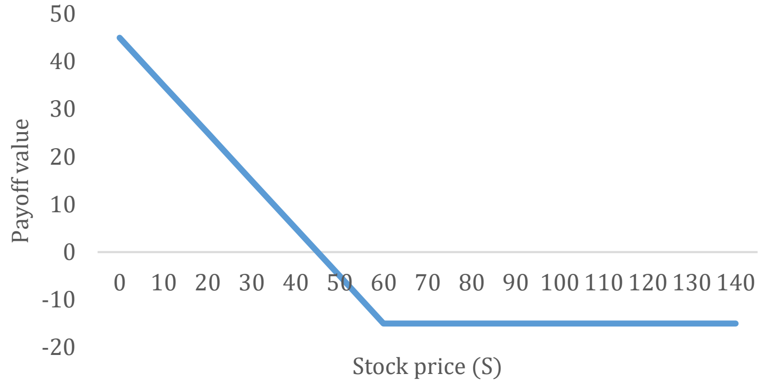
A payoff of a security that can be strictly zero or positive is of course not free. To buy a financial option, one needs to pay for a premium to the writer of an option. The writer of an option is a person who gets this premium in advance and promises to sell (call option) or buy (put option) the underlying asset of the option at the agreed strike price (Wilmott, 2007). If we want to compute the profit of an option that we bought, we take the payoff of the option, discount it to the time when we bought the option and subtract the premium from it. Due to the strictly zero or positive payoff of an option, losses can never exceed the premium, whereas profits are potentially unlimited. This can be observed from figures 1 and 2, where I take the data from the previous examples and assume an option price of 15 EUR for both.

Figure 1: Profit diagram of a call option



Source: Own work.

Figure 2: Profit diagram of a put option



Source: Own work.

In the figures 1 and 2 I am ignoring the time value of money as I am putting the option payoff that happens a year after we buy the option on the same chart with the 15 EUR of premium without discounting either of the numbers to the present or the future. However, since I want to build the intuition, these figures provide just that.

Imagine you want to invest in a risky project or for instance in a startup. There will first be an initial cost of investment and afterwards there can only be a positive payoff. Under the worst case scenario, the project fails or a startup goes bankrupt and you lose the initial investment. However, a startup can flourish and its value can explode. The potential positive side is limitless. The payoff of the holdings in a startup would be as in equation for the call option I showed and the profit diagram would look like figure 1. The premium would in this case be the cost of the initial investment. The premium is the price of the option. It is the price of a project or the price of a startup (or the share of startup ownership that you are buying). The idea of the thesis is to show that we can apply models for calculating the price of a financial option to real assets such as startups and thus calculate their value. The main models that I focus on are the binomial model and the Black-Scholes model, while I also mention other techniques like Monte-Carlo simulation.

Real options are valued in the same way as financial ones, so the general model derivations apply to both areas. There is criticism saying that unlike financial derivatives, real options do not have the efficiently traded market making sure the pricing of the underlying is arbitrage-free and containing all available information. The stock of the fictional company ABC from the previous example is traded on a stock exchange, whereas a startup or a risky corporate project (or other real options) is not. This would imply that the standard arbitrage-free valuation theory used to back the valuation tools for financial derivatives should not apply to real options. The counter argument was best structured by Copeland & Antikarov (2001) and the market asset disclaimer (MAD) assumption they presented. The MAD assumption claims that the most convenient way to form a replicable tracking portfolio to value the underlying asset is to use the illiquid asset itself and that the static present value of that asset without the flexibility is the best estimate of the market value (Copeland & Antikarov, 2001).

The two most general options are the call and the put option. We can further divide options into American and European-type options. The difference between them is in the possible option exercise time. The two ABC examples above were both European options, as the option holder was able to exercise the option only at a specific point in the future; precisely one year from now. If they were American options, the only difference would be that the possible exercise time would not be only at that date one year from now, but at any given point in time between now and that date – any time between today and expiration date.

Despite the MAD argument, the option pricing theory was originally developed to value financial options under the conditions that there exist actively traded underlying assets and it is normal to assume real options would run into additional challenges within the framework. Trigeorgis & Reuer (2017) discuss some of the issues arising from the differences between financial and real options:

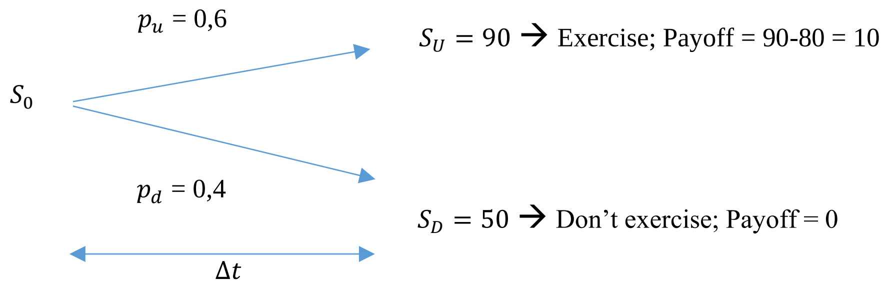
- real options are often illiquid or not traded in organized markets or do not even exist yet which can cause information asymmetries and issues with path dependencies;
- the specifics of real options such as option maturity are often vague and undefined;
- the benefits of exercising real options are often remote and difficult to predict and secure;
- exercising of real options can directly or indirectly affect other market participants such as rivals;
- numerous different uncertainty sources that can affect the value of a real option (exogenous, endogenous and behavioural).

1.3.2 Binomial model

The fundamentals of valuation remain the same when applying DCF techniques or the option pricing models. The value is derived by creating a financially traded portfolio that replicates the risk, return and time characteristics of the valuation object. Assuming there is no arbitrage opportunity at place, the value of such a portfolio should equal the valuation object's value. The first model I will introduce is the binomial model, where the underlying can only move to two possible future states in a single time step. The model has been introduced by Cox, Ross & Rubinstein (1979) and has gained popularity due to its simplicity and intuitive nature.

Imagine the ABC stock currently traded at 70 EUR that can after a period Δt either move up to 90 EUR or down to 50 EUR and there is a call option to buy it after that period at 80 EUR. The probability of an up move p_u is 60% and of a down move p_d is 40%.

Figure 3: Call option payoff example



Source: Own work.

Even though the expected value of this option is 6 EUR (0,6*10 EUR + 0,4*0 EUR = 6 EUR), the price of the option is different and has nothing to do with expectations. The reason for this is that the option can be hedged by short positions in the underlying asset and the development (rise or fall) of the stock is irrelevant (Wilmott, 2007). The condition necessary for having the perfectly hedged portfolio for the option is that the payoff is the same regardless of the outcome; in our case regardless if ABC stock goes to 90 EUR or to 50 EUR. The starting portfolio consists of 1 option and a short position of the stock:

$$V_0 - \Delta S_0 \tag{13}$$

Where,
V_0 = the value of the option today
Δ = delta or the share of the ABC stock that has been shorted (not to be confused with the length of a time period Δt)
ΔS_0 = the share of the ABC stock that has been shorted

The next step is to compare both possible future states of the stock and match them as equal via this equation:

$$V_U - \Delta S_U = V_D - \Delta S_D \tag{14}$$

$$\Delta = \frac{V_U - V_D}{S_U - S_D} \tag{15}$$

Delta is the range of option values divided by the range of asset prices after period Δt. By inserting the possible values of the ABC stock and payoffs of the option after period Δt, we can quickly calculate delta.

$$10\,EUR - \Delta 90\,EUR = 0\,EUR - \Delta 50\,EUR$$

$$\Delta = 1/4$$

In order to hold 1 option, we must short 0,25 of the underlying stock if we want a perfectly hedged portfolio whose payoff will be the same regardless of the move in the underlying stock. By plugging delta value into any of the two possible portfolio equations after Δt, we get the portfolio value equaling -12,5 EUR (0 – 0,25*50 EUR = -12,5 EUR). Assuming there are no arbitrage opportunities, this future value of the portfolio discounted by the risk-free rate must equal its value today.

$$V_0 - \Delta S_0 = \frac{V_D - \Delta S_D}{\left(1 + r_f\right)^{\Delta t}} = \frac{V_U - \Delta S_U}{(1 + r_f)^{\Delta t}} \tag{16}$$

Or in our case, where we assume risk-free rate to equal 5% and period $\Delta t = 1$:

$$V_0 - 0{,}25 * 70\ EUR = \frac{0\ EUR - 0{,}25 * 50\ EUR}{(1 + 0{,}05)^1}$$

$$V_0 = 5{,}60 \text{ EUR}$$

Note that the actual probabilities were not used once in calculating the value of the option. The option value would remain the same regardless of the probability of the next moves and need not be included in our calculations. These are predominantly based on the no arbitrage assumption and based on the current and possible prices of the underlying asset.

To take full advantage of the binomial model, the theory has developed risk-neutral probability calculations. The characteristics of the risk-neutral world are (Wilmott, 2007):

- no extra return is expected for risk-taking;
- no actual statistical probabilities are used for estimating chances of future events occurring;
- everything is priced by using expectations.

In order to calculate the risk-neutral probabilities q and (1-q) for the above case, we would simply plug in the actual values into a simple expectations' formula.

$$\frac{1}{\left(1 + r_f\right)^t} (q * S_U + (1 - q) * S_D) = S_0 \tag{17}$$

Where,
q = the risk-neutral probability of an up move of the underlying
(1-q) = the risk-neutral probability of a down move of the underlying

By doing so, we get the results: q = 0,5875 and (1-q) = 0,4125. To get to the option value, we calculate the option's expectations and discount it to the present using the risk-free rate.

$$V_0 = \frac{1}{\left(1+r_f\right)^t}\left(q * V_U + (1-q) * V_D\right) \tag{18}$$

Which in our case results in the present value of an option of 5,60 EUR, exactly the same result as before. This kind of operations work fine in a single time-step case like this one, but in order to build and operate with a tree of possible binomial prices, we need to implement a discrete-time version of the lognormal random walk as the option value is nothing else than the present value of the option payoff using a risk neutral random walk (Wilmott, 2007). This is clearly visible by comparing the equations for risk-neutral probability and actual probability of an up (or down) move.

$$q = \frac{1}{2} + \frac{r_f\sqrt{\Delta t}}{2\sigma} \tag{19}$$

$$p = \frac{1}{2} + \frac{\mu\sqrt{\Delta t}}{2\sigma} \tag{20}$$

Where,
μ is the drift rate or the average rate at which an asset increments
σ is the volatility or the measure of randomness of the asset increments

The only difference between the actual and risk-neutral probabilities is that the risk neutral ones use the risk-free rate for the drift rate.

For binomial model tree building, the most elegant way is to have an up move u and a down move d that are multiplied by the current value of an asset. Using the above terminology, we can phrase the potential up or down movements of an asset (Wilmott, 2007)

$$S_u = S_0 * u = S_0 * (1 + \sigma\sqrt{\Delta t}) \tag{21}$$

$$S_d = S_0 * d = S_0 * (1 - \sigma\sqrt{\Delta t}) \tag{22}$$

Where,
u = the incremental factor of an up move
d = the incremental factor of a down move
$0 < d < 1 < u$

In order to work within our binomial tree with most ease, Shockley (2007) recommends a shortcut, where u and d are reversed values of each other. Once we implement continuous compounding into the equations, as our assumption is that assets tend to follow the lognormal distribution, we can get to the following key equations (Shockley, 2007):

$$u = \frac{1}{d} = e^{\sigma\sqrt{\Delta t}} \quad (23)$$

$$q = \frac{e^{r_f \Delta t} - d}{u - d} \quad (24)$$

$$V_0 = \frac{1}{e^{r_f \Delta t}} (q * V_U + (1 - q) * V_D) \quad (25)$$

Equipped with this, all that is needed are the model inputs and the creation of the binomial tree can start. To value a European-type option on using annual time periods via the binomial tree, Shockley (2007) summarizes the process into 11 steps, while Kodukula & Papudesu (2006) narrow it down to 6. I mostly follow the methodology of Shockley (2007) in my examples and here I sum-up a generalized 8-step process for valuing the real option via the binomial model:

1. Determine the model inputs such as the starting value of the underlying asset S_0, the volatility of the asset's log returns (σ), time to expiration of the option (T), the number of time-steps N and the risk-free rate (r_f).
2. Calculate the length of a single time-step (Δt) by dividing T with N.
3. Calculate the size of an up move (u) using σ and Δt.
4. Calculate the size of a down move (d) by dividing 1 with u.
5. Build the binomial tree of possible asset's values. At the first step the value can either go to u*S_0 or to d*S_0. Keep multiplying by u or d for N periods of duration Δt.
6. At the last step estimate the payoff of the option based on the terminal value of the asset at each possible state.
7. Calculate the risk-neutral probabilities by solving the equation for q.
8. Start at the end of the binomial tree and move towards the beginning step by step. At each step calculate the value of the option using the equation of simple risk-neutral expectations discounted one time-step back. When you get to the very beginning, you obtain the value of the option.

The tricky part is the first point of course as any model is only as good as its inputs. I will get into that in upcoming chapters.

I started with the background for the binomial model because it is also the model, I am using the most in my practical examples. The binomial model can much better conceptualise a series of discrete events such as a R&D project (Copeland & Antikarov, 2001). Hence, while the Black-Scholes model is widely used in the financial industry, it is in fact the binomial model that can be best applied to real-option-like startups, at least as a starting point.

1.3.3 Black-Scholes model

While it seems that the binomial method should serve better at framing the problem of a real option valuation due to the time-discrete nature of most real options and the model itself, the end result should be very similar to the Black-Scholes model. In the limit of Δt approaching zero, the Black-Scholes model equals in fact the binomial model (Cox, Ross & Rubinstein, 1979). This is very intuitive since the shorter the time period of each time step is and the more time steps there are, less discrete and more continuous it gets.

Black-Scholes equation and the option valuation formula derivations were based on a principle proposed by Black & Scholes (1973), which states that if the options are correctly priced on the market, opportunities for creating profits by long and short portfolios of the underlying stock and the option derivatives should not exist. This is the same no arbitrage argument I used when deriving the binomial model. And while the binomial distribution allows only for two possible states in the immediate future's next step, Black and Scholes derived the formula using stochastic calculus in a continuous time frame. Apart from that the derivation for the European call option by Black and Scholes follows very similar process to the one that I used for the binomial model starting by creating a hedged portfolio where you long the stock and short the option.

Black & Scholes (1973) offer also an alternative derivation using the CAPM model. Again, they connect the option with the stock, here via the relation between the betas of the stock and the option.

Another possible derivation of the formula can be obtained by the martingale approach, which is based on risk-neutral expectations and forms the basis for simulation-type pricing (Wilmott, 2007).

The formula for valuing a European call option on the underlying stock producing no dividends first introduced by Black & Scholes (1973) is:

$$C = S_0 * N(d_1) - Ee^{-r_f T} * N(d_2) \tag{26}$$

with

$$d_1 = \frac{\ln\left(\frac{S_0}{E}\right) + \left(r_f + 0{,}5\sigma^2\right)T}{\sigma\sqrt{T}} \tag{27}$$

and

$$d_2 = d_1 - \sigma\sqrt{T} \tag{28}$$

Where,
C = the value of the call option

$N(d)$ = the cumulative normal density function
T = the time until expiration of the option
r_f = the risk-free rate
S_0 = the current value of the underlying asset
E = the exercise price of the option
σ = the volatility of the log returns of the underlying asset

The inputs for the formula above are the same inputs used also for the binomial model and while most of them are difficult to get, especially for real non-tradeable assets, it is important to understand them fully. These are the value drivers behind the option and looking at the formula, the directions of the effect of each driver can be observed. For some it is very intuitive, while for some the effect is a bit more indirect. The direction of the correlation between each driver and the option value is summed-up in the table 2. Apart from the above parameters I also added the dividend yield (D). This potential income from the underlying asset can be seen as opportunity cost of receiving income from exercising the option early. Hence, the direction in which it effects the option value is negative. The bigger the dividends, the lower the option value.

Table 2: Value drivers of option value

Value drivers of a call option value	
Value drivers	**Correlation with option value**
Underlying asset's current value (S_0)	Positive (+)
Volatility of log returns of the underlying asset (σ)	Positive (+)
Time until expiration of the option (T)	Positive (+)
Risk-free rate (r_f)	Positive (+)
Exercise price of the option (E)	Negative (-)
Dividend yield (D)	Negative (-)

Adapted from Damodaran (2012).

The Black-Scholes formula can be adjusted for other more complex options and to account for other attributes such as dividends. For example, a European put option formula with continuous dividend yield of the underlying asset can be written (Wilmott, 2007):

$$P = -S_0 e^{-DT} * N(-d_1) - Ee^{-r_f T} * N(-d_2) \qquad (29)$$

Where,
P = the value of the put option
D = the dividend yield of the underlying asset

While Black-Scholes equation is the method that gives the most accurate calculation given the input assumptions, I will start with the binomial model in my empirical examples. The biggest issue with real options is often framing of the problem and only once this is done properly can the basic assumptions and model inputs be revisited and adjusted. For that, a model such as the binomial model is much more suitable as it can follow the actual process in the timeline of the real option from the creation to the exercise point better.

Another useful tool to help with the assumptions and decrease the uncertainty of a real option problem are simulations. The Monte-Carlo simulation is one relatively easy way to calibrate your sensitivity analyses, but it can also be used as a principle to value options.

1.3.4 Monte Carlo simulation

The Monte Carlo method for valuing options was first introduced by Boyle (1977) when he outlined a process for valuing a European call option by producing a set of simulations for the starting stock price. One of the biggest advantages at the time was that it did not limit the valuation to strictly assume the lognormal distribution, but instead allowed for others as well, whereas one of the biggest disadvantages was that it was not very reliable for majority of the cases and demanded a lot of simulated trials to achieve precision (Boyle, 1977). This has changed dramatically with the steep rise in easily accessible computing power and strong software. Even a simple spreadsheet calculation can often be powerful enough to get to the result and it can be achieved without too complex mathematics, at least for the simple cases.

When running the simulation in order to get to the option value, it is crucial to use the assumptions of risk-neutrality and generate the possible future paths of the underlying using the risk-neutral expectations (Boyle, 1977). Hence, we should simulate the underlying movements with the risk-neutral drift rate; i.e. instead of the actual drift rate, we should use the risk-free rate.

While the Monte-Carlo simulations do a good job at valuing European options and the calculations are manageable on a mediocre open source software, the computations and computing power are a bigger issue with the American options. When dealing with American options, the Monte Carlo calculations become more complex as it is necessary to assess at every step not only the payoff of the option but also the value, so that the early-exercise constraint is met at all times (Wilmott, 2007). In order to compute this, it would require a lot of computational power as the number of computations grows exponentially with every new trial. An approximation is available as we can at every step use the least squares regression of exercising the option versus continue holding it across the underlying's prices (Longstaff & Schwartz, 2001).

1.3.5 A common view on valuation

In this chapter I wanted to derive the basic structures of the two key valuation models analyzed in the thesis. I first showed the main elements of DCF, where I explained how we estimate the future free cash flows and discount them back with the discount rate that reflects the required rate of return of the company's shareholders and debtholders. Afterwards I have shown how theory behind valuing financial instruments known as options can be used to value real assets and the parameters behind the models. To summarize the chapter, I would like to emphasize that both valuation techniques described so far are based on the same general idea and basic principle. This basic principle is arbitrage and the valuation based on replicating portfolio and linear pricing idea.

Arbitrage is known as an activity on the financial market, where one can profit without any investment (Shockley, 2007). Arbitrage opportunity allows for the so called free lunch cases and the market tends to eliminate such opportunities.

When we want to come up with a value using relative valuation, DCF or ROA, we are basically trying to get to the value of the asset as given by the financial markets. If we find a financial security that mimics the expected cash flows of a particular asset and their risks and we know the value of such a security, we can determine the value of this asset by applying linear pricing (Shockley, 2007). If the expected cash flows and risks associated with them are the same for the financial security and the other asset, then these two must be of the same value. If not, there would be an arbitrage opportunity.

Steps involved in every valuation using linear pricing:
- find a replicating portfolio for the underlying asset on the financial market;
- if such a portfolio of securities exists, apply linear pricing under no arbitrage assumption;
- the value of this portfolio equals the underlying asset.

DCF, ROA or even relative valuation are just different derivations used to value the situations they are most suited for, based on the above three steps and main assumptions.

In a theoretical example of a simplistic World with the state of market economy as the single common risk source, Shockley (2007) elaborates on how CAPM is only a shortcut to creating the replicating portfolio and points out that this shortcut is only applicable when the underlying conditional-mean cash flows are in linear relation with the market. The slope of this linear relation is the beta. As soon as you add an option-like feature to the asset being valued, the relation to the market is no longer linear and CAPM is not the appropriate tool (Shockley, 2007).

The key takeaway I want to stress is that there are various valuation techniques that all have merit in certain situations. The important step is to understand the asset being valued, the environment it functions in and the factors that influence the returns and risk of the underlying asset. Based on that, the method that fits into the story best and can meet its required assumptions while performing the valuation, should be the preferred choice. In the next chapter, I address the potential issues associated with applying standard valuation techniques to startup company valuations.

2 STARTUP COMPANY VALUATION

It is mainly the extreme uncertainty (unpredictability) of the future evolution of startups that makes valuations difficult, as new ventures with disruptive technology breaks are typically subject to high mortality risk (Shepherd, Douglas & Shanley, 2000; Stinchcombe, 1965). In fact, according to Giardino, Unerkalmsteiner, Paternoster, Gorschek & Abrahamsson (2014), sixty percent of startups fail in the first five years. With such high failure rate, it is obvious that adjusting for risk is of critical importance.

Another important component of valuation models, in particular the DCF, are cash flow projections. Since there are usually zero or close to no historical data for startups, obtaining projections from management is considered a good starting point. Management predictions for near-future financial results are a key input for valuation methodologies (Manigart et al., 2000). However, such predictions can quickly be overly optimistic, either due to attribution theory (Kelley, 1972) or due to the fact that they are aiming for a higher valuation with one of the external investors. An investor will anticipate the optimism and adjust the projections accordingly. This will again make the founders inflate their projections in the first place as they know the predictions will be downgraded anyway. Such behavior on both parts creates a vicious inflationary circle (Amram & Kulatilaka, 1999). This can lead to some founders with a potentially good business proposition, but badly advertised projection, not having beneficial terms for external financing (Amit, Glosten & Muller, 1990).

2.1 Startup company definition

Startup company is a relatively young term and not as clearly defined in academic literature. There are various definitions of a startup and it is often difficult to tell when a company like Amazon, Facebook, Tesla or Uber stops being a startup and is treated as an established company. On the other hand, it is just as tricky to decide whether a single-man company in pre-revenue stage with an idea already is a startup company and how to value it.

One popular definition of a startup company is that it is a company, partnership or temporary organization designed to search for a repeatable and scalable business model (Blank, 2010). A definition more applicable to early-stage startups is provided by Timmons and Spinelli (2004), as according to them startup companies are raw companies with an innovative idea that can develop into a high-growth company. Startups can also be thought of as newly created companies with little or no history facing high volatility in technologies and markets (Giardino, Unerkalmsteiner, Paternoster, Gorschek & Abrahamsson, 2014). A discussion about various startup definitions by a Forbes article (Robehmed, 2013) sums up that the key factor for determining a startup that everyone agrees upon is its ability to grow. Keeping also in mind that startup companies are predominantly young entities (or have just started as indicated by their name), they, at least in the beginning, usually rely on some sort of outside financing to start or scale their business. They suffer a structural lack of tangible and intangible resources (Wymar & Regan, 2005), which normally makes external financing a necessity.

To sum up, startup companies are generally:
- early stage/young companies;
- innovative and disruptive of the market;
- facing highly risky and volatile future;
- set for high growth and scalability;
- forced to finance externally.

These characteristics imply that startup companies have a payoff diagram consisting of a small initial cost for an investment and unlimited upside potential with downside limited at that initial cost of investment. Regardless of a startup definition one chooses to take, when considering applying ROA as a valuation tool, the more the underlying (startup company) possesses the characteristics of an option, the better the model will work and the more precise the valuation will be.

2.2 DCF application to startups

The traditional discounted cash flow approach (DCF) has many flaws when dealing with highly risky environment, but generally there are two major ones. One, DCF normally assumes a constant discount rate. Two, it does not include the value of future managerial flexibility (Hodder & Riggs, 1985). To elaborate further, DCF usually uses a single factor to adjust for risk, the discount rate. In most of actual new venture situations risk has a time-varying component and the discount rate in the DCF does not account for it (Steffens & Douglas, 2007). Trigeorgis (1996) has come to a similar conclusion, as he states that the traditional net present value method fails to properly capture management's flexibility to adapt and revise later decisions as they have the opportunity to respond to unexpected future events. Even if DCF is perfectly applied, it may fail in strategic applications (Myers, 1984). Academics and practitioners have long ago realised that DCF often undervalues corporate investment projects (Trigeorgis, 1993). Trigeorgis (1993) further concludes that in certain strategical questions (where future managerial flexibility is an important factor), managers rely more on intuition rather than passive NPV analysis (Trigeorgis, 1993).

Similarly, Mun (2003) recognises the main disadvantages of DCF as he focuses on the DCF assumptions that may not always hold. Some of these assumptions are:
- all decisions are made now with cash flow expectations fixed for the future;
- once launched, all projects are passively managed;
- future cash flows are highly predictable and deterministic;
- risk is dealt with entirely by the discount rate;
- intangibles and assets that cannot be measured or are unknown are worthless.

These assumptions may fail when considering the characteristics of a typical startup company which I described in chapter 3.1. A startup environment is very volatile and dynamic, meaning often pivoting from the initial direction after reevaluating new knowledge, information and market situation. With such an approach all projects must be very actively managed and cash flow expectations can quickly change in time. Given the huge fail-rate statistics of new ventures, risk is huge and key in the valuation of startups. Applying a single static rate to account for it can be imprecise. Furthermore, usually the largest value of a startup often lies within the unmeasurable aspects. Whether it is a new technology with undefined market and utilisation areas or the unknown scalability potential or any other sort of intangibles, these elements alone can be the reason investors would want to invest in a startup company but are also the elements that one will have difficulties evaluating using the DCF. The more unpredictable the future is, the more do the weaknesses of the DCF get exposed.

Regardless of the seemingly direct pitfalls, DCF is used the most in practice by venture capitalists (Timmons & Spinelli, 2004). Steffens and Douglas (2007) compare various sub-methods of the classical DCF valuation and ROA. They consider the decision tree applied with the DCF as the most optimal tool given the effort, transparency and accuracy.

The problem with simply taking the expected cash flows and discounting them to the present value is that since it is so difficult to forecast the cash flows of the future, a very large discount rate needs to be applied in order to account for that risk. Such a discount rate is extremely difficult to determine accurately using the tools I described for mature companies in chapter 2.2.1.

Roure and Keeley (1990) find that venture capitalists use a rate in excess of 30% to value startups. Studies from Ruhnka and Young (1987; 1991) and Timmons and Gumpert (1982) show that an annual rate of return between 40% and 50% is required by venture capitalists; Cochrane (2005) indicates that even higher rates are expected; Timmons and Spinelli (2004) give us a very wide range of the discount rate that is used by venture capitalists - between 20% and 100%; whereas Westland's results (2002) range from 40% to 75%. Such variability in the discount rate indicates that it is at least to some degree arbitrary.

In order to showcase the drawbacks of the DCF in startup environment I will introduce a fictional startup company XYZ. The very simplistic case of XYZ company will be used as the underlying for applying the various valuation methods later as well. Assume that XYZ is a technology startup in a pre-revenue phase but has an idea about a revolutionary new concept. In order to develop the concept, they need 2 million EUR for the preliminary development and market research and after two years they need an additional 50 million EUR and another two years to implement the new technology into commercial products. Afterwards, the company expects positive cash flows in years four, five and six with high growth rates and some terminal value going forward after that period. After year six, sales are expected to remain steady as the company reaches a certain market share and the competition starts catching up. The positive cash flows are dependent on the market demand for the product they will develop. Here the key factors are strength of the economy and the possible applications for their technology as the full portfolio of industries where the final products could be used is still unclear. To simplify, three possible states of the world are analysed at t_2 when the second investment is needed. If the demand strengthens, the cash inflows will be the highest. If it stagnates, they will be at the medium level, whereas they will be the lowest if the demand weakens. An overview is shown in the table 3.

Table 3: XYZ investment timeline and CF scenarios

Timeline of investment and cash flow scenarios							
in million EUR		**t=0**	**t=2**	**t=4**	**t=5**	**t=6**	**TV**
	Probability	**Initial investment**	**New investment**	**Exp. CFs**	**Exp. CFs**	**Exp. CFs**	**Exp. CFs**
Strong demand	0,3	-2	-50	20	40	60	161
Stagnant demand	0,4	-2	-50	12	24	36	97
Weak demand	0,3	-2	-50	3	6	9	24
Expected values		**-2**	**-50**	**12**	**23**	**35**	**94**

Source: Own work.

Looking forward, the company XYZ needs an initial capital investment to start with the development. After two years, a decision needs to be made as another bigger investment will be required to continue. The characteristics of such a startup are very similar to those of a corporate investment in a R&D project. This is understandable as many startups begin as a project. In the case of a corporate project, one is observing the incremental cash flows to the company made by this new project, while in the case of a startup, we are valuing the total future cash flows of the company and hence, the actual value of a startup. The way to value such a project or startup company via the DCF is to estimate a risk-adjusted discount rate and use it to discount the cash flows to NPV as of t=0. If the NPV of positive cash flows is greater than the NPV of the investment costs, then the project adds value to the project owners or startup shareholders and we should make the investment. Or, from another perspective, the value of the startup is the NPV of the sum of all future yearly cash flows. An overview of the quick valuation is shown in table 4. A discount rate of 40% was assumed and a growth rate of 2% for calculating the terminal value.

Table 4: XYZ NPV of FCF

Net present value of free cash flows at t=0					
in million EUR	**Probabilities**	**Initial investment**	**New investment**	**Positive CFs**	**Payoff**
Strong demand	0,3	-2	-26	42	14
Stagnant demand	0,4	-2	-26	25	-2
Weak demand	0,3	-2	-26	6	-21
Expected values		**-2**	**-26**	**25**	**-3**

Source: Own work.

The DCF discounts all future free cash flows to the present value and compares it with the present value of the investment costs. In this case the NPV value is negative (-3M EUR), which means that the initial investment in the company would be value-destructive for the investor and the startup company does not have any value today. Taking a step deeper, we can see from table 5 that even looking at the second investment alone, along with the cash flows it produces, it has a negative value.

Table 5: XYZ second investment decision expectations

Expectations at the point of the decision for the second investment				
in million EUR	**Probabilities**	**New investment**	**Positive CFs**	**Payoff of second investment**
Strong demand	0,3	-50	82	32
Stagnant demand	0,4	-50	49	-1
Weak demand	0,3	-50	12	-38
Expected values at t=2		**-50**	**48**	**-2**

Source: Own work.

Using the DCF, we would conclude that based on the information we have today, there is no sense of making the second large investment in two years as it has negative value. At that time, it is expected that it will be more beneficial not to make any investment at all compared to the one analysed. As the first initial investment brings no cash flows on its own, but merely allows for an option to make s subsequent investment which is expected to go unexercised, the first investment is also senseless and destroys value.

The DCF valuation shows that the startup is not investable and hence worth 0 EUR today. However, it is all based mainly on one large assumption, the risk-adjusted rate of return or the discount rate. Random changes in the discount rate show the value can differ greatly. In table 6, the spread of the value based on deviations of the two key assumptions, the discount rate and the growth rate for terminal value.

Table 6: XYZ sensitivity analyses at DCF valuation

Sensitivity analyses on final value			
discount rate \ g			
in million EUR	1%	2%	3%
15%	103	113	124
25%	26	28	31
40%	-3	-3	-2
55%	-11	-11	-11
70%	-13	-13	-13

Source: Own work.

As clearly seen, the value changes drastically if the discount rate changes towards bottom or top levels of discount rate range that is being used by venture capitalists. Given that a lot of the value depends on a partially arbitrary input, this is a potential risk for the accuracy of the method.

There have been attempts to set systematic rules for calculating the discount rates for startup companies rather than assuming targeted returns by investors or choosing them on a more arbitrary basis. Damodaran (2009) proposes the following measures:

- estimate the market beta based on the comparable public companies from the industry and unlever and relever in case of a private company;
- adjust the beta for the private risk by dividing the market beta with the correlation with the market parameter to get to the total beta instead. As the equity holders are not fully diversified, private risk needs to at least partially be considered (to the extent of the correlation with the market);
- build-up a synthetic rating for the cost of debt and add a premium to it if that is what banks would be expected to do;
- assign the debt-to-equity ratio based on public comparable companies as well as the strategic views of the management on the target ratio;
- allow for expected changes of the components in the discount rate throughout the timeline as the company matures.

These measurements can help with estimation of the required rate of return, but WACC can only properly tackle linear payoffs, whereas the flexibility part has the payoff of a call option and is hence, nonlinear.

Even if we were able to make a correct assumption of the static risk-adjusted rate of return as the discount factor, there would still be the conceptual flaw of the DCF in a case like that as DCF cannot account for the value of future flexibility. The above valuation would be correct if we needed to commit today to the total investment, meaning initial investment and the new investment after two years. But there is a choice after the first two years to continue with the second tranche of the investment (50 million EUR) or to abandon the project altogether. Through its research and other developments during these two years, the company will learn more about the demand for their products. They will have more knowledge than they do today. They have the flexibility to wait, learn and only then invest if it makes sense based on the information available at that time. I could not include the value of this flexibility in the DCF valuation.

The above calculation is a basic DCF model with three scenarios that were based on the upcoming states of the economy. In a case of a startup, scenario analysis can be a beneficial tool. It allows the appraiser to incorporate the extreme outcomes into the valuation. Another important set of assumption parameters in such cases are the probabilities of each scenario.

A further improvement is a decision tree. The decision tree acknowledges the value of flexibility and tries to value it by creating future decision points and evaluates what would the decisions be at that point. Decision tree can add a lot of value towards understanding the business and its future paths. It could even be used as an attempt to value real options in a manner similar to the binomial model. However, there are drawbacks. The discount rate is directly dependent on the subjective probabilities and it needs to be calculated differently for each node (Mun, 2003).

2.3 Other techniques

While ROA and DCF are the main subjects of the thesis, it is worth mentioning there are other techniques that are regularly used in valuations of startup companies. The main valuation technique that is not discussed thoroughly here is the relative valuation. Compared to the DCF or ROA, relative valuation does not pursue the path of obtaining the intrinsic value of the valuation asset, but rather tries to find the value by looking at how is the market valuing comparable assets. Similarly, the venture capital method also uses multiples observed from the market to get to the potential value of the valuation asset at a future time, but on the other hand discounts it back to the present similarly to how DCF does it. While relative valuation is a very commonly used valuation technique across the industries and heavily discussed in academic literature, the venture capital method does not have a strong reputation among academics while in practice it is mainly limited to startup companies and new ventures.

Aside from that there are multiple other criteria that practitioners, business angels and venture capitals are using when assessing the possibility of investing into a startup. These criteria are often very subjective, non-deterministic and rely on the intangibles instead of measurable data. That is why I name them “soft” methods. While they cannot be used to come up with a precise objective valuation, it is important to be aware of all the factors that can drive or diminish the value of startup companies. Similar factors, for example key man impact, apply also to established companies, but there are so many other value drivers that contribute to the value in a greater way, these usually become insignificant. When valuing startups, we can use these factors in various discount adjustments.

2.3.1 Relative valuation

The most popular valuation method in general is the relative valuation method. It is based on finding comparable companies with similar expectations in terms of risk, return and time. Usually, this is a set of companies from the same industry and similar maturity level as the company being valued. If these companies are publicly traded, the multiples based on accounting figures can be taken and applied to the accounting figures of the entity being valued. One could also look at the privately executed transaction multiples and apply those.

The two main criteria for a successful relative valuation are to value assets based on standardised figures and to find comparables that truly are reflective of the company being valued (Damodaran, 2012). Standardisation is the reason for the use of accounting figures and while standardisation of accounting reports can be a solid assumption for publicly traded companies with audited accounts, the books of young non-audited companies are not necessarily as trustworthy. The second criteria is an even bigger challenge as no two companies are the same even when looking at mature established ones. When trying to value a young, high-growth industry-disruptive startup, it is even more difficult to find comparable publicly traded companies or transactions of similar companies.

2.3.2 Venture capital method

Most startup companies turn to the venture capital market for their funding in order to grow their business. The venture capital method named after the venture capitalists that have been using it intensively is a valuation technique that looks at the valuation asset through a business model of a venture capital fund and its potential future returns. It is a mixture between the relative valuation and the intrinsic DCF method as it uses both to come up with the final value.

There are four basic steps of the venture capital method (Damodaran, 2009):

- estimating revenue and cash flows for the short-term forecast period a few years into the future;
- applying a relative multiple to the revenue, cash flows or earnings at the end of this forecast period and getting to the value at that point;
- discounting the value back to the present using the required rate of return and thus getting to the pre-money valuation of the company;
- adding the money invested to the pre-money valuation and getting to the final post-money valuation of the company.

In the aforementioned example of the XYZ startup, to apply the Venture capital method would mean to make the projections for the first four years and then apply a multiple on one of the accounting items as of t=4. Either a market or transactions multiple of comparable technological companies at that stage of a company's lifetime should be taken, the value then discounted back at the required rate of return of the venture capital to the pre-money valuation. Add the investment into the company to that value and this is the post-money valuation. The investor gets the amount of equity as per the relative share of the investment compared to the post-money valuation.

The method has gained popularity as it intuitively follows the possible outcomes of a venture capitalist's investment and offers a shortcut to the final value but did not get much support among academics. Damodaran (2009) outlines the key flaws of the model as follows:

- focusing solely on the top and the bottom line without looking at the substance of the business. As a result, revenue often becomes the sole line of importance and the valuation serves the negotiation and game theory more than the actual valuation;
- not properly estimating all the uncertainty after the period when a relative multiple is taken;
- the discount rate is theoretically not valid and turns out to be arbitrary and the sole factor for risk;

- there are often mistakes associated with the pre-money and post-money valuation.

Looking at the issues a bit more closely, the first remark points towards venture capitalists not looking closely at the operational expenses and reinvestment needs. This could be overcome by doing a proper technical analysis. At the end, the top and bottom line would still be the main subject of negotiations, but this is not so unusual as the main assumptions are the points of negotiations in every potential M&A deal. When it comes to a young startup, its ability to achieve revenue growth is often the key value driver.
By applying the multiple after the short-term period, one avoids estimating medium and long-term cash flows, which is extremely difficult for such a young company. However, the multiple that is applied after the forecast period should be based on cash flows at that time and should account for the uncertainties of those cash flows.
The discount rate is usually estimated by taking the required rate of return of the venture capitalist. However, the actual returns venture capitalists historically earn tend to be much lower than what they assume at valuations (Damodaran, 2009). Additionally, one needs to be careful when considering taking EV/EBITDA or EV/Sales multiples as the cost of equity (required rate of return of equity investors) has been used in the valuation and not cost of capital. This approach often leads to inconsistencies with valuations (Damodaran, 2009).
The last point refers to possibilities, when there are multiple classes of stock of the startup or part of the investment is being used to cash-out some of the previous shareholders. Such occasions should be treated specially.

Even if the above-mentioned flaws were overcome, the venture capital method would still withhold the negative elements of the DCF and relative valuation when it comes to the startup valuation.

2.3.3 "Soft" methods

Especially for startup companies in the early pre-revenue stages any type of precise valuation is extremely difficult as there are rarely any data to work with or to base your projections on. That is why angel investors and venture capitals sometimes turn also to alternative approaches to investments compared to the valuation tools I have described so far. While it is important to stress that this can in no way be a reliable way of company appraisal, the elements that drive the value at such an early stage should be considered.

Most of these alternative approaches use some sort of factor-based model, where they assign a standard value to each of the factors and then assess how strong each factor is specifically for the startup company at hand. Some of these methods are the so-called Berkus method, risk factor summation method and the scoreboard method, where some of the main criteria used are (Nasser, 2016):

- management team;
- market size;

- strategic relations;
- competition;
- technology;
- potential lucrative exit.

2.4 Real options framework for startup valuation

In this chapter I transpose the general characteristics of ROA I described in chapter 2.3. into startup environment and practical examples. I show how startup companies may be suitable for applications of ROA. There exist various types of real options and it all starts by identifying them. If there is significant value in one or multiple real options embedded in the company that is being valued, then it is useful to apply ROA. Startup companies usually hold some exclusive flexibility, which makes them very suitable for ROA approach. ROA enables us to value this part of the company's value – the flexibility – and adds it to the base static value, which can be valued using the DCF.

A lot of companies have some disruptive component to their business or technological innovation exclusive to them that can be exploited in the future. Startup companies normally develop in multiple stages where sequential capital investments are required to grow the business and shareholders at each breaking point have an option to invest more into the company or, in the absence of own funds, to raise capital on the market. Each subsequent investment round is normally considerably larger than the one before and generates new opportunities. If the new opportunities are exclusive to the startup companies, this type of business development through repeating fundraising rounds resembles a staging investment of corporations such as launching a new product or testing a new market.

2.4.1 Identifying real options

One of the issues with ROA is that one can start seeing options even where they do not exist. On the other hand, real options can be hidden at first sight. Identifying and defining the real options is a key step. Trigeorgis (1996) divides the option types to: option to defer or stage, option to grow, option to alter scale (either to expend or contract), option to switch certain operational aspects and option to abandon or to exit. Similarly, Amram and Kulatilaka (1999) recognise the following options: exit, timing, growth, operating, staging, flexibility and learning. Copeland and Antikarov (2001) identify a very similar set of basic real options, namely, options to defer, to abandon, to contract, to expand and to extend. At the same time, they refer to another type of classification. There are also more complicated options such as compound options, i.e. options on options and rainbow options, i.e. options driven by multiple uncertainty sources (Copeland & Antikarov, 2001). In a literature review of the real options theory, Trigeorgis and Reuer (2017) sum up contributions of various industry-leading authors about typology, taxonomy and practical implications. Some of the main complications associated with ROA are interactivity among options (option substitutability or complementarity), multiple sources of uncertainty (exogenous, endogenous, behavioural), trade-offs between competition and pre-emption against cooperation and learning effects (Trigeorgis & Reuer, 2017).

Sourcing from the above definitions, the stand-alone option types most relevant to startup companies are:

- option to expand (option to make additional investments and scale output potential);
- option to contract (option to scale down the current involvement in a project);
- option to defer (option to delay the start of a project);
- option to abandon (options to sell or shut down a project or a company).

While most large companies likely have a portfolio of real options, startup companies in early stage phases can be simpler and possess only one of the stand-alone options. Nonetheless, it is important to note that there may be multiple real options incorporated into a startup company that can affect each other's value upon exercise, which should be accounted for when evaluating them. Furthermore, exercising an option can not only affect other real options of the company, but also trigger other market events. Startup companies wish to get to the market first and scale fast – this is extremely important when proprietary tools are not officially protected – and a market entry move by a competitor could decrease (or diminish) the value of the option to defer. On the other hand an acquisition or merger with the only serious other competitor that was able to beat you to market, increases the value of such an option.

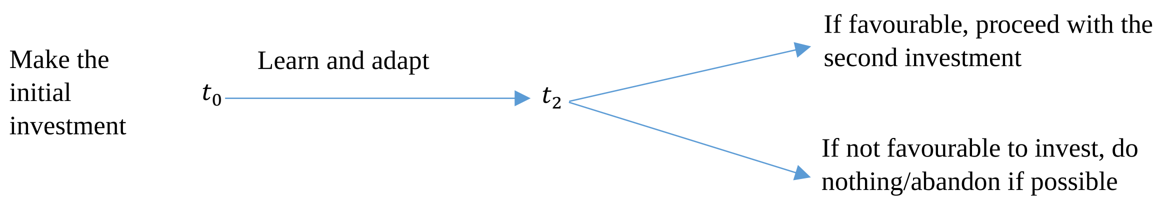
The key limitation to determining whether what you have identified as a real option can be valued as such, is exclusivity of the learning and adaptive behaviour (Damodaran, 2009). If there is no exclusivity, then there really is no optionality involved. The option that one is able to exercise must not be available to the public.

Another practical tip to test your understanding of the real option you are identifying is to draw it down into a few simple steps. If you are not able to do that quickly, chances are, the optionality is not truly there. Similar goes for the option's parameters. While it is often very difficult to assess them accurately, the descriptions of each parameter should intuitively be clear. The most important and case specific parameters for either the binomial model or the Black-Scholes equation, are the underlying asset, exercise price, the volatility, time to expiration and dividend yield if applicable.

To illustrate, I take the simple example of company XZY and identify the real options. When valued the company using the DCF, I assumed the three possible scenarios and calculated the expectations and the cash flows based on them. The calculations for all three cases included both, the first and the second investment. But the reality often is that a lot can happen in between t_0 and t_2 that can change future behaviour. New information may flow in about endogenous or exogenous factors influencing the project. The early tests for the product that company XYZ is developing may turn out better or worse than expected, additional applications for the final use that essentially increase the future expected cash flows may be discovered, a relevant move from a rival competitor can affect the decision making or the knowledge of the overall economy future conditions could be improved, to name a few. All this information can provide flexibility to the management or investor who can then decide whether to continue with the second investment and in what way, much better than they could have done today. The DCF model that I used assumed a commitment to the second investment based on the information obtained as of today, whereas the company does not need to decide about the second investment until t_2. Instead, they start with the first phase of development and gather new information. After two years, equipped with the new knowledge, they make a decision whether to proceed with the second investment. In fact, by making the initial investment, an option to make the second, larger investment or an option to expand is purchased. If there was an option to abandon the project and for example sell whatever is remaining of the company to a large corporation for a discounted price, this would be an option to abandon. At every step in the model, this would be evaluated and if it brought more value than further development, it would have been exercised. Another example of an option would be if the company also had a possibility to delay part of the second investment to a time with more suitable conditions. That would be an option to defer. Similarly, an option to contract would be to scale down in case of unfavourable events.

First, let us continue with the originally described option – option to expand. The real option is to decide at t_2 to invest the additional amount into the company or to do nothing. The underlying asset are the cash flows that are generated by the company if the option is exercised and the company finishes the second phase of development. The exercise price is the amount of the second investment needed at t_2. If we were to compare it to a financial option, it is a simple European call option and the initial investment at t_0 would be the price of the option. Time to expiration is two years and the volatility we need to calculate is the standard deviation of the lognormal returns of the underlying cash flows.
Since the technology behind the product of XYZ is revolutionary, it implies it is proprietary. Furthermore, the information they will receive from the first phase of development will be known solely to the company as well as how certain exogenous actions may affect it. The exclusivity criteria are met.

Figure 4: XYZ ROA diagram



Source: Own work.

In the next chapter I will use the methodology from chapters 2.3.2. and 2.3.3. to value the simple example of the XYZ company.

2.4.2 Valuing the option

I will focus mainly on the binomial model to value the option of the XYZ example. I will also apply the Black-Scholes equation to get to the precise price point. If applying the Monte-Carlo simulation as described earlier, using enough iterations, I would have come to similar end results.

When applying the binomial model technique to a startup valuation, the same standard real option process should be used. One of the key tasks has already been done in the previous chapter where I outlined the timeline, the optionality and the main parameters. Now, the parameters need to be estimated and applied into the model. After the estimation of the parameters, I simply plug them into the binomial model and assess in which scenarios it would be financially sensible to make the second investment at t_2. These scenarios show the possible expected values of future cash flows discounted to t_2, based on the additional information obtained throughout the year after the first tests. The binomial model shows how the expected present values of the underlying at t_2 can change from the starting expectations based on the knowledge we know today (at t_0) all the way to the possible expectations that we will be able to estimate with improved knowledge at t_2. How much these vary depends on the volatility of the underlying. I create a six-step binomial model for two years between t_0 and t_2 (T=2), so that the time step equals 1/3. Starting point of the binomial model or the current value of the underlying is the value of the future cash flows at t_0 (25 M EUR). Strike price is the second investment at t_2 (50 M EUR). Risk-free rate for these two years has been estimated at 1% and the volatility parameter set to 70%. For the purposes of the thesis I do not investigate what the risk-free rate should be as it is not the research subject, but rather use 1% across all cases and for all valuation methods. I will investigate volatility estimation in greater detail in the next chapter. From these parameters I compute the up and down moves and the risk-neutral probabilities as described in equations (23) and (24). In the next step the binomial model is outlined as in figure 5.

The possible movements of the underlying, i.e. the expected present value of the future free cash flows can vary as in the binomial model. The values at the end of step 6 are the possible expectations the company may have at t_2 when the decision about the second investment needs to be made. These are the values needed to be compared to the strike price and take the maximum value between the difference and 0.

Figure 5: XYZ binomial tree of the underlying movements

Value movements of the underlying
in million EUR

t=0	time steps between t=0 and t=2					t=2
	1	2	3	4	5	6
25	37	55	83	124	185	278
	16	25	37	55	83	124
		11	16	25	37	55
			7	11	16	25
				5	7	11
					3	5
						2

Source: Own work.

Figure 6: XYZ binomial tree of option values

Option value movements
in million EUR

t=0	time steps between t=0 and t=2					t=2
	1	2	3	4	5	6
4,48	9	19	38	74	136	228
	1	3	6	14	33	74
		0	0	1	2	5
			0	0	0	0
				0	0	0
					0	0
						0

Source: Own work.

Only the top three scenarios are in the money at the final step, whereas in the bottom four cases, the option would go unexercised. From there I move backwards with simple expectations using the risk-neutral probabilities and the exponent of the risk-free rate ($e^{r_f \Delta t}$). At the end we come back to the first step and the value of 4,5 M EUR. This is the break-even price to buy the option to make the second investment. If the initial investment is less than this amount, it creates value by buying it, otherwise it destroys it. Since the initial investment is 2 M EUR, the investment makes sense and is value-creating. Subtracting the initial cost from the option value, we get to the value of the startup company which in this case is 2,5 M EUR.

There is no particular reason why I used 6 number of steps. Generally, the more steps in the model, the greater the precision as the binomial model converges to the Black-Scholes as the number of steps goes towards infinity. To calculate the Black-Scholes equation, we only need to compute d1 and d2 from the existing parameters and get the normal distribution factors N(d1) and N(d2). The final value of the call option is 4,60 M EUR. Already with 6 steps, we get very close to the Black-Scholes value. In this case, the difference is 0,12 M EUR or undervalued by 2,6%. Were we to take one step less or one more, the binomial model would overvalue the option. The intuition behind is that there is either one up move more or one down move more. If we want to get closer to the Black-Scholes value, the average of the 6-step and 7-step binomial model can be taken. In this case, the average of 4,48 M EUR (6 steps) and 4,84 M EUR (7 steps) is 4,66 M EUR or 1,3% below the theoretical value. Generally, many authors (Shockley, 2007; Mun, 2002; Copeland & Antikarov, 2001) argue for the use of the binomial model as it provides clarity of possible movements of the underlying, which at the end can be more useful than the actual price point. Nevertheless, the Black-Scholes method is more precise and should be used for estimating the price point when needed.

The exercise of the company XYZ provides a clear example of a startup company where DCF failed to assign any value to it as the entire value sources from the future flexibility. ROA is able to account for that, but other flaws remain. The starting value is still based on the NPV of future free cash flows discounted back using the same discount rate as used in the DCF valuation. ROA does not solve that issue and the discount rate is still the key input factor that can change the end value drastically. Another key input is the annual volatility. In the next chapter I analyse the challenges with estimating it and I present the deviations of the valuation of the option value based on the sensitivity analyses of the discount rate and volatility parameters.

Table 7: XYZ sensitivity analyses at ROA valuation

Sensitivity analyses on final value					
discount rate \ volatility					
in million EUR	0,4	0,55	0,7	0,85	1
0,15	104	106	108	112	116
0,25	20	24	28	32	36
0,4	1	3	5	7	9
0,55	0	0	1	2	2
0,7	0	0	0	0	1

Source: Own work.

The ROA model is slightly less sensitive to the discount rate, but not by much. The volatility parameter, while significantly less sensitive to change than the discount rate, also changes the value at a moderate incremental change.

2.4.3 Estimating volatility

Volatility is one of the most difficult parameters to estimate in the real options model. Unlike with the financial options, it is usually impossible to get a large range of publicly traded prices of the underlying that would enable the same modelling technique. With financial options one quick way to calculate the volatility is to take the past traded prices of the underlying, calculate the lognormal returns of the prices and take the standard deviation of those changes. It is not the scope of the thesis to argue about the accuracy of volatility estimations in the financial markets, but such an approach is usually not available for ROA and especially not for its application to startups, where historical data is often close to non-existent. When estimating volatility as a parameter in the binomial model or Black-Scholes model, we are really estimating the volatility of the underlying future cash flows that the exercise of the option will bring.

There are several approaches used in ROA. Damodaran (2012) mentions the volatility of cash flows of similar projects the company has invested in, the volatility of the projected market scenarios that have been assigned with probabilities and the comparable public companies in the same field of business as the company with the underlying cash flows. Mun (2002) lists the logarithmic cash flow returns approach, logarithmic present value approach, GARCH approach, management assumption approach and market proxy approach.

There are mainly two sets of approaches: the ones looking to compare with some comparable, historical returns and the ones looking to project the future cash flows with different scenarios and calculating the volatility of the future possible cash flows. I describe the main approaches below and briefly discuss the advantages and disadvantages:

- Market proxy: Take the standard deviation of the market returns of publicly traded companies that resemble the underlying future cash flows. This usually means taking public companies from the industry of the company. This is fairly easy to do, but it is usually unreasonable to expect that publicly traded companies have identical risks and returns to the project in hand or a startup in question. Traded companies are usually well diversified, while individual projects have much higher risks (Shockley, 2007). Furthermore, the stock prices are subject to multiple market and other external factors that normally do not directly influence specific projects (Mun, 2002). Additionally, the market volatility is based on public companies that are levered, whereas the individual projects are usually unlevered (Mun, 2002). This applies even more to private-owned startup companies, which means that we need to unlever the market volatility using the debt-to-equity ratio of the public companies used in order to get to the final volatility estimate:

$$\sigma = \frac{\sigma_{MP}}{1 + \frac{D}{E}} \tag{30}$$

  Where,
  σ_{MP} is the volatility obtained via market proxy
  $\frac{D}{E}$ is the debt-to-equity ratio of the comparable public companies

- Logarithmic cash flow returns: Future cash flows are transformed into returns and the volatility is calculated with the following formula:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2} \tag{31}$$

Where,
x_i is the lognormal return of cash flow at time i compared to cash flow at time i-1
$\bar{x}$ is the average of the n lognormal cash flow returns
n is the number of cash flow returns

Normal distribution of the log returns is the assumption necessary for this solution. The problem can occur if cash flows in some period are negative as natural logarithm for null and negative numbers does not exist.

- Logarithmic present value approach: Similarly to the previous one, it uses future cash flow predictions, but instead of taking direct returns, it computes the present value to today's value at t_0 and to one period ahead at t_1 and computes the log return between the two. Copeland and Antikarov (2001) proposed such a solution using the Monte-Carlo simulation of the nominator for the present value at t_1 to get to the result.
- GARCH model is another way of forecasting the future returns. The problem with this approach, apart from being more technically rigorous than the others, is that a large amount of historical data is required to run a good volatility estimate (Mun, 2002).
- Utilisation of management assumptions: Managers may have difficulties estimating the volatility of the underlying, but they can provide intuition about the subjective probability of various scenarios. These probabilities of expected future cash flows can serve as a starting point to estimate the volatility. Assuming the normal distribution of log returns, the following equation can be used (Shockley, 2007):

$$Prob\left(\ln\left(\frac{x_T}{x_0}\right) > \ln\left(\frac{Z}{x_0}\right)\right) = N\left(\frac{\ln\left(\frac{x_0}{Z}\right) + (r - 0{,}5\sigma^2)T}{\sigma\sqrt{T}}\right) \tag{32}$$

Where,
x_0 = the current value of the underlying cash flows
x_T = the value of the underlying cash flows T years from today
Z = the value of the underlying cash flows at time T used for calculating the probability of achieving cash flows greater than it
r = the required rate of return on the underlying
$N()$ = the standard normal cumulative probability distribution function

By obtaining probabilities for different scenarios (i.e. best case, worst case, etc.), we can simulate outcomes using different volatility levels. The issue here is that management intuition is often biased. The model again relies heavily on the normal distribution of log returns and on the required rate of return, which I have pointed out is often hard to estimate precisely for startup companies.

Looking back at the XYZ example, due to the lack of any historical data, I could do a quick estimation using a market proxy by looking at the volatility levels of the comparable traded companies in the industry where the new technology will be used for. As mentioned, it is very important to use the comparable companies only if they resemble the risks and returns of the underlying.
Instead, what I do is to calculate the returns of the present values of the future expected cash flows to t_0 and t_2 when we need to decide for the second investment, compute the log returns for all three scenarios, compute the variance, divide it by the number of years between t_0 and t_2 and do the square root to get to the standard deviation which is our volatility estimate. Applying this method the volatility estimate is 54%. Normally, we would need to run a simulation of many different scenarios or take more possibilities into consideration to get to a more precise estimate but let us assume that the better estimate of the volatility would be 55% and not 70% as I used in previous chapter. The greater the volatility, the greater the value of the option, so this change should decrease the value of the startup. Looking at the sensitivity analyses in Table 7, I have already analysed the result at this level of volatility. The value of the startup company in this case is roughly 3 million EUR (2,65 million EUR to be precise), which is still greater than the initial investment of 2 million EUR.

Two other important volatility terms are the implied volatility and break-even volatility, which can be very useful when making practical decisions about the new investment. Implied volatility is usually used in financial options, where we take the current market prices and instead of the value of the option calculate the volatility implied by the market. Similarly, in the case of real options, we can use the break-even price of the option and calculate the volatility implied by the point when we are indifferent about the new investment or doing nothing. The implied break-even volatility can be computed by using iterative searches in the Black-Scholes equation. In the case of XYZ company, it is approximately 49,5%. This means that for the value of the option to be equal or higher than the initial investment, the volatility must be at least 49,5%.

2.5 Comparison summary

The critical task of any valuation method used to value startup companies is to account for the potential high returns in best case scenarios and to account for risk in worst case scenarios. Compared to regular established companies, both extremes are far more likely and far more significant with startups. The problem with the traditional DCF is that there are limited tools for combining extreme positive outcomes with extreme negative ones. The valuation is usually performed using multiple scenarios showcasing the upside and discounting future cash flows to the present using a very high discount rate. The discount rate is normally difficult to estimate and can significantly affect the value already at a small incremental change. The problems with the DCF are exposed further as the value of most startup companies lies in the uncertainty of the future. Usually, startup companies hold some degree of proprietary knowledge that they are not yet fully exploiting, but may be very valuable in the future; i.e., they have future flexibility to use assets that are exclusive to them. Future managerial flexibility is a concept that DCF was not originally designed to tackle and struggles with. ROA presents a solid complement to capture the value of this future flexibility or optionality. It indeed is a complement and not a supplement as ROA takes the static DCF value as the starting base and adds the value of optionality embedded. One needs to be careful not to include the best case scenarios, already accounted for in the DCF, again in the option's value. To avoid double-counting, only the incremental cash flows arising from this additional option need to be considered. As the ROA only adds the option value to the static value of the company, it only solves part of the problems associated with the DCF. It enables accounting for the optionality embedded in the companies and since most of the value of a startup company often lies precisely in that part, this is an important issue solved. The problem of the potentially imprecise discount rate remains and as I have shown, the end value is still most dependent on this risk factor even in case where almost all of the value arrives from the optionality. Other issues occur as well. Several assumptions need to be made to transfer the option pricing models from the financial markets to real assets in the first place. The challenge is also with obtaining reliable inputs for the model. One additional factor is the volatility of the underlying returns, where a certain level of creativity is needed.

As with any model, ROA is only as good as its inputs and in the case of startup companies, getting reliable and statistically significantly tested inputs is usually almost impossible. That is why getting to the point value is just one aspect of the startup valuation and investors have always been very interested in more detailed elements of the business and the company. Hence, soft methods have been developed and many investment decisions have been more based on the belief in the founding team than in any valuation model. I am not saying this is the approach that should be taken with startup valuation, but there is no denial that such elements are important, and it is important to consider them. Only with full understanding, one can add them as discounts or mark-ups into the valuation model or simply use the additional understanding of the key drivers to draw the possible future paths of the company. The latter is very important when performing the valuation as it not only allows you to identify the real options correctly, but also highlights the crucial points and decisions in the timeline and key elements that will drive these decisions.

Relative valuation is normally the most often used valuation method; however, applying it to startup companies is not so straight forward and should in most cases be avoided. It is difficult to find a set of comparable companies for public companies in established industries, it is much harder to do the same for smaller private companies, let alone startup companies. Startup companies are generally disruptive and still in the early-growth phase. Finding comparable publicly traded companies with similar risks and metrics to base multiples on, is extremely unlikely. An approach that encompasses the flaws of both, the

DCF and relative valuation, but tries to use the techniques in the way to mitigate their drawbacks, is the venture capital method. While mainly used by practitioners (venture capitalists), it should not be completely disregarded. With a careful approach, the model does not need to break any important theoretical rules and can in certain cases rely on less relaxed assumptions than DCF, relative valuation or ROA. It aims to solve the issues of relative valuation as it applies the multiples only at a possible future state of the startup company when the startup will be able to resemble the comparable public companies much better. It also uses the DCF only for the first few years, when DCF works better and future is easier to predict. Still, the intrinsic part holds most of the faults of the DCF and applying relative valuation to an unknown future scenario is just as tricky.

All in all, when trying to value a startup, all valuation methods should be considered as each startup has specific characteristics and business model that is best suited for different methods. Furthermore, the static value of the company and the option value should be treated separately. ROA in combination with the DCF for the static part should in most cases be the primary method, while others can serve as a comparison benchmark. Due to high uncertainty and importance of extreme scenarios, simulations are recommendable. Special consideration should be given to assessing the key inputs, starting from drawing out the decision trees, projecting future cash flows and estimating the discount rate and volatility.

3 EXAMPLES OF STARTUP VALUATION

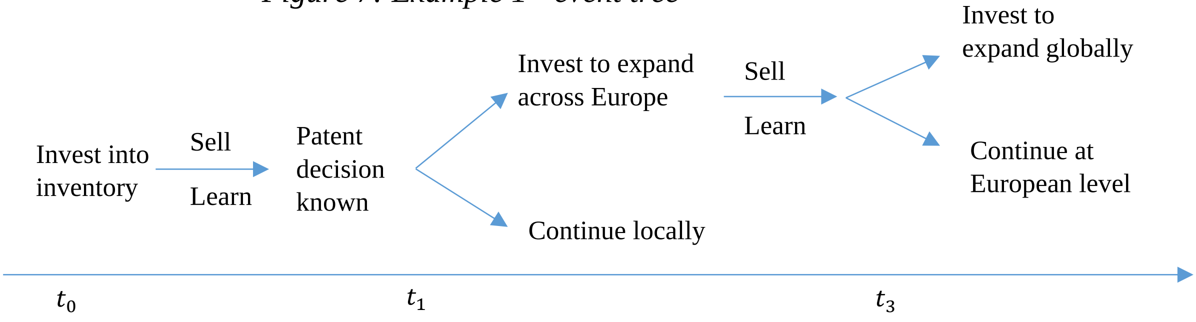
The best way to test a theory, in this case validity of using ROA on startup valuations, is to try it on real-life examples. I visited a startup accelerator and spoke with its managers and startup founders to gain first-hand information. I had the opportunity to interview some of the entrepreneurs developing their business ideas in there, hear their stories, plans and discuss projections. To respect the privacy of this information, I omit the name references and use fake or randomly altered numbers and, in some cases, adjust the business background and future opportunities. Still, I tried to keep as much of the business model intact and not change the event and decision trees of each business.

3.1 Example 1: Baby bottle product

The first case is a startup company with a patent pending for an appliance in the baby food industry. They are pre-revenue, but have designed a baby bottle that generates a baby formula to a perfectly heated temperature in under 10 seconds making it the only baby bottle to do so in such a short time and thus being far superior to the industry average of 3-7 minutes in this aspect. The company has a patent pending for their fast-heating and baby formula producing solution. The patent pending status lasts for 12 months, after which the patent bureau either grants or rejects it. After 12 months, if granted with the patent, the company needs to decide whether to pay an additional fee and prolong the patent term, both timewise and geographically. They would decide to do that if at that point they believe, their product's future cash flows can financially support the extra costs needed to pay for the patent and expand their operations to utilize from it. Founders have provided their projections. They plan to start producing inventory right away, so that they can start selling in the next quarter and have enough knowledge after 12 months to make further decisions. They will first start with sales in the local national market and two other neighbouring areas, Italy and Croatia. After 12 months, if the initial results are encouraging, they will likely pay for the patent and expand their sales. The expansion is planned in two steps. First, they would expand from the three local countries to other main European markets and later also globally (USA and Asia are especially attractive markets with high rates of baby formula usage over breastfeeding).

The company has already raised some money from angel investors and currently hold 100 thousand EUR on their bank account. They already have the base team assembled as well as connections with the local stores developed and basic e-commerce solution ready to go. As a local developing innovative project, they have also gained some publicity and brand recognition in the local markets, so no large initial investment is needed apart from the capital expenditures into bottle-producing equipment of 90 thousand EUR. They are seeking a seed round of 250 thousand EUR to cover the mentioned CAPEX and provide liquidity in the initial money-losing period.

Figure 7: Example 1 - event tree



Source: Own work.

The event tree in Figure 7 shows that there are two important decisions the company needs to make down the road, assuming they raise enough funds to start building the inventory and selling their product. The first one is in one year, when they need to decide on expanding into other markets. This decision will be based on the projected future incremental cash flows of investing into expansion and obtaining the inflows from the new markets. However, a lot can change in the next 12 months and new information will be brought to light to help cope with the decision. Namely, the two factors are:

- whether the patent is granted: (i) If yes, future projected free cash flows should increase due to limited competition in case of success; (ii) if no, future projected free cash flows should decrease due to stronger competition in case of success;
- how successful the product was locally: (i) if product has been a sales success and profitable by maintaining production costs, future projected free cash flows should

increase; (ii) if sales and profitability turn out worse than expected, future projected free cash flows should decrease.

If the product turns out good enough in the first preliminary sales period and the patent is granted, the company will surely decide to invest into expanding to larger markets. On the other hand, opposite results would shift the decision into the other direction as the company would not exercise the option to expand and would not invest further.

The second decision occurs only if the initial option to expand is exercised. Then, if the European expansion turns out successful, there is a possibility to scale up to a global player, especially into North American and Asian markets where the demand for baby formula is proportionally larger. Management estimates that they should make this decision no later than after three years. That is two years after potentially expanding to larger European markets, which should provide enough time to gather sufficient information about the last possible expansion. This option will only be exercised if the projections at some point will show value in doing so. These projections will naturally be adjusted after the results of the European expansion. If the product sells well and the team proves they are able to expand out of the local markets, the further expansion will take place, otherwise not.

To value the company, I first need to perform the static DCF valuation without the implied flexibility. I apply the MAD assumption and use this static valuation as my tracking portfolio and as the starting point of the value of the underlying. The DCF valuation is done on the basis of the projections made by the founders which are adjusted to account for several future scenarios. The scenarios are based on the patent grant, which was estimated at 70% probability of success. In addition, there are three other scenarios, the best case scenarios, the most likely scenario and the pessimistic scenario. There are three phases of the company's development. The first phase covers the local markets in which they already have some established partnerships and connections. The second phase covers the rest of the EU market and the third phase covers the rest of the global market. The decisions whether to start phases two or three are to be taken subsequently.

In the most likely scenario of phase one the company reaches 5% of the total three local markets in five years after the initiation in the case that the patent is not granted and 20% in the case that the patent is granted.

In the first year, there is no difference between the projections with or without the patent as the patent decision only takes place after the first year. In one year, the company has to make a decision about the phase II. Various scenarios are estimated for the incremental cash flows of the rest of EU markets. It should be noted that I present each phase as a separate project and denote PY1 as project year 1. PY1 for phase II in table 9 is thus actually FY2 from the entire company's perspective as phase II is only launched after one year.

Table 8: Example 1 - Sales projections for phase I

Sales projections for phase I							
in thousand EUR	**Probability**	**Scenario**	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**
Without patent							
	0,2	Best case	200	440	847	1.398	2.152
	0,6	Most likely	100	200	350	525	735
	0,2	Pessimistic	50	90	142	191	241
With Patent							
	0,2	Best case	200	660	1.650	3.300	5.775
	0,6	Most likely	100	300	750	1.500	2.625
	0,2	Pessimistic	50	135	304	547	861

Source: Own work.

Table 9: Example 1 - Sales projections for phase II

Sales projections for phase II							
in thousand EUR	**Probability**	**Scenario**	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**
Without patent							
	0,2	Best case	1.531	3.061	5.357	8.036	11.250
	0,6	Most likely	612	1.224	2.143	3.214	4.500
	0,2	Pessimistic	245	490	857	1.286	1.800
With Patent							
	0,2	Best case	6.122	12.245	21.429	32.143	45.000
	0,6	Most likely	2.449	4.898	8.571	12.857	18.000
	0,2	Pessimistic	980	1.959	3.429	5.143	7.200

Source: Own work.

In phase II, competition is expected to be tougher. This, in combination with the lack of free positive PR and domestic brand awareness present in the local markets, give slightly lower targets of the overall market share that can be captured. The percentages of target market share after five years in the most likely scenario are 3% (without patent) and 12% (with patent).

Table 10: Example I - Target market shares

Target market shares most likely to be achieved in 5 years			
	Phase 1 (domestic)	Phase 2 (rest of EU)	Phase 3 (global)
Target share with patent	5%	3%	2,5%
Target share without patent	20%	12%	10%

Source: Own work.

Similarly, in the global expansion plan of phase three, the target shares get even smaller. They are 2,5% without the patent and 10% with the patent under the most likely scenario. The most likely scenario has a probability of 60%, whereas the best case and pessimistic scenario both have 20%. The probabilities for the three scenarios are estimated at the same level for all three phases. Following same denotations as with phase II, PY1 for phase III in table 11 is actually FY4 of the company.

Table 11: Example 1 - Sales projections for phase III

Sales projections for phase III							
in thousand EUR	**Probability**	**Scenario**	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**
Without patent							
	0,2	Best case	18.707	37.415	65.476	98.214	137.500
	0,6	Most likely	7.483	14.966	26.190	39.286	55.000
	0,2	Pessimistic	2.993	5.986	10.476	15.714	22.000
With Patent							
	0,2	Best case	74.830	149.660	261.905	392.857	550.000
	0,6	Most likely	29.932	59.864	104.762	157.143	220.000
	0,2	Pessimistic	11.973	23.946	41.905	62.857	88.000

Source: Own work.

DCF valuation of Example 1

The way I perform the DCF valuation is to value each phase individually. This way, I am forced to decide today, based on the NPV, if each phase is even worth going for. If the NPV is negative, I simply assume zero value as the company is not expected to opt in for the expansion anyway. While I treat each phase as a separate subproject and hence use the notation of periods starting with PY1, it should again be noted that PY1 of phase II is actually FY2 of the company and PY1 for phase III is FY4 of the company. When performing the valuations, special attention is given to ensuring only the incremental cash inflows and outflows are considered for each of the phases. Furthermore, tax needs to be adjusted as the company is making a loss in the first years. The generated loss is transferred over to the future years and profit tax can be deducted for this amount as per the governing law. It is assumed that the transferred loss will cover up to 50% of the yearly profit (the rest is transferred further) and that for each phase a separate entity will be created, so that tax-recognisable costs only decrease the profit base within each phase. In the terminal value, the full 19% profit tax is always considered.

To initiate phase I, the founders are seeking an investment of 250 thousand EUR. This money will be used for the immediate capital expenditures of buying the machinery and to cover the liquidity issues over the first periods. To initiate phases II or III there are also some initial capital expenditures needed. Furthermore, a triggering kick-off investment is needed to launch each phase as the company will need to setup offices, logistics and operations in multiple countries and invest into marketing to launch a product in the new markets. Future incremental free cash flows for each phase are discounted to the beginning of the respective phase and compared with the kick-off cost to get to the NPV of the phase. NPV values of each phase – if positive – are then discounted to the present to get to the value of the company. Since the company clearly is a very risky early-phase startup, the required rate of return for the investors should be high. Calculating WACC the traditional way is not a solution as there is no possibility for debt financing, no comparable public companies with similar risk and returns and the company’s future clearly depends on the willingness of other investors (likely venture capitalists) to invest. For that reason I assume a risk-adjusted rate of return of 45% which is within the ranges of what venture capitalists normally take for an early phase startup as I have shown in the theoretical part of this research. It is a big assumption and that is why I perform the sensitivity analyses at the end to show how the value ranges if the discount rate changes.

Table 12: Example 1 - NPV phase I

NPV calculation of Phase I only							
in thousand EUR	PY0	PY1	PY2	PY3	PY4	PY5	TV
Expected values of Phase I							
Revenue		110	305	711	1.358	2.307	
COGS		39	107	249	475	808	
Fixed costs		185	185	185	185	185	
Variable costs		33	92	213	408	692	
EBITDA		-147	-78	64	290	623	
DCF valuation							
EBIT		-156	-89	50	271	594	594
Tax		0	0	5	26	97	113
Amortisation and depreciation		9	11	14	19	28	28
Kick-off CAPEX	90	0	0	0	0	0	0
Other CAPEX		2	6	14	27	46	46
Change in WC		-12	6	12	19	28	28
FCF	**-90**	**-136**	**-90**	**33**	**219**	**452**	**1.034**
Discount factor	1,00	0,69	0,48	0,33	0,23	0,16	0,16
PV of FCF	-90	-94	-43	11	49	70	161
NPV	**65**						

Source: Own work.

If the company committed to only sell its product on the three local markets, its expected future cash flows would be worth 65 thousand EUR today.

NPV of phase two project is negative, meaning that equipped with current info only, in expectations, the company would not choose to go into the first expansion. This project does not add any value according to the static DCF valuation. Based on the information given today and static analyses, the only reason why phase II would still make sense is if phase II is the prerequisite for phase III and the today's expected value of phase III is positive and greater than the expected loss of phase II.

Table 13: Example 1 - NPV phase II

NPV calculation of phase II only							
in thousand EUR	PY0	PY1	PY2	PY3	PY4	PY5	TV
Expected values of phase II							
Revenue		2.240	4.479	7.839	11.758	16.461	
COGS		784	1.568	2.744	4.115	5.761	
Fixed costs		500	500	500	500	500	
Variable costs		896	1.792	3.135	4.703	6.584	
Kick-off committed costs		2.000	1.500	0	0	0	
EBITDA		-1.940	-880	1.460	2.439	3.615	
DCF valuation							
EBIT		-2.049	-1.007	1.301	2.234	3.344	3.344
Tax		0	0	124	212	391	635
Amortisation and depreciation		109	127	158	205	271	271
Kick-off CAPEX	1.000	0	0	0	0	0	0
Other CAPEX		45	90	157	235	329	329
Change in WC		-162	88	195	82	98	98
FCF	**-1.000**	**-1.823**	**-1.058**	**984**	**1.910**	**2.797**	**6.055**
Discount factor	1,00	0,69	0,48	0,33	0,23	0,16	0,16
PV of FCF	-1.000	-1.257	-503	323	432	436	945
NPV at phase II kick-off	**-625**						

Source: Own work.

Table 14: Example 1 - NPV phase III

NPV calculation of phase III only							
in thousand EUR	PY0	PY1	PY2	PY3	PY4	PY5	TV
Expected values of phase III							
Revenue		27.373	54.746	95.805	143.707	201.190	
COGS		12.318	24.636	43.112	64.668	90.536	
Fixed costs		2.000	2.000	2.000	2.000	2.000	
Variable costs		10.949	21.898	38.322	57.483	80.476	
Kick-off committed costs		20.000	10.000	0	0	0	
EBITDA		-17.894	-3.788	12.371	19.556	28.179	
DCF valuation							
EBIT		-18.504	-4.617	11.159	17.770	25.587	0
Tax		0	0	1.060	1.688	3.217	0
Amortisation and depreciation		609	828	1.212	1.787	2.591	0
Kick-off CAPEX	5.000	0	0	0	0	0	0
Other CAPEX		547	1.095	1.916	2.874	4.024	0
Change in WC		-1.491	1.175	1.347	599	719	0
FCF	**-5.000**	**-16.950**	**-6.059**	**8.048**	**14.395**	**20.219**	**44.061**
Discount factor	1,00	0,69	0,48	0,33	0,23	0,16	0,16
PV of FCF	-5.000	-11.690	-2.882	2.640	3.256	3.154	6.874
NPV at phase III kick-off	**-3.647**						

Source: Own work.

NPV of phase three is also negative, so the company is not expected to go into global expansion nor into the European one.

Table 15: Example 1 - Static DCF all phases

NPV of all phases				
in thousand EUR	t=0	t=1	t=2	t=3
Phase I NPV				
Present	65			
Phase II NPV				
Start of the phase		-625		
Present	0			
Phase III NPV				
Start of the phase				-3.647
Present	0			
Total NPV value today	**65**			

Source: Own work.

To get to the total value of the company based on the static DCF valuation, we need to discount the NPV values of all the phases to the present and add them together. Then we should add or subtract any net debt currently on the balance sheet. Since phases two and three are negative, the projects would not get performed and the added value is zero. As the company is not indebted and holds 100 thousand EUR of cash, the company's total value is thus estimated at 165 thousand EUR.

ROA valuation of Example 1

The second method of valuing the startup is to apply ROA. I start with the same basis of static expectations, but instead of stopping there, I use these figures to build the binomial model and value the flexibility that the company holds relating with phases II and phases III. The company does not have to commit today to executing or not executing the expansions. Instead, they can test the market, wait and learn and with more knowledge and information make a revised decision at the two points mentioned in figure 7 of the event tree. At each decision point the company will compare the strike price for each phase with the expected future cash flows at that time (instead of doing so today).

When deciding about phase III, they will only go forward if the project's expected future cash flows at t_3 will be greater than the strike price of phase III at t_3.
When deciding about phase II, they will only go forward if the project's expected future cash flows at t_1 plus the option value of phase III at that time is greater than the strike price of phase II at t_1.

A quick summary of the static valuation for each phase is presented in table 16. As shown with the standard DCF valuation, based on today's information only phase I is expected to be executed.

Table 16: Example 1 - Static future CFs and options' strike prices

Executed options' CFs and strike prices				
in thousand EUR	t=0	t=1	t=2	t=3
Phase I				
CFs if exercised at phase start	155			
Initially committed expenses	90			
Phase II				
CFs if exercised at phase start		2.468		
	1.702	←		
Initially committed expenses		3.093		
	2.133	←		
Phase III				
CFs if exercised at phase start				19.903
	6.528	←	←	←
Initially committed expenses				23.549
	7.725	←	←	←
All phases NPV				
CFs if exercised	**8.386**			
Initially committed cost	**9.948**			

Source: Own work.

From the event tree it is apparent that there are two options the company has embedded. They are called compound options as the second one (option to expand in phase III) is dependent on making the first expenditure (phase II). With that in mind, there are a few rules one should follow when building the binomial model for such options:

- there is only one underlying: the CFs the company gets out if the last option is exercised;
- the same risk-neutral probabilities and volatility parameter is used throughout the binomial model for valuing both options;
- the starting point in the binomial model is the today's value (at t_0) of future CFs of phase III (the underlying);
- the strike price used at the last node of the binomial model is the strike price of phase III discounted to t_3 - the start of phase III (intuitively this means that we are developing the possible values of the underlying from how it is perceived today to how it can be at the phase III start when we will need to compare it with the strike price);
- the strike price and the positive CFs of phase II come into the model when working in the binomial model backwards: at the node when the decision for phase II needs to be made, the strike price of phase II needs to be subtracted and the CFs added.

The risk-free rate was chosen at 1% and T equals 3. The number of steps need to be a whole number multiplier of 3, so that the start of each year can easily be spotted at one of the nodes. This is important for the inclusion of phase II into the model. I will present the results done with a 9-step model. I have also performed the analyses with 12-step and 21-step models. The results are very similar and are presented at the end of this valuation.

The volatility parameter was estimated based on the projections. Since this startup is essentially a single pre-revenue project the risks associated with it would be impossible to find in a market-based proxy. Similarly, there is no strong dependency on a single market traded material, so that we would be able to perform a Monte-Carlo simulation on it. In fact, the volatility in this case does not come from the cost side, but rather the revenue side. Hence, the projections of revenue are broken down into 54 scenarios. Solely for the purpose of determining the true volatility I assume that all options get executed (even in the scenarios where that would not happen) and the observations from such circumstances are the basis for computing volatility. I am applying the lognormal returns technique, where I compute the expected value discounted at t_0 and 54 values discounted at t_1; that is one for each scenario, so that each scenario has total revenue and cumulative probability calculated. With these numbers I can calculate the lognormal returns and get to the probability-weighted average return. Afterwards, I compute the deviations, the total variance and standard deviation of the t_1 versus t_0 revenue returns. The standard deviation is 79%, so 80% was used as the estimation for volatility parameter. The numbers behind the calculations are presented in Appendix 2.

With all the inputs, all that is needed is to calculate the risk-neutral probabilities, the up and down move multipliers and to build the binomial model.

Figure 8: Example 1 – Binomial tree of the underlying movements

Value movements of the underlying
in thousand EUR

t=0			t=1			t=2			t=3
	time steps between t=0 and t=3								
	1	2	3	4	5	6	7	8	9
6.528	10.361	16.443	26.096	41.417	65.730	104.318	165.558	262.750	416.998
	4.114	6.528	10.361	16.443	26.096	41.417	65.730	104.318	165.558
		2.592	4.114	6.528	10.361	16.443	26.096	41.417	65.730
			1.633	2.592	4.114	6.528	10.361	16.443	26.096
				1.029	1.633	2.592	4.114	6.528	10.361
					648	1.029	1.633	2.592	4.114
						409	648	1.029	1.633
							257	409	648
								162	257
									102

Source: Own work.

Note that the starting point is in fact the today's value of phase III CFs. The next step is to go to the last node and compare the values with the strike price of phase III to see in what cases would the option get exercised and then work my way back node by node.

Figure 9: Example 1 – Binomial tree of option values

Option value movements

in thousand EUR

t=0			t=1			t=2			t=3
				time steps between t=0 and t=3					
	1	2	3	4	5	6	7	8	9
1.095	2.448	5.354	11.343	23.372	44.294	81.003	142.165	239.279	393.449
	235	604	1.554	4.741	10.122	21.065	42.337	80.847	142.009
		0	0	551	1.327	3.180	7.578	17.946	42.181
			0	23	58	150	385	990	2.547
				0	0	0	0	0	0
					0	0	0	0	0
						0	0	0	0
							0	0	0
								0	0
									0

Source: Own work.

Node 3 also represents another decision point. This is when the company will need to decide whether to proceed with phase II. In addition to the standard formula for determining whether the node has a positive value or 0, the strike price of phase II is subtracted and the incremental CFs are added back. The result of it is that two nodes at timestep 3 are turned to zero and the other two are decreased by the net static effect of phase II.

The total flexibility value is hence over 1 million EUR. It is clear that the large majority of the company's value is sourced from the optionality.

Table 17: Example 1 – Dynamic ROA valuation

Valuation summary (in thousand EUR)	
Option value including phase 2 and phase 3	**1.095**
Strike price of phase 1	-90
Positive incremental CFs of phase 1	155
True NPV of all future CFs	**1.160**
Current cash	100
Current net debt	0
Value of the company	**1.260**

Source: Own work.

The final valuation of the company amounts to 1,26 million EUR. Given that the company is seeking a venture-capital-type 250 thousand EUR investment, this valuation implies that a 17% equity share would be a fair deal.

I've mentioned at the beginning of this ROA valuation that I also I performed analyses using binomial model with 12 and 21 steps. The final value arriving from the 12-step model is 1,30 million EUR and from a 21-step model it is 1,29 million EUR.

Even though I have managed to come to an exact price point estimate, there are so many inputs that could have completely changed the valuation. Let me list some of the main assumptions and input factors used in the valuation:

- market size;
- future market share;
- patent grant decision probability;
- scenario probabilities;
- scenario outcomes;
- future cost structure;
- risk-adjusted discount rate;
- volatility parameter.

Some of them have been estimated well, for some the lack of data caused a bigger challenge. Just slightly changing some of them results in a brand new evaluation of the potential investment.

Table 18: Example 1 – Sensitivity analyses: risk-adjusted rate and volatility

Sensitivity analyses on total value					
discount rate \ volatility					
in thousand EUR	60%	70%	80%	90%	100%
35%	3.238	4.069	4.851	5.585	6.272
40%	1.213	1.899	2.559	3.174	3.744
45%	547	846	1.260	1.731	2.168
50%	250	389	609	890	1.278
55%	87	211	327	445	644

Source: Own work.

Table 19: Example 1 – Sensitivity analyses: growth factor and patent probability

Sensitivity analyses					
g \ prob. of patent approval					
in thousand EUR	30%	50%	70%	80%	90%
1%	152	467	1.211	1.669	2.305
2%	162	481	1.260	1.737	2.394
3%	172	501	1.310	1.815	2.488

Source: Own work.

Looking at the sensitivity analyses, the largest input factor is still the risk-adjusted rate of return. While it is difficult to rely completely on the point estimate of company's value from the above valuation, ROA definitely did a better job of valuing the flexibility, which in this case represented most of the value embedded. DCF failed to do this job. Furthermore, it highlighted very directly the key events on the future timeline and what the company or investor should be aware of. The internal KPIs should be set to meet the goals in the ROA analyses and to stimulate the value drivers. More than the actual number, ROA enables the company or the investor to see the possible future scenarios and at least to some extension understand what is the possible value the company can achieve in certain circumstances. It is then up to the investors to try and imagine such circumstances and decide how likely they are. ROA provides a model to test and estimate such various environments.

3.2 Example 2: Technology logistics company

The second example revolves around a company with existing business in the logistics industry. Unlike the traditional logistics companies, this company does not own any transport vehicles, warehouses or other similar fixed assets. Instead, it operates an online platform and utilises digital technology to integrate and manage its network of various suppliers on one hand and generate a user-friendly logistics booking portal for consumers and companies on the other. Logistics, while one of the largest industries in the world, is still primarily pen and paper driven industry and is as such very appealing for technologically advanced disruptors. The subject company searches to solve the problems of its customers by offering a unified, transparent and competitive product with instant pricing utilising aggregated demand, proprietary pricing and optimisation algorithms and existing digital integrations with its supply network. The company has started with offering bookings of shipments of parcels and pallets primarily within the EU which is how they went from 50 thousand EUR to 350 thousand EUR to 1,25 million EUR in their first three years. They are still loss-making as a result of increasing investments into growth, but expect to bring their core products to profitability in two years. However, management is planning to invest in new products as well. In addition to standard parcels and pallets, they want to launch Global Express shipments, an E-commerce product and Trucking to focus also on full-truck-loads and larger business clients. Management intuitively believes that adding new products would allow them to acquire new clients, which could then also use the other already existing service types of the company as well as to offer these new services to their existing pipeline of clients. In addition, there could be various optimisations in procurement of supply companies or within marketing and sales. While they admit there is a lot of uncertainties, they believe there are numerous synergies between the service types. Combining multiple logistics services within instant proprietary pricing algorithms and unified digital product would position them in a special spot in the market. They believe that such expansion is something that cannot be done from scratch and they have a unique opportunity to expand on their existing business model. Management wants to launch one new product each year, because of limited management capabilities and resources needed for network building and software development. Each new product would first go into a testing phase as a minimum viable product for one year at a limited investment amount. After the first year, the company could decide whether they want to continue with full launch and commit to additional expenses or stop. The founders have already been networking and preparing for such a launch, especially for the Global Express product. This product can be launched immediately, while the E-commerce product can only be launched after one year and the Trucking product after two years. Management would not like to postpone any of the launches much further as it could cause them to lose their unique advantage in the market. The company has no debt nor does it plan to acquire any. It has raised some money from the seed investment round and is looking for new equity financing to support their growth. The company has also had a standing interest from one of the traditional logistics companies to acquire them for 2 million EUR. This potential buyer is interested primarily in their technology and software, so the price is not driven by sales performance. Management believes that would have the opportunity to sell at this price for the next two years. The company currently holds 360 thousand EUR in cash, which they have obtained solely through equity financing and kept due to positive working capital.

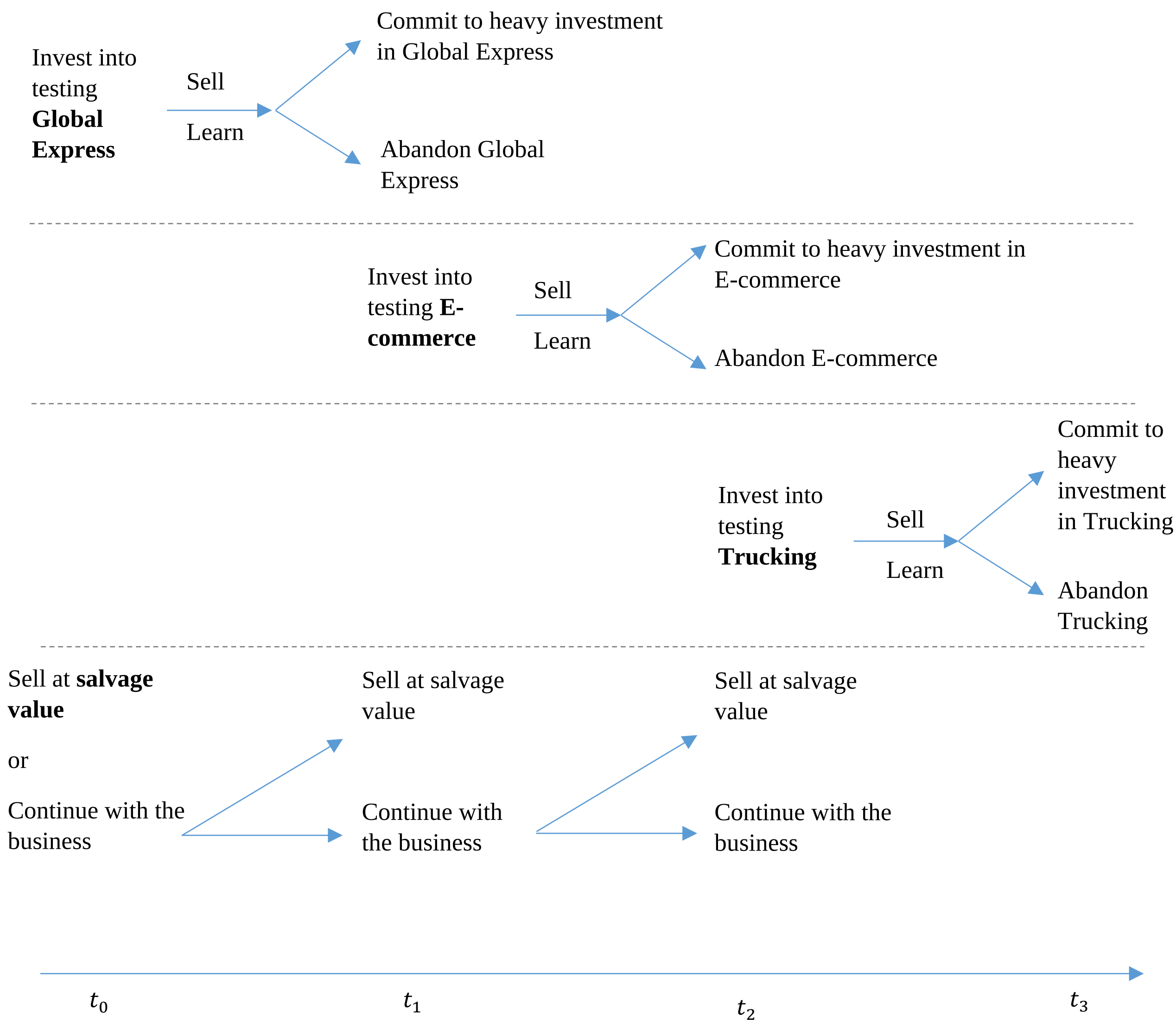
Key value drivers

With all this information, we can summarise the company's key value drivers:

- main established business channel of Parcels and Pallets;
- option to launch new product Global Express;
- option to launch new product E-commerce;
- option to launch new product Trucking;
- option to sell the company at salvage value.

The company will continue its core business of Parcels and Pallets and as there is seemingly no optionality involved with that product line, the value will depend on the standard DCF valuation. The other options have the characteristics as presented in the event tree in Figure 10. If the company decides to make the initial small investment into Global Express in year 1, after the first year it will have a choice to continue with a commitment to more heavy investments into the product or to stop investing or abandon the product. The same scenario also applies to the E-commerce and Trucking products, only that the exercise time is different for each of those. The company needs to decide whether to invest the initial small amount into E-commerce after year 1 and then needs to decide whether to commit to further heavy investments after another year. With Trucking both decisions are delayed for one more year. In contrast to the compound options in Example 1, the options here are assumed to be independent of each other. All three options are options to expand and are treated as European options. The last option is option to sell or to abandon and can be exercised at any point throughout the next two years. It is a put option and needs to be treated as an American option.

Figure 10: Example 2 – Event tree



Source: Own work.

DCF valuation

After discussions with management, I was provided with the basis for projections and historical numbers. As management tends to be biased with its projections, I further broke down the projections into three scenarios, “Best case”, “Pessimistic” and “Most likely”. I project each product line individually, calculate the NPV for each scenario of each product and calculate the final product NPV by multiplying the NPV of each scenario with its probability. The core segment of Parcels and Pallets is based on the historical data and further growth trendline is predicted. The balance sheet is also projected and the DCF calculation for the existing business alone is calculated. In the absence of substantial fixed assets, CAPEX is negligible, inventory is non-existent and the main accounts are open receivables and payables. Both are increasing as sales grows, but receivables are increasing faster with the customer split assumed to shift towards larger business customers who work only with longer payment terms. For the new products only the incremental inflows and outflows are considered. These are assumed to include any potential synergies that would arise after combining the new product with the existing services, acquisition channels and technology.

Estimating WACC is the trickiest part. All the issues associated with startup companies that I described in the theoretical part apply here as well, which is why I turn to a rough estimate and rather seek additional clarity from sensitivity analyses. In order to better account for the risk, I consider two different WACC rates, a slightly lower one for the established part of the business (Parcels and Pallets) and a higher one for all the new products, which are assumed to be riskier.

The main assumptions are:

- WACC is estimated to be 25% for Parcels and Pallets and 35% for all the new products;
- growth rate in the terminal value is 2%;
- probability of the Best case, Pessimistic and Most likely scenarios is 25%, 35% and 40%, respectively. The same scenario probabilities are applied for each of the product lines;
- profit tax is assumed to be 19% and is calculated in a manner to account for the accumulated losses. The full 19% of profit tax is always considered in the final year for the terminal value.

A DCF valuation for each product is calculated to give the value of the segment as at product launch. For Parcels and Pallets as well as for Global Express, this is t_0, while for E-commerce it is t_1 and for Trucking it is t_2. The DCF calculations for each segment for all scenarios are presented in Appendix 3. A summary of the results is shown in table 20.

Table 20: Example 2 – DCF summary by scenario

Summary of DCF valuation at project start date				
in thousand EUR	**Best case**	**Pessimistic**	**Most likely**	**Total NPV**
Parcels & Pallets	13.617	450	4.930	**5.534**
Global Express	3.811	-1.637	-785	**66**
E-commerce	4.310	-1.868	-1.175	**-46**
Trucking	4.888	-2.305	-1.109	**-28**

Source: Own work.

According to the DCF method, it is clear that E-commerce and Trucking products should not be considered and they would not have been launched if we were to decide about it today. The only two product lines that bring value are Parcels and Pallets and Global Express. I present the values at today's NPV in table 21.

Table 21: Example 2 – DCF summary by segment

DCF valuation summary			
in thousand EUR	**t=0**	**t=1**	**t=2**
Parcels & Pallets			
NPV	5.534		
Global Express			
NPV	66		
E-commerce			
Value at product launch		-46	
NPV	0		
Trucking			
Value at product launch			-28
NPV	0		
NPV value today	**5.600**		

Source: Own work.

Since the NPV is higher than 2 million EUR, which is what a potential buyer would be willing to pay today, this offer does not bring any added value to the DCF model. To get to the final value estimate of the company, I take the 5,6 million EUR of NPV value and as there is no net debt, I only add the current cash balance (360 thousand EUR) to it to get to the final value of 5,96 million EUR.

ROA valuation

Based on the business model, four real options can be identified. The first three are all very similar options to expand. The company has the opportunity to invest a small testing amount and thus buy an option to later expand further. This goes for each of the different new product opportunities. While intuitively, management believes the opportunity is there for all three of those, DCF supports only Global Express and rejects launching the other two if we are to decide today. The fourth real option is the option to abandon. Unlike the first three, this one is an American option meaning that it can be exercised at any point up until expiration date. This will require a slightly different methodology in calculating the option value. With the options identified, it is also fair to say that these options are unique for the company. Abandonment option is clearly not something accessible to anyone else and as the main advantages of launching the new products lie in utilising the already existing proprietary platform, technology, marketing and sales processes, customer base and supply-side base and offering a full array of logistics services to the customers, the same can be assumed for the expansion options. The Parcels and Pallets part assumes no optionality and the DCF valuation should be kept.

To start valuing the options, I source from the static DCF calculations of each product as the basis for the estimating the starting point of the underlying value. I separate the NPV calculation for the incremental committed outflows and all the rest for each product and each scenario. The incremental committed outflows of the first year after product launch are further taken out from the rest of the incremental committed outflows. By doing so I am able to calculate:

- option price as of today (the initial investment or the committed outflows of the first year of the product launch valued at today's NPV);
- current value of the underlying (predicted cash flows not including any of the committed fixed outflows valued at today's NPV);
- strike price of the option (committed outflows as at option decision time).

Table 22: Example 2 – Overview of options to expand

Options' CFs and strike prices if executed				
in thousand EUR	**t=0**	**t=1**	**t=2**	**t=3**
Global Express				
Initial investment at today's NPV	200			
Predicted CFs at today's NPV	2.253			
Committed outflows at decision time		2.683		
E-commerce				
Initial investment at today's NPV	225			
Predicted CFs at today's NPV	1.471			
Committed outflows at decision time			2.332	
Trucking				
Initial investment at today's NPV	325			
Predicted CFs at today's NPV	1.308			
Committed outflows at decision time				2.457

Source: Own work.

Table 22 summarises the option price, the current value of the underlying and the strike price for each of the three options to expand. Intuitively, we are interested in the value of the option today and need to compare this to the value of the option price as at today. The static valuation of the predicted future cash flows gives the current value of the underlying. From today until the point when the option can be purchased and then until the option needs to be exercised a lot can change either in the market or within the company and considering new information and knowledge, the value of the underlying may change. If the value of the underlying is higher than the incremental outflows that potentially need to be committed to at the exercise date, then the option will be exercised meaning that additional investments will take place.

Risk-free rate is assumed to be 1%, while volatility is estimated with the same method as for Example 1. To calculate volatility I use the projected NPV values of the future cash flows without the committed expenses for each product and each scenario. The volatility estimate is 78% as shown in Appendix 4. However, since the number of scenarios is somewhat limited using only the three new products, I perform the same volatility estimation by also including the core product Parcels and Pallets. This returns a 70% volatility estimate as shown in Appendix 5. I settle for 75% as a starting point and later perform sensitivity analyses on the volatility parameter. The binomial trees of the underlying movements and option values for each product can be found in Appendix 6. The basic Black-Scholes equation for valuing European type call option is also used. The summary of the results and the comparison with the DCF individual values is shown in Table 25. ROA shows that there is value originating from flexibility of the individual new products and supports management intuition to launch.

Table 23: Example 2 – Valuation overview per product

Products overview within ROA valuation			
in thousand EUR	**DCF**	**ROA**	**Difference**
Parcels & Pallets	5.534	5.534	0
Global Express	66	333	267
E-commerce	-34	184	218
Trucking	-16	109	124
Total	**5.550**	**6.159**	**609**

Source: Own work.

Once I have calculated the true dynamic value of the existing and potential new products, I can use this value as the current value of the underlying for calculating the fourth real option, the option to abandon. I use the binomial model and perform the movements of the underlying in exactly the same way as before. I use the same volatility estimate and risk-free rate assumptions. The strike price is 2 million EUR, which is what the potential buyer is willing to pay. The only differences in the process are in calculating the option values as we go backwards in the binomial tree. First, since it is a put option, the payoff at the last node is obviously reversed. It is calculated as the maximum between zero and strike price minus the value of the underlying. As we then start moving backwards towards the first node the same approach of risk neutral probabilities is applied with one important difference; since it is an American option, the possibility of an early exercise needs to be considered at every node. This means that at every node, if the value of the early exercise (strike price minus value of underlying) is greater than the option value, this greater value should be taken as the option value at that node. The binomial trees of the underlying movements, the early exercise option values and the final binomial tree of option values is presented in Appendix 7. This leads to the abandonment option being valued at 254 thousand EUR.

Table 24: Example 2 – Valuation comparison between DCF and ROA

Valuation comparison			
in thousand EUR	**DCF**	**ROA**	**Difference**
Parcels & Pallets	5.534	5.534	0
Global Express	66	333	267
E-commerce	0	184	184
Trucking	0	109	109
Abandonment option	0	254	254
True NPV	**5.600**	**6.413**	**813**

Source: Own work.

Table 24 shows an overview of the valuation using either DCF or ROA. It is clear that DCF was able to value the main part of the company accurately and ROA brought no added value there. However, the other drivers of the company's value could not have been properly valued using the DCF approach and ROA shows that there is flexibility value of 813 thousand EUR. To get to the final valuation amount of the company, I add the current cash balance, making the value of the company equal to 6,773 million EUR.

Table 25: Example 2 – Final valuation summary

Valuation summary	
in thousand EUR	
Parcels & Pallets	5.534
Global Express	333
E-commerce	184
Trucking	109
Abandonment option	254
Cash balance	360
Net debt	0
Total valuation	**6.773**

Source: Own work.

The actual final number is not the main result. It is much more insightful to look at the sensitivity analyses and see how the final value changes if the main assumptions have different values. As shown in tables 26 and 27, the value can change drastically if the discount rate or the volatility rate change only slightly.

Table 26: Example 2 – Sensitivity analyses: discount rates

Sensitivity analyses of discount rates on final value			
Discount rate for new products \ existing products			
in thousand EUR	20%	25%	30%
30%	10.245	7.684	6.175
35%	9.299	6.773	5.264
40%	8.844	6.318	4.835

Source: Own work.

Table 27: Example 2 – Sensitivity analyses: discount rate and volatility

Sensitivity analyses of discount rate and volatility on final value					
Discount rate for existing products \ volatility					
in thousand EUR	55%	65%	75%	85%	95%
15%	13.786	13.988	14.295	14.597	14.891
20%	8.768	8.995	9.299	9.657	10.006
25%	6.158	6.430	6.773	7.112	7.445
30%	4.676	4.933	5.264	5.593	5.927
35%	3.788	4.046	4.381	4.717	5.049

Source: Own work.

3.3 Summary of the examples from practice

Both examples were based on actual startup companies and actual financials and projections. Even though the numbers were slightly altered, the dynamics of the risks and returns were kept the same. In both cases I applied both DCF and ROA approaches and in both cases similar conclusions can be drawn. DCF is struggling with startup valuations. It does not have a reliable way of accounting for the risk associated with the potentially high returns and it cannot value future flexibility. ROA solves only the second struggle. Defining the discount rate remains an issue. In the second example I applied a lower discount rate to the existing business and a higher one to the new riskier projects in an attempt to better disperse the risk. I was also looking into applying a different discount rate through the years, but ended up not choosing this approach. It could have some validity to set the discount rate higher for the first years and lower for later years as the first years are subject to higher growth expectations and can as such be deemed as riskier compared to later more mature years. But in startup cases this approach can often lead to improper discounting and overvaluation as the first years could have predominantly negative returns and the majority of the value arises from the later years and terminal value. I could thus be undervaluing the initial loss-making beginning years and overvaluing the late potentially highly profitable years.

Optionality was inherent in both cases and a significant share of value originated from it. Even though the conclusions can be similar, both examples are very different. One is in a very early phase while the other has already had at least partially established business. In the first case, I had to value compounded call options, while in the second one, the call options were independent. There was also a separate put option. Both cases required a lot of thought, analyses and research to validate the projections and underlying assumptions. I did not explain every detail of what I analysed and simulated, but in order to believe in the probability of the projections, an in-depth analyses truly needs to be done when performing an actual valuation, which in the end gives you a lot of insight into the business and possible future scenarios. This is the true added value of performing a valuation of a startup company. There are too many unknowns and unpredictable events and unreliable data to be able to arrive to a precise value point of a startup. At best, a range can be generated and often it is not a very narrow one. Similar happened in my examples. However, by combining DCF and ROA (in particular by using multiple scenarios for DCF and the binomial model for ROA), I was able to identify the key value drivers and create frameworks for scenarios in which the value of the startup increases or decreases as well as to illustrate key decision points. This is what should matter to any founder, investor, or shareholder as it allows them to portray their own knowledge of the business and the market into the model and get some additional clarity.

3.4 Key findings

Working my way through the literature and especially practical examples of startup company valuation, ROA and other valuation techniques and applying all that on actual cases, I have come up with many findings. Some of them ended up as I was expecting them to be going into the research, some turned out as a surprise, and some were referring to brand new information for me. I am here listing the ones I believe are the most important and can be served as general guides when discussing and applying ROA or other valuation techniques to startup companies. I structure them in three general subgroups listed below.

Startup company characteristics and valuation:

- Startup companies work very similarly to a risky project. They almost always require multiple capital injections and expansions.
- A large part of the value often arrives from the optionality to execute these subsequent growth opportunities, so a model that can best capture such behaviour should be used.
- The biggest issue with startup company valuations are input estimations.
- The biggest challenge is to find a risk measure that can account for the high failure/mortality risk and at the same time allows the model to recognise the value, which is mainly arising from high growth possibilities, usually in the later years of the life cycle.
- Projecting the future for startups is very different than for established companies as there is no comparable past data. Potential, market size and scenario analyses of future projections play a key role.

DCF vs. ROA vs. "soft" methods:

- DCF in various types of form is still predominantly being used in startup valuations despite the clear drawbacks.
    - The main issue with it is that all risk needs to be incorporated in the risk-adjusted rate of return, which has an extremely large influence on the end value.
    - Using a single high discount rate, DCF normally overestimates the present value of the CFs in the first years (which are usually money-losing) and underestimates the present value of the CFs in the later years (which are usually money-generating after the tipping point).
    - DCF in its pure form is a linear model and cannot properly value flexibility.
- ROA serves as an improvement in both general DCF flaws mentioned above, but only solves the second one sufficiently.
    - The difference between ROA and DCF valuation is the embedded flexibility. With further input estimations, ROA provides an efficient valuation tool for it.
    - While the risk-adjusted rate of return has a slightly lower degree of influence on the final value, it is still the single most influential factor in the ROA model.
- Soft or qualitative valuation methods are important.
    - All the valuation issues associated with the sophisticated models like DCF and ROA give merit to the subjective qualitative methods, especially in the early phases.
    - DCF and ROA should be used to analyse the various future scenarios and show the appraiser the conditions and milestones needed for the company to achieve a certain value. Then, the qualitative methods should be assessed again to determine the

likelihood of the startup company living up to the expectations with the existing team, product, proprietary elements, market position, etc.

ROA process:

- The starting point of ROA valuation is the static “no flexibility” value estimation, so DCF (or similar) is a prerequisite.
- Event tree and timeline setup along with real options identification are key steps.
- One should first define the above key steps, then setup the binomial model and only lastly test it with Black-Scholes equation to get to a more exact point estimate if needed.
    - Binomial model allows solving for more flexible and complex option types and is the preferred practical choice over Black-Scholes equation
    - It also better mimics the actual event tree of startup company’s future.
    - Black-Scholes equation can provide a more exact answer and is especially useful for simpler options.
- Sensitivity analyses is very important: it helps one imagine the value of a startup company in various scenarios.
    - Future sales projections, discount rate and volatility are usually the most significant factors that can be used in such analyses.

CONCLUSION

The main research question of the thesis has been to investigate the usefulness of ROA in startup valuations. From a model fit perspective, ROA seems tailored specifically for valuing option-like cash flows exactly like a typical startup would have them. While there are limitations to applying option valuation theory to the real world and especially to a world of early-phase, private-owned startups, the true challenges lie in practical applications. The main issue is the lack of reliable data, which causes problems to the process of estimating the main model parameters such as the discount rate and the volatility rate. Discount rate is the factor that is particularly difficult to estimate for startups and often ends up being chosen - at least to some degree - arbitrarily. As the required rate of return is much higher than for an established stable company, the value of a startup is much more prone to changes in the discount rate. ROA does not include the discount rate in any of its formulars directly, but is indirectly dependent on it, especially since the starting value of the underlying is normally the DCF value. As such it does not avoid the potential discount rate inaccuracies. Having said that, I have shown how to successfully apply ROA to two real-life startup examples and it was clear ROA as a startup valuation approach provided added value. The direct dependency on the variability of the discount rate is smaller, but not by much and it would thus be unfair to say it solves this problem. However, my research as well as both practical cases have shown very clearly that ROA can serve as the model to account for future flexibility, which in the case of a startup company can represent a significant part of the value.

Sourcing also directly from my experience of being in and around the startup world and venture capital environment, many times it happens that the lack of reliable data is severe. In such cases it is difficult to definitively determine which valuation approach is best and what is the true value of the business. In the absence of comparable tradable peer companies that would resemble the startup company and reliable historical data, the financial models can often serve only as a framework to provide some range of value, while the predominant factors for decision making remain subjective "soft" valuation methods' factors such as entrepreneurial savviness and managerial skills of the founders and their team, the attractiveness of the business idea itself, the opportunity to scale, etc. The underlying investment concept of the venture capitalists is often guided by the willingness to let majority of their portfolio go bust on one side and the focus on making a new market leader with a few or even just one of their other investments.

To conclude, I believe ROA should be a vital analytical tool when considering startup company valuations and should be applied in combination with the DCF to solve the part of the value arising from future flexibility of the startup business. However, ROA does not solve all issues as the estimation of the discount rate remains a glaring problem. With that in mind, when it comes to startup valuations, the objective should not be to get to a precise point value, but rather to get some possible value ranges and investigate the frameworks and assumptions that are needed in order for a startup company to be above a certain value. ROA can be very useful in this regard, especially by applying the binomial model as it forces one to think of various future scenarios.

REFERENCES

1. Allen, F., Brealey, R.A. & Myers, S.C. (2011). *Principles of Corporate Finance*. New York: McGraw-Hill.
2. Amit, R., Glosten, L. & Muller, E. (1990). Does Venture Capital Foster the Most Promising Entrepreneurial Firms? *California Management Review, 32*(3), 102-111.
3. Amram, M. & Kulatilaka N. (1999). *Real Options*. Boston: Harvard Business School Press.
4. Baker Kent, H., Dutta, S. & Saadi, S. (2015). Management Views on Real Options in Capital Budgeting. *Journal of Applied Finance (Formerly Financial Practice and Education), 21*(1), 18-29.
5. Black, F. & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy, University of Chicago Press, 81*(3), 637-654.
6. Blank, S. (2010). *Why Startups are Agile and Opportunistic – Pivoting the Business Model.* Retrieved October 1st 2017 from web address https://steveblank.com/2010/04/12/why-startups-are-agile-and-opportunistic-%e2%80%93-pivoting-the-business-model/.
7. Block, S. (2007). Are "Real Options" Actually used in the Real World? *Engineering Economist, 52*(3), 255-267.

8. Brach, M. & Paxson, D. (2001). A Gene to Drug Venture: Poisson Options Analysis. *R&D Management, 31*(2), 203-214.
9. Brandao, L. E., Dyer, J. S. & Hahn, W. J. (2005). Using Binomial Decision Trees to Solve Real-Option Valuation Problems. *Decision Analysis*, *2*(2), 69–88.
10. Brealey, R. A., Myers, S. C. & Allen, F. (2011). *Principles of corporate finance (10th ed.)*. New York: McGraw-Hill/Irwin.
11. Boer, P. (2003). Risk-Adjusted Valuation of R&D Projects. *Research-Technology Management, 46*(5), 50-58.
12. Boyle, P.P. (1977). Options: A Monte Carlo Approach. *Journal of Financial Economics, 4*(3), 323-338.
13. Cochrane, J. H. (2005). The Risk and Return of Venture Capital. *Journal of Financial Economics, 75*(1), 3-52.
14. Copeland, T. & Antikarov, V. (2001). *Real Options: A Practitioner's Guide*. New York: Texere.
15. Cox, J., Ross, S. & Rubinstein, M. (1979). Option Pricing: A Simplified Approach. *Journal of Financial Economics, 7*(3), 229-263.
16. Damodaran.A. (2012). *Investment Valuation, 3e.* New York: John Wiley & Sons.
17. Damodaran, A. (2009). *Valuing Young, Start-up and Growth Companies: Estimation Issues and Valuation Challenges*. Stern School of Business, New York University.
18. Giardino, M., Unerkalmsteiner, N., Paternoster, T., Gorschek, P. & Abrahamsson, P. (2014). What do we know about Software Development in Startups? *IEEE Software., 31*(5), 28-32.
19. Graham, J. R. & Harvey, C. R. (2001). The theory and practice of corporate finance: evidence from the field. *Journal of Financial Economics, 60*(2-3), 187-243.
20. Gornall, W. & Strebulaev, I. A. (2017). *Squaring Venture Capital Valuations with Reality.* NBER Working Papers 23895, National Bureau of Economic Research, Inc.
21. Hodder, J. E. & Riggs, H. E. (1985). Pitfalls in Evaluating Risky Projects. *Harvard Business Review, 63*(1), 128-135.
22. Kelley, H. (1972). *Attribution: Perceiving the Causes of Behaviour*. Morristown, N.J.: General Learning Press.
23. Kellogg, D., Charnes, J. & Demirer, R. (1999). *Valuation of a Biotechnology Firm: An Application of Real-Options Methodologies.* Research paper presented at the 3rd Annual Real Options Conference: Theory meets Practice, The Netherlands.
24. Koller, T., Goedhart, M. & Wessels, D. (2010). *Valuation: Measuring and Managing the Value of Companies (5th ed.)*. Hoboken, NJ, USA: John Wiley & Sons, Inc.
25. Kodukula, P. & Papudesu, C. (2006). *Project Valuation Using Real Options: A Practitioner's Guide*. J. Ross Publishing, Inc.
26. Longstaff, F.A. & Schwartz, E.S. (2001). Valuing American Options by Simulation: A Simple Least-Squares Approach. *Review of Financial Studies, 14*(1), 113-47.
27. Manigart, S., De Waele, K., Wright, M., Robbie, K., Desbrières, P., Sapienza, H. &

Beekman, A. (2000). Venture Capitalists, Investment Appraisal and Accounting Information: A Comparative Study of the USA, UK, France, Belgium and Holland. *European Financial Management, 6*(3), 389-403.

28. Nasser, S. 2016. *Valuation For Startups – 9 Methods Explained*. Medium. Retrieved October 1st 2019 from web address https://medium.com/parisoma-blog/valuation-for-startups-9-methods-explained-53771c86590e.
29. Milanesi, G., Pesce, G. & El Alabi, E. (2013). Technology-Based Startup Valuation Using Real Options with Edgeworth Expension. *Journal of Finance and Accounting 1*(2), 54-61.
30. Mun, J. (2002). *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions*. John Wiley & Sons.
31. Myers, S.C. (1977). Determinants of Corporate Borrowings. *Journal of Financial Economics, 5*(2) 147-175.
32. Myers, S. & Majluf, N. (1984). Corporate Financing and Investment Decisions when Firms have Information Investors Do Not Have. *Journal of Financial Economics, 13*(2), 187-221.
33. Oliveira, F. B. & Zotes, L. P. (2018). Valuation methodologies for business startups: a bibliographical study and survey. *Brazilian Journal of Operations & Production Management, 15*(1), 96-111.
34. Roure, J. & Keeley, R. (1990). Predictors of Success in New Technology Ventures. *Journal of Business Venturing; Volume 5*(4), 201-220.
35. Ruhnka, J. & Young, J. (1987). A Venture Capital Model of the Development Process for New Ventures. *Journal of Business Venturing, 2*(2), 167-184.
36. Ruhnka, J. & Young, J. (1991). Some Hypotheses about Risk in Venture Capital Investing. *Journal of Business Venturing, 6*(2), 115-133.
37. Robehmed, N. (2013). *What Is A Startup?* Forbes. Retrieved October 1st 2017 from web address https://www.forbes.com/sites/natalierobehmed/2013/12/16/what-is-a-startup/#4d837cc44044.
38. Ryan, P.A. & Ryan, G.P. (2002). Capital Budgeting Tools of the Fortune 1000: How have things changed?. *Journal of Business and Management, 8*(4), 355-364.
39. Shepherd, D.A., Douglas, E.J. & Shanley, M. 2000. New venture survival – ignorance, external shocks, and risk reduction strategies. *Journal of Business Venturing, 15*(5-6), 393-410.
40. Shockley, R. L. (2007). *An Applied Course in Real Options Valuation*. Australia: Tomson South-Western.
41. Stinchcombe, A.L. 1965. Social structures and organisations. *Handbook of Organizations, Chicago: Rand McNally*, 142-193.
42. Steffens, P.R. & Douglas, E.J. 2007. Valuing technology investments: use real options thinking but forget real options valuation. *International Journal of Technoentrepreneurship, 1*(1), 58-77.
43. Timmons, J. & Gumpert, D. (1982). Discard Many Old Rules About Getting Venture Capital. *Harvard Business Review; Volume 60*(1), 273-280.

44. Timmons, J.A. & Spinelli, S. (2004). *New Venture Creation: Entrepreneurship for the 21st Century*. 6th edition, Boston: McGraw-Hill/Irwin.
45. Trigeorgis, L & Reuer, J. J. (2017). Real Options Theory in Strategic Management. *Strategic Management Journal, 38*(1), 42-63.
46. Trigeorgis, L. (1996). *Real options: Managerial flexibility and strategy in resource allocation*. Cambridge: The MIT Press.
47. Trigeorgis, L. (1993). Real Options and Interactions with Financial Flexibility. *Financial Management, 22*(3), 202-224.
48. Westland, C. (2002). *Valuing Technology*. Singapore: John Wiley & Sons (Asia).
49. Wilmott, P. (2007). *Paul Wilmott introduces quantitative finance*. John Wiley & Sons.
50. Wymar, S. & Regan, E. (2005). Factors Influencing e-commerce Adoption and Use by Small and Medium Businesses. *Electronic markets, 15*(4), 438-453.

APPENDICES

Appendix 1: Povzetek (Summary in Slovene language)

Medtem ko zagonska podjetja postajajo vedno večji del svetovnega gospodarstva, ocenjevanje vrednosti tovrstnih hitro rastočih podjetij, ostaja velik praktični izziv. V tej nalogi raziskujem možnosti uporabe analize realnih opcij kot metodo vrednotenja zagonskih podjetij. Uporabniki metode realnih opcij črpajo iz teorije vrednotenja finančnih inštrumentov in iščejo vzporedne povezave med tovrstnimi finančnimi inštrumenti ter podjetji ali drugimi sredstvi v realnem svetu, oziroma v gospodarstvu. V primeru, da so predvideni denarni tokovi, donosi in tveganja realnih objektov dovolj podobni tistim od finančnih inštrumentov, se v določenih primerih teorija vrednotenja finančnih inštrumentov lahko prenese v prakso ocenjevanja tovrstnih realnih objektov. Tipična struktura denarnih tokov zagonskih podjetij je zelo podobna strukturi denarnih tokov finančnega inštrumenta nakupne opcije. Mlada zagonska podjetja imajo običajno namreč podobno dinamiko visoko tveganemu projektu. Na začetku je potrebna določena začetna investicija, nato pa pridejo nova spoznanja o samem projektu, saj lahko le-ta ali nazaduje, oziroma propade ali pa odlično napreduje in raste. Na neki točki imamo nato možnost novih vlaganj in novega spodbujanja ideje. Če se bo izkazalo, da je ideja dobra, bomo možnost dodatnih vlaganj izkoristili in potencialno dosegli neomejene donose, v nasprotnem primeru pa v projekt ne bomo več vlagali in v najslabšem primeru zabeležili izgubo v vrednosti začetne investicije. Podobno je pri finančnih opcijah, ko moramo opcijo najprej kupiti (začetna investicija), čez čas pa se lahko odločimo ali jo bomo dejansko izkoristili ali ne (dodatno vlaganje). Zagonska podjetja imajo lahko tudi drugačno strukturo donosov in tveganj in vzporednice se lahko v teh primerih povežejo tudi z bolj kompleksnimi oblikami finančnih opcij. V nalogi primerjam metodo realnih opcij z več drugimi popularnimi metodami vrednotenja podjetij, glavni poudarek pa namenjam primerjavi z metodo diskontinuiranih denarnih tokov. Celotna procesa obeh metod sta analizirana drug ob drugem s posebno pozornostjo na visoko volatilna okolja, v katerih nastajajo zagonska podjetja. Poleg teoretične podlage, obe metodi primerjam tudi pri praktičnih primerih, kjer ovrednotim dve dejanski zagonski podjetji. Na podlagi celotne analize ugotovim, da je metoda realnih opcij dopolnilo in ne nadomestek bolj uveljabljeni metodi diskontinuiranih denarnih tokov. Medtem ko zadovoljivo rešuje problem vrednotenja prihodnje fleksibilnosti odločanja, ki je pri zagonskih podjetjih pomemben del vrednosti, analiza realnih opcij ne reši problema ocene ustrezne diskontne stopnje kot glavnega problem pri vrednostenju zagonskih podjetij. Navkljub temu je dodana vrednost očitna in analiza realnih opcij je tako priporočljiva kot ocenjevalna metoda pri vrednotenju zagonskih podjetij, kjer pa se je potrebno zavedati, da je sama analiza scenarijev in virov vrednosti podjetja bolj pomembna od ocenjevanja točnega zneska cenitve.

Appendix 2: Example 1 - Volatility estimation based on future scenarios

Volatility estimation based on future scenarios								
Patent grant	Phase 1	Phase 2	Phase 3	Cumulative prob. of the scenario	Total revenue (EUR)	Ln returns	Probability-weighted returns	Deviations*probability
Yes	Best case	Best case	Best case	0,6%	217.539.080	137,4%	0,8%	0,9%
Yes	Best case	Best case	Most likely	1,7%	108.133.218	67,5%	1,1%	0,6%
Yes	Best case	Best case	Pessimistic	0,6%	64.370.873	15,7%	0,1%	0,0%
Yes	Best case	Most likely	Best case	1,7%	198.718.785	128,4%	2,2%	2,4%
Yes	Best case	Most likely	Most likely	5,0%	89.312.923	48,4%	2,4%	0,8%
Yes	Best case	Most likely	Pessimistic	1,7%	45.550.579	-18,9%	-0,3%	0,1%
Yes	Best case	Pessimistic	Best case	0,6%	191.190.667	124,5%	0,7%	0,7%
Yes	Best case	Pessimistic	Most likely	1,7%	81.784.806	39,6%	0,7%	0,1%
Yes	Best case	Pessimistic	Pessimistic	0,6%	38.022.461	-37,0%	-0,2%	0,1%
Yes	Most likely	Best case	Best case	1,7%	215.459.724	136,5%	2,3%	2,7%
Yes	Most likely	Best case	Most likely	5,0%	106.053.862	65,6%	3,3%	1,6%
Yes	Most likely	Best case	Pessimistic	1,7%	62.291.518	12,4%	0,2%	0,0%
Yes	Most likely	Most likely	Best case	5,0%	196.639.429	127,3%	6,4%	7,0%
Yes	Most likely	Most likely	Most likely	15,1%	87.233.568	46,1%	7,0%	2,0%
Yes	Most likely	Most likely	Pessimistic	5,0%	43.471.223	-23,6%	-1,2%	0,6%
Yes	Most likely	Pessimistic	Best case	1,7%	189.111.312	123,4%	2,1%	2,2%
Yes	Most likely	Pessimistic	Most likely	5,0%	79.705.450	37,0%	1,9%	0,4%
Yes	Most likely	Pessimistic	Pessimistic	1,7%	35.943.105	-42,6%	-0,7%	0,5%
Yes	Pessimistic	Best case	Best case	0,6%	214.371.982	136,0%	0,8%	0,9%
Yes	Pessimistic	Best case	Most likely	1,7%	104.966.120	64,6%	1,1%	0,5%
Yes	Pessimistic	Best case	Pessimistic	0,6%	61.203.776	10,6%	0,1%	0,0%
Yes	Pessimistic	Most likely	Best case	1,7%	195.551.687	126,8%	2,1%	2,3%
Yes	Pessimistic	Most likely	Most likely	5,0%	86.145.826	44,8%	2,3%	0,6%
Yes	Pessimistic	Most likely	Pessimistic	1,7%	42.383.481	-26,1%	-0,4%	0,2%
Yes	Pessimistic	Pessimistic	Best case	0,6%	188.023.570	122,9%	0,7%	0,7%
Yes	Pessimistic	Pessimistic	Most likely	1,7%	78.617.708	35,7%	0,6%	0,1%
Yes	Pessimistic	Pessimistic	Pessimistic	0,6%	34.855.363	-45,7%	-0,3%	0,2%
No	Best case	Best case	Best case	0,2%	55.279.160	0,4%	0,0%	0,0%

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Volatility estimation based on future scenarios (cont.)								
Patent grant	Phase 1	Phase 2	Phase 3	Cumulative prob. of the scenario	Total revenue (EUR)	Ln returns	Probability-weighted returns	Deviations*probability
No	Best case	Best case	Most likely	0,7%	27.927.695	-67,8%	-0,5%	0,4%
No	Best case	Best case	Pessimistic	0,2%	16.987.109	-117,6%	-0,3%	0,4%
No	Best case	Most likely	Best case	0,7%	50.574.087	-8,5%	-0,1%	0,0%
No	Best case	Most likely	Most likely	2,2%	23.222.621	-86,3%	-1,9%	2,0%
No	Best case	Most likely	Pessimistic	0,7%	12.282.035	-150,0%	-1,1%	1,8%
No	Best case	Pessimistic	Best case	0,2%	48.692.057	-12,2%	0,0%	0,0%
No	Best case	Pessimistic	Most likely	0,7%	21.340.592	-94,7%	-0,7%	0,8%
No	Best case	Pessimistic	Pessimistic	0,2%	10.400.006	-166,6%	-0,4%	0,7%
No	Most likely	Best case	Best case	0,7%	54.170.444	-1,6%	0,0%	0,0%
No	Most likely	Best case	Most likely	2,2%	26.818.979	-71,9%	-1,6%	1,4%
No	Most likely	Best case	Pessimistic	0,7%	15.878.392	-124,3%	-0,9%	1,3%
No	Most likely	Most likely	Best case	2,2%	49.465.370	-10,7%	-0,2%	0,1%
No	Most likely	Most likely	Most likely	6,5%	22.113.905	-91,2%	-5,9%	6,6%
No	Most likely	Most likely	Pessimistic	2,2%	11.173.319	-159,4%	-3,4%	6,2%
No	Most likely	Pessimistic	Best case	0,7%	47.583.341	-14,5%	-0,1%	0,0%
No	Most likely	Pessimistic	Most likely	2,2%	20.231.875	-100,1%	-2,2%	2,6%
No	Most likely	Pessimistic	Pessimistic	0,7%	9.291.289	-177,9%	-1,3%	2,5%
No	Pessimistic	Best case	Best case	0,2%	53.724.369	-2,4%	0,0%	0,0%
No	Pessimistic	Best case	Most likely	0,7%	26.372.904	-73,6%	-0,5%	0,5%
No	Pessimistic	Best case	Pessimistic	0,2%	15.432.317	-127,2%	-0,3%	0,4%
No	Pessimistic	Most likely	Best case	0,7%	49.019.295	-11,6%	-0,1%	0,0%
No	Pessimistic	Most likely	Most likely	2,2%	21.667.830	-93,2%	-2,0%	2,3%
No	Pessimistic	Most likely	Pessimistic	0,7%	10.727.244	-163,5%	-1,2%	2,2%
No	Pessimistic	Pessimistic	Best case	0,2%	47.137.266	-15,5%	0,0%	0,0%
No	Pessimistic	Pessimistic	Most likely	0,7%	19.785.800	-102,3%	-0,7%	0,9%
No	Pessimistic	Pessimistic	Pessimistic	0,2%	8.845.214	-182,8%	-0,4%	0,9%
Sum				100,0%			10%	62%
Volatility								**79%**

Source: Own work.

Appendix 3: Example 2 – DCF valuations for each product for all scenarios as at product launch date

DCF for "Parcels & Pallets" within "Best case" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Net sales	**3.125**	**6.875**	**13.063**	**22.206**	**34.420**	**49.909**	
Gross margin	**1.219**	**2.681**	**5.094**	**8.883**	**13.768**	**20.463**	
OPEX	1.391	2.283	3.211	4.583	6.415	8.738	
EBITDA	**-172**	**398**	**1.883**	**4.300**	**7.353**	**11.724**	
EBIT	**-210**	**314**	**1.724**	**4.029**	**6.933**	**11.116**	
EBIT minus taxes	-210	314	1.487	3.263	5.616	9.004	
Change in working capital	-115	-195	-116	123	622	1.732	
CAPEX	43	79	128	185	244	301	
FCF	**-138**	**430**	**1.475**	**2.955**	**4.749**	**6.970**	**30.912**
WACC	25%	25%	25%	25%	25%	25%	25%
Discount factor	0,80	0,64	0,51	0,41	0,33	0,26	0,26
PV of FCF	-110	275	755	1.210	1.556	1.827	8.103
Sum of PV of FCF at t=0 **13.617**							

Source: Own work.

DCF for "Parcels & Pallets" within "Pessimistic" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Net sales	**2.375**	**3.800**	**5.320**	**6.916**	**8.299**	**9.129**	
Gross margin	**903**	**1.482**	**2.075**	**2.697**	**3.237**	**3.560**	
OPEX	1.182	1.752	1.980	2.220	2.427	2.552	
EBITDA	**-280**	**-270**	**95**	**478**	**810**	**1.009**	
EBIT	**-318**	**-317**	**30**	**393**	**708**	**897**	
EBIT minus taxes	-318	-317	30	393	708	743	
Change in working capital	-64	-99	-8	59	159	268	
CAPEX	32	44	52	58	59	55	
FCF	**-286**	**-261**	**-14**	**276**	**491**	**420**	**1.792**
WACC	25%	25%	25%	25%	25%	25%	25%
Discount factor	0,80	0,64	0,51	0,41	0,33	0,26	0,26
PV of FCF	-229	-167	-7	113	161	110	470
Sum of PV of FCF at t=0 **450**							

Source: Own work.

DCF for "Parcels & Pallets" within "Most likely" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Net sales	**2.625**	**4.988**	**8.479**	**13.142**	**18.399**	**23.919**	
Gross margin	**998**	**1.945**	**3.307**	**5.125**	**7.176**	**9.567**	
OPEX	1.252	2.000	2.524	3.223	4.012	4.840	
EBITDA	**-254**	**-55**	**783**	**1.902**	**3.164**	**4.728**	
EBIT	**-254**	**-116**	**680**	**1.742**	**2.939**	**4.436**	
EBIT minus taxes	-360	-94	551	1.411	2.381	3.593	
Change in working capital	-82	-144	-56	69	333	792	
CAPEX	36	58	83	110	131	144	
FCF	**-313**	**-7**	**523**	**1.232**	**1.918**	**2.657**	**11.781**
WACC	25%	25%	25%	25%	25%	25%	25%
Discount factor	0,80	0,64	0,51	0,41	0,33	0,26	0,26
PV of FCF	-251	-5	268	505	628	696	3.088
Sum of PV of FCF at t=0 **4.930**							

Source: Own work.

DCF for "Global Express" within "Best case" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Incremental project inflows	105	435	1.269	2.791	5.303	9.016	
Incremental committed outflows	270	2.049	1.422	551	535	0	
Incremental other outflows	28	116	290	638	1.212	3.382	
Incremental project returns	**-193**	**-1.730**	**-443**	**1.602**	**3.556**	**5.634**	**17.413**
WACC	35%	35%	35%	35%	35%	35%	35%
Discount factor	0,74	0,55	0,41	0,30	0,22	0,17	0,17
PV of FCF	-143	-949	-180	482	793	931	2.877
Sum of PV of FCF at t=0 **3.811**							

Source: Own work.

DCF for "Global Express" within "Pessimistic" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Incremental project inflows	6	50	138	316	594	928	
Incremental committed outflows	270	2.049	1.422	551	535	0	
Incremental other outflows	2	20	50	110	198	417	
Incremental project returns	**-266**	**-2.019**	**-1.334**	**-345**	**-139**	**511**	**1.580**
WACC	35%	35%	35%	35%	35%	35%	35%
Discount factor	0,74	0,55	0,41	0,30	0,22	0,17	0,17
PV of FCF	-197	-1.108	-542	-104	-31	84	261
Sum of PV of FCF at t=0 **-1.637**							

Source: Own work.

DCF for "Global Express" within "Most likely" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Incremental project inflows	21	121	327	747	1.392	2.161	
Incremental committed outflows	270	2.049	1.422	551	535	0	
Incremental other outflows	7	39	97	213	384	877	
Incremental project returns	**-255**	**-1.967**	**-1.192**	**-18**	**474**	**1.283**	**3.967**
WACC	35%	35%	35%	35%	35%	35%	35%
Discount factor	0,74	0,55	0,41	0,30	0,22	0,17	0,17
PV of FCF	-189	-1.079	-484	-5	106	212	655
Sum of PV of FCF at t=0	**-785**						

Source: Own work.

DCF for "E-commerce" within "Best case" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Incremental project inflows	44	600	1.625	3.850	7.838	14.212	
Incremental committed outflows	411	1.234	1.914	905	0	0	
Incremental other outflows	28	350	875	1.925	3.658	7.737	
Incremental project returns	**-395**	**-984**	**-1.164**	**1.020**	**4.180**	**6.475**	**20.015**
WACC	35%	35%	35%	35%	35%	35%	35%
Discount factor	0,74	0,55	0,41	0,30	0,22	0,17	0,17
PV of FCF	-292	-540	-473	307	932	1.070	3.306
Sum of PV of FCF at t=1	**4.310**						

Source: Own work.

DCF for "E-commerce" within "Pessimistic" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Incremental project inflows	6	49	140	308	624	936	
Incremental committed outflows	411	1.234	1.914	905	0	0	
Incremental other outflows	6	49	123	270	485	767	
Incremental project returns	**-411**	**-1.234**	**-1.897**	**-866**	**139**	**168**	**521**
WACC	35%	35%	35%	35%	35%	35%	35%
Discount factor	0,74	0,55	0,41	0,30	0,22	0,17	0,17
PV of FCF	-304	-677	-771	-261	31	28	86
Sum of PV of FCF at t=1	**-1.868**						

Source: Own work.

DCF for "E-commerce" within "Most likely" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Incremental project inflows	13	120	338	825	1.634	2.673	
Incremental committed outflows	411	1.234	1.914	905	0	0	
Incremental other outflows	11	105	263	578	1.040	1.771	
Incremental project returns	**-409**	**-1.219**	**-1.839**	**-657**	**594**	**902**	**2.788**
WACC	35%	35%	35%	35%	35%	35%	35%
Discount factor	0,74	0,55	0,41	0,30	0,22	0,17	0,17
PV of FCF	-303	-669	-747	-198	132	149	461
Sum of PV of FCF at t=1	**-1.175**						

Source: Own work.

DCF for "Trucking" within "Best case" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Incremental project inflows	270	731	1.733	3.527	6.395	10.223	
Incremental committed outflows	800	1.413	1.325	1.125	750	0	
Incremental other outflows	79	197	433	823	1.399	3.699	
Incremental project returns	**-609**	**-879**	**-25**	**1.579**	**4.246**	**6.524**	**20.165**
WACC	35%	35%	35%	35%	35%	35%	35%
Discount factor	0,74	0,55	0,41	0,30	0,22	0,17	0,17
PV of FCF	-451	-482	-10	475	947	1.078	3.331
Sum of PV of FCF at t=2 **4.888**							

Source: Own work.

DCF for "Trucking" within "Pessimistic" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Incremental project inflows	7	18	44	79	134	174	
Incremental committed outflows	800	1.413	1.325	1.125	750	0	
Incremental other outflows	4	9	19	35	52	88	
Incremental project returns	**-797**	**-1.404**	**-1.300**	**-1.080**	**-668**	**86**	**266**
WACC	35%	35%	35%	35%	35%	35%	35%
Discount factor	0,74	0,55	0,41	0,30	0,22	0,17	0,17
PV of FCF	-590	-770	-528	-325	-149	14	44
Sum of PV of FCF at t=2 **-2.305**							

Source: Own work.

DCF for "Trucking" within "Most likely" scenario							
in thousand EUR	**PY1**	**PY2**	**PY3**	**PY4**	**PY5**	**PY6**	**TV**
Incremental project inflows	75	188	454	891	1.337	1.737	
Incremental committed outflows	800	1.413	1.325	1.125	750	0	
Incremental other outflows	26	66	144	260	390	741	
Incremental project returns	**-751**	**-1.291**	**-1.015**	**-494**	**197**	**997**	**3.081**
WACC	35%	35%	35%	35%	35%	35%	35%
Discount factor	0,74	0,55	0,41	0,30	0,22	0,17	0,17
PV of FCF	-556	-708	-413	-149	44	165	509
Sum of PV of FCF at t=2 **-1.109**							

Source: Own work.

Appendix 4: Example 2 – Volatility estimation using only new products

Global Express	E-commerce	Trucking	Cumulative probability of the scenario	Probability-weighted NPV at t=0	NPV at t=1	LN returns	Probability-weighted returns	Deviations*probability
Best case	Best case	Best case	1,56%	230	19.846	137%	2%	3%
Best case	Best case	Pessimistic	2,19%	235	14.518	106%	2%	2%
Best case	Best case	Most likely	2,50%	285	15.404	112%	3%	3%
Best case	Pessimistic	Best case	2,19%	221	13.668	100%	2%	2%
Best case	Pessimistic	Pessimistic	3,06%	189	8.340	51%	2%	1%
Best case	Pessimistic	Most likely	3,50%	239	9.226	61%	2%	1%
Best case	Most likely	Best case	2,50%	266	14.361	105%	3%	3%
Best case	Most likely	Pessimistic	3,50%	234	9.033	59%	2%	1%
Best case	Most likely	Most likely	4,00%	294	9.919	68%	3%	2%
Pessimistic	Best case	Best case	2,19%	202	12.492	91%	2%	2%
Pessimistic	Best case	Pessimistic	3,06%	163	7.164	35%	1%	0%
Pessimistic	Best case	Most likely	3,50%	209	8.050	47%	2%	1%
Pessimistic	Pessimistic	Best case	3,06%	143	6.314	23%	1%	0%
Pessimistic	Pessimistic	Pessimistic	4,29%	31	986	-163%	-7%	12%
Pessimistic	Pessimistic	Most likely	4,90%	68	1.872	-99%	-5%	5%
Pessimistic	Most likely	Best case	3,50%	182	7.007	33%	1%	0%
Pessimistic	Most likely	Pessimistic	4,90%	61	1.679	-110%	-5%	6%
Pessimistic	Most likely	Most likely	5,60%	106	2.565	-67%	-4%	3%
Most likely	Best case	Best case	2,50%	253	13.642	100%	2%	2%
Most likely	Best case	Pessimistic	3,50%	216	8.314	50%	2%	1%
Most likely	Best case	Most likely	4,00%	273	9.200	60%	2%	1%
Most likely	Pessimistic	Best case	3,50%	194	7.464	39%	1%	0%
Most likely	Pessimistic	Pessimistic	4,90%	78	2.136	-86%	-4%	4%
Most likely	Pessimistic	Most likely	5,60%	125	3.022	-51%	-3%	2%
Most likely	Most likely	Best case	4,00%	242	8.157	48%	2%	1%
Most likely	Most likely	Pessimistic	5,60%	117	2.829	-58%	-3%	2%
Most likely	Most likely	Most likely	6,40%	176	3.715	-30%	-2%	1%
Sum			100,00%	5.032			4%	60%
Volatility								**78%**

Source: Own work.

Appendix 5: Example 2 – Volatility estimation using also existing products

Volatility estimation using also existing products									
Parcels & pallets	**Global Express**	**E-commerce**	**Trucking**	**Cumulative probability of the scenario**	**Probability-weighted NPV at t=0**	**NPV at t=1**	**LN returns**	**Probability-weighted returns**	**Deviations*probability**
Best case	Best case	Best case	Best case	0,39%	111	36.868	125%	0,49%	0,56%
Best case	Best case	Best case	Pessimistic	0,55%	133	31.540	109%	0,60%	0,59%
Best case	Best case	Best case	Most likely	0,63%	156	32.426	112%	0,70%	0,71%
Best case	Best case	Pessimistic	Best case	0,55%	130	30.689	107%	0,58%	0,56%
Best case	Best case	Pessimistic	Pessimistic	0,77%	152	25.361	88%	0,67%	0,51%
Best case	Best case	Pessimistic	Most likely	0,88%	179	26.247	91%	0,80%	0,64%
Best case	Best case	Most likely	Best case	0,63%	152	31.383	109%	0,68%	0,66%
Best case	Best case	Most likely	Pessimistic	0,88%	178	26.054	90%	0,79%	0,62%
Best case	Best case	Most likely	Most likely	1,00%	210	26.941	94%	0,94%	0,77%
Best case	Pessimistic	Best case	Best case	0,55%	125	29.514	103%	0,56%	0,51%
Best case	Pessimistic	Best case	Pessimistic	0,77%	145	24.186	83%	0,63%	0,45%
Best case	Pessimistic	Best case	Most likely	0,88%	171	25.072	86%	0,76%	0,57%
Best case	Pessimistic	Pessimistic	Best case	0,77%	140	23.336	79%	0,61%	0,41%
Best case	Pessimistic	Pessimistic	Pessimistic	1,07%	154	18.008	53%	0,57%	0,24%
Best case	Pessimistic	Pessimistic	Most likely	1,23%	184	18.894	58%	0,71%	0,34%
Best case	Pessimistic	Most likely	Best case	0,88%	165	24.029	82%	0,72%	0,51%
Best case	Pessimistic	Most likely	Pessimistic	1,23%	182	18.701	57%	0,70%	0,32%
Best case	Pessimistic	Most likely	Most likely	1,40%	217	19.587	62%	0,86%	0,44%
Best case	Most likely	Best case	Best case	0,63%	148	30.664	107%	0,67%	0,63%
Best case	Most likely	Best case	Pessimistic	0,88%	173	25.336	87%	0,77%	0,58%
Best case	Most likely	Best case	Most likely	1,00%	204	26.222	91%	0,91%	0,72%
Best case	Most likely	Pessimistic	Best case	0,88%	168	24.486	84%	0,74%	0,54%
Best case	Most likely	Pessimistic	Pessimistic	1,23%	186	19.158	60%	0,73%	0,35%
Best case	Most likely	Pessimistic	Most likely	1,40%	222	20.044	64%	0,90%	0,48%
Best case	Most likely	Most likely	Best case	1,00%	197	25.179	87%	0,87%	0,66%
Best case	Most likely	Most likely	Pessimistic	1,40%	220	19.851	63%	0,88%	0,46%
Best case	Most likely	Most likely	Most likely	1,60%	262	20.737	67%	1,08%	0,61%

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Volatility estimation using also existing products (cont.)									
Parcels & pallets	Global Express	E-commerce	Trucking	Cumulative probability of the scenario	Probability-weighted NPV at t=0	NPV at t=1	LN returns	Probability-weighted returns	Deviations*probability
Pessimistic	Best case	Best case	Best case	0,55%	83	20.409	66%	0,36%	0,20%
Pessimistic	Best case	Best case	Pessimistic	0,77%	86	15.081	36%	0,27%	0,07%
Pessimistic	Best case	Best case	Most likely	0,88%	104	15.967	41%	0,36%	0,11%
Pessimistic	Best case	Pessimistic	Best case	0,77%	81	14.231	30%	0,23%	0,04%
Pessimistic	Best case	Pessimistic	Pessimistic	1,07%	71	8.903	-17%	-0,18%	0,06%
Pessimistic	Best case	Pessimistic	Most likely	1,23%	89	9.789	-8%	-0,09%	0,02%
Pessimistic	Best case	Most likely	Best case	0,88%	97	14.924	35%	0,30%	0,07%
Pessimistic	Best case	Most likely	Pessimistic	1,23%	87	9.596	-10%	-0,12%	0,03%
Pessimistic	Best case	Most likely	Most likely	1,40%	109	10.482	-1%	-0,01%	0,01%
Pessimistic	Pessimistic	Best case	Best case	0,77%	74	13.055	21%	0,16%	0,02%
Pessimistic	Pessimistic	Best case	Pessimistic	1,07%	62	7.727	-31%	-0,34%	0,15%
Pessimistic	Pessimistic	Best case	Most likely	1,23%	79	8.614	-20%	-0,25%	0,08%
Pessimistic	Pessimistic	Pessimistic	Best case	1,07%	55	6.877	-43%	-0,46%	0,25%
Pessimistic	Pessimistic	Pessimistic	Pessimistic	1,50%	18	1.549	-192%	-2,88%	5,87%
Pessimistic	Pessimistic	Pessimistic	Most likely	1,72%	32	2.435	-147%	-2,52%	3,99%
Pessimistic	Pessimistic	Most likely	Best case	1,23%	69	7.570	-33%	-0,41%	0,19%
Pessimistic	Pessimistic	Most likely	Pessimistic	1,72%	29	2.242	-155%	-2,66%	4,43%
Pessimistic	Pessimistic	Most likely	Most likely	1,96%	46	3.128	-122%	-2,39%	3,18%
Pessimistic	Most likely	Best case	Best case	0,88%	92	14.205	30%	0,26%	0,05%
Pessimistic	Most likely	Best case	Pessimistic	1,23%	81	8.877	-17%	-0,21%	0,07%
Pessimistic	Most likely	Best case	Most likely	1,40%	102	9.763	-8%	-0,11%	0,03%
Pessimistic	Most likely	Pessimistic	Best case	1,23%	73	8.027	-27%	-0,34%	0,14%
Pessimistic	Most likely	Pessimistic	Pessimistic	1,72%	35	2.699	-136%	-2,34%	3,47%
Pessimistic	Most likely	Pessimistic	Most likely	1,96%	53	3.585	-108%	-2,12%	2,54%
Pessimistic	Most likely	Most likely	Best case	1,40%	91	8.720	-19%	-0,27%	0,09%
Pessimistic	Most likely	Most likely	Pessimistic	1,96%	50	3.392	-114%	-2,23%	2,79%
Pessimistic	Most likely	Most likely	Most likely	2,24%	72	4.278	-90%	-2,03%	2,07%
Most likely	Best case	Best case	Best case	0,63%	123	26.009	90%	0,56%	0,44%
Most likely	Best case	Best case	Pessimistic	0,88%	137	20.681	67%	0,59%	0,33%

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Volatility estimation using also existing products (cont.)									
Parcels & pallets	**Global Express**	**E-commerce**	**Trucking**	**Cumulative probability of the scenario**	**Probability-weighted NPV at t=0**	**NPV at t=1**	**LN returns**	**Probability-weighted returns**	**Deviations*probability**
Most likely	Best case	Best case	Most likely	1,00%	163	21.567	71%	0,71%	0,43%
Most likely	Best case	Pessimistic	Best case	0,88%	132	19.830	63%	0,55%	0,29%
Most likely	Best case	Pessimistic	Pessimistic	1,23%	136	14.502	32%	0,39%	0,08%
Most likely	Best case	Pessimistic	Most likely	1,40%	165	15.389	38%	0,53%	0,14%
Most likely	Best case	Most likely	Best case	1,00%	156	20.524	66%	0,66%	0,37%
Most likely	Best case	Most likely	Pessimistic	1,40%	163	15.195	36%	0,51%	0,13%
Most likely	Best case	Most likely	Most likely	1,60%	196	16.082	42%	0,67%	0,21%
Most likely	Pessimistic	Best case	Best case	0,88%	124	18.655	57%	0,50%	0,23%
Most likely	Pessimistic	Best case	Pessimistic	1,23%	125	13.327	23%	0,28%	0,04%
Most likely	Pessimistic	Best case	Most likely	1,40%	153	14.213	30%	0,42%	0,08%
Most likely	Pessimistic	Pessimistic	Best case	1,23%	118	12.477	17%	0,20%	0,01%
Most likely	Pessimistic	Pessimistic	Pessimistic	1,72%	97	7.149	-39%	-0,67%	0,34%
Most likely	Pessimistic	Pessimistic	Most likely	1,96%	124	8.035	-27%	-0,54%	0,22%
Most likely	Pessimistic	Most likely	Best case	1,40%	142	13.170	22%	0,31%	0,04%
Most likely	Pessimistic	Most likely	Pessimistic	1,96%	121	7.842	-30%	-0,58%	0,25%
Most likely	Pessimistic	Most likely	Most likely	2,24%	153	8.728	-19%	-0,43%	0,14%
Most likely	Most likely	Best case	Best case	1,00%	150	19.805	63%	0,63%	0,33%
Most likely	Most likely	Best case	Pessimistic	1,40%	155	14.477	31%	0,44%	0,09%
Most likely	Most likely	Best case	Most likely	1,60%	188	15.363	37%	0,60%	0,16%
Most likely	Most likely	Pessimistic	Best case	1,40%	146	13.627	25%	0,36%	0,05%
Most likely	Most likely	Pessimistic	Pessimistic	1,96%	128	8.299	-24%	-0,47%	0,18%
Most likely	Most likely	Pessimistic	Most likely	2,24%	161	9.185	-14%	-0,31%	0,09%
Most likely	Most likely	Most likely	Best case	1,60%	176	14.320	30%	0,49%	0,10%
Most likely	Most likely	Most likely	Pessimistic	2,24%	157	8.992	-16%	-0,36%	0,11%
Most likely	Most likely	Most likely	Most likely	2,56%	197	9.878	-7%	-0,17%	0,04%
Sum				100,00%	10.566			5,76%	49,37%
Volatility									**70,26%**

Source: Own work.

Appendix 6: Example 2 – Binomial tree of underlying movements of the underlying and of the call option values for each product

Binomial tree of the underlying movements - Express

t=0				**t=1**
	time steps between t=0 and t=1			
	1	2	3	4
2.253	3.278	4.770	6.940	10.097
	1.548	2.253	3.278	4.770
		1.064	1.548	2.253
			731	1.064
				503

Source: Own work.

Binomial tree of option values - Express

t=0				**t=1**
	time steps between t=0 and t=1			
	1	2	3	4
546	1.127	2.249	4.264	7.414
	143	350	855	2.087
		0	0	0
			0	0
				0

Source: Own work.

Binomial tree of the underlying movements - E-Commerce

t=0				**t=1**				**t=2**
	time steps between t=0 and t=2							
	1	2	3	4	5	6	7	8
1.471	2.140	3.113	4.530	6.590	9.589	13.952	20.300	29.537
	1.011	1.471	2.140	3.113	4.530	6.590	9.589	13.952
		695	1.011	1.471	2.140	3.113	4.530	6.590
			477	695	1.011	1.471	2.140	3.113
				328	477	695	1.011	1.471
					226	328	477	695
						155	226	328
							107	155
								73

Source: Own work.

Binomial tree of option values - E-Commerce

t=0				t=1				t=2
	time steps between t=0 and t=2							
	1	2	3	4	5	6	7	8
424	791	1.440	2.552	4.385	7.275	11.632	17.974	27.205
	171	342	672	1.287	2.390	4.270	7.263	11.620
		52	114	246	524	1.091	2.204	4.259
			9	22	54	131	320	781
				0	0	0	0	0
					0	0	0	0
						0	0	0
							0	0
								0

Source: Own work.

Binomial tree of the underlying movements - Trucking

t=0				t=1				t=2				t=3
	time steps between t=0 and t=3											
	1	2	3	4	5	6	7	8	9	10	11	12
1.308	1.903	2.769	4.029	5.862	8.530	12.411	18.058	26.274	38.228	55.621	80.928	117.750
	899	1.308	1.903	2.769	4.029	5.862	8.530	12.411	18.058	26.274	38.228	55.621
		618	899	1.308	1.903	2.769	4.029	5.862	8.530	12.411	18.058	26.274
			425	618	899	1.308	1.903	2.769	4.029	5.862	8.530	12.411
				292	425	618	899	1.308	1.903	2.769	4.029	5.862
					201	292	425	618	899	1.308	1.903	2.769
						138	201	292	425	618	899	1.308
							95	138	201	292	425	618
								65	95	138	201	292
									45	65	95	138
										31	45	65
											21	31
												15

Source: Own work.

Binomial tree of option values - Trucking

t=0				t=1				t=2				t=3
	time steps between t=0 and t=3											
	1	2	3	4	5	6	7	8	9	10	11	12
433	771	1.347	2.304	3.855	6.296	10.038	15.632	23.841	35.790	53.177	78.478	115.293
	199	373	685	1.234	2.170	3.717	6.184	9.979	15.619	23.829	35.777	53.165
		79	157	306	587	1.101	2.013	3.568	6.092	9.966	15.607	23.817
			25	53	112	231	471	939	1.824	3.418	6.079	9.954
				6	13	29	65	146	327	722	1.579	3.406
					1	1	4	9	21	52	128	313
						0	0	0	0	0	0	0
							0	0	0	0	0	0
								0	0	0	0	0
									0	0	0	0
										0	0	0
											0	0
												0

Source: Own work.

Appendix 7: Example 2 – Binomial tree of underlying movements of the underlying, of the early exercise option values and of the option values for the abandonment put option

Binomial tree of the underlying movements - abandonment put option

t=0				t=1				t=2
				time steps between t=0 and t=2				
	1	2	3	4	5	6	7	8
6.159	8.962	13.039	18.972	27.603	40.163	58.436	85.025	123.710
	4.233	6.159	8.962	13.039	18.972	27.603	40.163	58.436
		2.909	4.233	6.159	8.962	13.039	18.972	27.603
			2.000	2.909	4.233	6.159	8.962	13.039
				1.374	2.000	2.909	4.233	6.159
					945	1.374	2.000	2.909
						649	945	1.374
							446	649
								307

Source: Own work.

Binomial tree of early exercise option values

t=0				t=1				t=2
				time steps between t=0 and t=2				
	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0
			0	0	0	0	0	0
				626	0	0	0	0
					1.055	626	0	0
						1.351	1.055	626
							1.554	1.351
								1.693

Source: Own work.

Binomial tree of option values - abandonment put option

t=0				t=1				t=2
				time steps between t=0 and t=2				
	1	2	3	4	5	6	7	8
254	107	26	0	0	0	0	0	0
	357	164	44	0	0	0	0	0
		493	249	75	0	0	0	0
			666	371	127	0	0	0
				875	542	216	0	0
					1.110	771	368	0
						1.351	1.055	626
							1.554	1.351
								1.693

Source: Own work.