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OPTIMAL REINSURANCE: AN EXAMPLE OF A SELECTED LIFE INSURER

Ljubljana, July 2017 ECE KAYA

AUTHORSHIP STATEMENT

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TABLE OF CONTENTS

TABLE OF FIGURES

TABLE OF TABLES

INTRODUCTION

Every individual's goal is to minimize his potential losses during his life. In return for security and stability, he pays a small premium to an insurance company. Just like every individual seeks stability and security, every insurance company transfers part of its risk to other companies for stability and security, which are called reinsurance companies.

Firstly, marine commerce concluded the idea for insurance and reinsurance. The first marine insurance is back to before Christian era, whereas the first known reinsurance is from much later; in 1370, in Genoa (Swiss Re, 2002).

Fire insurance is the second insurance after marine insurance. The serious number of fires hit Hamburg between 1672 and 1676. This concluded as setting the *Hamburger Feuerkasse* (Hamburg Fire Fund) which is the oldest existing insurance company (Swiss Re, 2002).

The founding of the Amicable or Perpetual Assurance in London led serious actuarial development in 1706 with assessing risks and setting rates.

Other foundations from modern insurance industry are going back to the nineteenth century. Numerous of these foundations and insurance companies still exist in business today. Big developments in insurance industry included the rise of reinsurance. Reinsurance aimed at balancing portfolios on an international level.

Insurance companies insure their potential losses with the reinsurance companies under an agreement called reinsurance contract. This contract stipulates how the premium and the risk are split between two parties. One of the parties is the insurance company, called the insurer or the cedent. The other party is the reinsurance company, called the reinsurer. The transferred risk from the insurer to the reinsurer is cession.

Industrialisation created demand for reinsurance more than ever. Treaty reinsurance provides cover for portfolios (group of risks); facultative form of reinsurance provides coverage for a single-risk.

The first professional reinsurance company, Cologne Reinsurance Company, was emerged after another catastrophic fire event in Hamburg in 1842. 500,000 marks, the Hamburg Fire Fund had in total, was not enough to cover 18 million mark losses. As a result of this event, the insurance companies saw the need to distribute the risk arising from policies among risk carriers.

After Cologne Reinsurance Company, the following reinsurance companies were established:

- Aachen Reinsurance Company in 1853,
- Frankfurt Reinsurance Company in 1857,
- Swiss Reinsurance Company in 1863, and
- Munich Reinsurance Company in 1880.

Insurers have to estimate future losses in order to set premium rates. However, it is impossible to predict the exact loss amount. Insurance companies consider their insured persons as large group, and then assume that they are exposed to same risk and each loss is a separate event. In this case, it concludes the larger the group, the closer the average loss to a definite value. This result comes by “law of large numbers” (see Theorem 1) which is discovered by Jakob Bernouilli in 1700’s (Swiss Re, 2002).

A reinsurance contract is very important for both parties and must be constructed carefully. Both parties must take as much risk as they are able to carry and also maximize their profit to cover their expenses. One of the ways how they can maximize their profit is the right reinsurance contract (Liang & Guo, 2010).

A main purpose of reinsurance is to help insurance companies transfer their risk. Hence, we can say reinsurance is insurance for insurance companies. There are several reasons why an insurance companies need reinsurance. The first reason is the insurance company sells policies and gets more risk than it can keep. Then, the insurance company needs to transfer part of its risk to the reinsurance company. The second reason is that the insurance company develops products, but does not have enough technical knowledge to design or price them properly. The reinsurance company can provide assistance and in return, demands a share of the sales.

An insurance company can keep 100% of its risk arising from policies. The insurance company would be liable for all the claims. If the insurance company cedes all the risk, retains 0% of its risk arising from policies, the insurance company becomes a broker, sells policies and reinsures the whole portfolio covering the management expenses only. However, the insurance company can also retain some percentage of the risk. The challenge then becomes how much to keep. Every reinsurance and insurance company has a different risk appetite. The insurance companies are considering maximizing their profit by optimizing the reinsurance level.

If the insurance company keeps all the risk, in order to meet the future losses, insurance company is obliged to use its capital, the premium collected, and returns on the investment capital. Moreover, the behaviour of claims is a random, and fluctuates above and below the estimated probable loss. The insurance company can fix the level of liability per risk by reinsuring the portfolio. Then, the problem for the insurance company becomes how to set the retentions (Mapfre Re, 2013).

A reinsurance contract has significant benefits for insurance companies. The insurance company transfers part of risk arising from policies to the reinsurer. Therefore, it reduces the volatility of the underwriting results. Reduced volatility means less risk and less uncertainty for the insurance company, which results in lower required capital. The insurer uses the capital more efficiently with reinsurance, which allows offering a better price and a higher level of security to the policyholders (Raim & Langford, 2007). While both parties have the same goal to minimize greater losses, they both work together and help each other with providing information. Reinsurance companies have larger statistical databases; insurance companies provide feedback from the market. Insurance companies are helped by reinsurance companies' statistical data- if they do not have databases to estimate incident rates; and when the claim happens, insurance companies deliver claims' information to reinsurance companies which make reinsurance companies' data larger and more accurate.

Every insurance company must answer how much to reinsure. The answer depends on some factors such as willingness to take on risk, financial strength of the insurance company and market practice. However, reinsurance is not absolute solution against bankruptcy. Reinsurance is an instrument that helps insurance company to reduce the probability of ruin.

A life insurer buys a reinsurance contract to minimize its exposure to a high sum of claims and reduce mortality risk. The insurer is constantly looking for the opportunity to expand its business and increase its profit. The increment in the number of the policies will bring more risk to the insurance company. The main problem is to maximize profitability of the insurance company while having the optimal risk structure. The challenge regarding this contractual agreement is, however, to determine conditions which will ensure a less risky position with a higher profit to the insurer. If the insurer reinsures more than the optimal level, then the insurer will transfer a large amount of profit, and not expand its business as it wants. On the other hand, if the insurer reinsures less than it should, then the insurer will be exposed to catastrophic losses.

The first recorded reinsurance contract was written in Latin in July 1370 in Genoa. The cargo that was to be carried by ship from Cadiz to Sluis was insured. Due to the concern of possible huge claims, the insurer transferred part of the risk to another insurer, which today is called reinsurance (Mapfre Re, 2013).

The optimal reinsurance, from the insurer's perspective, was investigated in 1940's by the Italian mathematician de Finetti. De Finetti (1940) worked on optimal proportional reinsurance by minimizing the variance of the gain. Borch (1960) worked on optimal reinsurance by minimizing the variance of an insurer's retained loss, and assuming the reinsurance premium is calculated according to the expected value premium principle. His results show that the stop loss reinsurance is optimal Arrow (1963) shows that the stop loss reinsurance is optimal when the criterion is maximizing the expected utility of a risk-averse insurer. Gajek (2000), Zagrodny (2000) and Kaluzska (2001) consider the mean-variance premium principle additional to Borch’s (1960) research. Kaluzska and Okolewski (2008) show that stop-loss is optimal under the maximization of the expected utility, and the stability and the survival probability of the insurer. Cai and Tan (2007), Cai (2008), Tan (2009, 2011) and Cai and Tan (2011, 2013), analyzed the optimal reinsurance contract by minimizing the value at risk (hereinafter: VaR) and the conditional tail expectation (hereinafter: CTE) of the insurer’s total risk exposure.

Lampaert and Walhin (2005) worked on optimal proportional reinsurance, and its effect on the insurer’s balance sheet. Their research shows that quota-share reinsurance is suboptimal in comparison to the other types of proportional reinsurance for fire insurance. Verlaak and Beirlant (2003) have researched various combinations of reinsurance structures which are commonly used in practice. They show that the combination of reinsurance can change an insurer’s risk profile and profitability. Their research only allows variation of the sum insured in the portfolio, but not a range of different types of policies. In practice, insurers write policies with varying risk premiums, claims experience and reinsurance premiums. Reinsurance arrangements might be affected by this variability and the use of multiple retention levels. It may cause a different reinsurance agreement (Veprauskaite & Sherris, 2012).

Apart from relying on theoretical research, the desire in this study is to apply a practical and yet basic analysis of an optimal reinsurance contract for a life insurer. The research aims to seek higher profit with lower risk by using a proper reinsurance contract. The objective function will be to maximize the profit of the insurer with given risk premiums and incident rates, subject to different retention levels of each type of life insurance, and constrained by 99.5% Value at Risk of the expected loss that will not exceed some constant number.

In this study, the model will be constructed to show the reinsurance impact on the insurance company’s risk and profitability, for each type of life insurance, by changing the retention levels with obtained data. Then, the results will be compared in order to find the most suitable retention levels which bring the most profit to the selected life insurer. It is, however, possible that the best contract for the insurance company will not be accepted by the reinsurer.

1 LIFE INSURANCE

Every individual is exposed to unforeseen uncertain events. The person can be protected against these uncertain events by insurance. Insurance is financial compensation for future losses arising from such misfortunes.

According to International Financial Reporting Standard 4 (International Financial Reporting Standard 4, p. 10), “An insurance contract is a contract under which one party (an insurer) accepts significant insurance risk from another party (a policyholder) by agreeing to compensate a policyholder for his losses if a specified uncertain future event (an insured event) adversely affects the policyholder.", and “an insured event is an uncertain future event that is covered by an insurance contract.”

The agreement on the insurance policy is a contractual relationship between the insured person and the insurance company (the insurer). This contractual relationship guarantees to the insured person some amount of money (the sum insured) when the specified event in the contract occurs, in exchange for the payments, called the premium, from the insured person to the insurance company.

There are many effects of reinsurance on the insurance company. The reinsurance reduces the probability of the insurer’s ruin by assuming unexpected amount of claims, stabilizes the insurer’s balance sheet by taking part of its risk. When the reinsurance company takes part of the risk, the insurance company decreases the fluctuation of risk. Reinsurance enlarges the insurer’s underwriting capacity by accepting a proportional share of the risks and by providing part of the necessary reserves. It also increases the amount of available capital for the insurer by freeing equity that is tied up to cover risks (Swiss Re, 2002).

Insurance provides protection against various types of losses with different insurance types. One of them, life insurance, is the main subject of this study.

Life insurance is a protection that helps to reduce the effects of bad events while making sure income is not lost. Life insurance compensates the financial loss and gives income protection to families. Young families are protected against suffering from the death of the household’s head. Today, depending on the type of life insurance, life insurance provides payments after surviving a specified period, pays the debts after the insured person’s death (such as transferring the payment to the bank for the loan debt of the insured person), fulfils the economic goals of the family (such as sending children to the university), compensates for the loss of income during the insured person’s disability.

The insuree is a person on whose life the policy is based. It is not necessary that the insured person is also the beneficiary. The beneficiary is the person who receives the payment. For instance, an insured person who has a term policy is not the beneficiary. When the insuree dies, beneficiary will receive the payment. The policyholder is the person who is responsible for payments of premium. It is not necessary that the policyholder is the insuree. For instance, a parent can buy insurance policy for the child. In this case, a parent is the policyholder, and the child is the insured person. Coverage is provided by the insurer. The insurer is the insurance company that issues the policy.

The insurer needs to determine how much to pay when the insured dies, in other word, the value of the insured. Since human life value is difficult to determine, the sum insured is based on the individual's expected net future payments.

There are various types of life insurance such as whole life, term insurance, accidental death, critical illness insurance, short term (temporary) and long term (permanent) disability.

1.1 Term insurance

Term insurance is the most common life insurance type. It gives protection to the beneficiary against the financial loss arising from death of the insured person for a determined number of years. The sum insured is payable only if death occurs within the agreed period. If the insured person survives the agreed period, the insurance company does not have any obligation.

Term policies do not build cash value, which means the beneficiary cannot receive savings when the insured person dies. The maximum term policy period is 30 years whereas the usual term period is 20 years. In case the insured person wishes to renew the contract, it will be more expensive for him since the probability of death is going to be higher. For term insurance, the older the insured person, the higher the price of the policy.

1.2 Critical illness insurance

Critical illness insurance may be the most important type of life insurance. Health problems can cause big financial problems.

Critical illness provides a benefit with the sum insured to be paid if the insured person is diagnosed with specific illness that must be one of the predetermined illnesses from the insurance policy. It provides cash to pay medical treatments, in exchange for premium. The covered illnesses are usually:

- Heart attack,
- Stroke,
- Cancer,
- Deafness,
- Coronary artery by-pass surgery,
- Heart, lung, liver transplantation,
- Kidney failure.

The first critical illness insurance was invented by a physician, Marius Barnard in South Africa in 1985. Barnard saw the need for people to be protected from the effects of illnesses. Furthermore, South African insurers saw the problem and provided a service for persons who needed financial support such as having an unpaid mortgage or having a child to be sent to college (Lotter, 2010).

The insured person knows more about himself than the insurance company does. In order to avoid the anti-selection problem, the underwriting of critical illness focuses on personal medical history, family medical history and the present risk factors (smoking, diabetes, etc.). Hence, pricing factors become age, smoking status, gender, occupation, residence, family history, medical history.

When South Africa invented critical illness insurance in 1985, there was no data to estimate the incident rates. Fortunately, actuaries working for reinsurance companies had used techniques with available data for general population in order to calculate the incident rates for critical illnesses. However, those methods are still neither well-known nor widely understood (Lotter, 2000).

1.3 Accidental death and dismemberment

Accidental death insurance covers death that results from an accident or external violence. There might be some exclusion clauses for the causes of the death such as war, illegal activities and extreme hobbies.

Most of accidental death policies include coverage for dismemberment resulting from accident. Some of the usual causes covered under AD&D (accidental death and dismemberment) are burns, catastrophic accidents, concussion, eye injury, fracture, and loss of a finger or a toe, loss of a hand or a foot.

In order to price an AD&D contract, the analysis of the claim data is the best option. Low rates of the incidents on accidental death and dismemberment cause problems to the insurer. Therefore, many insurance companies are getting help from the reinsurers who have more data on claims.

1.4 Disability

Disability insurance is protection against loss of physical ability to work due to illness or injury. The number of payments, usually limited, depends on the contract.

Hospital cash is one of the types of coverage when the insured person has to stay in hospital for recovery. The insurance company covers expenses from the first day until the last day of hospital unless the cause is pregnancy, alcohol or drug rehabilitation treatment.

Income protection covers the loss of income due to inability to work. The problem with this coverage is moral hazard, which means the insured person may pretend being unable to work, and receive the benefit from the insurance company during the period of disability.

Total permanent disability is coverage for being unable to look after oneself. The insured person must provide a medical report proving that there will be no improvement throughout his/her life. The insured person is considered not to be able to take a shower (including getting into bath and out), feed himself, get between rooms, and move in and out of the bed.

2 REINSURANCE

Reinsurance is a contractual agreement that provides a service to the insurer transferring potential losses arising from the policies to the reinsurer. However, there is no contractual relationship between the insured person and the reinsurer.

The insurance company, the insurer, is called cedent or ceding company since it “cedes” part of the risk. The act of ceding risk is “cession”. The reinsurance company is called reinsurer. The reinsurer can also buy its own protection which is called “retrocession”.

The relationship between the cedent and the reinsurer is based on a contract which has all the negotiated details written down. Insurance company benefits from the reinsurance contract by stabilizing its own risk, reducing its expected loss and having a financial growth. The cedent can issue more policies after transferring part of its own risk, or the policy can be cheaper for the insured person, due to the decrement in capital requirements for the insurance company due to reduced risk (Raim & Longford, 2007).

The insurer receives the premium from the policyholder on the basis of the insurance contract (policy), which increases the assets of the insurance company and simultaneously causes an increment in the insurance company’s liability. In order to cover the losses, the insurance company is obliged to set aside reserves or reduce the liability by transferring part of the risk to the reinsurance company. By reinsuring risk, the insurer does not only transfer part of the liability, but also part of the premium that is received from the insurees. Then, the question becomes how much risk the insurance company should transfer.

Advantages of reinsurance are as follows:

- Statistical data: If the insurance company does not have enough historical claims to fit a distribution and estimate future expected losses; the reinsurer is a great help. Reinsurance companies have extensive databases that can assist the insurer to obtain more accurate incident rates. The insurer may use the reinsurer's incident rates based on their relationship through the reinsurance contract.
- Risk prevention: As mentioned before, the insurance company can prevent large expected losses and profit fluctuation. The insurance company becomes less risky, which makes the insurer more secure.
- Claim handling and premium rate determining: Reinsurance motivates the insurers to reduce pricing risk with the reinsurance company's larger expertise in the sector. The insurance company can calculate the risk of the insured person more quickly and economically with assistance of the reinsurance company, and then decide whether or not to take the risk.
- Providing advice, ideas, models, programs and rates: An insurance company can use reinsurance assistance for building models and also obtaining more accurate incident rates. The reinsurance company may advise on loss prevention based on its technology and expertise in the field. It can provide the insurer with education through conferences.

The main disadvantage of reinsurance is the cost of the contract to the insurance company. Then, the question becomes: is reinsuring the risk worth it? Small insurance companies tend to over-reinsure their portfolio since they cannot absorb large losses. If the insurance company purchases more reinsurance than needed, the insurer accepts transferring a large portion of the expected profit while transferring the risk to the reinsurer. This prevents the insurance company from growing. On the other hand, if the insurance company buys less reinsurance than it should, then it becomes more risky, which leads to larger capital requirements for the protection of the insured person. The insurance company has less economic stability since uncertainty -the future loss fluctuation- increases (Mapfre Re, 2013).

There are two different types of reinsurance contracts. One of them, called facultative reinsurance, depends on individual risk or group of specific risks. The second reinsurance type is depending on the class or the classes of risk and is called treaty reinsurance.

2.1 Facultative Reinsurance

This type of contract is used for contracts with a high sum insured. Facultative reinsurance is the oldest method of reinsurance. The insurance company defines all the details in a contract for the reinsurance company to transfer part of a single risk. If the reinsurance company agrees taking the risk, then the risk is shared. Large factories, ships, aircraft are the examples where individual losses are huge. Therefore, this type is not used for life business.

Facultative insurance can be proportional or non-proportional.

The insurance company transfers large risks; and facultative reinsurance significantly reduces the chance that the insurance company will become insolvent. The reinsurance company might take all of the risk, just a proportion of it, or decline to participate (Bellerose & Paine, 2003).

However, the disadvantages of this reinsurance type are as follows:

- Since the reinsurance companies are international, the insurer has to find a reliable reinsurance company, and give full information about the risk to them. The agreement is complex and expensive.
- The reinsurance company needs to measure the risk and decide whether or not it is worth taking it. The procedure takes time (Mapfre Re, 2003).

2.2 Treaty Reinsurance

Treaty reinsurance is the most common form of the reinsurance that can cover the loss proportionally or non-proportionally. If the treaty is proportional, the insurance company transfers the proportion of the premiums to the reinsurance company, and the reinsurance company covers the proportion of the loss. The treaty contract encompasses all types of risks.

2.2.1 Proportional treaty

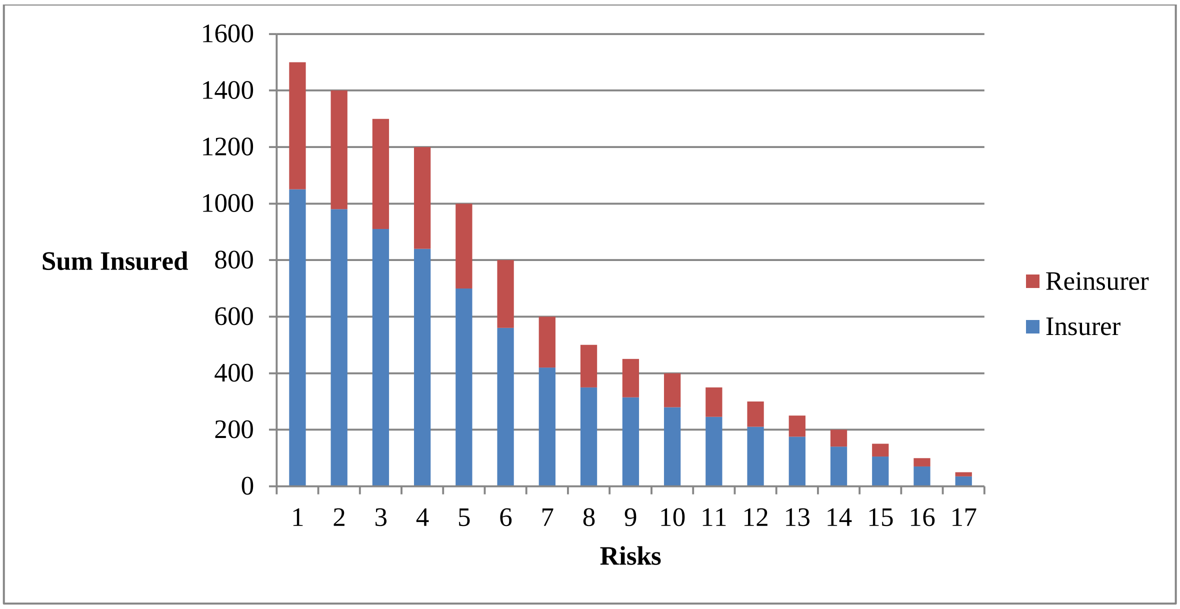
Proportional treaty reinsurance covers the proportion of claims that is between 0% and 100% of the loss for the whole risk ceded to it. This percentage is called cession.

The treaty creates a partnership between the cedent and the reinsurance as both of them participate in the loss. The participation amount depends on the cession rate. This rate is set by the insurer, depending on the risk appetite of insurance companies.

2.2.1.1 Quota Share

This type is the simplest reinsurance type among all. The ceded amount to the reinsurance is a percentage. The reinsurance company takes the same percentage of premium and covers the same percentage of the loss. Since both sides are affected by losses, it is quite often that the contract includes a profit sharing agreement, in order to motivate the insurer to conduct a better underwriting.

Figure 1. Quota share proportional reinsurance with 30% cession



Source: Swiss Re, *Proportional and non-proportional reinsurance*, 1997, p. 7.

Figure 1 illustrates how quota share reinsurance works for the reinsurer and the insurer. Each column represents the sum insured for the insured person. Blue part of the column is how much the insurer is liable to cover; whereas red part of the column is how much the reinsurer is liable to cover. The cession rate is 30% which means that 70% of the sum insured is covered by the insurance company, and the insurance company cedes 30% of the risk to the reinsurance company.

2.2.1.2 Surplus

Surplus is another proportional type of reinsurance treaty. Here, in order to calculate cession rate, the insurer sets his retention level as the maximum amount the insurance company can keep. The proportion of the retention level over the sum insured becomes the retention rate. Retention rate is a portion. Therefore, when the claim occurs, the insurer is obliged to cover the defined portion of the claim. Also, the cedent and the reinsurer share the premium with this rate. One minus retention rate is the cession rate.

For instance, Figure 2 illustrates how surplus reinsurance works. In the figure, each column represents the sum insureds for each insured person. The insurance company set the retention level to 300 euro, and the reinsurance agreement is with 3 lines. It means that the insurance company covers 300 euro of the loss. Then, the reinsurance company covers the entire loss up to 900 euro. If there is any retain loss, the insurance company is liable to cover the rest. As Figure 2 shows, the blue parts are how much the insurer cover; whereas the red part represents the reinsurer's portion in the coverage. In case the loss is more than 1200 euro, the insurance company has to cover the rest.

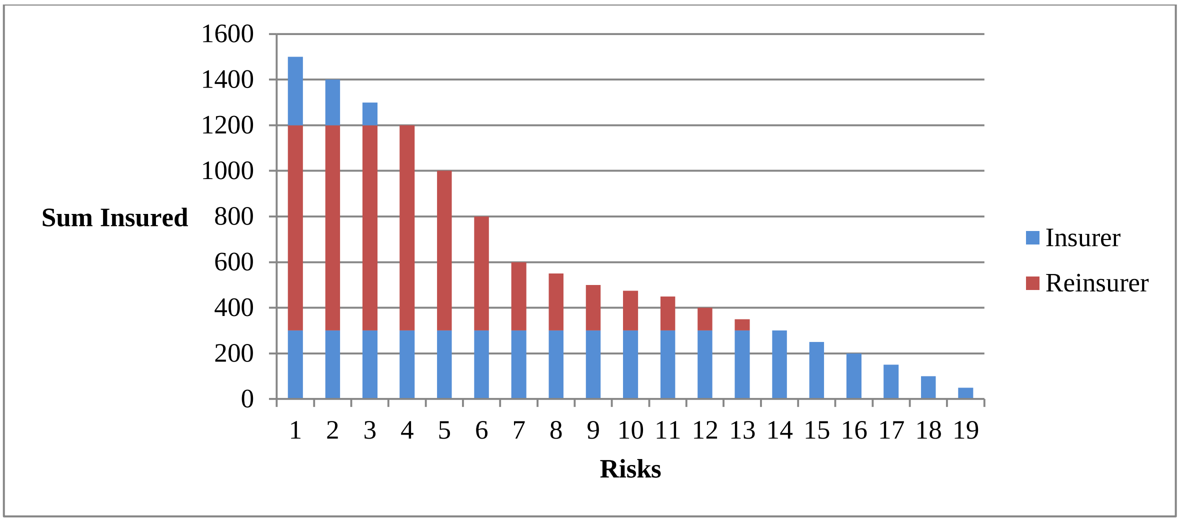
Surplus is a proportional reinsurance type. After finding how much the reinsurance company has to cover, the cession rates can be calculated. In order to achieving the cession rates, we take the proportion of how much the reinsurer covers out of the sum insured. This proportion defines the cession rate. The entire percentage is retention rate, which is the proportion of the sum insured that the insurance company is liable to cover.

Table 1. The cession rate for surplus proportional reinsurance

Sum Insured	**Insurer**	**Reinsurer**	**Insurer**	**Cession rate**
1500	300	900	300	0.6
1400	300	900	200	0.642857
1300	300	900	100	0.692308
1200	300	900	0	0.75
1000	300	700	0	0.7
800	300	500	0	0.625
600	300	300	0	0.5
550	300	250	0	0.454545
500	300	200	0	0.4
475	300	175	0	0.368421
450	300	150	0	0.333333
400	300	100	0	0.25
350	300	50	0	0.142857
300	300	0	0	0
250	250	0	0	0
200	200	0	0	0
150	150	0	0	0
100	100	0	0	0
50	50	0	0	0

Table 1 shows an example to illustrate the connection between the cession rate, what the insurer covers, what the reinsurer covers and the sum insured. The first column is the sum insured for each insured person, the second column is how much the insurer covers, and the third column is how much the reinsurer covers. After applying cession to reinsurance agreement, if there is any loss left to cover, then it becomes the insurer's liability. Those left losses are defined in the fourth column. Then, the cession rate is how much the reinsurer cover divided by the sum insured. The retention rate can be obtained as one minus cession rate, or how much the insurer covers over the sum insured. Both ways give same result.

Figure 2. Surplus proportional reinsurance, 3 lines with 300 euro



Source: Swiss Re, *Proportional and non-proportional reinsurance*, 1997, p. 8.

2.2.2 Non-proportional reinsurance

Non-proportional reinsurance is when the insurer decides not to cover the loss exceeding some amount. The claim can be individual risk, whole portfolio risk or specific event risk. Reinsurance is based on the claim amount not a ratio. Insurance companies buy this reinsurance when they want to avoid excessively high claims. Non-proportional reinsurance contracts are, contrary to the proportional contracts, not based on the sum insured, but the actual loss amount.

The difficulty of non-proportional reinsurance for the cedent is to set the retention level.

2.2.2.1 Excess of loss per risk

The basic type of non-proportional treaty is excess of loss. Here, the insurance company sets the retention- the maximum amount the cedent is willing to cover - if the claim is lower than the retention, the cedent covers all the loss. If the claim is higher than the retention, the cedent covers only the maximum amount (retention) and the reinsurance company covers the rest.

The reinsurance premium is not proportional to the premium charged by the insurer. It is calculated as the probability of having a claim above the retention level and the cost of capital instead.

2.2.2.2 Excess of loss per event (Catastrophic excess of loss)

The same event, most commonly earthquake, flood, windstorm and hail, may cause multiple losses arising from more than one policy. The cedent needs excess of loss per event treaty in order to avoid such a big loss.

2.2.2.3 Stop loss

The insurance company sets certain ratio limit and calculates its own loss ratio within a year; the reinsurance covers the excess of certain ratio limit. The loss ratio is

$$LR = \frac{Net\ incurred\ claim}{Net\ earned\ premium} \tag{1}$$

For instance, if the loss ratio in the end of a year is greater than the cedent's retention ratio, then the reinsurer covers the difference between the percentage of the retention ratio and loss ratio. Otherwise, the reinsurer is not obliged to cover any loss. This type of treaty is the most commonly used for livestock insurance.

Stop loss is the best and the most expensive reinsurance contract. It is used in non-life insurance sector.

3 THE MODEL

Data for optimal reinsurance analysis has been provided by the life insurer. The data has the sum insured, number of policies and the claim ratios for each type of life product, and in addition the safety margin and technical premium rates for accidental death policies. In order to analyze the disability portfolio, we have historical disability losses. The historical data include the sum insured and the loss amount. We take the ratio between the loss and sum insured. Therefore, we can estimate the severity of future claims.

For term, accidental death and critical illness policies, we have estimated the probability of a claim occurring. The incident rates are estimated differently by the insurer and the reinsurer. Additional to what we received from the life insurance company, by using sum insured and the insurer's and reinsurer's incident rates, we find the risk premiums for the insurer and the reinsurer for each line of business. Also, we calculate the expected losses with the Monte Carlo simulation by using the incident rates, the claim ratios and the sum insured. Bernoulli distribution (see Section 3.7) is used with the incident rates and the claim ratios for Monte Carlo simulation, in order to bring us the scenarios whether or not the claims happen.

For disability policies, we fit the claim data into distribution in order to find severity of the loss. After finding the severity distribution of disability (the distribution of how severe the future claims are. See Section 3.9), we also find the mean and the standard deviation of this distribution. Then, the expected loss for disability is modelled as the Bernoulli distribution of the incident rates and the claim ratios; the sum insureds and randomly generated values from the distribution of severity with the given mean and standard deviation. In the end, it is possible to show the distribution of the risk premium, the incident rates and the total expected loss arising from each business lines.

After collecting all we need, we can find the expected profit. The net income of the insurer is calculated as the risk premium of the insurer minus the risk premium of the reinsurer. The expected loss is the net income of the insurer minus the expected loss of the insurer. The risk premium of the reinsurer and the expected loss of the insurer depend on the retention levels. The retention level affects how much risk and premium the insurer retains

Before introducing the model, we need to define the parameters of different insurance types with different reinsurance contracts.

3.1 Term Insurance

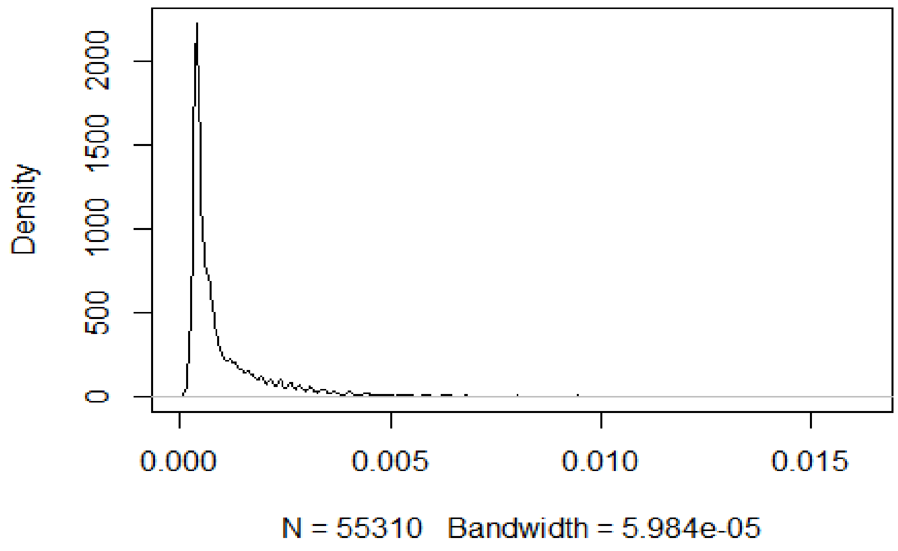
As the optimization problem in this study is for a life insurer, mortality and morbidity rates are the biggest concern.

The insurer and the reinsurer have their own mortality tables based on claim history. The mortality rates the insurance company is using can be denoted by q_{T_x} (for the distribution of it, see Figure 3), and the mortality rates from the reinsurance company are $q_{RE_{T_x}}$, for each person x who has a term policy. Those values are obtained by modeling mortality rates with the personal information of the insured people. The relevant factors for modeling the mortality are commonly age, gender and smoker/ non-smoker.

The incident rates for term insurance policies are actually the mortality rates (death rates), which is the probability of the insured person dying. The insurance company has its own mortality tables, and uses them in order to define the probability of insured's person dying. These probabilities are between 0 and 1.

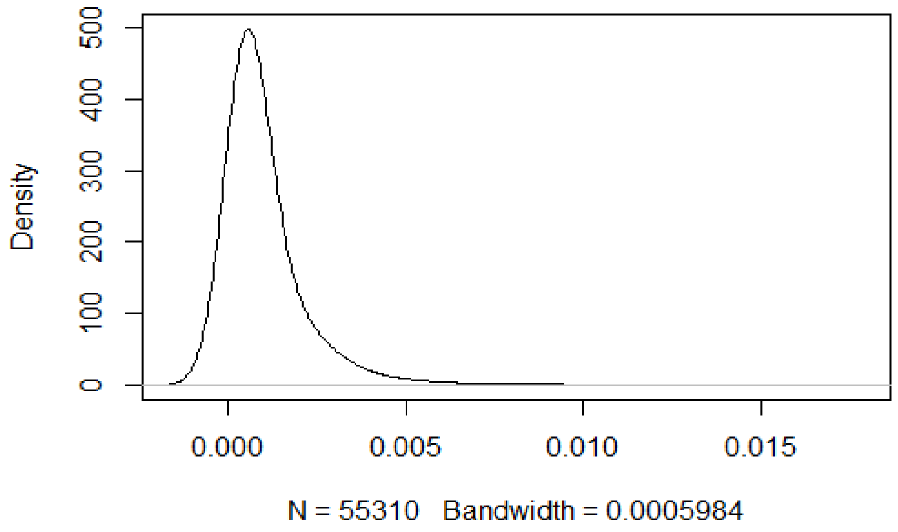
Figure 3 shows the distribution of the incident rates arising from term policies. The horizontal (x axis) values are the actual values of the data. The smaller the probability, the lower chance the insured person dying. The vertical (y axis) values are the density of the horizontal values. N is the number of term policies, and bandwidth is a compromise between smoothing enough to remove insignificant bumps and not smoothing too much to smear out real peaks (Venables & Ripley, 2002).
Plotting the distribution of mortality rates arising from term policies is to illustrate which kind of portfolio we have with the data. This distribution of mortality rates is not used in the following calculations.

Figure 3. Distribution of mortality rates arising from term policies



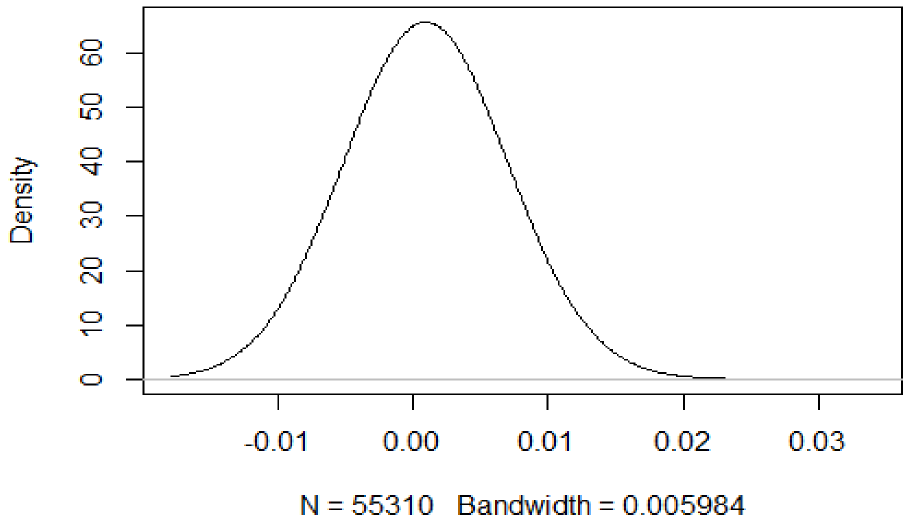
The program chooses the bandwidth value by itself. However, Figure 4 and Figure 5 show the effect of changing bandwidth. The distribution picture is smoother in Figure 5. However, we can see that mortality rates goes below zero. The probability of someone dying cannot be negative. That is why Figure 5 is not an improved version of Figure 4. Therefore, Figure 5 can look smoother, yet Figure 4 is better with bandwidth set by the program itself.

Figure 4. Adjusted bandwidth of mortality rates



Let us give an overview of the following calculations. We received the sum insured per policy. We need to find the portion of the reinsurer's liability to the insurer in case of loss. The entire portion goes to the insurer's liability to cover. We will define this liability amount with different reinsurance type. Also, we will calculate the risk premium of the insurer and the reinsurer. The risk premium will be considered as an income to both parties. Then, we will estimate the expected loss arising from the policies. The covered loss amount by the insurer will be different for different type of reinsurance.

Figure 5. Adjusted bandwidth of mortality rates



3.1.1 Sum insured with quota share treaty

For the calculations, we use the data that we received from the life insurer. As it was stated before, we have the sum insured for each policy for each business line. We do the following calculations based on quota share reinsurance type. In quota share, the reinsurance has to be stated in terms of the cession rate. This cession rate is the portion of ceded risk from the insurer to the reinsurer. We calculate how much the reinsurer is liable to cover in case of loss, with the cession rate and the sum insured of each policy. Then, the insurer is liable to cover the entire loss.

Let us define the cession rate as α (hereainfter α is the *cession rate)*, the sum insured (hereinafter: *SI*) as *SI*. Then, the reinsurer's sum insured is as follows:

$$SI_{RE} = \alpha \sum_{i=1}^{N} SI_i \tag{2}$$

Additionally, SI_i is the sum insured for a person i (hereinafter:SI_i), in where reinsurance participates with quota share reinsurance and N is the number of the policies in the portfolio. The retained sum insured part is as follows:

$$SI_I=(1-\alpha)\sum_{i=1}^{N}SI_i \quad (3)$$

Table 2 shows the sum insured for each type of insurances with quota share reinsurance.

Table 2. The sum insured for each type of insurances with quota share reinsurance

	Sum insured for the cedent	Sum insured for the reinsurer
Term	$SI_{I_T}=(1-\alpha_T)\sum_{i=1}^{N_T}SI_{T_i}$	$SI_{RE_T}=\alpha_T\sum_{i=1}^{N_T}SI_{T_i}$
Critical Illness	$SI_{I_{CI}}=(1-\alpha_{CI})\sum_{i=1}^{N_{CI}}SI_{CI_i}$	$SI_{RE_{CI}}=\alpha_{CI}\sum_{i=1}^{N_{CI}}SI_{CI_i}$
Accidental death	$SI_{I_{AC}}=(1-\alpha_{AC})\sum_{i=1}^{N_{AC}}SI_{AC_i}$	$SI_{RE_{AC}}=\alpha_{AC}\sum_{i=1}^{N_{AC}}SI_{AC_i}$
Disability	$SI_{I_D}=(1-\alpha_D)\sum_{i=1}^{N_D}SI_{D_i}$	$SI_{RE_D}=\alpha_D\sum_{i=1}^{N_D}SI_{D_i}$

3.1.2 Sum insured with surplus treaty

The insurance company decides how much risk the insurance company is willing to take. Let us denote the amount by *M*. The retention rate becomes the proportion between the retention level and sum insured. If the retention level is greater than the sum insured, it refers that the insurance company is liable for whole losses arising from i^{th} policy.

The retention and cession rate cannot be greater than 1. The insurance company cannot be liable to cover more than 100% of the loss. If the retention rate is zero, that means the retention level is 0, the insurance company is not willing to take part of the risk. Then, the insurance company's share can be denoted as follows:

$$\alpha_i=min(1,\frac{M}{SI_i}) \quad (4)$$

The cession rate becomes $1-\alpha_i$. The cession rate being 1 represents that the insurance company does not participate to cover the loss, which means the insurance company cedes all the risk. The cession rate being 0 is that the insurance company does not cede any part of risk, and is liable for the risk.

If the retention is larger than the sum insured, then all (100%) the loss will be covered by the insurance company, which means there is no reinsurance. Otherwise, the ratio that will be covered by the insurance company is going to be $\frac{M}{SI}$.

Table 3 shows sum insured of the insurance and the reinsurance company for each type of insurance.

Table 3. The sum insured for each type of insurance with surplus reinsurance

	Sum insured for the cedent	**Sum insured for the reinsurer**
Term	$SI_{I_T}=\sum_{i=1}^{N_T} \min(1, \frac{M_T}{SI_{T_i}})SI_{T_i}$	$SI_{RE_T}=\sum_{i=1}^{N_T} \max\left(0,1-\frac{M_T}{SI_{T_i}}\right)SI_{T_i}$
Critical Illness	$SI_{I_{CI}}=\sum_{i=1}^{N_{CI}} \min(1, \frac{M_{CI}}{SI_{CI_i}})SI_{CI_i}$	$SI_{RE_{CI}}=\sum_{i=1}^{N_{CI}} \max\left(0,1-\frac{M_{CI}}{SI_{CI_i}}\right)SI_{CI_i}$
Accidental death	$SI_{I_{AC}}=\sum_{i=1}^{N_{AC}} \min(1, \frac{M_{AC}}{SI_{AC_i}})SI_{AC_i}$	$SI_{RE_{AC}}=\sum_{i=1}^{N_{AC}} \max\left(0,1-\frac{M_{AC}}{SI_{AC_i}}\right)SI_{AC_i}$
Disability	$SI_{I_D}=\sum_{i=1}^{N_D} \min(1, \frac{M_D}{SI_{D_i}})SI_{D_i}$	$SI_{RE_D}=\sum_{i=1}^{N_D} \max\left(0,1-\frac{M_D}{SI_{D_i}}\right)SI_{D_i}$

If the ratio between the retention level and the sum insured exceeds 1, then the insurance company covers the whole loss and the cession rate becomes 0. Otherwise, the cession rate is $(1-\frac{M}{SI_i})$ for the person i.

In surplus reinsurance, the cession rate differs from contract to contract; whereas quota share reinsurance has a fixed rate for each contract in the portfolio.

3.1.3 Sum insured with excess of loss

The insurance company sets the retention, the largest amount the cedent is willing to cover. If the claim is lower than the retention, the cedent covers the whole loss. If the claim is higher than the retention, the cedent covers only the retention amount, and the reinsurance company covers the rest.

Equation (5) shows the sum insured for the cedent, minimum of retention or the sum insured. It means if the sum insured is more than the retention, the insurance company covers only retention, and the rest is covered by the reinsurer. Equation (6) shows the sum insured for the reinsurer with excess of loss reinsurance. The amount that the reinsurer is obliged to cover is the difference between the sum insured and the retention. If the sum insured is less than the retention, the reinsurer is not obliged to cover any loss. *M* is the retention level for term insurance that is set by the cedent.

$$SI_I=min(M,SI) \tag{5}$$

$$SI_{RE}=max(0,SI-M) \tag{6}$$

Table 4 shows the sum insured from the insurer's and the reinsurer's perspective for term, critical illness, accidental death and disability policies.

Table 4. The sum insured for each type of insurances with excess of loss

	Sum insured for the cedent	**Sum insured for the reinsurer**
Term	$SI_{I_T}=\sum_{i=1}^{N_T} \min(M_T,SI_T)$	$SI_{RE_T}=\sum_{i=1}^{N_T} \max(0,SI_T-M_T)$
Critical Illness	$SI_{I_{CI}}=\sum_{i=1}^{N_{CI}} \min(M_{CI},SI_{CI})$	$SI_{RE_{CI}}=\sum_{i=1}^{N_{CI}} \max(0,SI_{CI}-M_{CI})$
Accidental death	$SI_{I_{AC}}=\sum_{i=1}^{N_{AC}} \min(M_{AC},SI_{AC})$	$SI_{RE_{AC}}=\sum_{i=1}^{N_{AC}} \max(0,SI_{AC}-M_{AC})$
Disability	$SI_{I_D}=\sum_{i=1}^{N_D} \min(M_D,SI_D)$	$SI_{RE_D}=\sum_{i=1}^{N_D} \max(0,SI_D-M_D)$

3.1.4 Evaluating risk premium after determining the sum insured

After defining the sum insured for the insurer and the reinsurer, we can calculate the risk premium for the insurer and the reinsurer. Equation (7) and Figure 6 show the risk premium of the insurer which is a sum of the sum insured and the probability of a person dying within a year. Let us denote SI_{T_i} (see Table 2, Table 3 and Table 4, depending on a type of reinsurance) as the sum insured of the i^{th} person who has term contract. RP_{T_i} is the risk premium for the insurer and N_T is the number of policies for term insurance.

$$RP_{T_i}=\sum_{i=1}^{N_T} SI_{T_i}*q_{T_i} \tag{7}$$

Figure 6 shows the distribution of the risk premiums arising from term policies. The horizontal (x axis) values are the risk premiums. The vertical (y axis) values are the density of the horizontal values. *N* is the number of term policies.

Equation (8) shows the risk premium of the reinsurer which can be written as the sum of the reinsurer's sum insured and the probability of a person dying within a year.

$$RP_{RE_T} = \sum_{i=1}^{N_T} SI_{RE_{T_i}} * q_{RE_{T_i}} \tag{8}$$

Figure 6. Distribution of the insurer's risk premium

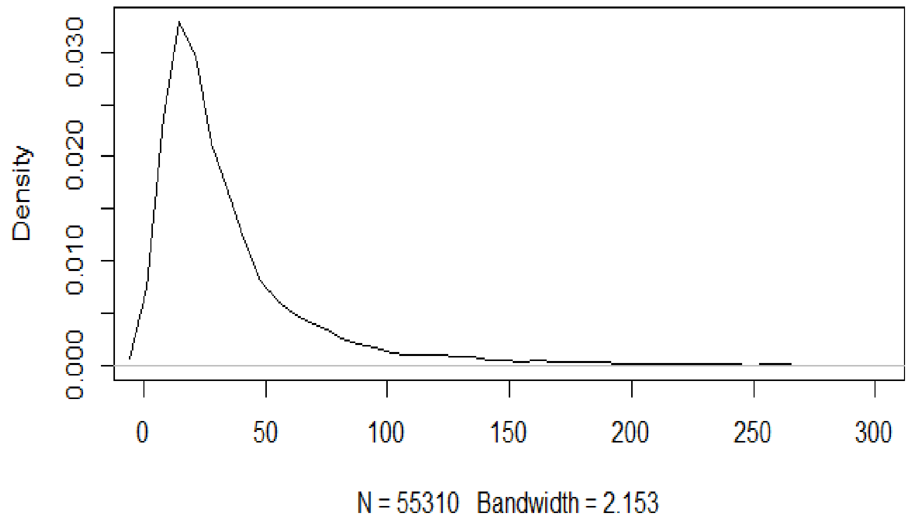
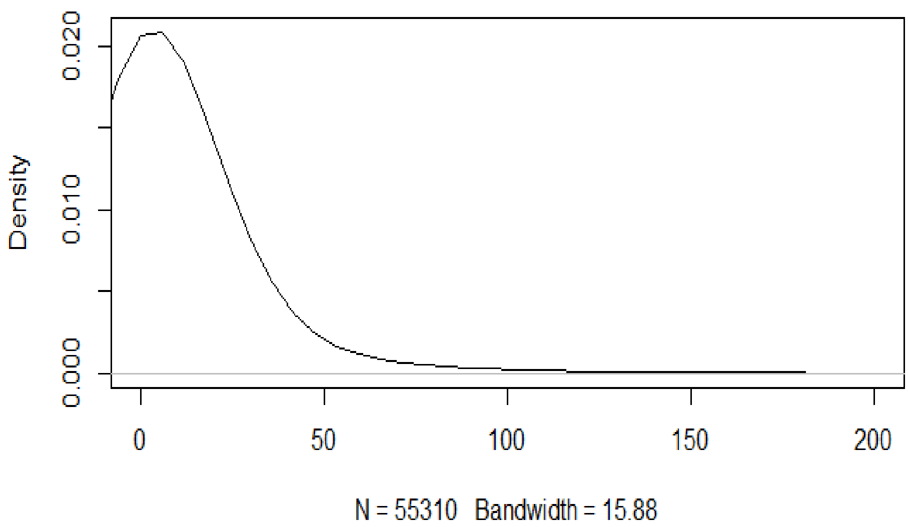


Figure 7 shows the distribution of the reinsurer's risk premium. The horizontal values are the risk premiums that the reinsurer receives from the each term policy, whereas the vertical values are the density of the reinsurer's risk premiums. *N* is the number of term policies.

Figure 7. Distribution of the reinsurer's risk premium



Equation (9) shows the net risk premium of the insurer for term insurance which can be obtained as the risk premium of the reinsurer subtracted from the risk premium of the insurer.

$$RP_{net_T}=RP_{I_T}\text{-}RP_{RE_T} \tag{9}$$

3.2 Critical Illness Insurance

For the model that we will construct in order to find optimal reinsurance, we will use the reinsurer's incident rates. Since the insurance company does not have enough claims in order to estimate incident rates, the reinsurer's rates are used for the insurer as well. The incident rates for critical illness can be denoted as q_{CI_x} and $q_{RE_{CI_x}}$, respectively for the insurer and the reinsurer, for each person x who has a critical illness policy. Those values are obtained by using personal information of the insured person such as age, gender and health status.

The risk premium of the insurer (see Equation (10)) is the sum of the sum insured multiplied by the probability of a person having illness.

$$RP_{I_{CI}}=\sum_{i=1}^{N_{CI}} SI_{CI_i} * q_{CI_i} \tag{10}$$

SI_{CI_i} is the sum insured of the i^{th} person that has a critical illness policy, depending on reinsurance type (see Table 2, Table 3, Table 4). Table 2, Table 3 and Table 4 show the sum insured from the insurer's and the reinsurer's perspective for term, critical illness, accidental death and disability policies with different reinsurance types.

$RP_{I_{CI}}$ is the risk premium for the insurer, N_{CI} is the number of the policies.

The risk premium of the reinsurer (see Equation (11)) is the sum of reinsurer's part of sum insured, depending on the reinsurance type (see Table 2, Table 3, Table 4), with the given incident rate.

$$RP_{RE_{CI}}=\sum_{i=1}^{N_{CI}} SI_{RE_{CI_i}} * q_{RE_{CI_i}} \tag{11}$$

$SI_{RE_{CI_i}}$ is the reinsurer's part of the sum insured from the i^{th} person's policy who has a critical illness contract, $RP_{RE_{CI}}$ is the risk premium for the reinsurer, N_{CI} is the number of the policies for critical illness insurance.

The net risk premium of the insurer (see Equation (12)) is the risk premium of the reinsurer subtracted from the risk premium of the insurer.

$$RP_{net_{CI}}=RP_{I_{CI}}-RP_{RE_{CI}} \tag{12}$$

3.3 Accidental Death and Dismemberment

For the model that we will construct in order to find optimal reinsurance, the insurer and the reinsurer have their own incident rates. The incident rates for AD&D can be denoted as $q_{x_{AC}}$ and $q_{x_{RE_{AC}}}$, respectively for the insurer and the reinsurer, for a person x. $q_{x_{AC}}$ is obtained by the technical premium rate, the safety margin and the claim loss ratio (see Equation (13)).

Insurance companies calculate the premium using the technical premium rate, the safety margin and the claim ratio. The safety margin ensures the long term viability to insurance companies, so that the insurer has some profit. The claim ratio is the payable claim amount as the percentage of the premium. It is calculated from the claim experience. The technical premium rate is the percentage that helps the insurance company to cover the expenses. Each business line has a different claim ratio, safety margin and technical premium rate, and all of them are in terms of percentage.

$$q_{x_{AC}}=q_{x_{tech}}(1+SM_x)CR_x \tag{13}$$

$q_{x_{tech}}$, SM_x, CR_x are respectively the technical premium rate, the safety margin and the claim loss ratio for the person x. The claim ratio is a constant percentage obtained from the claim history. The safety margin and the technical rate depend on the coverage type.

An increment in any of these three components will increase the insurer's risk premium as we can see in Equation (14).The risk premium of the insurer is the sum of the sum insured multiplied by the incident rate.

$$RP_{I_{AC}}=\sum_{i=1}^{N_{AC}} SI_{AC_i}*q_{AC_i} \tag{14}$$

$SI_{i_{AC}}$ is the sum insured, depending on the reinsurance type (see Table 2, Table 3 and Table 4), of the i^{th} person that has the accidental death and dismemberment policy, $RP_{I_{AC}}$ is the risk premium for the insurer, and N_{AC} is the number of the policies for accidental death of dismemberment insurance.

The risk premium of the reinsurer is the sum of the reinsurer's part of the sum insured with the incident rates (see Equation (15)). $q_{RE_{AC_x}}$ is determined by the reinsurance company.

$$RP_{RE_{AC}}=\sum_{i=1}^{N_{AC}} SI_{RE_{AC_i}} * q_{RE_{AC_i}} \tag{15}$$

$SI_{RE_{AC_i}}$ is the reinsurer's part of the sum insured, depending on the reinsurance type (see Table 2, Table 3,Table 4), of the i^{th} person that has AD&D policy, $RP_{RE_{AC}}$ is the risk premium for the reinsurer, N_{AC} is the number of the policies for AD&D insurance.

The net risk premium of the insurer is the risk premium of the reinsurer subtracted from the risk premium of the insurer (see Equation (16)).

$$RP_{net_{AC}}=RP_{I_{AC}}-RP_{RE_{AC}} \tag{16}$$

3.4 Disability

In the first three types of insurance, the risk premium rate is the same as the incident rates. However, disability has different risk premium and incident rates. The risk premium rates for the insurer are depending on the type of the disability product, and the rates are set by the insurance company as a feature of the product. The risk premium of the insurer (denoted as RP_{I_D}) is calculated by some loading with the sum insured of the insurer (SI_{D_x}), depending on reinsurance type (see Table 2, Table 3,Table 4; whereas the risk premium of the reinsurer (RP_{RE_D}) is evaluated from the determined rate and the sum insured of the reinsurer ($SI_{RE_{D_x}}$), depending on reinsurance type (see Table 2, Table 3, Table 4). The risk premium rate of the reinsurer is detailed by the reinsurance company under the reinsurance contract.

The net risk premium (see Equation (17)) of the insurer becomes the risk premium of the reinsurer subtracted from the risk premium of the insurer.

$$RP_{net_D}=RP_{I_D}-RP_{RE_D} \tag{17}$$

3.5 Estimating the Expected Loss for the Insurer

3.5.1 Law of large numbers

Law of large numbers theory is the fundamental theorem of probability. It has very important role in probability and statistics. The law says if you repeat an experiment with larger number of data, you will approach a definite value.

Theorem 1: (Law of Large Numbers): Let $X_1, X_2, \ldots, X_n$ be an independent trials process, with finite expected value $\mu = E(X_j)$ and finite variance $\sigma^2 = V(X_j)$. Let $S_n = X_1 + X_2 + \ldots + X_n$. Then for any $\varepsilon > 0$,

$$P\left(\left|\frac{S_n}{n} - \mu\right| \geq \varepsilon\right) \to 0 \tag{18}$$

as $n \to \infty$. Equivalently,

$$P\left(\left|\frac{S_n}{n} - \mu\right| < \varepsilon\right) \to 1 \tag{19}$$

as $n \to \infty$ (Snell & Grinstead, 2009).

Nowadays, the insurance companies make extensive database in order to use statistics to calculate the expected losses. Statistics are based on the past, but the laws of probability make it possible to apply data from the past to the present, and predict future trends (Swiss Re, 2002).

The law of large number theorem leads us to have large simulation number in Monte Carlo. Also, in the following sections, we will fit distribution into claims. Due to deficiency of claims, we will not have accurate statistical results.

3.5.2 Estimating the Expected Loss for the Insurer with Monte Carlo Simulation

Expected loss is value of loss multiplied by probability of loss occurring. It is calculated as probability of loss times the cost of the loss. For instance, the insurer’s expected loss for one term insurance policy is the probability of the insured person dying times how much the insurance company pays to the beneficiary (the present value of the sum insured). The expected loss of the whole term insurance portfolio is the sum of individual expected losses. The expected loss of the whole portfolio becomes the sum of all four types of insurance portfolios.

Let us define the total loss as Y and the total loss of the term policies as Y_T, critical illness policies as Y_{CI}, accidental death policies as Y_{AC} and disability policies as Y_D. Then, the total expected loss can be defined as in Equation (20):

$$E[Y] = E[Y_T + Y_{CI} + Y_{AC} + Y_D] \tag{20}$$

The expected value of sum can be written as the sum of expected values.

$$E[Y]=E[Y_T]+E[Y_{CI}]+E[Y_{AC}]+E[Y_D] \tag{21}$$

We assume that the probabilities of a claim are known for each insured person. Then, the expected loss, over one year, of term insurance is the sum of the incident rate and the sum insured for each policy (see Equation (22)).

$$Y_T=\sum_{i=1}^{N_T} q_{T_i} * SI_{T_i} \tag{22}$$

q_{T_i} is the probability of the i^{th} person dying and SI_{T_i} is the sum insured of the i^{th} person.

From the definition of expected loss, the expected loss of critical illness insurance is as follows:

$$Y_{CI}=\sum_{i=1}^{N_{CI}} q_{CI_i} * SI_{CI_i} \tag{23}$$

q_{CI_i} is the probability of the i^{th} person having one of the listed illnesses and SI_{CI_i} is the sum insured for the i^{th} person.

By the same reason, the expected loss of accidental death insurance can be obtained as in Equation (24).

$$Y_{AC}=\sum_{i=1}^{N_{AC}} q_{AC_i} * SI_{AC_i} \tag{24}$$

q_{AC_i} is the probability of the i^{th} person dying because of an accident and SI_{AC_i} is the sum insured of the i^{th} person.

Expected loss of disability insurance can be written as in Equation (25)

$$Y_D=\sum_{i=1}^{N_D} q_{D_i} * SI_{D_i} \tag{25}$$

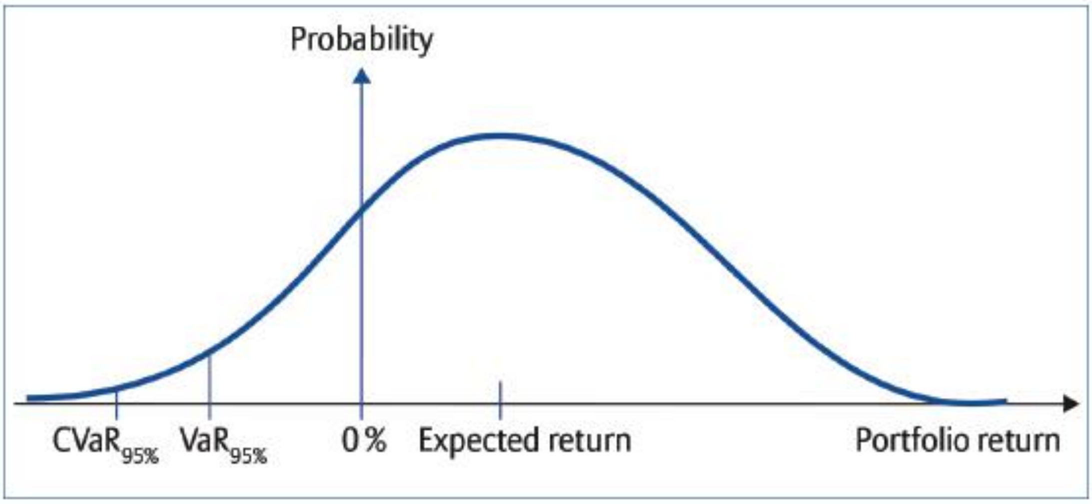
q_{D_i} is the probability of the i^{th} person becoming disabled and SI_{D_i} is the sum insured of the i^{th} person.

3.6 Value at Risk (VaR) and Conditional Value at Risk (CVaR)

Value at Risk (hereinafter: VaR) is a risk measure for financial companies to determine statistically how much they are likely to lose at a given confidence level. This confidence level means that we are α % sure that we will not lose more than $VaR_\alpha(Y)$ value.

Figure 8 shows the Value at Risk values in a sample of portfolio return.

Figure 8. Illustration of CVaR and VaR with a sample confidence interval of 95%



Source: N. Smith, *Volatility: Plotting a course*, 2013, p. 21.

Mathematical definition of VaR: Let Y be a random variable representing loss. Given a parameter $0<\alpha<1$,

$$VaR_\alpha(Y)=inf(y\in R : P(Y>y)\leq 1-\alpha) \tag{26}$$

Given the definition, VaR, can have several interpretations.

- $VaR_\alpha(Y)$ is the minimum loss that will not be exceeded with probability α.
- $VaR_\alpha(Y)$ is the α-percentile of the distribution of Y.
- $VaR_\alpha(Y)$ is the smallest loss in the $(1-\alpha)x100\%$ worst cases.
- $VaR_\alpha(Y)$ is the highest loss in the $\alpha\ x\ 100\%$ best cases.

The commonly used confidence levels (α) are 95%, 99%, 99.5%.

Mathematical definition of Conditional Value at Risk (hereinafter: CVaR): Let Y be a continuous random variable representing loss. Given the parameter $0<\alpha<1$,

$$CVaR_{\alpha}(Y)=E[Y|Y\geq VaR_{\alpha}(Y)] \tag{27}$$

CVaR is also known as Average Value at Risk, Expected Shortfall or Tail Conditional Expectation (Kisiala, 2015).

3.7 Bernoulli Distribution

When we toss the coin, we have two possible outcomes: having tail or head. Like tossing the coin, many of experiments have two outcomes. For instance a baby is a male or female, an egg is broken or not, a woman is pregnant or not. Bernoulli distribution is founded by Swiss mathematician Jakob Bernoulli (Scheaffer & Young, 2010).

Let us define X as a random variable.

$X=1$, if the claim occurs; $X=0$, if the claim does not occur. Then, the probability distribution of X is as follows:

$$p(x)=p^{x}(1-p)^{(1-x)}, \quad x=0,1 \tag{28}$$

Where $p(x)$ is the probability that $X=x$ (Scheaffer & Young, 2010).

3.8 Monte Carlo Simulation

The Monte Carlo method is repeated random sampling for modelling risk by computerized mathematical technique. It generates different scenarios with a given probability distribution. Accuracy of the simulation depends on the number of iterations; the larger number of repetitions the better the result. Each iteration result is recorded.

Monte Carlo simulation shows what can happen and how likely it is to happen. Because every iteration result is recorded, it is possible to graph the outcomes. It is also easy to examine the different combinations of the values with different inputs, in order to see how the changing values might affect the result. It may help to find out the correct path for the analysis.

3.9 Maximization Problem

As all companies, insurance companies' desire is to maximize their expected profit. Profit is a source for investors and the management; it has an important role for the insured person and the regulatory authorities, since it brings additional security against insolvency. Reinsurance has a significant role in profitability. Optimizing reinsurance will provide an increment in expected profitability. The insurance company transfers some part of the premium to the reinsurance company, and receives part of the claim amount if the claim occurs.

In order to obtain an optimal reinsurance contract we use a simple and practical approach. The risk premium of the insurance company is cost. The insurance company shifts some of the risk premium to the reinsurance company, in accordance with the reinsurance agreement. The ceded amount depends on the reinsurance contract. What is left with the insurance company is net income for the insurance company. Moreover, the expected total claim amount is an outgo. The subtraction of an outgo from a net income results in expected profit.

The objective function is maximizing the expected profit with the retention levels as variables. Equation (29) is our objective function.

$$\underset{st\ M_T, M_{CI}, M_{AC}, M_D}{Max} \{(RP_I - RP_{RE}) - E[Y]\} \tag{29}$$

M_T, M_{CI}, M_{AC}, M_D are the retention levels for term, critical illness, accidental death and disability policies, respectively. Those retention levels are the unknown parameters we are trying to find, which will result in maximum profit to the insurer. The risk premium of the cedent is the sum of the risk premiums arising from distinct insurances (see Equation (30)). The value is calculated from the policies and does not depend on the retention levels.

$$RP_I = RP_T + RP_{CI} + RP_{AC} + RP_D \tag{30}$$

The risk premium of the reinsurer is what the reinsurance company receives from the insurance company for reinsuring the portfolio. This amount relies on the retention level; it changes with different retention levels. The total risk premium of the reinsurer (see Equation (31)) is the sum of the risk premium from each insurances type.

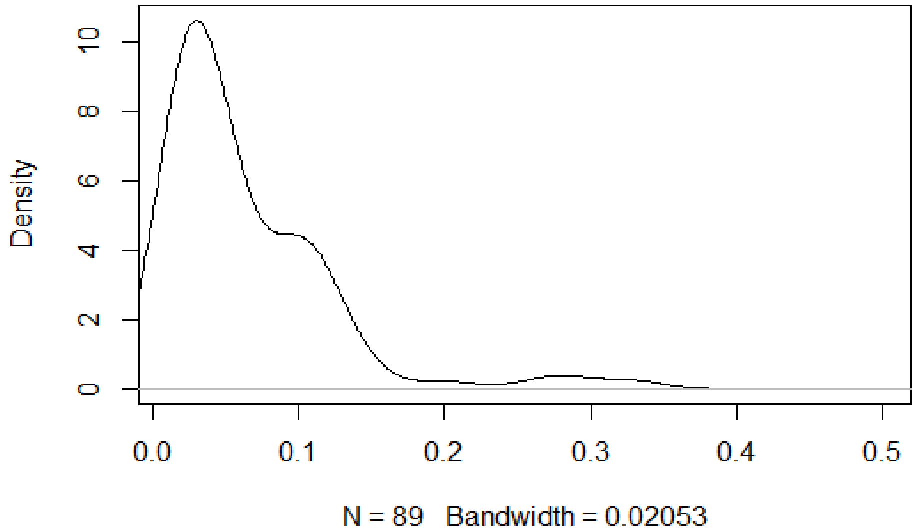
$$RP_{RE} = RP_{RE_T} + RP_{RE_{CI}} + RP_{RE_{AC}} + RP_{RE_D} \tag{31}$$

In order to obtain the expected loss of the insurance company, we generate 1000 different scenarios with Monte Carlo simulation. If a claim occurs, the cedent will pay the retained sum insured that the insurance company is obliged to cover in case of a claim (this amount depends on the retention level).The total loss amount is the sum insured for term, critical illness, accidental death and disability. Moreover, disability paid-out amount is dependent on the reason and how many days the insured suffers from it. We need to estimate how many percent of the sum insured (severity) is going to be paid. In order to do that, we take historical claim data and estimate the proportion of the paid amount.

Severity is how severe the claim is going to be. Since for the other type of insurances, when the claim occurs, the paid-out is the sum insured. With disability claims, the insurance company covers only the loss when the claim occurs.

Figure 9 shows the distribution of severity -in percentage- for disability insurance. Horizontal values are the percentages which mean how severe the loss will be. Vertical values are the density of the horizontal values. *N* is the number of historical claims arising from disability policies. Bandwidth is a compromise between smoothing enough to remove insignificant bumps and not smoothing too much to smear out real peaks (Venables & Ripley, 2002). Program sets the best bandwidth by itself.

Figure 9. The distribution of historical claims arising from disability policies



The next step is to fit a distribution to given claim data. The Akaike Information Criterion (hereinafter: AIC) says how good the distribution fits. A smaller value gives a better fit. Gamma, exponential, logistic, lognormal, Poisson and normal distribution are fitted, one by one. As a result, the lognormal distribution is the best fit for disability claim data (see Table 5).

After fitting the distribution, we need to estimate the mean and the standard variation, in order to generate random values with lognormal distribution.

Table 5. AIC values for each distribution

	Gamma	**Exponential**	**Logistic**	**Lognormal**	**Poisson**	**Normal**
AIC	-321.0569	-311.6915	-269.6084	-333.2036	-316.2553	-248.044

Table 6. The mean and the standard deviation of the lognormal distribution in terms of normal distribution values

Mean value for lognormal distribution	Standard deviation for lognormal distribution
-3.09718962	0.80561353

The program gives the result in Table 6 values in terms of normal distribution. The mean of normal distribution is -3.09718962; the standard deviation of normal distribution is 0.80561353. In order to find the mean and the standard variation in terms of lognormal distribution, we input mean and standard deviation values (in terms of normal distribution) into Equation (33) and Equation (34). In the end, we get the values in Table 7.

The probability density function of the lognormal distribution is as follows:

$$f(x)=\frac{1}{\sigma x\sqrt{2\pi}}\,e^{\left[-\frac{(\ln x-\mu)^2}{2\sigma^2}\right]} \quad (32)$$

Equation (33) shows the expected value (mean) of the lognormal distribution, and Equation (34) shows the variance of the lognormal distribution.

$$E(x)=e^{(\mu+\sigma^2/2)} \quad (33)$$

$$var(x)=(e^{\sigma^2}-1)e^{2\mu+\sigma^2} \quad (34)$$

Hence, the mean and the standard deviation of the lognormal distribution with the data are as follows:

Table 7. The mean and the standard deviation of the lognormal distribution in terms of lognormal distribution values

Mean value for lognormal distribution	Standard deviation for lognormal distribution
0.06249411	0.03869328

Table 7 values are in terms of lognormal normal distribution values. Finally, we obtain the mean and the standard deviation of the lognormal distribution with disability claim data. The mean is 0.06249411 whereas the standard deviation is 0.03869328. It means that the average severity of disability claim is 0.06249411 (6.2%). We will use these mean and standard variation values in order to generate random severity for the expected future claims number.

After obtaining the mean and the standard deviation of our assumed distribution, we can go one step forward in our expected loss calculation.

The possible scenarios are that a claim occurs or not. Therefore, we can take the Bernoulli distribution with the probability of a claim occurring for Monte Carlo. The probability of a claim occurring is known with data (q_T, q_{CI}, q_{AC} are known from the data and q_D is obtained by insurer's loadings).

Finally, the expected loss can be simulated. Equations (35), (36), (37) take basically the same approach; a claim occurs or not with a given probability, and it randomly generates Bernoulli values (1 or 0). 1 represents the claim occurring, whereas 0 represents no claim. When the randomly generated binomial value is 1, a proportion of the sum insured value is paid out for term, critical illness and accidental death policies. On the other hand, Equation (38) shows the calculation for disability. In Equation (38), we have also randomly generated lognormal values with mean and standard deviation of disability claim history with the lognormal distribution (Table 5 and Table 7 have already stated the distribution of the disability incident rates and mean and standard deviation value). If the Bernoulli random value is 1, randomly generated lognormal distribution value will model the severity in percentage. In the end, we have our expected loss arising from each type of life insurance products.

In Equation (35), (36), (37), (38), *rbinom* is randomly generating 1 or 0 with the probabilities of. q_T,q_{CI}, q_{AC}, q_D *rlnorm(mean,sd)* is randomly generating lognormal values with mean and standard deviation (*sd*).

$$Y_T = rbinom\left(q_T\right) * SI_{I_T} \tag{35}$$

$$Y_{CI} = rbinom\left(q_{CI}\right) * SI_{I_{CI}} \tag{36}$$

$$Y_{AC} = rbinom\left(q_{AC}\right) * SI_{AC} \tag{37}$$

$$Y_D = rbinom\left(q_D\right) * rlnorm(mean,sd) * SI_D \tag{38}$$

The calculation path starts with going through all policies 1.000 times. Each loop brings one value with the claims arising from policies. In the end, we have 1000 different scenarios for losses of term insurance. When we follow this path for each insurance type and sum them up, the total loss with 1.000 different scenarios is obtained. The mean value of these 1.000 scenarios is our estimate of the expected loss.

The constraint of the model (see Equation (39)) is that the VaR (Value at Risk) of the loss with that confidence level 99.5% shall not exceed a determined value that is set by the insurance company. This set value, let us denote it as *C*, is how much risk the insurer can bear. *C* is constant and known

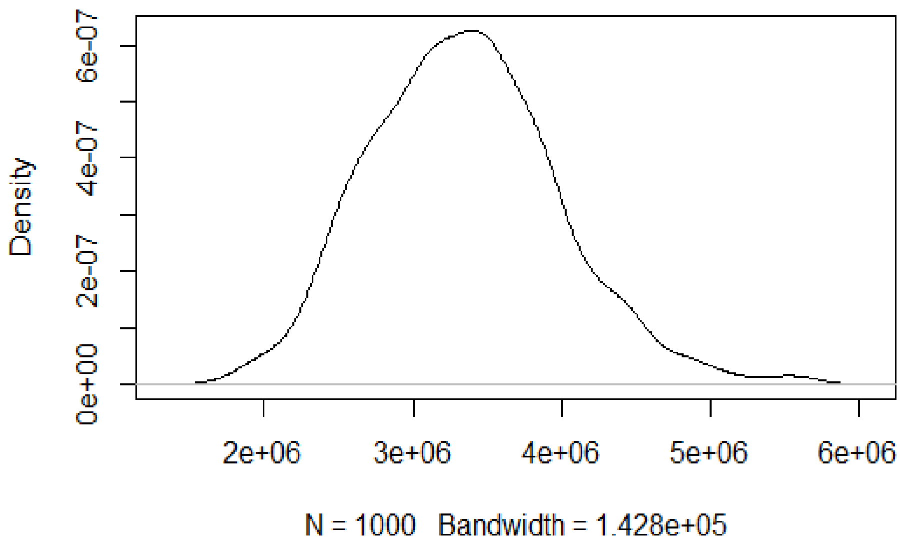
$$percentile(Y, 0.995) \leq C \tag{39}$$

3.10 Results without Reinsurance

The distribution of aggregate loss, obtained from Monte Carlo simulation, without any reinsurance is in Figure 10. Horizontal values are the losses of the insurance company without reinsurance. Vertical values are the density of the horizontal values. *N* is the number of simulations. Each simulation brings a different scenario with a different loss amount. The mean value of this distribution is the aggregate loss without reinsurance arising from the whole portfolio which is 3,346,382 euro, and the 99.5% of VaR is 5,332,806 euro. All the numbers hereon are in euros.

Let's assume that the insurance company cannot bear the risk with the current capital, and the maximum amount of risk that the insurance can take is 1,700,000 euro.

Figure 10. The distribution of aggregate loss without reinsurance



3.11 Comparison of Reinsurance Contracts

Limiting the risk appetite to 1,700,000 euro implies that the insurance company needs reinsurance. Table 8 shows the risk premium of the insurer and the reinsurer, the expected loss of the insurer, VaR (99.5%) of the loss from the insurance company's perspective with the insurance company's current retention levels.

The first column which has 40,000 euro retention level is term insurance, the second column which has 20,000 euro retention level is critical illness insurance, the third column which has 80,000 euro retention level is accidental death insurance, fourth column which has 80,000 euro retention level is disability insurance. The last column is total amount of values.

The income of the insurer is the risk premium of the insurer reduced by the risk premium of the reinsurer. The risk premium of the reinsurer depends on retention levels. An increment in retention level will result in a decrement in risk premium for the reinsurer since the insurance company will be covering a bigger expected claim amount. The outgo of the insurer is the loss arising from each life businesses. The percentile shows the worst case scenario that can happen once in 200 years for each insurance type. The 99.5% percentile is 1,988,608, which exceeds the maximum loss amount as 1,700,000. Table 8 clearly shows that the reinsurance level of the insurer is not sufficient.

Table 8. Insurer's profit from each insurance

	Term	**Critical Illness**	**Accidental death**	**Disability**	**Total**
Retentions (M)	40,000.00	20,000.00	80,000.00	80,000.00	
Income	967,462.00	447,476.00	626,653.10	1,618,639.30	3,660,230.40
Risk premium for the insurer	2,360,711.00	1,539,695.00	669,778.10	1,871,148.00	6,441,332.10
Risk premium for the reinsurer	1,393,249.00	1,092,219.00	43,125.00	252,508.70	2,781,101.70
Outgo	734,653.60	315,742.80	89,973.75	306,907.50	1,447,277.65
Expected loss	734,653.60	315,742.80	89,973.75	306,907.50	1,447,277.65
Percentile	1,154,466.00	534,379.40	360,006.20	446,580.00	1,988,608.00
Profit	234,808.40	131,733.20	536,679.35	1,311,731.80	2,212,952.75

3.11.1 Distributions of the losses for each business line

We fit a distribution to the losses in order to find the good-fit distribution of the losses. Figure 11 shows the distribution of the simulated losses arising from term policies. There are 1000 probable losses obtained by Monte Carlo simulation. Horizontal values are representing the expected losses. Vertical values are the density of the horizontal values. *N* is the number of simulations. The mean of this distribution estimates the expected aggregate loss for the term portfolio.

In order to see which distribution (Gamma, logistic, exponential, normal or lognormal) fits best, we look at the likelihood. We fit the distributions separately (gamma, logistic, exponential, normal and lognormal); check the log-likelihood result, and decide the closest distribution of our losses. The result of the log-likelihood assists us to determine the best fit. The biggest number is the best fit.

Log-likelihood values of fitting gamma, logistic, exponential, normal and lognormal distributions suggest that the loss of term insurance is normally distributed (see Table 9).

Table 9: The log-likelihood values of each fitted distribution.

	Gamma	Exponential	Logistic	Lognormal	Normal
Log-likelihood	-1,859,352	-14,507.15	-13,680.65	-13,432.92	-13,411.86

Taking the mean value of the losses arising from term insurance estimates the expected loss of term insurance as 734,653.600.

The 99.5% VaR is 1,154,466.00. The loss ratio, ratio between outgo and income, is 75.94%.

Figure 11. The distribution of the expected loss arising from term policies

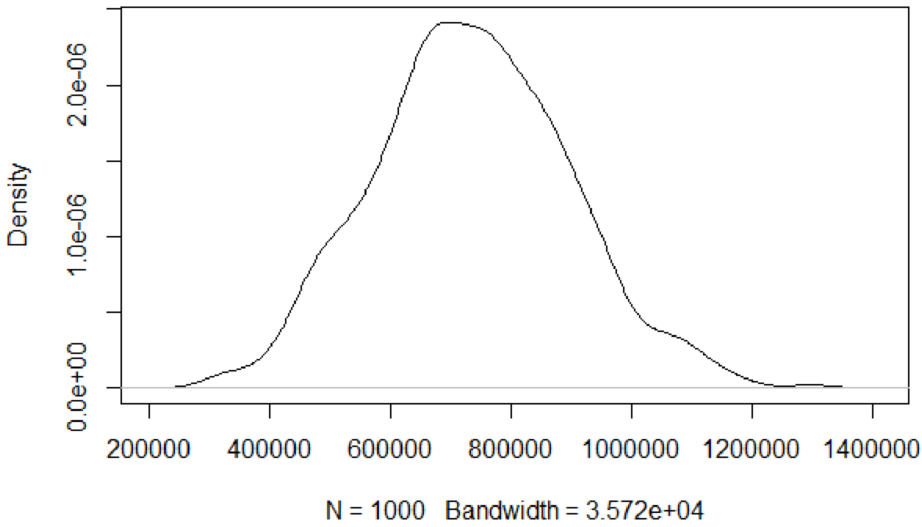


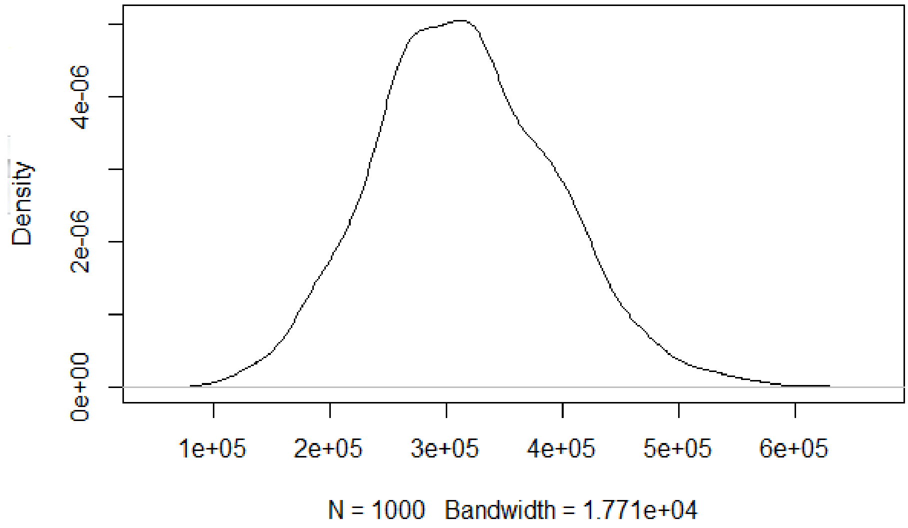
Figure 12 shows the distribution of the simulated losses arising from critical illness insurance policies. Horizontal values are the simulated losses; vertical values are the density of the horizontal values. *N* is the number of simulations.

Table 10. The log-likelihood values of each fitted distribution

	Gamma	Exponential	Logistic	Lognormal	Normal
Log-likelihood	-1,013,686	-13,662.68	-12,722.84	-12,702.53	-12,687.09

After fitting gamma, logistic, exponential, normal and lognormal distributions, log-likelihood values suggest that the expected loss of critical illness insurance is normally distributed.

Figure 12. The distribution of the expected loss arising from critical illness insurance

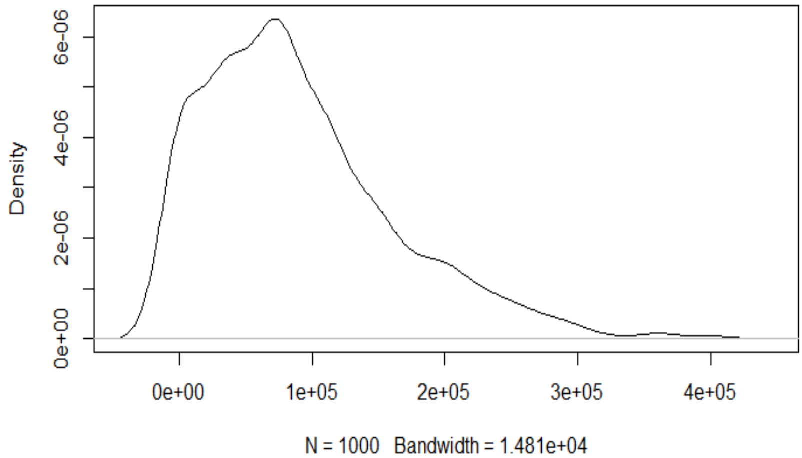


Taking the mean of the simulated losses arising from critical illness insurance estimates the expected loss for critical illness portfolio as 315,742.80.

The 99.5% VaR is 534,379.40. The loss ratio, between outgo and income is 70.56%.

Figure 13 shows the distribution of the simulated losses arising from accidental death insurance policies. Table 11 shows the log-likelihood values of fitting the distributions. Horizontal values are the simulated losses; vertical values are the density of the horizontal values. *N* is the number of simulations, in this case 1000 since it is number of simulations. Bandwidth is set by the program itself.

Figure 13. The distribution of the expected loss arising from accidental death insurance

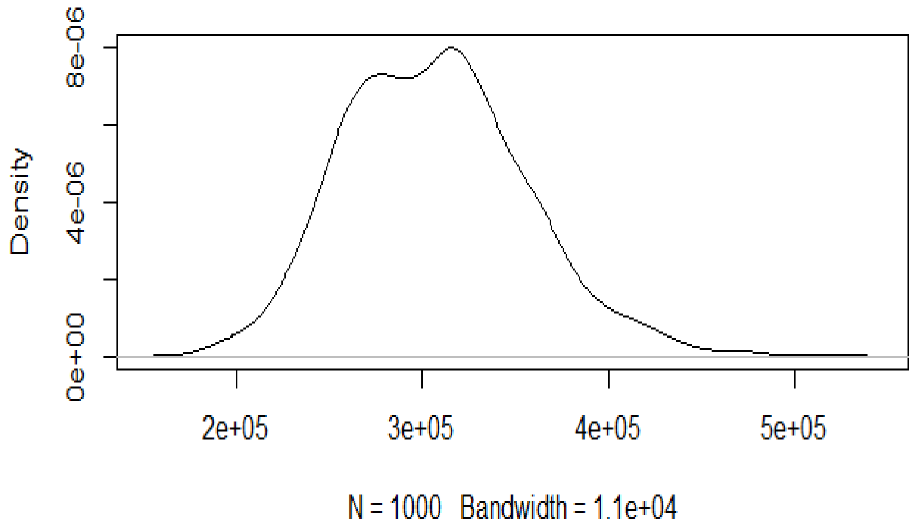


Log-likelihood values for fitting gamma, logistic, exponential, normal and lognormal distributions (see Table 11) suggest that the loss of accidental death insurance is exponentially distributed.

Taking the mean of the expected losses arising from accidental death insurance estimates the expected loss for accidental death as 89,973.75. The 99.5% VaR is 360,006.20. The loss ratio, between outgo and income is 14.36%.

Figure 14 shows the distribution of the simulated losses arising from disability insurance policies. Horizontal values are the simulated losses; vertical values are the density of the horizontal values. *N* is the number of simulations.

Figure 14. The distribution of the expected loss arising from disability insurance



Log-likelihood values for fitting gamma, logistic, exponential, normal and lognormal distributions (see Table 12) suggest that the loss of disability insurance is log normally distributed.

Table 11. The log-likelihood values of each fitted distribution

	Gamma	Exponential	Logistic	Lognormal	Normal
Log-likelihood	-1,013,686	-12,407.27	- 12,607.63	-12,702.53	-12,610.07

Table 12. The log-likelihood values of each fitted distribution

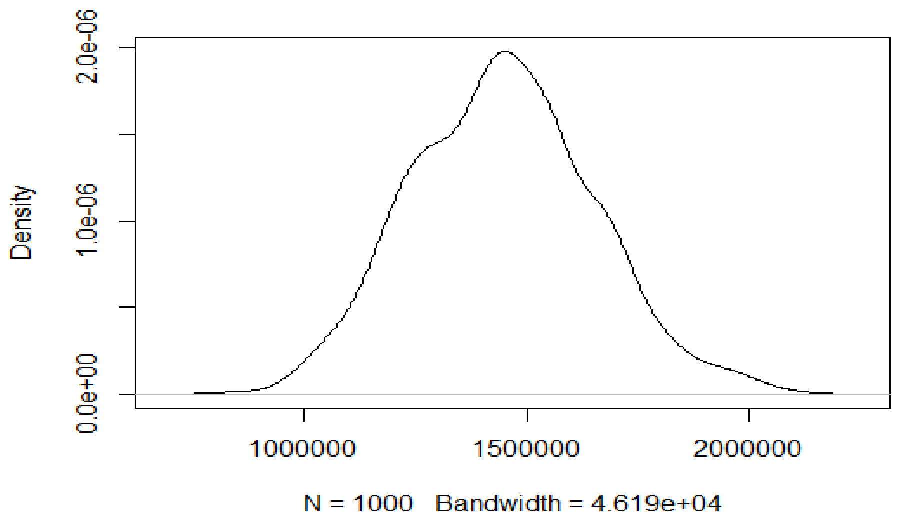
	Gamma	Exponential	Logistic	Lognormal	Normal
Log-likelihood	-389,148	-13,634.3	-12,238.36	-12,197.49	-12,210.69

Taking the mean value of the losses arising from disability insurance estimates the expected loss of disability as 306,907.50. The 99.5% VaR is 446,580.00. The loss ratio, between outgo and income, is 18.96%.

When we compare the loss ratios between portfolios, we see that disability and accidental death are the most profitable lines of business, since the loss ratios are significantly lower compared to term and critical illness insurances. This result comes from the lack of claims. The insurance company does not have enough claims to obtain statistically accurate results.

Figure 15 shows the distribution of the simulated aggregate losses arising from term, critical illness, accidental death, and disability insurance policies. Horizontal values are the simulated aggregate losses; vertical values are the density of the horizontal values. *N* is the number of simulations.

Figure 15. The distribution of aggregate expected losses



Next step after plotting the distribution of aggregate losses is to see which distribution is the best fit. In order to find out the best fit, we fit Gamma, Exponential, Logistic, Lognormal and normal distribution separately. Then, we obtain the log-likelihood values for each fitted distribution. The biggest log-likelihood brings the best fit for the aggregate losses. As we see in Table 13, the biggest log-likelihood value (-13,645.91) is by fitting normal distribution. Hence, the aggregate loss is normally distributed.

Table 13. The log-likelihood values of each fitted distribution

	Gamma	**Exponential**	**Logistic**	**Lognormal**	**Normal**
Log-likelihood	-1,462,346	-15,185.19	-13,680.65	-13,648.33	-13,645.91

3.11.2 Retention level effect on profitability

In order to improve profitability, we run the simulation with different retention levels. We will determine how to take less risk and have more profit.

Table 14 has retention levels; 20,000 euro for term and critical illness insurances, 300,000 euro for accidental death and 250,000 euro for disability. It means that the insurance company will keep the losses below these retention levels for each line of business. Total income is 3,515,768.78 euro. Most of the income is coming from disability insurance.

Table 14. Insurer's profit from each insurance with the retention levels

	Term	Critical Illness	Accidental death	Disability	Total
Retentions (M)	20,000.00	20,000.00	300,000.00	250,000.00	
Income	543,001.00	447,476.00	669,248.10	1,856,043.68	3,515,768.78
Risk premium for the insurer	2,360,711.00	1,539,695.00	669,778.10	1,871,148.00	6,441,332.10
Risk premium for the reinsurer	1,817,710.00	1,092,219.00	530.00	15,104.32	2,925,563.32
Outgo	385,240.30	315,847.10	104,848.80	404,761.60	1,210,697.80
Expected loss	385,240.30	315,847.10	104,848.80	404,761.60	1,210,697.80
Percentile	623,153.10	529,908.40	437,512.50	641,172.30	1,668,248.00
Profit	157,760.70	131,628.90	564,399.30	1,451,282.08	2,305,070.98

The total expected loss is 1,210,697.80 euro. Most of the loss is arising from the disability portfolio. However, if we compare the loss over the income, we see that term portfolio brings 543,001 euro income, and the expected loss is 385,240.30 euro. The loss ratio is 70.95% arising from term policies, which is the biggest, compared to the others.

Also, the critical illness portfolio is not as profitable as the disability and the accidental death. The income arising from the critical illness portfolio is 447,476 euro. The expected loss is 315,847.10 euro. The ratio between the loss and the income is 70.59%.

The disability portfolio brings 1,856,043.68 euro income. The expected loss arising from the disability is 404,761.60 euro. The ratio between the expected loss and the income is 21.80%. The result is quite low, compared to the term and the critical illness.

The accidental death portfolio seems the most profitable. The income from the accidental death policies is 669,248.10 euro. The expected loss is 104,848.80 euro. The ratio between the expected loss and the income is 15.67%. The loss ratio is the lowest, compared to the others.

Percentile is the risk the insurance company receiving. This risk is 99.5% of value at risk. In other word, percentile is the worst case scenario which can occur in 200 years.

The disability seems bringing the most risk. However, the term is almost as risky as disability. When we compare their incomes, the risk arising from the disability portfolio seems normal. This is due to low loss ratio of the disability portfolio. The accidental death portfolio brings 437,512.50 euro of the risk. The risk from the term portfolio is 623,153.10 euro. The risk from the critical illness is 529,908.40 euro. The aggregate risk exposure is 1,668,248 euro. The aggregate risk exposure shows that the insurer's risk exposure is below the admissible amount.

The aggregate expected profit is equal to 2,305,070.98. The disability brings 1,451,282.08 euro, the accidental death brings 564,399.30 euro, the critical illness brings 131,628.90 euro, the term brings 157,760.70 euro.

Table 15. Log-likelihood values of fitted distribution to the aggregate losses

	Exponential	**Lognormal**	**Normal**
Log-likelihood	-15,489.05	-13,554.58	-13,556.98

In order to see the best fitted distribution to the aggregate expected losses, we can check log-likelihood values. The biggest the log-likelihood values, the best fit. Table 15 shows the log-likelihood values by fitting the distribution into the aggregate losses. The best fitted distribution is lognormal.

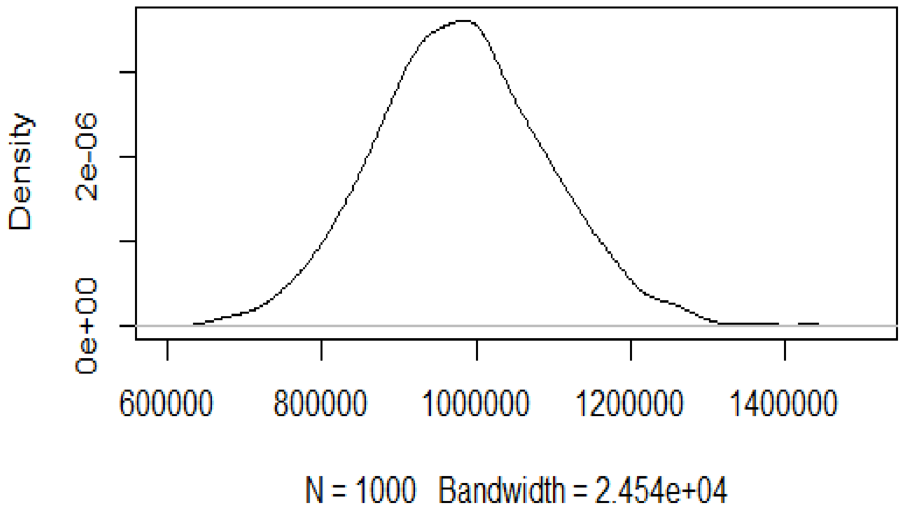
Table 16. Insurer's profit from each insurance with quota share reinsurance

	Term	**Critical Illness**	**Accidental death**	**Disability**	**Total**
Retentions (M)	0%	50%	100%	100%	
Income	74,058.00	729,477.90	669,778.10	1,871,148.00	3,344,462.00
Risk premium for the insurer	2,360,711.00	1,539,695.00	669,778.10	1,871,148.00	6,441,332.10
Risk premium for the reinsurer	2,286,653.00	810,217.10	0.00	0.00	3,096,870.10
Outgo	0.00	461,577.00	110,890.00	406,936.70	979,403.70
Expected loss	0.00	461,577.00	110,890.00	406,936.70	979,403.70
Percentile	0.00	822,590.60	468,775.00	608,097.90	1,470,531.00
Profit	74,058.00	267,900.90	558,888.10	1,464,211.30	2,365,058.30

Table 16 shows that the reinsurance type is quota share. The insurer cedes all the risk to the reinsurer with term policies. The cession rate is 50% with critical illness policies, whereas all the risks arising from accidental death and disability stay with the insurer. The aggregate profit is 2,365,058.30 euro. The risk exposure of the portfolio for the insurer is 1,470,531 euro, below the admissible amount of 1.7 million euro. It is even possible to see that the insurer's percentile of risk with this reinsurance is less than in Table 14. As the expected profit is shown in Table 16, it is possible to have more profit than in Table 14 while being exposed to the less risk. This shows us the importance of determining the right retention level for the insurance company.

Let us see the distribution of expected loss simulations. The distribution of 1000 simulated aggregate losses is shown in the following figure (Figure 16). *N* is the number of values, in this case the number of simulations. Bandwidth is determined by the program itself. The horizontal values are the expected losses and the vertical values are the density of the horizontal values.

Figure 16. The distribution of aggregate expected losses



Log-likelihood values show the best fitted distribution to the aggregate expected loss. We fit separately gamma, logistic, exponential, lognormal and normal distribution into aggregate loss. The bigger log-likelihood value gives us the best fitted distribution. Table 17 shows the log-likelihood values of fitted distribution. Hence, the aggregate loss is normally distributed.

Table 17. Log-likelihood values of fitted distribution to the aggregate losses

	Gamma	**Logistic**	**Exponential**	**Lognormal**	**Normal**
Log-likelihood	-645,414.4	-13,059.35	-14,795.33	-13,040.99	-13,040.66

Another interesting result is that the same expected profit can be attained with different risk exposure. It is important to find the maximum expected profit with our maximum acceptable risk amount.

Table 19 and Table 20 show the insurer's and the reinsurer's risk premiums, the insurer's expected loss and VaR 99.5%. Table 19 has the retention for term of 40,000, critical illness 40,000, accidental death 250,000 and disability 100,000. In Table 20, the insurance company cedes all the risk arising from term policies. The retention for critical illness is 50,000 and the insurance company keeps all the risk for the accidental death and disability.

In Table 19, the percentile is 2,397,999 with the expected profit 2,445,005, whereas in Table 20, the percentile is 1,735,801 with similar expected profit as in Table 19 which is 2,442,983. These results show the impact of reinsurance policy on the insurer's profitability.

Table 18. Log-likelihood values of fitted distribution to the aggregate losses

	Exponential	**Log normal**	**Normal**
Log-likelihood	-15,685.64	-13,794.61	-13,792.95

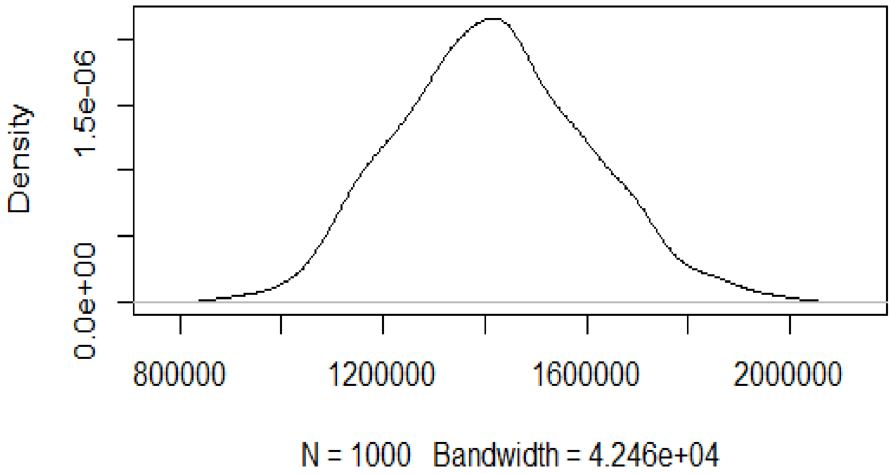
In order the see the distribution of aggregate losses with reinsurance as in Table 19, we use log-likelihood approach. We fit exponential, lognormal and normal distribution separately. The bigger log-likelihood value gives the best fitted distribution. The result is shown in Table 18 that the expected loss is normally distributed.

Table 19. Insurer's profit from each insurance with the retention levels

	Term	**Critical Illness**	**Accidental death**	**Disability**	**Total**
Retentions (M)	40,000.00	40,000.00	250,000.00	100,000.00	
Income	967,462.00	870,767.20	668,448.10	1,685,186.50	4,191,863.80
Risk premium for the insurer	2,360,711.00	1,539,695.00	669,778.10	1,871,148.00	6,441,332.10
Risk premium for the reinsurer	1,393,249.00	668,927.80	1,330.00	185,961.50	2,249,468.30
Outgo	737,096.40	570,291.60	106,262.50	333,207.60	1,746,858.10
Expected loss	737,096.40	570,291.60	106,262.50	333,207.60	1,746,858.10
Percentile	1,168,787.00	956,981.20	403,831.20	492,534.10	2,397,999.00
Profit	230,365.60	300,475.60	536,679.35	1,351,978.90	2,445,005.70

Figure 17 shows the distribution of the aggregate loss arising from the reinsurance given in Table 20. *N* is the number of the losses we fit, which is the number of simulations (1000). Bandwidth is determined by the program itself. The horizontal values are the expected losses arising from whole portfolio, and the vertical values are the density of the horizontal values.

Figure 17. The distribution of aggregate expected losses



All numbers in Table 20 are in euros.

Table 20. Insurer’s profit from each insurance with retention levels

	Term	Critical Illness	Accidental death	Disability	Total
Retentions (M)	0.00	50,000.00	100%	100%	
Income	74,058.00	1,241,245.20	669,778.10	1,871,148.00	3,636,229.30
Risk premium for the insurer	2,360,711.00	1,539,695.00	669,778.10	1,871,148.00	6,441,332.10
Risk premium for the reinsurer	2,286,653.00	298,449.80	0.00	0.00	2,805,102.80
Outgo	0.00	895,029.40	107,327.50	410,889.40	1,413,246.30
Expected loss	0.00	895,029.40	107,327.50	410,889.40	1,413,246.30
Percentile	0.00	1,084,469.00	500,037.50	635,580.80	1,735,801.00
Profit	74,058.00	346,215.80	562,450.60	1,460,258.60	2,442,983.00

In order to see the distribution of the expected loss with the reinsurance given in Table 20, we obtain the following results. Normal distribution has the greatest log-likelihood value (-13,486). Therefore, it is the best fit.

Table 21. Log-likelihood values of fitted distribution to the aggregate losses

	Gamma	Exponential	Lognormal	Normal
Log-likelihood	-1,093,363	-15,160.33	-13,491.14	-13,486

Eventually, we see that with the correct retention levels, it is possible to take less risk and have the same or larger expected profit. This possibility brings a big advantage to the insurance company. If the insurance company uses the right reinsurance for its portfolio, it will be easier to grow fast.

Table 22 illustrates different scenarios for the insurance company with different retention levels. All numbers are in euros. The first column says which type of life insurance products, the second column says the retention levels for each life insurance products, the third column shows the expected profit for the life insurer for each type of life insurance product, and the fourth column says the 99.5% risk percentile (VaR) for each type of life insurance product. All the numbers are obtained with the calculations defined previously.

We can see the effect of changing retention levels on profitability. The admissible risk amount has an influence on the strategy which affects profitability. Some of the retentions are set as a number, which means it is surplus reinsurance; whereas some of the retentions are set as a percentage, which means it is quota share reinsurance.

Table 22. Scenarios of outputs with different retention levels

	Retention	Expected profit	Risk percentile (99.5%)
Term	48,000	162,248	1,020,193
Critical illness	24,000	164,197	504,156
Accidental Death	96,000	225,780	198,538
Disability	96,000	796,783	244,738
Total		1,129,017	1,590,740
Term	56,000	171,915	1,037,827
Critical illness	28,000	185,088	572,588
Accidental death	112,000	223,196	172,105
Disability	112,000	581,901	250,341
Total		1,162,100	1,646,963
Term	58,000	168,039	1,086,199
Critical illness	34,800	184,899	556,533
Accidental death	116,000	224,000	193,015
Disability	116,000	581,923	245,343
Total		1,158,860	1,738,042

table continues

continued

	Retention	Expected profit	Risk percentile (99.5%)
Term	60,000	166,181	1,154,515
Critical illness	30,000	189,926	546,111
Accidental death	120,000	225,169	188,105
Disability	120,000	580,806	255,112
Total		1,162,082	1,743,714
Term	50,000	158,124	1,097,530
Critical illness	20,000	146,743	462,836
Accidental death	90,000	224,671	182,030
Disability	90,000	576,966	236,740
Total		1,106,504	1,581,181
Term	60,000	170,138	1,081,250
Critical illness	20,000	145,816	458,579
Accidental death	80,000	224,898	179,105
Disability	90,000	575,901	235,144
Total		1,116,752	1,633,083
Term	30,000	130,061	766,528
Critical illness	20,000	150,585	444,868
Accidental death	90,000	224,238	173,030
Disability	90,000	575,777	231,550
Total		1,080,661	1,297,701
Term	30,000	134,106	767,581
Critical illness	20,000	144,033	460,019
Accidental death	100,000	224,223	180,025
Disability	90,000	576,950	237,129
Total		1,079,313	1,299,923
Term	30,000	132,346	787,524
Critical illness	30,000	189,131	550,050
Accidental death	100,000	224,860	181,500
Disability	90,000	573,870	241,249
Total		1,120,207	1,376,981
Term	30,000	129,701	763,781
Critical illness	30,000	197,754	545,006
Accidental death	100,000	224,265	192,515
Disability	100,000	578,363	240,763
Total		1,130,084	1,380,905

table continues

continued

	Retention	Expected profit	Risk percentile (99.5%)
Term	30,000	133,319	781,274
Critical illness	100%	246,537	709,043
Accidental death	100%	223,652	172,035
Disability	100%	584,668	260,944
Total		1,188,176	1,509,948
Term	30,000	137,444	790,142
Critical illness	95%	231,285	705,413
Accidental death	95%	221,547	165,032
Disability	95%	572,627	245,221
Total		1,162,903	1,453,004
Term	30,000	137,444	776,265
Critical illness	100%	244,969	683,756
Accidental death	100%	224,748	177,503
Disability	95%	572,627	239,474
Total		1,179,788	1,460,231
Term	40,000	232,808	1,154,466
Critical illness	20,000	131,733	534,379
Accidental death	80,000	536,679	360,006
Disability	80,000	1,311,732	446,580
Total		2,212,953	1,988,608
Term	0	74,058	0
Critical illness	40,000	296,793	961,108
Accidental death	100%	562,848	482,500
Disability	100%	1,459,161	628,794
Total		2,392,860	1,658,750
Term	0	74,058	0
Critical illness	50,000	346,215.8	1,084,469
Accidental death	100%	562,451	500,038
Disability	100%	1,460,259	635,581
Total		2,442,983	1,735,801
Term	0	74,058	0
Critical illness	60,000	431,610	1,201,544
Accidental death	100%	563,256	437,531
Disability	100%	1,462,065	638,076
Total		2,530,989	1,781,762

table continues

continued

	Retention	Expected profit	Risk percentile (99.5%)
Term	20,000	157,761	623,153
Critical illness	20,000	131,629	529,908
Accidental death	300,000	564,399	437,513
Disability	250,000	1,451,282	641,172
Total		2,305,071	1,668,248
Term	0	74,058	0
Critical illness	80,000	473,959	833,389
Accidental death	100%	566,137	103,641
Disability	100%	1,462,888	408,260
Total		2,577,043	2,009,054
Term	0	74,058	0
Critical illness	60%	332,192	1,031,783
Accidental death	100%	566,783	441,469
Disability	100%	1,458,808	636,397
Total		2,431,841	1,664,274
Term	0	74,058	0
Critical illness	80%	477,288	1,290,212
Accidental death	100%	563,725	437,587
Disability	100%	1,461,938	621,191
Total		2,577,010	1,886,715
Term	0	74,058	0
Critical illness	50%	267,900.90	822,590
Accidental death	100%	558,888	468,775
Disability	100%	1,464,211	608,097
Total		2,442,983	1,470,531

If the insurer's risk appetite is different from 1.7 million euro, we can have an idea in which direction to go in order to find better reinsurance by seeing Table 22.

Table 22 shows how the portfolio is behaving. When we fix the retention for three different lines of business and change it for one, we see how much profit and risk change.

The last scenario in Table 22, ceding all the risk for term, 50% for critical illness and keeping all the risk retained, presents the possibility of making money with taking no risk. Although all the risk arising from term policies is ceded to the reinsurer, the insurer keeps some income. The insurer does not transfer the whole premium with reinsuring term portfolio while transferring all the risk. This is coming from the difference of the incident rates of the insurer and the reinsurer. The mortality rates of the insurer are greater than the reinsurer's.

While keeping term, accidental death and disability retention levels fixed (the term risk is fully ceded, the risk arising from accidental death and disability policies are fully retained with the insurer), and changing critical illness quota share cession rate from 50% to 60%, the insurer increases the profit by 2.8% and the risk by 13.18%. When the insurer changes the cession rate for only critical illness from 60% to 80% (keeping the others fixed same as the previous case), the insurer gets 6% more profit with 13.37% more risk. Another scenario is that the insurer has surplus reinsurance with the term risk is fully ceded, all the accidental death and the disability risks are retained, by changing retention for the critical illness risk from 50,000 euro to 60,000 euro, the risk increases by 2.6% and the profit increases by 2.8%. When the retention level of the critical illness changes from 40,000 euro to 50,000 euro, the profit increases by 4.2% and the risk increases by 4.6%. The comparison between quota share and surplus reinsurance shows us that the surplus reinsurance delivers a better result on the insurer's profitability.

Let us look at another case when the retention is 30,000 euro with surplus reinsurance for the term and the critical illness portfolio, and 100,000 euro retention level for the accidental death. While changing retention level from 95,000 euro to 100,000 euro for the disability portfolio, the insurer will increase the risk by 0.3% and the profit by 0.9%. Keeping the risk arising from the disability policies brings us more profit than risk. This result is because of estimating future losses with historical claim data. The insufficient amount of disability claims does not ensure reliable future estimation. According to the disability analysis, there will not be big losses for disability, so that there is no point in reinsuring the disability portfolio. However, this is not true. Therefore, the insurer still needs reinsurance for the disability portfolio in practice.

The accidental death portfolio has the same issue as disability. Due to lack of claim data, the loss ratio is low. In the calculation, the probability of a claim occurring is obtained by multiplication of the incident rates and the claim ratio. When the claim ratio is low, a loss is not so likely. In this case, the analysis suggests to keep all the premium arising from accidental death policies, and not to reinsure the accidental death portfolio because there will not be big losses.

Different scenarios have a significant impact on understanding the data. When we understand the data better, we go into a better direction to find reinsurance contracts. The retention levels are set, and the model shows whether or not the retention levels are better than the previous cases. It would be really helpful to solve optimum reinsurance contract with the model giving the best retention levels after numerical calculations. In that case, we would just give initial retention levels, and the model would result the best retention level with giving the biggest profit However, the distribution of the aggregate expected loss changes with different retention levels. Therefore, we have more analytical approach to optimum reinsurance problem with this study.

CONCLUSION

Insurance is a protection for the individual provided by insurance companies. Insurance is a contractual agreement between the insured person and the insurance company. The policyholder is liable to pay premiums to the insurance company; in return, the beneficiary receives the benefit.

Life insurance provides protection to the people with different lines of business. One of them is term insurance. Term insurance is one of the most common life insurance. In case of the insured person dying, the beneficiary receives the sum insured from the insurance company. Hence, the beneficiary does not suffer financially from the death of the insured person. Another life insurance is critical illness. Critical illness gives a protection for the beneficiary (the beneficiary can be also the insured person himself/herself) against the specified illnesses in the policy. Accidental death is a protection for the beneficiary in case of the death caused by an accident. Disability insurance protects the beneficiary’s income when the insured person becomes unable to work due to illness or injury.

Reinsurance is insurance for insurance companies. After the insurance companies receive the risk from the insured people, they also seek for the way to cede their risks to another party. The reinsurance companies are receiving some part of the insurance companies’ risk in return of some fee. Reinsurance is great help for the insurance companies’ illiquidity problem. Then, the problem becomes for the insurance company to answer the following question: How much reinsurance should the insurance company have? This study is focused on the usual problem from the insurer’s perspective and giving the practical approach to the solution.

The research topic is checking whether or not a selected life insurer has sufficient reinsurance coverage. If yes, the research problem turns into how to improve the current reinsurance contract with sample data from the life insurer. If not, the research problem is about the optimal reinsurance contract for the selected life insurer.

In Section 3.11, Table 8 shows that the current reinsurance level is not sufficient given the data of the insurer. The insurer cannot take the risk arising from the worst case scenario that can happen once in 200 years. In order to obtain that result, we consider the risk premium of the insurer and reinsurer, what stays with the insurer is taken into account as the insurer’s income. For the outgo, the losses arising from each insurance type are simulated with Monte Carlo simulation. The expected loss of the whole portfolio is the sum of the expected losses for each insurance type. The profit is then the difference between income and outgo from the insurer’s perspective.

In Section 3.11.2, it is shown how changing the retention levels affects profitability. This research also proves that, with the sample data, it is possible to have higher profit while taking lower risk. Moreover, this study finds that two different reinsurance contracts can give a similar amount of expected profit, and result in different risk exposures. Table 17contains the best reinsurance contract for the life insurer based on the data available.

It is important for an insurance company, as for all financial institutions, to take less risk and have higher profit. The data has an important role in optimizing profit. The insurance company must know its own portfolio. After understanding the portfolio's structure, the insurance company will be able to take better decisions, which will help to improve the insurance company's financial status.

This study gives a simple, yet practical approach to the insurance company how to improve profitability. All that the insurance company needs is accurate data. Larger databases assist us to obtain more accurate results. Due to insufficient historical claim data on accidental death and disability portfolios, the statistics imply low probability for future claims. This leads to the conclusion that the insurance company does not need reinsurance protection for accidental death and disability portfolios. If the insurance company understands its data, then it will see this problem, and it will buy reinsurance protection for those two portfolios. The study would derive more accurate and convenient reinsurance suggestions if there was larger data.

Following up, in the future we can have large data on claim history. Since the sample insurance company is small, they use the reinsurer's incident rates. It would be an improvement to obtain own incident rates for the optimization problem. Because the reinsurance company has large data from multiple different insurance companies which have different strategies on the market, it would be better for the insurance company to estimate its own incident rates. Additionally, it could be great help to the insurer if the optimization problem were to be solved by numerical methods with only initial retention level as input, and the result would be an optimal reinsurance contract without us setting the retention levels for each different scenario.

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APPENDIX

APPENDIX: List of abbreviation

The following table describes the significance of various abbreviations and acronyms used throughout the thesis. Nonstandard acronyms that are used to abbreviate the names of certain white matter structures are also in Table 1.

Table 1. Abbreviations

Abbreviation	Meaning
α	Cession rate
SI	Sum insured
SIi	Sum insured for person i
N	Number of policies
SI_{RE}	Sum insured for reinsurer
SI_I	Sum insured for insurer
SI_{I_T}	Sum insured for insurer for term insurance
$SI_{I_{CI}}$	Sum insured for insurer for critical illness insurance
$SI_{I_{AC}}$	Sum insured for insurer for accidental death insurance
SI_{I_D}	Sum insured for insurer for disability insurance
α_T	Cession rate for term insurance
α_{CI}	Cession rate for critical illness insurance
α_{AC}	Cession rate for accidental death insurance
α_D	Cession rate for disability insurance
N_T	Number of policies for term insurance
N_{CI}	Number of policies for critical illness insurance
N_{AC}	Number of policies for accidental death insurance
N_D	Number of policies for disability insurance
SI_{RE_T}	Sum insured for reinsurer for term insurance
$SI_{RE_{CI}}$	Sum insured for reinsurer for critical illness insurance
$SI_{RE_{AC}}$	Sum insured for reinsurer for accidental death insurance
SI_{RE_D}	Sum insured for reinsurer for disability insurance
M	Retention level
M_T	Retention level for term insurance
M_{CI}	Retention level for critical illness insurance
M_{AC}	Retention level for accidental death insurance
M_D	Retention level for disability insurance
RP_{T_i}	Risk premium for insurer for term insurance
q_{T_i}	Insurer’s incident rate for term insurance for person i
RP_{RE_T}	Risk premium for reinsurer for term insurance
$q_{RE_{T_i}}$	Reinsurer’s incident rate for term insurance for person i
$RP_{RE\,T}$	Risk premium of the reinsurer arising from term policies
q_{CI_x}	Incident rates of the insurer for critical illness for person x
$q_{RE_{CI_x}}$	Incident rates of the reinsurer for critical illness for person x

table continues

continued

Abbreviation	Meaning
$RP_{I_{CI}}$	Risk premium of the insurer for critical illness policies
$RP_{RE_{CI}}$	Risk premium of the reinsurer for critical illness policies
$RP_{net_{CI}}$	Net risk premium for critical illness insurance
$q_{x_{AC}}$	Incident rates of the insurer for accidental death policies for person x
$q_{x_{RE_{AC}}}$	Incident rates of the reinsurer for accidental death policies for person x
$q_{x_{tech}}$	Technical premium rate for person x
SM_x	Safety margin for person x
CR_x	Claim ratio depending on the insured's insurance type
$RP_{I_{AC}}$	Risk premium of the insurer arising from accidental death policies
$RP_{RE_{AC}}$	Risk premium of the reinsurer arising from accidental death policies
$RP_{net_{AC}}$	Net risk premium for accidental death insurance
RP_{I_D}	Risk premium of the insurer arising from disability policies
RP_{RE_D}	Risk premium of the reinsurer arising from disability policies
RP_{net_D}	Net risk premium for disability insurance
Y	Total loss
Y_T	Total loss arising from term insurance
Y_{CI}	Total loss arising from critical illness insurance
Y_{AC}	Total loss arising from accidental death insurance
Y_D	Total loss arising from disability insurance
RP_I	Total risk premium of the insurer
RP_{RE}	Total risk premium of the reinsurer