UNIVERSITY OF LJUBLJANA
FACULTY OF ECONOMICS

MASTER'S THESIS

CALULATION OF PREMIUMS AND PROVISIONS IN LIFE AND NON-LIFE INSURANCE COMPANY

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AUTHORSHIP STATEMENT

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LIST OF ABBREVATIONS

AER	Acquisition expense ratio
BE	Best estimate of premium provision
CLT	Central limit theorem
CR	Estimate of combined ratio
EIOPA	The European Insurance and Occupational Pensions Authority
EU	European Union
GDP	Gross domestic product
IBNR	Inccured but reported
IS	Insured sum
ISA	The Insurance Supervision Agency
OECD	The Organisation for Economic Co-operation and Development
PVFP	Present value of future premiums
RBNS	Reported but not settled
TIR	Technical interest rate
VM	Volume measure for unearned premiums
ZZavar	Insurance act

INTRODUCTION

Insurance premium is the price for the good »insurance« sold by the insurance industry. As with any other industry it is vital to charge the right price since too low a price level results in a loss, while too high a price is not competitive (Straub, 1997). In order to solve this problem, insurance companies employ actuaries who deal with different statistical and mathematical problems in insurance (Straub, 1997). Nevertheless, it is still important for economists and financial people working in insurance to understand basic premium and provision calculations. The purpose of this Master's thesis is to show basic calculations for life and non-life insurance premiums. Our goal is to construct practical examples and calculate premiums for different types of insurance, such as pure endowment, term, whole life and non-life insurance.

In life insurance, we focus on risks connected with mortality and not on financial risks. Insurers usually use different mortality tables for life insurance products than for annuities (Harrington & Niehaus, 1999) therefore we use separate tables when dealing with mortality and longevity risk. We use the equivalence principle which says that present value of expected premiums should equal expected present value of claims. In order to calculate expected present values made either on death of the insured person, or periodically, as long as person lives we use commutation functions, interest rates and expenses.

In non-life insurance present value of expected premiums that equals present value of expected claims is not sufficient. Ruin theory shows us that pure risk premium without any loading is not sufficient since, in the long run, ruin is inevitable even if the insurer has a large amount of initial reserves (Straub, 1997). In order to mitigate that risk, we introduce a loading using Variance Premium Principle. We show that when applying a fixed loading, ruin probability under Central Limit Theorem equals 0.

When calculating non-life insurance premiums, one has to consider moral hazard and adverse selection. Moral hazard is an effect of lower incentives of the insured to reduce expected losses. Adverse selection refers to situations in which customers have different expected losses, but the insurer is unable to distinguish between them and therefore charge them different premiums (Harrington & Niehaus, 1999). In practice insurers charge different rates to insured based on their characteristics. For example, an insurer builds homogeneous groups of drivers based on their age, region and type of car. Because each risk is unique such a classification is not sufficient, therefore we present and calculate credibility premium which is a combination of individual and collective premium. In order to further mitigate moral hazard and adverse selection we also present deductibles and solve a practical example.

The biggest liability on the balance sheet of an insurance company are provisions. There are different types of provisions, such as mathematical provisions, premium provisions, claim provisions, catastrophe provisions and others (Straub, 1997). In life insurance business, mathematical provisions are the most important whereas in non-life insurance claim provisions are the most important. Based on the type of provision the insurer needs to have a suitable portfolio of assets build in such a way that the maturity of assets matches the maturity of liabilities. If a particular provision is not calculated correctly it is possible the insurer won`t have available assets to cover the claims. On the other hand, too many short-term assets bring lower interest rates which is also not desirable. The purpose of this Master's thesis is also to show basic calculations of provision calculation in life and non-life insurance. Our goal is to solve practical examples for pure endowment, term and life insurance. In order to calculate mathematical provisions for a life insurance company we again use the equivalence principle. On the other hand, non-life insurance mainly deals with claim provisions that are provisions that are build, because there is a delay between claim occurrence and claim settlement. Its amount is given by the estimated amount of the claims which have already occurred, but have not yet been settled (Olivieri & Pitacco, 2010). We use the Chain Ladder method to predict future claims based on past claims, but because the Chain Ladder method has some downsides, such as method breaks down when cumulative claims equal zero, is very sensitive to changes even of a single number, disregards premiums, etc. (Straub, 1997), we also use the Cape Cod method to overcome those downsides.

Our work starts with examination of theory and research of data collected by Slovenian Insurance Supervision Agency and Annual reports of Slovenian insurance companies. We collect and analyse data using Excel spreadsheets and present our findings using graphs and tables. Figures in graphs and tables do not account for inflation, therefore value growth is partially attributed to the inflation. The goal is to get familiar with different products in life and non-life insurance and to see how important certain classes of products are by considering the amount of paid premiums and made provisions. Building upon existing theory we then present and use mathematical equations and methods used for calculation of premiums and provisions. We use German DAV 2008 T period mortality tables for calculating mortality risk and DAV 2004 R cohort mortality table for longevity risk. On the 1 of March 2011 the Court of Justice of the European Union (hereinafter: EU) made a ruling, based on which the insurers have to change their pricing in such a way that it does not use gender information as a risk factor (European Commission, 2012). In order to account for this fact, we calculate unisex mortality rates by combining mortality tables for men and women taking an average of both mortalities. Together with mortality rates we also calculate commutation functions which we use in our calculations. Mortality tables with calculated commutation functions are presented in the Appendixes. With all the needed ingredients, we construct practical examples of premium and provision calculations and solve them.

Research questions are: 1. What are the basic methods for the calculation of insurance premiums in life and non-life insurance? 2. What are the basic methods for the calculation of insurance provisions in life and non-life insurance? 3. How do provisions differ between life and non-life insurance company? 4. How are gross written premiums distributed between different lines of insurance business?

In the second chapter, we present different types of provisions and scale of different types of provisions and premiums on the Slovenian market. Third chapter contains information about different types of life insurance products especially about which risks they cover, how premiums and claim payments are paid and who they are meant for. In fourth chapter, we present tools which are needed in order to calculate insurance premiums and provisions for life insurance contracts, which are interest rates, costs and mortality tables. Furthermore, we introduce actuarial values and commutation functions which allow a calculation of most of the expected present values with a minimal number of arithmetical operations. At the end of the chapter we calculate premiums and provisions for whole life, term and endowment insurance contract. Fifth chapter is about different non-life insurance products classified by Organisation for Economic Co-operation and Development (hereinafter: OECD). We also make a comparison of non-life insurance classes by comparing the gross written premiums for Slovenian market. In the sixth chapter, we present the underwriting risk and ruin theory which give rise to a need for proportional loading. There are different rules for assigning a premium to an insurance risk. We describe and use Variance Premium Principle. In the second part of the chapter we calculate premiums by using credibility rating and by considering deductibles. In the last seventh chapter, we present the Chain Ladder method and the Cape Cod method as two methods for calculating claim provisions, which are commonly used in practice by the insurance companies. We also present two practical exercises in order to better present the two methods.

1 INSURANCE TECHNICAL PROVISION AND GROSS WRITTEN PREMIUM

In insurance business provisions represent the largest portion of total liabilities of an insurance company. According to 2016 Report from Slovenian Insurance Supervision Agency (hereinafter: ISA) provisions accounted for around 70 % of total liabilities of all Slovenian insurers (ISA, 2017). There are many different types of provisions, such as premium provisions, outstanding claim provisions, contingency provisions, IBNR (incurred but not reported) - provisions and some other additional provisions (Straub, 1997). In Slovenia, insurance technical provisions are prescribed by the Insurance act, Article 113 (hereinafter: ZZavar). ZZavar, Article 113, distinguishes between:

- premium provisions,
- provisions for bonuses, discounts and cancellations,
- mathematical provisions,
- claims provisions,

- equalization provisions
- provisions for unit-linked insurance contracts and
- other insurance technical provisions.

Premium provisions are made because premiums can be paid in advance for many years or more precisely more accounting periods and such premiums should not be recognized as revenue of a single accounting period. For example, if premium for a one-year policy is paid in the middle of the year (we consider a balance sheet at the end of the year), only the half of the premiums should be recognized as revenue in the current year and another half in the next year's income statement.

The European Insurance and Occupational Pensions Authority (hereinafter: EIOPA) has published Guidelines on the valuation of technical provisions with a goal to greater harmonization and coherent application of rules for financial institutions and markets across the European Union. According to EIOPA best estimate of premium provision is derived from the formula:

$$BE=CR*VM+(CR-1)*PVFP+AER*PVFP \tag{1}$$

Where: BE = best estimate of premium provision; CR = estimate of combined ratio = (claims + claim related expenses) / (earned premiums gross of acquisition expenses); VM = volume measure for unearned premiums. Volume measure represents premiums earned for business incepted at the valuation date less the premium that has already been earned against these contracts; PVFP = present value of future premiums; AER = estimate of acquisition expenses ratio for line of business (EIOPA, n.d.).

For VM calculation usually simple proportional method is used. Total gross premiums are multiplied by number of days from the start of new accounting period till the end of the contract and divided by the total days of insurance duration. Proportional method is not adequate for all types of risk, because for some risks, such as construction insurance, claim payments are not proportional and therefore other actuarial methods have to be used.

Provisions for bonuses, discounts and cancellations equal the amount to which policyholders are entitled because of the:

- right to receive bonuses which are part of their insurance contracts,
- right to premium reduction,
- and right to receive back the unearned premiums in case of premature termination of insurance contract.

Mathematical provisions are made by life insurer usually for each contract separately. Provisions should equal the difference between present value of future premiums and present value of future claims and costs. Actuaries calculate mathematical provisions from the data about mortalities, interest rate and cost. Mathematical provisions are mainly important for life insurers.

Claims provisions are made to cover claims for accidents that happened in the past till the end of accounting period and that have not been solved yet. We distinguish between claims that have already been notified (outstanding claims) and claims that have not been notified yet. Outstanding claim provisions or RBNS (hereinafter: Reported but not settled) provisions are provisions for claims already notified but not yet settled. Depending on the line of business, the time between claim notification and final claim regulation can range from a few weeks (e.g. small claims and in property insurance) to a few years (e.g. large claims and liability insurance) (Olivieri & Pitacco, 2010). IBNR-provisions stand for incurred but not reported provisions (hereinafter: IBNR-provisions). These are provisions that are formed for two types of claims. First are claims for which the damage has already been caused but the loss has not yet occurred. For example, asbestos has caused damage to a person's health but the person does not have to pay medical expenses for lung cancer yet. Second type are claims when damage was caused, loss has occurred but the claim has not been notified yet. For example, the insured is abroad and cannot notify the claim. We further denote outstanding claims provisions and IBNR-provisions as claim provisions. Actuaries calculate claims provisions with the help of different triangle methods, past data about claims and knowledge about future trends. Claims provisions are mostly important for non-life insurers.

Equalization provisions and **other insurance technical provisions** are made to cover risks for which premium provisions are not sufficient. These are risks that are large and the overall number of risks in the portfolio is not high enough to provide the sufficient amount of premiums. For example, claims connected with, nuclear damage, earthquake, floods, aviation, etc. At the same time, some of those risks vary over time and the insurer does not know when they will occur. Contingency provisions, as they are also called, are made because although actuaries can have great models for predicting future claim distribution, claims are contingent and therefore unpredictable. For example, there can be a catastrophic year with large floods and insurer has to pay a big claim amount. Contingency provisions should be built gradually and insurers often look for additional protection in re-insurance. How the insurer calculates equalization and other technical provisions is defined by statistical and accounting standards. Till 2005 Slovenian Insurance act allowed to form equalization provisions in following areas: railway rolling stock, aviation, goods in transit, fire and natural forces, general liability and ships. In 2006 amending act restricted equalization provisions only to credit insurance (Pavliha & Simoniti, 2007).

Provisions for unit-linked insurance contracts insurer that offers insurance policies, for which investment risks are taken by the insured, has to form special provisions.

In order to see how important or how large are individual type of insurance technical provisions we look at provision structure of 2 Slovenian insurance companies. Because structure of provisions depends mainly on the type of insurer, that is life or non-life insurer, we examine two insurers that are in both businesses. Table below includes data about insurance technical provisions for Zavarovalnica Triglav d.d. and Zavarovalnica Sava d.d. separately for life and property business. Namely, insurance companies prepare balance sheets separately for life and property business. Considering gross insurance premiums written both insurance companies have about 70 % of business in non-life insurance and 30 % in life insurance business. Rows include information about the size of premium, mathematical, claims provisions, provisions for bonuses, discounts and cancellations, provisions for unit-linked insurance contracts and other types of insurance technical provisions expressed in million € and percentage of total provisions. We expect that mathematical provisions will be relatively high for life insurance business. Claims provisions should be relatively higher for particularly non-life business of Zavarovalnica Triglav d.d. and Zavarovalnica Sava d.d.

Table 1: Structure of provisions in life and property insurance business in Zavarovalnica Triglav d.d. and Zavarovalnica Sava d.d.

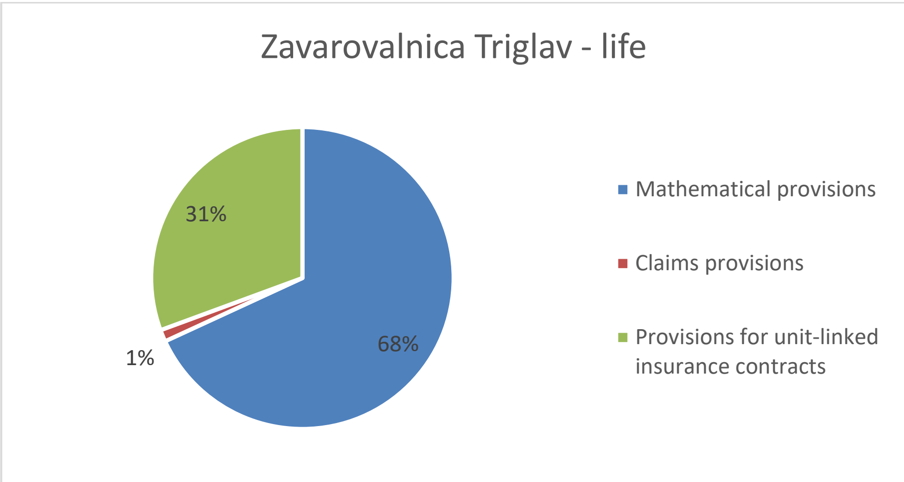
	Zavarovalnica Triglav d.d. (life, 2016)	Zavarovalnica Triglav d.d. (property, 2016)	Zavarovalnica Sava d.d. (life, 2016)	Zavarovalnica Sava d.d. (property,2016)
Premium provisions	0,42 million € / 1.412,59 million € = 0,03 %	188,07 million € / 663,72 million € = 28,34 %	0,89 million € / 503,03 million € = 0,18 %	110,30 million € / 415,35 million € = 26,55 %
Mathematical provisions	**959,27 million € / 1.412,59 million € = 67,91 %**	0 million € / 663,72 million € = 0 %	**259,14 million € / 503,03 million € = 51,52 %**	0 million € / 415,35 million € = 0 %
Claims provisions	17,86 million € / 1.412,59 million € = 1,26 %	**453,38 million € / 663,72 million € = 68,31 %**	16 million € / 503,03 million € = 3,18 %	**297,02 million € / 415,35 million € = 71,51 %**
Provisions for bonuses, discounts and cancellations	0 million € / 1.368,34 million € = 0 %	18,50 million € / 663,72 million € = 2,79 %	0 million € / 503,03 million € = 0 %	1,79 million € / 415,35 million € = 0,43 %
Provisions for unit-linked insurance contracts	**431,13 million € /** 1.412,59 **million € = 30,52 %**	0 million € / 663,72 million € = 0 %	**226,99 € / 503,03 million € = 45,13 %**	-

Table continues

	Zavarovalnica Triglav d.d. (life, 2016)	Zavarovalnica Triglav d.d. (property, 2016)	Zavarovalnica Sava d.d. (life, 2016)	Zavarovalnica Sava d.d. (property,2016)
Other insurance technical provisions	3,91 million € / 1.412,59 million € = 0,28 %	3,78 million € / 663,72 million € = 0,57 %	0 million € / 503,03 million € = 0 %	6,24 million € / 415,35 million € = 1,5 %
Combined Mathematical provisions and Provisions for unit-linked insurance contracts	**(959,27 million € + 431,13 million €) / 1.412,59 million € = 98,43 %**	-	**(259,14 million € + 226,99 €) / 503,03 million € = 96,64 %**	-

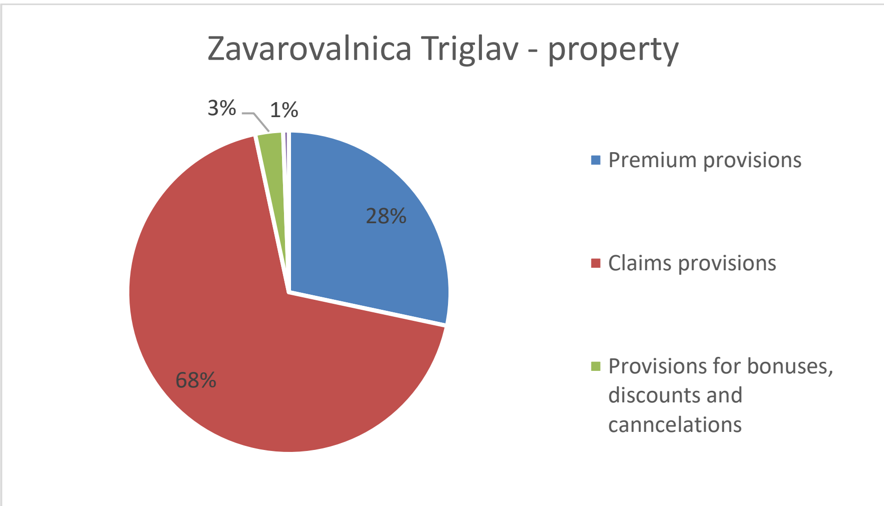
Source: Zavarovalnica Triglav d.d., 2016, p. 271; Zavarovalnica Sava d.d., 2016, p. 201, 202 & 204.

Figure 1: Composition of insurance technical provisions for life insurance business of Zavarovalnica Triglav d.d.



Source: Zavarovalnica Triglav d.d., 2016, p. 271; Zavarovalnica Sava d.d., 2016, p. 201, 202 & 204.

Figure 2: Composition of insurance technical provisions for property insurance business of Zavarovalnica Triglav



Source: Zavarovalnica Triglav d.d., 2016, p. 271; Zavarovalnica Sava d.d., 2016, p. 201, 202 & 204.

As expected both life insurers have majority of their insurance technical provisions in mathematical provisions. The share of mathematical provisions is around 60 %. Mathematical provisions, if we consider how they are calculated, are in fact much higher (around 95 %). Namely around 35 % of total provisions represent Provisions for unit-linked insurance contracts, from which according to 2016 Annual report of Zavarovalnica Sava d.d. more than 95 % are mathematical provisions. In property insurance business, claim provisions have the biggest share (around 70 %) of total technical insurance provisions.

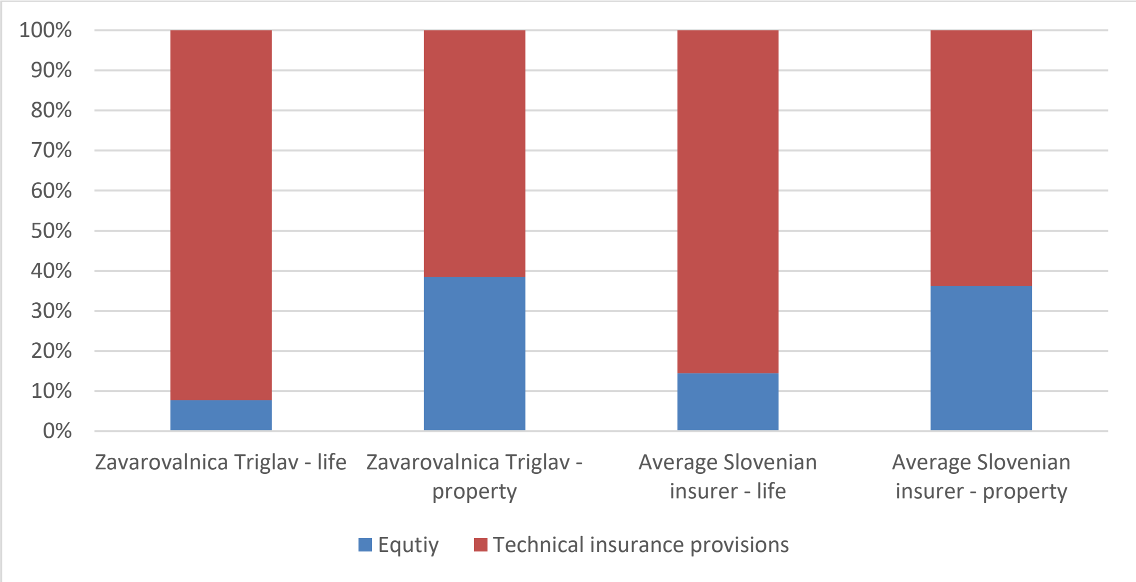
Another hypothesis we can make is that insurance technical provisions represent a higher part of all liabilities for life insurers compared to property insurers in other words life insurers have greater proportion of provisions and therefore smaller proportion of equity capital which means they have higher leverage compared to property insurer. The reason for that is greater volatility of property insurance business which needs to be compensated with higher buffer, that is, capital. The second reason is the nature of business. Life insurer has more long-term liabilities compared to property insurer therefore it also accumulates greater amount of liabilities. To test the hypothesis, we compare insurance technical provisions to total liabilities from 2015 Annual report of biggest Slovenian insurer Zavarovalnica Triglav d.d. separately for life and property business. In order to get a more general picture we also compare provisions to total liabilities for all Slovenian insurance companies from 2015 Report made by Slovenian Insurance Supervision Agency separately for life and property insurance.

Table 2: Comparison of technical insurance provisions to total liabilities and equity

	Slovenian life insurers (2015)	Slovenian property insurers (2015)	Zavarovalnica Triglav d.d. (life,2015)	Zavarovalnica Triglav d.d. (property,2015)
Technical insurance provisions (including Provisions for unit-linked insurance contracts) / Total liabilities	3.237,6 million € / 4.196,5 million € = 77,15 %	1.404,3 million € / 2.432 million € = 57,74 %	1.368,6 million € / 1.533,1 million € = 89,27 %	667,2 million € / 1.174,7 million € = 56,80 %
Equity / Total liabilities	544,4 million € / 4.196,5 million € = 12,97 %	797,5 million € / 2.432 million € = 32,79 %	113,7 million € / 1.533,1 million € = 7,42 %	416,9 million € / 1.174,7 million € = 35,49 %
Ratio between Technical insurance provisions and Equity	3.237,6 million € / 544,4 million € = 5,95	1.404,3 million € / 797,5 million € = 1,76	1.368,6 million € / 113,7 million € = 12,04	667,2 million € / 416,9 million € = 1,60

Source: Zavarovalnica Triglav d.d, 2015, p. 264 & 265; Insurance Supervision Agency, 2015, p. 37.

Figure 3: Comparison of ratio between technical insurance provision and equity for life and property insurance



Source: Zavarovalnica Triglav d.d, 2015, p. 264 & 265; Insurance Supervision Agency, 2015, p. 37.

According to Slovenian Insurance Supervision Agency average Slovenian life insurer in 2015 had around 77,15 % of all liabilities in Technical insurance provisions. The same ratio for property insurer was 57,74 %. Slovenian biggest insurer Zavarovalnica Triglav d.d. had even higher ratio of insurance technical provisions to total liabilities for life insurance business, that is, 89,27 %. The same ratio for property insurance business was 56,8 % which is similar to Slovenian average. Slovenian life insurers had 12,97 % equity in total liabilities, which is significantly lower compared to 32,79 % for Slovenian property insurers. The difference between equity to total liabilities ratio for life and property insurance business is more significant for biggest Slovenian insurer Zavarovalnica Triglav d.d. Company's equity amounted to just 7,42 % in life insurance business and 35,49 % in property insurance business. By comparing insurance technical provisions with equity, we get a better picture of the difference in leverage between life and property insurers. The ratio for average Slovenian life insurer is 5,95 and 12,04 for Zavarovalnica Triglav d.d. The same ratio for average Slovenian property insurer is 1,76 and 1,6 for property insurance business of biggest Slovenian insurer. Our results confirm our hypothesis, namely life insurers have higher leverage, that is, higher proportion of insurance technical provisions in total liabilities.

From the result we see, how important it is that insurers calculate the right amount of mathematical provisions. Mathematical provisions are calculated by actuaries therefore they play a vital part in business of an insurance company.

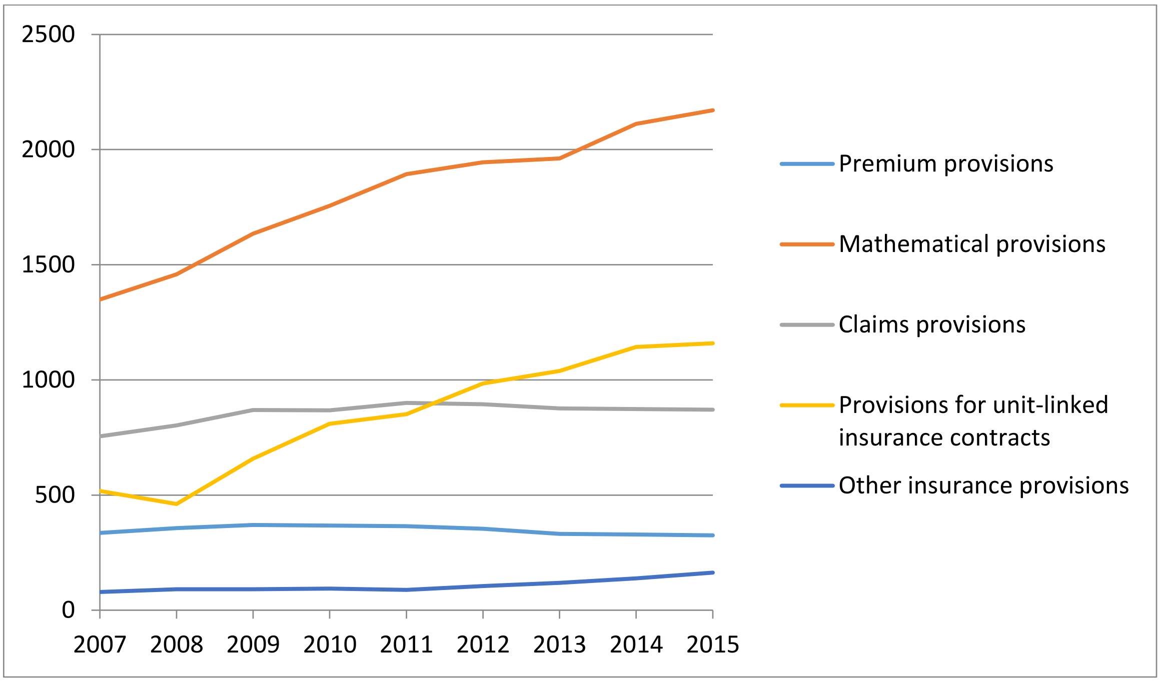
1.1 Insurance technical provisions development in Slovenia between 2007 and 2015

Usually insurance technical provisions are growing together with the size of a balance sheet. Between 2007 and 2015 balance sheet of Slovenian life insurers has grown from 2.299,5 million € in liabilities to 4.196,5 million € in liabilities or around 82,5 %. In the same period, Slovenian property insurers cumulative balance sheet has grown from 2.250,6 million € in liabilities to 2.432 million € in liabilities or around 8,06 % (ISA, 2008 & 2016).

In above calculations, we did not consider inflation. Throughout this and next chapter we consider annual inflation of 2 %. If we assume average annual inflation between 2007 and 2015 was 2 % than we can calculate approximate inflation between years 2007 and 2015 as $1{,}02^8 - 1 = 0{,}1717$ or 17,17 %. Accounting for inflation balance sheet of Slovenian life insurers has grown for about 56 % whereas balance sheet of property insurers has fallen for around 7 %.

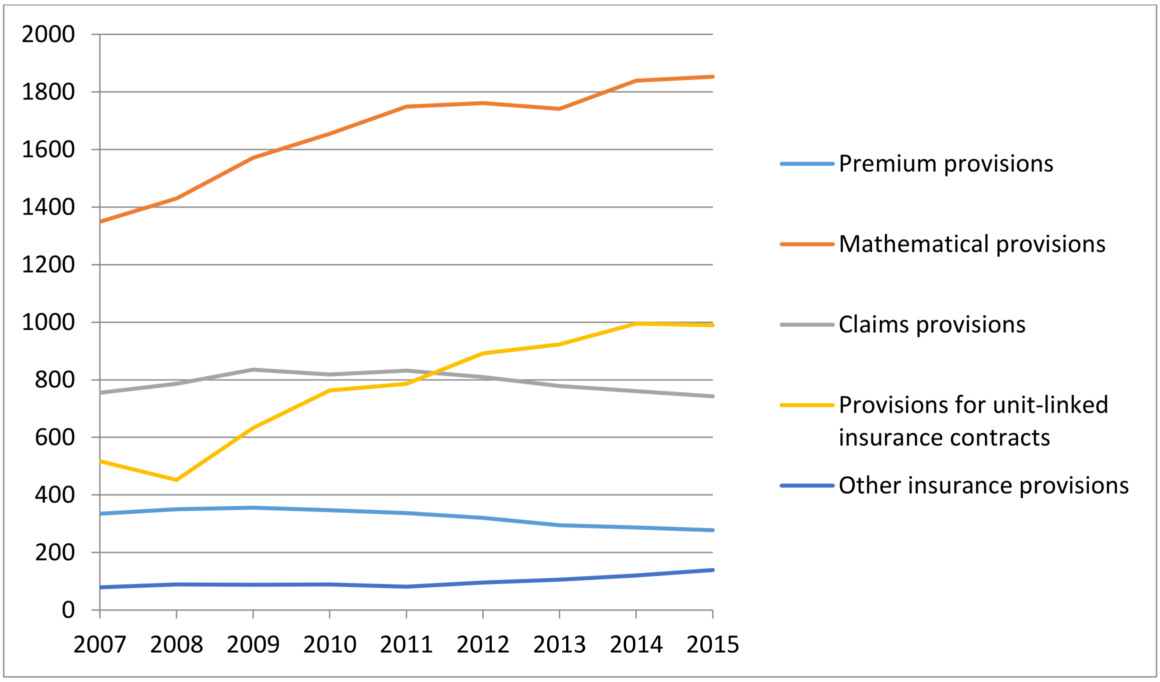
Gross written premiums have since 2007 slightly grown for property insurance (fallen if we consider inflation) business and even fallen for life insurance business. Average annual gross written premium for all Slovenian property insurers in years 2013, 2014 and 2015 was 1.395,67 million € which is 1,97 % increase compared to the 1.368,67 million €, which was average of gross written premiums for property insurers in years 2007, 2008 and 2009 (ISA, 2010 & 2016). The average gross written premium for life insurer between year 2013 and 2015 was 507,33 million €, which is 2,06 % decrease compared to the 518 million €, which was average gross written premium between years 2007 and 2009. Gross written premiums for both property and life insurers have in fact grown less if we consider inflation of 17 %. We see that the insurance business measured by gross written premiums has not grown much since 2007 but because premiums, especially in life insurance business, are accumulating we can expect mathematical provisions and provisions for unit-linked insurance contracts to grow with the balance sheet. On the other hand, claim and premium provisions are covering claims for shorter period therefore we expect they have not changed as significantly as mathematical provisions.

Figure 4: Movement of insurance technical provisions between 2007 and 2015 for all Slovenian insurance companies (in million €)



Source: Insurance Supervision Agency, 2015, p. 42; Insurance Supervision Agency, 2014, p. 42.

Figure 5: Movement of insurance technical provisions between 2007 and 2015 (in million €) for all Slovenian insurance companies considering adjustment for inflation. Estimated annual inflation is 2 %



Source: Insurance Supervision Agency, 2015, p. 42; Insurance Supervision Agency, 2014, p. 42.

As expected Mathematical provisions and Provisions for unit-linked insurance contracts have grown significantly more than Premium provisions and Claims provisions. From 2007 till 2015 Mathematical provisions have grown by 60,93 % and Provisions for unit-linked insurance contracts by 124,18 %. On the other hand, in the same period Claim provisions have grown by 15,23 % and Premium provisions have fallen by 2,99 %. If we consider inflation Claim provisions have remained similar (2 % fall) whereas Premium provisions have fallen for around 12 %.

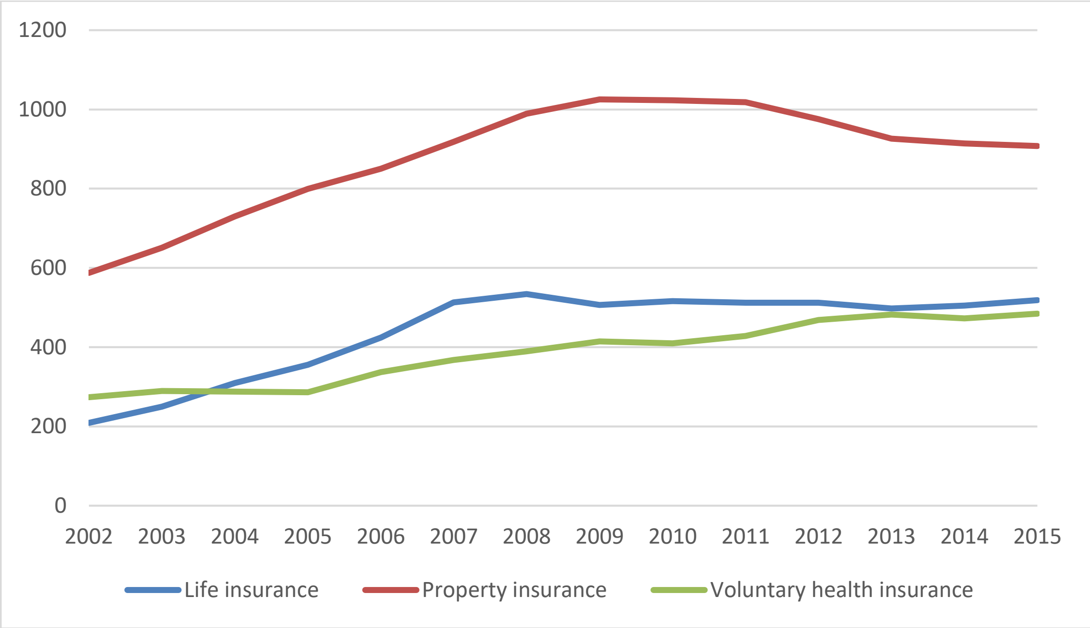
From the above graph, we see that insurance technical provisions are building up in life insurance business which also brings greater uncertainty about the future and greater need for transparency. Life insurers are exposed to the so-called longevity risk, which is a risk that the pay-outs will be higher than expected due to a trend of increasing life expectancy among policyholders. That risk is important especially for life insurers, who are paying supplementary pensions. Understanding of constantly changing life expectancy is also important for life insurers who offer claim payment in case of death. In order to maintain competitive prices insurer has to lower the premiums according to higher life expectancy.

From the above graph we also see, that insurance technical provisions are building up in life insurance business which also brings greater danger for losses in case of fraud or bad investments. Therefore, supervision and transparency of insurance business are very important. Namely, as long as the provisions are growing insurer could pay out claims without making any returns on investment or it could even make losses and still make payments.

1.2 Gross premiums written by Slovenian insurers

Insurance business can be divided into three major groups: 1. Life insurance, 2. Property insurance and 3. Voluntary health insurance. In 2015 gross premiums written by Slovenian insurance companies were 1.910,9 million € (ISA, 2016). Gross premiums written by Slovenian life insurers were 518,9 million €, which represents 27,15 % of total gross written premiums.

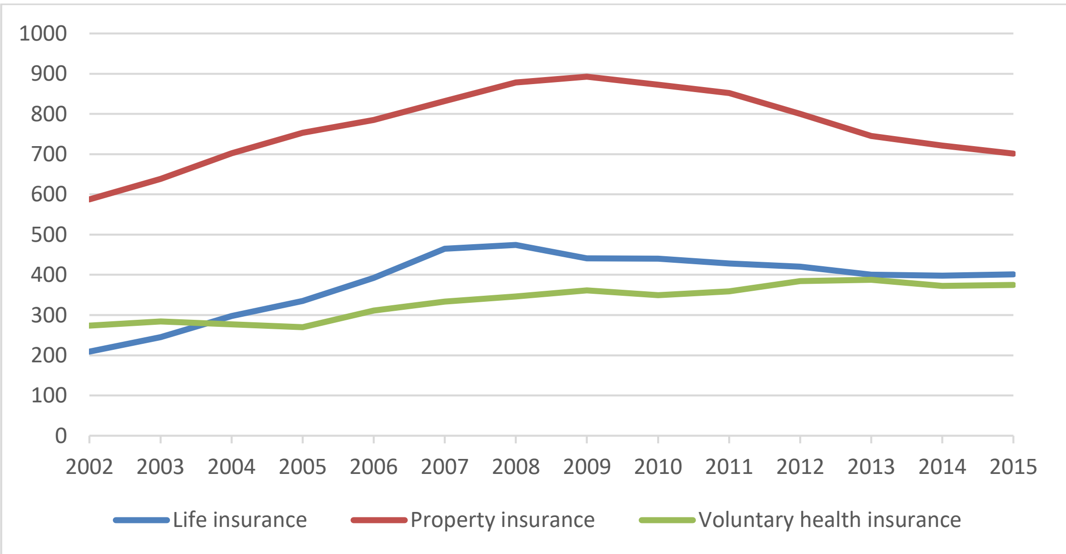
Figure 6: Gross premiums written by Slovenian Insurers between 2002 and 2015(in million €)



Source: Insurance Supervision Agency, 2015, p. 20 & 21; Insurance Supervision Agency, 2014, p. 21; Insurance Supervision Agency, 2011, p. 12; Insurance Supervision Agency, 2008, p. 5; Insurance Supervision Agency, 2005, p. 5; Insurance Supervision Agency, 2003, p. 4.

Gross insurance premium written by life insurance has grown from 209 million € in 2002 to 518,9 million € in 2015 or by 148,3 % which is the most among three major groups of insurance business. At the same period Property insurance gross premiums have grown for 54,37 % and Voluntary health insurance premiums by 77,03 % (ISA, 2002 & 2016).

Figure 7: Gross premiums written by Slovenian insurers (in million €) with adjustment for inflation considering inflation of 2 %



Source: Insurance Supervision Agency, 2015, p. 20 & 21; Insurance Supervision Agency, 2014, p. 21; Insurance Supervision Agency, 2011, p. 12; Insurance Supervision Agency, 2008, p. 5; Insurance Supervision Agency, 2005, p. 5; Insurance Supervision Agency, 2003, p. 4.

In our calculation, we did not account for inflation. If we say that average inflation between 2002 and 2015 was 2 % annually and we calculate $1{,}02^{13} - 1 = 29{,}36$ %, we can say around 29 % of premium growth can be attributed to inflation.

2 LIFE INSURANCE PRODUCTS

Life insurance products are generally bought for two reasons: to cover risk of death/occupational disability and to save money for future. If a person is an important contributor to family budget and if that person dies or is occupationally disabled, this can bring his/her family members to financial problems. In this case life insurance contract is meant to help close relatives cover: debts or loans, lost income, costs for raising children or care for other family members and burial costs. Therefore, life insurance is particularly suitable for families and persons who are in debt or want to take a loan. There are different suitable life insurance products that cover death/occupational disability risk such as: **term life insurance**, **occupational disability/severe illness/ long-term care insurance** and **whole life insurance**. Life insurance with focus on saving is suitable for people who want to save for the old age or for example want to finance child's education. Elderly can't work as hard as they worked in their most productive years but they are still used to the same standard of living. In order to smooth income distribution life insurance products that focus on savings can be bought such as: **endowment insurance** or **annuity insurance (**SIA - Types of Life Insurance, 2016).

Whole life insurance - insured sum is paid either on death or pre-determined age. A whole life insurance has a fixed premium no matter the future medical condition of the insured and fixed death benefit (Vijay & Tamilselvan, 2011).

Pure endowment insurance - insured sum is paid if insured is alive at the end of a policy period. In case insured dies within the policy period there is no payment. Policy is suitable for single people who are not planning to leave their money to anyone in case they die (Gerber, 1997).

Endowment insurance is a combination of whole life insurance and pure endowment insurance. Known insured sum is paid at the end of policy life time or death (Gerber, 1997). Thus, endowment insurance combines death protection as well as maturity benefit. Policyholder pays a fixed premium periodically during the premium paying period and he/she does not have any influence on the investment policy. Because the paid claim amount is fixed the insured does not take any investment risk but also the return is usually smaller compared to more aggressive policies. Law prescribes conservative investment strategy with focus on safety. There is a frequent lag between final premium and pay out (Vijay & Tamilselvan, 2011, pp. 1-4). The policy is suitable for policyholders who look especially for safety and less for high returns. Although there is a guarantee about a certain return, this return may not even cover the inflation. It is suitable for older people with children who are not yet financially independent (Vijay & Tamilselvan, 2011).

Annuity insurance - annuities are paid till the rest of the life of the insured. Insured pays a lump sum or a number of payments. Contributions earn interest, usually tax deferred, and after a period of time provide insured with a stream of income (What is an Annuity, 2016). There are different types of annuity insurance. Deferred annuity starts paying annuities after some time has passed since the last premium was paid. We also know immediate annuity where usually an insured pays a lump sum and soon after the first annuity is paid. Furthermore, we can distinguish between, fixed-term or lifelong annuity insurance, with or without term-life component annuity insurance and recurring or single premium annuity insurance. Annuity insurance is suitable as a complement to pensions which are because of negative demographic trend in Europe becoming lower and therefore insufficient for decent living. One of the main advantages of annuity insurance is government support (e.g. Slovenia, Germany) through tax reliefs (Lombardi, 2006).

Term life insurance - life insurance for certain number of years (term) (Gerber, 1997). Insured pays premiums to cover death event for either multi- or one-year duration. In case of death within the term there is a death payment, but if the insured those not die within the insured period he/she gets nothing, that is, there is no savings feature. If the insured wants to renew the contract he/she would usually have to provide evidence of good health. Besides the premiums increase with an increasing age (Nationwide, 2010). The policy is suitable for young people or for families with a limited budget that need a large amount of life insurance protection (Vijay & Tamilselvan, 2011, pp. 1-4).

Disability/severe illness/ long-term care insurance is an insurance against the risk of disability/long term nursing-care needs which usually doesn't include »savings« part. Policy can have one year or multiyear duration and the insured sum can be paid as one lump sum or long-term periodic payment. The most common severe illnesses are cancer, heart attack, stroke, blindness, kidney failure and others.

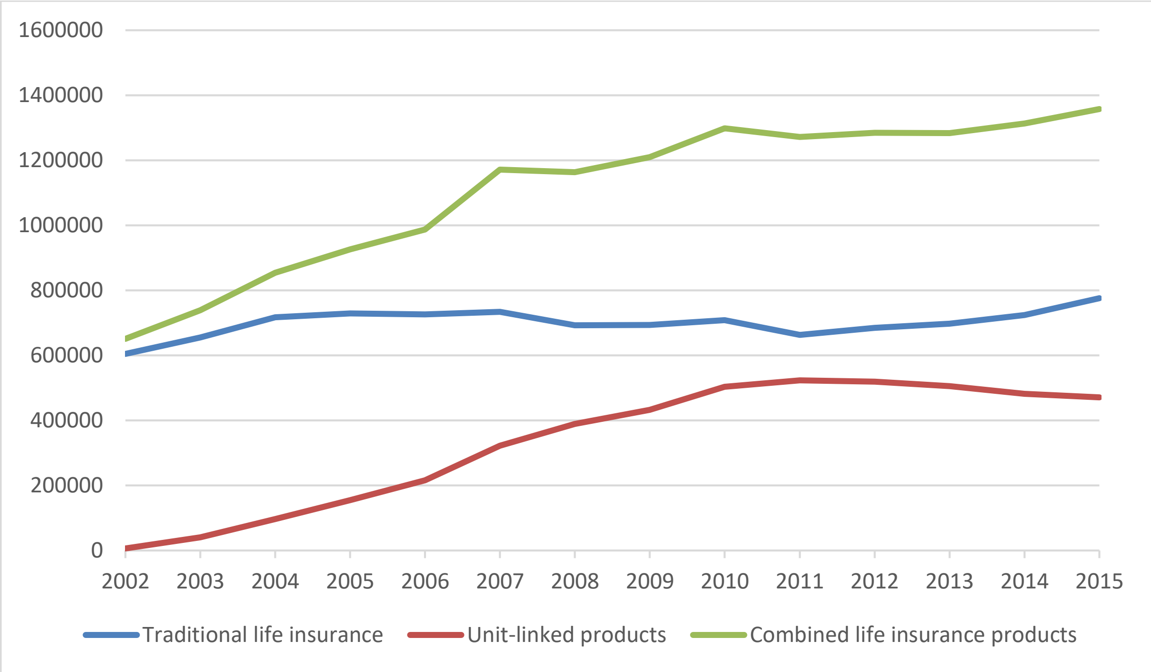
Insurance products with guarantees usually provide low returns which sometimes do not even cover the inflation. In recent years insurers have developed new products which offer higher returns but also higher risk. Such products are called **Unit-linked products**. In Slovenia Unit-linked products are allowed under the law ZPIZ-2 which was passed in December 2012.

Unit-linked products provide a combination of whole life insurance and investment in funds with different expected return and riskiness. In case of death during policy period, usually a fixed amount of insured sum is paid to beneficiaries independently of fund returns. Insurer runs a number of funds with different investment strategies. More aggressive funds contain a greater investment in stocks and bonds with medium safety. Less aggressive funds are largely composed of bonds with high and medium safety. Insured's assets are invested in funds based on his/her age. The idea is that younger policyholders can afford to take higher risk because they have longer time till their retirement. Young insured starts off in the riskiest fund and gradually passes through less risky funds. Before retirement all policyholder's assets are in fund with guaranteed return. Investment fund is initially suggested by the insurer but especially young policyholders have the flexibility of investing in riskier or less risky funds (Prva, 2017). With flexibility comes also investment risk, namely opposite to traditional saving products investment risk is with policyholder. Policy is suitable for insured who wants a combination of life insurance and decent returns. Because capital markets are prone to ups and downs positive returns are expected in the long run (SIA - Types of Life Insurance, 2016).

Life insurance products can be divided into four groups: 1. Traditional life insurance products, 2. Unit-linked products 3. Insurance with capitalization of payments and 4. Other life insurance. Gross premiums written by Slovenian life insurers in 2015 for all types of insurance products were 518,9 million €, of which, 255,4 million € or 49,22 % were attributed to traditional life insurance products, 224,3 million € or 43,23 % to unit-linked products, 38,9 million € or 7,5 % to insurance with capitalization of payments and 0,3 million € or 0,06 % to Other life insurance products. Traditional life insurance products and Unit-linked products are the most important types as they together represent 92,45 % of all gross premiums written by Slovenian life insurers (ISA, 2016).

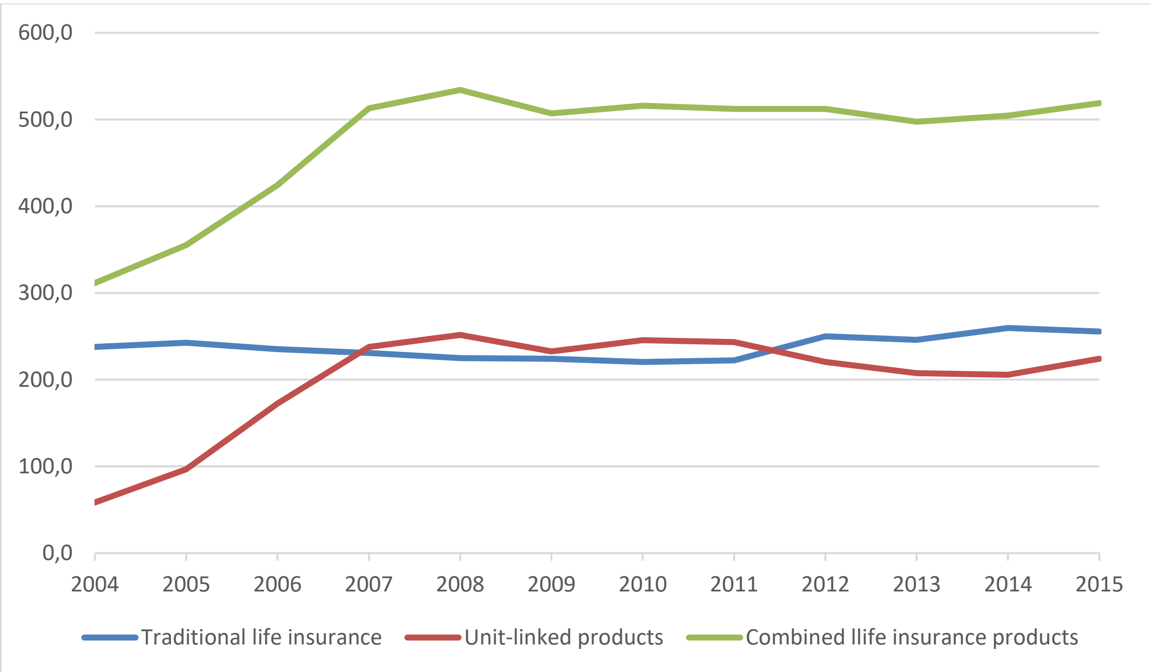
Number of life insurance policyholders has been growing in recent years, especially for unit-linked products. In year 2002 the number of life insurance policyholders was 650.954 of which only 6.393 were unit-linked policyholders and 604.640 or 92,89 % were policyholders of traditional insurance products. Five years later total number of life insurance policyholders has grown by 79,99 % to 1.171.657 policyholders but the number of unit-linked policyholders grew to 322.141 (around 50-times) from just 6.393 in 2002. In 2015 the total number of life policyholders was 1.357.610 of which 775.889 or 57,15 % were policyholders of traditional products and 471.202 or 34,71 % were unit-linked policyholders (ISA, 2015).

Figure 8: Number of traditional life, unit-linked and total number of life insurance policyholders



Source: Insurance Supervision Agency, 2016, p. 23; Insurance Supervision Agency, 2014, p. 24; Insurance Supervision Agency, 2012, p. 20; Insurance Supervision Agency, 2010, p. 11; Insurance Supervision Agency, 2008, p. 8; Insurance Supervision Agency, 2006, p. 10; Insurance Supervision Agency, 2005, p. 8; Insurance Supervision Agency, 2003, p. 8.

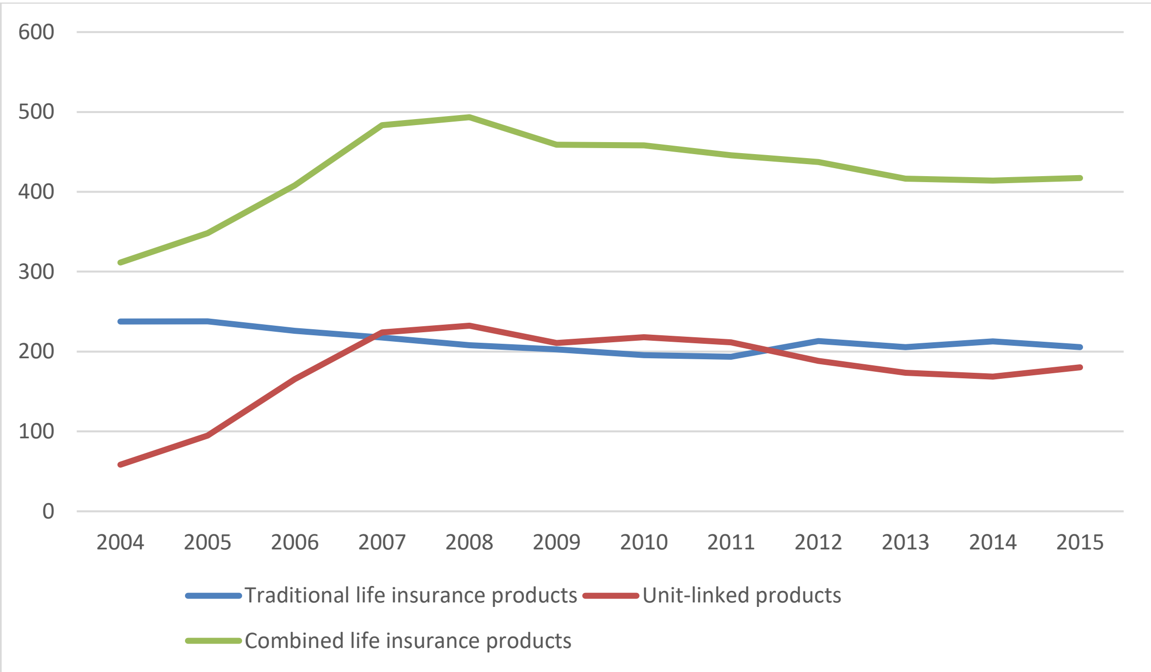
Figure 9: Gross premiums written for traditional life, unit-linked and combined life insurance products (in million €)



Source: Insurance Supervision Agency, 2016, p. 23; Insurance Supervision Agency, 2014, p. 24; Insurance Supervision Agency, 2012, p. 20; Insurance Supervision Agency, 2010, p. 11; Insurance Supervision Agency, 2008, p. 8; Insurance Supervision Agency, 2006, p. 10; Insurance Supervision Agency, 2005, p. 8.

From the two graphs (see Fig. (8 and 9)) we can see the development of purchase of different life insurance products. From 2002 the number of policyholders has been growing especially for unit-linked products. Between 2004 and 2015 number of unit-linked policyholders has increased by 387,77 % and gross written premiums of unit-linked products have grown by 284 % or 209 % if we consider annual inflation of 2%. Between 2007 and 2008 gross written premiums of unit-linked products even exceeded gross written premiums of traditional life insurance products but after 2011 the number of unit-linked products and gross written premiums has been falling. Traditional insurance products gross written premiums have grown from 2004 till 2015 by 7,49 %, which is less than inflation of 24,34 %, if we consider annual inflation growth of 2 %.

Figure 10: Gross premiums written for traditional, unit-linked and combined life insurance products adjusted for inflation (in million €). Annual inflation of 2 % is taken



Source: Insurance Supervision Agency, 2016, p. 23; Insurance Supervision Agency, 2014, p. 24; Insurance Supervision Agency, 2012, p. 20; Insurance Supervision Agency, 2010, p. 11; Insurance Supervision Agency, 2008, p. 8; Insurance Supervision Agency, 2006, p. 10; Insurance Supervision Agency, 2005, p. 8.

Unit-linked products are attractive because they offer potential higher returns compared to traditional life insurance. In times before crises, when capital markets were growing, more and more policyholders were deciding for unit-linked products. Since the crisis the market for unit-linked products has been cooling and it has not yet rebounded to its peak (ISA, 2016).

3 PREMIUM CALCULATION IN LIFE INSURANCE

In order to calculate premiums for life insurance contracts we need three key figures: interest rates, mortality rates and cost rates. Interest rates are needed in order to calculate present value of future premiums and benefits. Mortality rates tell us the probability that a person aged x will die in a year t. Cost rates include acquisition cost, premium collection cost and administration cost.

3.1 Interest rate

Interest rates that insurers use to calculate mathematical provisions are limited with so called maximal technical interest rates. Maximal technical interest rates (hereinafter: maximal TIR) are usually determined by the regulator (e.g. in Slovenia and Germany) who considers long-term rolling average of government bond yields. In recent years, interest rates in Europe have been falling and regulators have reacted by lowering maximal technical interest rates to adapt to new environment. In Slovenia and Germany many contracts are offered with guarantees and insurers normally match guaranteed rate with maximal TIR because that means prospectively calculated interest rates equal zero. If insurer would offer a higher guaranteed rate than maximal TIR than he would have positive initial provision and therefore he would need to prefinance it which is not desirable. If guaranteed rate would be lower than maximal TIR than insurer would expect more income than payments which is desirable but it might be uncompetitive (Moneyland.ch, 2017).

Table 3: Movement of maximal interest rate in Germany

Period	Maximal interest rate
from 01.07.1994 until 01.06.2000	4 %
01.07.2000 - 31.12.2003	3,25 %
01.01.2004 – 31.12.2006	2,75 %
01.01.2007 – 31.12.2011	2,25 %
01.01.2012 – 31.12.2014	1,75 %
from 01.01.2015	1,25 %

Source: Aktuare, 2017.

Because of low interest rates and therefore unattractive products for customers, insurers are trying to invent new products with less guarantees and higher potential gains but of course higher risk (Eling & Holder, 2011). In Slovenia, maximal TIR is determined by ISA. From 01.07.2015 maximal TIR is determined at 1,75 %.

3.2 Mortality rates and tables

Mortality rate is a measure of deaths in a particular population within a certain time horizon. Insurers are interested in mortality rates of the insured in order to predict future deaths, claims and to calculate premiums. There are different factors that affect mortality such as age, gender, health status, profession, smoking habits etc. (Olivieri & Pitacco, 2010). In the past insurers were particularly interested in mortality dependent on gender and age. They used mortality tables which show probability that a person at certain age will die within next year. On the 1 of March 2011 the Court of Justice of the EU made a ruling, based on which insurers have until 21st of December 2012 to change their pricing in such a way that it doesn't use gender information as a risk factor. The change is made according to the EU initiative to promote gender equality. Mortality tables which were in the past calculated separately for men and women needed to be changed and unisex mortalities have to be calculated (European Commission, 2012). Because of this change certain people now pay lower or higher premiums compared to a fair premium.

3.2.1 Types of mortality/life tables

In order to calculate premiums and provisions insurers use mortality and life tables which include data about mortality rates for men and women of different ages (under new regulation unisex mortality rates). Mortality tables show how a population is dying through ageing. They are used when insurance company is facing the risk of death (e.g. term insurance). Mortality tables include safety margin that makes mortality too high. When insurance company faces survival risk (e.g. annuities) it uses life tables which include mortality that is lower compared to reality. Deviations are made to build a safety margin.

There are many different mortality/life tables which differ in the observed data and methods of calculation. Mortality data can originate from observing a whole national population, specific part of population or an insurer's portfolio. Life tables that observe whole national populations are usually called population life tables. We also know market life tables which are constructed based on mortality of insurer's portfolio. Insurer uses different market life tables for different products as mortality rates may significantly differ. There are also life tables which are combination of population and market life tables, that is especially true when the insurer doesn't have sufficient data and therefore he uses data from population tables (Olivieri & Pitacco, 2010). Based on calculation method we distinguish between cohort life table and period life table.

Period mortality table – mortalities are calculated for each age group within a certain short observation period in other words it analyses existing population across the various ages. For example, if observation period is year 2008 than to get mortality in the first year of life we look at the mortality of new-borns in 2008 or if we want to get mortality of 30-year-old people we take mortality of 30-year olds in 2008. With the same method, we get mortalities for people aged from 0 to 100 and we can construct a mortality table.

Period mortality tables are used when insurance company faces mortality risk (e.g. term insurance). People who are born later are expected to live longer so period mortality tables provide a safety margin. In other words, mortalities are overestimated and mortality risk is smaller compared to what is shown in mortality table. For example, consider mortality table that was constructed based on observing period which is year 2008 and the mortality rate for an 80-year-old man in that mortality table which is 10,0261 %. That means in year 2008 10,0261 % of 80-year-old men died. We are in year 2017 now and 80-year-old men have lower mortality rate in other words 10,0261 % is overestimation.

Cohort mortality/life tables – mortalities are calculated for a group which was born in the same year. While period mortality table observes a population (e.g. 100.000 people) who lived in the same year, cohort mortality tables observe a population which was born in the same year. In order to construct such a table, we would theoretically have to wait until all people in a cohort die which means we would usually need to wait 100 years or more to construct a complete cohort table. To shorten the construction period, the missing values are estimated. That means the younger the cohort less precise is the table because more values have to be estimated. Another problem is that people can move to live in a foreign country and that means they can no longer be observed (if we consider mortality tables are made for certain nationality) (Sterbetafel, 2017).

Cohort mortality/life tables are used when insurance company faces a risk of survival (e.g. annuities). Basically, what insurance company is interested in is life expectancy of people born in different years. Cohort table is calculated for a particular group of people which were born in the same year (cohort). For example, we are in year 2000 and we construct mortality table in the same year 2000 which measures mortality of men born in 1970. We know exact mortality rates of men born in 1970 till their 30th year. For future year mortalities, we will have to make estimations. In any case these estimations should be lower compared to mortalities calculated with period mortality table constructed in 2000. In ten years (2010) now 30-year-old John will be 40 years old. Period mortality table from 2000 would predict his mortality rate by taking mortality rate for 40-year-old man in year 2000 but that is not right because that is estimation for a man born in 1960 and our John was born in 1970. In reality John has a lower mortality rate and that lower rate is estimated in cohort mortality tables.

Cohort mortality tables can be used also for people born in different years by taking into account the so-called age adjustment. Let's go back to our example of cohort mortality table with observing period year 2000 and calculated for men born in 1970. We are now in year 2000 and we bring George, 20 years old, born in 1980 into our discussion. Actuary in pension insurance company wants to know what is his mortality rate in year 2000. If we look for mortality rate for 20-year-old men in our cohort table constructed for cohort born in 1970 we will find a mortality rate that is too high in other words survival chance is greater in reality. That is because George was born in 1980 and not in 1970. In order to consider this fact age adjustment has to be made if we want to use our cohort mortality table. Although George is now 20 years old we consider him for example to be three years younger (17 years old). We take mortality rate of 17-year-old men born in 1970 to estimate George's mortality rate.

Slovenian ISA has determined that mortality tables have to be chosen by the insurer carefully and the probability of survival shouldn't be higher than survival probability in German life table DAV 1994 R (ISA, 2014). In our calculations, we will use newer German life table DAV 2004 R.

3.2.2 Structure of mortality tables

Mortality tables usually include more data than the one presented in the table below but all other information can be calculated from the mortality rates q_x. Mortality rate q_x tells us what is the expected proportion of people aged x who will die within next year.

Table 4: Example of mortality table

X	q_x	p_x	l_x	d_x
0	0,006113	0,993887	1.000.000	6.113
1	0,000423	0,999577	993.887	420
2	0,000343	0,999657	993.467	341
...	...	...	...	...
50	0,003981	0,996019	947629	3773
51	0,004371	0,995629	943856	4126
...	...	...	...	...
99	0,461101	0,538899	1335	615
100	1,0000	0,0000	719	719

Source: DAV, Herleitung der Sterbetafel Dav 2008 T für Lebensversicherung mit Todesfallcharakter, p. 38.

x = age of the insured
q_x = mortality rate at age x
p_x = survival probability at age x
l_x = estimated number of people alive at age x
d_x = number of deaths at age x

With mortality rates for different ages we can calculate all the numbers presented in table above. We assume our starting population has 1.000.000 people and maximum attainable age, denoted by ω, equals 100. That means $l_{101} = 0$. Here are some examples how we got the numbers:

- $p_1 = 1 - q_1 = 1 - 0{,}000423 = 0{,}999577$
- l_1 = size of starting population × p_0 = 1.000.000 × 0,993887 = 993.887
- $d_1 = q_1 \times l_1 = 0{,}000423 \times 993.887 = 420$

For our premium calculation, we use German mortality tables DAV 2008 T when there is a mortality risk and life tables DAV 2004 R when there is a survival risk. Because unisex mortality rates should be used under new regulation we calculate unisex tables (in Appendixes). In order to make things simpler we assume an equal portion of men and women in a portfolio, that is $q_{unisex}=0{,}5*(q_{men}+q_{women})$.

DAV 2008 T – is a period mortality table with observation period between 2001 and 2004. Mortalities were calculated from data collected from German reinsurance companies (Gen Re, der Münchener Ruck), Swiss Re and data from German Federal Statistical Office. Whole observation includes 104.029.858 of Total number of person years lived and 390.667 deaths. In order to calculate mortality, the following formula is used: $q_x=d_x/l_x$ for period between 2001 and 2004. With this calculation raw mortality rates were calculated and then graduated with Whittaker-Henderson methodology. Mortality rates are calculated for men and women separately (DAV, 2008).

DAV 2004 R – is a cohort mortality table with a base generation born in 1965 and observation period between 1995 and 2002. Again, data for mortality calculations was collected from German reinsurance companies Gen Re, Münchener Ruck and German Federal Statistical Office. Observation includes 20 cohorts and around 13,7 million of Total number of person years lived. DAV 2004 R tables include age adjustment tables which tell us for how many years should we consider a person born after 1965 to be younger and how many years older if the person was born before 1965. Mortality rates are calculated for men and women separately. DAV 2004 R substitute older mortality table DAV 1994 which underestimated the survival probability (DAV, 2004).

3.2.3 Mortality tables and probabilities

With the help of data in mortality tables we can calculate probabilities that help us by our premium and provision calculation. Probability that a person aged x will be alive at age x+h is expressed with following equation:

$$ {}_hp_x = p_x * p_{x+1} * \dots * p_{x+h-1} \qquad (2)$$

Probability that a person aged x will die before age x+h equals:

$$ {}_hq_x = 1 - {}_hp_x \qquad (3)$$

Now let's consider probability (see Eq. (3)) that an individual at age x lives till age h and then dies within next year. We need this probability to calculate actuarial present value of an amount payable whenever the death occurs (whole life insurance).

$$ {}_{h}p_{x} * q_{x+h} \tag{4} $$

h = time when the benefit is paid

3.3 Component of costs

When we calculate gross premium for an insurance policy we consider costs the insurer incurs. There are three main types of expenses associated with policies – initial expenses, renewal expenses and termination expenses (Dickson, Harvey & Waters, 2009).

Initial expenses are paid at inception of insurance contract. There are two major types of initial costs – compensation for agents selling a policy and underwriting expenses. Compensation for agents is usually paid as a percentage of the total gross premiums paid. Let α present percentage of total gross premiums paid, then compensation for agents = $\alpha \times n \times P$; where n = number of times premium is paid, P = premium. Underwriting expenses are usually related to insured sum. If insured some is greater the insurer has for example a greater motive to conduct more stringent medical tests compared with insured with lower insured sum (Dickson et al, 2009).

Renewal cots are paid each year as part of the premium and are linked to gross premium (P) or insured sum (IS). Renewal costs cover compensation for staff time and investment expenses as well as ongoing administration costs of the insurer such as staff salaries and rent for the insurer's premises, as well as specific costs such as annual statements to policyholders about their policies (Dickson et al, 2009). Let β denote costs for premium collection which are linked to the annual gross premium ($\beta \times P$). And let γ denote costs for administration which are linked to the insured sum ($\gamma \times IS$).

Termination costs occur when a policy expires, that is when an insured dies or on the maturity date of an endowment or term insurance. Usually these costs are small, and are largely connected with paperwork to finalize and pay a claim. When calculating gross premiums, specific allowance is often not made for termination expenses (Dickson et al, 2009).

3.4 Actuarial values

In insurance business many cash flows (premiums, benefits) are paid in the future and because of their nature are related to uncertainty. For example, the insured has agreed to pay an annual premium for the next ten years in order to finance his term life insurance contract. But what if he dies before he pays all the premiums. Insurer has to consider that possibility by taking into account mortality rates and calculation of the so called actuarial value.

Actuarial value is an expected present value of future cash flows and is determined by mortality rates and interest rate.

Survival benefit is the actuarial value of 1 unit that is paid at the maturity if the insured is alive at that time. If we multiply present value of the survival benefit with the insured sum we get the benefit for **pure endowment insurance**, that is a lump sum insured gets in case he is alive at contract maturity (Olivieri & Pitacco, 2010). Present value of the survival benefit is defined by the following equation:

$$ {}_{h}E_{x} = v^{h} * {}_{h}p_{x} \tag{5} $$

h = time when the benefit is paid

$v^h = (1+i)^{-h}$ = discount factor

${}_{h}p_{x}$ = h years survival probability of a x-year old person (see Eq. (3))

Whole life annuities are cash flows that are paid at the beginning of the year as long as the insured is alive. In order to calculate actuarial value of 1 unit for the whole life annuity we have to multiply each year survival probabilities from year x until the maximum attainable age (Olivieri & Pitacco, 2010):

$$ \ddot{a}_{x} = \sum_{h=0}^{\omega-x} {}_{h}E_{x} \tag{6} $$

ω = maximum attainable age

If the amounts are payable for m years the actuarial value of **temporary life annuity** equals each year survival probabilities from year x until year m (Olivieri & Pitacco, 2010):

$$ \ddot{a}_{x:m]} = \sum_{h=0}^{m-1} {}_{h}E_{x} \tag{7} $$

Deferred life annuities are annual payments payable as long as the insured is alive, beginning from time r. The present value of a r period deferred life annuity is given by (Olivieri & Pitacco, 2010):

$$ {}_{r|}\ddot{a}_{x} = \sum_{h=0}^{\omega-x} {}_{h}E_{x} = \ddot{a}_{x} - \ddot{a}_{x:r]} \tag{8} $$

Till now we were multiplying survival probabilities, because endowment insurance and annuity cash flows are connected with survival. **Whole life insurance** is an insurance of death from year x till maximum attainable age ω. In order to calculate actuarial value of an amount payable at the end of the period whenever the death occurs we use the following formula (Olivieri & Pitacco, 2010):

$$A_x = \sum_{h=0}^{\omega\text{-}x} v^{h+1} * {}_h p_x * q_{x+h} \tag{9}$$

If insurance period of life insurance is limited to time m than we talk about **term insurance**. Actuarial present value of a benefit paid at the end of that year, if this occurs within m years is (Olivieri & Pitacco, 2010):

$$_m A_x = \sum_{h=0}^{m-1} v^{h+1} * {}_h p_x * q_{x+h} \tag{10}$$

Endowment insurance benefit is obtained by combining the benefits of pure endowment and the term insurance. The unitary amount is paid at the end of the year of death if it occurs before m or at the time m at least (Olivieri & Pitacco, 2010).

$$A_{x,m]} = {}_m E_x + {}_m A_x \tag{11}$$

3.5 Commutation functions

Commutation functions are functions that allow actuaries to calculate present values of contingent payments. They are based on deterministic model of survival and a constant interest rate. Because of their deterministic nature they were mostly replaced by computers and calculators which allow actuaries to expand from deterministic to stochastic models. Although commutation functions are not necessarily needed anymore they are still commonly used to present actuarial calculations and can still be found in many books, computer programs and government regulation (Macdonald, 2004, pp. 300-302).

Actuarial values presented in the chapter above can be calculated with the help of commutation functions. Next, we present commutation functions that will be used in our calculations.

By discounting number of living people at age x, where $v = \frac{1}{1+i}$ represents discount factor, we get **discounted living of age x:**

$$D_x = l_x * v^x \tag{12}$$

Cumulative sum of $\boldsymbol{D_x}$ is written as follows:

$$N_x = \sum_{k=0}^{\infty} D_{x+k} \tag{13}$$

When discounting dead of age x, where $d_x = l_x * q_x$, we get **discounted dead of age x**:

$$C_x = d_x * v^{x+1} \tag{14}$$

Cumulative sum of $\boldsymbol{C_x}$ is written as follows (Macdonald, 2004, pp. 300-302):

$$M_x = \sum_{k=0}^{\infty} C_{x+k} \tag{15}$$

With the commutation functions mentioned above we can calculate actuarial values which will be used in our premium calculation. We calculate above commutation functions for each year in mortality tables separately for men, women and unisex which you find under Appendixes. Interest rate used is 1,75 % which is maximal technical interest rate determined by Slovenian Insurance Supervision Agency.

Actuarial value of pure endowment insurance for period m equals:

$${}_mE_x = \frac{D_{x+m}}{D_x} \tag{16}$$

Actuarial value of whole life annuity is written as follows:

$$\ddot{a}_x = \frac{N_x}{D_x} \tag{17}$$

Actuarial value for temporary life annuity for period m equals:

$$\ddot{a}_{x:m\rceil} = \frac{N_x - N_{x+m}}{D_x} \tag{18}$$

Actuarial value for whole life insurance is:

$$A_x = \frac{M_x}{D_x} \tag{19}$$

Actuarial value of term insurance is written as follows:

$$ {}_{m}A_x = \frac{M_x - M_{x+m}}{D_x} \tag{20} $$

3.6 Premium calculation examples

Essential cash flows in the insurance companies are premiums, losses, investment income and costs. Insurer has to charge a single premium or more premiums that will cover future benefits and expenses. In order to make calculation simpler we do not consider investment income. Also, investment income fluctuates and therefore premiums themselves should cover the benefits and costs. To calculate premiums, we will use *equivalence principle* which says that expected present value (shortly, the actuarial value) of premiums, denoted by E(P), should equal sum of expected present value of benefits, denoted by E(L), and expected present value of costs E(C).

Equivalence principal: E(P) = E(L) + E(C)

Premiums can be paid as a single premium, denoted by SP, or annually as annual premiums, denoted by AP. Also benefits can be either paid as a single benefit, denoted by IS (insured sum), or annuities, denoted by A. In our calculations, we make following assumptions about premiums and benefits:

- premiums are paid at the beginning of the year
- benefits are paid at the end of the year

3.6.1 Example 1 – Pure endowment insurance

Women aged 55, born in 1961, buys a pure endowment insurance. The survival benefit is 100.000 € and is payable at the age 80. Duration of the contract is 25 years and interest rate is 1,75 %. We will calculate: 1. probability that she is alive at age 80 2. premium if it is paid as a single premium 3. premium if it is paid annually 4. premium if it is paid annually and an acquisition cost rate of $\alpha = 1$ % is charged.

Pure endowment insurance is connected with the risk of survival therefore we use mortalities from life table DAV 2004 R for unisex. In DAV 2004 R mortalities are underestimated which gives us a buffer.

Input data: z = 55 (born in 1961); age adjustment = 0; IS = 100.000€; m = 25; i = 1,75 %;
$\alpha = 0{,}1$ %; acquisition cost = $\alpha \times m \times AP$

1. With the help of unisex mortality tables we calculate probability she is alive at age 80:

$$ {}_{25}p_{55} = \frac{l_{80}}{l_{55}} = \frac{869840}{968708} = 0{,}8979 \approx 90\ \% \tag{21} $$

2. Single premium she has to pay at the beginning of the first year:

$$
\begin{aligned}
&E(P)=E(L)\\
&SP=IS * {}_{25}E_{55}\\
&SP=IS * \frac{D_{80}}{D_{55}}\\
&SP=100.000 * \frac{217.113}{373.078}\\
&SP=58.195{,}07\ €
\end{aligned} \tag{22}
$$

3. Annual premium she has to pay until age 80:

$$
\begin{aligned}
&E(P)=E(L)\\
&AP* \ddot{a}_{55:\overline{25}|}=IS* {}_{25}E_{55}\\
&AP* \frac{N_{55}-N_{55+25}}{D_{55}} =IS* \frac{D_{80}}{D_{55}}\\
&AP=IS* \frac{D_{80}}{(N_{55}-N_{80})}\\
&AP=100.000* \frac{217.113}{(10.604.539-3.210.996)} =2.936{,}52\ €
\end{aligned} \tag{23}
$$

4. Annual premium if acquisition cost α is charged:

$$
\begin{aligned}
&E(P)=E(L)+E(C)\\
&AP* \ddot{a}_{55:\overline{25}|}= IS* {}_{25}E_{55}+\alpha*m*AP\\
&AP=IS* \frac{{}_{25}E_{55}}{\ddot{a}_{55:\overline{25}|}-\alpha*m}\\
&AP=100.000* \frac{\frac{D_{80}}{D_{55}}}{\frac{N_{55}-N_{55+25}}{D_{55}}-0{,}01*25}\\
&AP=100.000* \frac{\frac{217.113}{372.087}}{\frac{(10.604.539-3.210.996)}{372.087}-0{,}01*25}\\
&AP=2.974{,}04\ €
\end{aligned} \tag{24}
$$

3.6.2 Example 2 – Whole life insurance

Man aged 25, born in 1991, wants to buy a whole life insurance with annual premium payment. Insured sum is 200.000 €, maximal attainable age is ω = 100 and the interest rate equals 1,75 %. We will calculate: 1. annual premium if it is paid until the end of the contract 2. annual premium if it is paid in next 25 years.

Whole life insurance is connected with death risk therefore we use mortalities from mortality table DAV 2008 T for unisex. In this table mortalities are overestimated which gives us a certain buffer.

Input data: z = 25; IS = 200.000; ω = 100; i = 1,75%

1. Annual premium if it is paid until the end of the contract:

$$\begin{aligned}
&E(P)=E(L)\\
&AP*\ddot{a}_{25}=IS*A_{25}\\
&AP*\frac{N_{25}}{D_{25}}=IS*\frac{M_{25}}{D_{25}}\\
&AP=IS*\frac{M_{25}}{N_{25}}\\
&AP=200.000*\frac{259.403}{22.074.322}=2.350,27\text{ €}
\end{aligned} \tag{25}$$

2. Annual premium if it is paid in next 25 years:

$$\begin{aligned}
&E(P)=E(L)\\
&AP*\ddot{a}_{25:\overline{25}|}=IS*A_{25}\\
&AP*\frac{N_{25}-N_{50}}{D_{25}}=IS*\frac{M_{25}}{D_{25}}\\
&AP=IS*\frac{M_{25}}{N_{25}-N_{50}}\\
&AP=200.000*\frac{259.403}{22.074.322-9.105.013}\\
&AP=4.000,26\text{ €}
\end{aligned} \tag{26}$$

3.6.3 Example 3 – Term insurance

Man aged 30, wants to buy a term insurance with policy period 10 years and annual premium payment. The insured sum is 150.000 € and the interest rate is 1,75 %. Acquisition costs (α) are 0,004, compensation for premium collection (β) is 0,006 and compensation for administration (γ) is 0,002. Because insurer is exposed to risk of death unisex DAV 2008 T mortality table will be used. We will calculate: 1. annual premium without considering the costs 2. annual premium by also including the costs.

Input data: z = 30; IS = 150.000 €; m = 10; i = 1,75 %; α = 0,004; β = 0,006; γ = 0,002

1. Annual premium without considering the costs.

$$
\begin{aligned}
&E(P)=E(L)\\
&AP*\ddot{a}_{30:10\rceil}=IS*\ {}_{10}A_{30}\\
&AP=IS*\frac{M_{30}\text{-}M_{40}}{D_{30}}*\frac{D_{30}}{N_{30}\text{-}N_{40}}\\
&AP=150.000*\frac{257.763\text{-}254.074}{18.990.419\text{-}13.593.144}\\
&AP=102{,}52\ €
\end{aligned}
\tag{27}
$$

2. Annual premium considering also costs α, β and γ.

$$
\begin{aligned}
&E(P)=E(L)+E(C)\\
&AP*\ddot{a}_{30:10\rceil}=IS*\ {}_{10}A_{30}+\alpha*m*AP+\ddot{a}_{30:10\rceil}*(\beta*AP+\gamma*IS)\\
&AP*\ddot{a}_{30:10\rceil}-\alpha*m*AP-\beta*\ddot{a}_{30:10\rceil}*AP=IS*\ {}_{10}A_{30}+\ddot{a}_{30:10\rceil}*\gamma*IS\\
&AP*\left(\ddot{a}_{30:10\rceil}-\alpha*m-\beta*\ddot{a}_{30:10\rceil}\right)=IS*{}_{10}A_{30}+\ddot{a}_{30:10\rceil}*\gamma*IS\\
&AP=\frac{IS*{}_{10}A_{30}+\ddot{a}_{30:10\rceil}*\gamma*IS}{\left(\ddot{a}_{30:10\rceil}-\alpha*m-\beta*\ddot{a}_{30:10\rceil}\right)}\\
&AP=\frac{150.000*\frac{M_{30}-M_{40}}{D_{30}}+\frac{N_{30}-N_{40}}{D_{30}}*0{,}002*150.000}{\frac{N_{30}-N_{40}}{D_{30}}-0{,}004*10-\frac{N_{30}-N_{40}}{D_{30}}*0{,}006}\\
&AP=\frac{150.000*\frac{257.7\ 3-254.074}{584.379}+\frac{18.990.419-13.593.144}{584.379}*300}{\frac{18.990.419-13.593.144}{584.379}-0{,}04-\frac{18.990.419-13.593.144}{584.379}*0{,}006}\\
&AP=406{,}73\ €
\end{aligned}
\tag{28}
$$

3.7 Mathematical provisions

In life insurance company, there is a need to build mathematical provisions because premiums paid in a certain year do not equal benefits paid in the same year. Normally annual premiums remain equal throughout the policy period whereas risks that person will die on any given year increases with aging. We can distinguish between natural premiums and annual premiums. Natural premiums follow the actual risk of death and can be defined as an insured sum times the actuarial value of term insurance for the next year (${}_{x+1}A_x$). On the other hand, annual premiums are premiums that are paid each year by the insured and are calculated as an average of total risk of death. At the beginning of the multiyear contract annual premium is relatively high compared to natural premium and relatively low at the end of the multi period contract. Because of that insurer has to build a provision in the beginning to cover higher benefits in the future (Pešić – Andrijić, 2011).

Insured pays higher premiums at the beginning of the contract so he/she does not have to pay higher premiums at older age. Older people are usually less productive and it is therefore harder for them to pay higher premiums. Therefore, it is reasonable that premium payment is equally distributed through the life of the insured (Pešić – Andrijić, 2011).

Annual premium paid by the insured can be divided into two parts. First part is there to cover the risk of payment (current risk), that is expected benefit paid by the insurer due to claims on that year and the second part is excess premium income. Excess premium should not be regarded as a profit. Instead it should be put aside to build mathematical provisions which will cover future benefits that will surpass premiums. Mathematical provisions can be calculated by using two methods: 1. the retrospective method and 2. the prospective method. Both methods calculate provisions for a certain point in time. The retrospective method calculates provisions by deducting already paid benefits from the already paid premiums. The prospective method calculates premium reserve by deducting expected present value of future premium payments from expected present value of future benefits. Equation for prospective net reserve, denoted by V_i, is derived from $E(L)=E(P)$ which equals $V_t=E(L_t)$ - $E(P_t)$. Word prospective is used to stress that future benefits and premiums are considered and word net denotes that we are not considering expenses and related loadings. It should be noted that the above equation is used to calculate provision for single policy and not the whole portfolio (Olivieri & Pitacco, 2010).

3.7.1 Example 1 – Pure endowment insurance

Let's take our example of women aged 55, born in 1961, who bought a pure endowment insurance in our premium calculation exercise. The survival benefit is 100.000 € and is payable at age 80 and the contract duration is 25 years. We will calculate: 1. premium reserves in years t = 0, 10, 20 and 25 in case of annual premium payment. Commutation functions are calculated based on mortality rates of unisex DAV 2004 R mortality table.

1. Premium reserves in years t = 0, 10, 20, 25 in case of annual premium payment.

$$
\begin{aligned}
&{}_tV_x=E\big(L(t)\big)\text{-}E\big(P(t)\big)\\
&{}_tV_x=IS^*{}_{m-t}E_{x+t} - AP^*\ddot{a}_{x+t:\overline{m-t}|}\\
&{}_0V_{55}=IS^*{}_{25}E_{55} - AP^*\ddot{a}_{55:\overline{25}|}\\
&{}_0V_{55}=0
\end{aligned}
\tag{29}
$$

In the year t = 0 expected present value of future benefits (E(L(t)) equals expected value of future benefits (E(P(t)) therefore the premium reserve equals 0. Bellow we will calculate reserve in time (t = 10) and for annual premium we will take the premium that we got in the first premium calculation exercise without considering costs (see Eq. (21)).

$$
\begin{aligned}
&{}_{10}V_{65}=IS^*{}_{15}E_{65} - AP^*\ddot{a}_{65:\overline{15|}}\\
&{}_{10}V_{65}=IS^*\ \frac{D_{80}}{D_{65}} - AP^*\frac{N_{65}-N_{80}}{D_{65}}\\
&{}_{10}V_{65}=100.000^*\ \frac{217.113}{306.700}-2.936{,}52^*\frac{7.179.842-3.210.996}{306.700}\\
&{}_{10}V_{65}=32.790\ €
\end{aligned}
\tag{30}
$$

After ten years since contract was made ten premiums were already paid so expected future premiums have decreased. Furthermore, actuarial value of benefit is also higher compared to t = 0, because there is greater probability that a 65-year-old woman will live till 80 years compared to women aged 55 years. Therefore, premium reserve is now positive and is increasing each year. Next, we calculate premium reserve in year t = 20.

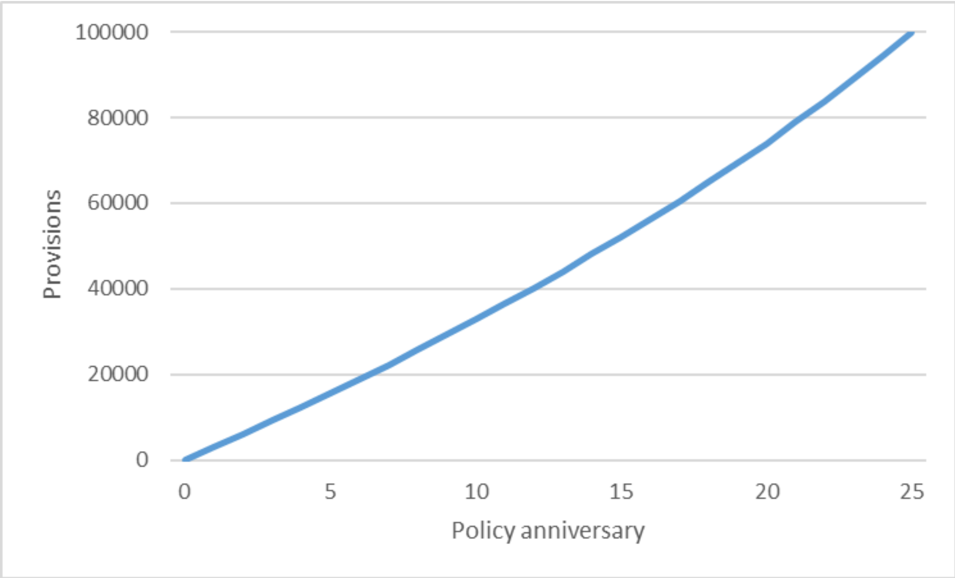
$$
\begin{aligned}
&{}_{20}V_{75}=IS^*\ {}_{5}E_{75}-AP^*\ddot{a}_{75:\overline{5|}}\\
&{}_{20}V_{75}=IS^*\ \frac{D_{80}}{D_{75}}-AP^*\frac{N_{75}-N_{80}}{D_{75}}\\
&{}_{20}V_{75}=100.000^*\ \frac{217.113}{246.745}-2.936{,}52^*\frac{4.385.977-3.210.996}{246.745}\\
&{}_{20}V_{75}=74.007\ €
\end{aligned}
\tag{31}
$$

In year t = 20 the reserve has again raised compared to year t = 10. Next, we calculate premium reserve at contract maturity (t=25).

$$
\begin{aligned}
&{}_{25}V_{80}=IS^*\ {}_{0}E_{80}-AP^*\ddot{a}_{80:\overline{0|}}\\
&{}_{25}V_{80}=IS^*\ \frac{D_{80}}{D_{80}}-AP^*\frac{N_{80}-N_{80}}{D_{80}}\\
&{}_{25}V_{80}=100.000^*\ 1-2.936{,}52^*\frac{0}{217.113}\\
&{}_{25}V_{80}=100.000\ €
\end{aligned}
\tag{32}
$$

At the contract maturity, premium reserve equals insured sum which is logical because insurer has to pay exactly 100.000 € to the insured.

Figure 11: Provisions (in €) pure endowment insurance



Source: Own work.

In Figure 11 we see that provisions for pure endowment insurance are growing more or less linearly till the contract maturity.

3.7.2 Example 2 – Whole life insurance

As in Example 2 in premium calculation exercise we are considering a 25-year-old man, who bought a whole life insurance with annual premium payment, maximal attainable age ω = 120 and insured sum of 200.000 €. We will calculate: 1. Premium reserves for years t = 0, 25, 50, and 75. Insurer is exposed to risk of death therefore we use unisex DAV 2008 T mortality table. Annual premium is taken from the Example 2 (see Eq. (23)).

1. Premium reserves for years t = 0, 25, 50, 75.

$$
\begin{aligned}
{}_tV_x &= E\big(L(t)\big) - E\big(P(t)\big) \\
{}_0V_{25} &= IS * A_{25} - AP * \ddot{a}_{25} \\
{}_0V_{25} &= IS * \frac{M_{25}}{D_{25}} - AP * \frac{N_{25}}{D_{25}} \\
{}_0V_{25} &= 200.000 * \frac{259.403}{639.060} - 2.350{,}27 * \frac{22.074.322}{639.060} \\
{}_0V_{25} &\approx 0\ €
\end{aligned}
\tag{33}
$$

At the issue date, present value of expected benefits equal expected present value of expected premiums therefore premium reserve equals 0.

$$
\begin{aligned}
&{}_{t}V_{x}=E\big(L(t)\big)\text{-}E\big(P(t)\big)\\
&{}_{25}V_{50}=IS*A_{50}\ \text{-}\ AP*\ddot{a}_{50}\\
&{}_{25}V_{50}=IS*\frac{M_{50}}{D_{50}}\text{-}AP*\frac{N_{50}}{D_{50}}\\
&{}_{25}V_{50}=200.000*\frac{245.775}{402.372}\ \text{-}\ 2.350{,}27*\frac{9.105.013}{402.372}\\
&{}_{25}V_{50}\approx 68.980{,}35\ €
\end{aligned}
\tag{34}
$$

When insured is 50 years old premium reserve increases to 68.980,35 €.

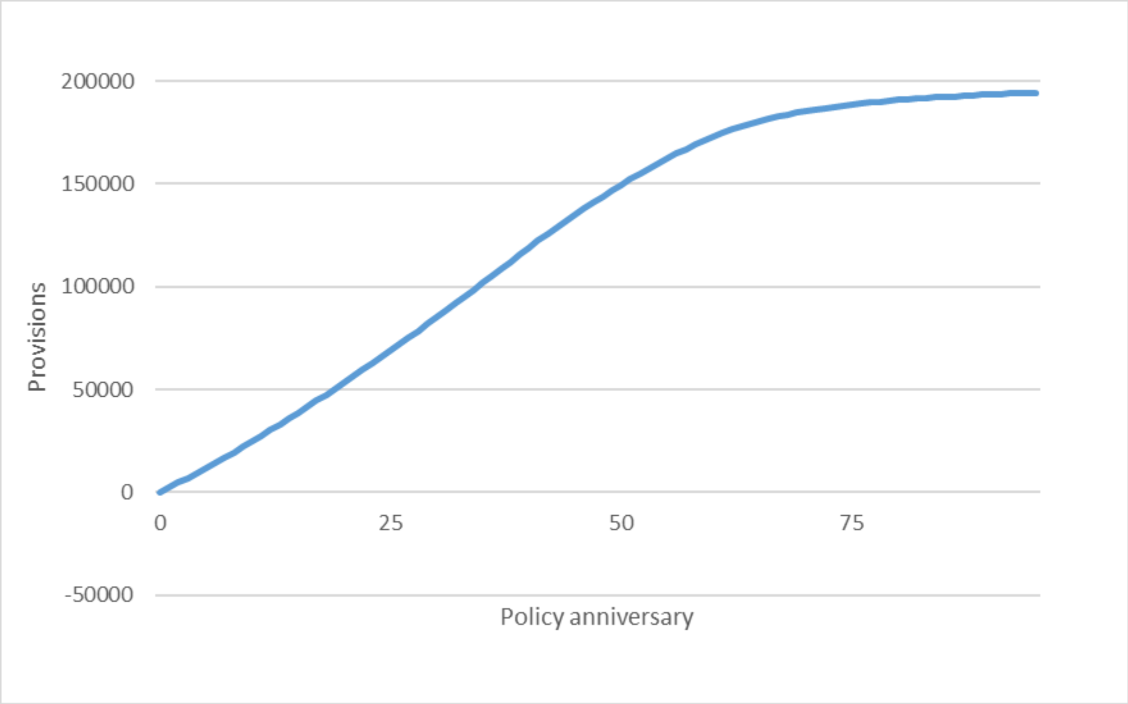
$$
\begin{aligned}
&{}_{t}V_{x}=E\big(L(t)\big)\text{-}E\big(P(t)\big)\\
&{}_{50}V_{75}=IS*A_{75}\text{-}AP*\ddot{a}_{75}\\
&{}_{50}V_{75}=IS*\frac{M_{75}}{D_{75}}\ \text{-}\ AP*\frac{N_{75}}{D_{75}}\\
&{}_{50}V_{75}=200.000*\frac{151.735}{178.467}\text{-}2.350{,}27*\frac{1.554.237}{178.467}\\
&{}_{50}V_{75}\approx 149.574{,}56\ €
\end{aligned}
\tag{35}
$$

When insured is 75 years old premium reserve is 149.574,56 €. We can see that premium reserve are increasing almost linearly.

$$
\begin{aligned}
&{}_{t}V_{x}=E\big(L(t)\big)\ \text{-}\ E\big(P(t)\big)\\
&{}_{75}V_{100}=IS*A_{100}\ \text{-}\ AP*\ddot{a}_{100}\\
&{}_{75}V_{100}=IS*\frac{M_{100}}{D_{100}}\ \text{-}\ AP*\frac{N_{100}}{D_{100}}\\
&{}_{75}V_{100}=200.000*\frac{236}{245}\ \text{-}\ 2.350{,}27*\frac{482}{245}\\
&{}_{75}V_{100}\approx 188.029{,}26\ €
\end{aligned}
\tag{36}
$$

At age 100 the premium reserve is 188.029,26 €. In the third 25-year interval (from age 75 to 100) the premium reserve has increased less than in first and second 25-year interval (from age 50 to 75). That is because according to DAV 2008 T mortality tables average person will die at age 80 and insurer has to prepare by building premium reserve before that age. The result can be nicely seen in the graph bellow. The curve is almost linear in first 50 years, after 50 years, it becomes more gradual. When insured reaches 120 years, which is maximal age we expect him to reach, reserve equals insured sum minus the last premium.

Figure 12: Provisions (in €) whole life insurance



Source: Own work.

3.7.3 Example 3 – Term insurance

We again consider the man aged 30 (example 3 under premium calculation), who is 30 years old and buys a 10-year term insurance with annual premium payment. We will calculate: 1. Premium reserve in time t = 0, 4, 8 and 10 without considering the costs.

$$\begin{aligned}
{}_tV_x &= E\big(L(t)\big) - E\big(P(t)\big)\\
{}_0V_{30} &= IS * {}_{10}A_{30} - AP * \ddot{a}_{30:10]}\\
{}_0V_{30} &= IS * \frac{M_{30}-M_{40}}{D_{30}} - AP * \frac{N_{30}-N_{40}}{D_{30}}\\
{}_0V_{30} &= 150.000 * \frac{257.763 - 254.074}{584.379} - 102{,}52 * \frac{18.990.419 - 13.593.144}{584.379}\\
{}_0V_{30} &= 0\ €
\end{aligned} \tag{37}$$

At the issue date reserve equals 0 € because expected future benefits equal expected future premiums.

$$\begin{aligned}
{}_tV_x &= E\big(L(t)\big) - E\big(P(t)\big)\\
{}_4V_{34} &= IS * {}_6A_{34} - AP * \ddot{a}_{34:6]}\\
{}_4V_{34} &= IS * \frac{M_{34}-M_{40}}{D_{34}} - AP * \frac{N_{34}-N_{40}}{D_{34}}\\
{}_4V_{34} &= 150.000 * \frac{256.504 - 254.074}{543.974} - 102{,}52 * \frac{16.714.347 - 13.593.144}{543.974}\\
{}_4V_{34} &= 81{,}83\ €
\end{aligned} \tag{38}$$

After four years premium reserve is 81,83 €. Reserve is low compared to insured sum because probability that the insured dies is low. Actuarial value for term insurance between age 34 and 40, denoted by ${}_6A_{34}$, is 0,004467.

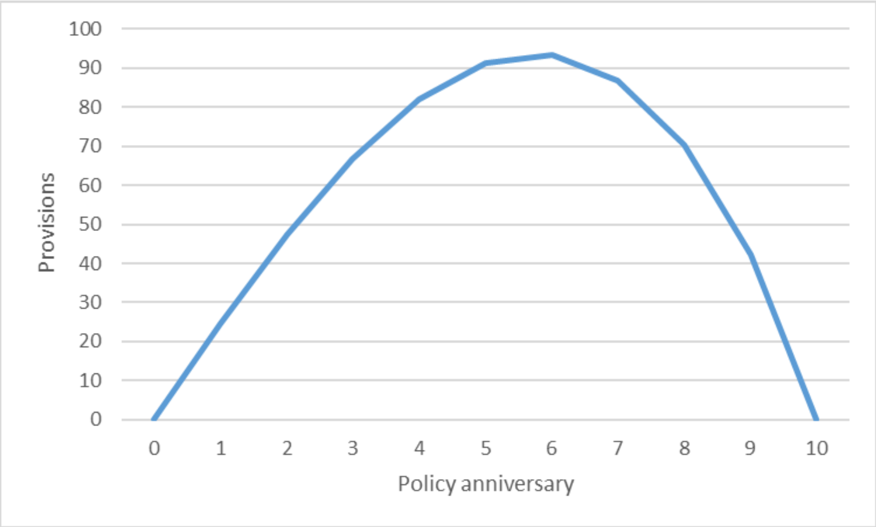
$$
\begin{aligned}
&{}_tV_x = E\big(L(t)\big) - E\big(P(t)\big) \\
&{}_8V_{38} = IS * {}_2A_{38} - AP * \ddot{a}_{38:2\rceil} \\
&{}_8V_{38} = IS * \frac{M_{38} - M_{40}}{D_{38}} - AP * \frac{N_{38} - N_{40}}{D_{38}} \\
&{}_8V_{38} = 150.000 * \frac{254.997 - 254.074}{506.035} - 102{,}52 * \frac{14.596.067 - 13.593.144}{506.035} \\
&{}_8V_{38} = 70{,}41 \text{ €}
\end{aligned}
\tag{39}
$$

In time t=8 premium has decreased to 70,41 €.

$$
\begin{aligned}
&{}_tV_x = E\big(L(t)\big) - E\big(P(t)\big) \\
&{}_{10}V_{40} = IS * {}_0A_{40} - AP * \ddot{a}_{40:0\rceil} \\
&{}_{10}V_{40} = IS * \frac{M_{40} - M_{40}}{D_{40}} - AP \times \frac{N_{40} - N_{40}}{D_{40}} \\
&{}_{10}V_{40} = 150.000 \times \frac{0}{487.862} - 102{,}52 \times \frac{0}{487.862} \\
&{}_{10}V_{40} = 0 \text{ €}
\end{aligned}
\tag{40}
$$

At the end of policy period premium reserve equals 0 €. Because by term insurance insured sum is paid only in case of death insurer has no obligation if the insured is alive when contract matures.

Figure 13: Provisions (in €) term insurance



Source: Own work.

From the figure above we see that provisions are initially growing, because the paid annual premium exceeds the corresponding natural premium, after it starts to decrease, and it equals zero at the end, because insurer has no obligation to the policyholder if he survives till the contract maturity. We also see that the provision is very small compared to the insured sum, because the probability that our example policyholder dies between age 30 and 40 is small.

4 NON-LIFE INSURANCE PRODUCTS

Non-Life insurance or property and casualty insurance include many different contracts but mostly health insurance, motor vehicle insurance and property insurance. Historically, many insurers have developed their own policies that can be quite varied in form and content (Williams, Smith & Young, 1995). In order to better understand different types of non-life insurance we will present classification used by OECD. OECD has organized non-life insurance products into 8 classes and for each class it has defined what it covers. For each type of class, we also write (bellow the name of the class) what is the amount of business in Slovenia. Amount is expressed in million Euros as average of three years starting from 2013 till 2015. We also present an annual amount in percentage of total non-life business in Slovenia between years 2013 and 2015. For comparison, we present sales numbers for USA. We have to consider that United States dollar grew for around 18 % between 2014 and 2015, which inflates USA numbers in comparison to Slovenian.

Table 5: Classes and coverage of non-life insurance products

CLASS	**COVERAGE**
1. Motor Vehicle Insurance • 448 million € (SLO) • 32,3 % • 176.984 million € (USA) • 19,8 %	LAND VEHICLES (other than railway rolling stock) All damage to or loss of: • Land motor vehicles, • Land vehicles other than motor vehicles. MOTOR VEHICLE LIABILITY All liability arising out of the use of motor vehicles operating on land (including carriers' liability).
2. Marine, Aviation and Other Transport Insurance • 7 million € (SLO) • 0,5 % • 1.330 million € (USA) • 0,1 %	RAILWAY ROLLING STOCK AND OTHER TRANSPORT All damage to or loss of railway rolling stock. AIRCRAFT All damage to or loss of aircraft. SHIPS (sea, lake, and river and canal vessels) All damage to or loss of: • River and canal vessels • Lake vessels • Sea vessels.

Table continues

	AIRCRAFT LIABILITY All liability arising out of the use of aircraft (including carrier's liability). LIABILITY FOR SHIPS (sea, lake, and river and canal vessels) All liability arising out of the use of ships, vessels or boats on the sea, lakes, rivers or canals (including carrier's liability).
3. Freight Insurance • 8 million € (SLO) • 0,6 % • 18.348 million € (USA) • 2,1 %	GOODS IN TRANSIT (including merchandise, baggage and all other goods) All damage to or loss of goods in transit or baggage, irrespective of the form of transport.
4. Fire and Other Property Damage Insurance • 226 million € (SLO) • 16,3 % • 108.064 million € (USA) • 12,1 %	FIRE AND NATURAL FORCES All damage or loss of property (other than land vehicles, railway rolling stock, aircraft, ships and goods in transit) due to: fire, explosion storm, natural forces other than storm, nuclear energy and land subsidence. OTHER DAMAGE TO PROPERTY All damage to or loss to property (other than land vehicles, railway rolling stock, aircraft, ships and goods in transit) due to hail or frost, and any event such as theft, other than those mentioned under FIRE AND NATURAL FORCES.
5. Pecuniary Loss Insurance • 48 million € (SLO) • 3,5 % • 23.131 million € (USA) • 2,6 %	CREDIT Includes: insolvency (general), export credit, instalment credit, mortgages and agricultural credit. SURETYSHIP MISCELLANEOUS FINANCIAL LOSS Includes: employment risk, insufficiency of income (general), bad weather, loss of benefits, continuing general expenses, unforeseen trading expenses, loss of market value, loss of rent or revenue, indirect trading losses other than those mentioned above, other financial loss (non-trading) and other forms of financial loss.
6. General Liability Insurance • 55 million € (SLO) • 4 % • 88.130 million € (USA) • 9,9 %	GENERAL LIABILITY All liability other than motor vehicle liability, aircraft liability and liability for ships.

Table continues

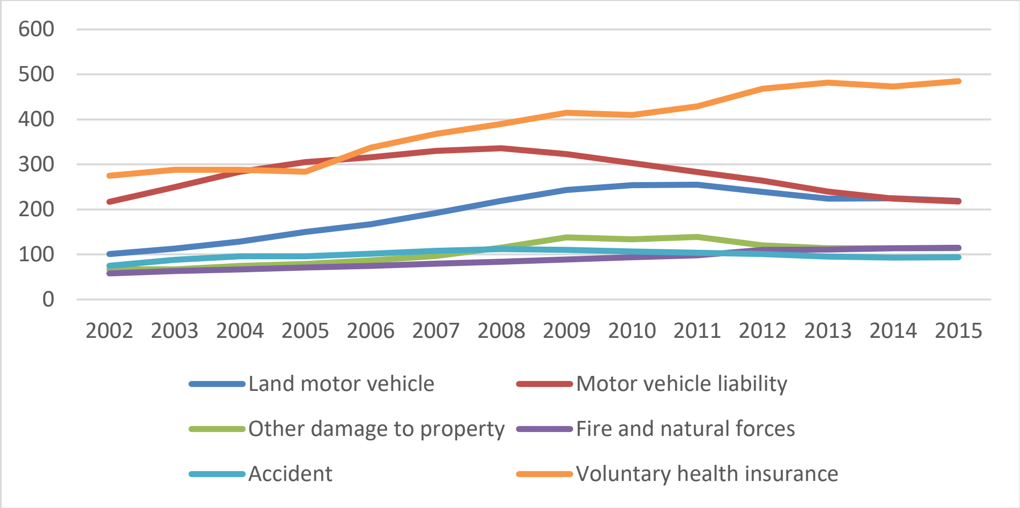
7. Accident and Health • 571 million € (SLO) • 41,1 % • 476.525 million € (USA) • 53,3 %	ACCIDENT (including industrial injury and occupational diseases) Includes: fixed pecuniary benefits, benefits in the nature of indemnity, combinations of the two and injury to passengers. SICKNESS Includes: fixed pecuniary benefits, benefits in the nature of indemnity and combinations of the two.
8. Other Non-Life Insurance • 25 million € (SLO) • 1,8 % • 2.114 million € (USA) • 0,2 %	LEGAL EXPENSES Legal expenses. ASSISTANCE MISCELLANEOUS

Source: OECD, Insurance business written in the reporting country, 2017.

Slovenian ISA monitors the amount of gross premiums written by Slovenian non-life insurers for different lines of business. ISA classifies non-life insurance products somewhat differently than OECD. Non-life insurance lines are divided into: 1. Property insurance and 2. Voluntary health insurance. As seen in Figure 6, Property insurance is the largest insurance type with 907 million € gross premiums written in 2015. That number represents 47,46 % of total 2015 gross premiums written in Slovenia. Voluntary health insurance gross premiums in 2015 were 484,6 million €, which is 25,36 % of total 2015 gross premiums written in Slovenia.

From year 2002 till 2009 there was a trend of increasing gross premiums written in property insurance line. Since 2010 the trend turned downwards. In order to find the reason for such movement we look at the premiums for the five biggest lines of property insurance from year 2002 till 2015.

Figure 14: Gross premium written by different lines of property insurance and voluntary health insurance



Source: Insurance Supervision Agency, 2015, p. 23; Insurance Supervision Agency, 2014, p. 22; Insurance Supervision Agency, 2011, p. 13; Insurance Supervision Agency, 2008, p. 6; Insurance Supervision Agency, 2005, p. 6; Insurance Supervision Agency, 2003, p. 5.

From the figure above we can see that gross premium written for all major non-life insurance lines had been growing till year 2008. That was the result of growing economy and increasing standard of living. Namely, between 2003 and 2008 Slovenian GDP was growing annually on average by around 8 %. After the crisis, between 2009 and 2015, Slovenian GDP has experienced on average negative annual growth of less than 1 %. People reduced their demand for insurance due to lower purchasing power. Reduction of demand was especially large in Motor vehicle liability line. Some large companies in construction and transport business went bankrupt, which additionally contributed to lower number of registered vehicles and less premiums. Number of newly registered cars in Slovenia was growing on average, from year 2002 till 2008, by 10,55 % annually. The same number, between 2009 and 2015, was – 2,12 % (Statistical Office RS, 2016). Increased price competition between motor vehicle insurers also contributed to lower gross written premiums.

Voluntary health insurance has experienced growth till year 2013 (see Figure 3) and a slight drop in 2014 due to high competition and therefore lower prices but it rebounded in 2015.

5 PREMIUM CALCULATION IN NON-LIFE INSURANCE

The premium is the price charged for the insurance sold by the insurance industry. As with any other industry pricing is important for insurance company to stay competitive. If price is too low company suffers loss and can go bankrupt. On the other hand, price level that is too high will probably not be competitive for a very long time. It is the actuary's job to find a method of premium calculation or so called premium calculation principles to charge the right premium (Straub, 1997).

5.1 Underwriting risk

In an ideal world, insurers could calculate premiums by using the above presented equivalence principle, where we equated expected present value of premiums with expected present value of benefits and costs. In reality insurer is exposed to so-called *underwriting insurance risk*, which represents a danger that for a certain period of time claim payments exceed the collected risk bearing funds (sum of risk premium payments and available risk capital). Underwriting risk can be mathematically expressed as:

$$\sum \text{claims} > \sum \text{premium} + \sum \text{risk capital} \tag{41}$$

Underwriting risk can be divided into two risks: 1. *contingency risk* and 2. *error risk*. Both risks appear because actuaries use historical data to predict the future. The *contingency risk* is connected with the fact that the distribution of claims is a priori unknown. The risk has to be estimated by statistical methods, based on past information. Even if insurers know the "true" distribution of claims, they are still contingent and therefore unpredictable. For example, usually flood claims move around the expected value but there are some above average floods as the one which happened in Ljubljana in 2010 when claims were meaningfully higher than expected. Actuaries can have great statistical models but they cannot predict the exact year a natural disaster will happen. The other reason for underwriting risk is the *error risk* which refers to the limits within the use of statistical methods. Error risk is a risk that actuary makes a mistake at predicting the "true" distribution of claims. Error risk can be further divided into risk diagnostic and risk prognosis. Risk diagnostic is a risk that error will be made due to errors in the historical data or incorrect choice of statistical models. That is especially true when actuaries are exposed to new risk. For example, there is very few historical data about risk connected with autonomous driving cars. Although actuaries can construct a claim's distribution it is very likely it won't be as correct as claim distribution for men driven cars. Diagnostic risk happens due to actuary's assumption that the observed distribution will be stable over time. Even if diagnostic is accurate it is not sure that the claim distribution is still valid for the future. For example, data about mortality from century ago hardly reflects modern mortality rates, that is, people live longer and longer due to better medical treatment and therefore new mortality tables have to be constructed regularly (The National Academies Press, 2017).

Because there is underwriting risk insurers have to cover that risk by including a security loading in their calculation of premium.

5.2 Ruin theory

In Life insurance exercises, we calculated premiums by equating expected present value of premiums with expected value of benefits. Ruin theory shows that such premiums are insufficient and that the so-called security loading should be introduced to prevent insurer from going bankrupt. The purpose of security loading is to cover the underwriting risk. Bellow we show that premium without a loading leads to ruin despite a large risk capital (Dickson, D.C.M., 2010).

One period ruin probability is defined as:

$$\Psi = P(C > \Pi(C) + RC) \tag{42}$$

Where: Ψ denotes ruin probability, P denotes probability, C denotes overall claim payment for the portfolio, Π denotes premium and RC denotes risk capital.

For our presentation, we use central limit theorem (hereinafter: CLT) which states that the distribution of average of a large number of independent, identically distributed claims will be approximately normal (The Central Limit Theorem, n.y.). First, we standardize the above formula for ruin probability.

$$\Psi = P\left(\frac{C - E(C)}{\sigma(C)} > \frac{\Pi(C) + RC - E(C)}{\sigma(C)}\right) \tag{43}$$

Where: E(C) denotes expected value of the overall claim payment and σ(C) denotes standard deviation of the distribution of C.

σ(C) can also be expressed as σ(C) = $\sqrt{Var * \sum_{i=1}^{n} C_i} = \sqrt{\sum_{i=1}^{n} Var(C_i)} = \sqrt{n \times Var(C_i)} = \sqrt{n} * \sigma(C_i)$. Where: n denotes number of individual risks in portfolio, $\sigma(C_i)$ denotes standard deviation of the distribution of C_i and C_i denotes individual claim payment of risk i.

$$\Psi = P\left(\frac{C - E(C)}{\sigma(C)} > \frac{\Pi(C) + RC - E(C)}{\sqrt{n} \times \sigma(C_i)}\right) \tag{44}$$

When applying CLT premiums equal expected claims Π(C)=E(C) and n goes towards infinity therefore second fraction equals 0.

$$\begin{aligned}
&\Psi = 1 - P\left(\frac{C - E(C)}{\sigma(C)} \leq \frac{RC}{\sqrt{n} \times \sigma(C_i)}\right) \\
&\Psi = 1 - P\left(\frac{C - E(C)}{\sigma(C)} \leq \frac{RC}{\infty}\right) \\
&\Psi = 1 - \phi(0) = 1 - 0,5 = 0,5
\end{aligned} \tag{45}$$

From the equation number 45 we can see that ruin probability equals 50 %, which means, on average insurer would go bankrupt every second year.

In our second case, we introduce safety loading for individual risk, denoted by *s*, and premium loading expressed by $n \times s$.

$$
\begin{aligned}
&\Psi = P(C > \Pi(C) + RC) \\
&\Psi = P(C > \Pi(C) + n \times s + RC) \\
&\Psi = P\left(\frac{C - E(C)}{\sigma(C)} > \frac{\Pi(C) + n \times s + RC - E(C)}{\sqrt{n} \times \sigma(C_i)}\right) \\
&\Psi = 1 - P\left(\frac{C - E(C)}{\sigma(C)} \leq \frac{n \times s + RC}{\sqrt{n} \times \sigma(C_i)}\right)
\end{aligned}
\tag{46}
$$

When *n* limits towards infinity second fraction limits towards plus infinity too.

$$\Psi = 1 - \phi(\infty) = 1 - 1 = 0 \tag{47}$$

When applying a fixed loading ruin probability under CLT equals zero.

5.3 Premium principles

Premium calculation principles are rules for assigning a premium to an insurance risk. Each premium principle is evaluated based on many different desirable properties the premium principle either satisfies or does not satisfy. Actuaries determine the right premium principle by using three different methods or combination of the three. The first method is called Ad-hoc-Method. Using this method actuary determines potentially reasonable premium principle and after checks which desirable properties are achieved. Second method is Characterization method by which actuary first specifies the list of properties that the premium principle has to satisfy. After getting a short list actuary determines the premium principle he/she wants to use. The third method is called the Economic method, within which, an actuary adopts a particular economic theory and then determines the resulting premium principle (Young, 2014).

The simplest and easiest to explain to policyholder are **Net premium principle** and **Expected Value Premium Principle**. Bellow we explain why those principles aren't suitable for serious discussion. Next, we use Ad-hoc-Method and chose Variance Premium Principle for premium calculation because it is a good mix between simplicity and usefulness. Besides, Variance Premium Principle is among the most widely used premium principles in practice (Straub, 1997).

Premium principles are rules for assigning a premium to an insurance risk. Insurance risk is represented by loss, which is a random variable. Let's denote, premium by Π, claim by C and premium calculation principle as H, then (Young, 2014):

$$\Pi = H(C) \tag{48}$$

The simplest premium principle, the so called **Net premium principle**, equals expected loss to expected premiums. Such approach was used in our calculation of Life premiums, where we used the so-called equivalence principal. The ruin theory shows that such premium calculation leads to 50 % probability of ruin, regardless of how big the reserve capital is (Melnikov, 2010). A first improvement to Net premium principle is to use proportional loading and the so called **Expected Value Premium Principle**, which is (Young, 2014):

$$\Pi = (1+\beta)\times E(C) \tag{49}$$

The principle is easy to understand and explain to policyholders but it does not depend on the degree of fluctuation of C. In other words, principle covers against error risk but it does not consider contingency risk. In order to account for contingency risk, we use **Variance Premium Principle** (Young, 2014):

$$\Pi = E(C)+\beta\times Var(C) \tag{50}$$

Beta is a variable of reliability. Higher beta means higher reliability and vice versa. If insurer has more data about the claims and good quality of data, she can lower the beta. On the other hand, less data and lower quality means the insurer must use higher beta because she can rely less on her data and she needs higher safety loading.

Next, we check which properties of premium principles does a Variance Premium Principle satisfy. There are at least 15 different properties but for simplicity reasons we are making a test just for 4 of them: security loading, translation invariance, scale invariance and additivity.

Security loading

$$\begin{gathered} \Pi(C) \geq E(C) \\ \boldsymbol{\Pi^V(C) = E(C) + \beta * Var(C) \geq E(C)} \end{gathered} \tag{51}$$

Where: $\beta>0$ and $Var(C)>0$

Security loading property **is fulfilled**.

Translation invariance

$$\begin{gathered} \Pi(C+a) = \Pi(C) + a \\ \Pi^V(C+a) = E(C+a)+\beta*Var(C+a)= \\ =\boldsymbol{E(C) + \beta * Var(C) + a = \Pi^V(C) + a} \end{gathered} \tag{52}$$

Translation invariance property **is fulfilled**.

Scale invariance

$$\begin{aligned}&\Pi(b*C)=b*\Pi(C)\\&\Pi^V(b*C)=E(b*C)+\beta*Var(b*C)=\\&=b*E(C)+\beta*b^2\,Var(C)=\\&=\boldsymbol{b*(E(C)+\beta*b*Var(C))\neq b*\Pi(C)}\end{aligned} \tag{53}$$

Scale invariance property **not** fulfilled.

Additivity

$$\begin{aligned}&\Pi(C+T)=\Pi(C)+\Pi(T)\\&\Pi^V(C+T)=E(C+T)+\beta*Var(C+T)=\\&=E(C)+E(T)+\beta*\left(Var(C)+Var(T)+2*Cov(C,T)\right)=\\&=E(C)+\beta*Var(C)+E(T)+\beta*Var(T)+\beta*2*Cov(C,T)=\\&=\boldsymbol{\Pi^V(C)+\Pi^V(T)+\beta*2*Cov(C,T)\neq\Pi(C)+\Pi(T)}\end{aligned} \tag{54}$$

Additivity property is **not** fulfilled. If C and T are independent and therefore Cov (C, T) = 0, then $\Pi^V(C+T)=\Pi^V(C)+\Pi^V(T)$ and property is fulfilled.

5.3.1 Example – Net and variance premium principle

In this example, we show the difference in calculation of premium when using net premium and variance premium principle. Policyholder insures his car for one year. There is 80 % probability he has a claim free year, 15 % probability he has claims worth 1.000 € and 5 % he has claims worth 10.000 €. We calculate: 1. premium using Net premium principle and 2. premium using Variance premium principle when intensity, denoted by β, is β = 0,0002 (Olivieri & Pitacco, 2010).

1. premium using Net premium principle

$$\Pi=E(C)=0{,}8*0+0{,}15*1.000+0{,}05*10.000=650€ \tag{55}$$

2. premium using Variance premium principle

We first need to calculate variance

$$\begin{aligned}&Var(C)=E\left(C^2\right)-\left(E(C)\right)^2\\&Var(C)=\left(0{,}8*0^2+0{,}15*1.000^2+0{,}05*10.000^2\right)-650^2\\&Var(C)=5.150.000-422.500=4.727.500\end{aligned} \tag{56}$$

Then we put variance into formula for Variance premium principle

$$\begin{aligned}&\Pi(C)=E(C)+\beta*Var(C)\\&\Pi(C)=650+0{,}0002*4.727.500\\&\Pi(C)=1.595{,}5€\end{aligned} \tag{57}$$

Security loading of Variance premium principle is the difference between 1.595,5 € and 650 €, which is 945,5 €.

5.4 Experience rating and credibility premium

In a perfect world, all risks would be homogeneous and actuaries would not have much problem calculating the expected claims. In reality risks are heterogeneous therefore actuaries need to classify risks into groups with similar risk characteristics to make them as homogeneous as possible. For example, insurer makes a classification of auto drivers based on variables, such as: driver's age, region, type of car etc. Based on this information and past observation (experience of the entire group) each member of this class is charged the same premium. But each risk is unique and therefore different from other risks in the class. For example, we can build a group of 20 to 30-year-old men drivers from Ljubljana but there are some risks we can initially not observe, such as how aggressive is individual driver. In order to account for individual risk insurers, use individual risk experience to calculate individual premium. If we only include individual risk experience in premium calculation we are in contradiction to the basic idea of insurance, which is to have a large collective of risks, each of them paying the same rate (Straub, 1997).

Somehow, we need to find a balance between individual premium and collective premium in order to calculate the right premium or so-called *credibility premium*. Credibility premium can be simply expressed as:

$$\textit{Credibility premium} = z * \textit{individual premium} + (1-z) * \textit{collective premium} \tag{58}$$

Where: z = credibility factor and z is between 0 and 1.

More past information insurer has about the insured the more credible the policyholder's own experience. On the other hand, larger the risk group the more credible the collective experience. Competitive considerations may force the insurer to give greater credibility to a policyholder in order to maintain a business. For example, insurer that can better detect a careful driver can offer him a lower premium not because of marketing but because careful driver means lower risk compared to aggressive one. That is a loss for insurer who gives greater weight on collective premium.

5.4.1 Example – Credibility premium

In this example, we calculate credibility premium for 5 different auto insurance contracts. In order to do so we need three numbers (see Eq. (58)): 1. credibility factor (z) 2. individual premium ($\bar{x}_j$) and 3. collective premium ($\hat{m}$). We have past information about the amount of accidents for 4 previous years. For each policy, we first calculate *individual premium,* which equals average claim for 4 years, using the formula:

$$\bar{x}_j = \frac{1}{4} \sum_{i=1}^{4} x_{ij} \tag{59}$$

Where: $\bar{x}_j$ = individual premium, i = year and j = policy.

Table 6: Past claims for five different policies

Policy y / Year i	1	2	3	4	5
2013	0	5	2	5	3
2014	0	1	1	3	0
2015	0	1	0	2	0
2016	0	1	1	2	1
$\bar{x}_j=\frac{1}{4}\sum_{i=1}^{4} x_{ij}$	0	2	1	3	1
$\frac{1}{n-1}\sum_{i=1}^{n}\left(x_{ij}-\bar{x}_j\right)^2$	0	4	0,6667	2	2

Source: Own work.

Next, we calculate *collective premium* using the equation:

$$\begin{aligned}\hat{m}&=\frac{1}{n*J}*\sum_{j=1}^{J}\sum_{i=1}^{n}x_{ij}=\frac{1}{J}\sum_{j=1}^{J}(\frac{1}{n}\sum_{i=1}^{n}x_{ij})\\ \hat{m}&=\frac{1}{5*4}*(0+8+4+12+4)\\ \hat{m}&=\frac{28}{20}=1{,}4\end{aligned} \tag{60}$$

Where: n = number of years, J = number of contracts and $\hat{m}$ = collective premium

We have individual premium and collective premium we only need to get *credibility factor,* denoted by $\hat{z}$, to calculate credibility premiums. Credibility factor is:

$$\hat{z}=\frac{n}{n+\frac{\hat{u}}{\hat{w}}} \tag{61}$$

Where: $\hat{z}$ = credibility factor, $\hat{u}$ = variance from year to year, or "noise", $\hat{w}$ = variance from risk policy to risk policy, or "heterogeneity" (Straub, 1997).

In order to come to credibility factor, we need to find out the values of the variances $\hat{u}$ and $\hat{w}$. As we can see from the equations for credibility factors and credibility premium the higher the variance of the individual premium, higher is denominator of credibility factor, which means smaller is credibility factor and therefore lower weight is assigned to individual premium. Therefore, we say $\hat{u}$ measures the "noise" of individual policy. On the other hand, higher the variance between the policies the lower is denominator of credibility factor and higher is credibility factor and therefore lower weight is assigned to collective premium. We say $\widehat{w}$ measures heterogeneity between policies. First, we calculate $\hat{u}$. To make our calculation easier we already calculated the variance for individual premium in the table above, therefore we only calculate average of the variances to get $\hat{u}$.

$$\begin{aligned}
\hat{u} &= \frac{1}{J}\sum_{j=1}^{J}\frac{1}{n-1}\sum_{i=1}^{n}\left(x_{ij}-\bar{x}_j\right)^2 \\
\hat{u} &= \frac{1}{5} * (0+4+0,6667+2+2) \\
\hat{u} &= \frac{1}{5} * 8,6667 = 1,7333
\end{aligned} \tag{62}$$

Now that we have $\hat{u}$ we can calculate $\widehat{w}$ as follows:

$$\begin{aligned}
\widehat{w} &= \frac{1}{J-1}\sum_{j=1}^{J}(\bar{x}_j-\widehat{m})^2 - \frac{\hat{u}}{n} \\
\widehat{w} &= \frac{1}{5-1} * ((0-1,4)^2+(2-1,4)^2+(1-1,4)^2+(3-1,4)^2+(1-1,4)^2) - \frac{1,7333}{4} = \\
\widehat{w} &= \frac{1}{4} * (1,96+0,36+0,16+2,56+0,16) - 0,4333 = 1,3 - 0,4333 = 0,8667
\end{aligned} \tag{63}$$

We have all the ingredients to calculate credibility factor:

$$\hat{z} = \frac{4}{4+\frac{1,7333}{0,8667}} = \frac{4}{5,9999} = 0,6667 \tag{64}$$

Credibility factor is 0,6667 which means higher weight is given to individual experience than to overall experience. That is due to the fact that the variance of the individual policy is smaller compared to the variance between policies. Using information about individual premium, collective premium and credibility factor we calculate credibility premium for each policy, using the following formula (Straub, 1997):

$$\hat{\mu}_j = \hat{z} + \bar{x}_j + (1-\hat{z}) * \widehat{m} = 0,6667 * \bar{x}_j + (1-0,6667) * 1,4 \tag{65}$$

Table 7: Credibility premium

Policy *j*	$\bar{x}_j$	$\hat{\mu}_j$
Policy 1	0	$\hat{\mu}_j = 0,6667*0+(1-0,6667)*1,4 = 0,4666$
Policy 2	2	$\hat{\mu}_j = 0,6667*2+(1-0,6667)*1,4 = 1,8$
Policy 3	1	$\hat{\mu}_j = 0,6667*1+(1-0,6667)*1,4 = 1,1333$
Policy 4	3	$\hat{\mu}_j = 0,6667*3+(1-0,6667)*1,4 = 2,4667$
Policy 5	1	$\hat{\mu}_j = 0,6667*1+(1-0,6667)*1,4 = 1,1333$

Source: Own work.

From the above table, we can see policyholders 1, 3 and 5 are paying higher premiums compared to their average claims and policyholders 2 and 4 are paying lower premiums than their average claim. We could say policyholders 2 and 4 are probably more aggressive drivers than the other three. If another insurer could somehow distinguish between aggressive and defensive drivers he could make different kind of classification. This classification would put aggressive and defensive drivers into different groups which would allow insurer to charge higher premiums to aggressive drivers and lower premiums to defensive drivers. Nowadays insurers actually use more and more data and analytics in order to better know the customer and better personalize insurance products.

5.5 Deductibles

Sometimes classification costs are higher than benefits of classifying policyholders. Credibility premium example that we did in previous chapter does not distinguish between aggressive and defensive drivers. Our example also shows that aggressive drivers pay premium that is lower compared to their average claims. The insurer can afford that due to the fact that defensive drivers pay higher premium than their average claims. If an insurer offers insurance at price which includes collective premium part, then the riskier policyholders will buy relatively more insurance coverage compared to the case, when insured are charged the premium based on their expected losses. On the other hand, less risky policyholders will buy less insurance coverage. When this happens, we talk about *adverse selection* (Harrington & Niehaus, 1999).

Another problem is *moral hazard*, which is an unobservable change in policyholder's behaviour due to the existence of an insurance contract. For example, driver will drive more aggressively after buying an insurance contract or a smoker will smoke twice as much as before. Credibility rating can "punish" such behaviour only after some time has passed. In other words, insurer has to collect enough past data in order to charge a higher premium. Deductibles present a tool for insurer to reduce adverse selection and moral hazard before they happen. Besides that, deductibles reduce claims processing costs and bring benefits to policyholders through premium reduction.

Deductible is a way to reduce the amount of coverage by eliminating coverage for relatively small losses. For example, Peter buys a motor liability insurance with a 500 € deductible. Peter has an accident with his car and a claim amounts to 2000 €. Peter pays 500 € and the difference (1500 €) is covered by the insurer. Because Peter has to pay the part of the costs he will probably drive more carefully, in other words deductible is reducing moral hazard. Without deductible insurer would have to charge Peter a higher premium, not just because higher coverage would be given, but also because Peter would drive more carelessly (Harrington & Niehaus, 1999).

Insurers can offer different deductibles in order to distinguish between good and bad risks and therefore mitigate adverse selection. Good risks are prepared to pay higher deductible in exchange for lower premium. That is not the case with bad risks which are willing to pay higher premium in exchange for lower deductible. In the end, good risks buy more deductible and by doing so help insurers distinguish between good and bad risk (Harrington & Niehaus, 1999).

Sometimes the costs of processing claims are unrelated to the size of the claim. That is especially true for small claims that occur relatively frequently. Imagine a 40 € claim with 20 % probability and 80 € fixed claim processing costs. In this case processing costs are higher than claim. Insurer would have to charge 8 € for expected claim and additional 16 € for expected claim processing costs. For insured it is wiser that he does not insure such a risk because the loading is relatively high.

5.5.1 Example - Deductibles

Peter lives on the Ljubljana Marshes and he insures his house against floods for 10 years. There is an 84 % probability he will have a claim free 10-year period, but there are, 10 % chance he will suffer minor damages which would amount to 1.000 €, 5 % chance there will be damages in total of 5.000 € and 1 % that damages will be worth 10.000 €. We calculate 1. Premium using Variance Premium Principle and 2. Premium using Variance Premium Principle with 500 € deductible and β = 0,0002.

1. Premium using Variance Premium Principle

First, we calculate variance.

$$\begin{aligned}&Var(C)=E(C^2)-(E(C))^2\\&Var(C)=0{,}84*0^2+0{,}1*1000^2+0{,}05*5000^2+0{,}01*10.000^2-\\&\quad -(0{,}84*0+0{,}1*1000+0{,}05*5000+0{,}01*10.000)^2\\&\quad Var(C)=2.350.000-202.500=2.147.500\end{aligned} \tag{66}$$

We put variance in our calculation of Variance Premium Principle.

$$\begin{aligned}&\Pi(C)=E(C)+0{,}0002*Var(C)\\&\Pi(C)=(0{,}84*0+0{,}1*1.000+0{,}05*5.000+0{,}01*10.000)*0{,}0002*2.147.500\\&\Pi(C)=450+429{,}5=879{,}5€\end{aligned} \tag{67}$$

2. Premium using Variance Premium Principle with 500 € deductible.

We calculate expected value similarly than before, but instead we deduct 500€ for all three scenarios where insurer has to pay a claim. New claim for scenario with 10 % probability is

500 €, 4.500 € for scenario with 5 % probability and 9.500 € for scenario with 1 % probability.

$$
\begin{aligned}
&Var(C)=E\left(C^2\right)-\left(E(C)\right)^2 \\
&Var(C) = 0{,}84*0^2+0{,}1*500^2+0{,}05*4.500^2+0{,}01* \\
&*9.500^2-(0{,}84*0+0{,}1*500+0{,}05*4.500+0{,}01*9.500)^2 \\
&Var(C) = 1.940.000 - 136.900 = 1.803.100
\end{aligned}
\tag{68}
$$

Again, we calculate the premium using the calculated variance.

$$
\begin{aligned}
&\Pi(C)=E(C)+0{,}0002*Var(C) \\
&\Pi(C)=(0{,}84*0+0{,}1*500+0{,}05*4.500+0{,}01*9.500)*0{,}0002*1.803.1 \\
&\Pi(C)=370+360{,}62=730{,}62€
\end{aligned}
\tag{69}
$$

With our example, we show that a premium is lower when a deductible is included. Lower premium will attract people who live in areas where floods make lesser damage compared to the average on Ljubljana Marshes (mitigation of adverse selection). Deductible will motivate policyholders to take some preventive measures, such as, sandbags or not keeping worthy objects in basement. Also, policyholder will not report damages that are worth up to 500 €.

6 NON-LIFE INSURANCE CLAIM PROVISIONS

In non-life insurance claim provisions are the biggest and most important provisions in insurance company's balance sheet. As mentioned already claim provisions should be formed for different scenarios: 1. damage was already caused, loss has occurred and the claim was notified but the final claim regulation has not been settled yet (e.g. there are still some open legal questions; RBNS = reported but not settled), 2. damage was already caused, loss has occurred but there has not been any notification (IBNR = incurred but not reported), 3. damage was already caused but losses did not already occur.

Methods for determining claims reserves can be divided into deterministic methods and stochastic methods. Stochastic methods make explicit reference to the randomness of the-pattern of a claim, while deterministic methods are based on average assessment of the time-pattern of a claim. Deterministic methods have an advantage of simplicity and are straightforward but they may lead to biased assessment (Olivieri & Pitacco, 2010). Next, we present two deterministic methods to calculate claim provisions, the so-called Chain-Ladder method and Cape Cod method.

6.1 Chain ladder method

First deterministic method to calculate provisions for outstanding claim payments is Chain Ladder method. In this approach actuaries extrapolate future expected claims from claims already paid or reported. Assumption of Chain Ladder Method is that the time pattern of claims is stable in time. As input data, we need a run-off triangle, which collects cumulative data about incurred claims in respect to accident year and so-called development year. Accident year is a year in which the accident has occurred and development year shows how claims are paid in years following the accident, there is namely delay between claim occurrence and claim settlement. Cumulative claims are denoted by S_{ij}, where i = accident year and j = development year.

Table 8: Cumulative run-off triangle

		Development years j				
		0	1	2	3	…f
Accident years i	2013	$S_{2013,0}$	$S_{2013,1}$	$S_{2013,2}$	$S_{2013,3}$	$S_{2013,f}$
	2014	$S_{2014,0}$	$S_{2014,1}$	$S_{2014,2}$	$S_{2014,3}$	
	2015	$S_{2015,0}$	$S_{2015,1}$	$S_{2015,2}$		
	2016	$S_{2016,0}$	$S_{2016,1}$			
	…f	$S_{f,0}$				

Source: Own work.

Year j represents the year when all the claims are paid. If we denote claim in each year as X_{ji} then $S_{2013,F}=X_{2013,0}+X_{2013,1}+X_{2013,2}+X_{2013,3}+\ldots+X_{2013,f}$. In the table above we already have cumulative claims but run-off triangle with incremental claims could also be used.

The unknown part of the triangle is estimated by using development factors. Development factor is defined as:

$$\hat{\lambda}_j=\frac{\sum_{i=0}^{f-1-j} S_{ij+1}}{\sum_{i=0}^{f-1-j} S_{ij}}$$

Development factor describes for any accident year i the increase of the cumulative aggregate claim from time j to time j+1. Assuming the claims are fully covered till year f, $\widehat{\lambda_f}=1$ (Olivieri & Pitacco, 2010). For each development year a final development factor is calculated by multiplication of estimated development factors. Final development factor is expressed as:

$$\hat{H}_j=\lambda_j*\lambda_{j+1}*\lambda_{j+2}*\ldots*\lambda_{f-1} \tag{70}$$

Final losses are estimated as follows:

$$S_{if} = S_{ij} * \widehat{H}_j \tag{71}$$

6.1.1 Example – Chain ladder method

Input data for our example is an incremental run of triangle which contains data about claims from year 2010 till 2016.

Table 9: Incremental run-off triangle

		Development year j						
		0	**1**	**2**	**3**	**4**	**5**	**6**
Accident year i	**2010**	110	88	70	65	90	60	18
	2011	122	100	50	48	40	20	
	2012	148	170	60	35	71		
	2013	200	180	70	41			
	2014	70	85	42				
	2015	95	79					
	2016	105						

Source: Own work.

We sum the claims in incremental run off triangle to calculate cumulative run of triangle.

Table 10: Cumulative run-off triangle

		Development year j						
		0	**1**	**2**	**3**	**4**	**5**	**6**
Accident year i	**2010**	110	198	268	333	423	483	501
	2011	122	222	272	320	360	380	
	2012	148	318	378	413	484		
	2013	200	380	450	491			
	2014	70	155	197				
	2015	95	174					
	2016	105						

$\hat{\lambda}_0$ $\hat{\lambda}_1$ $\hat{\lambda}_2$ $\hat{\lambda}_3$ $\hat{\lambda}_4$ $\hat{\lambda}_5$

Source: Own work

We calculate development factors ($\hat{\lambda}_j$) to forecast unknown part of the triangle.

$$\hat{\lambda}_0=\frac{\sum_{i=0}^{5} S_{i1}}{\sum_{i=0}^{5} S_{i0}}=\frac{198+222+318+380+155+174}{110+122+148+200+70+95}=1,94$$
$$\hat{\lambda}_1=\frac{\sum_{i=0}^{4} S_{i2}}{\sum_{i=0}^{4} S_{i1}}=\frac{268+272+378+450+197}{198+222+318+380+155}=1,23$$
$$\hat{\lambda}_2=\frac{\sum_{i=0}^{3} S_{i3}}{\sum_{i=0}^{3} S_{i2}}=\frac{333+320+413+491}{268+272+378+450}=1,14$$
$$\hat{\lambda}_3=\frac{\sum_{i=0}^{2} S_{i4}}{\sum_{i=0}^{2} S_{i3}}=\frac{423+360+484}{333+320+413}=1,19 \qquad (72)$$
$$\hat{\lambda}_4=\frac{\sum_{i=0}^{1} S_{i5}}{\sum_{i=0}^{1} S_{i4}}=\frac{483+380}{423+360}=1,10$$
$$\hat{\lambda}_5=\frac{S_{06}}{S_{05}}=\frac{501}{483}=1,04$$

Next, we calculate the final development factors ($\hat{H}_j$).

$$\hat{H}_0=\hat{\lambda}_0*\hat{\lambda}_1*\hat{\lambda}_2*\hat{\lambda}_3*\hat{\lambda}_4*\hat{\lambda}_5=1,94*1,23*1,14*1,19*1,1*1,04=3,70$$
$$\hat{H}_1=\hat{\lambda}_1*\hat{\lambda}_2*\hat{\lambda}_3*\hat{\lambda}_4*\hat{\lambda}_5=1,23*1,14*1,19*1,1*1,04=1,91$$
$$\hat{H}_2=\hat{\lambda}_2*\hat{\lambda}_3*\hat{\lambda}_4*\hat{\lambda}_5=1,14*1,19*1,1*1,04=1,55$$
$$\hat{H}_3=\hat{\lambda}_3*\hat{\lambda}_4*\hat{\lambda}_5=1,19*1,1*1,04=1,36 \qquad (73)$$
$$\hat{H}_4=\hat{\lambda}_4*\hat{\lambda}_5=1,1*1,04=1,14$$
$$\hat{H}_5=\hat{\lambda}_5=1,04$$

Afterwards we calculate final losses (S_{i6}) and reserves (R_i) for years 2011 till 2016. Final reserve is a difference between final loss and the last known claim. At the end, we sum final reserves to get the total chain ladder reserve.

Table 11: Estimation of losses and reserves

Final losses	Reserves	
$S_{16}=S_{15}*\hat{H}_5=380*1,04=395$ $S_{26}=S_{24}*\hat{H}_4=484*1,14=552$ $S_{36}=S_{33}*\hat{H}_3=491*1,36=668$ $S_{46}=S_{42}*\hat{H}_2=197*1,55=305$ $S_{56}=S_{51}*\hat{H}_1=174*1,91=332$ $S_{66}=S_{60}*\hat{H}_0=105*3,70=389$	$R_1=S_{16}-S_{15}=395 - 380=15$ $R_2=S_{26}-S_{24}=552 - 484=68$ $R_3=S_{36}-S_{33}=668 - 491=177$ $R_4=S_{46}-S_{42}=305-197=108$ $R_5=S_{56}-S_{51}=332 - 174=158$ $R_6=S_{66}-S_{60}=389 - 105=284$	(74)
	Total Chain Ladder reserve = $\sum_{i=1}^{6} R_i=810$	(75)

Source: Own work.

Total chain ladder reserve is 810 and equals the value of total expected future claims for accidents that happened between 2011 and 2016.

6.2 Cape cod method

Chain ladder method has an advantage of being straightforward and easy to understand but it also has some disadvantages, such as:

- method breaks down when $X_{f0}=0$ or $S_{if-i}=0$,
- very sensitive to changes even of a single number (especially S_{if-i} values),
- it disregards information given by the earned premium,
- $\hat{H}_j$ are estimated factor wise and this may lead to a serious bias (Straub, 1997).

Cape Cod method overcomes some of the shortcomings of the Chain Ladder method by introducing lag factors and information about the earned premium. The basic idea of Cape Cod method is to compare known losses with used-up premiums. Reserve (R_i) for year *i* is calculated with the equation:

$$R_i=P_i*\left(1-\hat{L}_{k-1}\right)*CF \tag{76}$$

Where: R_i = reserve in year *i*, P_i = paid premium in year i, L_{k-1} = lag factor and CF = correction factor

Lag factor tells us how much of the expected final loss of a given accident year is known by the end of the development year j. Mathematically lag factor is expressed as: $\hat{L}_j=\frac{1}{\hat{H}_j}$. Correction factor is a ratio of so far experienced claims and the so far used premiums for experienced development periods. Idea is to compare claims and premiums for more accident years which makes this method more robust compared to Chain Ladder method. Correction factor for Cape Cod method is:

$$CF=\frac{\sum_{i=1}^{f} S_{i,f-i}}{\sum_{i=1}^{f} \hat{L}_{f-1} *P_i} \tag{77}$$

If instead of using multiple accident years for correction factor calculation we would use only one accident year *i*, then the formula for reserve would be:

$$R_i=P_i*\left(1-\hat{L}_{k-1}\right)*\frac{S_{i,f-i}}{\hat{L}_{f-1}*P_i}=\left(\frac{1}{\hat{L}_{k-1}}-1\right)*S_{i,f-i}=\left(\hat{H}_{f-i}-1\right)*S_{i,f-i} \tag{78}$$

When we take into account only one accident year *i*, we see that reserve calculation equals the reserve calculation for Chain Ladder method, therefore we can say Cape Cod method is a type of Chain Ladder method which is less dependent on changes of a single observation (Straub, 1997).

6.2.1 Example – Cape cod method

We start off with the same cumulative run of triangle as in the previous example, but this time we add information about the paid premiums (P_i).

Table 12: Cumulative run-off triangle including paid premiums

		Premium	Development year j						
			0	**1**	**2**	**3**	**4**	**5**	**6**
Accident year i	**2010**	520	110	198	268	333	423	483	501
	2011	510	122	222	272	320	360	380	
	2012	535	148	318	378	413	484		
	2013	590	200	380	450	491			
	2014	500	70	155	197				
	2015	505	95	174					
	2016	520	105						

Source: Own work.

Next, we calculate lag factors using final development factors from previous example:

$$\begin{aligned}
&\hat{L}_j=\frac{1}{\hat{H}_j}\\
&\hat{L}_0=\frac{1}{\hat{H}_0}=\frac{1}{3,7}=0,27\\
&\hat{L}_1=\frac{1}{\hat{H}_1}=\frac{1}{1,91}=0,52\\
&\hat{L}_2=\frac{1}{\hat{H}_2}=\frac{1}{1,55}=0,65\\
&\hat{L}_3=\frac{1}{\hat{H}_3}=\frac{1}{1,36}=0,74\\
&\hat{L}_4=\frac{1}{\hat{H}_4}=\frac{1}{1,14}=0,88\\
&\hat{L}_5=\frac{1}{\hat{H}_5}=\frac{1}{1,04}=0,96
\end{aligned} \tag{79}$$

We use our lag factors to calculate correction factor:

$$\begin{aligned}
&CF=\frac{\sum_{i=1}^{f} S_{i,f-i}}{\sum_{i=1}^{f} \hat{L}_{f-1} * P_i}=\\
&=\frac{380+484+491+197+174+105}{0,96*510+0,88*535+0,74*590+0,65*500+0,52*505+0,27*520}=\\
&=0,86
\end{aligned} \tag{80}$$

At the end, we calculate Cape Cod reserve by multiplying correction factor with residual premium available for future losses: $P_i*\left(1-\hat{L}_{f-i}\right)*CF$.

Table 13: Cape Cod reserve

I	$1\text{-}\hat{L}_{f\text{-}i}$	P_i	$R_i = \left(1\text{-}L_{f\text{-}i}\right) * P_i * CF$
1 (2011)	0,04	510	0,04 * 510 * 0,86 = 17,54
2 (2012)	0,12	535	0,12 * 535 * 0,86 = 55,21
3 (2013)	0,26	590	0,26 * 590 * 0,86 = 131,92
4 (2014)	0,35	500	0,35 * 500 * 0,86 = 150,5
5 (2015)	0,48	505	0,48 * 505 * 0,86 = 208,46
6 (2016)	0,73	520	0,73 * 520 * 0,86 = 326,46
			Total Cape Cod reserve = $\sum_{i=0}^{6} R_i = 890{,}09$

Source: Own work.

By observing the methods, we can conclude that Chain Ladder is a special case of Cape Cod method. The letter is less dependent on changes of a single value in the triangle than Chain Ladder method and is therefore more robust in this respect, but it is still sensitive to changes in lag factors. Contrary to Chain Ladder method, Cape Cod method also contains information about earned premiums which among other things gives positive future claims estimates even when the final known cumulative claims equal zero (Straub, 1997).

CONCLUSION

Insurance provisions are very important for life and non-life insurance companies as they present around 75 % of life insurance company's liabilities and around 60 % of non-life insurance company's liabilities. The difference comes from the nature of business. Non-life insurers need higher capital buffer because they are exposed to greater volatility of claims. Higher equity means there is a lower portion of provisions. The second reason for difference in the size of provisions is, that life insurers have longer term liabilities compared to non-life insurers, which leads to greater accumulation of provisions for life insurers. Mathematical provisions are the most important provisions in life insurance business and Claim provisions are the most important provisions for non-life insurers. If we count Provisions for unit linked insurance contracts as Mathematical provisions, then the biggest and the second biggest insurer in Slovenia had around 97 % of all provisions in Mathematical provisions (looking only at the life insurance business). On the other hand, the same insurers had 70 % Claims provisions to Total provisions in their property insurance business. In recent years Mathematical provisions had been growing for two reasons. The first reason is, that Mathematical provisions are accumulating relatively fast due to undeveloped life insurance business two decades ago. That is especially true for unit linked products, for which in 2002 only 6.393 contracts had been written and in 2015 that number has grown to 471.202 contracts. The second reason for provision growth is that in life insurance business premiums are collected years before claims occur, therefore they need to be preserved for the future. Even if the yearly premiums are stagnant or decreasing Mathematical provisions will usually grow.

In 2015 there were 1.911 million € insurance gross premiums written by Slovenian insurance companies. 907 million € or 47,46 % of all premiums were written by property insurance business, 519 million € or 27,16 % were written by life insurance business and 485 million € or 25,38 % were written by voluntary health insurance business. The most important lines in property insurance business were Land motor vehicle (219 million €) and Motor vehicle liability (218 million €), which combined contributed 48,18 % to total property gross insurance premiums. That seems a lot but it is not compared to the past. For example, Motor vehicle insurance (Land motor vehicle and Motor vehicle liability combined) contributed 50,11 %, 51,54 %, 52,85 % and 54,45 % in years 2013, 2012, 2011 and 2010 to total gross premiums written in property insurance. On the third and fourth place in 2015 are Insurance against fire and natural forces and Other damage to property with 115 million € and 114 million € gross premium written or 12,68 % and 12,57 % of insurance premiums written by property insurance. On the fifth place is Accident insurance with 94 million € or 10,18 %. Other property insurance groups include: General liability (56 million € or 6,13 %), Credit (42 million € or 4,60 %), Other-non-life (38 million € or 4,16 %) and Goods in transit (8 million € or 0,88 %). Life insurance business had 519 million € gross written premiums, which is, 27,16 % of total gross premiums written in Slovenia in 2015. The two most important lines of products are Traditional life insurance products and Unit-linked products.

Premiums written due to Unit-linked were 58,4 million € in 2004 and have grown for 384 %, to 224,3 million € in 2015. In 2010 and 2011 Unit-linked gross written premiums were even higher then gross premiums written due to Traditional life insurance products. However, after 2011 Traditional life insurance products received more demand and in 2015 gross premiums written to traditional products were 255,4 million €. Other Traditional life insurance products have combined contributed 39,2 million € or 7,55 % to total gross premiums written by life insurance business. Voluntary health insurance business received 484,6 million € gross premiums written, which is 25 % of total gross premiums written in Slovenia in 2015. Since 2002 the collected premiums of Voluntary insurance business have been growing till year 2012, since then premiums have slightly fallen due to increased price competition.

Premium and provision calculations differ between life and non-life insurance. Premiums for life insurance are calculated with the help of equivalence principle, that is, by equating present value of expected future premiums with present value of expected future benefits plus present value of expected future costs. Costs are usually determined as a certain percentage of premiums or insured sum. Typical costs which are determined based on premiums times number of premiums are compensation for agents and some expenses connected to administration costs. On the other hand, underwriting costs and some administrations costs are expressed as a proportion of insured sum. In order to estimate present value of future premiums and benefits actuaries use actuarial value. Actuarial value is an expected present value of future cash flows determined by mortality rates and interest rate. Interest rates are usually determined by the regulator with so called maximal technical interest rates. Mortality rates can be found in mortality tables. Actuaries use different mortality rates when they deal with mortality and longevity risk. Namely, insurer needs a buffer, that is higher mortality than expected when premiums for life insurance are calculated and lower for pure endowment insurance. Calculating premiums with actuarial values is simpler if we use commutation functions. For every age commutation function is calculated based on predetermined mortality rate and interest rate. Because of their deterministic nature commutation functions were mostly replaced by computers and calculators which allow actuaries to expand from deterministic to stochastic models.

Premiums for non-life insurance are also calculated with the help of equivalence premium but ruin theory shows that a proportional fixed loading needs to be added in order to avoid ruin. Insurers use different premium principles, the one we present and is also commonly used by insurers is Variance Premium Principle. Non-life insurers try to group their clients in homogeneous groups and charge each group member similar premium. Because each risk is different insurers charge a premium that is a combination of "collective" and "individual" premium. Based on past experience, insurer determines the premium amount as a weighted average between the two premiums. If variance of particular insured is higher compared to variance of the whole group, then higher weight is given to "collective" premium and vice versa. When calculating non-life insurance premiums insurers also use deductibles.

Deductibles are used in order to mitigate moral hazard, adverse selection and administration costs for small claims, besides insurer does not have to pay for small claims.

Provisions in life insurance are calculated with the equivalence principle, that is, by equating provision with present value of expected future claims minus present value of expected future benefits. We also use commutation functions to determine expected present value of future premiums and claims.

The most challenging and important provisioning in non-life insurance is provisioning for claims. Namely there can be delay between loss occurrence or claim notification and claim settlement. Insurers estimate future claims by using different triangle methods. The idea is to predict future claim payments based on past claims. We have calculated claim provisions with Chain Ladder method, but because Chain Ladder method has some downsides we have also presented Cape Cod method as an alternative.

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APPENDIXES

Appendix 1: Slovenian summary

Eni od ključnih nalog za uspešno poslovanje zavarovalnice sta ustrezno zaračunavanje premij in izračunavanje zavarovalno-tehničnih rezervacij. Premije predstavljajo glavni vir sredstev zavarovalnicam. V primeru, da je premija za posameznika previsoka, bo zavarovanec odšel h konkurenci, v primeru, da je prenizka, bo zavarovalnica utrpela izgubo, saj bodo škodna izplačila presegla vrednost premij. Zavarovalno-tehnične rezervacije predstavljajo v bilanci stanja zavarovalnic, na strani obveznosti do virov sredstev, najvišjo postavko. Potreba po oblikovanju rezervacij izvira iz dejstva, da zavarovalnice dobijo sredstva v zameno za obljubo, da bodo v prihodnosti izplačale sredstva v primeru odškodninskega zahtevka. Zavarovalnice v prvi fazi akumulirajo sredstva, jih vlagajo in kasneje izplačajo. Namen magistrske naloge je predstaviti osnovne metode za izračun premij in zavarovalno-tehničnih rezervacij v življenjski in neživljenjski zavarovalnici. Za boljše razumevanje pomena premij in zavarovalno-tehničnih rezervacij, s pomočjo informacij iz letnih poročil Agencije za zavarovalni nadzor in letnih poročil dveh največjih slovenskih zavarovalnic, preverimo, kako pomembne so posamezne rezervacije in premije v slovenskih zavarovalnicah.

Izračunavanje premij in rezervacij se v življenjski in neživljenjski zavarovalnici razlikuje, zato izračune naredimo ločeno. V življenjski zavarovalnici je izračun premije odvisen od vrste zavarovanja. Poznamo več vrst življenjskih zavarovanj, najpomembnejše štiri vrste so: življenjsko zavarovanje, življenjsko zavarovanje za točno določen rok, zavarovanje preživetja, tem trem zavarovanjem rečemo tudi klasična zavarovanja in naložbeno zavarovanje (angl. *unit-linked insurance*). V zadnjem času, se kot alternativa za klasično življenjsko zavarovanje vedno več uporablja naložbeno življenjsko zavarovanje (angl. *unit-linked insurance*). Naložbeno življenjsko zavarovanje omogoča zavarovancu, da prevzame naložbeno tveganje in v zameno prejme potencialno višje donose od garantiranih. Do leta 2010 je v Sloveniji število zavarovancev v naložbena življenjska zavarovanja raslo iz leta v leto. V obdobju od leta 2007 do leta 2011 je število zavarovancev v naložbeno življenjsko zavarovanje celo preseglo število zavarovancev v klasično življenjsko zavarovanje. Tako je število zavarovancev v naložbena življenjska zavarovanja iz 6.393 v letu 2002 poskočilo na 471.202 v letu 2015, v istem obdobju je število zavarovancev v klasično življenjsko zavarovanje zraslo za 28 %.

Premijo v življenjski zavarovalnici izračunamo z uporabo 'načela ekvivalentnosti' (angl. *equivalence principle*). Načelo pravi, da je sedanja vrednost prihodnjih pričakovanih premij enaka sedanji vrednosti prihodnjih pričakovanih odškodninskih zahtevkov plus sedanji vrednosti bodočih pričakovanih stroškov. Stroške se da v veliki meri oceniti, poleg tega, so te velikokrat vezani na velikost vplačanih premij, večji izziv za zavarovalnico predstavljajo bodoča izplačila za škode, zato se v nalogi posvetimo predvsem slednjim. Bodoča izplačila za škode v zavarovalnicah ocenjujejo s pomočjo tablic smrtnosti. Za primere življenjskega zavarovanja oziroma zavarovanja smrti uporabimo nemško periodično tablico DAV 2008 T. Premijo za neživljenjska zavarovanja izračunamo s pomočjo nemške kohortne tablice DAV 2004 R. Periodične tablice smrtnosti precenjujejo možnost smrti, na drugi strani jo periodične podcenjujejo, kar zavarovalnicam daje varnostno rezervo oziroma možnost za zaslužek. 1. marca 2012 je Evropska komisija sprejela direktivo na podlagi katere morajo zavarovalnice zaračunavati premijo na takšen način, da pri tem ne upoštevajo spola. Tablice smrtnosti, ki so bile pred letom 2012 narejene ločeno za ženske in moške združimo, tako, da vzamemo povprečje obeh smrtnosti. Sedanjo vrednost prihodnjih premij in izplačil dobimo z uporabo 'aktuarskih vrednosti' (angl. *actuarial value*). Aktuarska vrednost je sedanja vrednost prihodnjih premij in škodnih izplačil, ki v izračunu vzame v obzir smrtnosti in obrestno mero. Za lažje izračunavanje aktuarskih vrednosti, brez uporabe računalnika, so aktuarji, predvsem včasih uporabljali komutacijske funkcije (angl. *commutation function*), katere tudi sami uporabimo pri izračunu premij. Pod prilogami se nahajajo uporabljene tablice smrtnosti, za vsako posamezno starost so zraven izračunane komutacijske funkcije. Upoštevali smo obrestno mero 1,75 %, kar je maksimalna možna obrestna mera določena s strani Agencije za zavarovalni nadzor.

Največje vrste neživljenjskih zavarovanj v Sloveniji, gledano po višini bruto vplačanih zavarovalnih premij v Sloveniji, so prostovoljno zdravstveno zavarovanje, zavarovanje kopenskih motornih vozil, zavarovanje odgovornosti pri uporabi motornih vozil ter zavarovanje požara in elementarnih nesreč. Vplačane premije v vseh vrstah zavarovanj so do leta 2010 rasle, od leta 2010 pa padale z izjemo prostovoljnega zdravstvenega zavarovanja, ki je še naprej raslo. Bruto vplačane premije v prostovoljno zdravstveno zavarovanje so se od leta 2002 do leta 2015 v Sloveniji povečale za približno 77 %, če upoštevamo letno inflacijo v višini 2 %, je rast znašala približno 37 %.

Premije v neživljenjski zavarovalnici se ne da izračunati zgolj z uporabo ekvivalenčnega principa. Matematično se da pokazati (angl. *ruin theory*), da je v primeru, ko zavarovalnica nima pribitka na premije, v primeru, velikega števila, neodvisnih zavarovanj, verjetnost, da bodo izplačila presegla premije enaka 50 %, ne glede na velikost kapitala. Zavarovalnice zato pričakovanim izplačilom dodajo pribitek (angl. *premium principle*), ki ga izračunajo na različne načine. V naših izračunih bomo uporabili fiksni pribitek variance pričakovanih škodnih izplačil (angl. *variance premium principle*), ki se pogosto uporablja v praksi.

Neživljenjske zavarovalnice skušajo razvrstiti svoje zavarovance v kar se da homogene skupine in jim zaračunavati enake premije. V resnici je z vidika tveganja vsak zavarovanec edinstven, poleg tega se morajo zavarovalnice soočiti z moralnim hazardom (angl. *moral hazard*) in neugodno izbiro (angl. *adverse selection*). Z uporabo lastnega deleža (angl. *deductible*) in tako imenovane kredibilnostne premije (angl. *credibility premium*), zavarovalnice zmanjšajo tveganje moralnega hazarda in neugodne izbire. Prikažemo praktična primera za izračun kredibilnostne premije in premije z uporabo lastnega deleža. Pri izračunu kredibilnostne premije ugotovimo, da nekateri zavarovanci plačujejo višjo premijo, kot bi bilo "pravično", nekateri na drugi strani nižjo. Zato je pomembno, da zavarovalnica čim bolje pozna svoje zavarovance z vidika tveganja, ki ga ta povzroča in mu na podlagi tega zaračuna primerno premijo ter se s tem izogne neugodni izbiri. Izračun premije z uporabo lastnega deleža pokaže, da zavarovalnica zaračuna nižjo premijo ne zgolj zato, ker zavarovanec plača del škode, temveč zato, ker se zavarovanec vede manj tvegano, oziroma se zmanjša moralni hazard.

Zavarovalno-tehnične rezervacije predstavljajo, na strani obveznosti, najvišjo postavko v bilanci stanja zavarovalnice. Na podlagi letnega poročila Agencije za zavarovalni nadzor za leto 2016 so zavarovalno tehnične rezervacije predstavljale približno 70 % obveznosti do virov sredstev v bilancah slovenskih zavarovalnic. Poznamo različne vrste zavarovalno-tehničnih rezervacij, Zakon o zavarovalništvu predpisuje oblikovanje naslednjih zavarovalno-tehničnih rezervacij: rezervacije za prenosne premije, rezervacije za bonuse, popuste in storno, škodne rezervacije, izravnalne rezervacije, matematične rezervacije, posebne rezervacije za zavarovance, ki prevzemajo naložbeno tveganje (v večini gre za matematične rezervacije) in druge zavarovalno tehnične rezervacije. V življenjski zavarovalnici so najpomembnejše matematične rezervacije, saj so v letu 2016 predstavljale 96 % vseh zavarovalno-tehničnih rezervacij v slovenskih zavarovalnicah. V neživljenjskih zavarovalnicah so najvišje škodne rezervacije, ki so v letu 2016, po podatkih AZN, predstavljale okrog 70 % vseh zavarovalno-tehničnih rezervacij. Matematične rezervacije se oblikujejo za več let v naprej, zato se z leti nabirajo. Čeprav so vplačane premije v slovenskih življenjskih zavarovalnicah od leta 2008 do 2015 nekoliko padle, so matematične rezervacije v istem obdobju zrasle za 24 %. Škodne rezervacije, se na drugi strani ponavadi oblikujejo za krajše obdobje, ponavadi eno leto, kar pomeni, da se vsako leto porabijo in nato znova oblikujejo. Od leta 2007 do leta 2015 so škodne rezervacije v slovenskih zavarovalnicah stagnirale.

Matematične rezervacije izračunamo z uporabo ekvivalenčnega načela. Višina rezervacije se spreminja s potekom časa od sklenitve pogodbe. V posameznem trenutku so matematične rezervacije enake razliki med sedanjo vrednostjo prihodnjih pričakovanih škodnih izplačil in sedanjo vrednostjo prihodnjih pričakovanih premij. Matematične rezervacije, pri življenjskem zavarovanju za primer doživetja, rastejo približno enakomerno, linearno skozi obdobje zavarovanja. Pri življenjskem zavarovanju rezervacije sprva rastejo hitreje, do obdobja, ko naj bi umrl povprečen zavarovanec. Po tem obdobju, se rezervacije povečujejo vedno počasneje, krivulja rezervacij v odvisnosti od števila let od podpisa zavarovalne pogodbe, je vedno bolj položna. Rezervacije za življenjsko zavarovanje za določeno obdobje sprva rastejo, nato pa se zmanjšujejo in so na koncu zavarovalnega obdobja enake nič.

Rezervacije v neživljenjski zavarovalnici so v večini škodne rezervacije. Škodne rezervacije se izračuna s pomočjo podatkov o preteklih izplačilih škod. Pri neživljenjskih zavarovanjih namreč pogosto pride do zamika med trenutkom, ko je nastal škodni dogodek in trenutkom, ko je izplačana zavarovalnina. Ta zamik je lahko večleten. Sami smo izračunali škodne rezervacije z uporabo Chain Ladder metode. Po tej trikotniški metodi, najprej izračunamo rast izplačil za posamezno leto od škodnega dogodka (angl. *development factor*), nato rast pomnožimo z zadnjo znano kumulativno vsoto izplačil in dobimo pričakovano skupno končno izplačilo. Vsota razlik med pričakovanimi skupnimi končnimi izplačili in zadnjimi znanimi kumulativnimi izplačili, predstavlja skupno rezervacijo. Chain Ladder metoda ima nekatere pomanjkljivosti, ki so: nemogoče je izračunati pričakovano skupno izplačano zavarovalnino v primeru, da so robna kumulativna izplačila enaka nič, ne upošteva informacije o vplačanih premijah in je zelo odvisna od spremembe posamezne vrednosti. Kot alternativo predstavimo in rešimo nalogo z uporabo druge trikotniške metode, imenovane Cape Cod, ki predstavlja rešitev, za pomanjkljivosti Chain Ladder metode.

Appendix 2: Mortality table DAV 2008 T for men with commutation functions

DAV 2008 T Men							
x	q_x	l_x	d_x	D_x	N_x	C_x	M_x
0	0,006113	1000000	6113	1000000	41709752	6008	282633
1	0,000423	993887	420	976793	40709752	406	276625
2	0,000343	993467	341	959587	39732959	323	276219
3	0,000275	993126	273	942760	38773371	255	275896
4	0,00022	992853	218	926290	37830612	200	275641
5	0,000182	992634	181	910159	36904321	163	275441
6	0,000155	992454	154	894342	35994162	136	275278
7	0,000139	992300	138	878824	35099820	120	275142
8	0,000129	992162	128	863589	34220996	109	275022
9	0,000125	992034	124	848627	33357407	104	274912
10	0,000129	991910	128	833927	32508780	106	274808
11	0,000143	991782	142	819479	31674853	115	274702
12	0,000173	991640	172	805269	30855375	137	274587
13	0,000222	991469	220	791282	30050105	173	274450
14	0,000303	991248	300	777500	29258823	232	274277
15	0,000417	990948	413	763897	28481323	313	274046
16	0,000557	990535	552	750445	27717426	411	273733
17	0,000709	989983	702	737128	26966980	514	273322
18	0,00085	989281	841	723936	26229853	605	272808
19	0,000953	988440	942	710880	25505917	666	272204
20	0,001012	987498	999	697988	24795036	694	271538
21	0,001022	986499	1008	685289	24097048	688	270844
22	0,001004	985491	989	672815	23411759	664	270155
23	0,000963	984501	948	660579	22738944	625	269491
24	0,000911	983553	896	648592	22078366	581	268866
25	0,000856	982657	841	636857	21429773	536	268286
26	0,000808	981816	793	625367	20792917	497	267750
27	0,000772	981023	757	614115	20167549	466	267253
28	0,000752	980265	737	603087	19553434	446	266787
29	0,000745	979528	730	592269	18950347	434	266341
30	0,000752	978799	736	581649	18358078	430	265908
31	0,000768	978063	751	571215	17776429	431	265478
32	0,000791	977311	773	560960	17205214	436	265047
33	0,00082	976538	801	550876	16644255	444	264611
34	0,000855	975738	834	540957	16093379	455	264167
35	0,000895	974903	873	531199	15552422	467	263712
36	0,000945	974031	920	521595	15021223	484	263245
37	0,001005	973110	978	512140	14499628	506	262761
38	0,001083	972132	1053	502826	13987488	535	262255
39	0,001181	971079	1147	493642	13484663	573	261719

DAV 2008 T Men							
x	q_x	l_x	d_x	D_x	N_x	C_x	M_x
40	0,001301	969933	1262	484579	12991020	620	261147
41	0,001447	968671	1402	475625	12506441	676	260527
42	0,001623	967269	1570	466769	12030815	745	259851
43	0,001833	965699	1770	457996	11564047	825	259106
44	0,002082	963929	2007	449294	11106050	919	258281
45	0,002364	961922	2274	440647	10656756	1024	257362
46	0,002669	959648	2561	432045	10216109	1133	256338
47	0,002983	957087	2855	423481	9784064	1242	255205
48	0,003302	954232	3151	414956	9360583	1347	253963
49	0,00363	951081	3452	406472	8945627	1450	252616
50	0,003981	947629	3773	398031	8539155	1557	251166
51	0,004371	943856	4126	389628	8141124	1674	249609
52	0,004812	939731	4522	381253	7751495	1803	247935
53	0,005308	935209	4964	372893	7370242	1945	246132
54	0,005857	930244	5448	364534	6997349	2098	244187
55	0,00646	924796	5974	356166	6632815	2261	242089
56	0,007117	918822	6539	347779	6276648	2433	239827
57	0,007831	912283	7144	339365	5928869	2612	237395
58	0,008604	905138	7788	330917	5589503	2798	234783
59	0,009454	897351	8484	322427	5258587	2996	231985
60	0,010404	888867	9248	313886	4936160	3210	228989
61	0,011504	879619	10119	305278	4622274	3452	225779
62	0,012818	869500	11145	296576	4316996	3736	222328
63	0,014429	858355	12385	287739	4020420	4080	218592
64	0,016415	845970	13887	278710	3732681	4496	214511
65	0,018832	832083	15670	269420	3453971	4986	210015
66	0,021704	816413	17719	259800	3184552	5542	205028
67	0,025016	798694	19980	249790	2924752	6141	199487
68	0,028738	778714	22379	239352	2674962	6760	193345
69	0,032822	756335	24824	228475	2435610	7370	186585
70	0,037219	731511	27226	217176	2207135	7944	179215
71	0,04188	704285	29495	205497	1989959	8458	171271
72	0,046597	674789	31443	193504	1784462	8862	162813
73	0,051181	643346	32927	181314	1590958	9120	153951
74	0,05611	610419	34251	169076	1409644	9324	144831
75	0,061477	576168	35421	156844	1240568	9476	135507
76	0,067433	540747	36464	144670	1083724	9588	126031
77	0,07416	504283	37398	132594	939054	9664	116443
78	0,081806	466885	38194	120650	806460	9700	106779
79	0,090478	428691	38787	108874	685811	9681	97079
80	0,100261	389904	39092	97321	576936	9590	87398
81	0,111193	350812	39008	86057	479616	9404	77808
82	0,123283	311804	38440	75173	393559	9108	68404

DAV 2008 T Men							
x	q_x	l_x	d_x	D_x	N_x	C_x	M_x
83	0,136498	273364	37314	64772	318386	8689	59296
84	0,150887	236050	35617	54968	253614	8151	50607
85	0,1665	200433	33372	45872	198646	7506	42455
86	0,183344	167061	30630	37576	152774	6771	34949
87	0,201323	136432	27467	30159	115198	5967	28178
88	0,220284	108965	24003	23673	85039	5125	22211
89	0,240073	84962	20397	18141	61365	4280	17086
90	0,260556	64565	16823	13549	43224	3469	12805
91	0,281602	47742	13444	9846	29676	2725	9336
92	0,303079	34298	10395	6952	19830	2071	6611
93	0,324872	23903	7765	4762	12878	1520	4540
94	0,346887	16137	5598	3159	8116	1077	3020
95	0,369051	10540	3890	2028	4957	736	1943
96	0,391305	6650	2602	1258	2929	484	1207
97	0,413938	4048	1676	752	1671	306	724
98	0,437313	2372	1037	433	919	186	417
99	0,461101	1335	615	240	486	109	231
100	0,485304	719	349	127	246	61	123
101	0,509924	370	189	64	119	32	62
102	0,534957	181	97	31	55	16	30
103	0,560407	84	47	14	24	8	14
104	0,586265	37	22	6	10	4	6
105	0,612529	15	9	2	4	1	2
106	0,639188	6	4	1	1	1	1
107	0,666233	2	1	0	0	0	0
108	0,693651	1	0	0	0	0	0
109	0,721425	0	0	0	0	0	0
110	0,749533	0	0	0	0	0	0
111	0,77795	0	0	0	0	0	0
112	0,806647	0	0	0	0	0	0
113	0,835585	0	0	0	0	0	0
114	0,864722	0	0	0	0	0	0
115	0,894008	0	0	0	0	0	0
116	0,923382	0	0	0	0	0	0
117	0,952778	0	0	0	0	0	0
118	0,982113	0	0	0	0	0	0
119	1	0	0	0	0	0	0

Appendix 3: Mortality table DAV 2008 T for women with commutation functions

DAV 2008 T Women							
y	q_y	l_y	d_y	D_y	N_y	C_y	M_y
0	0,005088	1000000	5088	1000000	43143156	5000	257980
1	0,000387	994912	385	977800	42143156	372	252980
2	0,000318	994527	316	960611	41165355	300	252608
3	0,000255	994211	254	943790	40204744	237	252308
4	0,000202	993957	201	927321	39260954	184	252071
5	0,000163	993756	162	911188	38333634	146	251887
6	0,000134	993594	133	895370	37422446	118	251741
7	0,000115	993461	114	879853	36527076	99	251623
8	0,000105	993347	104	864621	35647223	89	251524
9	0,000099	993243	98	849661	34782602	83	251434
10	0,000102	993144	101	834965	33932941	84	251352
11	0,000111	993043	110	820521	33097976	90	251268
12	0,000127	992933	126	806319	32277456	101	251178
13	0,000153	992807	152	792350	31471137	119	251078
14	0,000188	992655	187	778604	30678786	144	250959
15	0,000228	992468	226	765069	29900183	171	250815
16	0,000271	992242	269	751739	29135114	200	250643
17	0,00031	991973	308	738609	28383375	225	250443
18	0,000324	991666	321	725681	27644766	231	250218
19	0,00033	991344	327	712969	26919085	231	249987
20	0,000328	991017	325	700475	26206116	226	249756
21	0,000322	990692	319	688202	25505641	218	249530
22	0,000314	990373	311	676148	24817439	209	249312
23	0,000304	990062	301	664310	24141291	198	249104
24	0,000297	989761	294	652686	23476981	191	248905
25	0,000293	989467	290	641270	22824295	185	248715
26	0,000292	989177	289	630056	22183025	181	248530
27	0,000292	988888	289	619039	21552969	178	248349
28	0,000296	988600	293	608214	20933930	177	248171
29	0,000302	988307	298	597577	20325716	177	247995
30	0,000311	988009	307	587122	19728139	179	247817
31	0,000327	987701	323	576844	19141017	185	247638
32	0,000351	987378	347	566738	18564173	196	247452
33	0,000386	987032	381	556795	17997435	211	247257
34	0,000433	986651	427	547007	17440640	233	247046
35	0,00049	986224	483	537367	16893632	259	246813
36	0,000555	985740	547	527866	16356266	288	246554
37	0,000624	985193	615	518499	15828400	318	246266
38	0,000701	984578	690	509263	15309901	351	245948
39	0,000783	983888	770	500154	14800638	385	245597

DAV 2008 T Women							
y	q_y	l_y	d_y	D_y	N_y	C_y	M_y
40	0,000872	983118	857	491167	14300484	421	245212
41	0,000972	982261	955	482298	13809317	461	244791
42	0,001084	981306	1064	473542	13327019	504	244331
43	0,001213	980242	1189	464893	12853477	554	243826
44	0,001359	979053	1331	456343	12388584	610	243272
45	0,001524	977723	1490	447885	11932240	671	242662
46	0,001706	976232	1665	439511	11484355	737	241992
47	0,001903	974567	1855	431215	11044843	806	241255
48	0,002109	972712	2051	422992	10613628	877	240448
49	0,002324	970661	2256	414840	10190636	948	239572
50	0,002546	968405	2466	406758	9775796	1018	238624
51	0,002782	965940	2687	398744	9369038	1090	237606
52	0,003035	963252	2923	390796	8970293	1166	236516
53	0,003306	960329	3175	382909	8579497	1244	235350
54	0,003593	957154	3439	375079	8196588	1324	234106
55	0,003898	953715	3718	367304	7821508	1407	232782
56	0,004228	949997	4017	359580	7454204	1494	231375
57	0,004585	945981	4337	351901	7094625	1586	229880
58	0,004974	941643	4684	344263	6742724	1683	228295
59	0,005402	936960	5061	336659	6398461	1787	226612
60	0,005884	931898	5483	329081	6061802	1903	224824
61	0,006449	926415	5974	321519	5732720	2038	222921
62	0,007126	920441	6559	313951	5411202	2199	220884
63	0,007935	913881	7252	306353	5097251	2389	218685
64	0,008898	906630	8067	298695	4790898	2612	216296
65	0,010025	898563	9008	290945	4492204	2867	213684
66	0,011323	889555	10072	283075	4201258	3150	210817
67	0,012797	879482	11255	275056	3918184	3459	207667
68	0,01446	868227	12555	266866	3643128	3793	204208
69	0,016332	855673	13975	258484	3376262	4149	200415
70	0,01844	841698	15521	249889	3117778	4529	196266
71	0,020813	826177	17195	241062	2867889	4931	191738
72	0,023475	808982	18991	231985	2626827	5352	186807
73	0,027035	789991	21357	222643	2394842	5916	181454
74	0,030413	768634	23376	212898	2172198	6364	175539
75	0,034287	745257	25553	202873	1959300	6836	169175
76	0,038749	719704	27888	192548	1756427	7333	162339
77	0,043937	691817	30396	181903	1563879	7855	155006
78	0,049993	661420	33066	170920	1381976	8398	147151
79	0,057024	628354	35831	159583	1211056	8944	138754
80	0,065113	592523	38581	147894	1051473	9464	129810
81	0,074288	553942	41151	135886	903579	9921	120346
82	0,08459	512791	43377	123628	767692	10278	110425

DAV 2008 T Women							
y	q_y	l_y	d_y	D_y	N_y	C_y	M_y
83	0,096095	469414	45108	111224	644064	10504	100147
84	0,109028	424305	46261	98807	532840	10587	89643
85	0,123611	378044	46730	86520	434033	10511	79055
86	0,140022	331314	46391	74521	347513	10255	68544
87	0,158257	284922	45091	62984	272992	9796	58289
88	0,178185	239832	42734	52105	210007	9125	48493
89	0,199669	197097	39354	42084	157903	8258	39368
90	0,222504	157743	35098	33102	115819	7239	31110
91	0,246453	122645	30226	25294	82717	6127	23871
92	0,271195	92418	25063	18732	57423	4993	17745
93	0,295584	67355	19909	13417	38690	3898	12752
94	0,319362	47446	15152	9289	25273	2915	8854
95	0,343441	32294	11091	6214	15984	2097	5939
96	0,367818	21203	7799	4009	9771	1449	3841
97	0,392493	13404	5261	2491	5761	961	2392
98	0,41746	8143	3399	1487	3270	610	1431
99	0,442716	4744	2100	852	1783	371	821
100	0,468258	2644	1238	466	931	215	450
101	0,494075	1406	695	244	465	118	236
102	0,520164	711	370	121	221	62	117
103	0,546514	341	186	57	100	31	55
104	0,573114	155	89	25	43	14	25
105	0,599953	66	40	11	17	6	10
106	0,627014	26	17	4	6	3	4
107	0,654283	10	6	2	2	1	2
108	0,681741	3	2	1	1	0	1
109	0,709364	1	1	0	0	0	0
110	0,73713	0	0	0	0	0	0
111	0,765011	0	0	0	0	0	0
112	0,792974	0	0	0	0	0	0
113	0,820987	0	0	0	0	0	0
114	0,849009	0	0	0	0	0	0
115	0,876998	0	0	0	0	0	0
116	0,904905	0	0	0	0	0	0
117	0,932675	0	0	0	0	0	0
118	0,960249	0	0	0	0	0	0
119	0,987564	0	0	0	0	0	0
120	1	0	0	0	0	0	0

Appendix 4: Mortality table DAV 2008 T for unisex with commutation functions

DAV 2008 T Unisex							
z	q_z = (q_x+q_y)/2	l_y	d_y	D_y	N_y	C_y	M_y
0	0,0056005	1000000	5601	1000000	42373730	5504	271092
1	0,000405	994400	403	977297	41373730	389	265650
2	0,0003305	993997	329	960099	40396433	312	265293
3	0,000265	993668	263	943275	39436334	246	264996
4	0,000211	993405	210	926806	38493059	192	264758
5	0,0001725	993195	171	910673	37566254	154	264569
6	0,0001445	993024	143	894856	36655581	127	264415
7	0,000127	992881	126	879338	35760724	110	264289
8	0,000117	992754	116	864105	34881386	99	264179
9	0,000112	992638	111	849144	34017281	93	264080
10	0,0001155	992527	115	834446	33168137	95	263987
11	0,000127	992412	126	820000	32333691	102	263892
12	0,00015	992286	149	805794	31513692	119	263789
13	0,0001875	992138	186	791816	30707898	146	263671
14	0,0002455	991952	244	778052	29916082	188	263525
15	0,0003225	991708	320	764483	29138030	242	263337
16	0,000414	991388	410	751092	28373547	306	263095
17	0,0005095	990978	505	737868	27622455	369	262789
18	0,000587	990473	581	724808	26884587	418	262420
19	0,0006415	989891	635	711924	26159779	449	262002
20	0,00067	989256	663	699231	25447855	460	261553
21	0,000672	988594	664	686744	24748624	454	261092
22	0,000659	987929	651	674479	24061880	437	260639
23	0,0006335	987278	625	662442	23387401	412	260202
24	0,000604	986653	596	650636	22724958	386	259789
25	0,0005745	986057	566	639060	22074322	361	259403
26	0,00055	985490	542	627708	21435262	339	259042
27	0,000532	984948	524	616573	20807554	322	258703
28	0,000524	984424	516	605646	20190982	312	258381
29	0,0005235	983909	515	594917	19585336	306	258069
30	0,0005315	983393	523	584379	18990419	305	257763
31	0,0005475	982871	538	574023	18406040	309	257457
32	0,000571	982333	561	563842	17832016	316	257149
33	0,000603	981772	592	553828	17268175	328	256832
34	0,000644	981180	632	543974	16714347	344	256504
35	0,0006925	980548	679	534274	16170373	364	256160
36	0,00075	979869	735	524722	15636098	387	255796
37	0,0008145	979134	798	515310	15111377	413	255409
38	0,000892	978336	873	506035	14596067	444	254997
39	0,000982	977464	960	496888	14090032	480	254553

DAV 2008 T Unisex							
z	q_z = (q_x+q_y)/2	l_y	d_y	D_y	N_y	C_y	M_y
40	0,0010865	976504	1061	487862	13593144	521	254074
41	0,0012095	975443	1180	478951	13105282	569	253553
42	0,0013535	974263	1319	470144	12626331	625	252983
43	0,001523	972944	1482	461432	12156188	691	252358
44	0,0017205	971463	1671	452806	11694755	766	251667
45	0,001944	969791	1885	444252	11241950	849	250902
46	0,0021875	967906	2117	435763	10797698	937	250053
47	0,002443	965789	2359	427331	10361935	1026	249116
48	0,0027055	963429	2607	418955	9934604	1114	248090
49	0,002977	960823	2860	410636	9515649	1201	246976
50	0,0032635	957962	3126	402372	9105013	1291	245775
51	0,0035765	954836	3415	394161	8702641	1385	244484
52	0,0039235	951421	3733	385996	8308480	1488	243099
53	0,004307	947688	4082	377869	7922484	1599	241610
54	0,004725	943606	4459	369771	7544615	1717	240011
55	0,005179	939148	4864	361694	7174844	1841	238293
56	0,0056725	934284	5300	353632	6813151	1971	236452
57	0,006208	928984	5767	345578	6459519	2108	234481
58	0,006789	923217	6268	337526	6113940	2252	232373
59	0,007428	916949	6811	329469	5776414	2405	230121
60	0,008144	910138	7412	321397	5446945	2572	227715
61	0,0089765	902726	8103	313297	5125547	2764	225143
62	0,009972	894623	8921	305145	4812250	2991	222379
63	0,011182	885702	9904	296906	4507105	3263	219388
64	0,0126565	875798	11085	288537	4210199	3589	216125
65	0,0144285	864713	12477	279985	3921662	3970	212536
66	0,0165135	852237	14073	271199	3641677	4401	208566
67	0,0189065	838163	15847	262134	3370478	4871	204165
68	0,021599	822317	17761	252754	3108344	5365	199294
69	0,024577	804555	19774	243042	2855590	5871	193929
70	0,0278295	784782	21840	232991	2612548	6373	188058
71	0,0313465	762942	23916	222612	2379557	6858	181686
72	0,035036	739026	25893	211925	2156945	7297	174827
73	0,039108	713134	27889	200983	1945020	7725	167530
74	0,0432615	685244	29645	189801	1744038	8070	159805
75	0,047882	655600	31391	178467	1554237	8398	151735
76	0,053091	624208	33140	166999	1375770	8714	143337
77	0,0590485	591068	34902	155413	1208771	9019	134623
78	0,0658995	556167	36651	143721	1053358	9308	125604
79	0,073751	519516	38315	131941	909637	9563	116296
80	0,082687	481201	39789	120108	777696	9761	106733
81	0,0927405	441412	40937	108282	657588	9869	96972
82	0,1039365	400475	41624	96550	549306	9862	87103

DAV 2008 T Unisex							
z	q_z = (q_x+q_y)/2	l_y	d_y	D_y	N_y	C_y	M_y
83	0,1162965	358851	41733	85027	452756	9718	77240
84	0,1299575	317118	41212	73846	367728	9432	67522
85	0,1450555	275906	40022	63145	293882	9002	58090
86	0,161683	235884	38138	53057	230737	8431	49088
87	0,17979	197746	35553	43713	177681	7724	40657
88	0,1992345	162193	32314	35237	133968	6900	32933
89	0,219871	129879	28557	27732	98730	5993	26034
90	0,24153	101322	24472	21262	70999	5047	20041
91	0,2640275	76850	20290	15849	49736	4113	14994
92	0,287137	56559	16240	11464	33887	3235	10881
93	0,310228	40319	12508	8032	22423	2449	7646
94	0,3331245	27811	9265	5445	14391	1783	5197
95	0,356246	18546	6607	3569	8947	1249	3415
96	0,3795615	11939	4532	2258	5378	842	2165
97	0,4032155	7408	2987	1377	3120	546	1323
98	0,4273865	4421	1889	807	1744	339	777
99	0,4519085	2531	1144	454	936	202	438
100	0,476781	1387	662	245	482	115	236
101	0,5019995	726	364	126	237	62	122
102	0,5275605	362	191	62	111	32	60
103	0,5534605	171	95	29	49	16	28
104	0,5796895	76	44	13	21	7	12
105	0,606241	32	19	5	8	3	5
106	0,633101	13	8	2	3	1	2
107	0,660258	5	3	1	1	0	1
108	0,687696	2	1	0	0	0	0
109	0,7153945	0	0	0	0	0	0
110	0,7433315	0	0	0	0	0	0
111	0,7714805	0	0	0	0	0	0
112	0,7998105	0	0	0	0	0	0
113	0,828286	0	0	0	0	0	0
114	0,8568655	0	0	0	0	0	0
115	0,885503	0	0	0	0	0	0
116	0,9141435	0	0	0	0	0	0
117	0,9427265	0	0	0	0	0	0
118	0,971181	0	0	0	0	0	0
119	0,993782	0	0	0	0	0	0
120	1	0	0	0	0	0	0

Appendix 5: Mortality table DAV 2004 R for men with commutation functions

DAV Men 2004 R							
x	q_x (Basic table 1965)	l_x	d_x	D_x	N_x	C_x	M_x
0	0,000083	1000000	83	1000000	45580443	82	216061
1	0,000083	999917	83	982719	44580443	80	215980
2	0,000083	999834	83	965737	43597724	79	215899
3	0,000083	999751	83	949049	42631986	77	215821
4	0,000083	999668	83	932649	41682937	76	215743
5	0,000083	999585	83	916532	40750288	75	215667
6	0,000083	999502	83	900694	39833756	73	215592
7	0,000083	999419	83	885129	38933062	72	215519
8	0,000083	999336	83	869834	38047933	71	215447
9	0,000083	999253	83	854803	37178099	70	215376
10	0,000083	999170	83	840031	36323297	69	215306
11	0,000098	999087	98	825515	35483266	80	215238
12	0,000104	998989	104	811237	34657751	83	215158
13	0,000114	998886	114	797202	33846514	89	215075
14	0,00014	998772	140	783401	33049312	108	214986
15	0,000192	998632	192	769820	32265910	145	214878
16	0,000276	998440	276	756435	31496090	205	214733
17	0,000364	998165	363	743219	30739656	266	214528
18	0,000596	997801	595	730171	29996436	428	214262
19	0,000598	997207	596	717185	29266266	422	213834
20	0,000598	996610	596	704429	28549081	414	213412
21	0,000598	996014	596	691899	27844652	407	212998
22	0,000598	995419	595	679592	27152753	399	212592
23	0,000598	994823	595	667505	26473160	392	212192
24	0,000598	994228	595	655632	25805656	385	211800
25	0,000598	993634	594	643970	25150024	378	211415
26	0,000598	993040	594	632516	24506053	372	211036
27	0,000598	992446	593	621266	23873537	365	210665
28	0,000598	991852	593	610216	23252271	359	210299
29	0,000598	991259	593	599362	22642055	352	209941
30	0,000598	990666	592	588701	22042693	346	209589
31	0,000605	990074	599	578230	21453992	344	209243
32	0,000626	989475	619	567941	20875762	349	208899
33	0,000663	988856	656	557824	20307821	363	208549
34	0,000713	988200	705	547866	19749997	384	208186
35	0,000754	987495	745	538060	19202130	399	207802
36	0,000805	986751	794	528407	18664070	418	207403
37	0,000871	985957	859	518901	18135664	444	206985
38	0,00094	985098	926	509532	17616763	471	206541
39	0,001008	984172	992	500298	17107231	496	206070

DAV Men 2004 R							
x	q_x (Basic table 1965)	l_x	d_x	D_x	N_x	C_x	M_x
40	0,001073	983180	1055	491198	16606933	518	205575
41	0,001137	982125	1117	482231	16115735	539	205057
42	0,001197	981008	1174	473399	15633504	557	204518
43	0,001259	979834	1234	464700	15160105	575	203961
44	0,001325	978600	1297	456132	14695405	594	203386
45	0,001395	977304	1363	447693	14239273	614	202792
46	0,001473	975940	1438	439380	13791580	636	202178
47	0,001557	974503	1517	431187	13352200	660	201542
48	0,001644	972985	1600	423111	12921013	684	200882
49	0,001735	971386	1685	415150	12497902	708	200199
50	0,001826	969700	1771	407302	12082752	731	199491
51	0,001924	967930	1862	399566	11675450	756	198760
52	0,002023	966068	1954	391938	11275884	779	198004
53	0,002121	964113	2045	384418	10883945	801	197225
54	0,002212	962068	2128	377005	10499527	820	196424
55	0,002294	959940	2202	369701	10122522	834	195604
56	0,00237	957738	2270	362509	9752821	844	194771
57	0,002451	955468	2342	355430	9390311	856	193926
58	0,00254	953126	2421	348461	9034881	870	193070
59	0,002649	950705	2518	341598	8686420	889	192200
60	0,002781	948187	2637	334834	8344822	915	191311
61	0,002957	945550	2796	328160	8009988	954	190396
62	0,003176	942754	2994	321562	7681829	1004	189442
63	0,003432	939760	3225	315028	7360267	1063	188438
64	0,003707	936535	3472	308547	7045239	1124	187376
65	0,00398	933063	3714	302116	6736692	1182	186252
66	0,00427	929349	3968	295738	6434576	1241	185070
67	0,004631	925381	4285	289411	6138838	1317	183829
68	0,004995	921096	4601	283116	5849428	1390	182512
69	0,005363	916495	4915	276857	5566312	1459	181122
70	0,005744	911580	5236	270636	5289455	1528	179662
71	0,00615	906343	5574	264453	5018819	1598	178135
72	0,006605	900769	5950	258307	4754366	1677	176536
73	0,007122	894820	6373	252187	4496059	1765	174859
74	0,007722	888447	6861	246085	4243872	1868	173094
75	0,00846	881586	7458	239985	3997787	1995	171227
76	0,009337	874128	8162	233862	3757802	2146	169231
77	0,010403	865966	9009	227694	3523941	2328	167085
78	0,011693	856958	10020	221450	3296247	2545	164757
79	0,013259	846937	11230	215096	3074797	2803	162212
80	0,015167	835708	12675	208594	2859701	3109	159410
81	0,01745	823033	14362	201897	2651108	3463	156300
82	0,020162	808671	16304	194962	2449211	3863	152838

DAV Men 2004 R							
x	q_x (Basic table 1965)	l_x	d_x	D_x	N_x	C_x	M_x
83	0,023324	792366	18481	187745	2254249	4304	148975
84	0,02697	773885	20872	180213	2066504	4777	144671
85	0,031142	753013	23450	172336	1886291	5275	139894
86	0,035854	729563	26158	164098	1713955	5782	134620
87	0,041159	703405	28951	155493	1549857	6290	128837
88	0,04709	674454	31760	146529	1394364	6781	122547
89	0,053666	642694	34491	137227	1247835	7238	115766
90	0,060681	608203	36906	127629	1110607	7611	108528
91	0,067908	571297	38796	117823	982978	7864	100917
92	0,075209	532501	40049	107933	865155	7978	93053
93	0,082462	492452	40609	98099	757222	7950	85075
94	0,089515	451844	40447	88461	659123	7782	77125
95	0,096209	411397	39580	79157	570662	7485	69343
96	0,102378	371817	38066	70311	491505	7075	61858
97	0,107876	333751	36004	62027	421194	6576	54783
98	0,113045	297747	33659	54384	359166	6042	48207
99	0,118108	264088	31191	47407	304782	5503	42165
100	0,121553	232897	28309	41089	257375	4909	36662
101	0,126442	204588	25869	35473	216286	4408	31754
102	0,131302	178720	23466	30455	180813	3930	27345
103	0,13613	155253	21135	26001	150357	3479	23415
104	0,140927	134119	18901	22075	124356	3058	19937
105	0,14569	115218	16786	18638	102281	2669	16879
106	0,150416	98432	14806	15649	83642	2313	14210
107	0,155105	83626	12971	13066	67993	1992	11897
108	0,159752	70655	11287	10850	54927	1703	9905
109	0,164354	59368	9757	8960	44077	1447	8202
110	0,168907	49611	8380	7358	35117	1222	6754
111	0,173407	41231	7150	6010	27759	1024	5533
112	0,177848	34081	6061	4883	21748	853	4509
113	0,182224	28020	5106	3945	16865	707	3655
114	0,186528	22914	4274	3171	12920	581	2949
115	0,190752	18640	3556	2535	9749	475	2367
116	0,194887	15084	2940	2016	7214	386	1892
117	0,198923	12145	2416	1595	5198	312	1506
118	0,202848	9729	1973	1256	3603	250	1194
119	0,206649	7755	1603	984	2347	200	944
120	0,210311	6153	1294	767	1363	159	744
121	1	4859	4859	595	595	585	585

Appendix 6: Mortality table DAV 2004 R for women with commutation functions

DAV Women 2004 R							
y	q_y (Basic table 1965)	l_y	d_y	D_y	N_y	C_y	M_y
0	0,000066	1000000	66	1000000	46684774	65	197068
1	0,000066	999934	66	982736	45684774	64	197003
2	0,000066	999868	66	965770	44702038	63	196939
3	0,000066	999802	66	949097	43736268	62	196877
4	0,000066	999736	66	932712	42787170	61	196815
5	0,000066	999670	66	916610	41854458	59	196754
6	0,000066	999604	66	900786	40937848	58	196695
7	0,000066	999538	66	885235	40037062	57	196637
8	0,000066	999472	66	869952	39151828	56	196579
9	0,000066	999406	66	854933	38281876	55	196523
10	0,000066	999340	66	840174	37426942	54	196467
11	0,000071	999274	71	825669	36586768	58	196413
12	0,000075	999203	75	811411	35761099	60	196355
13	0,000079	999128	79	797396	34949688	62	196295
14	0,000092	999049	92	783619	34152293	71	196233
15	0,00012	998958	120	770071	33368673	91	196163
16	0,000144	998838	144	756736	32598602	107	196072
17	0,000166	998694	166	743613	31841867	121	195965
18	0,000201	998528	201	730703	31098253	144	195843
19	0,000201	998327	201	717991	30367550	142	195699
20	0,000201	998127	201	705500	29649559	139	195557
21	0,000201	997926	201	693227	28944059	137	195418
22	0,000201	997725	201	681167	28250832	135	195281
23	0,000201	997525	201	669317	27569664	132	195146
24	0,000222	997324	221	657674	26900347	143	195014
25	0,000225	997103	224	646219	26242673	143	194871
26	0,000225	996879	224	634962	25596455	140	194728
27	0,000235	996654	234	623900	24961493	144	194587
28	0,000258	996420	257	613026	24337593	155	194443
29	0,00028	996163	279	602327	23724567	166	194288
30	0,000291	995884	290	591802	23122240	169	194122
31	0,000302	995594	301	581454	22530438	173	193953
32	0,000318	995294	317	571281	21948984	179	193780
33	0,000344	994977	342	561277	21377703	190	193602
34	0,000385	994635	383	551434	20816426	209	193412
35	0,000423	994252	421	541741	20264992	225	193203
36	0,000464	993831	461	532199	19723251	243	192978
37	0,000508	993370	505	522803	19191052	261	192735
38	0,00055	992866	546	513550	18668250	278	192474
39	0,000593	992320	588	504440	18154700	294	192197

DAV Women 2004 R							
y	q_y (Basic table 1965)	l_y	d_y	D_y	N_y	C_y	M_y
40	0,000642	991731	637	495470	17650260	313	191903
41	0,000693	991094	687	486636	17154790	331	191590
42	0,000743	990408	736	477935	16668155	349	191259
43	0,000788	989672	780	469366	16190220	363	190910
44	0,00083	988892	821	460929	15720855	376	190546
45	0,000874	988071	864	452626	15259925	389	190170
46	0,000921	987207	909	444452	14807299	402	189781
47	0,000971	986298	958	436406	14362847	416	189379
48	0,001022	985341	1007	428484	13926441	430	188963
49	0,001069	984334	1052	420684	13497957	442	188532
50	0,001111	983281	1092	413006	13077274	451	188090
51	0,001149	982189	1129	405452	12664267	458	187639
52	0,001182	981060	1160	398021	12258815	462	187181
53	0,001218	979901	1194	390713	11860794	468	186719
54	0,001259	978707	1232	383525	11470081	475	186251
55	0,001306	977475	1277	376455	11086555	483	185777
56	0,001363	976198	1331	369497	10710101	495	185294
57	0,00143	974868	1394	362647	10340604	510	184799
58	0,001504	973474	1464	355900	9977957	526	184289
59	0,001585	972010	1541	349253	9622057	544	183763
60	0,001674	970469	1625	342702	9272804	564	183219
61	0,001771	968845	1716	336244	8930102	585	182655
62	0,001876	967129	1814	329876	8593858	608	182070
63	0,001986	965314	1917	323594	8263982	632	181462
64	0,002096	963397	2019	317397	7940388	654	180830
65	0,002229	961378	2143	311284	7622991	682	180176
66	0,002345	959235	2249	305248	7311707	703	179494
67	0,00252	956986	2412	299295	7006459	741	178791
68	0,002732	954574	2608	293406	6707164	788	178050
69	0,002959	951966	2817	287572	6413758	836	177262
70	0,003199	949149	3036	281790	6126186	886	176425
71	0,003478	946113	3291	276057	5844396	944	175539
72	0,00378	942822	3564	270366	5568339	1004	174596
73	0,00409	939258	3842	264711	5297973	1064	173591
74	0,004446	935417	4159	259095	5033261	1132	172527
75	0,004864	931258	4530	253506	4774167	1212	171395
76	0,005328	926728	4938	247934	4520661	1298	170183
77	0,005823	921791	5368	242372	4272726	1387	168885
78	0,006429	916423	5892	236816	4030354	1496	167498
79	0,007203	910532	6559	231247	3793538	1637	166002
80	0,008215	903973	7426	225633	3562291	1822	164365
81	0,009536	896547	8549	219930	3336658	2061	162543
82	0,011237	887997	9978	214087	3116728	2364	160482

DAV Women 2004 R							
y	q_y (Basic table 1965)	l_y	d_y	D_y	N_y	C_y	M_y
83	0,013343	878019	11715	208040	2902642	2728	158118
84	0,015844	866304	13726	201734	2694601	3141	155389
85	0,018792	852578	16022	195123	2492867	3604	152248
86	0,022273	836556	18633	188163	2297744	4119	148644
87	0,026353	817924	21555	180808	2109581	4683	144526
88	0,031049	796369	24726	173016	1928773	5280	139843
89	0,036369	771642	28064	164760	1755757	5889	134563
90	0,042123	743579	31322	156038	1590997	6460	128674
91	0,048071	712257	34239	146894	1434959	6940	122214
92	0,054145	678018	36711	137428	1288065	7313	115274
93	0,060268	641307	38650	127751	1150637	7567	107961
94	0,066351	602656	39987	117987	1022886	7694	100394
95	0,072275	562669	40667	108264	904899	7690	92701
96	0,077904	522003	40666	98712	796635	7558	85010
97	0,083095	481336	39997	89456	697923	7306	77453
98	0,087727	441340	38717	80612	608467	6950	70147
99	0,091681	402622	36913	72275	527855	6512	63197
100	0,100158	365710	36629	64520	455580	6351	56685
101	0,104765	329081	34476	57059	391060	5875	50333
102	0,109394	294605	32228	50203	334000	5397	44458
103	0,114045	262377	29923	43942	283797	4925	39061
104	0,118719	232454	27597	38261	239855	4464	34136
105	0,123417	204857	25283	33139	201594	4020	29672
106	0,128138	179574	23010	28549	168455	3595	25652
107	0,132883	156564	20805	24463	139906	3195	22057
108	0,137652	135759	18688	20847	115443	2820	18862
109	0,142443	117072	16676	17669	94596	2473	16042
110	0,147255	100396	14784	14891	76927	2155	13568
111	0,152087	85612	13020	12480	62036	1865	11413
112	0,156935	72592	11392	10400	49556	1604	9548
113	0,161796	61199	9902	8617	39156	1370	7944
114	0,166665	51298	8550	7099	30539	1163	6573
115	0,171536	42748	7333	5814	23440	980	5411
116	0,176401	35415	6247	4734	17627	821	4431
117	0,18125	29168	5287	3832	12893	683	3610
118	0,186074	23881	4444	3083	9061	564	2927
119	0,190855	19438	3710	2466	5978	463	2363
120	0,195579	15728	3076	1961	3512	377	1901
121	1	12652	12652	1551	1551	1524	1524

Appendix 7: Mortality table DAV 2004 R for unisex with commutation functions

DAV Unisex 2004 R							
z	q_z (Basic table 1965)	l_z	d_z	D_z	N_z	C_z	M_z
0	0,0000745	1000000	75	1000000	46132609	73	206564
1	0,0000745	999926	74	982728	45132609	72	206491
2	0,0000745	999851	74	965754	44149881	71	206419
3	0,0000745	999777	74	949073	43184127	69	206349
4	0,0000745	999702	74	932681	42235054	68	206279
5	0,0000745	999628	74	916571	41302373	67	206211
6	0,0000745	999553	74	900740	40385802	66	206144
7	0,0000745	999479	74	885182	39485062	65	206078
8	0,0000745	999404	74	869893	38599880	64	206013
9	0,0000745	999330	74	854868	37729987	63	205949
10	0,0000745	999255	74	840102	36875119	62	205887
11	0,0000845	999181	84	825592	36035017	69	205825
12	0,0000895	999096	89	811324	35209425	71	205757
13	0,0000965	999007	96	797299	34398101	76	205685
14	0,000116	998911	116	783510	33600802	89	205610
15	0,000156	998795	156	769945	32817292	118	205520
16	0,00021	998639	210	756585	32047346	156	205402
17	0,000265	998429	265	743416	31290761	194	205246
18	0,0003985	998165	398	730437	30547345	286	205053
19	0,0003995	997767	398	717588	29816908	282	204766
20	0,0003995	997368	398	704965	29099320	277	204485
21	0,0003995	996970	398	692563	28394355	272	204208
22	0,0003995	996572	398	680380	27701792	267	203936
23	0,0003995	996174	398	668411	27021412	262	203669
24	0,00041	995776	408	656653	26353001	264	203407
25	0,0004115	995368	409	645095	25696348	261	203143
26	0,0004115	994959	409	633739	25051254	256	202882
27	0,0004165	994550	414	622583	24417515	255	202626
28	0,000428	994136	425	611621	23794932	257	202371
29	0,000439	993711	436	600844	23183311	259	202114
30	0,0004445	993275	441	590252	22582467	258	201855
31	0,0004535	992834	450	579842	21992215	258	201598
32	0,000472	992384	468	569611	21412373	264	201339
33	0,0005035	991916	499	559550	20842762	277	201075
34	0,000549	991417	544	549650	20283211	296	200799
35	0,0005885	990874	583	539900	19733561	312	200503
36	0,0006345	990291	628	530303	19193661	330	200191
37	0,0006895	989663	682	520852	18663358	353	199860
38	0,000745	988982	736	511541	18142506	374	199508
39	0,0008005	988246	790	502369	17630965	395	199133

DAV Unisex 2004 R							
z	q_z (Basic table 1965)	l_z	d_z	D_z	N_z	C_z	M_z
40	0,0008575	987455	846	493334	17128597	415	198739
41	0,000915	986610	902	484434	16635263	435	198323
42	0,00097	985708	955	475667	16150829	453	197888
43	0,0010235	984753	1007	467033	15675163	469	197435
44	0,0010775	983746	1059	458531	15208130	485	196966
45	0,0011345	982687	1113	450160	14749599	501	196481
46	0,001197	981574	1173	441916	14299439	519	195980
47	0,001264	980400	1237	433796	13857523	538	195461
48	0,001333	979163	1303	425797	13423727	557	194922
49	0,001402	977860	1369	417917	12997930	575	194365
50	0,0014685	976491	1432	410154	12580013	591	193790
51	0,0015365	975059	1495	402509	12169858	607	193200
52	0,0016025	973564	1557	394980	11767349	621	192593
53	0,0016695	972007	1619	387566	11372370	635	191972
54	0,0017355	970388	1680	380265	10984804	647	191337
55	0,0018	968708	1739	373078	10604539	658	190690
56	0,0018665	966968	1800	366003	10231461	670	190032
57	0,0019405	965168	1868	359039	9865457	683	189362
58	0,002022	963300	1943	352181	9506419	698	188679
59	0,002117	961358	2030	345425	9154238	717	187982
60	0,0022275	959328	2131	338768	8808813	739	187265
61	0,002364	957197	2256	332202	8470045	769	186525
62	0,002526	954941	2404	325719	8137843	806	185756
63	0,002709	952537	2571	319311	7812125	847	184950
64	0,0029015	949966	2746	312972	7492814	889	184103
65	0,0031045	947220	2928	306700	7179842	932	183214
66	0,0033075	944292	3109	300493	6873142	972	182282
67	0,0035755	941183	3349	294353	6572649	1029	181310
68	0,0038635	937835	3604	288261	6278296	1089	180281
69	0,004161	934230	3866	282214	5990035	1148	179192
70	0,0044715	930364	4136	276213	5707820	1207	178044
71	0,004814	926228	4432	270255	5431607	1271	176837
72	0,0051925	921796	4757	264336	5161352	1341	175566
73	0,005606	917039	5107	258449	4897016	1415	174225
74	0,006084	911932	5510	252590	4638567	1500	172811
75	0,006662	906422	5994	246745	4385977	1604	171311
76	0,0073325	900428	6550	240898	4139231	1722	169707
77	0,008113	893879	7188	235033	3898333	1858	167985
78	0,009061	886690	7956	229133	3663301	2021	166128
79	0,010231	878734	8894	223171	3434168	2220	164107
80	0,011691	869840	10051	217113	3210996	2466	161887
81	0,013493	859790	11456	210914	2993883	2762	159422
82	0,0156995	848334	13141	204524	2782970	3114	156660

DAV Unisex 2004 R							
z	q_z (Basic table 1965)	l_z	d_z	D_z	N_z	C_z	M_z
83	0,0183335	835193	15098	197893	2578445	3516	153546
84	0,021407	820094	17299	190973	2380553	3959	150030
85	0,024967	802796	19736	183730	2189579	4439	146071
86	0,0290635	783060	22395	176131	2005850	4951	141632
87	0,033756	760664	25253	168151	1829719	5486	136681
88	0,0390695	735411	28243	159772	1661568	6030	131195
89	0,0450175	707168	31277	150994	1501796	6563	125165
90	0,051402	675891	34114	141834	1350802	7036	118601
91	0,0579895	641777	36517	132359	1208968	7402	111565
92	0,064677	605259	38380	122680	1076610	7645	104164
93	0,071365	566879	39629	112925	953930	7759	96518
94	0,077933	527250	40217	103224	841005	7738	88760
95	0,084242	487033	40124	93711	737780	7587	81022
96	0,090141	446910	39366	84511	644070	7316	73434
97	0,0954855	407544	38000	75742	559558	6941	66118
98	0,100386	369543	36188	67498	483817	6496	59177
99	0,1048945	333355	34052	59841	416318	6008	52681
100	0,1108555	299303	32469	52804	356477	5630	46673
101	0,1156035	266834	30172	46266	303673	5142	41044
102	0,120348	236662	27847	40329	257406	4664	35902
103	0,1250875	208815	25529	34972	217077	4202	31238
104	0,129823	183286	23249	30168	182106	3761	27036
105	0,1345535	160037	21034	25889	151937	3344	23275
106	0,139277	139003	18908	22099	126049	2954	19931
107	0,143994	120095	16888	18765	103950	2593	16977
108	0,148702	103207	14987	15849	85185	2262	14384
109	0,1533985	88220	13217	13314	69336	1960	12122
110	0,158081	75003	11582	11125	56022	1688	10161
111	0,162747	63421	10085	9245	44897	1445	8473
112	0,1673915	53336	8727	7641	35652	1229	7028
113	0,17201	44610	7504	6281	28011	1038	5799
114	0,1765965	37106	6412	5135	21730	872	4761
115	0,181144	30694	5444	4174	16595	728	3889
116	0,185644	25250	4594	3375	12420	603	3161
117	0,1900865	20656	3851	2713	9045	497	2558
118	0,194461	16805	3209	2170	6332	407	2061
119	0,198752	13596	2656	1725	4162	331	1654
120	0,202945	10940	2185	1364	2437	268	1322
121	1	8755	8755	1073	1073	1055	1055

Appendix 8: DAV 2004 R Age Adjustment

DAV 2004 R Age Adjustment			
born in	men	women	unisex
1910	12	11	11,5
1911	12	11	11,5
1912	12	11	11,5
1913	12	11	11,5
1914	12	11	11,5
1915	12	11	11,5
1916	12	11	11,5
1917	12	10	11
1918	11	10	10,5
1919	10	9	9,5
1920	9	8	8,5
1921	8	8	8
1922	7	7	7
1923	7	7	7
1924	7	7	7
1925	7	6	6,5
1926	6	6	6
1927	6	6	6
1928	6	6	6
1929	6	6	6
1930	6	6	6
1931	6	6	6
1932	6	6	6
1933	6	6	6
1934	6	5	5,5
1935	5	5	5
1936	5	5	5
1937	5	5	5
1938	5	5	5
1939	5	5	5
1940	5	4	4,5
1941	5	4	4,5
1942	4	4	4
1943	4	4	4
1944	4	4	4
1945	4	4	4
1946	4	3	3,5
1947	4	3	3,5
1948	3	3	3
1949	3	3	3
1950	3	3	3

DAV 2004 R Age Adjustment			
born in	men	women	unisex
1951	3	2	2,5
1952	3	2	2,5
1953	2	2	2
1954	2	2	2
1955	2	2	2
1956	2	1	1,5
1957	1	1	1
1958	1	1	1
1959	1	1	1
1960	1	1	1
1961	0	0	0
1962	0	0	0
1963	0	0	0
1964	0	0	0
1965	0	0	0
1966	-1	-1	-1
1967	-1	-1	-1
1968	-1	-1	-1
1969	-1	-1	-1
1970	-2	-2	-2
1971	-2	-2	-2
1972	-2	-2	-2
1973	-2	-2	-2
1974	-3	-2	-2,5
1975	-3	-3	-3
1976	-3	-3	-3
1977	-3	-3	-3
1978	-4	-3	-3,5
1979	-4	-3	-3,5
1980	-4	-4	-4
1981	-4	-4	-4
1982	-5	-4	-4,5
1983	-5	-4	-4,5
1984	-5	-4	-4,5
1985	-5	-5	-5
1986	-6	-5	-5,5
1987	-6	-5	-5,5
1988	-6	-5	-5,5
1989	-6	-5	-5,5
1990	-7	-6	-6,5
1991	-7	-6	-6,5
1992	-7	-6	-6,5
1993	-7	-6	-6,5

DAV 2004 R Age Adjustment			
born in	men	women	unisex
1994	-7	-7	-7
1995	-8	-7	-7,5
1996	-8	-7	-7,5
1997	-8	-7	-7,5
1998	-8	-7	-7,5
1999	-9	-8	-8,5
2000	-9	-8	-8,5
2001	-9	-8	-8,5
2002	-9	-8	-8,5
2003	-10	-8	-9
2004	-10	-8	-9
2005	-10	-9	-9,5
2006	-10	-9	-9,5
2007	-10	-9	-9,5
2008	-11	-9	-10
2009	-11	-9	-10
2010	-11	-10	-10,5
2011	-11	-10	-10,5
2012	-12	-10	-11
2013	-12	-10	-11
2014	-12	-10	-11
2015	-12	-11	-11,5
2016	-12	-11	-11,5
2017	-13	-11	-12
2018	-13	-11	-12
2019	-13	-11	-12
2020	-13	-12	-12,5