UNIVERSITY OF LJUBLJANA
SCHOOL OF ECONOMICS AND BUSINESS

MASTER'S THESIS

SVIT KUNAVER

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MANAGEMENT OF LONGEVITY RISK WITH DERIVATIVES AND REINSURANCE

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AUTHORSHIP STATEMENT

The undersigned Svit Kunaver, a student at the University of Ljubljana, School of Economics and Business, (hereafter: SEB LU), author of this written final work of studies with the title Management of Longevity Risk with Derivatives and Reinsurance, prepared under supervision of izr. prof. dr. Mihael Perman.

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APC	Age Period Cohort
CAPM	Capital Asset Pricing Model
CBD	Cairns-Blake-Dowd
CMI	Continuous Mortality Investigation
EIOPA	European Insurance and Occupational Pensions Authority
EU	European Union
GAAP	Generally Accepted Accounting Principles
GDP	Gross Domestic Product
HMD	Human Mortality Database
IFRS	International Financial Reporting Standards
IMF	International Monetary Fund
LC	Lee-Carter
LLMA	Life and Longevity Markets Association
NIG	Normal Inversed Gaussian
NPA	Net Payoff Amount
OECD	Organization for Economic Co-Operation and Development
ORSA	Own Risk and Solvency Assessment
PCFA	Pensions Combined Female Amounts
PCMA	Pensions Combined Male Amounts
RGA	Reinsurance Group of America
RH	Renshaw-Haberman
SAPS	Self-Administered Pension Scheme
SCR	Solvency Capital Requirement
SPC	Special Purpose Company
SPVs	Special Purpose Vehicles

INTRODUCTION

Life expectancy has been increasing rapidly in the last 100 years. This progress can be attributed to the reduction in infant mortality and medical advancements which make older people live longer. Life expectancy at age 60 in advanced European economies was 15 years in 1910 and rose to 24 years in 2010 according to the IMF (International Monetary Fund). United Nations project that this number will increase to 26 years by 2050. This trend positively affects our lives, knowing we live longer. However, governments, pension providers and insurance companies providing life annuities must evaluate the financial consequences of an ageing population. From their point of view, unexpected longevity improvements represent longevity risk. Mortality forecasting and predicting the trend of ageing into the future is, therefore, necessary for longevity risk mitigation. Forecasting has consistently underestimated the trends in the past, which had a significant impact on the financial stability of pension and annuity providers. An IMF report from 2012 projects that if everyone in the United States lives three years longer than expected, the living expenses of everyone during those unexpected additional years amount to between 25% and 50% of the 2010 U.S. GDP (gross domestic product). On the global scale, such underestimation means an increase of liabilities by trillions of U.S. dollars. This is especially concerning considering the fact that 20-year forecasts made in recent decades by countries such as the United States, Canada, Japan, and others have, on average, under-estimated longevity by three years (International Monetary Fund, 2012, pp. 136-139).

The accuracy of mortality and longevity tables is of crucial importance not only for insurance companies providing annuities but also for the governments which run their own pension schemes that might depend on them. There have been many attempts to project future mortality in the literature. The most successful method so far is extrapolation which means we assume past trends will continue into the future. The most known extrapolation model is the Lee-Carter model, which is regarded as the benchmark in mortality forecasting. Recently many new methods and extensions have been developed which eliminate some of the flaws of the original Lee-Carter model. One of the most significant weaknesses of this model is that it does not include cohort effects which means some cohorts are over and some under-estimated, which decreases the accuracy of predictions. I will use the Lee-Carter model as the basis for mortality forecasting and fit it to Slovenian data to obtain the forecasts. I will also use some extensions of the Lee-Carter model, which can include cohort effects for the purposes of comparison (Booth & Tickle, 2008, pp. 1-5).

Mortality forecasting is not the only tool used for the protection or prevention against longevity risk. Those models can yield expected future mortality rates, but the Solvency II regulation also requires that insurance companies are adequately prepared for unexpected changes with a sufficient Solvency Capital Requirement (SCR). Pension funds and insurance companies that provide annuities can also participate in longevity risk transfer markets which enable companies to partially or entirely eliminate unexpected risk through products such as q-forwards, longevity swaps, bonds and others which can decrease the amount of SCR. This market is still nascent, and its size is insignificant compared to its potential. I will present some of the products used in this market and how insurance companies can use them together with reinsurance to mitigate longevity risk.

Risk management has evolved a lot since the 2008 financial crisis. There has been an extensive focus on financial companies and how they should manage their risk exposure. Exposure to financial risks demonstrates itself quickly due to the volatile environment of equity and commodity prices. Therefore, financial derivatives such as options, futures, swaps and others are commonly used to mitigate risk. Longevity risk is different. It does not become evident as quickly, and has not received as much attention in the past. I believe it is essential to emphasize the importance of longevity risk and ways to mitigate it due to massive exposures not only for insurance companies but also for the governments. This research aims to use up-to-date literature and explain how insurance companies protect themselves against longevity risk using mortality forecasting models, risk transfer markets and reinsurance.

With regards to mortality forecasting, the purpose is to learn and emphasize its importance because predictions those models give are used as the base for constructing mortality and longevity tables on which many actuarial calculations rely; furthermore, those models can also be used to determine the SCR under Solvency II when an internal model is used, and play a crucial role in pricing longevity derivatives where mortality forecasts are necessary. For these reasons, I find it meaningful to explore those models. I will start with the Lee-Carter model, which is regarded as the benchmark, and compare it with other extended models to see which one fits the Slovenian data best and how much difference there is between their predictions.

The use of derivatives to mitigate financial risks is known and well-established. However, longevity risk transfer markets have not received that much attention, and the volume traded is still surprisingly low, given that the global size of annuity and pension-related longevity risk exposure is between 15 and 25 trillion U.S. dollars. Studying how to price and use longevity derivatives might therefore be a valuable contribution and raise awareness of the so far underutilized way of managing this risk (Bank for International Settlements, 2013).

My goals are:

- Explain the importance of mortality in insurance,
- Fit the Lee-Carter model to Slovenian data and compare it to other forecasting models popular in the literature,
- Review of the products on the longevity risk transfer market and the ways insurance companies use those to mitigate longevity risk,
- Research the role of reinsurance in longevity risk mitigation,
- Investigate optimal hedging strategies under Solvency II.

The master's thesis will be primarily theoretical in nature. Therefore, I will focus on the literature review by reading, analyzing and evaluating its relevance to my research area. The literature will consist of scientific papers, articles and some textbooks that I have used in some of my master's degree courses. I will then synthesize and write about the key findings from different sources to achieve the goals of the thesis. This part of the thesis will therefore be based on the descriptive method and the method of compilation, where I will present longevity risk exposure and its magnitude, as well as possible scenarios assuming governments and insurance companies do not mitigate this risk. I will use quantitative data analysis to fit the Lee-Carter model to Slovenian data and compare it to other forecasting models. First, I will use R as a programming language and collect the Slovenian mortality data from the Human mortality database (HMD). Then, I will use R packages StMoMo, Lifecontingencies and Demography to fit the model to Slovenian data.

The master's thesis will be divided into three main parts: mortality forecasting, longevity derivatives and the use of reinsurance for longevity risk mitigation. All of them will consist of multiple subparts. In the first part, I will explain the general role of mortality in insurance and give some theoretical underpinnings needed for understanding the importance of mortality forecasting. After this, I will proceed with mortality forecasting, its usage and compare different models. In the second part, I will focus on presenting longevity derivatives that are the most popular in the literature and most often used in practice. Lastly, I will focus on the role of reinsurance in managing longevity risk.

1 ROLE OF MORTALITY IN LIFE INSURANCE

Life insurance is constantly evolving, and many new products have emerged due to the increase of computational power and development of mathematics in the field of options and guarantees by Black, Scholes, and Merton. As a result, actuaries can now design more attractive products and give their customers a more comprehensive range of insurance contracts to fit their needs. Some of those products include investment components and return guarantees. However, the role of mortality is present in all life insurance products, and we can understand it better by focusing on simpler products such as whole-life insurance and whole life annuities.

When we talk about longevity risk, it is essential to emphasize that longevity improvement does not represent a risk in itself; it is the underestimation of it which leads insurance companies to set an inadequate price for the product, which in turn causes losses in the future. The insurance mechanism uses the contributions of many to finance losses of the few. When an insurance company calculates the premium for a life insurance product, the expected present value of benefits paid by the insurance company and the expected present value of premiums paid by the policyholder must be equal. This is called the principle of equivalence which assures that the insurance company can expect to receive as much value from policyholders as it will give back to them. Through this process, the technical premium is determined (Bowers, Gerber, Hickman, Jones & Nesbitt, 1997, pp. 167-170).

In whole life insurance, the policyholder is paying the premium, and in return, the beneficiary is paid a predetermined amount in case the insured dies. The longer the insured lives, the longer the premium payments will be for the insurance company. In the case of whole life annuities, people make a single payment or a series of payments, and in return, they receive annuity payments later in the future up until they die. To determine the technical premium of a whole life insurance product, the insurance company views premiums paid by the policyholder as receiving a life annuity since they are paid only if the insured is alive. Therefore, insurance companies can expect to receive more premiums due to decreased mortality rates. When the insurance company provides a whole life annuity, it has to pay annuities as long as the insured lives, where a decrease in mortality rates results in more payments by the insurance company. Regardless of which product we look at, the valuation of annuities is, therefore, a key ingredient in the pricing of all life insurance products, and it also carries the effect of mortality. For this reason, I will explain the role of mortality in life insurance through the context of life annuities (Dickson, Hardy & Waters, 2009, pp. 142-143).

1.1 Effects of mortality on pricing and reserving

Mortality determines the value of annuities used in pricing and reserving for all life insurance products. Depending on the product, changes in mortality can increase or decrease the longevity risk exposure of an insurance company. A decrease in mortality rates would benefit insurance companies focusing on life insurance products instead of life annuities. With annuities, a decrease in mortality rates represents a risk, whereas in life insurance this increases the probability of premiums being paid. When combining the two lines of business, selling life insurance products can be a natural hedge for the longevity risk exposure of the annuity business. On the contrary, if mortality rates increase due to an external factor, the roles would be reversed. Even though this has not been the case, it would mean less liability from the annuity business and less premium income from life insurance. Therefore, the annuity line of business would serve as a natural hedge (Dowd, 2003, p. 342).

1.1.1 The use of mortality tables

To use mortality in calculations, actuaries rely on mortality and annuity tables. The choice of tables used depends on the line of business. When the insurance company provides life annuities or when it relies on receiving them through premium payments in life insurance products, the adverse selection impacts the choice of tables chosen in calculations. This is the case because we expect that purchasers of annuities are likely to live longer than the average population. Therefore, for life annuities, we use tables with lower mortality rates (annuity tables) than those we use in pricing life insurance products (mortality tables). Longevity, therefore, represents risk regarding annuities and mortality when it comes to a product such as whole life insurance. Because my focal point is longevity risk, I will focus on annuity tables used in pricing liabilities of life annuities, not those used in general life insurance products where the primary risk is mortality. Although there are different types of annuities, all of them serve the purpose of converting asset accumulation into a regular flow of income. Longevity represents one of the main risks for annuity providers, and because of this, annuity tables are a necessary tool for actuaries when pricing those products. One of the problems is that many developing countries lack national mortality statistics and have to use the mortality data of other countries. This is the case in Latin America and Asia and has been the case in Slovenia in the past. Another issue is that changes in mortality make it impossible to rely on the exact data for an extended period. This is why actuaries estimate a so-called period table from past data and create a forward-looking table by extrapolating future trends in mortality (McCarthy & Mitchell, 2000, pp. 1-5).

Even though mortality improvements have been consistently underestimated in the past, many countries still do not have a regulatory framework to ensure that pension plans and annuity providers use appropriate tables for pricing and reserving that would account for future mortality development. The regulation dealing with annuity tables varies a lot across countries; where some require a minimum annuity table for valuing annuity liabilities, and others do not. Regulation can also require incorporating mortality improvements even if the minimum annuity table is not required. In the U.K., annuity providers are not required to use a minimum annuity table set by the regulation but need to include mortality improvements in their calculations. In the U.S., the opposite is true: annuity providers are required to use a minimum annuity table but do not have to include mortality improvements in their calculations. When this is the case, a minimum annuity table usually already accounts for a certain amount of future mortality improvement. There are countries where both are required, as is the case in France (OECD, 2014).

In the case of Slovenia, the Insurance Supervision Agency set the German annuity tables as a minimum standard in the past. However, in 2010 Slovenian annuity tables were created and became a standard for valuing annuity liabilities in Slovenia. The table also accounts for future mortality improvements through stochastic models using past data (Ahčan, Medved, Pittaco, Sambt & Sraka, 2012).

1.1.2 Quantifying the size of longevity risk

Using a particular annuity table significantly impacts the valuation of liabilities. Many countries have developed numerous tables in the past which have different mortality rates due to different assumptions and models used in their creation. To emphasize how vital the assumptions and models used in developing annuity tables are, we need to quantify the size of the risk it serves to manage.

Private sector longevity risk exposure arising from annuity and pension liabilities is enormous. As an example, in the U.S. ($14,46 trn), the U.K. ($2,685 trn), and the

Netherlands ($1,282 trn). Globally this exposure is between 15 in 25 trillion U.S. dollars (Pigott & Walker, 2016, p. 6). Michaelson and Mulholland (2014) estimated that the potential size of global longevity exposure is between $60 and $80 trillion, where around 32 trillion were linked to private pension systems, including pension funds, banks, investment companies, and insurance companies. An interesting estimation they made is that each additional unexpected year of life at the age of 65 amounts to an increase in liabilities by 4-5%. This is equivalent to a 0.8% increase in annual mortality improvement over ten years. The annual improvement of a rate of 0.8% is usually expected, so when they estimated the consequences in the tail event being 2.5 standard deviations away or an annual mortality improvement of 2% over a decade, the longevity-related liabilities could increase by 10-12.5%. Going back to the massive longevity risk exposure, this would mean that the global liability of pension systems could be more than $6 trillion higher in case mortality improves faster than expected (Blake, Cairns, Dowd & Kessler, 2019).

1.1.3 Annuity values and life expectancies under different annuity tables

As stated earlier, countries use different annuity tables, which evolve over time. As a result, they have a significant impact both on pricing and reserving for annuity liabilities. For example, the U.K. pension funds use tables called Self-Administered Pension Scheme (SAPS) tables for funding and accounting purposes, published in 2008. On the other hand, insurers tend to use older tables for reserving and calculating the premium, where they usually used tables PCMA (Pensioners Combined males amounts) or PCFA (Pensioners Combined female amounts) published in 2006, which can include mortality improvements published by the Continuous Mortality Investigation (CMI), owned by the U.K. Institute and Faculty of Actuaries.

When comparing the annuity factors and life expectancies of different tables across different countries, it is evident that assuming mortality improvements dramatically decreases the longevity risk and the chance of a shortfall in provisions. Annuity values were based on a 4.5% discount rate in all cases below.

Using the PCMA 2000 table with a 1.75% rate of improvement, the 2012 life expectancy of a male at the age of 65 is 24.1 years, and the annuity factor is 14.5. The SAPS table that uses data from 2004 to 2011, and has a 1.5% rate improvement gives us the life expectancy of the same-aged male to 23 years and the annuity factor of 14.2. Using the SAPS table instead of PCMA 2000 would result in underestimating the life expectancy of a male aged 65 for more than one year. Therefore, given the annuity factors, this means that the SAPS 2 table would underestimate future liabilities by around 3% compared to PCMA 2000, and we get similar results focusing on women aged 65. Even though the difference is subtle, it is still significant given the longevity risk exposure in the U.K. stated previously.

The differences are even more pronounced in other countries such as Switzerland, where using the ERM 2000 table for 65-year-old males results in a life expectancy of 26 years and an annuity factor of 15.2 and using the EVK 2000 table in the same example results in a life expectancy of 17.6 years and an annuity factor of 12. The difference between estimated future liabilities would be even higher in such a case. For this reason, the law prescribes an additional supplementary reserve in case EVK 2000 table is used. We can see similar results in other countries as well, which applies to both genders. The main reason for changes between different annuity tables is our assumption about the future longevity improvement rate.

Because of the trend of decreasing mortality rates, insurance companies need to select the optimal rate of longevity improvement that will provide the best estimates for the future. This is crucial to prevent setting aside additional reserves in the future after realizing mistakes about mortality assumptions made in the past. Additional reserves are the result of underpricing due to wrong mortality assumptions, which means the insurance company needs to compensate for the difference to fulfil its obligations. Doing so affects its profitability and financial stability. The competitive environment and achieving the needs of shareholders force insurance companies and other life annuity providers to seek the correct development of future mortality rates to prevent both under and over-estimating reserves. Before discussing the models for forecasting future longevity improvements, I will focus on essential mortality functions and notations (OECD, 2014).

1.2 Mortality functions and notations

To understand mortality forecasting methods, including mortality functions and notations, it is helpful to describe these before presenting the models.

1.2.1 The lifetime distribution $[F_x(t)]$

It represents the probability that T_x, or future years lived by a person aged *x*, will be less than *t*. In actuarial notation, this is denoted as ${}_tq_x$ and is called the mortality rate at age *x*. T_x + *x*, therefore, represents the age-at-death random variable.

$$F_x(t) = Pr[Tx \leq t] = {}_tq_x \tag{1}$$

1.2.2 The survival function $[S_x(t)]$

This represents the probability that T_x, or future years lived by a person aged *x*, will be larger than *t*. In actuarial notation, this is denoted as ${}_tp_x$.

$$S_x(t) = 1 - F_x(t) = Pr[T_x > t] = {}_tp_x \tag{2}$$

1.2.3 The force of mortality [μ_x]

This can be interpreted as the probability that a person aged *x* dies before reaching age $x+dx$

$$\mu_x = \lim_{dx \to \infty} \left(\frac{1}{dx}\right) \cdot Pr[1 - S_x(dx)] \tag{3}$$

(Dickson, Hardy & Waters, 2009, pp. 17-18).

1.2.4 Central mortality rate [m_x]

The Lee-Carter model uses the central morality rate which is the rate of probability that a life aged anywhere on the interval [*x*, $x+t$] dies before reaching age [$x+t$]. This is not the same as a probability of death ${}_tq_x$ which can be described as the initial mortality rate. We use central mortality because it considers that some lives will die during the year, which means m_x is always higher than ${}_tq_x$. The central mortality rate, therefore, represents the ratio between deaths during the period, denoted with d_x and the average population over that period, denoted with L_x below. We will use the logarithmic form in the models because of the exponential shape of mortality.

$$L_x = \int_0^1 l_{x+t} \cdot dt = \int_0^1 l_x \cdot {}_tp_x \cdot dt \tag{4}$$

Therefore:

$$m_x = \frac{d_x}{L_x} \tag{5}$$

If we assume the constant force of mortality to hold between [*x*, $x+t$], the central death rate m_x is generally very close to the force of mortality μ_x in the middle of the interval (Thatcher, Kannisto & Vaupel, 1999).

1.2.5 Probability of death [q_x] as a function of m_x

Under the assumption that the force of mortality is constant on the intervals [*x*, $x+t$] and that the number of deaths follows a Poisson distribution then, we can express ${}_tq_x$ as follows:

$$q_x = 1 - e^{-m_x} \sim 1 - e^{-\mu_x} \tag{6}$$

The assumption of the constant force of mortality and Poisson distribution will later be used in the Lee-Carter model (Spedicato, 2013a, p. 3).

1.2.6 Curtate life expectancy [e_x]

Life expectancy represents the additional number of years an individual of a given age *x* can expect to live at time *t*. This is usually denoted as e_x. When the life expectancy is in whole years, we call it the curtate life expectancy, which can be expressed as follows:

$$e_x = \sum_{k=1}^{\infty} {}_k p_x \tag{7}$$

I use curtate life expectancy for model comparisons below (Dickson, Hardy & Waters, 2009, p. 33).

2 MORTALITY FORECASTING

In the previous chapter, I have defined the importance of mortality through life annuities which are a crucial building block for pricing life insurance products. Moreover, because of the longevity improvements, the need for forecasting mortality is necessary for longevity risk mitigation. Insurance companies recognize this, and in Slovenia and other European Union (EU) member countries, they state their exposure to longevity and ways to mitigate it in their Own Risk and Solvency Assessment (ORSA). Most insurance companies in Slovenia are not exposed to this risk to a large degree, but a few are by providing annuities. They typically use stochastic mortality forecasting models to manage their longevity risk exposure. In this chapter, I will apply the Lee-Carter model to Slovenian data and compare the results to those by newer extended models. The purpose of this chapter is to explain how we can forecast mortality by using different models and how the estimates differ depending on the one we use. I will compare the results of the Lee-Carter model because it is used in some Slovenian insurance companies dealing with longevity risk. This chapter might also serve as an insight later when the pricing of longevity derivatives is presented (Insurance Supervision Agency, 2022).

2.1 Historical development of mortality models

We have been trying to model mortality for ages. Many models have been proposed since Gompertz published his law of mortality in 1825. However, in the last few decades, the methods have become more sophisticated and steered away from subjective judgement. The most recent introduction of stochastic methods improved the process further, where we can produce a forecast probability distribution rather than a deterministic point forecast. There are three main approaches to mortality forecasting: extrapolation, explanation and expectation.

The explanation method tries to make forecasts based on specific causes of death involving different disease processes and known risk factors. The expectation is based on experts' opinions, and actuaries have relied heavily on this method in the past but are now moving to more sophisticated extrapolative methods on which most new models rely. The main idea of the extrapolative method is that it assumes that historical trends will continue in the future. These models take advantage of regularities in age patterns and trends over time. This is usually a reasonable assumption, but exceptions do occur, such as temporary increases in young adult male mortality due to deaths from AIDS in the past or the recent pandemic (Booth & Tickle, 2008).

Because mortality forecasting has become more critical in the recent past, new models are constantly being developed, and extensions of established models are being performed, all in order to predict future mortality rates as accurately as possible. In the past, most methods were based on the method of explanation and expectation, which were highly subjective. However, after Lee and Carter presented their model in 1992, it became a leading statistical mortality model and has served as a benchmark of extrapolative mortality models (Lee, 2000, pp. 80-81).

Many models, following the publication of the original Lee-Carter (LC) model, also referred to as the M1 model, were trying to improve the goodness of fit by adding additional terms. After 2000 the age period cohort model became popular even though it has been previously used in demography, sociology and epidemiology for a long time. Renshaw and Haberman extended the LC model, to include the cohort effects. This model, referred to as the M2 model, will not be discussed here since I will focus on its extended version (M3). The age period cohort model (APC or M3 model) encompasses the vast majority of mortality models, which can be expressed in terms of generalized linear models or generalized non-linear models, as shown by Currie in 2016. The APC model includes cohort effects and is a special case of the M2 model.

One of the most prominent variants of the LC model is the Cairns-Blake-Dowd (CBD or M5) model, which uses an entirely different approach compared to the LC model. This model can also incorporate the cohort effect and a quadratic age term and is then called the M7 model in the literature. This improves the model fit on ages of a broader range and captures the cohort effects, which we will see in the residual analysis later. We expect that the models which include cohort effects (APC and M7 model) will better fit Slovenian data than those which do not (LC and CBD model) (Cairns et.al, 2008, p. 2).

2.2 The Lee-Carter model

The model is specified in the following way:

$$\log(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t} \tag{8}$$

We can interpret the parameters of the model as:

- $m_{x,t}$ is the central mortality rate at age *x* in year *t*,
- α_x is average (over time) log-mortality at age *x* and it captures the general shape of $\log(m_{x,t})$,
- κ_t is the time-variant variable, and it captures how the mortality has been decreasing over time,
- β_x shows how different ages respond to declining mortality over time,
- $\varepsilon_{x,t}$ are independent and identically distributed error terms expected to be distributed normally.

The original Lee-Carter model is a two-component model with a single random process κ_t which drives all the dynamics. The mortality forecasts are obtained using univariate ARIMA processes where κ_t follows a one-dimensional random walk with drift.

$$\kappa_t = \delta + \kappa_{t-1} + \xi_t \qquad \xi_t \sim N\left(0, {\delta_\kappa}^2\right) \tag{9}$$

Here δ represents the drift parameter and ξ_t the Gaussian white noise process with variance ${\delta_\kappa}^2$. To ensure the identifiability of the model, Lee and Carter proposed the following parameter constraints:

$$\sum_x \beta_x \;=\; 1 \quad and \quad \sum_t \kappa_t \;=\; 0 \tag{10}$$

(Villegas, Millossovich & Kaishev, 2017, p. 6).

In practice, insurance companies use the Poisson log-bilinear variant of the Lee-Carter model, which is popular since it is computationally more stable, and it does not assume the homoscedasticity of the random errors. Therefore, I will also use this variant later. Assuming the Poisson distribution for the number of deaths in a given interval, the parameters are obtained by an iterative procedure (Insurance Supervision Agency, 2022).

The Lee-Carter model is prevalent due to its simplicity and the fact that it provides an excellent fit for historical data in many countries. The term α_x allows the model to be used for all ages, even younger ones, where the shape of a mortality table can be complex. The term $\beta_x\kappa_t$ captures the trend in the evolution of mortality across different ages. There have been models with more than one term of $\beta_x\kappa_t$. Renshaw and Haberman proposed one such model in 2003. However, their proposed model is not widely used because it adds more complexity than the fit with historical data. Another benefit of the Lee-Carter method is the linear trend in κ_t which is common in most datasets where random walk with drift time series structure is used to yield estimates of future central mortality rates.

Even though the model is easy to use and has the benefits stated above, it still has some severe limitations. One limitation is that the model only has a one-period term κ_t. This implies that projections through all future years will have the exact change in mortality rates for a given age. This assumption that changes in mortality rates are perfectly correlated in all future years is unrealistic and presents a problem when determining the riskiness of liabilities. Another significant issue is that β_x can give wrong projections. As said earlier, β_x captures how different ages respond to mortality improvements. When we look at the extended range of historical data for most countries, β_x will be high for ages 0 to 50 and much lower for ages 60 to 90. This means that most mortality improvements in the past occurred due to improvements at younger ages. When using more recent data, β_x changes to reflect the recent patterns of mortality improvements, especially at older ages. This means that fitting the LC model to an extended range of historical data will continue to project very high rates of improvement at younger ages and lower rates at older ages (OECD, 2014).

Another issue specific to the Lee-Carter model is that it does not include cohort-specific effects. Those are the effects which depend on the individual year of birth. This model assumes that the sensitivity of log-mortality rates at each age (β_x) remains constant, but it has been observed that age-time interaction is highly likely and that incorporation of cohort effects (age-time interactions) improves the fit of the model (Li, 2019).

2.3 The APC model

The APC model is an extension, or a special case, of the original Renshaw-Haberman (RH) model, which is basically a LC model with added cohort effects developed in 2003. The RH model is specified in the following way:

$$\log(m_{x,t}) = \alpha_x + \beta_x^{(1)}\kappa_t + \beta_x^{(0)}\gamma_{t-x} \tag{11}$$

Where we assume that κ_t follows a one-dimensional random walk with drift, mortality projections of this model are obtained by using time series forecasts of κ_t and γ_{t-x} which are generated by using univariate ARIMA processes assuming that there is independence between the period and cohort effects. The APC model differs from the original RH model because it sets both $\beta_x^{(1)}$ and $\beta_x^{(0)}$ to the value of 1. The APC model is therefore specified as follows:

$$\log(m_{x,t}) = \alpha_x + \kappa_t + \gamma_{t-x} \tag{12}$$

The identifiability of the model is ensured by imposing the following constraints:

$$\sum_t \kappa_t = 0\,, \quad \sum_{c=t_1-x_\kappa}^{t_n-x_1} \gamma_c = 0, \quad \sum_{c=t_1-x_\kappa}^{t_n-x_1} c\gamma_c = 0 \tag{13}$$

The last two constraints ensure that cohort effects fluctuate around zero and that there is no noticeable linear trend present (Villegas, Millossovich & Kaishev, 2017, pp. 7-8).

These identifiability constraints allow us to interpret the demographic significance of the parameters. The first two constraints mean that α_x can be interpreted as the average level of mortality at age x in a given period where κ_t and γ_{t-x} would represent deviations from that average. The first would capture the effect of mortality improvement, and the second would capture the cohort effect. The last constraint ensures that no deterministic trend is randomly assigned to the age and period effects (Hunt & Blake, 2020).

The APC model uses cohort effects which means that, in theory, it should provide a better basis for forecasting than the model, which does not. In practice, their use is somewhat limited because they require a lot of data which is sometimes hard to obtain. The main disadvantage of cohort models is that if the entire age range is of interest, data covering one hundred years provides estimates for one cohort only, so we need a much longer series of annual data for forecasting. We usually focus on older ages when forecasting mortality which to some extent eliminates this issue but not entirely (Booth & Tickle, 2008, pp. 20-21).

2.4 The CBD model

Cairns, Blake and Dowd introduced the CBD model in 2006 to model survivor bonds proposed by the European Investment Bank. The purpose was to resolve issues regarding the LC model by taking a completely different approach. The CBD model assumes that probabilities of death can be modelled as follows:

$$\operatorname{logit}\left(q_{x,t}\right) = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} \tag{14}$$

Where $\bar{x}$ represents the average age in the data. It assumes that the logit of probabilities of death is a linear function of age. This can be a reasonable assumption when considering higher ages of 50 and above, but it does not hold at younger ages. This is because we have high probabilities of death observed for ages between 15 and 25 due to accidents. Mortality in ages below 2 is also high due to infant mortality.

This means the CBD model can only be used at older ages. The $\kappa_t^{(1)}$ in the model represents the level of mortality in a given year and across all ages and $\kappa_t^{(2)}$ determines the increase in mortality between one age and the next for the year.

The CBD model is widely used to mitigate risks related to liabilities linked to probabilities of death at high ages, such as annuities. The model has similar advantages as the LC model, such as ease of fit and good interpretability; however, the disadvantages are that below the age of 50, the linearity of $\operatorname{logit}\left(q_{x,t}\right)$ is no longer a reasonable assumption. Another issue it has in common with the LC model is that it does not include cohort effects which might decrease the fit of the model (OECD, 2014, pp. 69-70).

2.5 The M7 model

The M7 model is one of the extensions of the CBD model, which includes cohort and the quadratic age effect.

The model is specified in the following way:

$$\text{logit}\left(q_{x,t}\right) = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + \kappa_t^{(3)}((x - \bar{x})^2 - \sigma_x^2) + \gamma_{t-x} \tag{15}$$

Where σ_x^2 is the average value of $(x - \bar{x})^2$. To identify the model, Cairns et. al (2008) imposed the following restrictions:

$$\sum_{c=t_1-x_\kappa}^{t_n-x_1} \gamma_c = 0, \quad \sum_{c=t_1-x_\kappa}^{t_n-x_1} c\gamma_c = 0, \quad \sum_{c=t_1-x_\kappa}^{t_n-x_1} c^2\gamma_c = 0 \tag{16}$$

This ensures, just as in the APC model previously, that cohort effects fluctuate around zero and that there is no noticeable linear trend or quadratic trend present. The three coefficients $\kappa_t^{(1)}$, $\kappa_t^{(2)}$, $\kappa_t^{(3)}$ are modelled as a 3-dimensional random walk with drift. The γ_{t-x} is a cohort effect and is modelled as an AR(1) process. (Dowd et.al, 2010). "The third index, $\kappa_t^{(3)}$, measures the curvature of the logit-transformed mortality curve." (Tan, Li, Li & Balasooriya, 2014, p. 9).

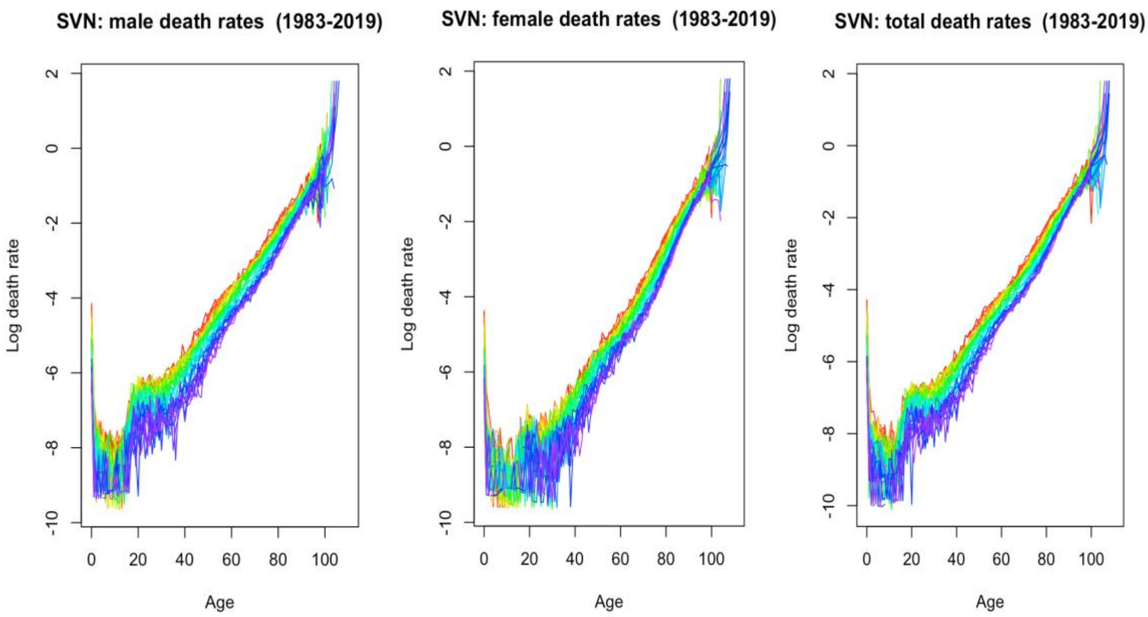
3 MODEL COMPARISONS

In order to compare the models, I obtained the data from the Human mortality database (HMD), which is publicly available online. In this database, I obtained mortality rates of the Slovenian population from 1983 and 2019 for both males, females and total rates, which include both genders. The main objective of comparison is to see which model fits the data best and how their estimates for a life expectancy of a cohort born in 1954 differ. Therefore, I chose the cohort born in 1954 since those people reached the age of 65 in 2019, and this is usually the age for which insurance companies are interested in forecasting life expectancy. It is important to understand that the tables obtained by using those models are based on population mortality and cannot be used for valuing annuities. For that certain loading factors would have to be applied to the predictions since mortality rates in annuity tables are lower than the general population mortality rates due to abovementioned reasons.

3.1 Slovenian mortality from 1983 to 2019

Slovenian mortality rates have been decreasing in the period from 1983 and 2019. However, the level of mortality improvement has not been the same at all ages. We can see that in figure 1 where the vertical distance between lines is different across ages.

Figure 1: Slovenian death rates



Source: own work

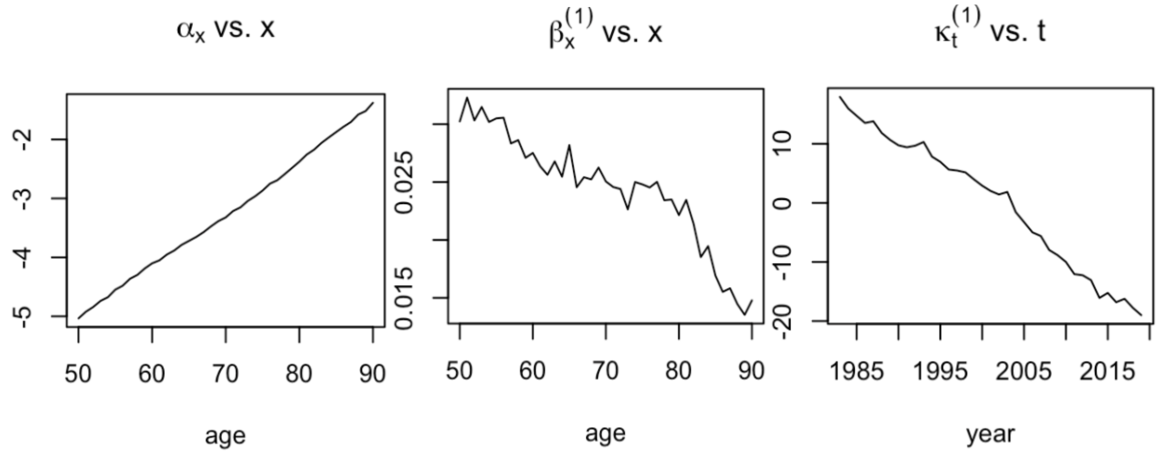
The figure above shows log-central death rates, which increase as people get older. The colour of the lines represents the year in which the data was taken (the red lines are closer to data from 1983, and the purple are closer to 2019). Unfortunately, the data after the age of 90 is unreliable due to the tiny sample of people that reach this age.

Insurance companies are usually only interested in the development of mortality at older ages, so I will fit the model to the data from 50 to 90. However, I opt not to include the ages beyond 90 due to the abovementioned problem. This means that I assume everyone dies at the age of 90 or before. From here onwards I will only present results obtained by using data for males, the ones obtained by using the data for females are included in the Appendices.

3.2 Parameters of the LC model

Mortality forecasting models try to capture the shape and movement of mortality rate over time and model that process. For example, in the data used in figure 2, we can see the parameters of the Lee-Carter model described earlier. The Lee-Carter model I used was the Poisson log-bilinear variant, where parameters are obtained by an iterative process.

Figure 2: Parameters of the LC model (males)



Source: Villegas, Milossovich & Kaishev (2017, p. 17).

We can see that κ_t is decreasing, showing how mortality has declined over time, however, the improvements have not been equal at all ages. The parameter β_x is stable from age 70 to 80 and then falls sharply. This means that the longevity of those between 70 and 80 improved faster than those aged above 85, for example. This makes sense since mortality can only be improved up to a certain point, after which the improvement is less significant due to human limitations. The parameter α_x increases almost linearly between ages 50 and 90 because it captures the general shape of $\log(m_{x,t})$, and we know older people have a higher chance of dying (both $m_{x,t}$ and $\log(m_{x,t})$ increase with age).

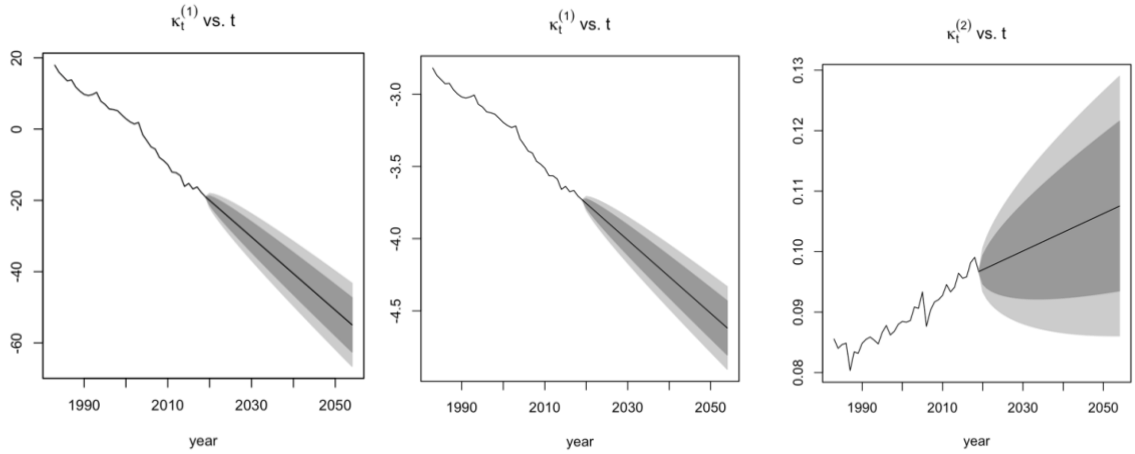
If we want to predict how the central rate of mortality will develop in the future, we need to simulate the time-dependent variable or variables depending on the model. All other variables are assumed to follow the same pattern as they did in the past, which means we assume different ages will respond to longevity improvements in the same manner as they did in the past.

3.3 Forecasting time dependent variables

Obtaining the future mortality rates requires forecasting variables that are time dependent. The assumptions made in forecasting are that κ_t follows a random walk with drift and that cohort effects follow an ARIMA processes. Cohort effects are only present in the APC and M7 model. Therefore, we use the ARIMA (1,1,0) with zero mean for the APC model and ARIMA (0,0,0) for the M7 model which is done by default in StMoMo package. The projections are for 35 years into the future. In figure 3, we can see the obtained forecasts of time dependent parameters of LC, CBD and M7 model for males.

Figure 3: Forecasting time-depending variables (males)

LC CBD

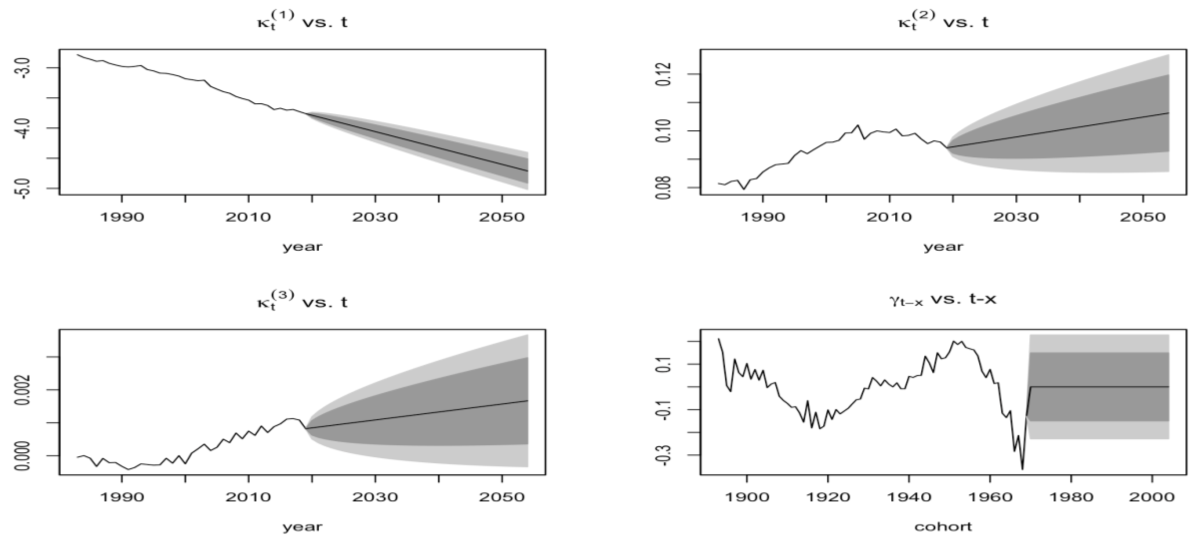


Source: Villegas, Milossovich & Kaishev (2017, p. 23).

The LC and CBD models do not include the cohort effect; therefore, we only perform the forecast using the parameter κ_t. The parameter κ_t has a linear trend in figure 2, which is expected to continue in the future, meaning mortality rates across all ages will decrease. In the CBD model, the $\kappa_t^{(1)}$ has the same interpretation as in the LC model; therefore, the forecasts look very similar, and we expect that mortality rates will decrease in the future. The $\kappa_t^{(2)}$ determines the rate of ageing and can, therefore, be interpreted as the rate of improvement in longevity.

Figure 4: Forecasting time-dependent variables and cohort effects (males)

M7



Source: Villegas, Milossovich & Kaishev (2017, p. 23).

Figure 4 shows the forecasts for the M7 model, which also includes cohort effects. Because the StMoMo package uses the ARIMA (0,0,0) without a constant, we got a time series with the mean 0. Forecasting of time-dependent variables for APC model and using data for females is in the Appendices 5 and 6.

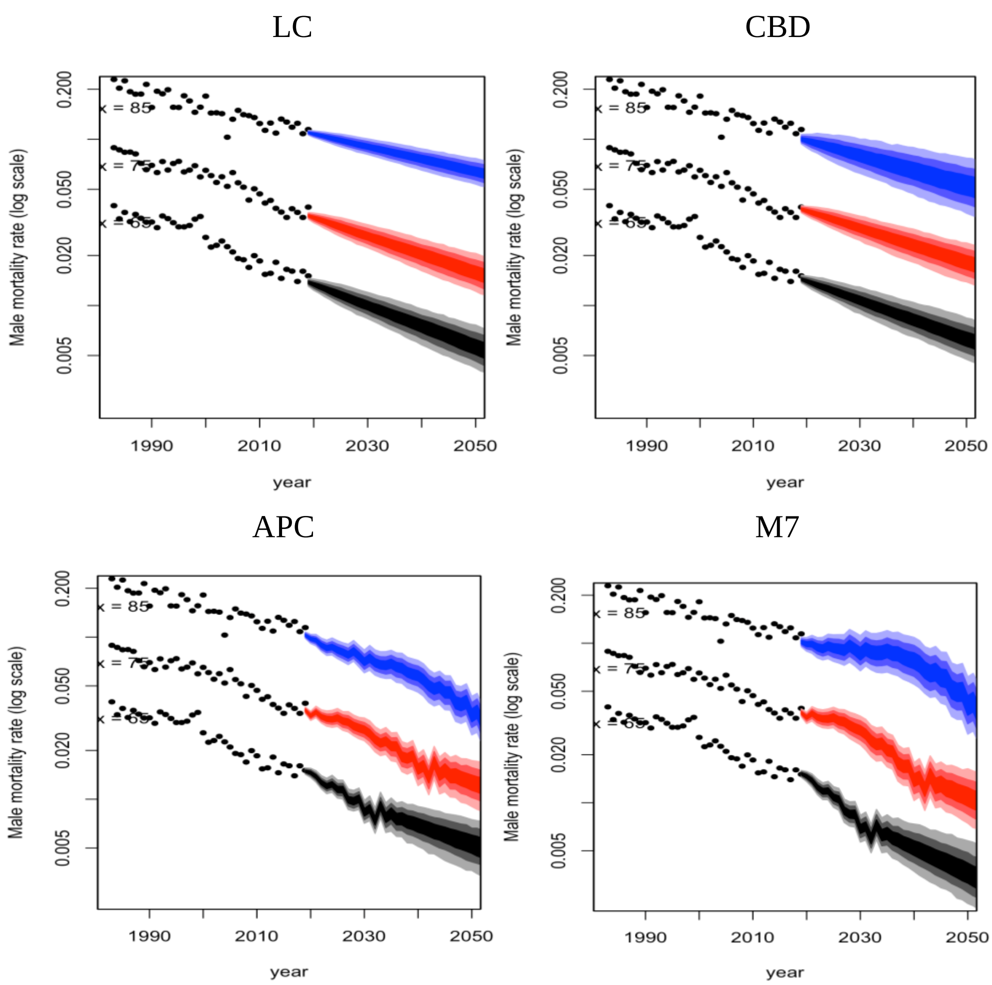
3.4 Forecasting future mortality rates and obtaining life expectancies

We can obtain the forecasts for different models' log-central death rates with the estimated parameters and construct cohort mortality tables used for actuarial calculations.

3.4.1 Forecasts of log-central mortality rates

Below are the 35-year forecasts of different models for males aged 65, 75 and 85. The trend of a decrease in mortality rates is common in all the models, but there are differences in confidence intervals.

Figure 5: Forecasts of log-central death rates (males)



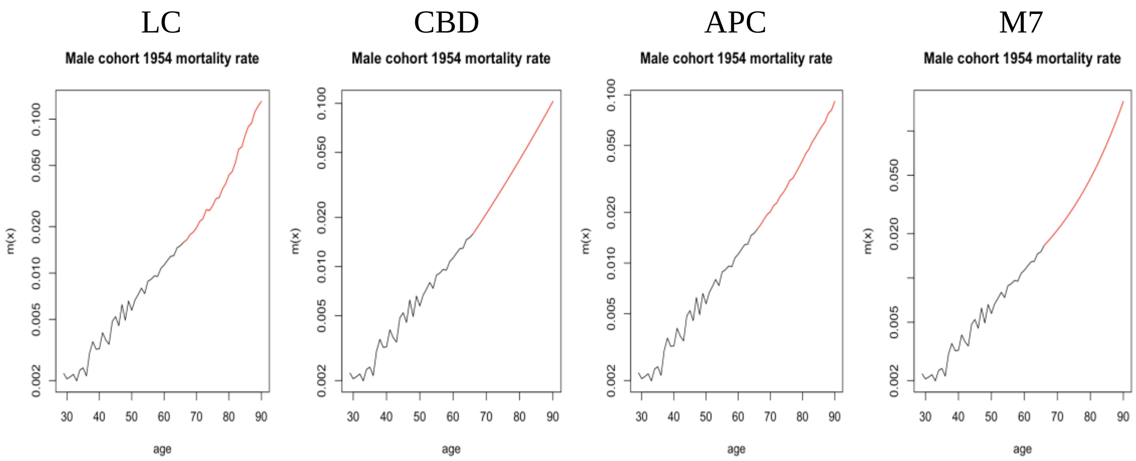
Source: Villegas, Milossovich & Kaishev (2017, p. 29).

Shades of the colours present confidence intervals at 50%, 80% and 95% levels. The dots represent historical mortality rates. In figure 5, we can see that the LC model has the narrowest projection intervals. This is also one of the drawbacks of the LC, which poses challenges when using the model to assess longevity risk at extreme percentiles. With the APC and M7 model, which include cohort effects, we can see a change in the pattern where mortality rates slightly increase in some years, which might indicate that cohort effects are present in the Slovenian male data (Villegas, Milossovich & Kaishev, 2017).

3.4.2 Forecasts of central mortality rates of a male cohort born in 1954

To calculate life expectancies and annuity values, we need to project the mortality rates of a specific cohort. For example, below are the projected central mortality rates of a male cohort born in 1954. StMoMo package yields results of m_x for APC and M7 model as well. The relation between m_x and q_x which used in these two models is shown in equation 6.

Figure 6: Forecasts of central mortality rates for the 1954 male cohort



Source: Villegas, Milossovich & Kaishev (2017, p. 30).

Historical mortality rates are in black, starting from the age of 29 since the data obtained from HMD started in 1983 when a chosen cohort was 29 years of age and ended in 2019 when this cohort was 65 years old. The red colour starts in 2020 and represents the forecast central death rates for 35 years when the cohort reaches the age of 90.

3.4.3 The life expectancy of a male cohort born in 1954 in 2019

From figure 6, we can calculate the life expectancy of a 65-year-old male in 2019. The results of different models for males and females are shown below. The mortality tables produced by different models can be found in the Appendix 13.

Table 1: Life expectancies of a 1954 cohort

Gender	**LC**	**CBD**	**APC**	**M7**
Male 65	18.146	18.027	18.408	17.715
Female 65	21.193	20.739	20.726	20.115

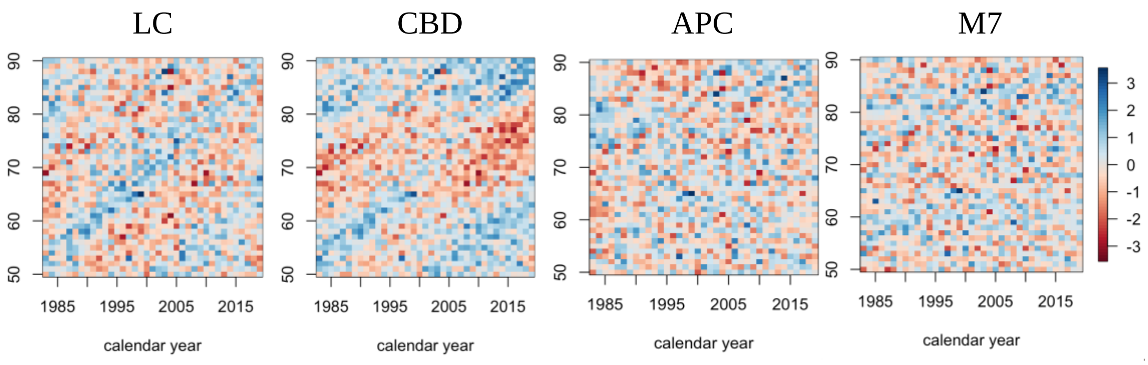
Source: Spedicato (2013b, p. 18).

The results of the models are expected to be lower than the actual values because I simplified the data and assumed everyone dies at the age of 90 or sooner. Furthermore, insurance companies use annuity tables when the risk is longevity and apply selection factors which I did not. Because I used the same data for all the models, we can still compare the results and recognize that the LC model gives the highest results. One of the reasons for this might be that the APC and the M7 include cohort effects which might increase mortality rates at certain ages compared to other models, which do not include cohort effects. This is visible in figure 5. The differences between the models are not significant but might increase if we used them on younger cohorts. The choice of the model is usually determined ad hoc, where we compare the models based on their goodness of fit.

3.5 Residual analysis

The choice of the model depends on how well it fits the data, which can vary depending on the underlying population. The choice also depends on the presence of cohort effects. In the U.K. population, cohort effects are present, and the choice of a model which includes them might fit the data better than the LC or CBD model. In Slovenia, the cohort effects are not as significant as seen in figure 7 below, where the deviance residuals for males between 50 and 90 are shown.

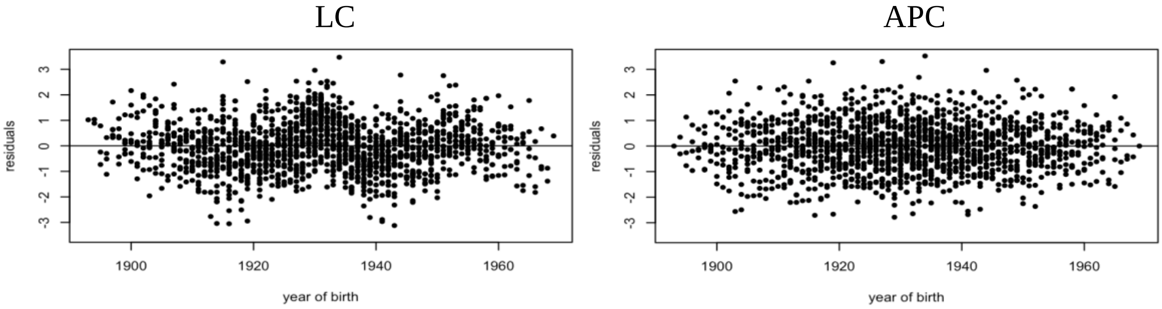
Figure 7: Deviance residuals in a colourmap (males)



Source: Villegas, Milossovich & Kaishev (2017, p. 20).

Figure 7 illustrates the difference between the two models, which include cohort effects (APC and M7) and those that do not (LC and CBD). The cohorts are on the diagonals, and we can see that some cohorts in the LC and CBD model are over and some under-estimated. The errors are spread much more randomly in the APC and M7 model, which is expected. After all, additional variables that capture cohort effects can explain the variability that the other two models cannot.

Figure 8: Deviance residuals in a scatter-plot (males)



Source: Villegas, Milossovich & Kaishev (2017, p. 21).

The cohort effects can also be seen when plotting the residuals with a scatter plot as seen in figure 8 above. We can see a pattern in the LC model, whereas the APC model has more random residuals. Because the cohort effects seem to be subtle, we can also compare the models based on the information criteria AIC and BIC. The male data was used in the table below.

Table 2: Information criteria (males)

Inf. criteria	**LC**	**CBD**	**APC**	**M7**
AIC	12702.67	13234.25	12432.73	12403.19
BIC	13325.63	13628.26	13242.05	13388.22

Source: Villegas, Milossovich & Kaishev (2017, p. 22).

The AIC and BIC criteria penalize more complex models with more parameters. We can see that the lowest AIC and BIC belong to the M7 and APC model. All of the results are very close, which means that adding to the complexity of the model by introducing the cohort effect may be unnecessary, and the LC model may be the right choice. This is because the cohort effects present in the Slovenian male data from 1983 to 2019 obtained by HMD might not be significant enough to justify using a more complex model that captures these effects. In addition, similar results are obtained by using data for females (see Appendix 10). This also explains why many insurance companies in Slovenia prefer the LC model.

3.6 Drawbacks of mortality forecasting models

The choice of the model depends heavily on the population we are working with. Even though stochastic mortality models give us an insight into how future mortality rates might improve, they cannot provide complete answers. Therefore, the results obtained from any of the models should not be accepted blindly and must be placed in context with the recent evolution of mortality. For example, if the mortality improvements have accelerated rapidly in the recent decade due to medical advances or other external factors, then forecasts based on models likely underestimate future life expectancies because they reflect a more general average improvement over the entire historical period. Currie et al. proposed one answer to such a problem in 2004 in the P-spline model, which puts more weight on the recent improvements that can help mitigate this issue.

Therefore, the data used for the calculations is vital in forecasting mortality. The OECD (Organization for Economic Co-operation and Development) estimated that the life expectancy of 65-year-olds has been increasing steadily over the past few decades and accelerated since 1990, which means that models such as the LC and CBD, which they compared, would lead to a shortfall in provisions for longevity. Another issue is a significant difference in mortality for white-collar and higher-income pensioners. Therefore, the socioeconomic profile of the population for which standard tables are used should be adjusted for the level of mortality. Managing longevity risk with forecasting models only cannot provide us with all the answers but can be the first step in managing longevity risk. Those models provide us with the answers about expected mortality improvements. However, unexpected improvements still represent a risk which needs to be addressed (OECD, 2014).

3.7 Application of mortality forecasting models

Mortality forecasting methods do not give us perfect predictions about future mortality; hence longevity risk cannot be managed through this tool alone. Unexpected increases in mortality are a risk that those models do not hedge against, and insurance companies and annuity providers need to evaluate the financial impact of those unexpected improvements. A sound risk management strategy for longevity risk is based on correct mortality assumptions with future improvements obtained by the stochastic model that fits the data best, and evaluating the financial impact of significantly under-estimating future mortality improvements, which is done by following Solvency II regulation in the EU.

Under Solvency II, insurance companies need to calculate the Solvency Capital Requirement (SCR), which ensures that companies can meet all their obligations in 99.5% of the cases in a period of one year. In addition, the SCR requirement considers many risks the insurance company is exposed to, which includes longevity risk. This means insurance companies need to set aside additional capital, which ensures the company's solvency even if the future mortality improvements using stochastic models have been underestimated.

Solvency II offers two methods to calculate SCR. Companies can use the standard formula as it is written in the regulation and use a one-off shock on all mortality rates in the amount of 20%. This means investigating the financial impact of underestimating mortality improvements for all ages by 20% in a period of one year. Companies can also use internal models, which are the stochastic models discussed earlier which are able to estimate future mortality rates and the uncertainty in their forecasts. We have seen that those models produce confidence intervals that can quantify the uncertainty with respect to death rates. In figure 5, we can see that the LC model has the narrowest confidence intervals, which would produce the lowest SCR result out of the models discussed. The size of the SCR differs significantly between the standard formula and using stochastic models, where stochastic models yield a lower SCR compared to the standard formula, which is also a part of the reason many insurance companies use those models for their SCR calculations (Wu, 2015, pp. 6-7).

Mortality forecasting models also play a crucial role in determining the risk premium in longevity derivatives. This is because those products are used as a hedge against unexpected mortality changes in the future, which is usually estimated by first calculating expected changes via the stochastic models and then adding a risk premium based on the volatility of those predictions. Therefore, the forecasting model greatly influences longevity derivatives' prices (Blake, Cairns, Dowd & Kessler, 2019, pp. 42-44).

4 LONGEVITY DERIVATIVES

New risk management solutions are needed because of the increasing capital required due to longevity risk. One solution for companies who try to mitigate unexpected longevity improvements might be to use capital markets to hedge those risks. Insurance companies could fully or partially eliminate their longevity exposure with financial instruments depending on their risk preference.

4.1 Development of longevity risk transfer market

The longevity risk transfer market started in 2006 in the U.K. To follow up on this development, the British Actuarial Journal published an article, "Living with mortality", focusing on how insurance companies can use mortality-linked securities and over-the-counter contracts to manage their longevity risk exposures. This paper included a detailed analysis of two such securities: A mortality catastrophe bond issued by Swiss Re in December 2003 and a longevity bond issued by BNP Paribas in November 2004. They further investigated the potential use of hypothetical mortality-linked securities such as bonds, swaps, futures and options. The paper mainly focused on the issues concerned with the construction of mortality indices and possible barriers to market development; however, further research and discussions followed as this new market emerged. The evidence for the existence of global market in longevity risk transfers came when Goldman Sachs announced that the best way of dealing with pension liabilities is to remove them altogether, for which they set up a buyout company Rothesay Life. As with many other economic activities, the progression of this market did not follow a smooth path and instruments such as the one proposed by BNP Paribas did not attract sufficient investor interest, and were withdrawn in late 2005. A great deal was learned from this failed issue about the requirements needed to launch a financial product intended to mitigate longevity risk (Blake & Cairns, 2020, p. 220; Blake, Cairns, Dowd & Kessler, 2019, pp. 1-2).

The world's first longevity market derivative transaction was a *q*-forward transaction executed in January 2008 between J.P. Morgan and the U.K. pension fund buyout company

Lucida. The first longevity swap was executed in July 2008, where Canada Life hedged £500 million of its U.K.-based annuity liabilities. This was a 40-year swap customized to the insurer's longevity exposure of 125.000 annuitants. All the longevity risk was transferred to investors (Blake & Cairns, 2020, p. 220). Almost all activity on this market has occurred in the U.K., and only few non-U.K. transactions have occurred in the past, some of which are a $26 billion pension buyout deal between General Motors and Prudential Insurance, a longevity swap between Aegon and Deutsche Bank worth $12 billion and a pension buy-out between Verizon Communications and Prudential Insurance in the amount of $7 billion. Even though these volumes are impressive, they still represent only a tiny amount in comparison to the multi-trillion dollar size of longevity risk (Bank for International Settlements, 2013, p. 2).

In Europe, the data shows that longevity risk transfer markets are concentrated in just a few countries. This market mainly exists in countries with privately defined benefit pension schemes. Countries with a predominant state pension scheme have less activity in this market since governments do not tend to transfer longevity risk in this manner.

European Insurance and Occupational Pensions Authority (EIOPA) has analyzed Europe's longevity risk transfer market and found that only 5 out of 26 countries participating in a questionnaire have reported a sale of longevity derivatives. Those countries are; France, Ireland, Liechtenstein, the Netherlands and the U.K. The Netherlands and the U.K. had the most significant activity, with $25.4 billion and $52.7 billion, respectively, between 2011 and 2014, which can be explained by the fact that these two countries have the largest share of the European pension market. In addition, participation in this market is generally limited. For example, in the Netherlands, only three insurance companies have been active, and three transactions have been made, all of which were longevity swaps.

When countries were asked about their impressions of the market prospects for the future, three of the five countries expected that the longevity risk transfer market would grow further in the coming years. The majority of the countries which are not active in this market did not show any interest in it, and did not expect a development in the future. Some countries even doubt that this market will exist in their environment since they believe that insurance companies and annuity providers know the health and mortality of their customers better than players participating in the capital market. Another reason for their doubts is that the market is illiquid, and only a few prominent participants are present. It is important to mention that this questionnaire was conducted almost ten years ago and that the current situation might be different. However, the activity on the market is still not as high as it was expected (Dujim, 2015).

There are several financial arrangements which insurance companies, annuity providers and pension funds can use to hedge longevity risk. Different types of structures for this arrangement are possible. We have insurance-based solutions where a buyout or buy-in structure is the most common. This type removes the longevity and investment risks and transfers them to the counterparty which is usually a reinsurance company. This type of contract maximizes the risk transfer but requires significant upfront premiums. An alternative structure is the use capital market solutions through longevity derivatives which pass only the longevity risk to the third party while retaining the investment risk. Longevity derivatives are a more economical solution to hedging longevity risk as they typically do not require significant upfront premiums and involve more market participants (investment banks and other non-insurance related companies or investors). In this chapter I focus on longevity derivatives where insurance-based solutions and the use of reinsurance is discussed later (OECD, 2014, pp. 176-177).

Before discussing how insurance companies can use those products to mitigate longevity risk it is useful to learn how they are priced. Many different research papers discuss the best pricing strategy for longevity derivatives, and there is no consensus on the best method. Some pricing methods are based on the risk-neutral principle where the underlying martingale process is the development of future mortality rates. This method tries to determine a risk premium, given that all investors have a different level of risk aversion. This method proposes that it is impossible to price derivatives under the original physical measure. A solution for this problem that is used in finance is a risk-neutral pricing methodology which creates fair prices for derivative products. To obtain this price, we use a martingale pricing technique which ensures that the expected rate of return under martingale pricing is the risk-free rate one could get when buying a risk-free bond. The mortality rate under the martingale measure means that the volatility of the mortality rate is considered, and the expected mortality rate change is zero. Another popular method to convert expectations under a physical measure into its risk-neutral equivalent is by using distortion approach such as the Wang's transformation discussed in the next section (Chuang, 2013).

4.2 Wang's transformation

The Wang's transformation is a method which incorporates some of the major pricing theories such as the CAPM (Capital Asset Pricing Model), actuarial standard deviation approach and option pricing theory. This method uses a distortion function which changes the survival function to create suitable risk-adjusted expected values, which we can discount with a risk-free rate. This method can transform the best estimates obtained with mortality forecasting and get their risk-neutral counterparts. This approach determines the z scores as if they were normally distributed and then shifts them uniformly by the amount λ which stands for the market price of risk. The shifted z scores are then transformed back by using the standard normal distribution (Wanyama, 2017).

Wang's transformation is based on the idea that the annuity market considers the uncertainty of the annuity table obtained with forecasting models discussed earlier. The higher the λ, the lower the probability of death for all ages. This implies that under the distorted mortalities, people live longer.

We can obtain risk-adjusted mortalities by applying the Wang's transformation to a given mortality or annuity table, which can further be used to price longevity derivatives. I will apply this method to pricing longevity bonds, where I will apply the Wang's transformation to the Slovenian annuity tables (SIA 65, 2010). Firstly, I will describe the Wang transformation.

Let $\Phi(x)$ be a standard normal cumulative distribution function with the probability density function ϕ for all *x*:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \tag{17}$$

Wang defines the distortion operator as follows:

$$g_\lambda(u) = \Phi[\Phi^{-1}(u) - \lambda] \tag{18}$$

For $0<u<1$. The distorted cumulative density function $F^*(t)$ is determined by λ.

$$F^*(t) = g_\lambda(F)(t) = g_\lambda\big(F(t)\big) \tag{19}$$

Let's now consider an insurer's liability *X* over a time horizon [*0*, *T*]. The fair price of liability *X* is the discounted expected value under the distorted distribution obtained by Wang's transformation. For simplicity, omitting the discounting yields the following formula for the price of liability *X*.

$$H(X, \lambda) = \mathbb{E}^*(X) = \int x dF^*(x) \tag{20}$$

Where:

$$F^*(x) = g_\lambda(F)(x) = \Phi\big[\Phi^{-1}\big(F(x)\big) - \lambda\big] \tag{21}$$

(Lin & Cox, 2005).

In general insurance pricing, the distortion operator g should meet the following criteria:

- $0 < g_\lambda(u) < 1$, $g_\lambda(0) = 0$, and $g_\lambda(1) = 1$,
- $g_\lambda(u)$ is increasing (where it exists, $g'_\lambda(u) \geq 0$),
- $g_\lambda(u)$ is concave $g''_\lambda(u) \leq 0$ (where it exists),
- $g'_\lambda(0) = \infty$.

Under this new probability measure, we can define a risk-adjusted fair value of X and discount it back to time zero with the risk-free rate. In terms of annuity, the formula for the price in a discrete setting can be written as:

$$H(X,\lambda) = \mathbb{E}^*(X) = s\sum_{k=0}^{n-1} d^k \, {}_kp^*_{l0} \tag{22}$$

Where ${}_kp^*_{l0}$ is the risk-adjusted survival probability obtained from Wang's transformation and s is a yearly annuity payment. Combining the formulas (20) and (21), we can get the following:

$$\begin{aligned} {}_kp^*_{l0} &= g_\lambda({}_kp_{l0}) \\ &= g_\lambda(u) = \Phi[\Phi^{-1}({}_kp_{l0}) - \lambda] \\ &= \Phi[\Phi^{-1}(1 - {}_kq_{l0}) - \lambda] \end{aligned} \tag{23}$$

Which implies:

$$H(X,\lambda) = \mathbb{E}^*(X) = s\sum_{k=0}^{n-1} d^k \, \Phi[\Phi^{-1}(1 - {}_kq_{l0}) - \lambda] \tag{24}$$

(Torske, 2015).

This equation will later be used to obtain the market price of risk. Before discussing the pricing of longevity bonds, we have to determine the market price of risk by applying Wang's transformation. In this example, I will closely follow the work by Torske (2015), where the inputs I will use will be the same, but I will use the Slovenian annuity tables in my calculation for the market price of risk and compare it with the results obtained using 1996 IAM U.S. annuity 2000 table used by Torske (2015).

Let's assume the insurance company issues a single premium immediate annuity for 100.000 EUR (π) where for the discount rate, *d=1/(1+r)*, we will use *r* = 3%. The monthly payouts for different ages and both genders are assumed as follows:

Table 3: Monthly payouts for different ages

Age	Male (EUR/month)	Female (EUR/month)
55	671.7	627.13
60	726.44	669.96
65	804.02	729.13
70	911.69	812.49
75	1060.03	936.41
80	1265.68	1118.95

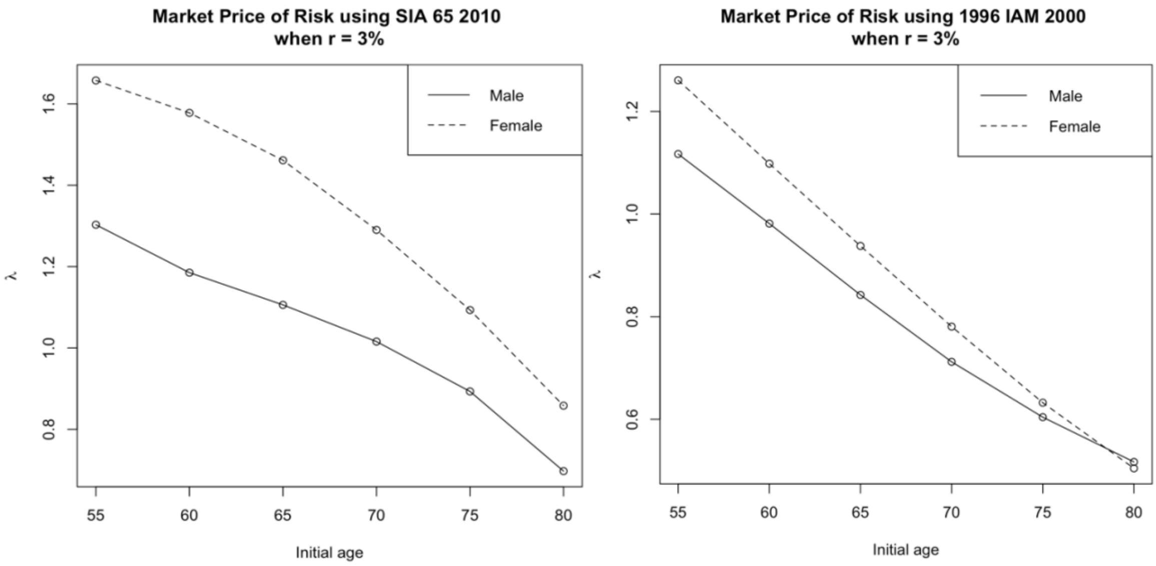
Adapted from Torske (2015, p. 31).

Let's use the equation below to calculate the risk market price. We can solve the equation bellow in R with the code listed in the Appendix 11.

$$\pi = s \times 12 \sum_{k=0}^{n-1} d^k \, \Phi[\Phi^{-1}(1 - {}_{k}q_{l0}) - \lambda] \tag{25}$$

Here, *s* represents the monthly payment (multiplied by 12 since we have yearly mortality data) and π a single premium of 100.000 EUR, the data for monthly payments is in table 3. In this example, I will calculate the market price of risk (λ) by using the Slovenian annuity tables (SIA 65, 2010), which were designed to value annuity liabilities in Slovenia. Of course, I could also use the tables obtained with the forecasting done in previous chapters, but since those are mortality and not annuity tables, it would be hard to compare the results with the ones obtained by Torske (2015).

Figure 9: Market price of risk



Source: Torske (2015, p. 27). *Adapted from Torske (2015, p. 27).*

In figure 9, we can see the results obtained by applying Wang's transformation to SIA 65, 2010 annuity table on the left and the results obtained from Torske (2015) on the right. The results are similar, where the market price of risk decreases as people get older. This is obvious since most "risky" annuitants (those who could live longer than expected) have already died; therefore, the selection bias will be small as the mortalities for the group of annuitants will not deviate too much from the average; hence less risk premium would be charged at those ages. We can also see that females have a higher market price of risk, which is because females live longer, and more future payouts are expected. The exact results are listed below.

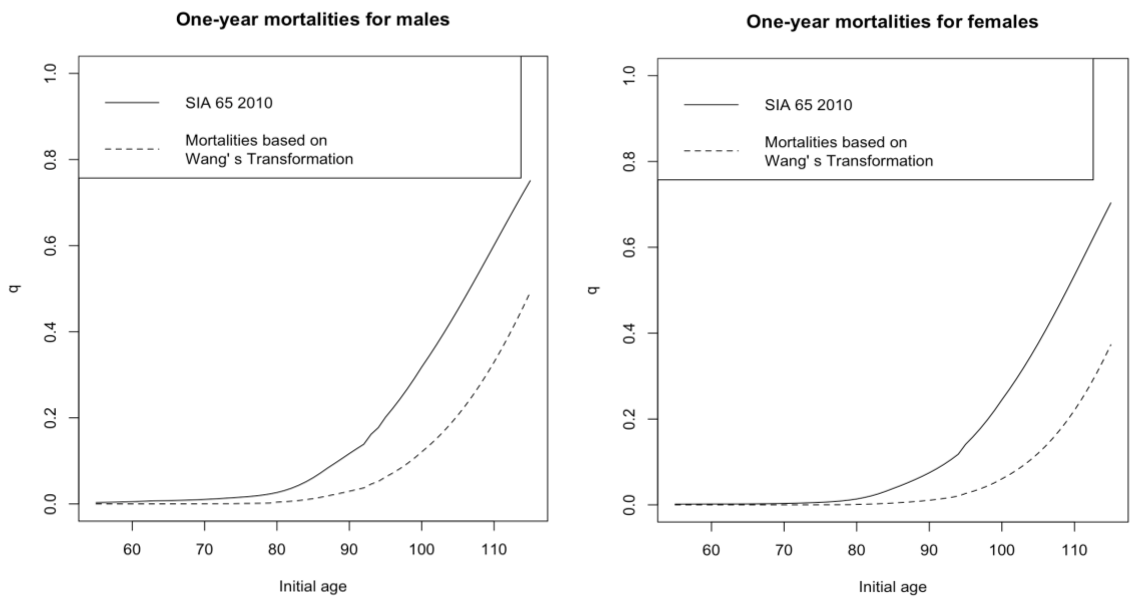
Table 4: Market price of risk using the Wang's transformation

Age	λ (male)_SIA65	λ (female)_SIA 65	λ (male)_IAM U.S	λ (male)_IAM U.S
55	1.3029	1.6576	1.117	1.261
60	1.1851	1.5778	0.981	1.098
65	1.1059	1.4614	0.842	0.938
70	1.0157	1.2902	0.712	0.781
75	0.8931	1.0932	0.604	0.632
80	0.6973	0.8581	0.517	0.504

Source: Torske (2015, p. 27).

When comparing the results obtained using the SIA 65, 2010 table and the 1996 IAM U.S. 2000 table, we can see that the market price of risk is higher using the former. This can be explained by the fact that SIA 65, 2010 have slightly higher mortality rates at older ages compared to the 1996 IAM U.S. 2000 table, which means that comparing the annuity payments listed above with equal single premium would lead to higher λ when SIA 65, 2010 is used. In other words, using Slovenian annuity tables and performing Wang's transformation would lead to higher distorted mortality rates (fewer annuities expected to be paid) compared to the U.S. tables, and if the single premium and monthly annuities are equal, this means using Slovenian table charges a higher risk premium (λ) (same price for less expected payouts). To see how Wang's transformation changes the mortality rates of an annuity table into its transformed counterparts, we can plot them side by side.

Figure 10: Mortalities based on Wang's transformation



Source: Torske (2015, p. 32).

We can see that the transformed mortalities are much lower at higher ages than those found in SIA 65, 2010 table. The higher the λ, the higher the difference between the two lines.

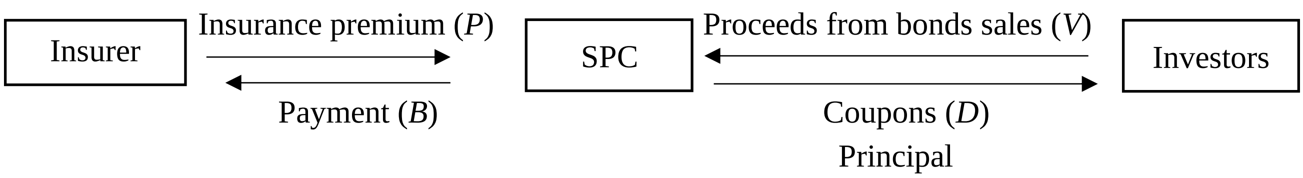
4.3 Longevity bonds

The distorted mortality rates and the market price of risk obtained from Wang's transformation can now be used to price longevity bonds. This has been done by Lin and Cox (2005) and Torske (2015).

4.3.1 Introduction to longevity bonds

A longevity bond is an instrument insurance companies can use to hedge longevity risk. Bonds are securities under which the insurer (the debtor) owes the holder (the creditor) a debt and is obligated to repay it at maturity with interest (coupons). When we talk about longevity bonds, the insurer buys insurance from the special purpose company (SPC) for a premium *P*, which issues bonds to investors. The SPC invests the money collected from sales and premium in risk-free securities (Lin & Cox, 2005).

Figure 11: Longevity bond



Adapted from Lin & Cox (2005, p. 5).

4.3.2 Pricing longevity bonds using Wang's transformation

Suppose that the insurer must pay an immediate life annuity to n_x annuitants all aged *x* initially. If the payment rate is 1000 EUR per year and we denote the number of annuitants alive in year *k* as n_{x+k} then the insurer needs to pay $1000 \cdot n_{x+k}$ at time *k*. Because of the longevity risk, the insurer would benefit from arranging a pre-determined level of n_{x+k} so that in case fewer people than anticipated die, the insurer does not suffer the losses.

In the case of longevity bonds, such an insurer can buy a contract from a SPC for a premium. Such a contract has a fixed trigger level at X_k where the SPC pays the insurer the excess of payments over that trigger. Let us assume the maximum amount the SPC is willing to cover over the trigger X_k is *C*. This implies that the insurer collects the benefit from the SPC as follows:

$$B_k = \begin{cases} 1000C, & if \ \ n_{x+k} > X_k + C \\ 1000(n_{x+k} - X_k), & if \ \ X_k < n_{x+k} \leq \ \ X_k + C \\ 0, & if \ \ n_{x+k} \leq X_k \end{cases} \tag{26}$$

The insurer's cash flow to annuitants at k is now offset by positive cash flow from the insurance:

$$1000n_{x+k} - B_k = \begin{cases} 1000(n_{x+k} - C), & if \;\; n_{x+k} > X_k + C \\ 1000X_k, & if \;\; X_k < n_{x+k} \leq \;\; X_k + C \\ 1000n_{x+k}, & if \;\; n_{x+k} \leq X_k \end{cases} \tag{27}$$

The cash flow from SPC to investors is as follows:

$$D_k = \begin{cases} 0, & if \;\; n_{x+k} > X_k + C \\ 1000(C + X_k - n_{x+k}), & if \;\; X_k < n_{x+k} \leq \;\; X_k + C \\ 1000C, & if \;\; n_{x+k} \leq X_k \end{cases} \tag{28}$$

Here the D_k is the total coupon paid to the investors. The maximum value of n_{x+k} is n_x which implies that no one has died; therefore, n_{x+k} is a random variable on the interval [*0*, n_x]. We can denote the market price of a longevity bond as *V*. The aggregate cash flow out of the SPC is therefore:

$$B_k + D_k = 1000C \tag{29}$$

For the SPC to fulfil the obligation towards the insurance company and investors, the SPC can buy a default-free fixed coupon bond for a price *W* with an annual coupon of 1000*C* and a principal of 1000*F*. In other words, SPC can buy a straight bond to receive the required cashflows to meet all their obligations:

$$P + V \geq 1000Fd(0,K) + \sum_{k=1}^{K} 1000 \cdot C \cdot d(0,k) \tag{30}$$

The *d(0,k)* is the discount factor that can be taken from the bond market when insurance is issued. The idea is that the SPC can use the money it generates from the sale and premium to buy the straight bond and have precisely the required coupon at each time *k* to fulfil its obligations towards both parties (Torske, 2015). In the following example, I will use the same strike levels as Lin and Cox (2005), as well as their mortality improvement forecasts.

Table 5: Change in force of mortality

Age group	Change in force of mortality
65-74	-0.007
75-84	-0.0093
85-94	-0.013

Adapted from Lin & Cox (2005, p. 11).

These improvement levels determine the strike levels:

$$X_k = \begin{cases} n_x \cdot p_x \cdot e^{0.0070t}, & for \;\; k = 1,\ldots,10, \\ n_x \cdot p_x \cdot e^{0.0070t} \cdot e^{0.0093(t-10)}, & for \;\; k = 11,\ldots,20, \\ n_x \cdot p_x \cdot e^{0.0070t} \cdot e^{0.013(t-20)}, & for \;\; k = 21,\ldots 30 \end{cases} \tag{31}$$

Where ${}_kp_x$ is the survival probability from SIA 65, 2010 table.

Now we need to calculate the coupon payment $\mathbb{E}^*[D]$ from the equation stated previously:

$$\frac{D_k}{1000} = \begin{cases} 0, & if \;\; n_{x+k} > X_k + C \\ (C + X_k - n_{x+k}), & if \;\; X_k < n_{x+k} \leq \;\; X_k + C \\ C, & if \;\; n_{x+k} \leq X_k \end{cases} \tag{32}$$

Therefore:

$$\frac{D_k}{1000} = C - (n_{x+k} - X_k)_+ + (n_{x+k} - X_k - C)_+ \tag{33}$$

Here $(n_{x+k} - X_k)_+$ and $(n_{x+k} - X_k - C)_+$ can only take positive values to ensure result for $D_k/1000$ is between 0 and C.

Hence:

$$\frac{1}{1000}\mathbb{E}^*[D_k] = C - \mathbb{E}^*[(n_{x+k} - X_k)_+] + \mathbb{E}^*[(n_{x+k} - X_k - C)_+] \tag{34}$$

For the calculation, we have that the distribution of n_{x+k} is the distribution of the number of survivors from n_x who survive to age $x+k$, which happens with the probability ${}_kp^*_x$. This means that the n_{x+k} has a binomial distribution with parameters n_x and ${}_kp^*_x$. Because n_x is a large value we have that n_{x+k} is distributed approximately normally with the mean $\mathbb{E}^*[n_{x+k}] = \mu^*_k = n_x \cdot {}_kp^*_x$, and variance $V^*[n_{x+k}] = \sigma^{*2}_k = n_x \cdot {}_kp^*_x \cdot (1-{}_kp^*_x)$. For random variable X with $\mathbb{E}[X] < \infty$ we can obtain the following by integrating “by parts”:

$$\mathbb{E}[(X-a)_+] = \int_a^\infty [1 - F(t)]dt$$

$$\mathbb{E}[(X-a)_+] = \int_a^\infty [1 - \Phi(t)]dt \tag{35}$$

We can also write this as:

$$\Psi(a) = \int_a^\infty [1 - \Phi(t)]dt$$

$$= \phi(a) - a[1 - \Phi(a)] \quad (36)$$

Here $a = (X_k - \mu^*{}_k)/\ \sigma^{*2}_k$. We can now express:

$$\mathbb{E}^*[D_k] = 1000 \cdot \{C - \sigma^*{}_k [\Psi(a_k) - \Psi(a_k + C/\sigma^{*2}_k)]\} \quad (37)$$

Using equation (37), the bond price *V* can be calculated, which is the discounted face value with added discounted coupon payments under distorted mortality rates:

$$V = Fd(0, K) + \sum_{k=1}^{K} \mathbb{E}^*[D_k] d(0, k) \quad (38)$$

This equation determines the value *V* of the bond for the investors (Torske, 2015).

Let us use the information from Lin and Cox (2005) and calculate the price of a longevity bond. In order to do this, we also need to have a set of values for λ that represent the market price of risk. We have calculated this using the SIA 65, 2010 tables in the previous section, so I will use those values. In this calculation, I will again use the SIA 65, 2010 table. All other information will be the same as in the example done by Lin and Cox (2005) and is listed below:

- $\lambda_{65,male}$ = 1.1059 and $\lambda_{65,female}$ = 1.4614,
- n_{65} = 10.000 for each gender of age 65,
- *s* = 1000 EUR per year,
- *F* = 10.000.000 EUR,
- *C* = 700,
- The risk-free rate is 3%,
- *W* = 10.000.000 EUR with a coupon rate of 7%,
- *K* = 30 years (duration of the contract),
- Changes in force of mortality from table 5.

(Lin & Cox, 2005).

Before calculating the price, I want to clarify why Lin and Cox (2005) used the above data since it is essential in understanding how this product is priced. This data is selected in such a way that the investors are willing to take the longevity risk of a portfolio of 10.000 annuitants for each gender of age 65.

C = 700 means that in case longevity improves in a given year, the hedger (investors) is willing to pay the yearly annuity worth 1000 EUR for a maximum of 700 people who were not expected to survive. As mentioned in equation (29), the SPC needs sufficient funds to cover both parties in all scenarios. They need to invest 10.000.000 EUR with a yearly coupon of 7%, which means they receive 700.000 EUR each year.

If longevity improves rapidly and the number of annuitants at the end of a given year is higher by 700 or more, then the SPC has the 700.000 EUR needed to cover its obligation. On the other hand, if the mortality does not improve in a given year and the number of annuitants alive in the end is lower or equal than expected (threshold not hit), then the SPC has the 700.000 EUR, which it pays to the investors. This means the SPC also has sufficient funds to cover the obligations of all possible scenarios between the two extremes explained.

4.3.3 Results using the SIA 65, 2010 table

The results are: V_{male} = 9.701.000 EUR and V_{female} = 9.456.010 EUR. The code for this calculation can be found in the Appendix 12. This is the maximum price investors are willing to pay the SPC for the longevity bond, given the values of λ. However, we already know the SPC needs 10.000.000 EUR to fulfil its obligations to both parties during the contract. Therefore, the insurance company needs to cover the difference, which is precisely the price of a longevity bond or a premium the insurance company pays the SPC to mitigate longevity risk for 30 years in this example. In symbols, this means: $P_{male} = 299.000\, EUR$ and $P_{female} = 543.990\, EUR$.

These are the premiums the insurance company needs to pay the SPC at time 0, so the liabilities in all 30 years will be covered towards both parties depending on the development of mortality. Lin and Cox (2005) did not consider any fee charged by the SPC, which might increase the price. We can see that the premium for females is much higher since they live longer; hence the risk is more significant. Results by Lin and Cox (2005) are lower since they used lower values for the market price of risk and a different annuity table, to begin with. These results mean that the insurance company has relatively high initial costs for such a hedge which only eliminates the risk partially since the number of annuitants covered by the investors is set to a maximum of 700, where for the additional survivors the insurance company needs to pay the annuities. They also explain one of the main disadvantages of this instrument which is a large up-front payment by the investors to the SPC which makes this product less attractive especially when we account for the counterparty risk.

This section presents the longevity bond and how this instrument can be priced using Wang's transformation. This transformation can be used in pricing other longevity derivatives discussed in the subsequent chapters; however, the methods used can be complex and unique to specific hedge providers; therefore, I will present the other longevity derivatives in a more theoretical context and use simplified examples to show how insurance companies can use them for longevity risk mitigation.

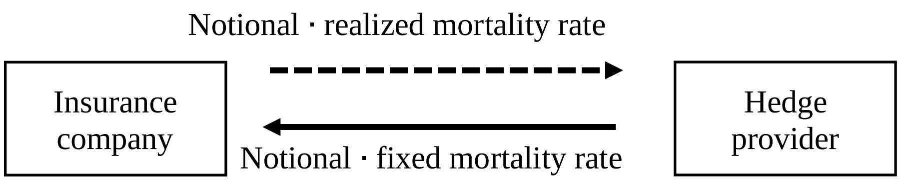
4.4 *q*-Forwards

A mortality forward, also known as a *q*-forward because the letter *q* is the standard actuarial symbol for mortality rates, is another instrument used for transferring longevity risk. Moreover, this instrument has significant importance in practice since it forms the basic building block from which other more complex insurance-related derivatives can be constructed.

4.4.1 Introduction to *q*-forwards

A *q*-forward contract is an agreement between two parties where they agree that the realized mortality rate for a given population is exchanged for the fixed mortality rate at the expiry. This contract is similar to an interest rate swap where the counterparties exchange realized and fixed mortality rates instead of a floating and fixed interest rate.

Figure 12: q-forward contract

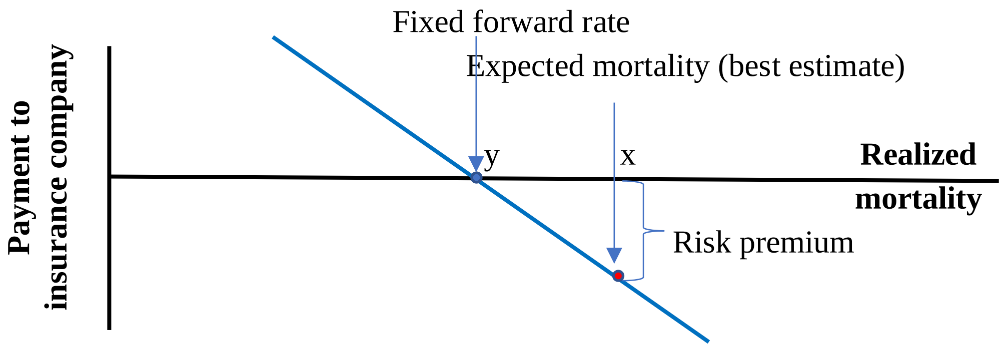


Source: Coughlan, Blake, McMinn, Cairns & Dowd (2013, p. 18).

Figure 12 shows how the insurance company can sell this instrument to hedge longevity risk. When a *q*-forward is fairly priced, there is no payment at the inception of the trade. At maturity, one of the two counterparties will make a net payment. There will be no payment if the fixed and actual mortality rates are the same, which is usually not the case. The settlement, which happens at maturity, is based on the net amount payable and is proportional to the difference between the fixed and realized rates. When the realized mortality rate in a year is lower than the fixed rate, the settlement is favorable, and the insurance company receives the payment, which offsets the increase in the value of its liabilities.

On the other hand, if mortality rates increase and, at the end of the contract, the fixed rate is lower than the realized rate, then the insurance company needs to make a payment to the hedge provider, which will be offset by a fall in the value of liabilities. Using this instrument, the net liability value of an insurance company is locked in and will stay the same regardless of whether the mortality rates increase or decrease; therefore, the insurance company is protected from unexpected changes in mortality. In figure 12, we consider an insurance company as a seller of a *q*-forward and a hedge provider as a buyer (Coughlan, Blake, McMinn, Cairns & Dowd, 2013, pp. 11-18).

Figure 13: Payout from a q-forward



Source: Coughlan, Blake, McMinn, Cairns & Dowd (2013, p. 18).

In figure 13 above, we can see the payout structure of a *q*-forward. Forward mortality rates are not quite the same as the market's expected mortality rates. The reasoning is that there are more market participants with exposure to longevity risk (those that lose if mortality rates decline) than those with the opposite exposure, who would benefit from a decline in mortality. When a risk hedger enters such a contract, he expects to be compensated for it through a risk premium that compensates him for taking the longevity risk of the insurance company; therefore, the mortality forward is below the expected mortality rate, which is a risk premium incentivizing the hedger to enter into such a contract. We could also say that the insurance company selling *q*-forwards would have to charge a negative risk premium to make buyers interested in purchasing this contract.

In figure 13, the expected mortality rate, which can be produced by the stochastic models discussed earlier, is at the *x* level. Because the hedger expects a risk premium, the (fixed) forward mortality rate will be lower and is at the *y* level in the figure above (Coughlan, Epstein, Sinha & Honig, 2007).

4.4.2 Pricing based on the LLMA structure

In section 3, I have presented the mortality forecasting models which yield future mortality rates. Those models would have to be transformed to yield mortality rates under a risk-neutral or martingale measure for pricing longevity derivatives. We have previously applied Wang's transformation to obtain risk-neutral mortality rates for this purpose. For pricing *q*-forwards a framework has been established in the past which provides a pricing structure for this instrument where previously obtained risk-neutral mortality rates could be applied.

Life and Longevity Markets Association (LLMA) has proposed a standardized framework for pricing longevity derivatives. The pricing structure used for *q*-forwards was the one used by J.P. Morgan in 2007.

We can see a realized mortality rate as $m_{realized}$ and fixed mortality rate as m_{fixed}. The net settlement amount is as follows:

$$notionalamount \cdot \left(\mathrm{m}_{\text{realized}} - m_{fixed}\right), \quad for\ the\ hedge\ provider \tag{39}$$

$$notionalamount \cdot \left(m_{fixed} - \mathrm{m}_{\text{realized}}\right), \quad for\ the\ insurance\ company \tag{40}$$

The fixed rate is pre-determined in the contract. Therefore, the hedge provider should discount the net settlement with a required rate of return, where the discounted net settlement would be the premium the hedger would set. Because the realized mortality is unknown, we replace it with the expected mortality rate.

Ideally, the risk settlement is zero, where both realized and fixed rates are equal. However, this is usually not the case since the mortality rate varies across contract duration. The hedge provider is exposed to this risk and wants to be compensated through a risk premium. For this reason, the fixed rate is lower than the expected mortality rate at maturity. It is calculated based on the hedge provider's expected mortality rate and risk tolerance and is called a forward rate.

The LLMA structure of a *q*-forward provides a simple way of evaluating this contract which is done as follows:

$$m_{x,t} = m_{x,0} \times \prod_{i=1}^{t} \left(1 - \left(\widehat{m}_{x,i} + \xi\right)\right) \tag{41}$$

Where *x* is the age of the group, $m_{x,t}$ is the forward (fixed) rate at time *t*, $\widehat{m}_{x,I}$ is the best estimate of mortality improvement and ξ is the adjustment term for risk appetite. $\widehat{M}_{x,i}$ explains how future mortality improves. We can briefly drop the ξ from the equation to understand this LLMA structure better.

$$m_{x,t} = m_{x,0} \times \prod_{i=1}^{t} \left(1 - \widehat{m}_{x,i}\right) \tag{42}$$

This is similar to estimating the mortality rate with the improvement rate shown below.

$$M_{x,t} = m_{x,0} \times \prod_{i=1}^{t} \left(1 - r_{x,i}\right) \tag{43}$$

Here:

$$r_{x,i} \coloneqq 1 - \frac{m_{x,i}}{m_{x,i-1}} \tag{44}$$

The LLMA structure suggests an average mortality improvement rate as the best estimate of mortality improvement rate. If the reference mortality rate is the rate of an age group or group, such as males aged 65 to 69, the best estimate can be the average predicted mortality improvement of those who belong to this group. If the reference is for a specific age, the best estimate can be obtained by the average predicted mortality improvement rate over a contract duration. This can be simplified as follows:

$$m_{x,t} = m_{x,0} \times (1 - \widehat{m}_x^t)^t \tag{45}$$

Where $\widehat{m}_x^t$ is the average predicted mortality improvement rate over the contract duration *t*. In this equation, the forward rate is the expected mortality rate at maturity and contains no information about the hedge provider's risk appetite. If the hedge provider uses an equation above zero payment is expected. The hedge provider is taking all the risk in such a contract since a decrease in mortality rates means he will receive less payment from the insurance company and pay fixed payment meaning the difference counts as a loss. On the other hand, an insurance company is not effectively taking a risk even in case of a mortality rates increase and resulting higher payments to the hedge provider since a decrease in liabilities can compensate for this. For this reason, a hedge provider sets a forward rate he pays below the expected floating rate he receives.

In a standardized q-forward contract such as the one proposed by J.P. Morgan in 2007, a forward rate is set to 1.2%, and we can calculate ξ to measure the risk appetite of the transaction. For example, if the hedge provider expects higher mortality improvement (receiving less at maturity), he increases the value of ξ.

The present value of the net settlement or premium can be calculated as follows:

$$PV\ of\ net\ settlement = notional\ amount \cdot \frac{\left(m_{x,0} \times (1 - (\widehat{m}_x^t + \xi)\right)^t - m_{x,t})}{(1+r)^t} \tag{46}$$

From the equation above, we can see that if the hedger assumes higher mortality improvement and increases ξ he expects to receive less payment at maturity and, therefore, also needs to determine a lower $m_{x,t}$ or forward fixed rate before entering the contract to expect a positive cash flow for taking the risk (Chuang, 2013).

For determining the correct level of ξ and calculating the resulting premium refer to Chuang (2013) where an extended Lee-Carter model with the normal inversed Gaussian (NIG) Lévy processes is used together with the Esscher transformation to obtain risk-neutral mortality rates and the level of ξ.

4.4.3 Pricing q-forwards with classical methods

We have mentioned previously that pricing longevity derivatives can be done by using a distortion function to obtain adjusted mortality rates or standard risk neutral pricing used in other financial instruments such as options. Barrieu and Veraart (2016) wrote a paper published in the Scandinavian actuarial journal arguing that the standard risk-neutral pricing methods cannot be used mindlessly in q-forwards or other longevity derivatives given that the underlying mortality rate is not tradable and hence standard arbitrage-free valuation formula for forward prices does no longer hold. Furthermore, they point out that the longevity risk transfer market, in general, is still immature and lacks liquidity; therefore, a classical arbitrage-free pricing methodology is inapplicable as it relies upon the idea of risk replication which can only be done when we have high liquidity.

"In a complete market, the price of the contingent claim is the expected future discounted cash-flows, calculated by the unique risk-neutral probability measure. In contrast, in an incomplete market, such as a longevity-linked securities market, there will be no universal pricing probability measure, making the choice of pricing probability measure crucial". (Barrieu & Veraart, 2016, p. 6). Due to the abovementioned reasons, they propose some different methods, which I will present here.

- Net premium principle

$$Value_0\big(NPA(T)\big) = \mathbb{E}_P[exp(-rT)z(q(T) - K)] = 0 \qquad (47)$$

Here *NPA(T)* stands for "net payoff amount" at time *T*, *exp(-rt)* is the discount rate *(r≥0)*, *z* represents the notional amount agreed at time 0, *q(T)* is the realized mortality at time *T* and *K* is the forward mortality rate.

The formula implies that *K* (forward mortality rate) is:

$$K = \mathbb{E}_P[q(T)] \qquad (48)$$

Here the time 0 value of the NPA can be derived by the expectation of the NPA under the physical probability measure ($\mathbb{E}_P$). The authors state that using this expectation for pricing can be justified by appealing to the strong law of large numbers where many q-forward contracts would make such a limit result applicable. This will only work if the amount of q-forward contracts is sufficient, which in their opinion seems a less restrictive assumption than assuming the replicability of these derivatives.

- Standard deviation principle

$$V_0\big(NPA(T)\big) = \mathbb{E}_P[exp(-rT)z(q(T) - K)] + \lambda\sqrt{V_p[exp(-rT)z(q(T) - K)]} = 0 \quad (49)$$

The V_0 represents time zero value and V_p denotes the variance under the physical probability measure; therefore, its square root is the corresponding standard deviation. This implies that:

$$K = \mathbb{E}_P[q(T)] + \lambda\sqrt{V_P[q(T)]} \tag{50}$$

This approach can be interpreted as a market risk premium where λ can be related to the Sharpe ratio of the risky mortality rate *q(T)*. The choice of λ depends on the various risks the hedger is exposed to.

- Principle of zero utility

$$\mathbb{E}_P\left[U\left(W_0 exp(rT) + z(q(T) - K)\right)\right] = \mathbb{E}_P\left[U\left(W_0 exp(rT)\right)\right] \tag{51}$$

Where *U* represents the utility function where exponential utility is assumed, i.e., *U(y)* = -*exp(-Ɣy)* where *Ɣ > 0* is the constant coefficient of absolute risk aversion. W_0 represents initial wealth which we assume to be constant in order to obtain:

$$K = -\frac{1}{Ɣz}\log\left(\mathbb{E}_P\left[exp\left(-Ɣzq(T)\right)\right]\right) \tag{52}$$

This principle of zero utility is also referred to as indifference pricing in a financial context. However, the authors state that a more appropriate strategy involves utility maximization in an incomplete market where perfect replication is no longer possible. In this context, the maximum price depends on the individual preference and is chosen such that the agent is indifferent between paying this price and obtaining the *q*-forward or not.

As discussed by Loeys, Panigirtzoglou and Ribeiro (2007), longevity risk transfer market is net short longevity, which means that more participants try to hedge longevity risk than overtaking it. This implies that prices of *q*-forwards will include a risk premium, making such products attractive to investors. As already mentioned, this means that the forward rate will be lower than the expected forward rate predicted by forecasting models.

We can denote the risk premium as *R*:

$$K = \mathbb{E}_P[q(T)] + R \tag{53}$$

The equation above explains that the forward rate *K* is the expected mortality rate at time *T* under physical probability measure with added *R*. Here, we assume the insurance company is a seller of *q*-forwards and needs to set a negative risk premium to attract potential investors (*R<0*). The standard deviation principle already has a similar structure where the risk premium depends solely on the term λ.

For insurance companies selling *q*-forwards to hedge longevity risk λ needs to be sufficient to make investors interested in purchasing this product. Therefore, a certain Sharpe ratio denoted as *S* is necessary, which can be determined as follows:

$$S = \frac{\mathbb{E}_P[q(T)] - K}{\sqrt{V_P[q(T)]}} \tag{54}$$

This means the forward mortality rate is determined as follows:

$$K = \mathbb{E}_P[q(T)] - S\sqrt{V_P[q(T)]} \tag{55}$$

We can see that this approach is related to the standard deviation principle in which λ is replaced by *S* that ensures the forward mortality rate is below the expected mortality rate by a margin large enough to attract buyers of q-forwards who demand a certain risk premium (Sharpe ratio).

Authors also use this principle in pricing the q-forward contract where the *q(T)* is based on the LC and the CBD model. The prices those two models yield are very similar. Their main conclusion is that the estimation window significantly affects the price of a q-forward obtained by this method. They compared the results of both the LC and CBD model based on the data of the most recent six years or 21 years, using a Sharpe ratio of 0.1.

The results for the corresponding prices were very different due to the differences in confidence intervals depending on the estimation window. They argue that using a linear forecasting model such as CBD and LC might be inappropriate and that one would need to allow for random changes in this linear trend (Barrieu & Veraart, 2016).

There are many other methods to price q-forwards and longevity derivatives in general. However, because longevity derivatives such as a q-forward are used to reduce the amount of SCR, other researchers such as Zeddouk and Devolder (2019) have taken a different approach to price these contracts. They propose a cost-of-capital approach. The idea of this method is that the price of a longevity derivative should not be higher than the cost of holding the additional SCR. Otherwise, investors would not be interested in buying it.

4.4.4 A hypothetical example of a q-Forward

Let us assume in a simplified example where the insurance company wants to hedge longevity risk with a ten-year q-forward on a population aged between 58 and 60. The cash flow at maturity depends on the average mortality rate of ages 68 and 70. I am assuming that $q_{x,t}$ in a given year for the corresponding population are (1.8%, 2%, and 2.2%) respectively. This means that the average one-year mortality rate is 2%. To simplify the example, I will assume that the best estimate of a one-year mortality decrease is 1.5% per year. This implies that the one-year death probability is 98.5% of the rate in the previous year. As a risk premium, an additional 1% decrease in mortality rate is considered. The overall improvement in mortality is, therefore, 2.5% per year.

The forward mortality rate of the group can be calculated as follows:

$$2\% \times (1 - 2.5\%)^{10} = 1.553\%$$

The best estimate of the mortality rate is:

$$2\% \times (1 - 1.5\%)^{10} = 1.72\%$$

The corresponding risk premium is the difference and is 0.165% which compensates the hedger for taking the risk.

Table 6: q-forward settlements for various outcomes

Realized rate (%)	Fixed-rate (%)	Notional amount (in million €)	Settlement (in million €)
1.2	1.553	50	17.65
1.553	1.553	50	0
1.8	1.553	50	-12.35
2	1.553	50	-22.335

Source: Coughlan, Epstein, Sinha & Honig (2007, p. 3).

This example is simplified and only meant to outline how insurance companies can use those derivatives in practice, not how the actual risk premium is determined. In table 6, we can see that the net settlement at maturity of this contract is positive for the hedge provider when the realized rate at maturity is lower than the fixed rate, which means the hedge provider needs to pay the settlement to the insurance company, which uses this payment to offset the negative impact this has on its annuity liabilities. When the reverse is true, the hedge provider has a negative settlement which means he receives the payment from the insurance company, which is offset by a decrease in annuity liabilities (Coughlan, Epstein, Sinha & Honig, 2007).

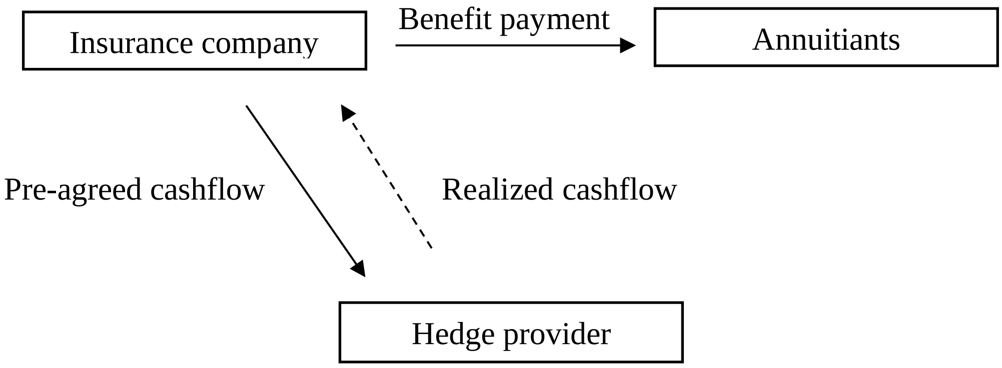
4.5 Longevity swaps

Longevity or survivor swaps have been one of the most used longevity derivatives in the past. This instrument involves exchanging actual pension or annuity payments for a series of pre-agreed fixed payments. Each payment is weighted on the survival rate.

4.5.1 Introduction to longevity swaps

The difference between a *q*-forward and a longevity swap is that counterparties regularly exchange realized and pre-agreed cashflows instead of exchanging realized for a fixed mortality rate at expiry based on the notional amount.

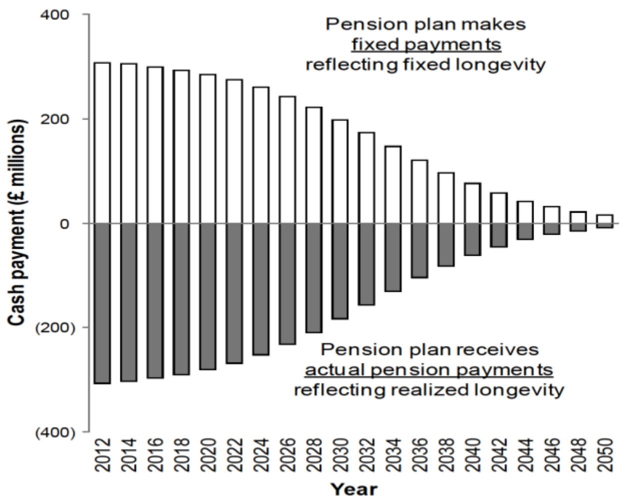
Figure 14: Structure of longevity swap transaction



Source: Bank for International Settlements (2013, p. 7).

Figure 14 shows that the insurance company is responsible for providing benefit payments to its annuitants. If the longevity improves in the future years, the insurance company has to pay annuities to more annuitants, increasing the liability. The insurance company is hedged with the longevity swap since it receives a realized cash flow and pays the fixed one. This means that the insurance company can make a loss in case mortality increases and the realized cash flow is lower than the fixed one. This can be represented in the next figure (Blake, 2018).

Figure 15: Exchange of cash flows in a longevity swap



Adapted from Blake (2018, p. 15).

With this contract, the insurance company relies on the payments made by a hedge provider. Because of this, longevity swaps are usually collateralized by both parties in the agreement. This means the hedge provider and the insurance company need to post collateral depending on whether the value of a swap is positive or negative (Biffis, Blake, Sun & Pitotti, 2016, p. 3).

4.5.2 Pricing a longevity swap

Pricing of a longevity swap can be described by assuming that the insurance company pays a fixed rate $\bar{p}^N \in (0,1)$ against the realized survival rate experienced by the population between time 0 and *T*, *T>0* on the initial population of *n* annuitants. The hedger's liability is, therefore $n - N_T$ where the N_T represents the number of deaths from [*0*, *T*]. If the population has a certain force of mortality, we can write the expected number of survivors at time *T*.

$$\mathbb{E}^P[n - N_T] = np_T^P \tag{55}$$

Where the p_T^P represents the probability of survival under the real-world probability measure:

$$p_T^P := \mathbb{E}^P\left[e^{-\int_0^T \mu_t dt}\right] \tag{56}$$

We can obtain the p_T^P with the forecasting models discussed earlier. We know that for pricing longevity derivatives, we need to transform death rates into their risk-neutral counterparts. We assume this can be done by using a risk-neutral measure Q. The liability, therefore, has a time zero price:

$$\mathbb{E}^Q\left[e^{-\int_0^T r_t dt}(n - N_T)\right] = n\mathbb{E}^Q\left[e^{-\int_0^T (r_t + \mu_t)}\right] \tag{57}$$

Where the r_t is the risk-free rate process used for discounting in continuous time. We will ignore the default risk and, for simplicity, consider a single payment instrument.

The hedger's payout can be written as:

$$hedger's\ payout = n\left(\frac{n - N_T}{n} - \bar{p}^N\right) \tag{58}$$

In this equation, we see that the hedger pays the difference between the realized number of survivors and the fixed rate $n\bar{p}^N$ which is agreed on at the contract's inception.

Therefore, the value of the swap at inception is:

$$S_0 = n\mathbb{E}^Q\left[e^{-\int_0^T r_t dt}\left(\frac{n - N_T}{n} - \bar{p}^N\right)\right]$$

$$= n\mathbb{E}^Q\left[e^{-\int_0^T (r_t + \mu_t) dt}\right] - nB(0,T)\bar{p}^N \tag{59}$$

Where $B(0, T)$ denotes the time-zero price of a zero-coupon bond maturing at T. Setting $S_0 = 0$ we get:

$$\bar{p}^N = p_T^Q + B(0,T)^{-1} Cov^Q \left(e^{-\int_0^T r_t dt}, e^{-\int_0^T \mu_t dt} \right) \tag{60}$$

Where the p_T^Q represents the risk-adjusted survival probability we can obtain using Wang's transformation discussed earlier (Biffs, Blake, Sun & Pitotti, 2016).

4.5.3 A hypothetical example of a longevity swap

Let us assume a in simplified example of an annuity provider with 100.000 annuitants of age 65, each receiving 100 EUR per month. The annuity provider (insurance company) enters a longevity swap on January 1st. We assume the following:

Table 7: Outcomes of a longevity swap

Date	Actual pension payment (in million €)	Predefined cashflow (in million €)	Payment to the insurer (in million €)
Feb. 1st	10	9.8	0.2
March 1st	9.6	9.5	0.1
April 1st	9.3	9.35	-0.05
May 1st	9.15	9.15	0

Source: OECD (2014, p. 179).

This hypothetical example aimed to show how the insurance company can use this instrument to hedge longevity risk. The insurance company and the hedge provider agree on the predefined cash flows at time 0. During the contract, the actual number of annuity survivors is revealed, and the annuity provider either receives the money or pays the hedge provider. In the example above, the annuity provider predicted that 2000 annuitants would die between January 1st and February 1st. Therefore, the realized deaths were 0, and they had to pay 200.000 EUR more, which they received from the hedge provider. From January 1st to April 1st, they assumed that 7500 annuitants will die, where the realized deaths were 7000; therefore, the insurance company paid 50.000 EUR to the hedge provider. We can see that the longevity swap is essentially a series of forward contracts discussed previously (OECD, 2014).

4.6 Drawbacks of longevity derivatives

Longevity derivatives can transfer the longevity risk from the insurance companies or annuity providers to the counterparty, which accepts to carry this risk for a certain price. As discussed previously, these products can help mitigate unexpected mortality improvements that the forecasting models do not capture. However, even though derivatives provide solutions for mitigating longevity risk, they have unique issues I have not discussed so far.

I have already mentioned that longevity derivatives can be index based or customized. To have a liquid market, index-based longevity derivatives are preferred. This means we need to set a base population under which we will price a specific product that might differ from the population to which a particular insurance company is exposed. For example, we can imagine an index-based *q*-forward contract where the population mortality used in pricing is the Slovenian population. However, if the insurance company is exposed to annuitants of a particular group which are not well represented by Slovenian population mortality, such a hedge would not be as effective. This basis risk can be reduced by having many different contracts unique to specific insurance companies (customized contracts); however, this leads to lower liquidity. Blake explains this as a tradeoff between having a liquid market and decreasing the basis risk. This means that for higher hedge effectiveness, we need to accept a higher basis risk, and in pursuit of a liquid market, we need to accept that the hedge effectiveness of these products will be lower. The choice of a mortality model is also crucial, where there needs to be a joint agreement between the market participants on which mortality forecasting model to use in the design and pricing of longevity derivatives.

There is also a regulatory issue. As mentioned, insurance companies might want to purchase such products to decrease the amount of SCR needed. However, this is not always the case when the regulator has to be persuaded to offer a capital release when such products are added to the balance sheet of an insurance company. Blake (2019) suggests that in the design phase of longevity derivatives, it is crucial to work with the regulator from the beginning so there is no uncertainty about whether the regulator will accept the product and enable a capital release when it is purchased. As mentioned earlier, the longevity bond proposed by BNP Paribas, which was withdrawn in late 2005, suffered from this exact reason: it did not attract sufficient investors because it did not lead to a capital release which would benefit insurance companies. This failure happened because BNP Paribas kept the design process private without discussing it with the regulator or potential buyers.

These drawbacks, to an extent, explain why the longevity risk transfer market has not evolved as much as it was initially anticipated. Another reason is that insurance companies can use reinsurance for the same purpose, which is a much older and a more familiar way of transferring risk discussed in the next chapter (Blake, 2019).

4.7 Optimal usage of longevity derivatives

The decision on which longevity derivative product to choose depends a lot on the abovementioned drawbacks and the goals of the individual insurance company regarding longevity risk. Insurers purchase longevity derivatives first to reduce the uncertainty in future cashflows and second to reduce the capital charges imposed by Solvency II regulation. Standardized or index-based products offer lower capital release due to the basis risk. Still, they may, on the other hand, provide cost advantages due to being cheaper than customized deals which offer higher capital release for a higher price. Therefore, optimal hedging decisions must consider both the hedge effectiveness and capital efficiency of the specific hedge.

4.7.1 Hedged liability

Let us assume the insurance company wants to hedge a liability of life annuities where all premiums have been paid upfront to the insurer. These annuities start at the age x_R and pay one unit of currency at the beginning of every year until the beneficiary dies. We can define N_{sub} as distinct and sufficiently large subpopulations of different socioeconomic status where all individuals within this subpopulation have the same force of mortality where $\rho \in \{1, \ldots, N_{sub}\}$.

The time-*t* random present value of all future unhedged liabilities is as follows:

$$L(t) := \sum_{\rho=1}^{N_{sub}} L^{\rho}(t)$$

$$:= \sum_{\rho=1}^{N_{sub}} \sum_{s>t} (1+r)^{-(s-t)} B^{\rho}_{x_{0+s,s}}, \quad t \geq 0 \tag{61}$$

Here the $B^{p}_{x_{0+s,s}}$ represents the number of survivors from population ρ aged $x_0 + s$ at time $s > 0$. Therefore, the time-*t* present value of best-estimate unhedged liabilities is:

$$\tilde{L}(t) := \sum_{\rho=1}^{N_{sub}} B^{\rho}_{x_0+t,t} \sum_{s>t} (1+r)^{-(s-t)} \times \prod_{u=t}^{s-1} (1 - \tilde{q}^{\rho}_{x_0+u,u+1}(t)), \qquad t \geq 0 \tag{62}$$

By using longevity derivatives, we can express time-*t* present value of hedged liability as:

$$L_H(t) := L(t) - H(t), \qquad t \geq 0 \tag{63}$$

And the time-*t* best-estimate hedged liabilities as follows:

$$\tilde{L}_H(t) := \tilde{L}(t) - \widetilde{H}(t), \qquad t \geq 0 \tag{64}$$

Where $\widetilde{H}(t)$ represents the time-*t* best estimate of all future cash flows coming from a chosen longevity derivative instrument. This value does not represent the market value of the derivative product. Because insurers also need to hold sufficient regulatory capital, the adjusted unhedged liabilities can be written as follows:

$$\Pi_L^M \coloneqq L(0) + CoC_L^M, \qquad M \in \{IM, SF\} \tag{65}$$

Where $\mathrm{CoC}_\mathrm{L}^\mathrm{M} := \sum_{t\geq 0} \frac{r_{CoC} SCR_L^M(t)}{(1+r)^{t+1}}$ which denotes time zero random present value of all costs of capital for the hedged liabilities, which can be done by using the internal model (*M=IM*) or standard formula (*M=SF*) in Solvency II regulation. r_{CoC} reflects the return in excess of the risk-free rate shareholders demand for providing equity.

The adjusted hedged liabilities are defined as follows:

$$\Pi_{L_H}^M \coloneqq L_H(0) + CoC_{L_H}^M, \qquad M \in \{IM, SF\} \tag{66}$$

4.7.2 Capital efficiency

Capital efficiency reflects the net cost of capital relief:

$$NReCoC^M(H) \coloneqq \mathbb{E}(\Pi_L^M) - \mathbb{E}(\Pi_{L_H}^M)$$

$$\coloneqq \mathbb{E}(CoC_L^M) - \mathbb{E}\big(CoC_{L_H}^M\big) + \mathbb{E}\big(H(0)\big), \qquad M \in \{IM, SF\} \tag{67}$$

Here two opposing effects come into play:

- The hedge usually reduces the insurance company's SCR, which generates a positive cost of capital relief:

$$ReCoC^M(H) \coloneqq \mathbb{E}(CoC_L^M) - \mathbb{E}\big(CoC_{L_H}^M\big) \geq 0 \tag{68}$$

- On the other hand, the present value of all hedging instrument cashflows is usually negative because the hedge provider wants to be compensated for taking the risk: $\mathbb{E}\big(H(0)\big) < 0$.

In this setting, the insurance company could be hedged entirely, which would result in a zero SCR for the longevity risk, and the cost of capital would reduce to zero. Furthermore, if this hedge was provided on the best-estimate basis or $\mathbb{E}\big(H(0)\big) = 0$, this would provide a maximum net cost of capital relief since the hedger would charge no premium.

With this, we can define the capital efficiency of the hedge *H* as follows:

$$CE^M(H) \coloneqq \frac{NReCoC^M(H)}{\mathbb{E}(CoC_L^M)}, \qquad M \in \{IM, SF\} \tag{69}$$

The condition for a hedge to be capital efficient is $CE^M(H) > 0$, or simply, the capital savings exceed the hedging costs. *H1* is more capital efficient than *H2* if $CE^M(H1) > CE^M(H2)$.

4.7.3 Hedge effectiveness

We can define the hedge effectiveness of a hedge as the relative reduction of longevity risk under a risk measure ρ as follows:

$$HE_\rho^M(H) := 1 - \frac{\rho(\bar{\Pi}_{L_H}^M)}{\rho(\bar{\Pi}_L^M)}$$
$$:= 1 - \frac{\rho\left(\Pi_{L_H}^M - \mathbb{E}\left(\Pi_{L_H}^M\right)\right)}{\rho\left(\Pi_L^M - \mathbb{E}(\Pi_L^M)\right)}, \qquad M \in \{IM, SF\} \qquad (70)$$

Where a perfect hedge yields the maximum hedge effectiveness of one. *H1* is more effective hedge than *H2* if $HE_\rho^M(H1) > HE_\rho^M(H2)$. It is important to mention that defining hedge effectiveness in this way considers the reduction in uncertainty regarding the future cost of capital.

Hedge effectiveness and capital efficiency are essential factors that can help insurers decide the optimal hedging strategy in a given situation. For example, high hedge effectiveness might reduce SCR significantly but can be low in capital efficiency when the hedge provider charges a non-zero premium for taking on the risk. On the other hand, capital-efficient hedges might offer a lower reduction in SCR, making them less hedge effective. The insurance company must, therefore, find an optimal trade-off between hedge effectiveness and capital efficiency to achieve the hedging objective, which can differ depending on the specific insurance company (Borger, Freiman & Ruß, 2021).

Meyricke and Sherris (2014) have shown that by using longevity swaps, the cost of hedging for earlier ages is lower than the cost of capital required under Solvency II. On the other hand, using longevity swaps at ages higher than 90 results in higher costs than savings in regulatory capital costs. “The Solvency II capital regulations for longevity risk generates an incentive for life insurers to hold longevity tail risk on their own balance sheets, rather than transferring this to the reinsurance or the capital markets.” (Meyricke & Sherris, 2014, p. 154).

Cairns and Boukfaoui (2021) have researched the regulatory capital relief in the presence of basis risk and concluded that even though index-based hedges suffer from this drawback, their lower hedging costs have a positive impact where the loss of hedge effectiveness due to population basis risk is close to zero under Solvency II regulation meaning there is no difference between index-based and customized hedge under this regime.

Borger, Freimann and Ruß (2021) argue that different hedging instruments have structurally different impacts on the hedger's economic capital. They also consider the difference in capital relief resulting from using the standard formula in Solvency II with one-off longevity stress and a risk-based internal model, as indicated in the abovementioned equations. They conclude that the Solvency II standard formula overestimates the efficiency and assigns relatively high capital reliefs. They also show that hedge effectiveness can change if the uncertainty of future capital charges for longevity risk is considered.

Market-based solutions such as longevity derivatives can help insurers in longevity risk mitigation but suffer from various drawbacks, which is why this market has not developed as initially expected. Insurers can also use insurance-based solutions where insurers and reinsurers act as hedge providers. These solutions have been dominant in longevity risk transfer and will be discussed next.

5 LONGEVITY RISK MITIGATION USING REINSURANCE

In the previous chapter, we have seen three of the most popular capital market solutions in managing longevity risk (longevity bond, swap and a *q*-forward). However, those longevity derivatives are not the only tools insurance companies and pension providers can use to manage this risk. There are also insurance-based solutions where an insurance or reinsurance company acts as a hedge provider instead of other market participants. This leads to significant implications given that in most jurisdictions, investment banks and other financial entities are not allowed to issue or take on longevity risk in the form of annuities. This means that hedging strategies, which I will discuss in this chapter, such as pension buy-outs and buy-ins, are possible only when the hedge provider is either an insurance or reinsurance company. This is because those companies follow a stricter regulation than entities such as investment banks and have higher capital requirements, which makes them suitable to carry the longevity risk of annuities directly. In contrast, other financial entities can only do it indirectly through the derivatives discussed earlier. In this chapter, I will focus on the abovementioned insurance-based solutions where reinsurance companies act as hedge providers (Bank for International Settlements, 2013, pp. 1-5).

5.1 Introduction to longevity risk mitigation with reinsurance

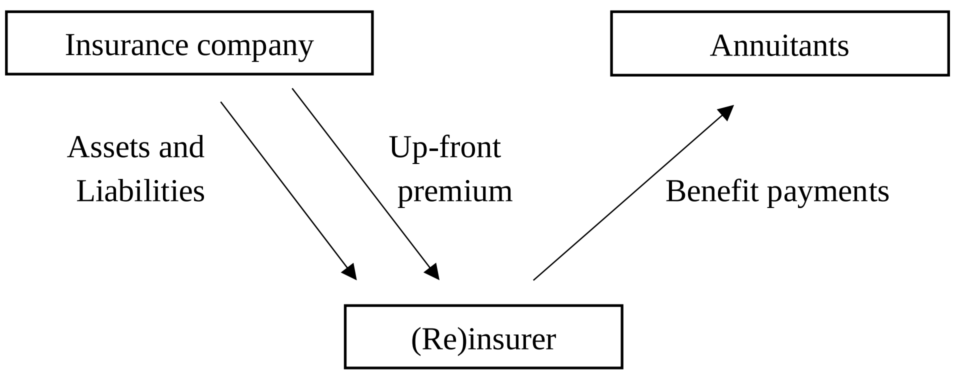
An essential distinction between hedging solutions involving reinsurance companies as hedge providers and longevity derivatives where other financial entities can replace this role is in the type of contract between the two counterparties. We have financial contracts when we talk about longevity derivatives, where hedgers are investment banks or other non-insurance-related entities. However, we have an insurance contract when the reinsurance takes the longevity risk. This structuring is essential and has implications for an applicable regulatory regime concerning insurance and financial contracts, which are regulated under different laws. This is especially important when considering capital release in Solvency II regulation (Society of Actuaries, 2014).

The legal form of the contract creates significant differences between longevity derivatives and a reinsurance contract, where the latter is recognized differently by the regulator, which leads to a capital release under the Solvency II regulation. As discussed earlier, this has not always been the case with longevity derivatives. Therefore, regulatory acceptance is one of the major advantages of using reinsurance as a longevity hedge. For example, we can look at the recent longevity reinsurance between Aegon and Reinsurance Group of America (RGA) in 2021, where 7 billion EUR of Dutch pension liabilities was transferred from Aegon to the RGA. This reinsurance protects Aegon against the potential financial impact of longevity improvement over the entire life of the policies. The benefit of this reinsurance is expected to be a lower Solvency II capital requirement of around 15% points for the Dutch business or 5% points on a Group's level (Aegon, 2021). When the risk is transferred via a reinsurance agreement, these reinsurers are usually large global companies which hedge their longevity risk against their mortality risk portfolios. This is one of the reasons why the global reinsurance industry has been leading in developing products for pension plans and insurance companies that try to hedge longevity risk (Society of Actuaries, 2014).

5.2 Buy-outs

The traditional insurance-based solution for dealing with unwanted longevity risk is the sale of all or a part of annuity books to a reinsurer. This is also known as a pension buy-out or bulk annuity transfer and is usually used by pension providers and insurance companies that provide annuities. This hedging solution removes the pension or annuity liability from the insurance company's balance sheet. Buy-outs do not only hedge the longevity risk, as was the case with longevity derivatives, as they involve transferring both assets and liabilities to the reinsurer. This contract is usually settled with an upfront premium. In this transaction, the reinsurer takes full responsibility for making payments to pensioners or annuitants, as seen in the next figure.

Figure 16: Buy-out structure

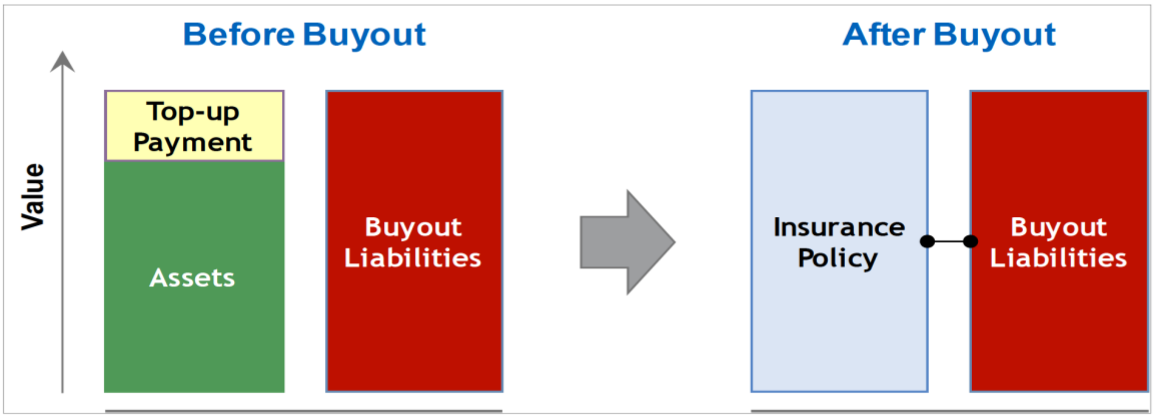


Source: Bank for International Settlements (2013, p. 6).

The figure above represents a buy-out structure where the insurer or reinsurer takes over full responsibility for making payments to the annuitants. The advantage of buy-outs is that annuity liabilities are entirely removed from the insurance company's balance sheet, which provides a hedge for other risks such as investment, interest rate, inflation and sometimes operational risk, which are then transferred along with the longevity risk. (Bank for International Settlements 2013). Even though this agreement is usually paid up by the insurance company with an up-front premium, it can also be settled with a loan which is (unlike fluctuating annuity liabilities that depend on future longevity) readily understood by investment analysts and shareholders. Using this hedging structure enables the insurance company to avoid or decrease the volatility in the profit and loss account due to some of the annuity liabilities no longer being present on the company's balance sheet (Blake, Cairns, Dowd & Kessler, 2019, p. 9).

The reason why only insurance and reinsurance companies can participate in this agreement has already been addressed and is also visible from figure 16 above, where the hedge provider ((re)insurer) is responsible for benefit payments made to annuitants, which has not been the case with longevity derivatives. In addition, because there is a potential risk that the hedge provider becomes insolvent (counterparty risk), in which case the pensioners or annuitants could be left with no income, it is imperative that those hedge providers are heavily regulated entities and maintain sufficient capital levels, which is the case for insurance and reinsurance companies. A buy-out is more expensive than market-based solutions due to insurance and reinsurance companies being subject to more stringent regulation than other financial entities on the risk transfer market. The buy-out liability is, therefore, typically more extensive than the accounting liability under International Financial Reporting Standards (IFRS) or Generally Accepted Accounting Principles (GAAP) used in the U.S. as it reflects higher and often more realistic longevity assumptions, expenses and risk premium for taking over investment, inflation, interest rate and longevity risks. According to one U.K. pension consultant, the 2011 buy-out premium was approximately 15% of the accounting liability for pensioners and 25% for non-pensioners (Coughlan, Blake, McMinn, Cairns & Dowd, 2013, p. 9).

Figure 17: Buy-out effect on the insurer's balance sheet



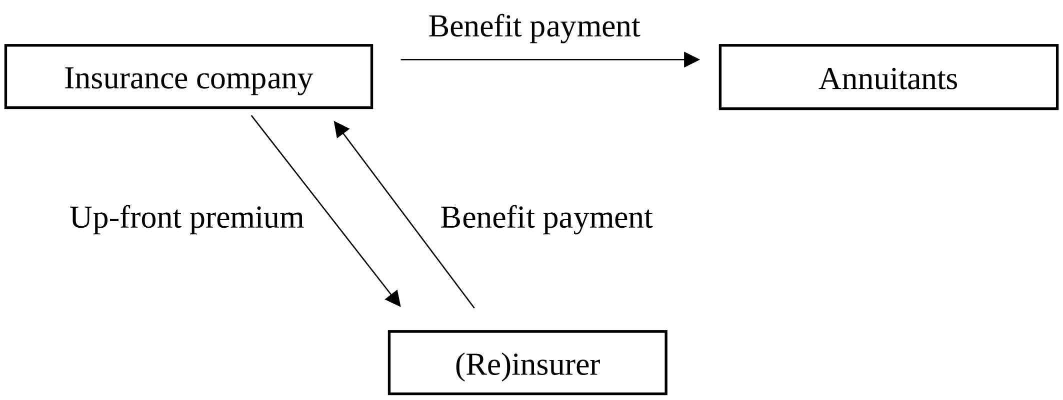
Adapted from Coughlan, Blake, McMinn, Cairns & Dowd (2013, p.16).

The figure above represents the difference between the buy-out liabilities, which reflects more realistic longevity assumptions and insufficient assets for the reinsurer to take over this risk. For this reason, a top-up payment or premium needs to be paid for a buy-out to happen. One of the main disadvantages of this structure is its sensitivity to timing due to interest rates. A buy-out price depends on the level of interest rates when the policy is signed. If the long-term interest rates increase in the future, this leads to a higher discount rate used to value annuity liabilities; hence the value of those liabilities would decrease, as would the price of the hedge. This means that the cost of hedging longevity in this way dramatically depends on timing and the associated interest rate at the time of agreement which makes it difficult for insurers and pension providers to decide when it is optimal to enter into such a contract due to unknown and often volatile future interest rates. For example, with the recent trend of higher interest rates, these agreements can be purchased for a lower premium than two or three years ago when interest rates were significantly lower than today. The timing of entering into such an agreement is, therefore, essential and must be chosen with due diligence since those contracts are usually non-renegotiable after the deal is complete (Coughlan, Blake, McMinn, Cairns & Dowd, 2013).

5.3 Buy-ins

Another insurance-based solution for managing longevity is a buy-in structure. When an insurance company or a pension fund enters into a buy-in, it purchases a bulk of annuities to hedge the risks associated with their own annuity or pension liabilities. The annuities purchased, therefore, become an asset and cover specific mortality characteristics of the portfolio of annuitants or pensioners regarding age, gender and size of payments. This hedging strategy can eliminate the issue of timing that buy-outs suffer from since they can be purchased in phases, enabling the insurance company to smooth out annuity rates over time and avoid spikes in pricing when it decides to proceed with the buy-out (Blake, Cairns, Dowd & Kessler, 2019, p. 9).

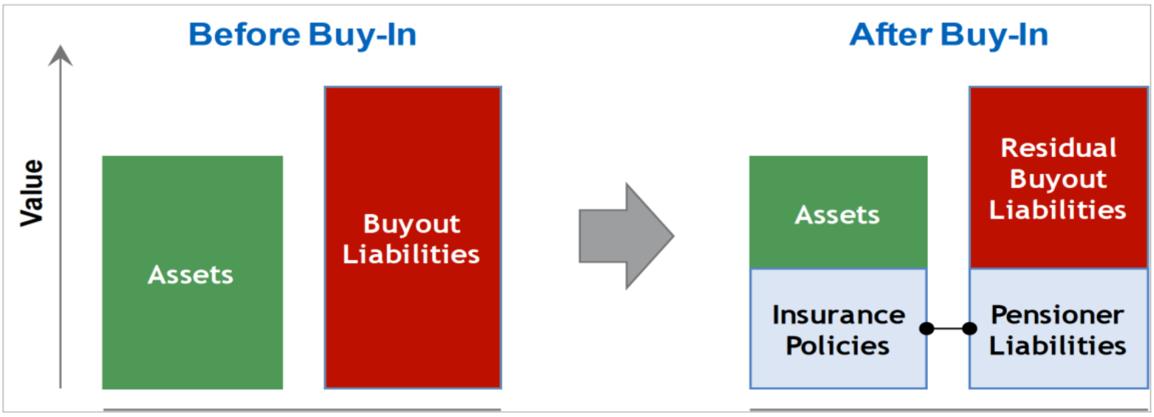
Figure 18: Buy-in structure



Source: Bank for International Settlements (2013, p. 6)

Figure 18 represents the structure of a buy-in where the insurance company pays an up-front premium to the reinsurer, who then makes periodic payments to the insurance company, which are equal to those made by the insurer to annuitants. This policy is therefore held as an asset by the insurance company, which guarantees payments even if annuitants have a longer life than expected (Bank for International Settlements, 2013, p. 6). The difference between a buy-in and a buy-out can be more evident when looking at the insurance company's balance sheet, as seen in the next figure.

Figure 19: Buy-in effect on the insurer's balance sheet



Adapted from Coughlan, Blake, McMinn, Cairns & Dowd (2013, p.17).

Figure 19 shows that the buy-in liabilities and buy-in assets remain on the insurance company's balance sheet. This structure offers an insurance company or pension provider a complete hedge of a portion of the pension liabilities for a much lower or even zero up-front cash payment relative to a buy-out. Because the annuity or pension liability remains on the company's balance sheet and is not transferred to the reinsurer, as was the case with buy-outs, the insurer remains responsible for making payments to annuitants or pensioners and assures those with benefit payments it receives from the reinsurer. However, the annuitants or pensioners are still exposed to the risk of the reinsurer's insolvency indirectly if the buy-in deal has not been fully collateralized (Coughlan, Blake, McMinn, Cairns & Dowd, 2013, pp. 9-10; Blake, Cairns, Dowd & Kessler, 2019, p. 10).

The most common arrangements for transferring the longevity risk in the past have been buy-outs and buy-ins. These structures maximize the risk transfer from the insurance company to the reinsurer. This hedge's effectiveness, however, comes with the price where these structures are usually more expensive than longevity derivatives. Nevertheless, even though their price is higher, their acceptance by the regulator and, consequently, a lower SCR still make them a viable solution for managing longevity risk. This is why they have dominated the longevity risk transfer market in the past (OECD, 2014, p. 177). Solvency II regulation which came into force in January 2016, led to higher capital requirements for insurers and reinsurers, which has many positive effects since it leads to a more financially stable and secure insurance sector. Consequently, hedges, where insurance entities take over longevity risk, are more reliable due to a lower counterparty risk arising from stricter regulation, as discussed previously. However, this means risk transfer where reinsurers take over longevity risk will be more expensive due to the higher capital required by reinsurers. This represents an issue because the high capital constraint imposed by the regulation limits the capacity of reinsurers to mitigate the longevity risk. The main problem of using traditional reinsurance for transferring longevity risk is, therefore, the capacity to which reinsurers can underwrite this business. Furthermore, due to the enormous size of the annuity liabilities and longevity risk in general, reinsurers cannot collectively manage these risks alone. Therefore, a popular solution to increase the capacity of reinsurers for taking on a larger size of longevity risk is using reinsurance sidecars (D'Amato, Di Lorenzo, Haberman, Sagoo & Sibillo, 2018, p. 125).

5.4 Reinsurance sidecars

Because of the enormous size of annuity liabilities and longevity risk in general, insurers and reinsurers cannot collectively manage these risks alone. Traditional reinsurance cannot be effectively used for longevity risk mitigation due to the limited financial capacity to satisfy the risk protection demand fully. For this reason, reinsurance sidecars were developed to increase the capacity and capability of managing the longevity risk and to open the longevity risk transfer market discussed earlier to a broader spectrum of investors and participants. This cooperation of private investors and reinsurers through sidecars can provide an adequate capacity for longevity risk mitigation and suggests that market and insurance-based solutions must come together.

5.4.1 The development of reinsurance sidecars

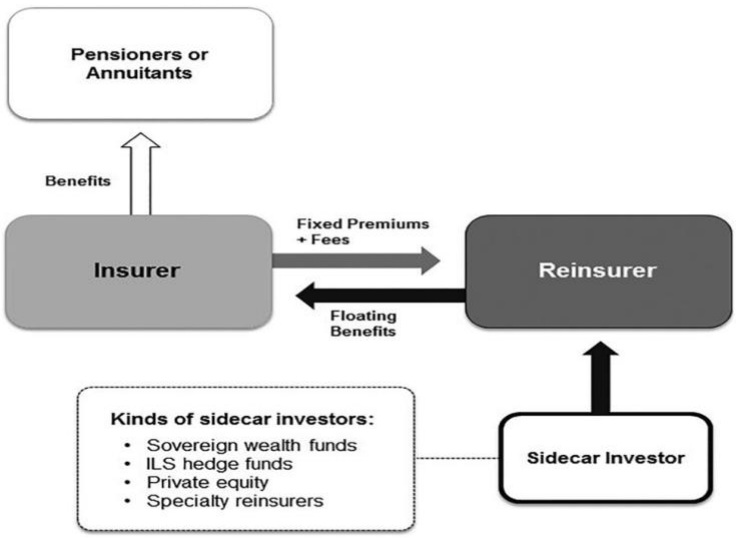
A sidecar is a term which denotes the concept of a separate entity which is a part of the main reinsurance company and can be easily removed once no longer needed. Sidecars were developed to supply additional short-term capital to traditional reinsurance companies and provide coverage to their clients without additional capital requirements. They were first successfully applied in the U.S. natural catastrophe market, where private investors provided the reinsurance companies with the capital needed for coverage and exposed themselves to the property-catastrophe insurance risk. This means sidecars enabled private investors to invest directly in the insurance risk of their choice. Even though sidecars were developed to provide additional short-term capital, many investors have typically reinvested or rolled over their investments from year to year; this is because funds that invest in sidecars are managed by sophisticated investors with extensive knowledge of the insurance market and can engage in a dialogue with the corresponding reinsurer regarding the expected returns. As a result, in times of low investment yields in other markets, those investors can achieve better returns on reinsurance risks than other investments, leading to a longer-term relationship between investors and reinsurance companies. Furthermore, these insurance risks are desirable to investors since they are not correlated with other risks facing their portfolios. This opens up possibilities of using sidecars in the longevity risk transfer market where risk is long-term.

Reinsurance sidecars are special purpose vehicles (SPVs) which usually have to be fully funded (the maximum amount of payments, including expenses that may occur, cannot exceed its assets). One of the significant benefits of reinsurance sidecars is that Solvency II regulation provides a modified and lighter regulation for SPVs that are fully funded. However, because the usage of reinsurance sidecars was usually short term and it is often challenging to find investors who are willing to invest for a period of 30 or 40 years, the reinsurer will need to be sufficiently well capitalized to bear the exposure that exceeds the limit of liability under the reinsurance agreement using a sidecar as well as to cover the risks which exist after the maturity of a sidecar arrangement. Regardless, there will still be benefits to using a sidecar in a long-term transaction due to a greater capacity of a reinsurer to cover longevity risks and offer the hedge for a lower price due to no additional capital requirement when the sidecar is fully funded (Bugler, Maclean, Nicenko & Tedesco, 2020).

5.4.2 A typical reinsurance sidecar structure

The typical reinsurance sidecar structure in the figure below offers many advantages. In many transactions, this structure enables private investors to take a proportionate share of profits and losses alongside a reinsurer with its own capital at risk under the same conditions. These structures, therefore, enable private investors to co-invest in longevity risk alongside insurers and reinsurers.

Figure 20: Typical reinsurance sidecar structure



Adapted from Kessler (2021, p. 22).

Given that market-based solutions have been slow to materialize in practice, sidecar arrangements may be a preferred strategy going forward. Private and third-party investors who would not have been able to take on longevity risk can partner up with existing reinsurers and benefit from the return of specific books of insurance and reinsurance business. This offers excellent opportunities to these investors who can benefit from this venture without insurance or actuarial expertise by relying on big data on historical mortality experience that reinsurers use to analyze these opportunities (Kessler, 2021).

On the other hand, the reinsurers would benefit from additional funds to increase the reinsurance capacity without needing additional permanent capital. This means reinsurers could leverage their underwriting expertise which would provide an additional source of income.

5.4.3 Drivers and issues of reinsurance sidecar development

One of the main reasons why these structures are more relevant now than in the past goes back to the Solvency II regulation. After its implementation in 2016, the reinsurers have had increasingly higher capacity constraints meaning they cannot take advantage of all the longevity risk opportunities in the insurance and pension market. The only long-term solution to this capacity constraint is to bring in new investors from the capital markets.

These investors can be hedge funds, private equity investors, sovereign wealth funds, endowments, family offices and other investors who seek asset classes with low correlation to their existing portfolio. Even though this structure provides a solution to the limited capacity of reinsurers, two issues need to be resolved for its future development. Firstly, the hedger needs some assurance that the solution sold to investors using sidecars provides an effective hedge. Secondly, these investors need proof that they are not sold a “lemon”, which can be done by the reinsurer having some “skin in the game” by agreeing to share risks with the sidecar investors (Blake, Cairns, Dowd & Kessler 2019, pp. 66-67). It is also important that investors participating in this market are well educated and informed about potential dangers related to investing in insurance risks. This is crucial to prevent events such as the massive losses of Lloyds of London investors in the early 1990s, where most investors exposed to insurance risk lost their money due to fire on the Piper Alpha oil rig in the North Sea and asbestos claims; therefore, participants on this market need to have sufficient expertise related to insurance risks (Tuohy & Mulligan, 1994).

With the development of Solvency II and the corresponding limited capacity to which reinsurers can take on longevity risk, reinsurance sidecars are gaining momentum, and an early sign of the success of this structure came at the beginning of 2019 when RGA and ReinsuranceRe announced a new start-up named Langhorne Re that targets in-force life and annuity business. The company has secured $780 million of equity capital from RGA, ReinsuranceRe and third-party sidecar investors, including pension funds and other life companies as of 2019 (Blake, Cairns, Dowd & Kessler, 2019, pp. 66-67).

CONCLUSION

Increasing life expectancy in the last 100 years positively affects our lives, knowing we live longer; however, longevity improvement presents a challenge for annuity and pension providers, which have to evaluate and manage the financial consequences of an ageing population. Managing longevity risk is difficult since it does not demonstrate itself as quickly as financial risk, and has been consistently underestimated in the past. Perhaps the biggest challenge is the size of longevity risk exposure arising from annuity and pension liabilities which ranges between $60 and $80 trillion globally. The extent of exposure and the fact that each unexpected additional year of life at the age of 65 amounts to an increase in liabilities by 4-5% means that global liability arising from longevity risk can be more than $6 trillion higher if longevity improves faster than expected.

Insurance companies can forecast the development of future mortality rates by using different mortality models. The Lee-Carter model is the simplest model and comparing it to other extended models did not lead to major differences in terms of life expectancies for 65-year-olds. Because M7 and APC model uses cohort effects, their residuals were spread more randomly, which was expected. The AIC and BIC criteria are close in all the models, which can explain why the original Lee-Carter model remains popular in practice since it is the simplest of them all and easy to interpret. Even though these forecasting models give us some insight into future mortality development, their predictions can never be certain and depend on the population and timeframe of the data we are working with. While existing literature focuses on mortality forecasting and pricing longevity derivatives separately, this thesis aims to describe their relationship and how predictions obtained by forecasting models can later be transformed to price longevity derivatives.

Because we cannot rely on mortality forecasting models with sufficient certainty, insurance companies cannot manage longevity risk with this tool alone. Unexpected increases in longevity still represent a risk for which insurers in the EU need to set aside additional SCR. To decrease the amount of SCR arising from longevity risk, insurers can also use insurance and market-based solutions to transfer the risk.

Insurance-based solutions have dominated the longevity risk transfer market since insurers and reinsurers follow stricter regulation. The regulator recognizes the transfer of annuities through a buy-out or purchasing bulk annuities in a buy-in, which leads to a capital release under Solvency II regulation. Even though these solutions have been successful in the past, one of their disadvantages is the capacity to which insurance and reinsurance companies can take on longevity risk.

Market-based solutions were proposed to achieve the same result for a lower price. Even though these solutions are more capital efficient, the market for them did not grow as anticipated. This failure is due to the tradeoff between a liquid market and basis risk. In pursuit of a liquid market, we need a lot of standardized contracts, which leads to basis risk where the base population under which we price a specific product might differ from the population to which a particular insurance company is exposed to. When the objective is to eliminate basis risk, we have many standardized contracts, leading to lower liquidity. Another reason for failure is regulatory in nature, where usage of some longevity derivatives, such as the one proposed by BNP Paribas, was not recognized favorably by the regulator and did not lead to a capital relief.

Even though both market and insurance-based solutions have flaws outlined in this work, they can eliminate some of them by working together through a reinsurance sidecar. This structure enables participants in the capital markets to invest in longevity risk. These investments increase reinsurers' capacity, which has the capital required to take on more considerable longevity risk. By managing longevity risk in this manner, insurers do not suffer the basis, liquidity and regulatory risk related to longevity derivatives. Questions such as: "How to provide the hedger with the assurance that the solution sold to investors using sidecars provides an effective hedge?" and "Does using sidecars to take on more longevity risk lead to the emergence of new risks?" still remain open for future research.

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APPENDICES

Appendix 1: Povzetek vsebine (Summary in Slovene language)

Pričakovana življenska doba se v zadnjih letih konstantno podaljšuje. Ta trend pozitivno vpliva na življenje vseh, vendar pa morajo zavarovalnice, ki ponujajo rente, oceniti finančne posledice staranja prebivalstva in posledično izplačevanja večjega števila rent. Z vidika zavarovalnic predstavlja nepričakovano izboljšanje življenske dobe tveganje, ki ga je potrebno nadzirati in čim bolj zmanjšati. Svetovne obveznosti iz naslova izplačevanja rent in pokojnin znašajo med $60 in $80 biljonov, vsako nepričakovano dodatno preživeto leto človeka pa poveča obveznost za 4-5%. Po nekaterih študijah so lahko te obveznosti za okoli $6 biljonov večje, če se v prihodnje življenska doba podaljša nad pričakovanji.

S tem namenom sem v tem magistrskem delu preučeval vpliv dolgoživosti na zavarovalnice in načine kako se te pred tem zaščitijo. Sprva sem opisal problematiko dolgoživosti z vidika zavarovalnic, nato pa modele, ki se uporabljajo za napovedovanje smrtnosti v prihodnost. Lee-Carterjev model, ki zaradi svoje popularnosti med modeli smrtnosti služi kot merilo, sem primerjal z nekaterimi novejšimi modeli, ter prišel do ugotovitev, da med rezultati različnih modelov ne prihaja do velikih razlik. Bolj kompleksna modela kot sta APC in M7 model, imata sicer ostanke bolj neodvisne od kohort, kar je zaželjeno, a je zaradi njune kompleksnosti in težje interpretacije Lee-Carterjev model še vedno primeren in posledično pogosto uporabljan v slovenskih zavarovalnicah.

V naslednjih poglavjih sem se posvetil raziskovanju uporabe izvedenih finančnih instrumentov pri zmanjševanju tveganja dolgoživosti. Najprej sem predstavil problematiko vrednotenja teh instrumentov in predstavil Wangovo transformacijo kot način, ki nam to omogoča. Nadaljeval sem s predstavitvijo treh izvedenih finančnih instrumentov (longevity bonds, q-forwards in longevity swaps). V predstavitvi sem se osredotočil na strukturo inštrumenta, izračun njegove cene in ga ponazoril na hipotetičnem primeru. Z raziskovanjem sem prišel do ugotovitev, da se trg ni razvil po pričakovanjih, predvsem zaradi slabe likvidnosti in neprepoznanje teh produktov s strani regulatorja za zmanjšanje zahtevanega solventnostnega kapitala.

V delu sem se na koncu osredotočil tudi na bolj tradicionalne metode zmanjševanja tveganja zavarovalnic z uporabo pozavarovanja. Tudi ta način ima omejitev, ki pa je v obsegu, v katerem so pozavarovalnice zmožne sprejeti tveganje dolgoživosti. Z razvojem novih posebnih oblik pozavarovanja, ki vključujejo tudi ostale investitorje na trgu (“reinsurance sidecars”) pa se ta slabost zmanjša, saj imajo pozavarovalnice več kapitala, ki jim omogoča sprejemati večji obseg tveganja dolgoživosti.

Appendix 2: R-code used for stochastic mortality modelling

```
#we start by downloading the following packages
install.packages("StMoMo")
install.packages("Demography")
install.packages("Lifecontingencies")
#we activate them
library(StMoMo)
library(demography)
library(lifecontingencies)
#we get the data from human mortality database: https://www.mortality.org
#we install Slovenian data for population size and death rates in .txt
#we delete the 2020 data in population .txt file since both population and
mortality need to have the same data size
#we import the data set in the global environment via .txt
#we obtain the data by using the package demography
SVNdata <- read.demogdata("Mx_1x1.txt","Population.txt", type="mortality",
label="SVN")
#now we have the data to work with
#Firstly we will plot the data to know what we are working with
plot(SVNdata, series = "total")
plot(SVNdata, series = "female")
plot(SVNdata, series = "male")
```

#1. THE LEE CARTER MODEL

```
#we define the model
LC_model <- lc()
#we fit the model to the data
SLO_Female <- StMoMoData(SVNdata, series = "female")
ages_fit <- 50:90
LC_model_fit_Female <- fit(LC_model, data = SLO_Female, ages.fit = ages_fit)
plot(LC_model_fit_Female)
#we look the goodness of fit trough the residuals
LC_model_residuals_Female <- residuals(LC_model_fit_Female)
plot(LC_model_residuals_Female, type = "colourmap", reslim = c(-3.5, 3.5))
plot(LC_model_residuals_Female, type ="scatter", reslim = c(-3.5, 3.5))
AIC(LC_model_fit_Female)
BIC(LC_model_fit_Female)
#now we start with forecasting
LC_model_forecast_Female <- forecast(LC_model_fit_Female, h = 35)
plot(LC_model_forecast_Female)
#simulation of different mortality projections
LC_model_sim_Female <- simulate(LC_model_fit_Female, nsim = 1000, h=35)
library(fanplot)
mxt <- LC_model_fit_Female$Dxt/LC_model_fit_Female$Ext
probs <- c(2.5, 10, 25, 50, 75, 90, 97.5)
qxt <- LC_model_fit_Female$Dxt/LC_model_fit_Female$Ext
matplot(LC_model_fit_Female$years, t(qxt[c("65", "75", "85"), ]), xlim = c(1983,
2055), ylim = c(0.0025, 0.2), pch = 20, col = "black", log = "y", xlab = "year",
ylab = "Female mortality rate (log scale)")
fan(t(LC_model_sim_Female$rates["65",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("black", "white")), ln = NULL)
fan(t(LC_model_sim_Female$rates["75",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("red", "white")), ln = NULL)
fan(t(LC_model_sim_Female$rates["85",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("blue", "white")), ln = NULL)
text(1985, qxt[c("65", "75", "85"), "1990"], labels = c("x = 65", "x = 75", "x =
85"))
#now we calculate life expectancy
chosen_cohort_Female <- 1954
LC_historical_rates_Female <- extractCohort(SLO_Female$Dxt/SLO_Female$Ext, cohort
= chosen_cohort_Female)[1:37]#observed values
LC_forecasted_rates_Female <- extractCohort(LC_model_forecast_Female$rates,
cohort = chosen_cohort_Female)
LC_54_cohort_rates_Female <- c(LC_historical_rates_Female,
LC_forecasted_rates_Female)
```

```
plot(29:90, LC_54_cohort_rates_Female, type = "l", log = "y", xlab = "age", ylab
= "m(x)", main = "Female cohort 1954 mortality rate")
lines(66:90, LC_forecasted_rates_Female, col = "red")
LC_mortality_rate_1954_Female <- mx2qx(LC_54_cohort_rates_Female)
LC_lifetable_1954_Female <- probs2lifetable(probs =LC_mortality_rate_1954_Female,
type = "qx", name = "LC-1954_Female") #We can obtain the life table and export it
in excel
exn(LC_lifetable_1954_Female, x = 36, type = "curtate") #life expectancy of a
female born in 1954 in 2019 under LC, age = 65 = (29+36)-data starts at 29
#We do the same for males
LC_model <- lc()
#we fit the model to the data
SLO_Male <- StMoMoData(SVNdata, series = "male")
ages_fit <- 50:90
LC_model_fit_Male <- fit(LC_model, data = SLO_Male, ages.fit = ages_fit)
plot(LC_model_fit_Male)
#we look the goodness of fit trough the residuals
LC_model_residuals_Male <- residuals(LC_model_fit_Male)
plot(LC_model_residuals_Male, type = "colourmap", reslim = c(-3.5, 3.5))
plot(LC_model_residuals_Male, type ="scatter", reslim = c(-3.5, 3.5))
AIC(LC_model_fit_Male)
BIC(LC_model_fit_Male)
#now we start with forecasting
LC_model_forecast_Male <- forecast(LC_model_fit_Male, h = 35)
plot(LC_model_forecast_Male)
#simulation of different mortality projections
LC_model_sim_Male <- simulate(LC_model_fit_Male, nsim = 1000, h=35)
library(fanplot)
mxt <- LC_model_fit_Male$Dxt/LC_model_fit_Male$Ext
probs <- c(2.5, 10, 25, 50, 75, 90, 97.5)
qxt <- LC_model_fit_Male$Dxt/LC_model_fit_Male$Ext
matplot(LC_model_fit_Male$years, t(qxt[c("65", "75", "85"), ]), xlim = c(1983,
2049), ylim = c(0.0025, 0.2), pch = 20, col = "black", log = "y", xlab = "year",
ylab = "Male mortality rate (log scale)")
fan(t(LC_model_sim_Male$rates["65",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("black", "white")), ln = NULL)
fan(t(LC_model_sim_Male$rates["75",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("red", "white")), ln = NULL)
fan(t(LC_model_sim_Male$rates["85",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("blue", "white")), ln = NULL)
text(1985, qxt[c("65", "75", "85"), "1990"], labels = c("x = 65", "x = 75", "x =
85"))
#now we calculate life expectancy
chosen_cohort_Male <- 1954
LC_historical_rates_Male <- extractCohort(SLO_Male$Dxt/SLO_Male$Ext, cohort =
chosen_cohort_Male)[1:37]#observed values
LC_forecasted_rates_Male <- extractCohort(LC_model_forecast_Male$rates, cohort =
chosen_cohort_Male)
LC_54_cohort_rates_Male <- c(LC_historical_rates_Male, LC_forecasted_rates_Male)
plot(29:90, LC_54_cohort_rates_Male, type = "l", log = "y", xlab = "age", ylab =
"m(x)", main = "Male cohort 1954 mortality rate")
lines(66:90, LC_forecasted_rates_Male, col = "red")
LC_mortality_rate_1954_Male <- mx2qx(LC_54_cohort_rates_Male)
LC_lifetable_1954_Male <- probs2lifetable(probs =LC_mortality_rate_1954_Male,
type = "qx", name = "LC-1954_Male")#We can obtain the life table and export it in
excel
exn(LC_lifetable_1954_Male, x = 36, type = "curtate") #life expectancy of a male
born in 1954 in 2019 under LC, age = 65 (29+36)
```

#2. THE CBD MODEL

```
#we define the model
CBD_model <- cbd(link = "log")
#we fit the model to the data
CBD_model_fit_Female <- fit(CBD_model, data=SLO_Female, ages.fit = ages_fit)
plot(CBD_model_fit_Female, parametricbx = FALSE)
#we look the goodness of fit trough the residuals
CBD_model_residuals_Female <- residuals(CBD_model_fit_Female)
plot(CBD_model_residuals_Female, type = "colourmap", reslim = c(-3.5, 3.5))
plot(CBD_model_residuals_Female, type ="scatter", reslim = c(-3.5, 3.5))
```

```
AIC(CBD_model_fit_Female)
BIC(CBD_model_fit_Female)
#now we start with forecasting
CBD_model_forecast_Female <- forecast(CBD_model_fit_Female, h = 35)
plot(CBD_model_forecast_Female, parametricbx = FALSE)
#simulation of different mortality projections
CBD_model_sim_Female <- simulate(CBD_model_fit_Female, nsim = 1000, h=35)
library(fanplot)
mxt <- CBD_model_fit_Female$Dxt/CBD_model_fit_Female$Ext
probs <- c(2.5, 10, 25, 50, 75, 90, 97.5)
qxt <- CBD_model_fit_Female$Dxt/CBD_model_fit_Female$Ext
matplot(CBD_model_fit_Female$years, t(qxt[c("65", "75", "85"), ]), xlim = c(1983,
2049), ylim = c(0.0025, 0.2), pch = 20, col = "black", log = "y", xlab = "year",
ylab = "Female mortality rate (log scale)")
fan(t(CBD_model_sim_Female$rates["65",,]), start = 2019, probs = probs, n.fan =
4, fan.col = colorRampPalette(c("black", "white")), ln = NULL)
fan(t(CBD_model_sim_Female$rates["75",,]), start = 2019, probs = probs, n.fan =
4, fan.col = colorRampPalette(c("red", "white")), ln = NULL)
fan(t(CBD_model_sim_Female$rates["85",,]), start = 2019, probs = probs, n.fan =
4, fan.col = colorRampPalette(c("blue", "white")), ln = NULL)
text(1985, qxt[c("65", "75", "85"), "1990"], labels = c("x = 65", "x = 75", "x =
85"))
#now we calculate life expectancy
chosen_cohort_Female <- 1954
CBD_historical_rates_Female <- extractCohort(SLO_Female$Dxt/SLO_Female$Ext,
cohort = chosen_cohort_Female)[1:37]#observed values
CBD_forecasted_rates_Female <- extractCohort(CBD_model_forecast_Female$rates,
cohort = chosen_cohort_Female)
CBD_54_cohort_rates_Female <- c(CBD_historical_rates_Female,
CBD_forecasted_rates_Female)
plot(29:90, CBD_54_cohort_rates_Female, type = "l", log = "y", xlab = "age", ylab
= "m(x)", main = "Female cohort 1954 mortality rate")
lines(66:90, CBD_forecasted_rates_Female, col = "red")
CBD_mortality_rate_1954_Female <- mx2qx(CBD_54_cohort_rates_Female)
CBD_lifetable_1954_Female <- probs2lifetable(probs
=CBD_mortality_rate_1954_Female, type = "qx", name = "CBD-1954_Female")#We can
obtain the life table and export it in excel
exn(CBD_lifetable_1954_Female, x = 36, type = "curtate") #life expectancy of a
female born in 1983 in 2019 under CBD
#we can do the same for male
#we define the model
CBD_model <- cbd(link = "log")
#we fit the model to the data
CBD_model_fit_Male <- fit(CBD_model, data=SLO_Male, ages.fit = ages_fit)
plot(CBD_model_fit_Male, parametricbx = FALSE)
#we look the goodness of fit trough the residuals
CBD_model_residuals_Male <- residuals(CBD_model_fit_Male)
plot(CBD_model_residuals_Male, type = "colourmap", reslim = c(-3.5, 3.5))
plot(CBD_model_residuals_Male, type ="scatter", reslim = c(-3.5, 3.5))
AIC(CBD_model_fit_Male)
BIC(CBD_model_fit_Male)
#now we start with forecasting
CBD_model_forecast_Male <- forecast(CBD_model_fit_Male, h = 35)
plot(CBD_model_forecast_Male, parametricbx = FALSE)
#simulation of different mortality projections
CBD_model_sim_Male <- simulate(CBD_model_fit_Male, nsim = 1000, h=35)
library(fanplot)
mxt <- CBD_model_fit_Male$Dxt/CBD_model_fit_Male$Ext
probs <- c(2.5, 10, 25, 50, 75, 90, 97.5)
qxt <- CBD_model_fit_Male$Dxt/CBD_model_fit_Male$Ext
matplot(CBD_model_fit_Male$years, t(qxt[c("65", "75", "85"), ]), xlim = c(1983,
2049), ylim = c(0.0025, 0.2), pch = 20, col = "black", log = "y", xlab = "year",
ylab = "Male mortality rate (log scale)")
fan(t(CBD_model_sim_Male$rates["65",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("black", "white")), ln = NULL)
fan(t(CBD_model_sim_Male$rates["75",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("red", "white")), ln = NULL)
fan(t(CBD_model_sim_Male$rates["85",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("blue", "white")), ln = NULL)
```

```
text(1985, qxt[c("65", "75", "85"), "1990"], labels = c("x = 65", "x = 75", "x =
85"))
#now we calculate life expectancy
chosen_cohort_Male <- 1954
CBD_historical_rates_Male <- extractCohort(SLO_Male$Dxt/SLO_Male$Ext, cohort =
chosen_cohort_Male)[1:37]#observed values
CBD_forecasted_rates_Male <- extractCohort(CBD_model_forecast_Male$rates, cohort
= chosen_cohort_Male)
CBD_54_cohort_rates_Male <- c(CBD_historical_rates_Male,
CBD_forecasted_rates_Male)
plot(29:90, CBD_54_cohort_rates_Male, type = "l", log = "y", xlab = "age", ylab =
"m(x)", main = "Male cohort 1954 mortality rate")
lines(66:90, CBD_forecasted_rates_Male, col = "red")
CBD_mortality_rate_1954_Male <- mx2qx(CBD_54_cohort_rates_Male)
CBD_lifetable_1954_Male <- probs2lifetable(probs =CBD_mortality_rate_1954_Male,
type = "qx", name = "CBD-1954_Male")#We can obtain the life table and export it
in excel
exn(CBD_lifetable_1954_Male, x = 36, type = "curtate") #life expectancy of a male
born in 1983 in 2019 under CBD
```

#3. THE AGE PERIOD COHORT (APC) MODEL

```
#we define the model
APC_model <- apc()
#we fit the model to the data
APC_model_fit_Female <- fit(APC_model, data = SLO_Female, ages.fit = ages_fit)
plot(APC_model_fit_Female, parametricbx = FALSE)
#we look the goodness of fit trough the residuals
APC_model_residuals_Female <- residuals(APC_model_fit_Female)
plot(APC_model_residuals_Female, type = "colourmap", reslim = c(-3.5, 3.5))
plot(APC_model_residuals_Female, type ="scatter", reslim = c(-3.5, 3.5))
AIC(APC_model_fit_Female)
BIC(APC_model_fit_Female)
#now we start with forecasting
APC_model_forecast_Female <- forecast(APC_model_fit_Female, h = 35, gc.order =
c(1,1,0), gc.include.constant = FALSE)
plot(APC_model_forecast_Female, parametricbx = FALSE)
#simulation of different mortality projections
APC_model_sim_Female <- simulate(APC_model_fit_Female, nsim = 1000, h=35)
library(fanplot)
mxt <- APC_model_fit_Female$Dxt/APC_model_fit_Female$Ext
probs <- c(2.5, 10, 25, 50, 75, 90, 97.5)
qxt <- APC_model_fit_Female$Dxt/APC_model_fit_Female$Ext
matplot(APC_model_fit_Female$years, t(qxt[c("65", "75", "85"), ]), xlim = c(1983,
2049), ylim = c(0.0025, 0.2), pch = 20, col = "black", log = "y", xlab = "year",
ylab = "Female mortality rate (log scale)")
fan(t(APC_model_sim_Female$rates["65",,]), start = 2019, probs = probs, n.fan =
4, fan.col = colorRampPalette(c("black", "white")), ln = NULL)
fan(t(APC_model_sim_Female$rates["75",,]), start = 2019, probs = probs, n.fan =
4, fan.col = colorRampPalette(c("red", "white")), ln = NULL)
fan(t(APC_model_sim_Female$rates["85",,]), start = 2019, probs = probs, n.fan =
4, fan.col = colorRampPalette(c("blue", "white")), ln = NULL)
text(1985, qxt[c("65", "75", "85"), "1990"], labels = c("x = 65", "x = 75", "x =
85"))
#now we calculate life expectancy
chosen_cohort_Female <- 1954
APC_historical_rates_Female <- extractCohort(SLO_Female$Dxt/SLO_Female$Ext,
cohort = chosen_cohort_Female)[1:37]#observed values
APC_forecasted_rates_Female <- extractCohort(APC_model_forecast_Female$rates,
cohort = chosen_cohort_Female)
APC_54_cohort_rates_Female <- c(APC_historical_rates_Female,
APC_forecasted_rates_Female)
plot(29:90, APC_54_cohort_rates_Female, type = "l", log = "y", xlab = "age", ylab
= "m(x)", main = "Female cohort 1954 mortality rate")
lines(66:90, APC_forecasted_rates_Female, col = "red")
APC_mortality_rate_1954_Female <- mx2qx(APC_54_cohort_rates_Female)
APC_lifetable_1954_Female <- probs2lifetable(probs
=APC_mortality_rate_1954_Female, type = "qx", name = "APC-1954_Female")#We can
obtain the life table and export it in excel
```

```
exn(APC_lifetable_1954_Female, x = 36, type = "curtate") #life expectancy of a
female born in 1983 in 2019 under APC
#we can do the same for males
#we define the model
APC_model <- apc()
#we fit the model to the data
APC_model_fit_Male <- fit(APC_model, data = SLO_Male, ages.fit = ages_fit)
plot(APC_model_fit_Male, parametricbx = FALSE)
#we look the goodness of fit trough the residuals
APC_model_residuals_Male <- residuals(APC_model_fit_Male)
plot(APC_model_residuals_Male, type = "colourmap", reslim = c(-3.5, 3.5))
plot(APC_model_residuals_Male, type ="scatter", reslim = c(-3.5, 3.5))
AIC(APC_model_fit_Male)
BIC(APC_model_fit_Male)
#now we start with forecasting
APC_model_forecast_Male <- forecast(APC_model_fit_Male, h = 35, gc.order =
c(1,1,0), gc.include.constant = FALSE)
plot(APC_model_forecast_Male, parametricbx = FALSE)
#simulation of different mortality projections
APC_model_sim_Male <- simulate(APC_model_fit_Male, nsim = 1000, h=35)
library(fanplot)
mxt <- APC_model_fit_Male$Dxt/APC_model_fit_Male$Ext
probs <- c(2.5, 10, 25, 50, 75, 90, 97.5)
qxt <- APC_model_fit_Male$Dxt/APC_model_fit_Male$Ext
matplot(APC_model_fit_Male$years, t(qxt[c("65", "75", "85"), ]), xlim = c(1983,
2049), ylim = c(0.0025, 0.2), pch = 20, col = "black", log = "y", xlab = "year",
ylab = "Male mortality rate (log scale)")
fan(t(APC_model_sim_Male$rates["65",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("black", "white")), ln = NULL)
fan(t(APC_model_sim_Male$rates["75",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("red", "white")), ln = NULL)
fan(t(APC_model_sim_Male$rates["85",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("blue", "white")), ln = NULL)
text(1985, qxt[c("65", "75", "85"), "1990"], labels = c("x = 65", "x = 75", "x =
85"))
#now we calculate life expectancy
chosen_cohort_Male <- 1954
APC_historical_rates_Male <- extractCohort(SLO_Male$Dxt/SLO_Male$Ext, cohort =
chosen_cohort_Male)[1:37]#observed values
APC_forecasted_rates_Male <- extractCohort(APC_model_forecast_Male$rates, cohort
= chosen_cohort_Male)
APC_54_cohort_rates_Male <- c(APC_historical_rates_Male,
APC_forecasted_rates_Male)
plot(29:90, APC_54_cohort_rates_Male, type = "l", log = "y", xlab = "age", ylab =
"m(x)", main = "Male cohort 1954 mortality rate")
lines(66:90, APC_forecasted_rates_Male, col = "red")
APC_mortality_rate_1954_Male <- mx2qx(APC_54_cohort_rates_Male)
APC_lifetable_1954_Male <- probs2lifetable(probs =APC_mortality_rate_1954_Male,
type = "qx", name = "APC-1954_Male")#We can obtain the life table and export it
in excel
exn(APC_lifetable_1954_Male, x = 36, type = "curtate") #life expectancy of a male
born in 1983 in 2019 under APC.
```

#4. THE M7 MODEL

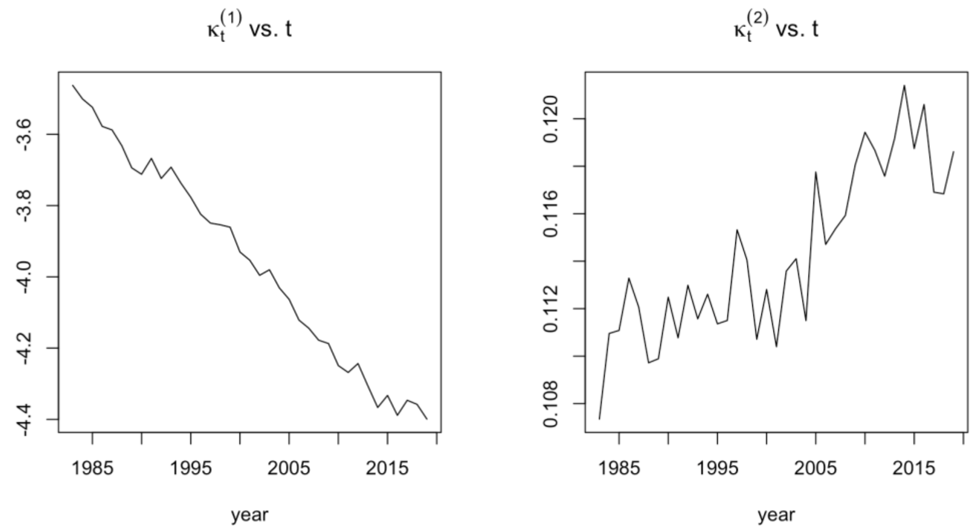
```
#we define the model
M7_model <- m7(link = "log")
#we fit the model to the data
M7_model_fit_Female <- fit(M7_model, data = SLO_Female, ages.fit = ages_fit)
plot(M7_model_fit_Female, parametricbx = FALSE)
#we look the goodness of fit trough the residuals
M7_model_residuals_Female <- residuals(M7_model_fit_Female)
plot(M7_model_residuals_Female, type = "colourmap", reslim = c(-3.5, 3.5))
plot(M7_model_residuals_Female, type ="scatter", reslim = c(-3.5, 3.5))
AIC(M7_model_fit_Female)
BIC(M7_model_fit_Female)

#now we start with forecasting
M7_model_forecast_Female <- forecast(M7_model_fit_Female, h=35, gc.order =
c(0,0,0), gc.include.constant = FALSE)
```

```
plot(M7_model_forecast_Female, parametricbx = FALSE)
#simulation of different mortality projections
M7_model_sim_Female <- simulate(M7_model_fit_Female, nsim = 1000, h=35)
library(fanplot)
mxt <- M7_model_fit_Female$Dxt/M7_model_fit_Female$Ext
probs <- c(2.5, 10, 25, 50, 75, 90, 97.5)
qxt <- M7_model_fit_Female$Dxt/M7_model_fit_Female$Ext
matplot(M7_model_fit_Female$years, t(qxt[c("65", "75", "85"), ]), xlim = c(1983,
2049), ylim = c(0.0025, 0.2), pch = 20, col = "black", log = "y", xlab = "year",
ylab = "Female mortality rate (log scale)")
fan(t(M7_model_sim_Female$rates["65",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("black", "white")), ln = NULL)
fan(t(M7_model_sim_Female$rates["75",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("red", "white")), ln = NULL)
fan(t(M7_model_sim_Female$rates["85",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("blue", "white")), ln = NULL)
text(1985, qxt[c("65", "75", "85"), "1990"], labels = c("x = 65", "x = 75", "x =
85"))
#now we calculate life expectancy
chosen_cohort_Female <- 1954
M7_historical_rates_Female <- extractCohort(SLO_Female$Dxt/SLO_Female$Ext, cohort
= chosen_cohort_Female)[1:37]#observed values
M7_forecasted_rates_Female <- extractCohort(M7_model_forecast_Female$rates,
cohort = chosen_cohort_Female)
M7_54_cohort_rates_Female <- c(M7_historical_rates_Female,
M7_forecasted_rates_Female)
plot(29:90, M7_54_cohort_rates_Female, type = "l", log = "y", xlab = "age", ylab
= "m(x)", main = "Female cohort 1954 mortality rate")
lines(66:90, M7_forecasted_rates_Female, col = "red")
M7_mortality_rate_1954_Female <- mx2qx(M7_54_cohort_rates_Female)
M7_lifetable_1954_Female <- probs2lifetable(probs =M7_mortality_rate_1954_Female,
type = "qx", name = "M7-1954_Female")#We can obtain the life table and export it
in excel
exn(M7_lifetable_1954_Female, x = 36, type = "curtate") #life expectancy of a
female born in 1983 in 2019 under M7.
#we can do the same for male
#we define the model
M7_model <- m7(link = "log")
#we fit the model to the data
M7_model_fit_Male <- fit(M7_model, data = SLO_Male, ages.fit = ages_fit)
plot(M7_model_fit_Male, parametricbx = FALSE)
#we look the goodness of fit trough the residuals
M7_model_residuals_Male <- residuals(M7_model_fit_Male)
plot(M7_model_residuals_Male, type = "colourmap", reslim = c(-3.5, 3.5))
plot(M7_model_residuals_Male, type ="scatter", reslim = c(-3.5, 3.5))
AIC(M7_model_fit_Male)
BIC(M7_model_fit_Male)
#now we start with forecasting
M7_model_forecast_Male <- forecast(M7_model_fit_Male, h=35, gc.order = c(0,0,0),
gc.include.constant = FALSE)
plot(M7_model_forecast_Male, parametricbx = FALSE)
#simulation of different mortality projections
M7_model_sim_Male <- simulate(M7_model_fit_Male, nsim = 1000, h=35)
library(fanplot)
mxt <- M7_model_fit_Male$Dxt/M7_model_fit_Male$Ext
probs <- c(2.5, 10, 25, 50, 75, 90, 97.5)
qxt <- M7_model_fit_Male$Dxt/M7_model_fit_Male$Ext
matplot(M7_model_fit_Male$years, t(qxt[c("65", "75", "85"), ]), xlim = c(1983,
2049), ylim = c(0.0025, 0.2), pch = 20, col = "black", log = "y", xlab = "year",
ylab = "Male mortality rate (log scale)")
fan(t(M7_model_sim_Male$rates["65",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("black", "white")), ln = NULL)
fan(t(M7_model_sim_Male$rates["75",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("red", "white")), ln = NULL)
fan(t(M7_model_sim_Male$rates["85",,]), start = 2019, probs = probs, n.fan = 4,
fan.col = colorRampPalette(c("blue", "white")), ln = NULL)
text(1985, qxt[c("65", "75", "85"), "1990"], labels = c("x = 65", "x = 75", "x =
85"))
#now we calculate life expectancy
```

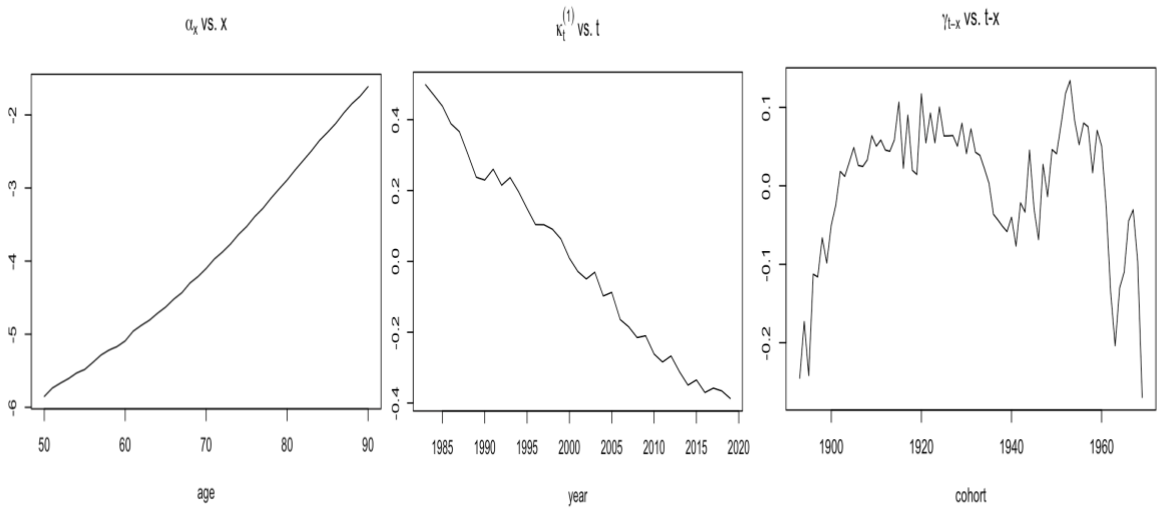
```
chosen_cohort_Male <- 1954
M7_historical_rates_Male <- extractCohort(SLO_Male$Dxt/SLO_Male$Ext, cohort =
chosen_cohort_Male)[1:37]#observed values
M7_forecasted_rates_Male <- extractCohort(M7_model_forecast_Male$rates, cohort =
chosen_cohort_Male)
M7_54_cohort_rates_Male <- c(M7_historical_rates_Male, M7_forecasted_rates_Male)
plot(29:90, M7_54_cohort_rates_Male, type = "l", log = "y", xlab = "age", ylab =
"m(x)", main = "Male cohort 1954 mortality rate")
lines(66:90, M7_forecasted_rates_Male, col = "red")
M7_mortality_rate_1954_Male <- mx2qx(M7_54_cohort_rates_Male)
M7_lifetable_1954_Male <- probs2lifetable(probs =M7_mortality_rate_1954_Male,
type = "qx", name = "M7-1954_Male")#We can obtain the life table and export it in
excel
exn(M7_lifetable_1954_Male, x = 36, type = "curtate") #life expectancy of a male
born in 1983 in 2019 under M7.
```

Appendix 3: Parameters of forecasting models (males)

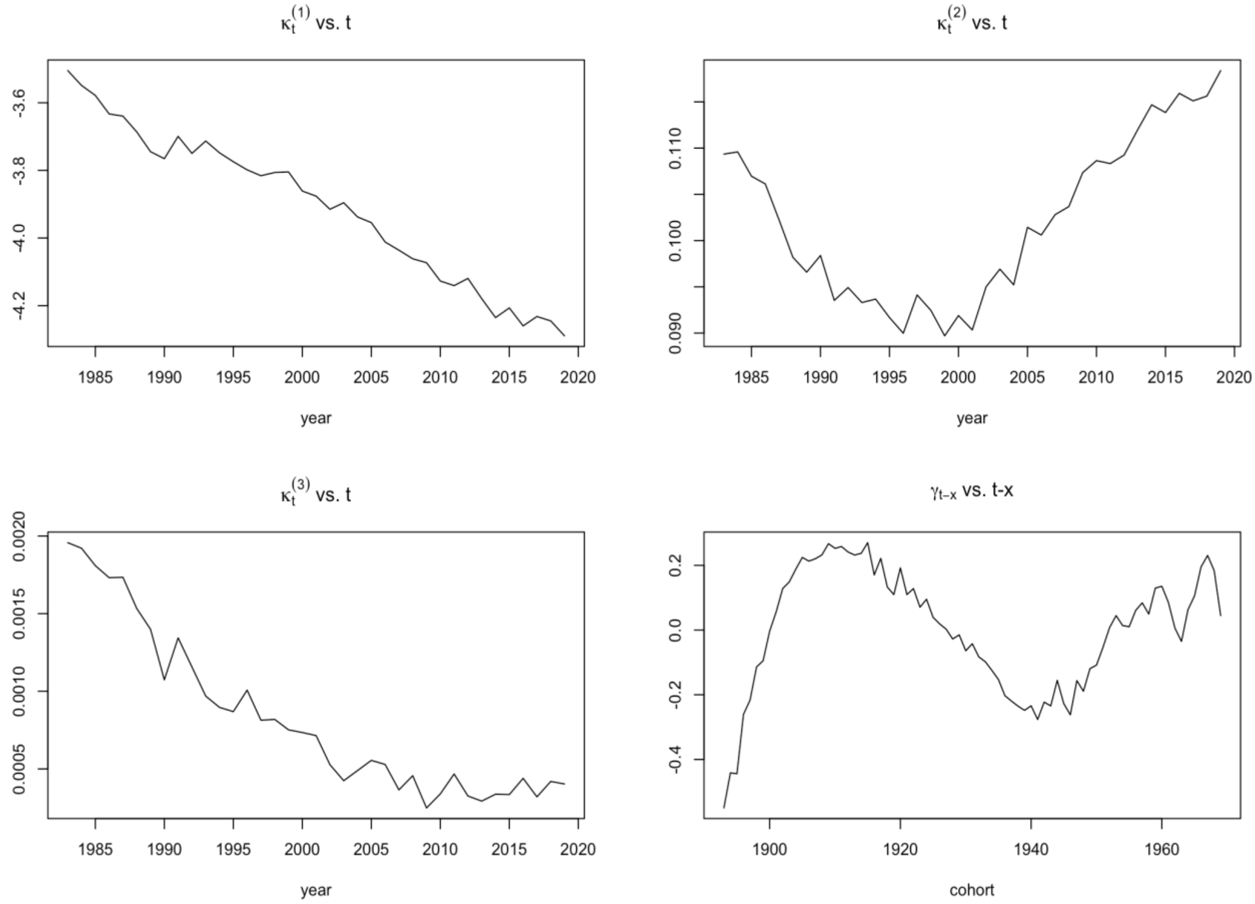
CBD



APC

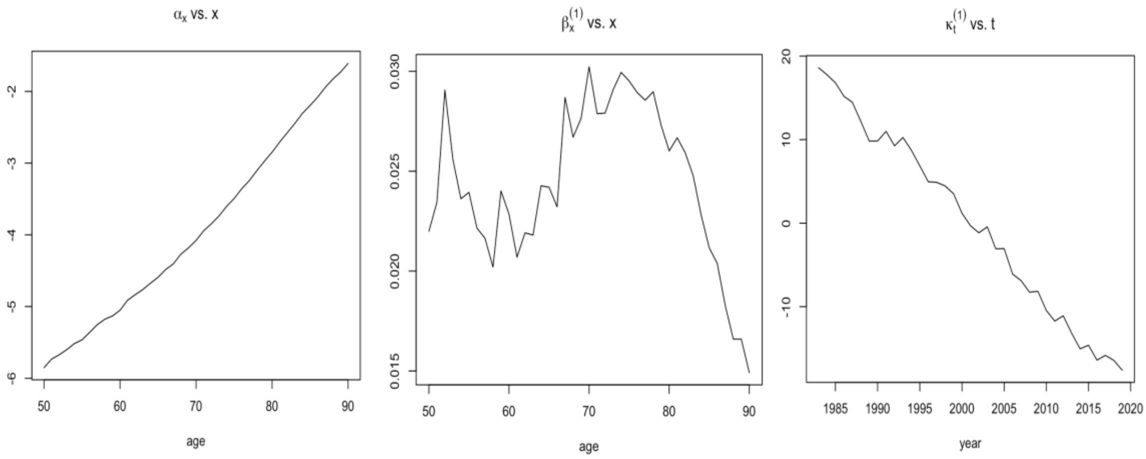


M7

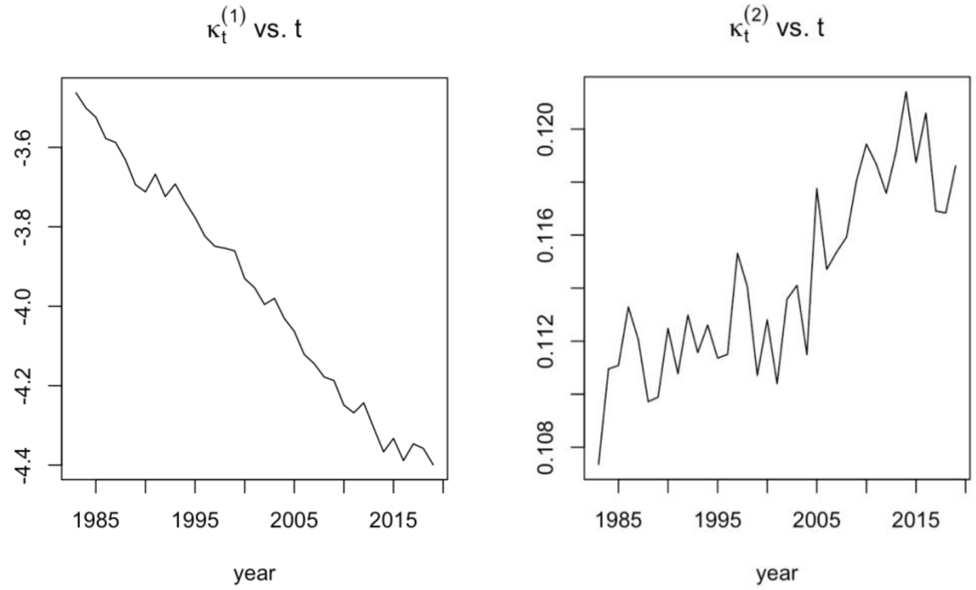


Appendix 4: Parameters of forecasting models (females)

LC

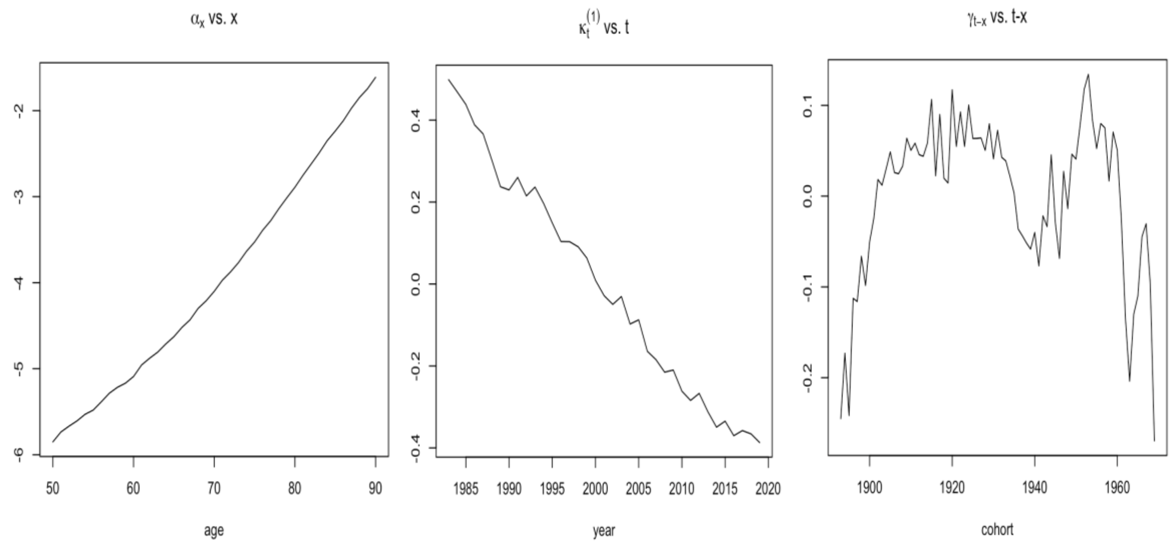


CBD



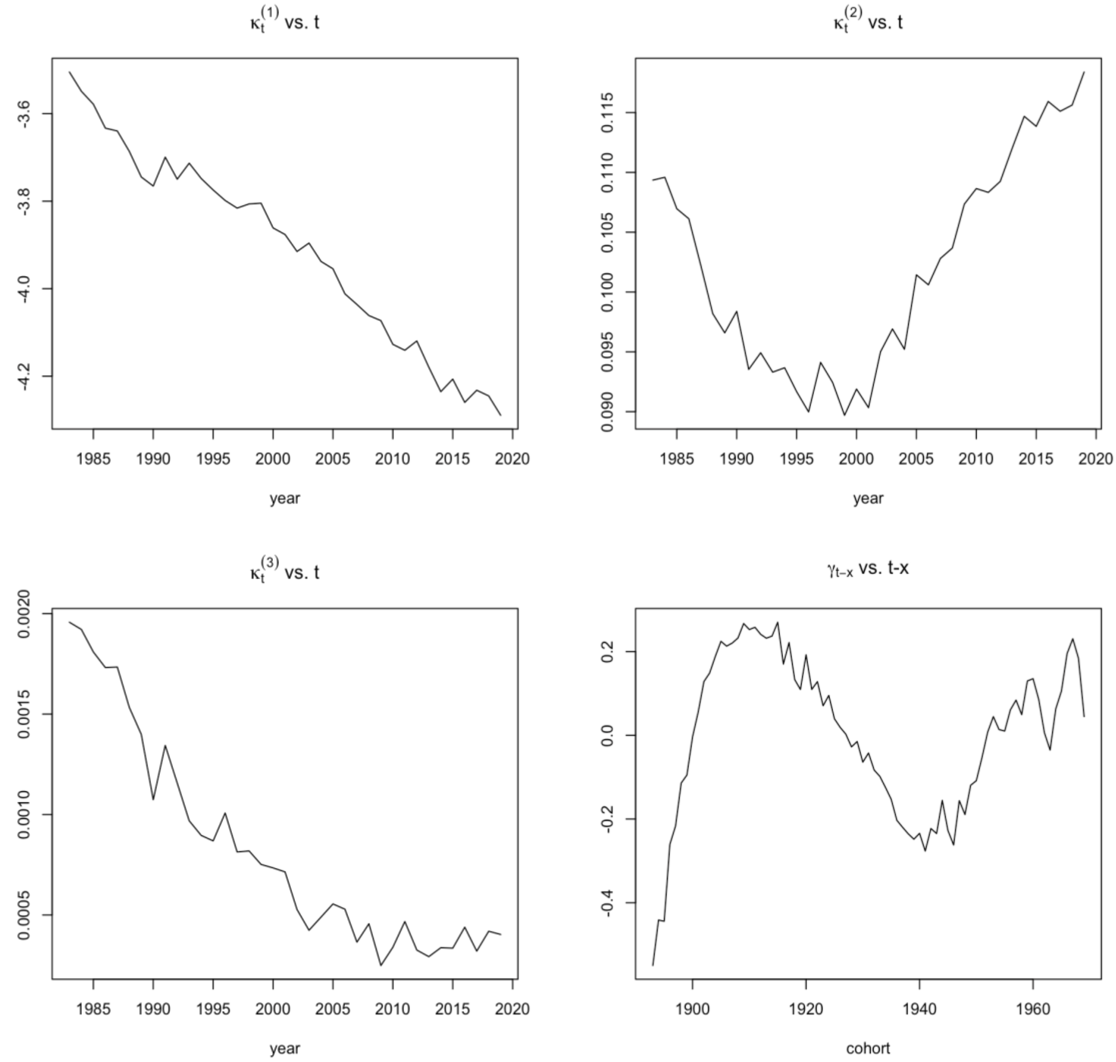
$\kappa_t^{(1)}$ vs. t
-3.6
-3.8
-4.0
-4.2
-4.4
1985
1995
2005
2015
year
$\kappa_t^{(2)}$ vs. t
0.120
0.116
0.112
0.108
1985
1995
2005
2015
year

APC

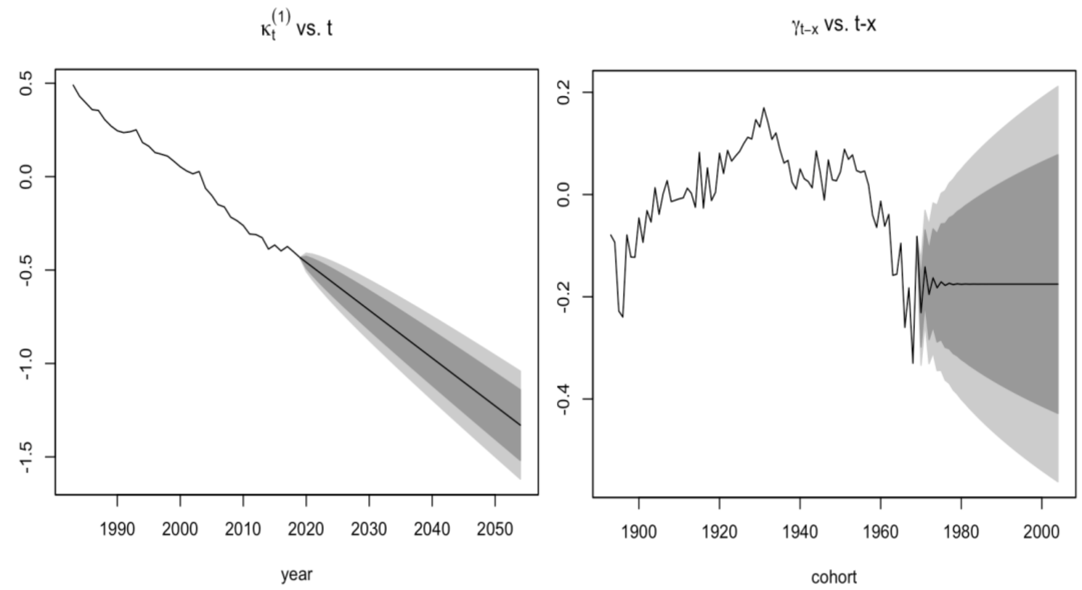


α_x vs. x
-2
-3
-4
-5
-6
50
60
70
80
90
age
$\kappa_t^{(1)}$ vs. t
0.4
0.2
0.0
-0.2
-0.4
1985
1990
1995
2000
2005
2010
2015
2020
year
γ_{t-x} vs. t-x
0.1
0.0
-0.1
-0.2
1900
1920
1940
1960
cohort

M7

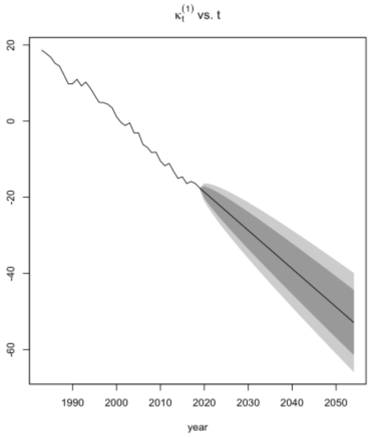


Appendix 5: Forecasting time-dependent variables for males (APC)

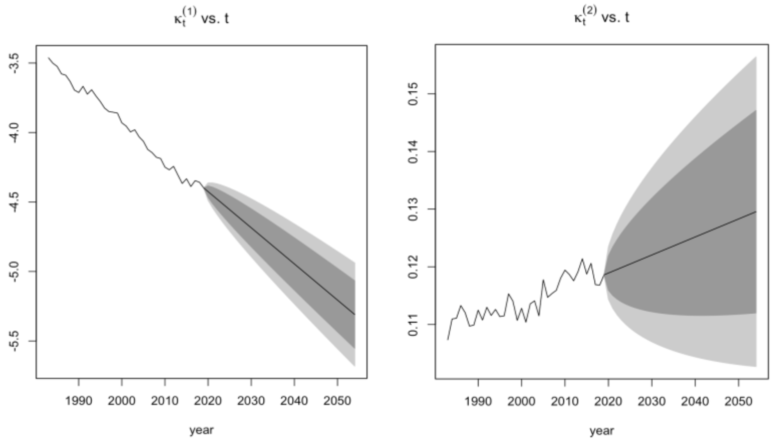


Appendix 6: Forecasting time-dependent variables (females)

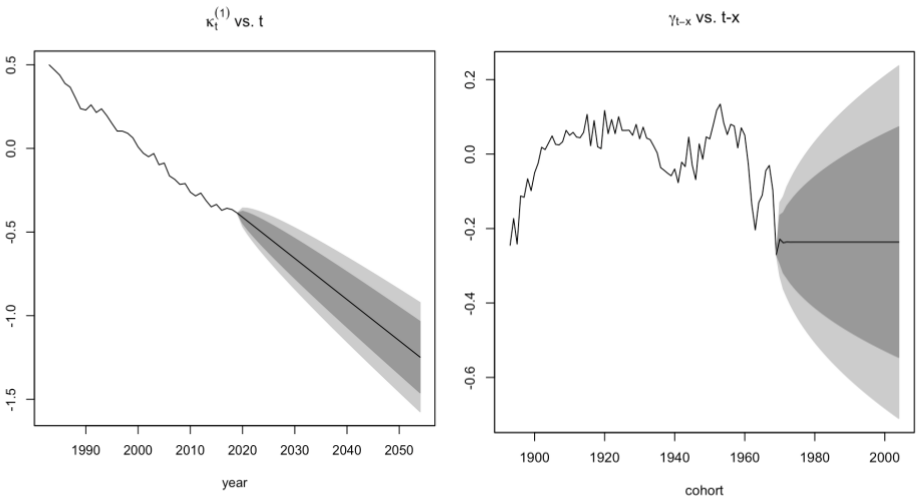
LC



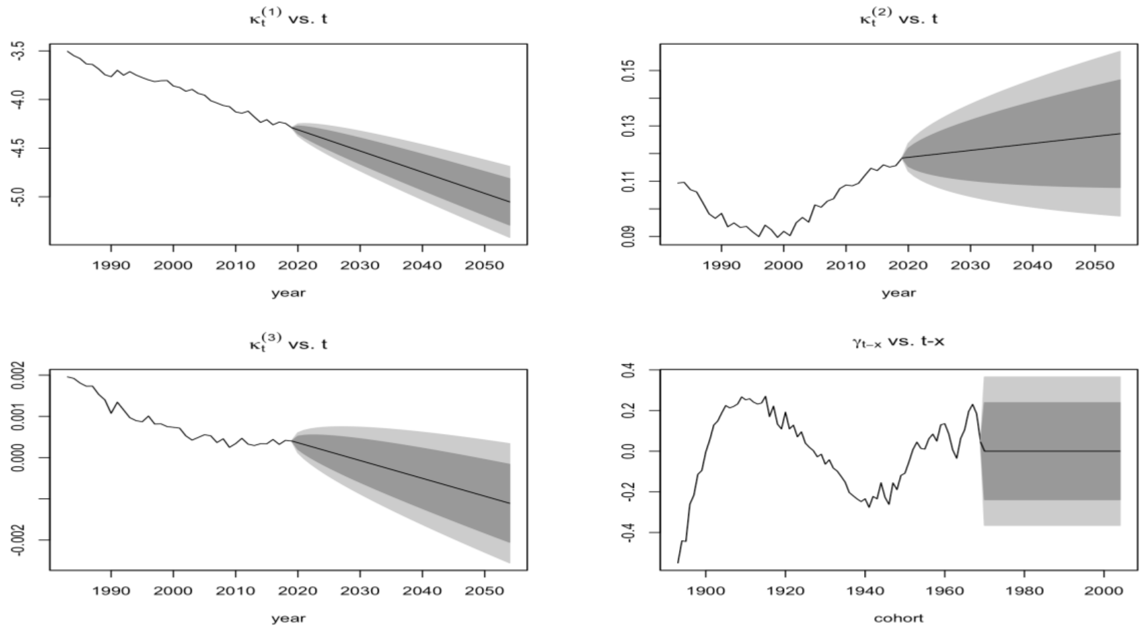
CBD



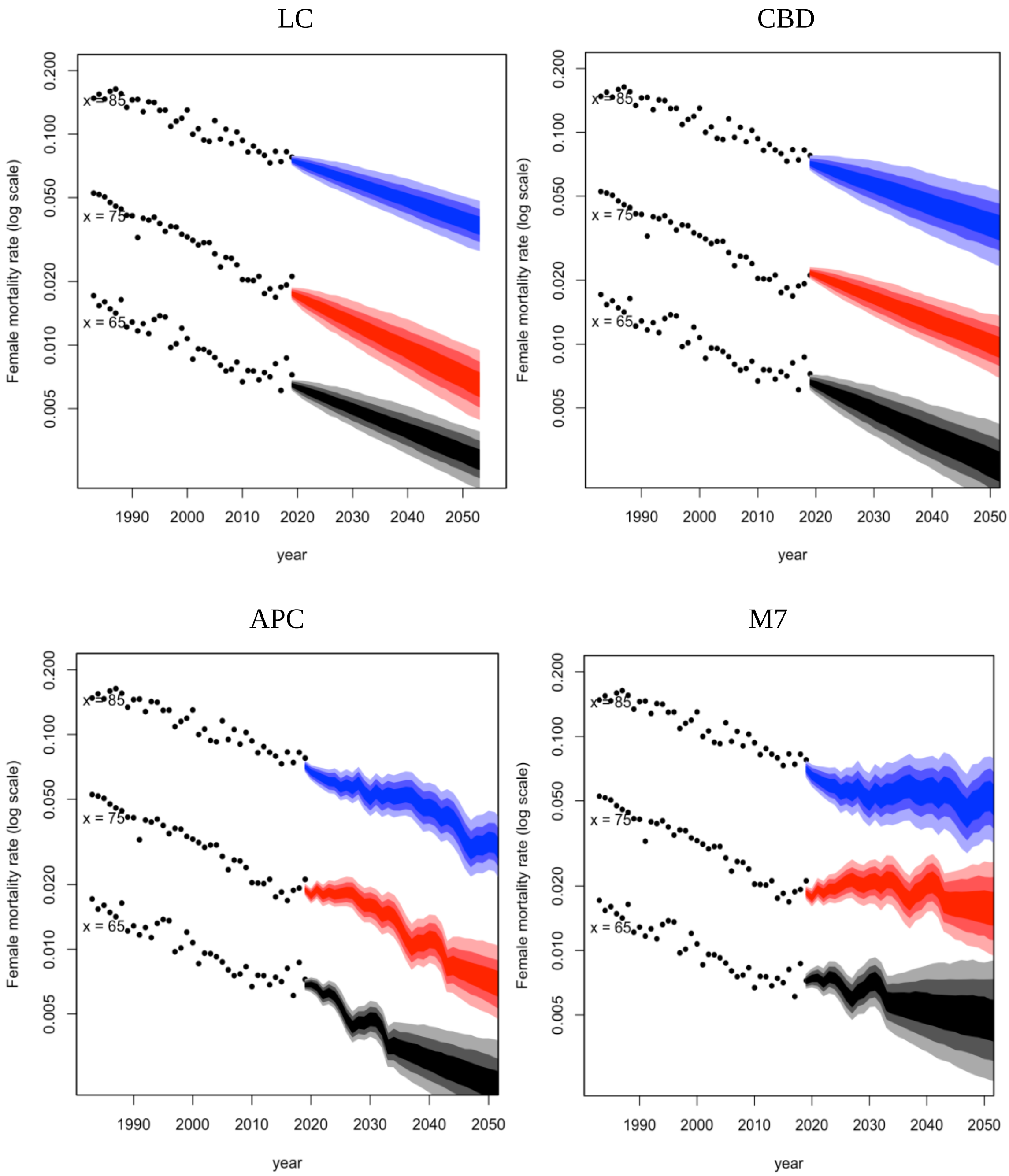
APC



M7

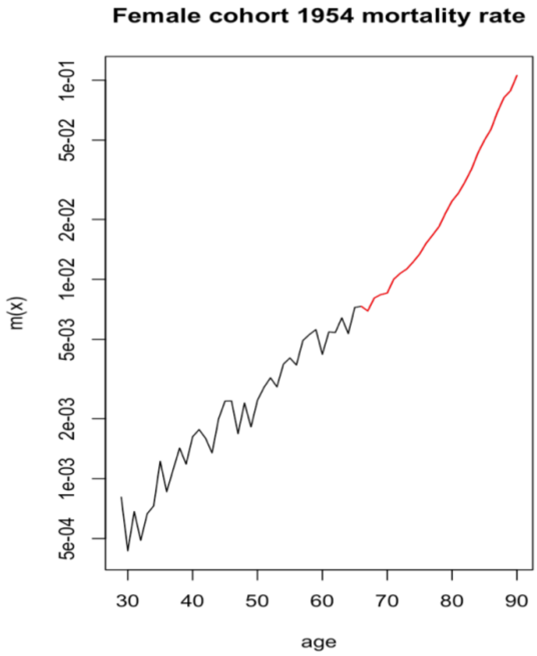


Appendix 7: Forecasts of log-central death rates (females)

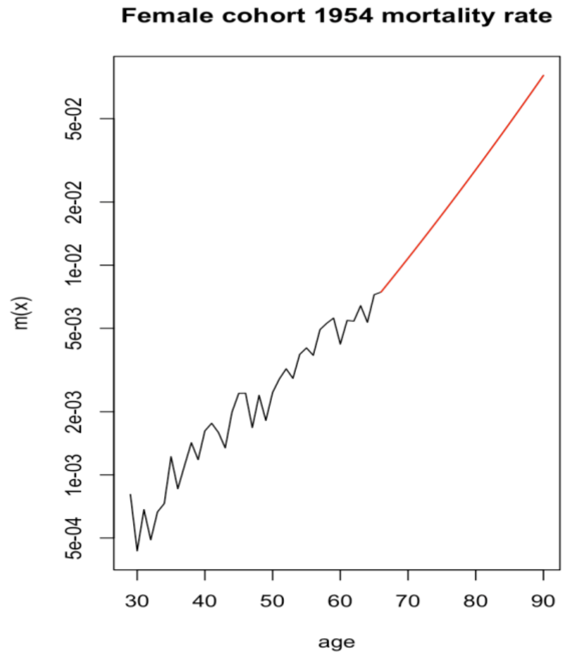


Appendix 8: Forecasts of central mortality rates for the 1954 cohort (females)

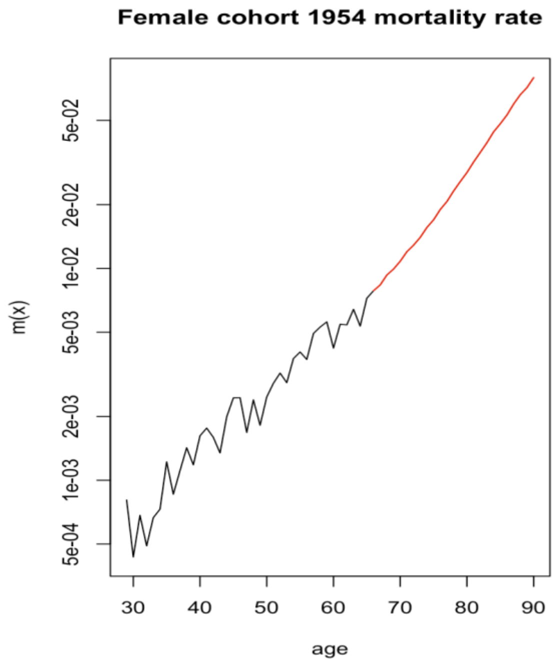
LC



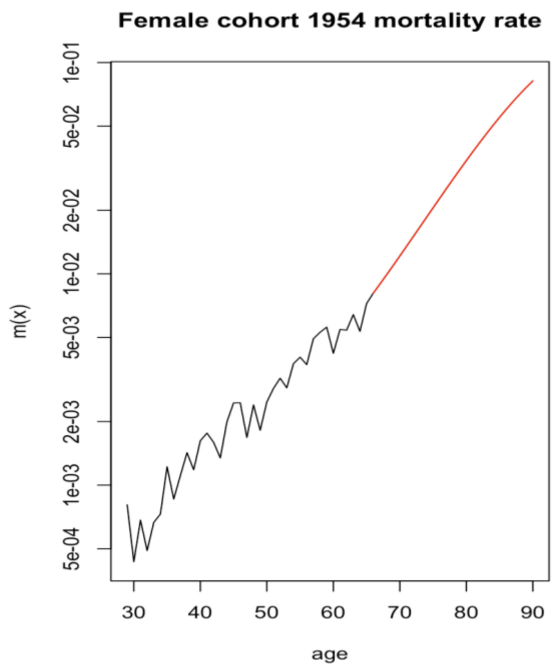
CBD



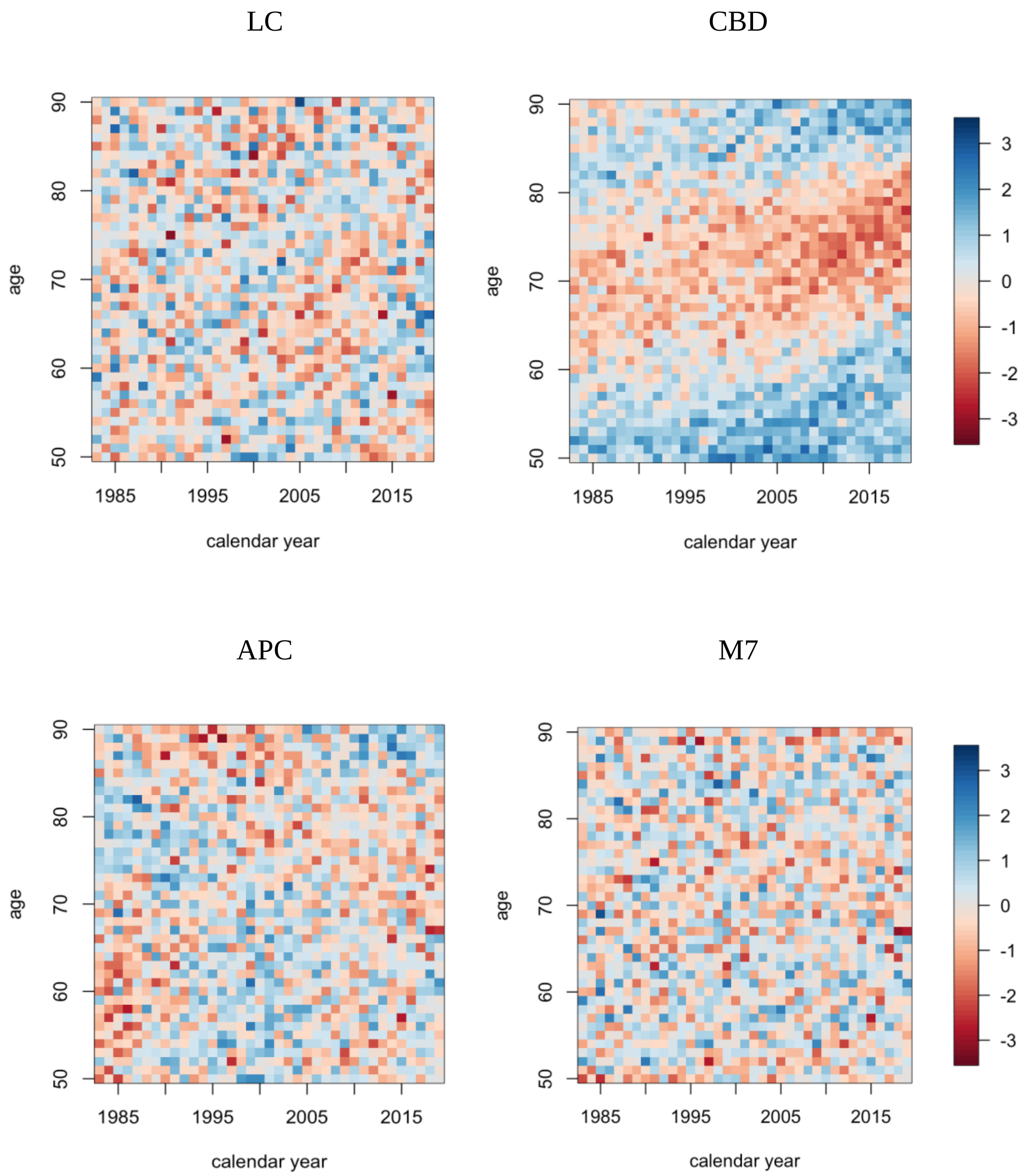
APC



M7



Appendix 9: Deviance residuals in a colormap (females)



Appendix 10: Information criteria (females)

Inf. criteria	**LC**	**CBD**	**APC**	**M7**
AIC	12277.4	15110.12	12348.01	12157.88
BIC	12900.37	15504.13	13157.33	13142.91

Appendix 11: R code for applying the Wang's transformation on SIA 65, 2010 table

```
q_x = read.table("/Users/svit/Desktop/JMD.txt", header = T)
L=function (lambda , r , q , s , x0 , le , male)
{
# to be optimalized wrt lambda
# r = fixed interest rate
# q = mortality table
# s = monthly payout from SPIA
# x0 = initial age
# le = maximum age set to 115
# male : TRUE/FALSE
# dividing with 1000 to get the mortalities
q_male=q$Male
q_female=q$Female
# K = number of time periods
K=le-x0
# discount
d=1 / (1+r)
# s is monthly , q is in years
s=s*12
# calculating k_q_x0 and inserting them in a matrix
 if (male) q=q_male else
    q=q_female
  q_=c ( q , rep (1 ,le) )
  kq=matrix (0 ,K+1 ,le)
  for (l in 0:le)
  {
    kq [ 1 : K+1 ,l]=1-cumprod(1-q_ [l : ( l+K-1) ] )
  }
  # the Wang transform
  A=s*sum(d** ( 0 : K) * (pnorm(qnorm(1-kq [ 1 : ( K+1) ,x0 ] )-lambda) ) )
  list (A=A)
}
f=function (lambda , r=0.03 ,q=q_x , s=680,x0=65,le=115,male=FALSE) L(lambda , r
, q , s , x0
           , le , male) $A
fzero=function (lambda , pi_x0) f(lambda)-pi_x0
uni=uniroot (fzero , c (-10 ,10) ,pi_x0=100000)

lambda=uni$root
lambda
# initial age
age=c (55 ,60 ,65 ,70 ,75 ,80)

# row 1=male , row 2=female
# payouts based on Troske, 2015
sCL=matrix ( c(671.7 ,726.44 ,804.02 ,911.69 ,1060.03 ,1265.68 ,627.13 ,669.96
,729.13 ,812.49 ,936.41 ,1118.95)
             ,byrow=T , ncol =6)

# estimating the Wang transform
l_male=1:6*0
l_female=1:6*0
gender=c (TRUE , FALSE)

for (i in 1: length (age) )
{
  for (j in 1:2)
  {
    f=function (lambda , r=0.03 ,q=q_x , s=sCL[j , i] ,x0=age[i]
,le=115,male=gender[j ] ) L(
      lambda , r , q , s , x0 , le , male) $A
    fzero=function (lambda , pi_x0) f(lambda)-pi_x0
    uni=uniroot (fzero , c (-10 ,10) ,pi_x0=100000)
    if (gender[j ] ) l_male[i]=uni$root else
      l_female[i]=uni$root
```

```
  }
}

# plotting the Wang transform
plot(age,l_male, "o" ,lty=1 ,main="Market Price of Risk using SIA 65 2010 \nwhen
r =
3%" ,xlab="Initial age" ,ylab=expression (lambda) ,ylim=c(min(l_male , l_female)
,
                                                          max(l_male , l_female)
) )
lines(age , l_female ,"o",lty=2)
legend ( "topright" , c ( "Male" , "Female " ) ,lty=c (1 ,2) , col =1)

### Basic mortalities versus the transformed mortalities
q_male=q_x$Male
q_female=q_x$Female
Age=q_x$Age

# Wang transform on Males
q_starm=55:115*0

l_male2=c(rep (l_male [ 1 ] , 5 ) , rep (l_male [ 2 ] , 5 ) , rep (l_male [ 3 ] ,
5 ) , rep (l_male [ 4 ] , 5 ) ,
          rep (l_male [ 5 ] , 5 ) , rep (l_male[ 6 ] , 36 ) )

for (i in 1: length ( q_starm) )
{
  q_starm[i]=pnorm(qnorm( q_male[Age[54+i ] ] )-l_male2[i ] )
}

# Plotting SIA 65 2010 against Wang's transformed rates
plot(Age[55:115],q_male[55:115], "l" ,ylim=c(min(q_starm) ,max(q_male)),main=
"One-year mortalities for males" ,xlab="Initial age" ,ylab="q")
lines(Age[55:115], q_starm, "l",lty=2)
legend ( "topleft" , c( "SIA 65 2010 " , "Mortalities based on
Wang' s Transformation " ) ,col =1,lty=c (1,2))

#Wang transform on Females
q_starf=55:115*0
l_female2=c (rep(l_female[ 1 ],5) , rep(l_female [2], 5) , rep(l_female [3], 5) ,
rep (l_female [ 4 ] , 5 ) , rep (l_female [ 5 ] , 5 ) , rep (l_female[ 6 ] ,36 )
)

for (i in 1: length ( q_starf) )
{
  q_starf[i]=pnorm(qnorm( q_female[Age[54+i ] ] )-l_female2[i ] )
}
# Plotting SIA 65 2010 against Wang's transformed rates
plot(Age[55:115],q_female[55:115] ,"l",ylim=c(min(q_starf),max(q_female)),main=
        "One-year mortalities for females",xlab="Initial age",ylab="q")
lines(Age[55:115],q_starf,"l",lty=2)
legend ("topleft",c("SIA 65 2010" ,"Mortalities based on
Wang' s Transformation"), col=1 ,lty=c (1,2))
```

Appendix 12: R code for calculating the price of a longevity bond

```
q_x = read.table("/Users/svit/Desktop/JMD.txt", header = T)

#FEMALE
q_male=q_x$Male
q_female=q_x$Female
le=115;K1=115
p_=c(1-q_male , rep (0 ,le+1) )
kp=matrix (1 ,K1+1 ,le+1)
for (l in 0:le+1)
{
  kp [ 1: K1+1 ,l]=cumprod (p_ [l : ( l+K1-1) ] )
```

```
}

l_male=1.1059
l_female=1.4614

q_starm=1: length ( q_male) *0

q_starf=1: length ( q_female) *0
for (i in 1: length ( q_starm) )
{
  q_starm[i]=pnorm(qnorm( q_male[i ] )-l_male)
  q_starf[i]=pnorm(qnorm( q_female[i ] )-l_female)
}

p_star=c(1-q_starf , rep (0 ,le+1) )
kp_star=matrix (1 ,K1+1 ,le+1)
for (l in 0:le+1)
{
  kp_star [ 1 : K1+1 ,l]=cumprod (p_star[l : ( l+K1-1) ] )
}

K=30
x0=65; n_x=10000
X=1:K*0
for (k in 1:10)
{
  X[k]=n_x*kp[k+1 ,x0] *exp (0.0070 *k)
}
for (k in 11:20)
{
  X[k]=n_x*kp[k+1 ,x0] *exp ( 0.07 ) *exp (0.0093 * (k-10) )
}
for (k in 21:30)
{
  X[k]=n_x*kp[k+1 ,x0] *exp (0.163) *exp (0.0103 * (k-20) )
}

mu=1:K*0
sigma=1:K*0
for (k in 1:K)
{
  mu[k]=n_x*kp_star[k+1 ,x0]
  sigma[k]= sqrt (n_x*kp_star[k+1 ,x0] *(1-kp_star[k+1 ,x0 ] ) )
}

psi=function (a)
{
  dnorm(a)-a*(1-pnorm(a) )
}

C=700
E_D=1:K*0
for (k in 1:K)
{
  a=(X[k]-mu[k ] ) / sigma[k]
  E_D[k]=1000* (C-sigma[k] * (psi(a)-psi(a+C/ sigma[k ] ) ) )
}
r=0.03
d=1/(1+r)

F=10000000
V=F*d**K+sum(d** ( 1 : K) *E_D)

P=F-V

#MALE
q_male=q_x$Male
q_female=q_x$Female
```

```
le=115;K1=115
p_=c(1-q_male , rep (0 ,le+1) )
kp=matrix (1 ,K1+1 ,le+1)
for (l in 0:le+1)
{
  kp [ 1: K1+1 ,l]=cumprod (p_ [l : ( l+K1-1) ] )
}

l_male=1.1059
l_female=1.4614

q_starm=1: length ( q_male) *0

q_starf=1: length ( q_female) *0
for (i in 1: length ( q_starm) )
{
  q_starm[i]=pnorm(qnorm( q_male[i ] )-l_male)
  q_starf[i]=pnorm(qnorm( q_female[i ] )-l_female)
}

p_star=c(1-q_starm , rep (0 ,le+1) )
kp_star=matrix (1 ,K1+1 ,le+1)
for (l in 0:le+1)
{
  kp_star [ 1 : K1+1 ,l]=cumprod (p_star[l : ( l+K1-1) ] )
}

K=30
x0=65; n_x=10000
X=1:K*0
for (k in 1:10)
{
  X[k]=n_x*kp[k+1 ,x0] *exp (0.0070 *k)
}
for (k in 11:20)
{
  X[k]=n_x*kp[k+1 ,x0] *exp ( 0.07 ) *exp (0.0093 * (k-10) )
}
for (k in 21:30)
{
  X[k]=n_x*kp[k+1 ,x0] *exp (0.163) *exp (0.0103 * (k-20) )
}

mu=1:K*0
sigma=1:K*0
for (k in 1:K)
{
  mu[k]=n_x*kp_star[k+1 ,x0]
  sigma[k]= sqrt (n_x*kp_star[k+1 ,x0] *(1-kp_star[k+1 ,x0 ] ) )
}

psi=function (a)
{
  dnorm(a)-a*(1-pnorm(a) )
}

C=700
E_D=1:K*0
for (k in 1:K)
{
  a=(X[k]-mu[k ] ) / sigma[k]
  E_D[k]=1000* (C-sigma[k] * (psi(a)-psi(a+C/ sigma[k ] ) ) )
}
r=0.03
d=1/(1+r)
F=10000000
V=F*d**K+sum(d** ( 1 : K) *E_D)

P=F-V
```

Appendix 13: Mortality tables of a 1954 cohort under different models

Lee-Carter (Female_1954)

year	age	lx	px	ex	qx
1983	29	10000	0.9991933	54.184624	0.0008067
1984	30	9991.933	0.9995661	53.2283686	0.0004339
1985	31	9987.598	0.9993172	52.2514747	0.0006828
1986	32	9980.778	0.9995101	51.2871747	0.0004899
1987	33	9975.889	0.9993342	50.3123116	0.0006658
1988	34	9969.247	0.9992703	49.3458307	0.0007297
1989	35	9961.972	0.9987787	48.3818663	0.0012213
1990	36	9949.806	0.9991414	47.4410251	0.0008586
1991	37	9941.263	0.9988906	46.4817945	0.0011094
1992	38	9930.234	0.998577	45.5334179	0.001423
1993	39	9916.104	0.9988187	44.5983037	0.0011813
1994	40	9904.39	0.9983793	43.6510501	0.0016207
1995	41	9888.338	0.9982386	42.7219095	0.0017614
1996	42	9870.92	0.9984083	41.7972947	0.0015917
1997	43	9855.208	0.9986559	40.8639309	0.0013441
1998	44	9841.962	0.998009	39.9189299	0.001991
1999	45	9822.367	0.997554	38.9985676	0.002446
2000	46	9798.341	0.997551	38.0941922	0.002449
2001	47	9774.345	0.9983204	37.1877138	0.0016796
2002	48	9757.928	0.9976069	36.250279	0.0023931
2003	49	9734.576	0.9981837	35.3372388	0.0018163
2004	50	9716.895	0.9975301	34.4015404	0.0024699
2005	51	9692.894	0.9971341	33.4867207	0.0028659
2006	52	9665.116	0.9967991	32.5829657	0.0032009
2007	53	9634.179	0.9971172	31.6875944	0.0028828
2008	54	9606.405	0.996252	30.7792087	0.003748
2009	55	9570.401	0.9959761	29.8950021	0.0040239
2010	56	9531.89	0.9962899	29.0157822	0.0037101
2011	57	9496.526	0.9950801	28.1238347	0.0049199
2012	58	9449.805	0.9947269	27.2628843	0.0052731
2013	59	9399.975	0.9944206	26.4074053	0.0055794
2014	60	9347.529	0.9958038	25.5555692	0.0041962
2015	61	9308.305	0.9945628	24.6632568	0.0054372
2016	62	9257.694	0.9946006	23.7980884	0.0053994
2017	63	9207.708	0.9936085	22.927281	0.0063915
2018	64	9148.857	0.9946742	22.0747635	0.0053258
2019	65	9100.132	0.9927921	21.1929583	0.0072079
2020	66	9034.539	0.9926981	20.3468247	0.0073019
2021	67	8968.57	0.9930801	19.4964882	0.0069199
2022	68	8906.509	0.9919723	18.6323413	0.0080277
2023	69	8835.009	0.9916476	17.7831274	0.0083524
2024	70	8761.216	0.991496	16.9329109	0.008504
2025	71	8686.711	0.990041	16.0781428	0.009959
2026	72	8600.2	0.9893368	15.2398753	0.0106632
2027	73	8508.494	0.9887937	14.4041321	0.0112063
2028	74	8413.146	0.9878464	13.5673784	0.0121536
2029	75	8310.896	0.9866834	12.7342992	0.0133166
2030	76	8200.223	0.9849401	11.9061655	0.0150599
2031	77	8076.729	0.9834432	11.0882127	0.0165568
2032	78	7943.004	0.9817449	10.2748885	0.0182551
2033	79	7798.004	0.9786946	9.4659456	0.0213054
2034	80	7631.864	0.9755555	8.6720117	0.0244445
2035	81	7445.307	0.9732194	7.889306	0.0267806
2036	82	7245.917	0.969559	7.1064003	0.030441
2037	83	7025.344	0.9648759	6.3295184	0.0351241
2038	84	6778.585	0.9577459	5.5599299	0.0422541
2039	85	6492.162	0.9511897	4.8052244	0.0488103
2040	86	6175.278	0.9448887	4.0518042	0.0551113
2041	87	5834.951	0.9331143	3.2881286	0.0668857
2042	88	5444.676	0.9215037	2.5238219	0.0784963
2043	89	5017.289	0.915289	1.7388083	0.084711
2044	90	4592.269	0.8997368	0.8997368	0.1002632

Lee-Carter (Male_1954)

year	age	lx	px	ex	qx
1983	29	10000	0.9977785	47.9446575	0.0022215
1984	30	9977.785	0.9979491	47.0514051	0.0020509
1985	31	9957.321	0.9978852	46.1481009	0.0021148
1986	32	9936.264	0.9978054	45.2459	0.0021946
1987	33	9914.458	0.998011	44.3454145	0.001989
1988	34	9894.738	0.9976577	43.4337942	0.0023423
1989	35	9871.562	0.9975829	42.535766	0.0024171
1990	36	9847.702	0.9978553	41.6388273	0.0021447
1991	37	9826.581	0.9969726	40.7283219	0.0030274
1992	38	9796.832	0.9964244	39.8519977	0.0035756
1993	39	9761.803	0.9968061	38.9950037	0.0031939
1994	40	9730.624	0.9967902	38.1199485	0.0032098
1995	41	9699.391	0.9959104	37.2427015	0.0040896
1996	42	9659.724	0.9963367	36.3956354	0.0036633
1997	43	9624.338	0.9965639	35.529453	0.0034361
1998	44	9591.268	0.9951657	34.6519562	0.0048343
1999	45	9544.901	0.9947986	33.8202874	0.0052014
2000	46	9495.253	0.9954633	32.9971213	0.0045367
2001	47	9452.176	0.9937794	32.1475011	0.0062206
2002	48	9393.378	0.9950762	31.3487294	0.0049238
2003	49	9347.127	0.9934277	30.5038495	0.0065723
2004	50	9285.694	0.9942903	29.7056573	0.0057097
2005	51	9232.676	0.9933452	28.8762402	0.0066548
2006	52	9171.235	0.9927593	28.0696927	0.0072407
2007	53	9104.829	0.9920418	27.2744191	0.0079582
2008	54	9032.371	0.9926789	26.4932158	0.0073211
2009	55	8966.244	0.991186	25.6886058	0.008814
2010	56	8887.215	0.9909224	24.9170382	0.0090776
2011	57	8806.541	0.9904518	24.1452974	0.0095482
2012	58	8722.454	0.990535	23.378064	0.009465
2013	59	8639.896	0.9893184	22.6014515	0.0106816
2014	60	8547.608	0.9887773	21.8454788	0.0112227
2015	61	8451.681	0.9880083	21.0934261	0.0119917
2016	62	8350.331	0.987235	20.3494415	0.012765
2017	63	8243.739	0.9871037	19.6125609	0.0128963
2018	64	8137.425	0.9855097	18.8687949	0.0144903
2019	65	8019.512	0.9850466	18.1462285	0.0149534
2020	66	7899.593	0.9842789	17.4216947	0.0157211
2021	67	7775.403	0.9836682	16.6999571	0.0163318
2022	68	7648.417	0.9823959	15.9772249	0.0176041
2023	69	7513.774	0.981699	15.2635294	0.018301
2024	70	7376.264	0.9804868	14.5480751	0.0195132
2025	71	7232.33	0.9785703	13.8376036	0.0214297
2026	72	7077.343	0.9776766	13.1406337	0.0223234
2027	73	6919.353	0.9745451	12.4406755	0.0254549
2028	74	6743.221	0.9748119	11.7656226	0.0251881
2029	75	6573.373	0.9728554	11.0696336	0.0271446
2030	76	6394.941	0.970067	10.3784979	0.029933
2031	77	6203.521	0.9695211	9.6987436	0.0304789
2032	78	6014.445	0.965407	9.0036432	0.034593
2033	79	5806.387	0.96259	8.3262668	0.03741
2034	80	5589.17	0.9575552	7.6498583	0.0424448
2035	81	5351.939	0.955381	6.9889476	0.044619
2036	82	5113.141	0.9491441	6.3153515	0.0508559
2037	83	4853.107	0.9382429	5.6537335	0.0617571
2038	84	4553.393	0.9359217	5.0258742	0.0640783
2039	85	4261.619	0.9243276	4.369973	0.0756724
2040	86	3939.132	0.9143182	3.7277319	0.0856818
2041	87	3601.62	0.9100184	3.0770621	0.0899816
2042	88	3277.541	0.8943402	2.3813185	0.1056598
2043	89	2931.236	0.8855123	1.6626541	0.1144877
2044	90	2595.646	0.8776182	0.8776182	0.1223818

Cairns-Blake-Dowd (Female_1954)

year	age	lx	px	ex	qx
1983	29	10000	0.9991933	53.772414	0.0008067
1984	30	9991.933	0.9995661	52.815826	0.0004339
1985	31	9987.598	0.9993172	51.838753	0.0006828
1986	32	9980.778	0.9995101	50.874171	0.0004899
1987	33	9975.889	0.9993342	49.899106	0.0006658
1988	34	9969.247	0.9992703	48.932349	0.0007297
1989	35	9961.972	0.9987787	47.968083	0.0012213
1990	36	9949.806	0.9991414	47.026736	0.0008586
1991	37	9941.263	0.9988906	46.067149	0.0011094
1992	38	9930.234	0.998577	45.118312	0.001423
1993	39	9916.104	0.9988187	44.182606	0.0011813
1994	40	9904.39	0.9983793	43.234861	0.0016207
1995	41	9888.338	0.9982386	42.305045	0.0017614
1996	42	9870.92	0.9984083	41.379695	0.0015917
1997	43	9855.208	0.9986559	40.445665	0.0013441
1998	44	9841.962	0.998009	39.500101	0.001991
1999	45	9822.367	0.997554	38.578903	0.002446
2000	46	9798.341	0.997551	37.673499	0.002449
2001	47	9774.345	0.9983204	36.765988	0.0016796
2002	48	9757.928	0.9976069	35.827843	0.0023931
2003	49	9734.576	0.9981837	34.91379	0.0018163
2004	50	9716.895	0.9975301	33.977321	0.0024699
2005	51	9692.894	0.9971341	33.061451	0.0028659
2006	52	9665.116	0.9967991	32.156473	0.0032009
2007	53	9634.179	0.9971172	31.259733	0.0028828
2008	54	9606.405	0.996252	30.35011	0.003748
2009	55	9570.401	0.9959761	29.464289	0.0040239
2010	56	9531.89	0.9962899	28.583329	0.0037101
2011	57	9496.526	0.9950801	27.689771	0.0049199
2012	58	9449.805	0.9947269	26.826675	0.0052731
2013	59	9399.975	0.9944206	25.968883	0.0055794
2014	60	9347.529	0.9958038	25.114587	0.0041962
2015	61	9308.305	0.9945628	24.220416	0.0054372
2016	62	9257.694	0.9946006	23.352827	0.0053994
2017	63	9207.708	0.9936085	22.479602	0.0063915
2018	64	9148.857	0.9946742	21.624205	0.0053258
2019	65	9100.132	0.9927921	20.739987	0.0072079
2020	66	9034.539	0.992588	19.890565	0.007412
2021	67	8967.575	0.9918769	19.039095	0.0081231
2022	68	8894.73	0.9910923	18.195019	0.0089077
2023	69	8815.499	0.9902262	17.358552	0.0097738
2024	70	8729.338	0.9892698	16.529885	0.0107302
2025	71	8635.67	0.9882129	15.709178	0.0117871
2026	72	8533.88	0.9870446	14.896553	0.0129554
2027	73	8423.32	0.9857524	14.092077	0.0142476
2028	74	8303.308	0.9843227	13.295757	0.0156773
2029	75	8173.134	0.98274	12.507519	0.01726
2030	76	8032.066	0.9809873	11.727191	0.0190127
2031	77	7879.355	0.9790455	10.954478	0.0209545
2032	78	7714.247	0.9768935	10.188936	0.0231065
2033	79	7535.997	0.9745075	9.429936	0.0254925
2034	80	7343.886	0.9718613	8.676617	0.0281387
2035	81	7137.239	0.9689256	7.927834	0.0310744
2036	82	6915.453	0.9656679	7.182088	0.0343321
2037	83	6678.031	0.9620519	6.43743	0.0379481
2038	84	6424.613	0.9580375	5.691354	0.0419625
2039	85	6155.02	0.9535802	4.940638	0.0464198
2040	86	5869.305	0.9486304	4.181146	0.0513696
2041	87	5567.802	0.9431337	3.407561	0.0568663
2042	88	5251.181	0.9370296	2.61302	0.0629704
2043	89	4920.512	0.9302518	1.78862	0.0697482
2044	90	4577.315	0.922727	0.922727	0.077273

Cairns-Blake-Dowd (Male_1954)

year	age	lx	px	ex	qx
1983	29	10000	0.9977785	47.8490676	0.0022215
1984	30	9977.785	0.9979491	46.9556023	0.0020509
1985	31	9957.321	0.9978852	46.0521013	0.0021148
1986	32	9936.264	0.9978054	45.1496969	0.0021946
1987	33	9914.458	0.998011	44.2489998	0.001989
1988	34	9894.738	0.9976577	43.3371874	0.0023423
1989	35	9871.562	0.9975829	42.4389324	0.0024171
1990	36	9847.702	0.9978553	41.541759	0.0021447
1991	37	9826.581	0.9969726	40.631045	0.0030274
1992	38	9796.832	0.9964244	39.7544254	0.0035756
1993	39	9761.803	0.9968061	38.8970812	0.0031939
1994	40	9730.624	0.9967902	38.0217123	0.0032098
1995	41	9699.391	0.9959104	37.144149	0.0040896
1996	42	9659.724	0.9963367	36.2966782	0.0036633
1997	43	9624.338	0.9965639	35.4301319	0.0034361
1998	44	9591.268	0.9951657	34.5522927	0.0048343
1999	45	9544.901	0.9947986	33.7201398	0.0052014
2000	46	9495.253	0.9954633	32.89645	0.0045367
2001	47	9452.176	0.9937794	32.046371	0.0062206
2002	48	9393.378	0.9950762	31.2469663	0.0049238
2003	49	9347.127	0.9934277	30.4015829	0.0065723
2004	50	9285.694	0.9942903	29.602714	0.0057097
2005	51	9232.676	0.9933452	28.7727059	0.0066548
2006	52	9171.235	0.9927593	27.9654647	0.0072407
2007	53	9104.829	0.9920418	27.1694309	0.0079582
2008	54	9032.371	0.9926789	26.3873854	0.0073211
2009	55	8966.244	0.991186	25.5819949	0.008814
2010	56	8887.215	0.9909224	24.8094792	0.0090776
2011	57	8806.541	0.9904518	24.0367531	0.0095482
2012	58	8722.454	0.990535	23.2684733	0.009465
2013	59	8639.896	0.9893184	22.4908137	0.0106816
2014	60	8547.608	0.9887773	21.7336465	0.0112227
2015	61	8451.681	0.9880083	20.9803244	0.0119917
2016	62	8350.331	0.987235	20.234967	0.012765
2017	63	8243.739	0.9871037	19.4966063	0.0128963
2018	64	8137.425	0.9855097	18.7513254	0.0144903
2019	65	8019.512	0.9850466	18.0270318	0.0149534
2020	66	7899.593	0.9842716	17.3006885	0.0157284
2021	67	7775.345	0.9831285	16.577149	0.0168715
2022	68	7644.163	0.9818919	15.8616308	0.0181081
2023	69	7505.742	0.9805536	15.1541522	0.0194464
2024	70	7359.782	0.9791047	14.45469	0.0208953
2025	71	7205.998	0.9775353	13.7631708	0.0224647
2026	72	7044.117	0.9758347	13.0794614	0.0241653
2027	73	6873.894	0.9739912	12.4033575	0.0260088
2028	74	6695.112	0.9719919	11.734569	0.0280081
2029	75	6507.594	0.9698228	11.0727029	0.0301772
2030	76	6311.213	0.9674688	10.4172431	0.0325312
2031	77	6105.902	0.964913	9.7675239	0.035087
2032	78	5891.664	0.9621373	9.122699	0.0378627
2033	79	5668.59	0.9591218	8.4817016	0.0408782
2034	80	5436.868	0.9558448	7.8431953	0.0441552
2035	81	5196.802	0.9522827	7.205511	0.0477173
2036	82	4948.825	0.9484097	6.5665671	0.0515903
2037	83	4693.514	0.9441977	5.9237666	0.0558023
2038	84	4431.605	0.9396162	5.2738628	0.0603838
2039	85	4164.007	0.934632	4.6127842	0.065368
2040	86	3891.815	0.9292092	3.9354014	0.0707908
2041	87	3616.31	0.9233087	3.2352156	0.0766913
2042	88	3338.97	0.9168882	2.5039372	0.0831118
2043	89	3061.463	0.9099021	1.7309078	0.0900979
2044	90	2785.631	0.9023012	0.9023012	0.0976988

Age-Period-Cohort (Female_1954)

year	age	lx	px	ex	qx
1983	29	10000	0.9991933	53.7593675	0.0008067
1984	30	9991.933	0.9995661	52.8027689	0.0004339
1985	31	9987.598	0.9993172	51.8256902	0.0006828
1986	32	9980.778	0.9995101	50.8610993	0.0004899
1987	33	9975.889	0.9993342	49.8860273	0.0006658
1988	34	9969.247	0.9992703	48.9192625	0.0007297
1989	35	9961.972	0.9987787	47.9549866	0.0012213
1990	36	9949.806	0.9991414	47.0136234	0.0008586
1991	37	9941.263	0.9988906	46.0540255	0.0011094
1992	38	9930.234	0.998577	45.1051738	0.001423
1993	39	9916.104	0.9988187	44.1694493	0.0011813
1994	40	9904.39	0.9983793	43.2216885	0.0016207
1995	41	9888.338	0.9982386	42.291851	0.0017614
1996	42	9870.92	0.9984083	41.3664773	0.0015917
1997	43	9855.208	0.9986559	40.4324266	0.0013441
1998	44	9841.962	0.998009	39.4868448	0.001991
1999	45	9822.367	0.997554	38.5656206	0.002446
2000	46	9798.341	0.997551	37.6601836	0.002449
2001	47	9774.345	0.9983204	36.7526397	0.0016796
2002	48	9757.928	0.9976069	35.8144729	0.0023931
2003	49	9734.576	0.9981837	34.9003873	0.0018163
2004	50	9716.895	0.9975301	33.9638939	0.0024699
2005	51	9692.894	0.9971341	33.0479906	0.0028659
2006	52	9665.116	0.9967991	32.1429747	0.0032009
2007	53	9634.179	0.9971172	31.2461905	0.0028828
2008	54	9606.405	0.996252	30.3365287	0.003748
2009	55	9570.401	0.9959761	29.4506566	0.0040239
2010	56	9531.89	0.9962899	28.5696415	0.0037101
2011	57	9496.526	0.9950801	27.6760326	0.0049199
2012	58	9449.805	0.9947269	26.8128682	0.0052731
2013	59	9399.975	0.9944206	25.9550036	0.0055794
2014	60	9347.529	0.9958038	25.1006292	0.0041962
2015	61	9308.305	0.9945628	24.2063998	0.0054372
2016	62	9257.694	0.9946006	23.3387338	0.0053994
2017	63	9207.708	0.9936085	22.4654327	0.0063915
2018	64	9148.857	0.9946742	21.6099443	0.0053258
2019	65	9100.132	0.9927921	**20.7256504**	0.0072079
2020	66	9034.539	0.9921859	19.876124	0.0078141
2021	67	8963.943	0.9916794	19.0326611	0.0083206
2022	68	8889.358	0.9907335	18.1923521	0.0092665
2023	69	8806.984	0.9901201	17.3625082	0.0098799
2024	70	8719.973	0.9892483	16.535759	0.0107517
2025	71	8626.218	0.9880803	15.7154788	0.0119197
2026	72	8523.396	0.9872079	14.9050623	0.0127921
2027	73	8414.364	0.9860828	14.0982005	0.0139172
2028	74	8297.259	0.9845157	13.2971777	0.0154843
2029	75	8168.782	0.9831831	12.506313	0.0168169
2030	76	8031.408	0.9812096	11.7202282	0.0187904
2031	77	7880.495	0.9795214	10.9446729	0.0204786
2032	78	7719.113	0.9770738	10.1734908	0.0229262
2033	79	7542.143	0.9746269	9.4122036	0.0253731
2034	80	7350.776	0.9720485	8.6572373	0.0279515
2035	81	7145.31	0.9687106	7.9061785	0.0312894
2036	82	6921.738	0.9653114	7.1615485	0.0346886
2037	83	6681.632	0.9615286	6.4189002	0.0384714
2038	84	6424.581	0.9568156	5.6757244	0.0431844
2039	85	6147.139	0.9529459	4.9318894	0.0470541
2040	86	5857.891	0.9483597	4.1754139	0.0516403
2041	87	5555.388	0.9419976	3.4027742	0.0580024
2042	88	5233.162	0.936012	2.6122962	0.063988
2043	89	4898.302	0.9310593	1.790879	0.0689407
2044	90	4560.61	0.9234855	0.9234855	0.0765145

Age-Period-Cohort (Male_1954)

year	age	lx	px	ex	qx
1983	29	10000	0.9977785	48.1543649	0.0022215
1984	30	9977.785	0.9979491	47.2615795	0.0020509
1985	31	9957.321	0.9978852	46.3587072	0.0021148
1986	32	9936.264	0.9978054	45.4569526	0.0021946
1987	33	9914.458	0.998011	44.5569313	0.001989
1988	34	9894.738	0.9976577	43.6457326	0.0023423
1989	35	9871.562	0.9975829	42.748202	0.0024171
1990	36	9847.702	0.9978553	41.8517779	0.0021447
1991	37	9826.581	0.9969726	40.9417303	0.0030274
1992	38	9796.832	0.9964244	40.0660541	0.0035756
1993	39	9761.803	0.9968061	39.2098282	0.0031939
1994	40	9730.624	0.9967902	38.3354614	0.0032098
1995	41	9699.391	0.9959104	37.4589083	0.0040896
1996	42	9659.724	0.9963367	36.6127301	0.0036633
1997	43	9624.338	0.9965639	35.7473458	0.0034361
1998	44	9591.268	0.9951657	34.8706003	0.0048343
1999	45	9544.901	0.9947986	34.0399937	0.0052014
2000	46	9495.253	0.9954633	33.2179764	0.0045367
2001	47	9452.176	0.9937794	32.3693627	0.0062206
2002	48	9393.378	0.9950762	31.5719797	0.0049238
2003	49	9347.127	0.9934277	30.7282045	0.0065723
2004	50	9285.694	0.9942903	29.9314966	0.0057097
2005	51	9232.676	0.9933452	29.1033764	0.0066548
2006	52	9171.235	0.9927593	28.2983506	0.0072407
2007	53	9104.829	0.9920418	27.5047446	0.0079582
2008	54	9032.371	0.9926789	26.725389	0.0073211
2009	55	8966.244	0.991186	25.9224913	0.008814
2010	56	8887.215	0.9909224	25.1530035	0.0090776
2011	57	8806.541	0.9904518	24.3834243	0.0095482
2012	58	8722.454	0.990535	23.6184865	0.009465
2013	59	8639.896	0.9893184	22.8441714	0.0106816
2014	60	8547.608	0.9887773	22.0908194	0.0112227
2015	61	8451.681	0.9880083	21.3415512	0.0119917
2016	62	8350.331	0.987235	20.6005782	0.012765
2017	63	8243.739	0.9871037	19.8669448	0.0128963
2018	64	8137.425	0.9855097	19.1265023	0.0144903
2019	65	8019.512	0.9850466	**18.407725**	0.0149534
2020	66	7899.593	0.9841604	17.6871608	0.0158396
2021	67	7774.467	0.9832094	16.9718282	0.0167906
2022	68	7643.929	0.9819849	16.2616622	0.0180151
2023	69	7506.222	0.9807482	15.5599922	0.0192518
2024	70	7361.714	0.9800041	14.8654297	0.0199959
2025	71	7214.511	0.9782874	14.1687418	0.0217126
2026	72	7057.865	0.9774648	13.4832096	0.0225352
2027	73	6898.814	0.9754542	12.7940619	0.0245458
2028	74	6729.477	0.9740866	12.1160049	0.0259134
2029	75	6555.094	0.9721963	11.4383243	0.0278037
2030	76	6372.838	0.9695472	10.7654472	0.0304528
2031	77	6178.767	0.9685316	10.1035824	0.0314684
2032	78	5984.331	0.9659878	9.4318558	0.0340122
2033	79	5780.791	0.9631979	8.7639489	0.0368021
2034	80	5568.046	0.9600286	8.0988041	0.0399714
2035	81	5345.483	0.9561839	7.4360032	0.0438161
2036	82	5111.265	0.9534625	6.7767502	0.0465375
2037	83	4873.399	0.9489889	6.1075163	0.0510111
2038	84	4624.802	0.9453782	5.4358144	0.0546218
2039	85	4372.187	0.9412173	4.7498835	0.0587827
2040	86	4115.178	0.9369234	4.0465321	0.0630766
2041	87	3855.606	0.9330953	3.3189573	0.0669047
2042	88	3597.648	0.9253006	2.5569327	0.0746994
2043	89	3328.906	0.9219671	1.7633534	0.0780329
2044	90	3069.142	0.9125991	0.9125991	0.0874009

M7 (Female_1954)

year	age	lx	px	ex	qx
1983	29	10000	0.9991933	53.2033549	0.0008067
1984	30	9991.933	0.9995661	52.2463074	0.0004339
1985	31	9987.598	0.9993172	51.2689872	0.0006828
1986	32	9980.778	0.9995101	50.3040158	0.0004899
1987	33	9975.889	0.9993342	49.3286709	0.0006658
1988	34	9969.247	0.9992703	48.3615347	0.0007297
1989	35	9961.972	0.9987787	47.3968515	0.0012213
1990	36	9949.806	0.9991414	46.4548059	0.0008586
1991	37	9941.263	0.9988906	45.4947277	0.0011094
1992	38	9930.234	0.998577	44.5452549	0.001423
1993	39	9916.104	0.9988187	43.6087325	0.0011813
1994	40	9904.39	0.9983793	42.6603085	0.0016207
1995	41	9888.338	0.9982386	41.7295597	0.0017614
1996	42	9870.92	0.9984083	40.8031938	0.0015917
1997	43	9855.208	0.9986559	39.8682451	0.0013441
1998	44	9841.962	0.998009	38.921904	0.001991
1999	45	9822.367	0.997554	37.9995527	0.002446
2000	46	9798.341	0.997551	37.0927277	0.002449
2001	47	9774.345	0.9983204	36.1837907	0.0016796
2002	48	9757.928	0.9976069	35.2446669	0.0023931
2003	49	9734.576	0.9981837	34.3292144	0.0018163
2004	50	9716.895	0.9975301	33.3916817	0.0024699
2005	51	9692.894	0.9971341	32.4743615	0.0028659
2006	52	9665.116	0.9967991	31.5676969	0.0032009
2007	53	9634.179	0.9971172	30.6690654	0.0028828
2008	54	9606.405	0.996252	29.757735	0.003748
2009	55	9570.401	0.9959761	28.8696855	0.0040239
2010	56	9531.89	0.9962899	27.9863232	0.0037101
2011	57	9496.526	0.9950801	27.0905421	0.0049199
2012	58	9449.805	0.9947269	26.2244829	0.0052731
2013	59	9399.975	0.9944206	25.3634992	0.0055794
2014	60	9347.529	0.9958038	24.5058061	0.0041962
2015	61	9308.305	0.9945628	23.6090701	0.0054372
2016	62	9257.694	0.9946006	22.7381386	0.0053994
2017	63	9207.708	0.9936085	21.8615771	0.0063915
2018	64	9148.857	0.9946742	21.0022044	0.0053258
2019	65	9100.132	0.9927921	20.1146564	0.0072079
2020	66	9034.539	0.9919377	19.260694	0.0080623
2021	67	8961.7	0.9911004	18.4172416	0.0088996
2022	68	8881.945	0.9901604	17.582619	0.0098396
2023	69	8794.55	0.9891066	16.7573436	0.0108934
2024	70	8698.748	0.987927	15.9418984	0.012073
2025	71	8593.728	0.9866088	15.1367175	0.0133912
2026	72	8478.647	0.9851386	14.3421678	0.0148614
2027	73	8352.643	0.9835026	13.5585274	0.0164974
2028	74	8214.846	0.9816862	12.7859605	0.0183138
2029	75	8064.401	0.9796751	12.0244883	0.0203249
2030	76	7900.492	0.977455	11.2739551	0.022545
2031	77	7722.376	0.9750122	10.5339888	0.0249878
2032	78	7529.41	0.9723339	9.8039564	0.0276661
2033	79	7321.101	0.969409	9.0829116	0.030591
2034	80	7097.141	0.9662284	8.3695355	0.0337716
2035	81	6857.459	0.9627861	7.6620672	0.0372139
2036	82	6602.266	0.9590792	6.9582242	0.0409208
2037	83	6332.096	0.9551096	6.2551089	0.0448904
2038	84	6047.845	0.950884	5.5491008	0.049116
2039	85	5750.799	0.946415	4.8357287	0.053585
2040	86	5442.643	0.941722	4.1095225	0.058278
2041	87	5125.456	0.9368315	3.3638383	0.0631685
2042	88	4801.689	0.9317777	2.5906547	0.0682223
2043	89	4474.106	0.9266029	1.7803357	0.0733971
2044	90	4145.72	0.9213578	0.9213578	0.0786422

M7 (Male_1954)

year	age	lx	px	ex	qx
1983	29	10000	0.9977785	47.5987563	0.0022215
1984	30	9977.785	0.9979491	46.7047337	0.0020509
1985	31	9957.321	0.9978852	45.8007171	0.0021148
1986	32	9936.264	0.9978054	44.8977799	0.0021946
1987	33	9914.458	0.998011	43.9965288	0.001989
1988	34	9894.738	0.9976577	43.0842132	0.0023423
1989	35	9871.562	0.9975829	42.1853643	0.0024171
1990	36	9847.702	0.9978553	41.2875766	0.0021447
1991	37	9826.581	0.9969726	40.3763162	0.0030274
1992	38	9796.832	0.9964244	39.4989231	0.0035756
1993	39	9761.803	0.9968061	38.6406621	0.0031939
1994	40	9730.624	0.9967902	37.7644716	0.0032098
1995	41	9699.391	0.9959104	36.8860799	0.0040896
1996	42	9659.724	0.9963367	36.0375494	0.0036633
1997	43	9624.338	0.9965639	35.1700503	0.0034361
1998	44	9591.268	0.9951657	34.2913144	0.0048343
1999	45	9544.901	0.9947986	33.4578937	0.0052014
2000	46	9495.253	0.9954633	32.6328328	0.0045367
2001	47	9452.176	0.9937794	31.7815524	0.0062206
2002	48	9393.378	0.9950762	30.9804899	0.0049238
2003	49	9347.127	0.9934277	30.133788	0.0065723
2004	50	9285.694	0.9942903	29.3331475	0.0057097
2005	51	9232.676	0.9933452	28.5015913	0.0066548
2006	52	9171.235	0.9927593	27.6925339	0.0072407
2007	53	9104.829	0.9920418	26.8945094	0.0079582
2008	54	9032.371	0.9926789	26.1102585	0.0073211
2009	55	8966.244	0.991186	25.3028241	0.008814
2010	56	8887.215	0.9909224	24.527826	0.0090776
2011	57	8806.541	0.9904518	23.7525197	0.0095482
2012	58	8722.454	0.990535	22.9814998	0.009465
2013	59	8639.896	0.9893184	22.2010981	0.0106816
2014	60	8547.608	0.9887773	21.4408028	0.0112227
2015	61	8451.681	0.9880083	20.6841569	0.0119917
2016	62	8350.331	0.987235	19.9352049	0.012765
2017	63	8243.739	0.9871037	19.1929682	0.0128963
2018	64	8137.425	0.9855097	18.4437203	0.0144903
2019	65	8019.512	0.9850466	17.714904	0.0149534
2020	66	7899.593	0.9835047	16.9838225	0.0164953
2021	67	7769.287	0.982544	16.268674	0.017456
2022	68	7633.667	0.9814885	15.5577046	0.0185115
2023	69	7492.357	0.9803252	14.8511324	0.0196748
2024	70	7344.946	0.979039	14.14919	0.020961
2025	71	7190.988	0.9776125	13.4521216	0.0223875
2026	72	7030	0.9760255	12.7601782	0.0239745
2027	73	6861.459	0.9742546	12.0736116	0.0257454
2028	74	6684.808	0.9722723	11.392666	0.0277277
2029	75	6499.453	0.9700465	10.7175676	0.0299535
2030	76	6304.771	0.9675394	10.0485097	0.0324606
2031	77	6100.115	0.9647068	9.3856332	0.0352932
2032	78	5884.823	0.9614963	8.7290006	0.0385037
2033	79	5658.235	0.9578457	8.0785591	0.0421543
2034	80	5419.716	0.9536817	7.4340921	0.0463183
2035	81	5168.684	0.9489168	6.7951502	0.0510832
2036	82	4904.651	0.9434467	6.160955	0.0565533
2037	83	4627.277	0.9371472	5.530263	0.0628528
2038	84	4336.439	0.9298697	4.9011679	0.0701303
2039	85	4032.323	0.9214362	4.2708117	0.0785638
2040	86	3715.529	0.9116342	3.6349508	0.0883658
2041	87	3387.203	0.9002088	2.9872912	0.0997912
2042	88	3049.19	0.8868559	2.3184425	0.1131441
2043	89	2704.192	0.871213	1.6142267	0.128787
2044	90	2355.928	0.8528497	0.8528497	0.1471503

Appendix 14: SIA 65, 2010 Annuity table

	qx		px	
Age	Male	Female	Male	Female
0	1.842%	1.860%	98.158%	98.140%
1	0.149%	0.147%	99.851%	99.853%
2	0.070%	0.081%	99.930%	99.919%
3	0.049%	0.052%	99.951%	99.948%
4	0.042%	0.038%	99.958%	99.962%
5	0.035%	0.029%	99.965%	99.971%
6	0.027%	0.023%	99.973%	99.977%
7	0.021%	0.018%	99.979%	99.982%
8	0.017%	0.014%	99.983%	99.986%
9	0.016%	0.012%	99.984%	99.988%
10	0.018%	0.011%	99.982%	99.989%
11	0.020%	0.011%	99.980%	99.989%
12	0.022%	0.012%	99.978%	99.988%
13	0.025%	0.013%	99.975%	99.987%
14	0.029%	0.016%	99.971%	99.984%
15	0.035%	0.019%	99.965%	99.981%
16	0.042%	0.023%	99.958%	99.977%
17	0.048%	0.025%	99.952%	99.975%
18	0.052%	0.027%	99.948%	99.973%
19	0.056%	0.028%	99.944%	99.972%
20	0.059%	0.027%	99.941%	99.973%
21	0.062%	0.027%	99.938%	99.973%
22	0.064%	0.027%	99.936%	99.973%
23	0.065%	0.026%	99.935%	99.974%
24	0.064%	0.026%	99.936%	99.974%
25	0.063%	0.026%	99.937%	99.974%
26	0.061%	0.027%	99.939%	99.973%
27	0.060%	0.028%	99.940%	99.972%
28	0.058%	0.028%	99.942%	99.972%
29	0.056%	0.029%	99.944%	99.971%
30	0.055%	0.030%	99.945%	99.970%
31	0.055%	0.031%	99.945%	99.969%
32	0.057%	0.033%	99.943%	99.967%
33	0.061%	0.034%	99.939%	99.966%
34	0.068%	0.036%	99.932%	99.964%
35	0.076%	0.037%	99.924%	99.963%
36	0.083%	0.039%	99.917%	99.961%
37	0.089%	0.041%	99.911%	99.959%
38	0.093%	0.043%	99.907%	99.957%
39	0.095%	0.046%	99.905%	99.954%
40	0.097%	0.049%	99.903%	99.951%
41	0.099%	0.053%	99.901%	99.947%
42	0.104%	0.057%	99.896%	99.943%
43	0.110%	0.063%	99.890%	99.937%
44	0.119%	0.069%	99.881%	99.931%
45	0.129%	0.077%	99.871%	99.923%
46	0.140%	0.086%	99.860%	99.914%
47	0.153%	0.096%	99.847%	99.904%
48	0.166%	0.107%	99.834%	99.893%
49	0.180%	0.118%	99.820%	99.882%
50	0.198%	0.128%	99.802%	99.872%
51	0.220%	0.139%	99.780%	99.861%
52	0.245%	0.149%	99.755%	99.851%
53	0.271%	0.158%	99.729%	99.842%
54	0.299%	0.166%	99.701%	99.834%
55	0.328%	0.169%	99.672%	99.831%
56	0.358%	0.173%	99.642%	99.827%
57	0.391%	0.177%	99.609%	99.823%
58	0.431%	0.183%	99.569%	99.817%
59	0.477%	0.189%	99.523%	99.811%
60	0.531%	0.195%	99.469%	99.805%
61	0.590%	0.195%	99.410%	99.805%
62	0.652%	0.197%	99.348%	99.803%
63	0.717%	0.200%	99.283%	99.800%
64	0.753%	0.205%	99.247%	99.795%
65	0.779%	0.214%	99.221%	99.786%
66	0.816%	0.226%	99.184%	99.774%
67	0.859%	0.243%	99.141%	99.757%
68	0.913%	0.266%	99.087%	99.734%
69	0.979%	0.294%	99.021%	99.706%
70	1.055%	0.328%	98.945%	99.672%
71	1.138%	0.367%	98.862%	99.633%
72	1.238%	0.410%	98.762%	99.590%
73	1.345%	0.460%	98.655%	99.540%
74	1.455%	0.518%	98.545%	99.482%
75	1.571%	0.588%	98.429%	99.412%
76	1.701%	0.676%	98.299%	99.324%
77	1.857%	0.788%	98.143%	99.212%
78	2.059%	0.936%	97.941%	99.064%
79	2.318%	1.125%	97.682%	98.875%
80	2.656%	1.366%	97.344%	98.634%
81	3.093%	1.695%	96.907%	98.305%
82	3.648%	2.093%	96.352%	97.907%
83	4.331%	2.582%	95.669%	97.418%
84	5.141%	3.162%	94.859%	96.838%
85	6.091%	3.811%	93.909%	96.189%
86	7.170%	4.452%	92.830%	95.548%
87	8.346%	5.138%	91.654%	94.862%
88	9.441%	5.866%	90.559%	94.134%
89	10.561%	6.643%	89.439%	93.357%
90	11.701%	7.484%	88.299%	92.516%
91	12.805%	8.404%	87.195%	91.596%
92	13.873%	9.423%	86.127%	90.577%
93	16.180%	10.556%	83.820%	89.444%
94	17.717%	11.817%	82.283%	88.183%
95	20.145%	14.103%	79.855%	85.897%
96	22.199%	15.835%	77.801%	84.165%
97	24.386%	17.729%	75.614%	82.271%
98	26.712%	19.797%	73.288%	80.203%
99	29.185%	22.044%	70.815%	77.956%
100	31.804%	24.474%	68.196%	75.526%
101	34.284%	26.840%	65.716%	73.160%
102	36.864%	29.342%	63.136%	70.658%
103	39.537%	31.978%	60.463%	68.022%
104	42.296%	34.742%	57.704%	65.258%
105	45.133%	37.626%	54.867%	62.374%
106	48.038%	40.623%	51.962%	59.377%
107	51.000%	43.721%	49.000%	56.279%
108	54.006%	46.909%	45.994%	53.091%
109	57.044%	50.172%	42.956%	49.828%
110	60.100%	53.495%	39.900%	46.505%
111	63.159%	56.859%	36.841%	43.141%
112	66.205%	60.247%	33.795%	39.753%
113	69.221%	63.636%	30.779%	36.364%
114	72.191%	67.007%	27.809%	32.993%
115	75.096%	70.336%	24.904%	29.664%
116	77.920%	73.599%	22.080%	26.401%
117	80.645%	76.774%	19.355%	23.226%
118	83.252%	79.835%	16.748%	20.165%
119	85.726%	82.759%	14.274%	17.241%
120	100.000%	100.000%	0.000%	0.000%