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MASTER'S THESIS

**STOCK VALUATION MODEL ERRORS AND EFFICIENT  
FRONTIERS ON CAPITAL MARKETS**

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## INTRODUCTION

Which strategies should be used when creating an investment decision? A rational, profitable and well-constructed investment decision is considered the one which provides the highest return while keeping the risk at minimum. This statement can also be expressed differently as given the desired level of risk, acquire the highest level of return.

The simplification of this procedure came with the constitution of the “Modern Portfolio Theory”. The founder is the Nobel Prize winner Dr Henry Markowitz (1952), who in his paper “Portfolio Selection” presented a solution for creating an efficient portfolio.

In order to reduce the risk, it is crucial to diversify. The graphical representation for choosing the optimal portfolio is achieved through the efficient frontier graph, where the investor can clearly separate efficient from inefficient portfolios, choose the risk-return ratio and select the portfolio which provides the desired return.

This model helps investors to create efficient portfolios and choose the best one which suits their two basic needs:

- they want the return to be high;
- they want the return to be dependable, stable and not subject to uncertainty.

One of the main assumptions, which the whole portfolio theory is based on, is that the market is efficient and investors use all available information to derive their decision. This assumption is criticized, because even if every information is understood and used correctly, the systematic risk still cannot be fully diversified. Before making a decision and creating a portfolio, investors should choose assets and get acknowledged with their behaviour, current situation, price, predictions etc. In other words, they need to analyse and value the assets which are later included in the portfolio. Since the inputs for the analysis are sensitive and unpredictable, the analysis (valuation) is prone to errors.

The purpose of this research is to present the connection between the valuation errors in stock prices and the efficient frontiers. This research also points out the procedures of creating efficient frontier, separating the efficient from inefficient portfolios, choosing the optimal one, valuating the stocks, identifying and explaining the errors in the stocks. It will help the potential investor to choose the optimal portfolio and utilize his resources in such a manner that given the risk, the maximum output or gain is generated.

The research is based on 50 randomly selected stocks within the UK's FTSE 100 index for a time period of five years (01.01.2009–31.12.2014). The structure of stocks on the efficient frontiers, shows what would be the best strategy for investors which will satisfy the needs. Each stock is individually valuated using three different valuation models

(Dividend Discount model, Free Cash Flow to Firm and the Residual Income). Based on the magnitude of the deviation<sup>1</sup> from the current price (absolute percentage valuation error), the stocks are divided in 5 categories (small, medium, medium-high, high and large errors) and an efficient frontier is created for each of these groups. The list of stocks with the price, industry, ticker and the price obtained from the valuation results can be seen in appendix F (Table 17).

This supports the research question with numerical evidence and shows, through the portfolio selection theory and the efficient frontier, that taking more risk can be profitable if the portfolio is assembled properly. Simultaneously, it shows that by using the modern portfolio theory, the amount of data needed in order to derive a rational decision is reduced.

## **1 THEORY OF VALUATION AND MODERN PORTFOLIO THEORY**

### **1.1 Valuation process**

Everyone in the investment business is trying to determine and forecast the right stock value in order to make rationally backed up decision. Based on the market information and the methodologies available one is to make a reasonable valuation for most stocks. In reality it is a complicated process. Even though the techniques still remain the same, each stock has its own characteristics and some stocks are easier to be valued than others. This points out the role that asset valuation plays in portfolio creation and the problems and challenges that come along the process (Damodaran, 2002).

No one will pay for a stock more than it is worth, unless the individual knows something that the others do not know. Some investors think that any price can be justified if there is someone willing to pay that price and Damodaran (2002, p. 3) finds this absurd. This is based on perception, but we invest in a stock to receive some kind of benefit from it. The price of the stock should reflect the potential return. That is exactly what valuation models try to do: connect the risk and the expected return of the particular stock or portfolio.

The valuation process is a large field always open for debates. Some think that valuation methods are solid and if executed right, they give the intrinsic value of a stock. On the other hand, some believe that valuating an asset is an individual ability because the inputs can be easily manipulated to get the wanted result. Neither of these statements are neglectable. The right answer is a mixed product of the two.

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<sup>1</sup> The deviation is calculated based on the average value of the calculated price of all three models used for valuating.



## 1.2 Bias in valuation

The company's stock that is chosen for investing in, is not randomly selected. Maybe it is that particular company because of something we heard in the day, without knowing that we heard it. Maybe we saw a commercial and created our perception about the situation of the company based on that commercial. This is one simple example of how the bias can start. Once the company is chosen the process of data selection and gathering occurs.

The first three information that are obtained for a stock price are:

- the annual reports of the company;
- third party estimates;
- the market price (which is the markets own estimate about the value).

Analysts think that the closer their value is to the market value the better, since it is more convenient to say that the market is almost always right. In other words, analysts are happier with smaller valuation errors (no matter in which direction). Up to this stage, the bias is already included in the calculation but there are still factors that can interfere, such as institutional factors and the reward and punishment structure (Damodaran, 2002, p. 3).

Valuation models and inputs are vulnerable and exposed to manual improvements to get the desired value. These models rely on many assumptions which can be easily biased by the analyst's opinion. The final value is also vulnerable to biases because if the desired or close to the market assumption is not acquired, the analyst can go back and manually adjust the inputs in order to obtain a "good" valuation result. Some include qualitative factors in order to explain the difference between their assumption and the market value.

Damodaran (2002) believes that although bias exists in the human nature and it is not possible to entirely eliminate it, there are certain ways which it can be minimized with:

- no institutional pressures for analysts;
- no connection between reward and punishments;
- not publicly speaking their opinion before the analysis is finalized;
- self-control;
- honest valuation process.

If these simple rules are followed the bias should be significantly minimized and consequently the more realistic value should be obtained.

### **1.3 Uncertainty in valuation**

Valuation is nothing than predicting the future value of an asset (stock) based on many assumptions and data available in the present (Penman, 2009). There is not enough data to predict what the future might bear so there is a lot of room for uncertainty embedded in the analysis. Damodaran (2006a) suggests that the inputs (assumptions) used in every valuation are exposed to three kinds of uncertainty:

- Hazy estimations: Analyst's view on the company's fundamentals;
- Company-specific risk: Complete misinterpretation of the current situation in the company;
- Country-specific risk: Unpredictable macroeconomic changes in the country.

Each company has different level of exposure and is differently vulnerable to the above-mentioned factors. Each information that generates an input for an analysis, has to be checked and updated constantly in order to get the best results, because new information constantly overflows the market (Damodaran, 2006a).

### **1.4 Managing uncertainty**

When making a valuation, dubious inputs arise. It is up to the analyst to confront those inputs and manage them properly. In his book Damodaran (2006b) says that there are few proper ways to deal with ambiguous results such as:

- Choose different, more suitable model;
- Use other models to value the same asset;
- Create a range of probable results;
- Assign a weight of probability to each result;
- Use simplified models in order to avoid hazy inputs.

On the other hand, if the analyst is not incentivised to act properly in order to elude the valuation error, he will most probably copy inputs from third parties and then blame them if the valuation turns out unreliable.

Previously, three types of uncertainties are mentioned. Some of them can be controlled and some of them cannot. Using and creating better models will maybe minimize the threat of hazy estimations but it won't affect company and country risk. Proper way to do better is to focus and stay away from information which are out of reach or be neutral about them. Convenient method is to use the best models, update them as new information come and concentrate on the best legally obtained company specific data, whilst staying indifferent on country specific inputs.

Valuation errors will always be present in the process. Henceforward, valuation is different for each company. Mature companies within stable industries are more easy to be valued because of the simplicity present in their future. Also, less assumptions are necessary (Damodaran, 2002). On the other hand, when valuing a start-up company in a new industry more inputs are needed and consequently the room for errors increases. An investor looking at the latter mentioned assumptions will conclude that the mature company is safer and a better investment. What is interesting in this example is the return embedded in each valuation. Because the start-up company has enhanced volatility, its return is higher, so analysts who are willing to value such companies are more interesting and have an edge against other analysts simply because their asset is more attractive (Henschke, 2009, pp. 35–44).

## **1.5 Risk and return**

Risk is not having enough information about certain events. If all information about everything are available, then risk would have not existed. Risk examination can be approached qualitatively and quantitatively. It can be defined as the possible variation in outcomes or deviation from the expected (Amenc and Le Sourd, 2003, p. 51). In the investment world, it is simple to understand: any action which has potential danger, implemented has potential benefit for taking on that danger. According to this, there is a positive relation between risk and return. Each additional information obtained is considered as an opportunity to better understand the future occurrences and thus minimize the potential risk by acting rationally and in accordance with the new obtained information (Rasmussen, 2003, p. 24). Risk is embedded in every company and it takes many forms: growth risk, operating risk, financing risk, fundamental risk, liquidity risk etc.

Rasmussen (2003, pp. 24–25) says that in order to quantify risk, it is necessary to take into consideration the attributes that define risk:

- Frequency;
- Magnitude;
- Confidence (Quality of information).

In every deviation from the expected, there is a potential opportunity. This is also one of the goals of asset managers, not to focus too much on the damage because in that way the opportunity that arises can be easily overlooked. Simply because future events cannot be predicted it is crucial to use the best tools possible in order to be able to estimate and manage risk. By doing so, there is a good chance of predicting the variability in the occurrences which are about to happen in the future.

One mistake among asset managers is that they are way too much focusing on the returns and not on the risk-management (Markowitz, 1952, p. 77). This is due to the incentive

schemes which are connected to the obtained return. In order for asset managers to be successful on the long term, it is very important to track their portfolio consistently and try to minimize variability as much as possible, because in that way the portfolio looks much more attractive.

- **Measuring risk**

Many areas are dedicated to measuring risk. In addition, many new tools and theories are developed to measure the price volatility. The core of risk is uncertainty about future events and the asset's reaction to those events. According to Rasmussen (2003, pp. 23–28) every risk-measuring methodology tries to form a scenario analysis in which the following questions could be answered:

- What are the possible risky situations?
- What is the probability of their occurrence?
- How badly they affect us?

Some of these questions can be answered numerically and others cannot. The questions which cannot be quantified can be arranged in order of probability and assign a number to each scenario, which will represent a probability of that scenario to become real (Damodaran, 2002).

- Systematic and non-systematic risk

Systematic (undiversifiable) or market risk has macroeconomic nature and affects the market as a whole (Saunders and Cornett, 2007, p. 273). Even the best risk manager using the best tools and diversification strategies cannot completely avoid this risk. It is possible to try to minimize it but never to completely eliminate it (Carol, 1999, p. 11). The best way for obtaining a measure of a particular asset's (or portfolio's) exposure to systematic risk is its *beta* ( $\beta$ ). The beta measures how much the particular asset (or portfolio) is volatile compared to its benchmark (the market as a whole) (Ross, Westerfield, and Jordan, 2010, pp. 415–419). Beta is calculated in two ways:

- With linear regression analysis where the dependent variable ( $y$ ) is the particular asset's return (or portfolio's return) and the independent variable ( $x$ ) is the benchmark's return (the market's return).
- With the following equation:

$$\beta_a = \frac{Cov(r_a, r_m)}{Var(r_m)} \quad (1)$$

Where:

$r_a$  = return of the asset

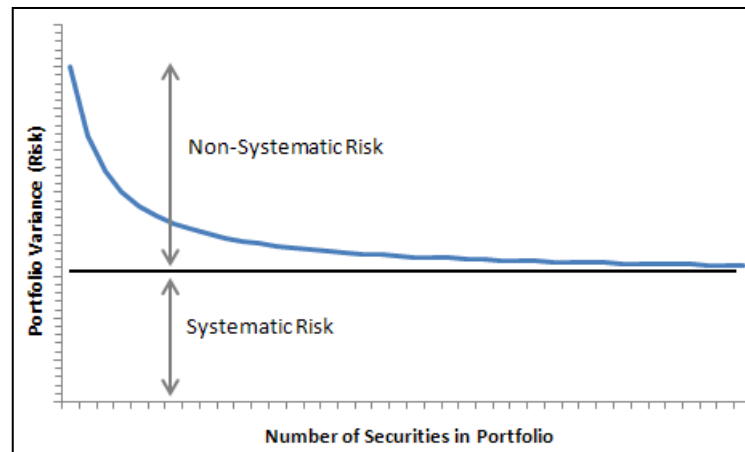
$r_m$  = return of the market

Based on the results from the above described methods, beta values are interpreted as:

- If  $\beta > 1$  means that the asset (or portfolio) has more systematic risk than the market;
- If  $\beta < 1$  means that the asset (or portfolio) has less systematic risk than the market;
- If  $\beta = 1$  means that the asset (or portfolio) has the same systematic risk as the market.

Following this, portfolio managers according to their calculations and expectations of the market in the future will add stocks (assets) with high betas in the portfolios if the market shows bullish activity and add stocks (assets) with lower betas if the market demonstrates bearish activity (Damodaran, 2002).

*Figure 1. Systematic and Non-Systematic Risk*



Source: *How Warren Buffett thinks about risk*, 2016.

On the other hand, company, sector (or industry) specific risk which each security possesses is the non-systematic (diversifiable) risk (Saunders and Cornett, 2007, p. 273). Unlike the systematic risk the non-systematic risk can be diversified.

In Figure 1 the number of stocks (assets) is presented with the blue line. By adding securities, the non-systematic risk reduces until it reaches a point where adding more securities does not make any difference (Goetzmann, n.d.). The systematic risk is not influenced by the number of securities and it stays the same. It is highly recommended to include assets from different industries in the portfolio since one specific occurrence in one asset or industry won't affect the whole portfolio but just that particular asset.

- **Variance and standard deviation**

One stock with a good performance in the past does not forecast anything about its future performance, but it is the best data that is available. Even if the sample satisfies all statistical rules and proves that it is significant, it is not enough. Risk is basically probability of deviations from the expected return (Bodie, Kane, and Marcus, 2010, p. 132). In the background of the variance is the averaged squared deviations from the expected return. Variance is always positive number. The smaller the variance value the better.

$$\sigma^2 = \frac{1}{n} \sum_{s=1}^n [r_{(s)} - E_{(r)}]^2 \quad (2)$$

Because the variance is a squared number in order to obtain interpretable result the Standard Deviation (hereinafter: *STD*) is calculated. This is the square root of the variance.

$$STD = \sqrt{\frac{1}{n} \sum_{s=1}^n [r_{(s)} - E_{(r)}]^2} \quad (3)$$

- **Covariance and correlation**

Co-variation is the variation over time of the individual assets relative to each other (Rasmussen, 2003, p.75). The covariance measures how the return of two assets in the same portfolio is moving. The covariance and correlation are very useful for measuring the diversification potential. Covariance can be obtained from the following equation:

$$Cov_{(r_a, r_b)} = \frac{1}{T-1} \sum_{t=1}^T (r_a - \bar{r}_a)(r_b - \bar{r}_b) \quad (4)$$

A very important part when constructing a portfolio is not to look at the risk of the individual asset itself but to look for the risk it brings to the whole portfolio (Bodie et al., 2010, p. 186). Following this statement, if the covariance is negative, it implies that the return of those two assets is moving in the opposite direction which consequently lowers the portfolio's expected risk. An investment which has perfect negative covariance with the initial portfolio can be claimed as a perfect hedge position. On the other hand, if the covariance is positively correlated, it implies that the return of the two assets in question is moving in the same direction thus increasing the portfolio risk. The covariance (and correlation) matrixes are presented in appendix G (Tables 18 through 23) and appendix H (Tables 24 through 29).

Because the covariance is an absolute measure between two assets it does not contain the individual's asset return deviation from the mean. To fix this, the covariance is standardized (Damodaran, 2002, p. 16–17). Since the deviation from the mean is quantified through the standard deviation (*STD*), in order to standardize the covariance, it is divided with the multiple of each assets individual standard deviation.

By doing this, the correlation coefficient is calculated. It is worth mentioning that the covariance of the assets return with itself is the assets variance. The correlation coefficient is implying that the two assets are directionally correlated (Rasmussen, 2003, p. 78). Its value can be between -1 and 1, the latter meaning that the assets are perfectly correlated. Because the correlation coefficient does not respect the magnitude of the moves, it cannot be concluded based on the correlation coefficient that one asset follows the other. In addition, the equations for calculating the correlation coefficient are presented.

$$Corr(r_a, r_b) = \frac{\frac{1}{T-1} \sum_{t=1}^T (r_a - \bar{r}_a)(r_b - \bar{r}_b)}{\sqrt{\frac{1}{T-1} \sum_{t=1}^T (r_a - \bar{r}_a)^2 * \frac{1}{T-1} \sum_{t=1}^T (r_b - \bar{r}_b)^2}} \quad (5)$$

Or the simplified version of equation (5):

$$Corr(r_a, r_b) = \rho_{a,b} = \frac{Cov(r_a, r_b)}{\sigma_a * \sigma_b} = \frac{\sigma_{a,b}}{\sigma_a * \sigma_b} \quad (6)$$

- **Standardizing risk**

The desired result is to estimate the expected volatility of returns. In order to do that the risk is standardized. After calculating the variance and the standard deviation of each asset in order to standardize them to be consistent with the other inputs, the values are multiplied by the number of months in one year which is 12. The final number is the annualized one which is further used in the calculations. The results are in the appendix A (as Table 1).

- **Portfolio risk and diversification effect**

The whole portfolio theory is based on incorporating the risk in investment decisions. The process of calculating the portfolio risk is more complicated. The risk of a single asset is quite straightforward. However, when constructing a portfolio with more assets the procedure for calculating risk is harder. The complication emerges from the fact that each asset has different characteristics and behaves differently. An asset which seems risky can bring stability to the return of the portfolio. Based on that behaviour, the portfolio can yield

unexpected results (Bodie et al., 2010, pp. 182–183). The general equation for calculating portfolio variance (risk) is:

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i * w_j * Cov(r_i, r_j) \quad (7)$$

The risk of the particular portfolio is obtained through its volatility. The volatility depends on the covariance between the assets within the portfolio (Sanders and Cornett, 2007, p. 175). The essence of diversification is to find correlated assets and include them with appropriate weights in the portfolio in order to somehow reduce and control the volatility (Penman, 2009, p. 648). This relies on the assumption that asset returns tend to move in consistent patterns to each other.

Another important aspect of portfolio creation is the number of assets which are to be added in the portfolio. It is crucial not to look at the assets individual risk but the risk it contributes to the whole portfolio when added. The right number is highly debatable but thanks to the modern portfolio theory it is possible to calculate the returns, volatilities and correlations for any number of assets thus optimizing and choosing the best combination. The type of the asset which is in the portfolio also has great role in optimizing. As a rule of a thumb, a satisfactory number of assets is between 15-20. After that, each additional item tends to slightly reduce the portfolio volatility. Adding additional assets bears transactional costs and the investor at a certain point in time will suffer overall loss. But, if the minimum-variance portfolio is calculated when each new stock is added in the portfolio the results are better off approximately by 3% (Rasmussen, 2003, p. 126).

In this context, it is also worth mentioning the term **international diversification**. It means exactly how it sounds. By diversifying across other markets which are not perfectly correlated the volatility of the portfolio is reduced. The dispersion of returns between assets (and indexes) persists and as long it is that way, the diversification benefits are possible. Investors have been cautious about investing outside their country of residence. Reasons for this occurrence are that the investors perceive the rest of the markets as less efficient and transparent (Bodie et al, 2010, pp. 901–908). Also, the currency risk plays a great role in the decision for international diversification (Bodie et al., 2010, p. 904).

- **Portfolio return**

Calculating the portfolio return is similar to calculating the return of a single asset. The difference is that the return of an asset is multiplied by the weight with which the asset contributes in the portfolio. The weight should constantly be  $w=1$ . The summed-up value of all assets weights multiplied by their return gives the total portfolio return.



$$r_p = \sum_{n=1}^N (w_n * r_n) \quad (8)$$

The calculation is straight forward for simple portfolios consisting of small number of assets. When it comes to calculating large portfolios return, the matrix notation is used. The portfolios return in this research is obtained using the matrix notation. Using the linear matrix notation, it is suggested that portfolio returns are linearly related to individual assets weight and returns (Rasmussen, 2003, p.74).

$$r_p = [w_1 \dots w_2 \dots w_n] * \begin{bmatrix} r_1 \\ r_2 \\ r_n \end{bmatrix} \quad (9)$$

- **Required and expected return**

Investors are not interested in the price of the asset but in its performance. The measure for performance is the return that it produces over time. Calculating the return is the starting point for portfolio creation. The other core question is the risk of those returns or in other words how the return varies over time. The calculation of returns can be applied to a single asset as well as on a portfolio of assets.

Cost of capital is used as a synonym for required return. This is what the investor asks for, in return for the risk that he is willing to take by making the particular investment. The CAPM approach assesses what is the rational required rate of return for the particular risk. Markets are not efficient meaning that asset price does not always ideally yield the required return (Machado, 2012). This is one of the goals of active asset managers, finding mismatches in asset prices where expected return is different from the required one (Amenc and Le Sourd, 2003, pp. 6–8). Both, expected and required returns would ideally be equal only if the markets were efficient (Rasmussen, 2003, p. 138).

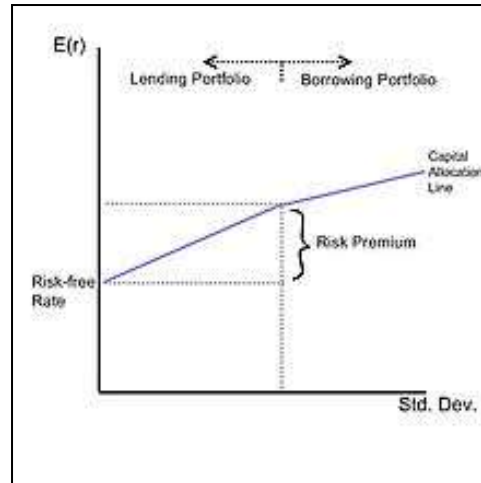
## 1.6 Capital Allocation Line (CAL)

The expected return is also expressed as a function of the standard deviation. Its slope is known as the reward-to-variability ratio. The Capital Allocation Line (CAL) is a graph which helps to assess the risk of risky and riskless assets (Viswanath, 2000). The line is mathematically obtained from the equation (10) and it displays the expected return for each level of risk taken by investors (Bodie et al., 2010, p. 212).

$$CAL = \frac{E(r_p) - r_f}{\sigma_p} \quad (10)$$

The slope of the *CAL* depends on the investors risk aversion. If the investor is more risk inclined the slope would be steeper (Capital allocation between a risk-free asset and a risky asset, 2015). Consequently, if the investor is completely risk averted the slope of the *CAL* will be 0. The slope of the *CAL* is defined by the Sharpe ratio (19).

*Figure 2. Capital Allocation Line*



Source: *Capital allocation between a risk-free asset and a risky asset*, 2015.

## 2 VALUATION, VALUATION ERROR AND EFFICIENT FRONTIER

### 2.1 Data selection

The data used in this research are 50 randomly selected stocks within the UK's FTSE 100 index. Each of these stocks is listed on the London Stock exchange. They are chosen randomly, in order to avoid the bias, because some of the companies are popular and some of them are not so popular. Not each industrial sector is included in the analysis. The time period is from 01.01.2009 to 31.12.2014 and the returns are presented and calculated using the total return index.

The inputs for the models are obtained from the Bloomberg platform, Datastream as well as Yahoo finance, Google Finance UK, FTSE 100 Index (2015) and from a forecasting web page 4-traders (2015). The data was collected on 24.04.2015 and the real prices used for comparison to the model results, are the prices of the individual stock on that day. For each model and stock, the inputs are individually inserted and adjusted to the company's current situation. The prices are in United States Dollar (USD). Monthly returns are calculated and then each input relevant for the analysis is annualized in order to achieve more comparable and presentable results.

Each of the 50 stocks is valued using 3 models. Based on their valuation errors the stocks are divided into 5 categories and efficient frontier is created for each category. At the end, one efficient frontier for all stocks is created.

## 2.2 Excess return and risk free rate

Important part in the asset management and valuation process is to include and determine the risk-free rate. It is defined as the return of a riskless asset. The most liquid government bond is a good indicator. The risk-free rate used in this research is the UK's 30-years government bond which in the period when the data for this research is conducted ranges from 2.10% - 2.45% (UK bond rate, 2015). The risk-free rate is highly correlated with the particular countrys economy situation, future growth projections, change in fiscal and monetary policies and many more macroeconomic factors. Corresponding to this, the excess return is defined as return above the risk-free rate of return (Rasmussen, 2003, p. 18). In other words, the return generated by the skills of the asset manager.

$$r_a^0 = r_a - r_f \quad (11)$$

Where:

$r_a^0$  = Excess return;

$r_a$  = Asset return;

$r_f$  = Risk free rate.

For harmonization with the overall inputs, equation (11) is slightly modified because the risk-free rate has been changing over the 5-year time frame.

$$Excess\ return_a^0 = Total\ Return\ Index_{asset\ a} - Mean_{asset\ a} \quad (12)$$

## 2.3 Total return index

To avoid problems whether dividends and stock splits are included in the asset price and to evaluate the returns as correctly as possible, the Total Return Index (method 1) is used. It is calculated for individual assets. This index represents growth in the asset value held over particular time period, simultaneously assuming that the dividends received within that time frame are used to buy additional assets at the closing price on the ex-dividend date (Thomson Reuters, 2015). This index is being calculated from 1988 onwards using two methods. In both methods gross dividends, no tax and no re-investment costs are applied.

The Return index is calculated using the annualized dividend yield. This method assumes that there are 260 working days per year and accordingly adds  $1/260^{\text{th}}$  part of the dividend yield to the asset price each weekday (Thomson Reuters, 2015).

- Method 1: Annualized dividend yield

$$RI_t = RI_{t-1} * \frac{PI_t}{PI_{t-1}} * (1 + \frac{DY_t}{100} * \frac{1}{N}) \quad (13)$$

Where:

Return index on the base date = 100

$RI_t$  = Return Index on day t

$RI_{t-1}$  = Return Index on the previous day

$PI_t$  = Price index on day t

$PI_{t-1}$  = Price index on previous day

$DY_t$  = Dividend Yield in % on day t

$N$  = Number of working days per year (260)

- Method 2: Ex-dividend date

$$Return\ Index_t = Return\ Index_{t-1} * \frac{P_t + D_t}{P_{t-1}} \quad (14)$$

Where:

$P_t$  = Price on ex-date

$P_{t-1}$  = Price on previous day

$D_t$  = Dividend payment associated with ex-date t

- **Total return index standardization**

Assets have different number of trading days. In order for the total return index (for the time period from 01.01.2009 to 31.12.2014 obtained from DataStream) to be representative it is standardized (annualized). Everything is calculated over the 5-year-course on monthly basis, so in order to prepare the input data for further analysis the mean and the standard deviation of each asset is multiplied by 12.

Standardization is helpful for investors when comparing the performance of two assets with different returns which have been owned for different time periods (Bodie et al., 2010, p. 117). In the appendix A (as Table 1) is the table with each stock information such

as the ticker, the industry in which operates, historical standardized (annualized) average return and standardized (annualized) standard deviation.

## 2.4 Valuation error

Frequently debated area when valuing a company's stock is the valuation margin. In other words, which is the highest acceptable margin of error. The valuation margin (or valuation error) is the disparity between a company's valuation estimate and current market price (Ogunba and Iroham, 2010, p. 54). It is important to understand that the value of the company is perceived differently from different parties depending on their interest to the company. It is also crucial to differentiate between company's value and company's price (Fernandez, 2002, p. 2). The latter is the negotiated and accepted price between the buyer and the seller and can substantially deviate from the company's value. This difference can be from numerous reasons which are complicated.

Valuation is used for many purposes such as: mergers and acquisitions, initial public offerings, compensation schemes, strategic decisions. In this research valuation is intended for listed companies in order to know how to proceed regarding the company's stock based on the valuations. According to Henschke (2009, p. 27), there are different benchmarks for comparing the punctuality of the model which are roughly divided into two categories analytical and empirical benchmarks.

Analytical are further divided in sensitivity analysis and simulations, and empirical benchmarks are further divided into observed prices and ex-post returns. This research is focused on empirical benchmarks, more specifically on observed prices because it avoids unknown or hard to estimate inputs such as future payoffs which are crucial for valuation. Empirical analysis directly compares visible values like returns and current stock prices. Consequently, it is easier to reveal where the mistake is. By using this method, when presented with large deviations between estimated and market price there are two possible views. First, that the market is efficient and always right, consequently something is wrong with our estimations and vice versa that the market is inefficient and that it is possible for the price to be abnormally high or low. The intrinsic methods in this research came up with different valuation errors which are later presented.

The three models which are used in this research (*DDM*, *RI* and *FCFF*) are supposed to deliver the same results but under the ideal valuation conditions. In the real world, ideal inputs are hard to find (predict) simply due to the uncertainties and risks which follow the everyday occurrences.

This has been the basis for authors like Penman and Sougiannis, 1998; Francis, Olsson, and Oswald, 2000; Courteau, Kao, and Richardson, 2001) to conduct a research. What is interesting is their results all came across similar results that *RI* is better model than *FCFF*

and *DDM* based on the velocity of the valuation error. The above-mentioned authors had concluded that the error depends on many inputs but mostly on payoff conditions, steady state assumptions, dividend and terminal value growth rates. They had also stated that it is possible for these models to present equal results if the inputs are properly adjusted.

According to Hess, Homburg, Lorenz, and Sievers (2008, p. 5) there are two identified problems connected with forecasts: discount rates and cash flows. They are considered as a problem because discount rates are determined separately in the valuation process and different inputs may be entered in order to determine the discount rates. There is also a difficulty with the estimation of correct free cash flows, because of accounting standards.

Dittmann and Maug (2006) tried to explain if the ranking of some model gives the lowest valuation error, depends on the error measure.

Percentage errors are stricter when it comes to overvaluations than undervaluation, because undervaluation larger than 100% are impossible (Dittmann and Maug, 2008, p. 1). The accuracy of each valuation model depends on the error measure used to compare. There are four error measures by which the error of the valuation methods is measured in this research:

- Relative percentage errors;
- Absolute percentage errors;
- Squared errors;
- Logarithmic errors.

According to Penman and Sougiannis (1998), out of the three methods which are used in this research, the residual income model should show best results when back tested to percentage and logarithmic error measures.

On the other hand, if only percentage error measure is used, the *DDM* model is better than the *FCFF* model and if the logarithmic error measure is used, the *FCFF* model gives better results than the *DDM* model. The latter statement is true because when using the *DDM* model the value of the company is underestimated and consequently the logarithmic error measure is more sensitive to it.

Nevertheless, if clean surplus relation (*CSR*) between *DDM* and *RI* model holds, it is expected both methodologies to yield equal results. *CSR* suggests that changes in book value of equity between two given periods can be explained only from differences between earnings and net dividends (Hess et al., 2008, p. 6).

This research is not aimed to determine which error measurement should be chosen, but to emphasize the characteristics on each valuation error technique.

As mentioned above, in order to compare the findings with the real values and document the results there are different error measures. Henschke (2009, p. 29) states that each error measure is more or less inclined to certain valuation method depending on the inputs which it favours. It is beneficial to determine which error measure suits most for which valuation technique because if not so, different conclusions and rankings will come up. All error measures agree that the estimated value should be as close as possible to the observed market price. Also, that the error measure reflects (or should reflect) the ability of the analyst to precisely assemble and present the intrinsic valuation model.

In addition, the valuation error measures are described. In all of the following equations  $P_v$  is the valuation price whereas  $P_m$  is the observed market price.

- **Percentage Error or bias** is the most frequently used method and it simply represents the percentage difference in either direction of the valuation model compared to the observed market price:

$$\text{Percentage error} = \frac{P_v - P_m}{P_m} \quad (15)$$

- **Absolute percentage error or accuracy** gives the same results like the percentage error only in absolute terms.

$$\text{Absolute Percentage error} = \left| \frac{P_v - P_m}{P_m} \right| \quad (16)$$

- **Squared error**

$$\text{Squared error} = \left( \frac{P_v - P_m}{P_m} \right)^2 \quad (17)$$

- **Logarithmic error (Log error)**

$$\text{Log error} = \ln \left( \frac{P_v}{P_m} \right) \quad (18)$$

As mentioned earlier, different error models assign different weight on deviations. The percentage and absolute percentage error put equal weights on deviations. Squared error punishes deviations more and puts more weight on values which deviate further from the market value. At the end, logarithmic error puts more weight on stronger overvaluations.

- **Valuation error measures**

According to the error measures, *FCFF* has the smallest overall valuation error. It has overvaluation of 2% based on the percentage error (Table 1). Results from the log errors (Table 2) also show that *FCFF* has the smallest undervaluation of -10%. According to squared error (Table 3) results, the residual income model has the smallest error of 16% overvaluation, followed by *DDM* and *FCFF* model, both of which have 20% overvaluation. The best valuation model according to the absolute percentage error (Table 4) is the residual income model with 35%.

The average value of all valuation models is calculated and included in the tables as analysis average. Squared error (Table 3) shows the best result with overvaluation of 8% followed by the percentage undervaluation of -18% (Table 1). Log and absolute percentage errors show the same valuation error of -24% and 24% consequently.

*Table 1. Calculated percentage errors*

<b>Percentage Errors</b>				
<b>Method</b>	<b>Mean</b>	<b>Median</b>	<b>STD</b>	<b>Skewness</b>
DDM	-24%	-37%	37%	0.93
FCFF	<b>2%</b>	-6%	44%	0.51
RI	-33%	-34%	22%	0.58
Analysis Average	-18%	-17%	23%	0.23

*Table 2. Calculated log errors*

<b>Log Errors</b>				
<b>Method</b>	<b>Mean</b>	<b>Median</b>	<b>STD</b>	<b>Skewness</b>
DDM	-39%	-47%	49%	0.01
FCFF	<b>-10%</b>	-6%	57%	-2.21
RI	-45%	-42%	35%	-0.65
Analysis Average	-24%	-19%	31%	-1.05

*Table 3. Calculated squared errors*

<b>Squared Errors</b>				
<b>Method</b>	<b>Mean</b>	<b>Median</b>	<b>STD</b>	<b>Skewness</b>
DDM	<b>20%</b>	17%	17%	1.11
FCFF	<b>20%</b>	11%	29%	2.50
RI	<b>16%</b>	12%	14%	1.36
Analysis Average	8%	4%	10%	2.46



Table 4. Calculated absolute percentage errors

Absolute Percentage Errors				
Method	Mean	Median	STD	Skewness
DDM	39%	41%	22%	-0.05
FCFF	36%	32%	26%	1.29
RI	<b>35%</b>	34%	18%	0.13
Analysis Average	24%	20%	16%	0.87

The results show that the best error measure is the percentage error followed by the logarithmic error, because they have the smaller error margins. Also, the *FCFF* model is the most suitable one to use, because it also has the smallest errors.

Investors tend to choose simpler error valuation models which they can understand. However, regardless of the error measure, the investor's goal is to include possibly all undervalued companies in his portfolio and since both models rank the companies in the same order practically the investor is indifferent which error model he uses. Nevertheless, the acknowledgement of the errors in the eyes of the investor depends on the role he is in. An acquiring company, looking to make a bid based on these errors, will look at the errors differently and assign different weights than an average investor who is simply willing to earn some extra income via investing.

The calculations and the graphical representation of each valuation error are presented in appendix C (Tables 7 through 10) and appendix D (Figure 1 through 4).

## 2.5 Efficient frontiers and portfolios

Adding more assets to a portfolio in order to reduce risk is not always a good idea. When the volume of the assets increases, it is harder for the portfolio manager to control the portfolio and shift it to reach the desired goals. The goal of the portfolio manager is to find assets which form low correlations between each other. In that way, the assets with undesired characteristics are substituted in the portfolio by assets which will help diversify the potential risk. In order to do this, the portfolio manager should be familiar with the investors requests and to establish guidance objectives which will explain the desired asset characteristics.

"Efficiency is the utilisation of resources in such a manner that the maximum output or gain is generated" (Rasmussen, 2003, p. 98). The term efficiency in the portfolio context is defined as: "Maximum generated return for a given level of volatility or minimum volatility for a given level of return" (Rasmussen, 2003). Synonym term when speaking about portfolios is the mean-variance efficiency (M-V). With the help of the portfolio theory the investor can re-design the portfolio's constitution basically at no cost in order to

obtain more efficient results. The quantitative portfolio optimisation helps quantify the return, the risk and the efficient frontier helps to visualize that connection.

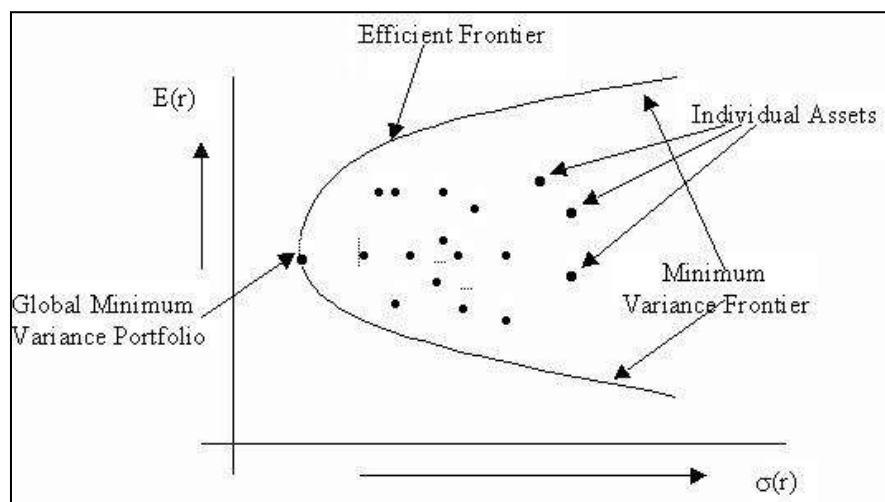
In the following sections, an efficient mean-variance portfolio will be referred as an optimal portfolio. In order for the portfolio to be efficiently tailored, it is crucial to assign different weights to each asset. Random selection of weights consequently returns inefficient portfolios. With the help of mathematical algorithms logical weight is assigned to each asset. The latter mentioned concept is known as portfolio optimisation and it consist of a variety of different techniques. According to Rasmussen (2003, p. 100), the main goal of the portfolio optimisation techniques is to satisfy the investor's needs (in terms of desired return) while simultaneously doing one of the following three things among which the last two yield the same result:

- Determine the Minimum-Variance portfolio;
- Determine the minimum level of volatility for a given level of return;
- Determine the maximum level of return for a given level of volatility.

### 2.5.1 Efficient frontier

"It is the line between the Minimum variance portfolio and the maximum variance portfolio that traces out all attainable portfolios (asset combinations) that produce the optimal/efficient portfolios" (Rasmussen, 2003, p. 112).

Figure 3. Efficient frontier graph



Source: P. V. Viswanath, *Optimal Portfolio Construction and Selection*, 2000.

Everything is presented on the efficient frontier graph. On the x-axis is the volatility and on the y-axis, is the expected return. In Figure 3, the minimum variance frontier of risky assets represents the lowest variance for a portfolio with a given return. The black dots are

individual assets and all lie right of the minimum variance frontier which means that by itself, each asset is inefficient. The candidates for the optimal portfolio lie on the minimum variance frontier above the global minimum variance portfolio. This part is also called the efficient frontier.

Fifty randomly selected stocks are divided into 5 groups based on their absolute percentage valuation error calculated from the average price of all three models (DDM, FCFF and RI) compared to the current price:

- Group 1 – Valuation Error 0% - 10%
- Group 2 – Valuation Error 10% - 17%
- Group 3 – Valuation Error 17% - 26%
- Group 4 – Valuation Error 26% - 40%
- Group 5 – Valuation Error 40% - 76%

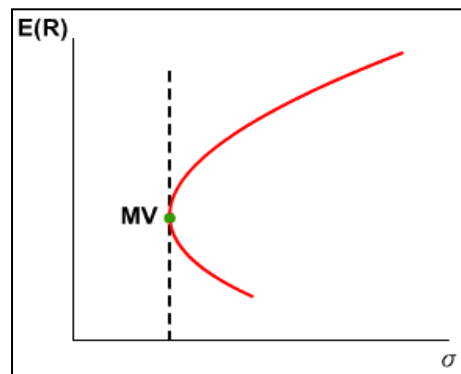
For each of these groups an efficient frontier is created and the optimal, M-V and equally weighted portfolio is calculated. At the end, from each asset is created one efficient frontier and the same abovementioned parameters are obtained. The list of stocks in each group is in appendix B (Tables 2 through 6).

The whole concept of the modern portfolio theory and efficient frontier is based on assumptions. The investor is not obligated to pay transaction costs, the return is tax-free etc. It is also assumed that upon every dividend pay out, the value of the stock decreases. Because of this, the correlation between stock prices is affected and consequently the form of the efficient frontier. But since the dividend pay outs are discrete and happen maximum of 2 times per calendar year, the impact is negligible.

### **2.5.2 Minimum variance portfolio**

The M-V portfolio is obtained using complex mathematical algorithms where the weight of each asset in the portfolio is tested and changed as long as the minimum volatility of the portfolio is obtained without further return minimization. In Figure 4, this portfolio is represented by the green dot. By doing so, some assets are left out of the portfolio or are present with a very small weight. The line above the dot is the efficient frontier and all rational portfolios lie on that area. Portfolios below the M-V portfolio are inefficient. The result is that the portfolio risk is lower than the overall market's risk. This is due to the possibility to adjust the weights and even leave out assets.

Figure 4. Minimum Variance Portfolio



Source: E. J. Elton, S. J. Brown, M. J. Gruber and W. J. Goetzmann, *Modern Portfolio Theory and Investment Analysis*, 2009.

While searching for the M-V portfolio, there are two constraints: the weight of the assets should be 1 in order to fulfill the goal to invest completely and the restriction of short sales. Once the M-V portfolio is determined, it is possible to obtain the expected return which corresponds with that particular volatility.

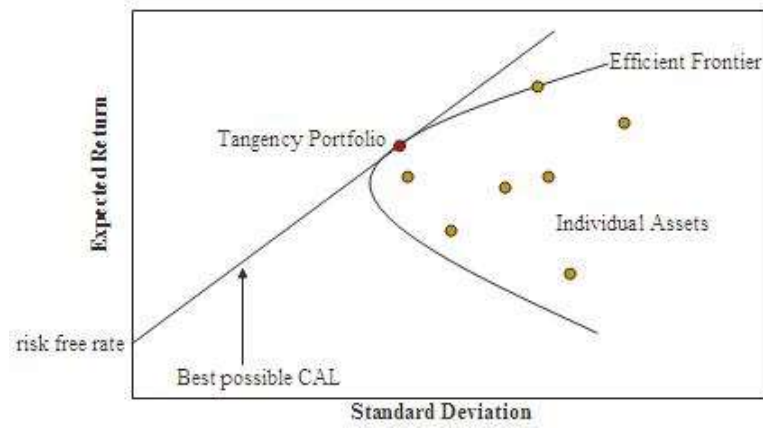
### 2.5.3 Optimal risk (tangency) portfolio and the capital market line (CML)

While the goal in the M-V portfolio is to find the minimum risk, in the optimal risk (tangency) portfolio the goal is to find the best portfolio which dominates all other risky portfolios. It is the market portfolio where investors only mix the risk-free asset with the market portfolio according to their risk preferences (Rasmussen, 2003).

In addition, the Sharpe ratio in this portfolio is the highest, meaning that the optimal risk portfolio is the *CAL*, which is tangent on the efficient frontier with a starting point at the risk-free rate, which in this research is 2.45%. In this case that tangent is referred as Capital Market Line (*CML*). Every rational investor will choose the best *CML* and allocate his investment between the risk-free asset and the optimal risk set. The optimal risk portfolio is basically maximizing the Sharpe ratio while keeping the sum of the weights to 1.

Every rational investor will choose to invest on the *CML* line. The only difference is in the risk aversion amongst investors, because some of them will allocate more in risk free assets and the rest in the optimal portfolio. This is known as Tobin's two fund separation theorem.

Figure 5. Optimal risk (tangency) portfolio and the Capital market line (CML)



Source: *Efficient frontier*, n.d.

On the y-axis is the expected return and on the x-axis, is the risk depicted as standard deviation. The red dot in Figure 5 is the market portfolio, meaning that in that point all rational investors will have portfolio consisting of risky assets in the same proportion as their weights. The slope of the CML is an indicator of the change in the return based on the change of the risk (standard deviation). With the help of CML investor maximize their return, given a certain level of risk (Bodie et al., 2010, pp. 210–213).

## 2.6 Sharpe ratio

Originally created in 1966 by the Nobel prize winner William F. Sharpe, the Sharpe ratio is one of the most frequently used indicators for risk-return measure. It measures the average return in excess of the risk-free rate per unit of volatility (Sharpe ratio, 2015). This ratio is used in the modern portfolio theory. William F. Sharpe (1994) in his paper presents the logic behind it, in connection with portfolio creation. He states that when adding assets with correlations  $<1$  in a well-diversified portfolio, it can decrease volatility without influencing the return. Consequently, this will increase the Sharpe ratio of a portfolio. In conclusion, the portfolios with higher Sharpe ratios are more attractive. The equation (19) is used to calculate the Sharpe ratio:

$$Sharpe = \frac{E_{r_p} - r_f}{\sigma_p} \quad (19)$$

Where:

$E_{r_p}$  = Expected return of the portfolio

$r_f$  = Risk Free rate

$\sigma_p$  = Portfolio Volatility

## 2.7 Inputs for the valuation

Valuation is a sensitive process which consists of three key inputs:

- Cash flows – the cash coming from owning the asset;
  - Time - the time periods of receiving the cash flows;
  - Required return – which is used to discount the future cash flows to present time. It also represents the level of risk incorporated.
- 
- **Weighted average cost of capital (WACC)**

The result which the valuation model shows is highly dependent on the inputs which are used in that model. Consequently, the valuation error is dependent on those inputs too. The Weighted Average Cost of Capital (WACC) in this research is used for discounting the free cash flows of the company to their present value. Since in the WACC are included all sources of capital (stocks, bonds, any debt), a slight increase in the WACC value leads to greater risk and decrease in the final valuation result. In the core, it is right to explain the cost of capital as “The opportunity cost of all capital invested in an enterprise.” (Damodaran, 2002, p. 53). The equation to calculate the WACC is the following:

$$WACC = \frac{Equity}{Equity + Debt} * r_e + \frac{Debt}{Equity + Debt} * r_d * (1 - Corporate Tax) \quad (20)$$

Where:

$r_e$  = Cost of Equity

$r_d$  = Cost of Debt

Cost of Debt is relatively easy to calculate because the interest rate under which the debt is acquired is pre-determined and known. In this research the WACC value is obtained from the Bloomberg terminal and then compared to own calculations.

- **Cost of equity**

In the WACC calculation it is important to determine the cost of equity. It is not satisfactory to assign certain cost to the capital because capital does not have any determined value. Shareholders, in order to invest in the particular company, require some compensation for the risk they are taking (Chartered Financial Analyst Institute, 2015). From the company's point of view, the return that the investors require from the company is the cost of equity. If this criterion is not met it leads to destabilization and unsatisfied investors who are more likely to sell their share. The best way to calculate the cost of equity is the CAPM model (Capital Asset Pricing Model).

$$CAPM = r_f + (r_m - r_f) * \beta \quad (21)$$

Where:

*CAPM* or  $r_e$  = Cost of Equity

$r_f$  = risk free rate

$r_m - r_f$  = Market risk premium

$\beta$  = Beta (unsystematic risk)

In addition, each component of the *CAPM* model is explained.

- The risk-free rate is the rate of the securities such as government treasury bills which are considered free of credit risk (Bodie et al., 2010). In the research is used the UK's 30-year government bond with value of 2.45%.
- Beta - With beta interpretation it is compared how much the particular company is connected to the market. If the beta value is 1, it means that it is perfectly correlated with the market's movement and follows the market trend. If it is under 1, it means that the company is more stable and does not fluctuate as fast as the market does and finally a beta larger than 1 means that the company reacts stronger than the market movement (Damodaran, 2002). The beta for each company is different. It can be calculated by regressing the stock returns with the benchmark's market returns. The slope is the company's beta. Another way of calculating beta is the bottom up procedure, but it requires very good knowledge and the company's business mix in order to calculate it. In this thesis, the unlevered bottom up beta taken from Bloomberg terminal is used for each company and then compared to the slope of the regression.
- Equity Market Risk Premium ( $r_m - r_f$ ) is the excess return that the investors plan to acquire for investing in a security with return rate greater than the risk-free rate.

According to Watson and Head (2007, p. 222), the *CAPM* model is fairly simple to calculate but it relies on many assumptions connected with the market efficiency:

- Investors are price takers meaning that they can't affect the market prices;
- Investors have only one (and the same) holding period;
- Investors can borrow and lend at the risk-free rate and are limited to publicly traded assets;
- Investors are tax exempt;
- All investors are mean-variance optimizers meaning they use and stick to the Markowitz's portfolio model;

- All investors have the perfect information and process that information in the same way;

In the above mentioned assumptions, many IFs are present and once the cost of equity is calculated some adjustments which account for risk can be implemented and small corrections can be assigned.

- **Cost of debt**

The cost of debt comes from the lender's exposure to the company. If the lenders (banks), after carefully examining the company's financial records believe that the particular company is potentially safe, consequently the interest rate of the loan will be lower and vice versa. For company valuation, the after-tax cost of debt is used. Damodaran (2002, p. 286) believes that in order to calculate the cost of debt three components are crucial:

- Risk Free Rate - This is the same value used for calculating the cost of equity. Thumb rule is that if the cost of equity relies on the risk-free rate, then the cost of debt should too;
- Default spread - Where three approaches are used in order to determine the reliability of a company. It is based on the company's bonds (if there are any) or on credit ratings from companies such as Moody's. Also, to get a clearer picture the synthetic rating based on the company's financial records is used;
- Marginal Tax Rate - It is used because interest payments on a loan are tax exempt.

The general equation about calculating the cost of debt is:

$$\text{After Tax Cost of Debt} = (\text{Riskfree rate} + \text{Default spread}) * (1 - \text{Marginal Tax Rate}) \quad (22)$$

For the purpose of this research, the cost of debt for each company is obtained from Bloomberg terminal.

- **Corporate and effective tax rate**

Corporate tax is the tax that the particular company pays, based on its income for the current year (KPMG, n.d.). The tax rate is determined by the government and in the UK for 2015 is 20% and the average rate from 2009-2015 is 24.28% (Tax rates and allowances, n.d.). The corporate tax rate is used in the calculations for WACC.

For the needs of this research the effective tax rate is also used. In the terms of a company, the effective tax rate is explained as the average rate at which its pre-tax profits are taxed (Damodaran, 2002).



$$\text{Effective Tax Rate} = \frac{\text{Total tax expenses}}{\text{Earnings before taxes}} \quad (23)$$

This rate is better than the marginal tax rate because it gives the average rate meaning that companies with the same marginal tax rates can have different effective tax rates, because different incomes are taxed differently. The effective tax rate in this research is obtained from the Bloomberg Terminal.

- **Growth rate**

Important input in all models is the expected growth rate in earnings and in cash flows (CFs). Growth comes from within companies. Companies focus on two types of growth (Damodaran, 2006a, p. 2):

- Sustainable growth - generated with investing in new assets;
- Efficiency growth - exploiting the maximum potential growth from current assets in place.

When valuing the company's growth, analysts look in the past performance of the company. It is doubtful whether the past performance is sufficient to predict the future performance of a company including all the risks which the market brings. Growth is applied on *EBIT*, *EBITDA*, *EPS*, Revenue and Net Income. This research uses growth predictions for EPS as a measure of company's potential growth.

Analysts are better growth predictors than historical patterns because in their analysis are included the newest occurrences on the market and have feeling of how much and what the company is doing to grow. On the other hand, equity analysts, can be biased towards the particular company therefore giving more optimistic forecasts.

Another way of estimating growth rates rather than analysts and historical growth is to look at the company's fundamentals. Companies in the UK's FTSE 100 Index are in mature industries with limited potential to exponential growth, so it is only fair to suppose that most of their growth is generated by exploiting the maximum growth potential from assets in place. It is hard for these companies to maintain high returns from re-investments. For the purposes of this research the growth rates in *EPS* is used. The *EPS* forecasts are obtained from different sources and then implemented in the models (Index detail: FTSE 100 Index, 2015).

The hardest question to answer is how long one company can maintain its growth. It is not long because potentially smaller companies which are present in the industry can become more competitive and even rule out the mature ones. A rule of the thumb is that larger companies have lower growth rates. One of the key roles of valuation is to determine the

length of high growth value (Penman, 2009). As time passes, large companies face a pressure to reduce growth because of the reinvestment rates and return on capital. The period of 5 years high growth rate is used for all companies in this research.

Another crucial input is the terminal value. Since nobody can predict the cash flows indefinitely, it is logical to determine the end of the high growth rate and from that point on to establish a terminal growth rate (Penman, 2009, p. 591). This rate cannot exceed the growth rate of the country's economy in which it operates (Damodaran, 2002). By doing that, it is ensured that the terminal growth rate is lower than the discount rate. There is a reference that also the terminal growth rate should not be larger than the risk-free rate. If the company is multinational and operates all around the world, then the referent growth rate which cannot be exceeded is the world growth rate. The UK's economy is expected to grow around 2% annually (Pricewaterhouse Coopers, 2015), the terminal growth for most of the companies in the analysis is set around that number and used in the model.

## **2.8 Equity and enterprise valuation**

The starting point for valuation is the hypothesis that the value of an asset is equal to its cash flows discounted to present time. For a good asset valuation, certain set of conditions are followed. The most important one is to know the nature of the company in question. Each company is different and has its own business model. The first job of the analyst is to identify that model and put himself "inside the company", understand its plans, workflow, current situations, competitive advantage, how the macroeconomic factors can affect the company, etc. (Damodaran, 2006a). The process of valuation is different, therefore there is not a strict procedure to follow. When conducting such a valuation certain parameters are important:

- Macroeconomic factors within the industry in which the company operates - By understanding both the company and the industry the analyst will be able to compare the company's numbers with its benchmark (the industry) more accurately, simultaneously assessing if the comparison is meaningful and applicable.
- Forecast the company's performance – how the company will adjust to the future events;
- Choose the suitable valuation model;
- Derive a result from the valuation;
- React relying on the valuation.

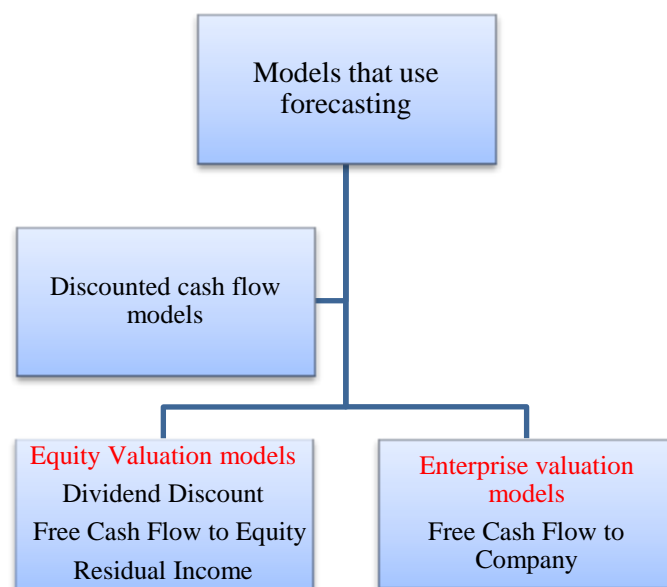
This takes time and experience to master and use it properly. It is up to the analyst to choose the best model which suits the particular company's business model.

Two types of valuation models are present:

- Methods that do not use forecasting (comparables, multiple screening and asset-based valuation);
- Methods that use forecasting (discounted cash flows methods).

This research is conducted based on the models that include forecasting. The stocks are valued using both equity valuation models (Dividend discount and Residual Income) and enterprise valuation models (Free cash flow to the company). In Figure 6 it is clear to see how the models are divided.

*Figure 6. Diagram of the forecasting models used in the research*



### **2.8.1 Equity valuation**

Equity is a partnership in a business. Consequently, it is an attempt to value and predict uncertain events which influence the cash flows in the future. Equity valuation models offer insight of the current and future value of the company by valuing just the equity share in the business. These models are used by owners to assess the possible future outcomes, because in these models are included factors which may have potential future impact on the value (Chartered Financial Analyst Institute, 2015). The methodology is used for companies with stable leverage and when a stock is being valued. Equity valuation is considered as the core of the financial system. With its help, enterprises with competitive and stable models enjoy a premium, whereas enterprises with weak models and results face a decline in value. It has many applications such as:

- Stock picking;

- Estimating the market sentiment;
- Listing of private businesses;
- Mergers and acquisitions;

The equity value is derived from discounting the future expected cash flows to the cost of equity. Since equity valuation is guessing the value of an organization, in the future it has many assumptions integrated. A valuation is only good as the assumptions that it is built upon (Damodaran, 2006b). According to the blog: managementstudyguide (Dividend Discount Model: Disadvantages, 2016) the most common assumptions are:

- Going concern;
- Re-investment;
- Dividend pay out;
- Macroeconomic assumptions;
- Industry assumptions.

It is very important when using equity valuation models to discount at the cost of equity. Otherwise, the result with the cash flows discounted with WACC will be biased upwards (Damodaran, 2006b). Since the companies included in the analysis regularly pay dividend, then the two-stage dividend discount model (*DDM*) and residual income model as representatives for the equity valuation models are used.

#### 2.8.1.1 Dividend discount model (two stage)

This model is set up to calculate the stock price based on the future dividends paid. Since dividends are cash flows that shareholders get from the company, they are discounted to the present value by using the cost of equity (Damodaran, 2002, p. 322).

$$Price_A = \sum_{t=1}^{\infty} \frac{Dividend_t}{(1 + cost\ of\ equity)^t} \quad (24)$$

The arguable part in this model is that we have to do as much as accurate forecast for all future dividend payments and do that into perpetuity. Equation (24) is the basic representation of this model and it presumes that the company does not grow and consequently its dividends do not grow either.

The dividend discount model has advantages and disadvantages which are worth mentioning (Dividend Discount Model: Disadvantages, 2016):

- Advantages of the *DDM* model:
  - Justification: easy to explain and justify the logic;

- Consistency: dividends are usually paid regularly and companies do not set up high dividend expectations because it can hurt the company on the long run;
- No subjectivity: the definition of dividends is so clear that any analyst using this model will come up with very similar results;
- No need of control;
- Mature businesses: regular pay out of dividends proves that the business is well established and is expected to continue to work good in the future (unless something unusual happens).
- Disadvantages of the *DDM* model:
  - Limited use: mostly suitable for mature, stable growing and regular dividend paying companies;
  - Not necessarily connected to earnings: it is not always the case that high earnings equal to high dividends, because companies tend to keep steady dividends regardless the variation of earnings;
  - Too many assumptions;
  - Tax efficiency;
  - Control: big shareholders can influence the dividend pay out policy.

If the analyst's opinion is that the growth rate is higher than the cost of equity, than the multistage *DDM* models are applicable. These companies achieve high growth (larger than the economy's growth) for particular time period and after that period they settle down and grow at a stable rate indefinitely (Damodaran, 2002). Equation (25) represents the following statement, assuming that the high growth rate and pay out ratio remain the same for the high growth years.

$$Price = \frac{Div_0 * (1 + g) * \left[1 - \frac{(1 + g)^n}{(1 + c_{e,hg})^n}\right]}{c_{e,hg} - g} + \frac{Div_{n+1}}{(c_{e,sg} - g_n)(1 + c_{e,hg})^n} \quad (25)$$

Where:

$c_e$  = cost of equity (hg=high growth, sg= stable growth)

$hg$  = high growth rate for n years

$g_n$  = stable growth rate into infinity

Companies which are valued with this particular model have the following characteristics (Damodaran, 2002):

- Have moderate growth rate (yet not stable);
- Pay out dividends that roughly approximate *FCFE*;
- It is hard to estimate *FCFE*;
- Are protected by a patent or have an entry barrier.

Damodaran (2006a) points out certain problems which arise when using this model. It is very hard to predict the length of the high growth phase (hg) which can significantly affect the stock's price. The second problem is the transition period between the high growth and stable growth. If there is a large gap between the growth rates, it can impact the price. Last but not least, because its focus is on dividends, this model can give different results for companies with lower pay out ratios.

In this research the *DDM* 2-stage model is used, since all of the companies within the FTSE 100 index fit in the group where this model can evaluate their stock. Common for all stocks is that the growth in stable growth period is not greater than 2%, which is considered as the rate at which the UK economy is growing (Pricewaterhouse Coopers, 2015).

*Table 5.* Percentage error for 50 stocks of the FTSE 100 index calculated with the two-stage dividend discount model

No.	Ticker	Price	DDM	% Error	No.	Ticker	Price	DDM	% Error
1	AAL.LN	\$ 15.50	\$ 15.81	2.00%	26	KGF.LN	\$ 4.90	\$ 4.96	1.22%
2	ANTO.LN	\$ 11.57	\$ 5.69	-50.82%	27	MKS.LN	\$ 8.38	\$ 4.89	-41.65%
3	ARM.LN	\$ 15.05	\$ 3.39	-77.48%	28	LAND.LN	\$ 18.03	\$ 17.11	-5.10%
4	ABF.LN	\$ 46.86	\$ 16.44	-64.92%	29	MGGT.LN	\$ 7.86	\$ 3.58	-54.45%
5	AZN.LN	\$ 70.11	\$ 26.71	-61.90%	30	NXT.LN	\$104.00	\$112.85	8.51%
6	BG.LN	\$ 12.12	\$ 6.13	-49.42%	31	PSO.LN	\$ 20.56	\$ 9.72	-52.72%
7	BP.LN	\$ 6.00	\$ 3.28	-45.33%	32	NG.LN	\$ 13.95	\$ 12.33	-11.61%
8	BTI.LN	\$ 53.05	\$ 46.54	-12.27%	33	RB.LN	\$ 80.59	\$ 43.87	-45.56%
9	BRBY.LN	\$ 25.11	\$ 23.16	-7.77%	34	REL.LN	\$ 16.63	\$ 21.18	27.36%
10	CUK.LN	\$ 46.20	\$ 17.10	-62.99%	35	REX.LN	\$ 6.72	\$ 4.03	-40.03%
11	CNA.LN	\$ 3.94	\$ 2.11	-46.45%	36	RR.LN	\$ 14.64	\$ 22.08	50.82%
12	CPG.LN	\$ 16.67	\$ 9.66	-42.05%	37	SAB.LN	\$ 50.87	\$ 21.81	-57.13%
13	CRDA.LN	\$ 40.00	\$ 44.37	10.93%	38	SGE.LN	\$ 7.04	\$ 2.94	-58.24%
14	DGE.LN	\$ 27.78	\$ 16.48	-40.68%	39	SVT.LN	\$ 31.30	\$ 35.22	12.52%
15	GFS.LN	\$ 4.57	\$ 2.00	-56.24%	40	SHPG.LN	\$ 79.95	\$ 85.50	6.94%
16	GKN.LN	\$ 5.25	\$ 5.63	7.24%	41	SNN.LN	\$ 17.19	\$ 6.71	-60.97%
17	GSK.LN	\$ 21.33	\$ 17.77	-16.69%	42	SMIN.LN	\$ 18.00	\$ 8.15	-54.72%
18	HMSO.LN	\$ 10.22	\$ 14.06	37.57%	43	SSE.LN	\$ 23.00	\$ 7.31	-68.22%
19	ITRK.LN	\$ 40.49	\$ 21.24	-47.54%	44	TATE.LN	\$ 9.10	\$ 12.86	41.32%
20	IHG.LN	\$ 39.82	\$ 24.71	-37.95%	45	TSCO.LN	\$ 3.57	\$ 0.92	-74.23%
21	IMI.LN	\$ 18.17	\$ 27.62	52.01%	46	UU.LN	\$ 14.17	\$ 9.23	-34.86%
22	IMT.LN	\$ 44.92	\$ 34.50	-23.20%	47	VOD.LN	\$ 3.38	\$ 6.37	88.46%
23	ITV.LN	\$ 3.21	\$ 3.91	21.81%	48	WOS.LN	\$ 61.11	\$ 40.41	-33.87%
24	SBRY.LN	\$ 4.09	\$ 3.50	-14.43%	49	WTB.LN	\$ 78.58	\$ 49.50	-37.01%
25	JMAT.LN	\$ 53.00	\$ 30.87	-41.75%	50	WPP.LN	\$ 23.81	\$ 15.36	-35.49%

The conclusion which is derived from the analysis is that according to this model, most of the stocks are overvalued meaning that their current value does not represent the company's real value. This conclusion is based on the percentage error which ranges from -77.48% to 88.46%. In Table 5, the % error column, it is obvious that the stocks with (–) are overvalued. The average absolute percentage error is 38.69 %.

### 2.8.1.2 Residual income model

Residual income (*RI*) is the income generated by a company after accounting for the true cost of its capital (Efficient Frontier, 2015). On the income statement companies show the interest expense only for the cost of debt, whilst ignoring the equity cost which on the other hand can be explained as the opportunity cost for the shareholders. This is precisely the “problem” which the residual income model aims to fix. It projects adjusted future earnings in which the cost of equity is incorporated. For the average investor, it is important that the company in which he invests has a positive residual income at the end.

For the purpose of calculating the residual income (in future referred as *RI*) model, 3 values are needed:

- Book value per share (*BPS*);
- Present value of future residual income (*PV RI*);
- Present value of continuous value of residual income.

This model relies on the book value per share. It measures the return to shareholders above the required return on capital. Frequently *RI* results in a smaller forecasting error compared to other models. Residual income is the money left after all the debt and obligations have been paid out (Penman, 2009, pp. 161–162). Residual income model is a very useful tool for measuring internal corporate performance and estimation of the common stock.

For calculating the *RI* model the following data is needed:

- Earnings per share (*EPS*);
- Dividend per share (*DPS*);
- Book value per share (*BPS*);

The period used for valuation is the same as in the previous models from year 2009-2015. The data is obtained from Bloomberg and Data Stream, and the forecasts for the period of 2015-2019 are obtained from expert analysts forecasts and *EPS* growth reference on Yahoo Finance. The same sources are used for *DPS*. The Book Value per share (*BPS*) is calculated using equation (26) and also compared to the Bloomberg values in case there are some serious deviations.

$$BPS_t = BPS_{t-1} + EPS_t - DPS_t \quad (26)$$

$BPS = \text{Equity attributable to equity shareholders of the company} / \text{Diluted number of shares}$

After collecting the necessary data for the beginning of the calculations, equation (27) is used to calculate the return on investment (ROE).

$$ROE = \frac{EPS_t}{BPS_t} \quad (27)$$

Residual income is calculated using equation (28).

$$RI_t = (ROE_t - r_e) * BPS_{t-1} \quad (28)$$

In order to calculate the present value of residual income (PV of RI), Residual income is discounted to present time using the cost of equity as a discount factor with equation (29)

$$PV_t = \frac{RI_t}{(1 + r_e)^t} \quad (29)$$

The next step is to calculate the sum of all present values of residual incomes. The terminal growth rate ( $g$ ) used is around 2% (depending on the company), considering the fact that growth is similar to average GDP growth rate of markets where the particular company operates (Pricewaterhouse Coopers, 2015). In the calculations, it is assumed that GDP growth rate will stay low for next decade because most industries in which the selected companies operate are stable and with limited investment opportunities and radical exponential growth or changes are not likely to happen. Using  $g$  as a growth rate the horizon value ( $HV$ ) is calculated. To sum, the following equation (30) is used:

$$V_0^E = BPS_0 + \frac{RI_1}{(1 + re)^1} + \dots + \frac{RI_5}{(1 + re)^5} + \frac{\frac{RI_6}{(re - g)}}{(1 + re)^5} \quad (30)$$

To get the final estimation about the share price I summed all residual incomes plus horizon value. In Table 6 below we can see the results for all 50 companies. When the percentage error is applied to this model, we can see that it ranges from -79.47% to 29.12%. The average absolute error for this model is 35.08%.

According to Penman (2009, p. 161), when using this model the valuation error tends to get smaller, but unlike any other method this one too has advantages and disadvantages. In addition, a list of the most important allegations of this model is presented:

- Advantages:
  - Data can be found easily right from the company's financial statements;
  - Can be used for companies which do not pay dividends regularly;



- Can be used for companies which do not generate positive free cash flow;
- Looks at the economic profitability of a company;
- Shorter forecast period can be used;
- Any accounting principle applies;
- Does not give too much weight on growth and consequently protects the investor not to pay too much for growth;
- Residual income can be calculated from analyst's forecasts;
- Residual income is not affected by dividends, share issues/repurchases.
- Disadvantages:
  - The result is highly dependent on future estimates about the company's statements;
  - Relies on accounting principles which in certain cases can be misleading;
  - Assumes that cost of debt equals interest expense;
  - More value is recognized in the near future than in the uncertain continuous value.

*Table 6. Percentage error for 50 stocks of the FTSE 100 index calculated with the residual income model*

No.	Ticker	Price	RI	% Error	No.	Ticker	Price	RI	% Error
1	AAL.LN	\$ 15.50	\$ 19.28	24.39%	26	KGF.LN	\$ 4.90	\$ 3.17	-35.31%
2	ANTO.LN	\$ 11.57	\$ 8.60	-25.67%	27	MKS.LN	\$ 8.38	\$ 5.65	-32.58%
3	ARM.LN	\$ 15.05	\$ 3.09	-79.47%	28	LAND.LN	\$ 18.03	\$ 23.28	29.12%
4	ABF.LN	\$ 46.86	\$ 15.28	-67.39%	29	MGGT.LN	\$ 7.86	\$ 3.97	-49.49%
5	AZN.LN	\$ 70.11	\$ 30.28	-56.81%	30	NXT.LN	\$ 104.00	\$ 71.01	-31.72%
6	BG.LN	\$ 12.12	\$ 8.93	-26.32%	31	PSON.LN	\$ 20.56	\$ 12.94	-37.06%
7	BP.LN	\$ 6.00	\$ 5.78	-3.67%	32	NGL.LN	\$ 13.95	\$ 11.16	-20.00%
8	BTI.LN	\$ 53.05	\$ 37.27	-29.75%	33	RB.LN	\$ 80.59	\$ 49.86	-38.13%
9	BRBY.LN	\$ 25.11	\$ 24.69	-1.67%	34	REL.LN	\$ 16.63	\$ 9.65	-41.97%
10	CUK.LN	\$ 46.20	\$ 36.77	-20.41%	35	REX.LN	\$ 6.72	\$ 5.90	-12.20%
11	CNA.LN	\$ 3.94	\$ 2.96	-24.87%	36	RR.LN	\$ 14.64	\$ 11.63	-20.56%
12	CPG.LN	\$ 16.67	\$ 9.00	-46.01%	37	SAB.LN	\$ 50.87	\$ 18.74	-63.16%
13	CRDA.LN	\$ 40.00	\$ 23.50	-41.25%	38	SGE.LN	\$ 7.04	\$ 4.78	-32.10%
14	DGE.LN	\$ 27.78	\$ 14.93	-46.26%	39	SVT.LN	\$ 31.30	\$ 15.92	-49.14%
15	GFS.LN	\$ 4.57	\$ 4.38	-4.16%	40	SHPG.LN	\$ 79.95	\$ 33.21	-58.46%
16	GKN.LN	\$ 5.25	\$ 2.00	-61.90%	41	SNN.LN	\$ 17.19	\$ 7.26	-57.77%
17	GSK.LN	\$ 21.33	\$ 15.13	-29.07%	42	SMIN.LN	\$ 18.00	\$ 10.22	-43.22%
18	HMSO.LN	\$ 10.22	\$ 10.33	1.08%	43	SSE.LN	\$ 23.00	\$ 21.10	-8.26%
19	ITRK.LN	\$ 40.49	\$ 20.01	-50.58%	44	TATE.LN	\$ 9.10	\$ 6.58	-27.69%
20	IHG.LN	\$ 39.82	\$ 26.48	-33.50%	45	TSCO.LN	\$ 3.57	\$ 1.12	-68.63%
21	IMI.LN	\$ 18.17	\$ 12.91	-28.95%	46	UU.LN	\$ 14.17	\$ 8.52	-39.87%
22	IMT.LN	\$ 44.92	\$ 32.43	-27.80%	47	VOD.LN	\$ 3.38	\$ 2.20	-34.91%
23	ITV.LN	\$ 3.21	\$ 2.70	-15.89%	48	WOS.LN	\$ 61.11	\$ 65.27	6.81%
24	SBRY.LN	\$ 4.09	\$ 2.44	-40.34%	49	WTB.LN	\$ 78.58	\$ 41.45	-47.25%
25	JMAT.LN	\$ 53.00	\$ 30.11	-43.19%	50	WPP.LN	\$ 23.81	\$ 14.74	-38.09%

## 2.8.2 Enterprise valuation

Company valuation models approach the company's value from a different angle. The other two models are more oriented in valuing the equity in the company, while this approach allows to put the weight and value the enterprise as a whole, including the

claimholders. With this method, the debt is not included in the cash flows, so consequently the cost of capital does not fluctuate (Damodaran, 2002). The residual cash flows after operating expenses and taxes, but before debt payments are discounted at the weighted average cost of capital (*WACC*). It is very similar to the *DDM* model. One of the key assumptions is the growth rate and the terminal growth rate which is used in the calculations. Crucial input for this method is calculating the *WACC* value. One of the weak points of this model is that it requires information about debt ratios and interest rates in order to determine the discount rate.

#### 2.8.2.1 Free cash flow to the firm (two stage) model

This model is used when a company has high leverage and is trying to lower it or vice versa. Companies which have negative free cash flow to equity but have positive free cash flow to the company are also suitable for this model. Similarly like in the 2-stage *DDM* model, up to certain point in the future the company grows at an accelerated growth rate and after that continues to grow at a constant rate (Elton, Brown, Gruber, and Goetzmann, 2009, pp. 460–462). This often happens when the company is in the transition period between the new player and the mature company within the industry or simply because its competitiveness is no longer shining. Equation (31) presents this model. The first part of the equation is the present value of the cash flow in the high growth phase. The second part of the equation is the terminal value of the company in maturity (which is not more than the growth rate of the economy of the country in which the company operates).

$$Firm\ Value = \sum_{t=1}^{t=0} \frac{FCFF_t}{(1 + WACC)^t} + \frac{\frac{[FCFF_n(1 + g)]}{(WACC - g)}}{(1 + WACC)^n} \quad (31)$$

All of the companies in the research are part of the FTSE 100 Index and there is an assumption that they are stable companies in mature industries because somehow, they represent the UK's economical core. In addition to the latter statement for evaluating the companies the 2-stage *FCFF* model is used. The high growth phase is 5 years. The results are in the Table 7. Some of the stocks are under-valuated and some are over-valuated. The percentage error ranges from -94.03% to 117.93%. In absolute terms the price error ranges from 1.77% to 117.93%. The average absolute percentage error is 35.67%.

Table 7. Percentage error for 50 stocks of the FTSE 100 index calculated with the two stage FCFF model

No.	Ticker	Price	FCFF	% Error	No.	Ticker	Price	FCFF	% Error
1	AAL.LN	\$ 15.50	\$ 20.32	31.10%	26	KGF.LN	\$ 4.90	\$ 5.13	4.69%
2	ANTO.LN	\$ 11.57	\$ 19.95	72.43%	27	MKS.LN	\$ 8.38	\$ 9.46	12.89%
3	ARM.LN	\$ 15.05	\$ 4.53	-69.90%	28	LAND.LN	\$ 18.03	\$ 8.00	-55.63%
4	ABF.LN	\$ 46.86	\$ 33.71	-28.06%	29	MGGT.LN	\$ 7.86	\$ 5.24	-33.33%
5	AZN.LN	\$ 70.11	\$ 68.09	-2.88%	30	NXT.LN	\$104.00	\$ 92.47	-11.09%
6	BG.LN	\$ 12.12	\$ 16.26	34.16%	31	PERSON.LN	\$ 20.56	\$ 13.35	-35.07%
7	BP.LN	\$ 6.00	\$ 3.74	-37.67%	32	NG.LN	\$ 13.95	\$ 15.56	11.54%
8	BTI.LN	\$ 53.05	\$ 49.81	-6.11%	33	RB.LN	\$ 80.59	\$ 53.94	-33.07%
9	BRBY.LN	\$ 25.11	\$ 18.16	-27.68%	34	REL.LN	\$ 16.63	\$ 22.00	32.29%
10	CUK.LN	\$ 46.20	\$ 43.48	-5.89%	35	REX.LN	\$ 6.72	\$ 9.46	40.77%
11	CNA.LN	\$ 3.94	\$ 4.81	22.08%	36	RR.LN	\$ 14.64	\$ 19.51	33.27%
12	CPG.LN	\$ 16.67	\$ 11.40	-31.61%	37	SAB.LN	\$ 50.87	\$ 27.23	-46.47%
13	CRDA.LN	\$ 40.00	\$ 33.95	-15.13%	38	SGE.LN	\$ 7.04	\$ 4.70	-33.24%
14	DGE.LN	\$ 27.78	\$ 18.70	-32.69%	39	SVT.LN	\$ 31.30	\$ 27.23	-13.00%
15	GFS.LN	\$ 4.57	\$ 8.27	80.96%	40	SHPG.LN	\$ 79.95	\$ 59.73	-25.29%
16	GKN.LN	\$ 5.25	\$ 2.50	-52.38%	41	SNN.LN	\$ 17.19	\$ 13.54	-21.23%
17	GSK.LN	\$ 21.33	\$ 17.04	-20.11%	42	SMIN.LN	\$ 18.00	\$ 21.16	17.56%
18	HMSO.LN	\$ 10.22	\$ 0.61	-94.03%	43	SSE.LN	\$ 23.00	\$ 28.15	22.39%
19	ITRK.LN	\$ 40.49	\$ 55.34	36.68%	44	TATE.LN	\$ 9.10	\$ 18.05	98.35%
20	IHG.LN	\$ 39.82	\$ 51.65	29.71%	45	TSCO.LN	\$ 3.57	\$ 7.78	117.93%
21	IMI.LN	\$ 18.17	\$ 24.33	33.90%	46	UU.LN	\$ 14.17	\$ 12.48	-11.93%
22	IMT.LN	\$ 44.92	\$ 86.66	92.92%	47	VOD.LN	\$ 3.38	\$ 5.11	51.18%
23	ITV.LN	\$ 3.21	\$ 2.08	-35.20%	48	WOS.LN	\$ 61.11	\$ 62.19	1.77%
24	SBRY.LN	\$ 4.09	\$ 6.22	52.08%	49	WTB.LN	\$ 78.58	\$ 74.13	-5.66%
25	JMAT.LN	\$ 53.00	\$ 27.65	-47.83%	50	WPP.LN	\$ 23.81	\$ 28.77	20.83%

## 2.9 Risk return trade off

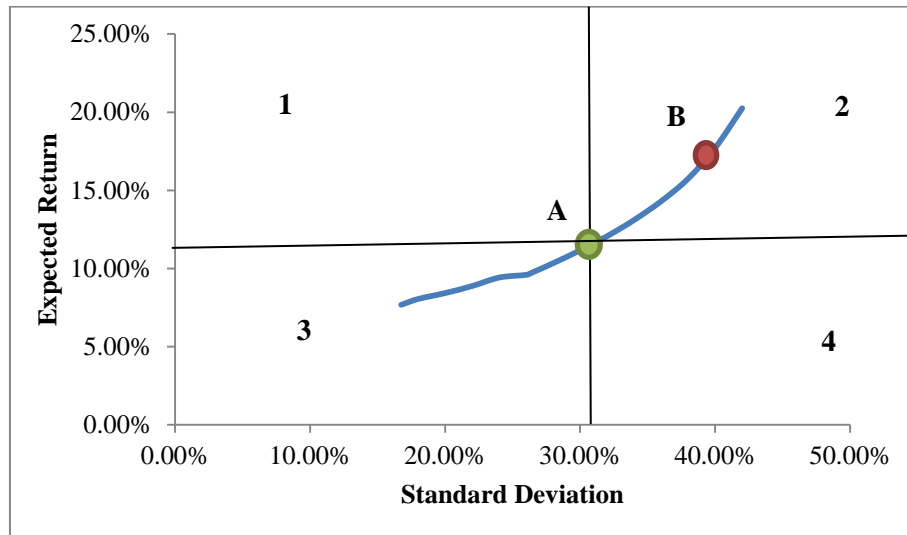
The portfolio creation process is consisted of two divided but closely connected actions. Selecting the assets which are to be included in the portfolio and deciding how much to invest in each of those assets (Markowitz, 1952, p. 1). The investor cannot solely decide in what proportion to invest the funds in the risky portfolio and the risk-free asset without calculating and perfectly understanding the return and the risk of that particular portfolio (Bodie et al., 2010, p.172). It is crucial to get acknowledged with the risk-return trade off of the portfolio. Asset managers help to create the optimal risk portfolio, but it is up to the investor himself to decide how much is he willing to invest in that portfolio.

The expression “investing in the market is gambling and speculating” is present in the financial world. There is a major difference in these two terms. “A gamble is the assumption of risk with no purpose but enjoyment of the risk itself, whereas speculation is undertaken in spite of the risk involved because the investor perceives a favourable risk-return trade-off ” (Bodie et al., 2010, p. 161). In order to transform the gamble into speculation, risk-premium is required from the investors to buffer the risk they are taking.

Investors are risk averse, meaning that they will not accept an investment which is risky and has a risk premium of zero. To conclude, the risk averse investor requires greater rate of return when the risk is higher. On the other hand, there are risk-neutral investors to whom is only important how much is the return of the portfolio whilst ignoring the risk that comes with it. Lastly, risk-lovers gladly invest in the so called fair game portfolios which are risky portfolios with risk premium of zero. In other words, this type of investor likes to gamble.

The graphical way of representing the investor's trade-off is to draw the characteristics of the particular portfolio on a single graph where on the X-axis is plotted the risk (standard deviation) and on the Y-axis, is plotted the expected return (Basics of Investments – Financial concepts, n.d.).

Figure 7. Types of portfolios based on the risk return trade-off



The point A in Figure 7 is the portfolio which will be chosen by any risk-averse investor. Quadrant 4 is the least attractive and irrational area where the portfolios are plotted because there is the same return by taking greater risk and vice versa. On the other hand, any portfolio which lies in area 1 is better than the portfolio A because it has return equal or greater than point A and risk equal or smaller than point A. Consequently, any rational investor will move and try to create his portfolio in area 1. All of the above explained is known as the **Mean-variance criterion (M-V)** (Bodie et al., 2010, p. 172), which suggests that portfolio A is better than portfolio B if:

$$\begin{aligned} E(r_a) &\geq E(r_b) \\ \sigma_a &\leq \sigma_b \end{aligned} \quad (32)$$

It is up to the investors risk aversion to choose whether he will position himself in area 2 and 3. The blue line represents the **indifference curve** (Bodie et al., 2010, p. 172). Along

the indifference curve from point A towards point B, the return increases but consequently the risk also increases. Towards area 3 the risk is getting smaller but also the return is getting smaller. The simplest way for asset managers or for investors to estimate their risk aversion is through various questionnaires which at the end derive a coefficient of risk aversion.

## 2.10 Security market line (SML)

In terms of reward-risk ratio, it is safe to use the synonym known as **expected return to beta** relation. This is graphically presented with the help of the Security Market Line (*SML*) (Bodie et al., 2010, p. 198). The *SML* line presents the trade-off between systematic risk and return for an individual asset or portfolio. When an asset is part of a portfolio, it is important how the asset affects the total portfolio variance. The core of the *SML* is the *CAPM* model and both models more or less graphically present the same results.

Fairly priced securities should lie exactly on the *SML* line, which means that their expected return is in accordance with the risk they possess (Ross et al., 2010, pp. 419–427). What is attractive for investors are the securities that lie above the *SML*. This applies that they are undervalued and cheap for their riskiness. In other words, investors, would invest in these assets because it is expected to receive higher return than they should for the level of risk they are taking (Rasmussen, 2003, pp. 63–66). On the other hand, assets that lie below the *SML* are not attractive for investors because they are overvalued and are expensive for the level of risk they possess. The **alpha** ( $\alpha$ ) measures directly the difference between fair and expected rate of return of an asset (Elton et al., 2009, p. 213).

Figure 8 is the graphical presentation of the *SML* line for the 50 FTSE 100 stocks used in this research. In order to draw the *SML* line, a couple of things are calculated. On the X axis is the beta, which as mentioned above represents the systematic risk. On the Y axis is the expected return. The *SML* line equation is the following:

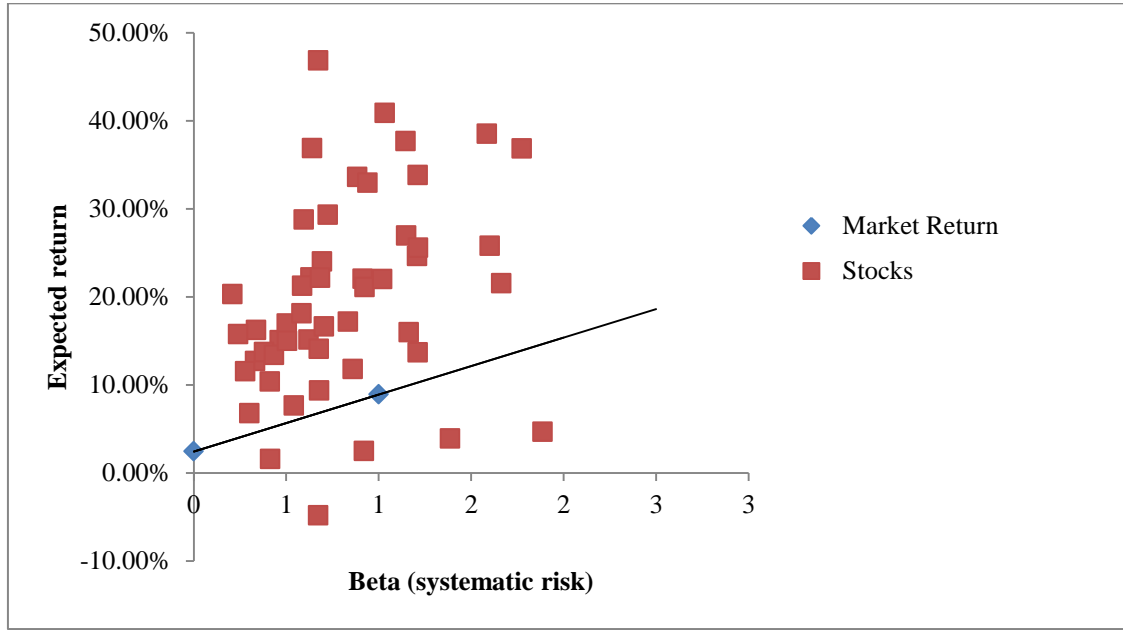
$$E(r_i) = r_f + \beta[E(r)_m - r_f] \quad (33)$$

Where:

$E(r)_m$  = Expected market return

$r_f$  = Risk free rate

Figure 8. Security market line (SML)



In the calculation, the 30-years UK government bond rate of 2.45% is used. This security has beta value of 0. On the other hand, the market return which is derived from the FTSE 100 index for the period of 5 years is 11.36% minus the risk-free rate equals 8.91 %. The beta assigned to this return is 1. These are the primary inputs to draw the *SML* line in Figure 8.

The blue mark placed on the Y axis is the beginning of the *SML* line which is the risk-free rate of 2.45%. The other blue mark in the middle of the *SML* line represents the market portfolio and the distance to the value 1 on the X axis is the slope which is defined as:

$$Slope = E(r)_m - r_f \quad (34)$$

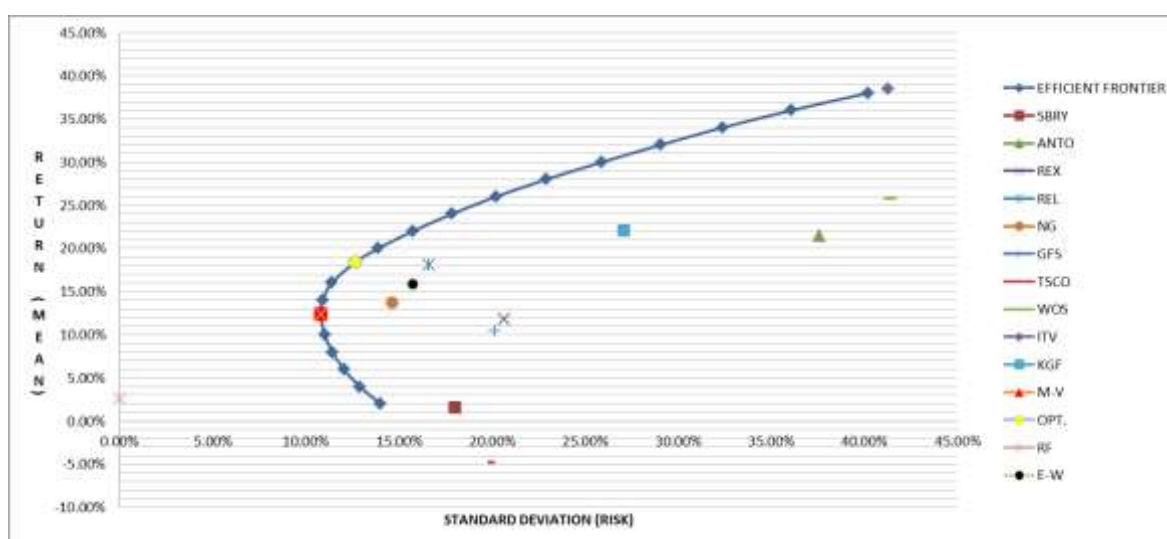
In Figure 8 all red rectangles represent single asset included in the portfolio. As mentioned above, it is obvious that the assets in this portfolio are mostly undervalued which makes the stocks attractive for investors.

### 3 EFFICIENT FRONTIERS FOR THE GROUPS OF STOCKS

#### 3.1 Efficient frontier (group 1 with absolute percentage valuation error between 0% - 10%)

In this group the stocks which have an absolute percentage valuation error between 0 and 10 % are included.

Figure 9. Efficient frontier of stocks with 0-10% absolute percentage error



There are 10 stocks included and 20 different return scenarios which create the efficient frontier are calculated. Each asset in this group is separately presented on the graph. The equal weights portfolio (black dot in Figure 9) in this group is created by setting the weight of each of the 10 assets to 1/10. On the graph, it is visible as E-W and it has return of 15.87% with standard deviation of 15.73%. Following the minimum variance criteria, we can conclude that this portfolio is not efficient since right above it there is differently constructed portfolio which offers higher return given the same risk.

In Figure 9 (the red square) and in Table 8 is marked the M-V portfolio which in this group has an expected return of 12.42% with standard deviation of 10.80%. This portfolio is created from 5 out of 10 stocks and the stock of National Grid (NG) has the largest weight of 33.52%. Following it is the stock of Reed Elsevier (REL) contributing to this portfolio with 22.51%. Half of the stocks in this group are not included in this portfolio because of the covariance between them.

The starting point for the CAL line is the risk-free rate of 2.45% which is 30-year UK government bond. In the yellow point where the CAL becomes a tangent to the efficient frontier it is the optimal risk portfolio. It has expected return of 18.42%, standard deviation of 12.68% and a maximized Sharpe ratio of 1.453.

This portfolio is created of 6 stocks where the stock of Reed Elsevier has a weight of 37.89% followed by the stock of National Grid.

Considering this fact, we can conclude that both the M-V and the optimal portfolio are fairly diversified. In Table 11 in appendix E is the list of all portfolios which are on the efficient frontier for this group.

Table 8. Minimum variance, optimal risk and equal weights portfolio from the stocks with absolute percentage valuation error of 0-10%

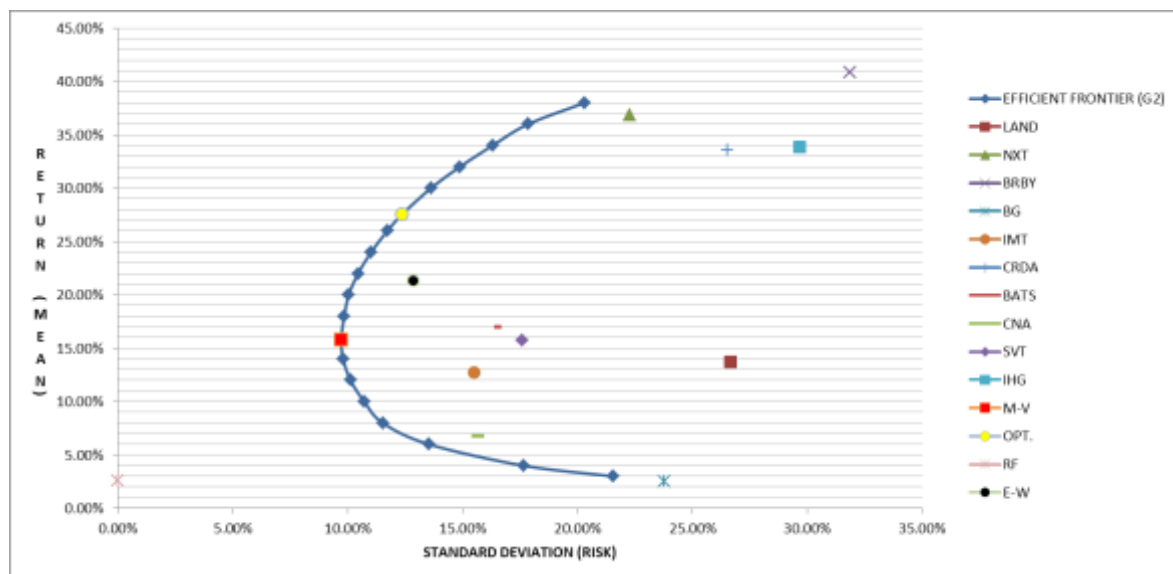
	M-V	OPT.	E-W
Mean	12.42%	18.42%	15.87%
STDEV	10.80%	12.68%	15.73%
Ratio	1.1491647	1.4523298	1.0085987
No.Stocks	5	6	10

W E I G H T S	SBRY	19.03%	0.00%	10.00%
	ANTO	0.00%	0.00%	10.00%
	REX	0.00%	0.00%	10.00%
	REL	22.51%	37.89%	10.00%
	NG	33.52%	27.86%	10.00%
	GFS	17.72%	8.45%	10.00%
	TSCO	0.00%	0.00%	10.00%
	WOS	0.00%	2.97%	10.00%
	ITV	0.00%	6.35%	10.00%
	KGF	7.23%	16.48%	10.00%

### 3.2 Efficient frontier (group 2 with absolute percentage valuation error between 10% - 17%)

The next efficient frontier is created from stocks where the absolute percentage valuation error compared to the average of the three models is between 10% - 17%. This group also contains 10 stocks.

Figure 10. Efficient frontier of stocks with 10-17% absolute percentage error





Similarly, in this group, 20 scenarios are calculated in order to create the efficient frontier. The equally weighted portfolio is the black spot in Figure 10 and as in the previous case each asset contributes to 1/10 to the portfolio. It has expected return of 21.36% and risk of 12.86% followed by a Sharpe ratio of 1.66 as presented in Table 9. This portfolio is not efficient since slightly above it marked with a yellow dot in Figure 10 is the optimal risk portfolio.

The M-V portfolio (red square in Figure 10) has expected return of 15.82% and standard deviation of 9.72% with a Sharpe ratio of 1.62. This portfolio is created from 6 out of 10 available stocks where the stock of Imperial Tobacco Group (IMT) has a weight of 30.16% closely followed by the stock of Centrica (CNA) which has a weight of 28.89% (Table 9).

*Table 9.* Minimum variance, optimal risk and equal weights portfolio from the stocks with absolute percentage valuation error of 10-17%

	<b>M-V</b>	<b>OPT.</b>	<b>E-W</b>
<b>Mean</b>	15.82%	27.56%	21.36%
<b>STDEV</b>	9.72%	12.38%	12.86%
<b>Ratio</b>	1.62796	2.2269181	1.6605683
<b>No.Stocks</b>	6	7	10

<b>W E I G H T S</b>	<b>LAND</b>	0.00%	0.00%	10.00%
	<b>NXT</b>	18.52%	34.00%	10.00%
	<b>BRBY</b>	0.00%	5.96%	10.00%
	<b>BG</b>	5.38%	0.00%	10.00%
	<b>IMT</b>	30.16%	3.24%	10.00%
	<b>CRDA</b>	2.15%	10.08%	10.00%
	<b>BATS</b>	0.00%	20.32%	10.00%
	<b>CNA</b>	28.89%	0.00%	10.00%
	<b>SVT</b>	14.90%	19.88%	10.00%
	<b>IHG</b>	0.00%	6.52%	10.00%

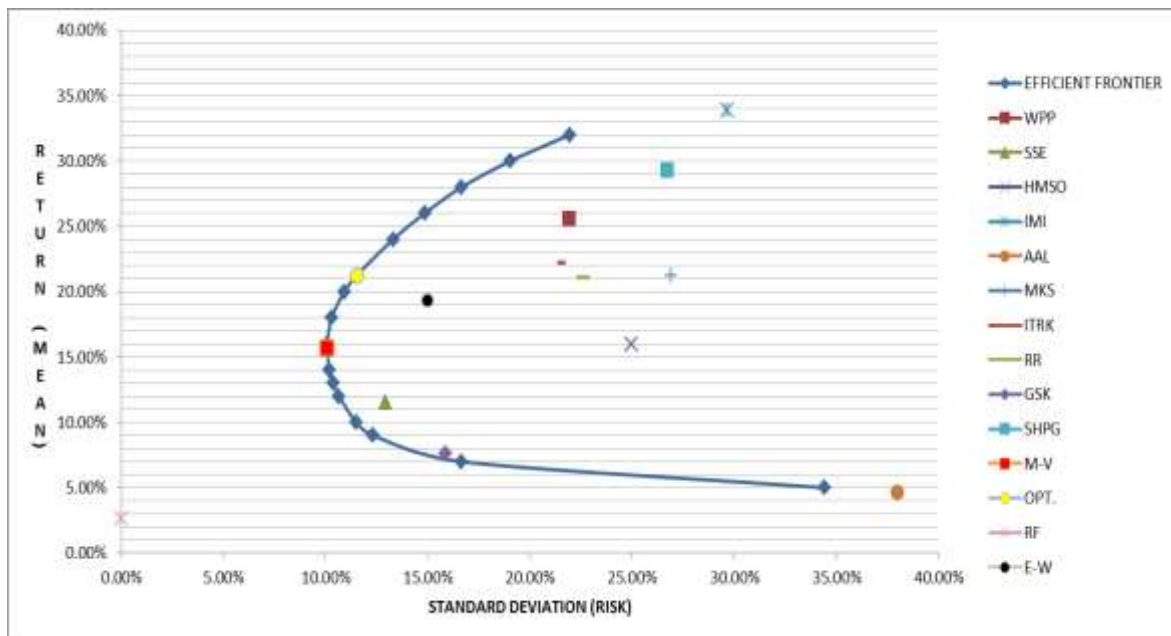
The optimal risk portfolio marked with a yellow dot on the efficient frontier in Figure 10, has an expected return of 27.56% which is almost 11% higher than the M-V portfolio and a standard deviation of 12.38% which is only 2.65% higher than the M-V. Also, this portfolio contains 7 out of 10 stocks where the stock of Next (NXT) has the highest weight of 34% followed by British American Tobacco (BATS) with weight of 20.32% (Table 9). This is very well diversified portfolio. In Table 12 in appendix E is the list of all portfolios which are on the efficient frontier for this group.

### 3.3 Efficient frontier (group 3 with absolute percentage valuation error between 17% - 26%)

In this group, there are stocks which according to the calculations have an absolute percentage valuation error compared to the average values of the analysis, between 17%-26%. This group like the two previous ones consists of 10 stocks.

Unlike the other three groups where 20 possible scenarios are calculated, for this group, 18 possible scenarios are calculated (appendix E, Table 13). The equally weighted portfolio as usually is the black dot in Figure 11, where each of the 10 stocks contributes to 10% in the portfolio, thus creating an expected return of 19.32% with a standard deviation of 14.98% and a Sharpe ratio of 1.28.

Figure 11. Efficient frontier of stocks with 17-26% absolute percentage error



In Figure 11, the red square on the efficient frontier is the M-V portfolio with an expected return of 15.71%, standard deviation of 10.06% and Sharpe ratio of 1.56. The highest contributor in this portfolio is the stock of SSE (SSE) with 48.44% followed by the stock of Intertek (ITRK) with 15.97%. The most interesting portfolio is the optimally diversified one and it is the yellow dot on the efficient frontier in Figure 11. It is created from 7 out of 10 stocks and has an expected return of 21.24% with a standard deviation of 11.52% and highest Sharpe ratio of 1.84. Like in the M-V portfolio the largest weight in this portfolio has the stock of SSE with 29.41% followed by the stock of Shire (SHPG) with weight of 24.73%.

Table 10. Minimum variance, optimal risk and equal weights portfolio from the stocks with absolute percentage valuation error of 17-26%

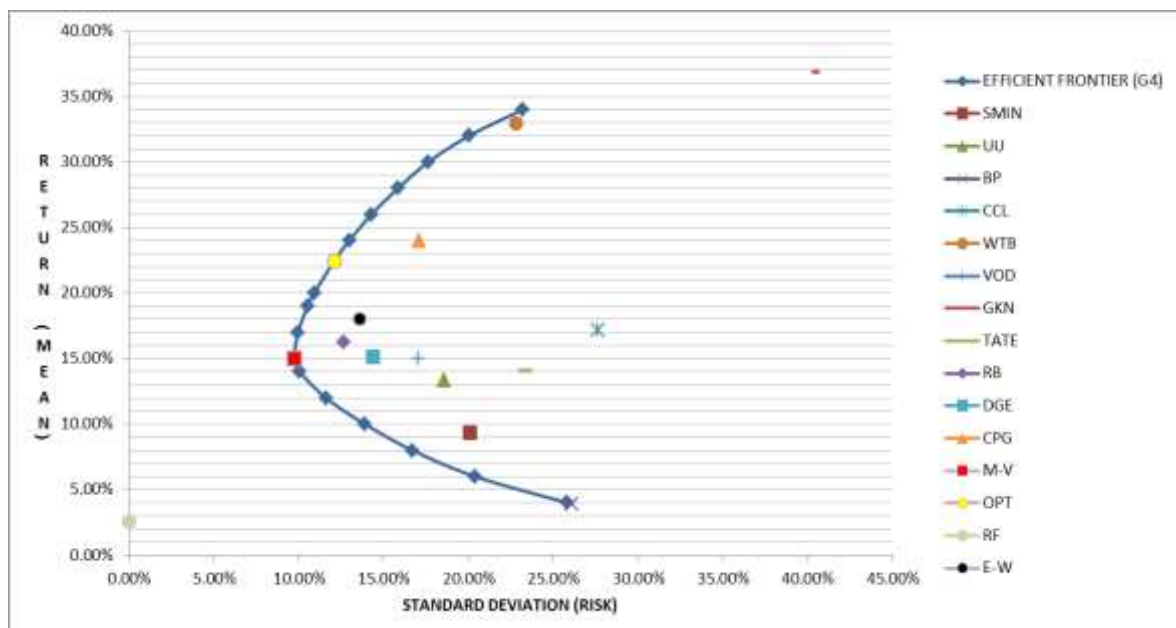
	M-V	OPT.	E-W
Mean	15.71%	21.24%	19.32%
STDEV	10.06%	11.52%	14.98%
Ratio	1.5614353	1.8435865	1.2895218
No.Stocks	6	7	10

W E I G H T S	WPP	0.00%	1.32%	10.00%
	SSE	48.44%	29.41%	10.00%
	HMSO	0.00%	0.00%	10.00%
	IMI	0.00%	4.47%	10.00%
	AAL	0.00%	0.00%	10.00%
	MKS	7.97%	12.66%	10.00%
	ITRK	15.97%	24.66%	10.00%
	RR	4.87%	2.74%	10.00%
	GSK	13.03%	0.00%	10.00%
	SHPG	9.72%	24.73%	10.00%

### 3.4 Efficient frontier (group 4 with absolute percentage valuation error between 26% - 40%)

The stocks that deserve a place in this category are those which based on the calculations have an absolute percentage valuation error compared to the average values of the analysis, between 26% - 40%. This group is consisted of 11 stocks.

Figure 11. Efficient frontier of stocks with 26-40% absolute percentage error



For this group, 18 possible scenarios are calculated (appendix E, Table 14). The equally weighted portfolio is the black dot (Figure 12), where each of the 11 stocks contributes to 9.0909% in the portfolio, thus creating an expected return of 18.01% with a standard deviation of 13.57% and a Sharpe ratio of 1.32 (Table 11).

*Table 10.* Minimum variance, optimal risk and equal weights portfolio from the stocks with absolute percentage valuation error of 26-40%

	<b>M-V</b>	<b>OPT.</b>	<b>E-W</b>
<b>Mean</b>	15.03%	22.46%	18.01%
<b>STDEV</b>	9.74%	12.10%	13.57%
<b>Ratio</b>	1.54	1.8560168	1.3267912
<b>No. Stocks</b>	7	7	11

<b>W E I G H T S</b>	<b>S MIN</b>	2.67%	0.00%	9.09%
	<b>UU</b>	21.67%	5.31%	9.09%
	<b>BP</b>	0.00%	0.00%	9.09%
	<b>CCL</b>	0.76%	0.00%	9.09%
	<b>WTB</b>	0.00%	23.21%	9.09%
	<b>VOD</b>	15.96%	6.78%	9.09%
	<b>GKN</b>	0.00%	3.17%	9.09%
	<b>TATE</b>	7.97%	0.44%	9.09%
	<b>RB</b>	46.80%	36.33%	9.09%
	<b>DGE</b>	4.16%	0.00%	9.09%
	<b>CPG</b>	0.00%	24.76%	9.09%

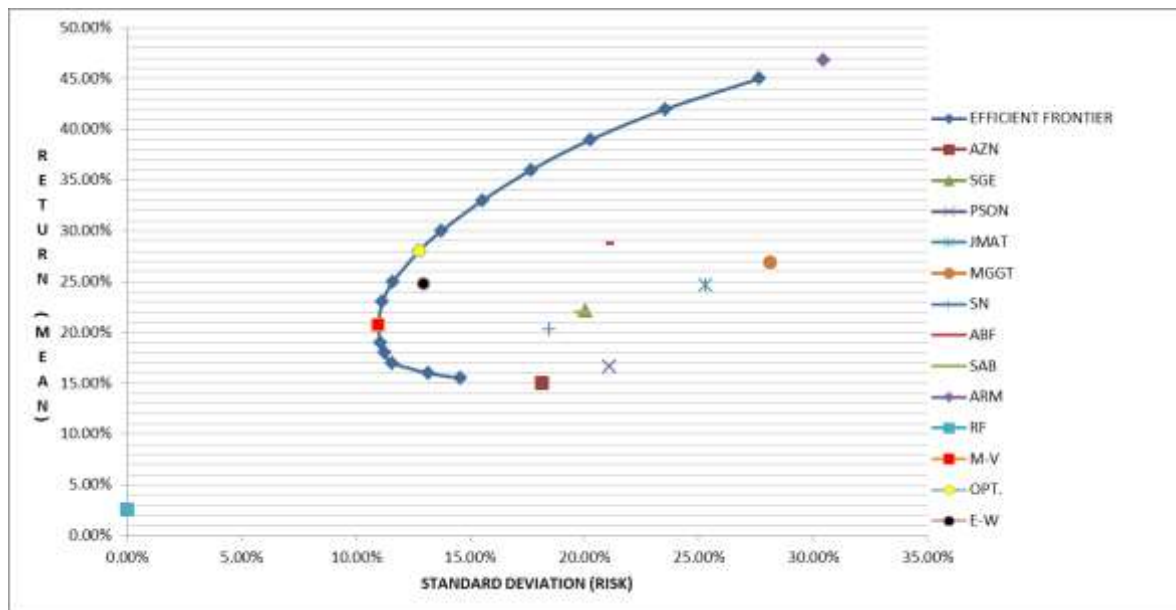
From Table 11 and from Figure 12 the M-V portfolio is visible. It is consisted of 7 out of 11 stocks and has an expected return of 15.03%, standard deviation of 9.74% and a Sharpe ratio of 1.54. The largest weight belongs to the Reckitt Benckiser (RB) with 46.8% followed by considerably smaller weight of 21.67% of United Utilities (UU). The optimal portfolio is also consisted of 7 stocks, where as in the M-V the largest weight of 36.33% belongs to Reckitt Benckiser, followed by 24.76% weight of Compass Group (CPG). This results in 22.46% expected return, 12.10% standard deviation and a Sharpe ratio of 1.85.

The shape of each efficient frontier is slightly different. This is due to covariance and correlation coefficients between the stocks in the portfolio. Even though the shape is different, the main characteristics of the efficient frontier are present.

### **3.5 Efficient frontier (group 5 with absolute percentage valuation error between 40% - 70%)**

The last group is the one where the absolute percentage valuation error is between 40%-70%. This group contains 9 stocks.

Figure 12. Efficient frontier of stocks with 40-70% absolute percentage error



Sixteen possible scenarios are calculated for this group. Since there are 9 stocks the equally weighted portfolio (black dot in Figure 13) is obtained with a weight of 11.11% from each stock, thus obtaining expected return of 24.82% with a standard deviation of 12.96% and a Sharpe ratio of 1.91 (table 12).

Table 11. Minimum variance, optimal risk and equal weights portfolio from the stocks with absolute percentage valuation error of 40-70%

	M-V	OPT.	E-W
<b>Mean</b>	20.76%	28.07%	24.82%
<b>STDEV</b>	10.99%	12.80%	12.96%
<b>Ratio</b>	1.89	2.1937671	1.9155351
<b>No. Stocks</b>	9	8	9

<b>W E I G H T S</b>	<b>AZN</b>	31.91%	15.20%	11.11%
	<b>SGE</b>	0.22%	0.00%	11.11%
	<b>PSON</b>	16.27%	3.76%	11.11%
	<b>JMAT</b>	5.07%	3.82%	11.11%
	<b>MGGT</b>	0.81%	2.88%	11.11%
	<b>SN</b>	31.12%	25.29%	11.11%
	<b>ABF</b>	6.05%	20.35%	11.11%
	<b>SAB</b>	1.32%	3.99%	11.11%
	<b>ARM</b>	7.23%	24.71%	11.11%

Table 12 presents the parameters for the three portfolios. In this group the M-V and the optimal risk portfolios are highly diversified. The M-V portfolio contains each of the 9

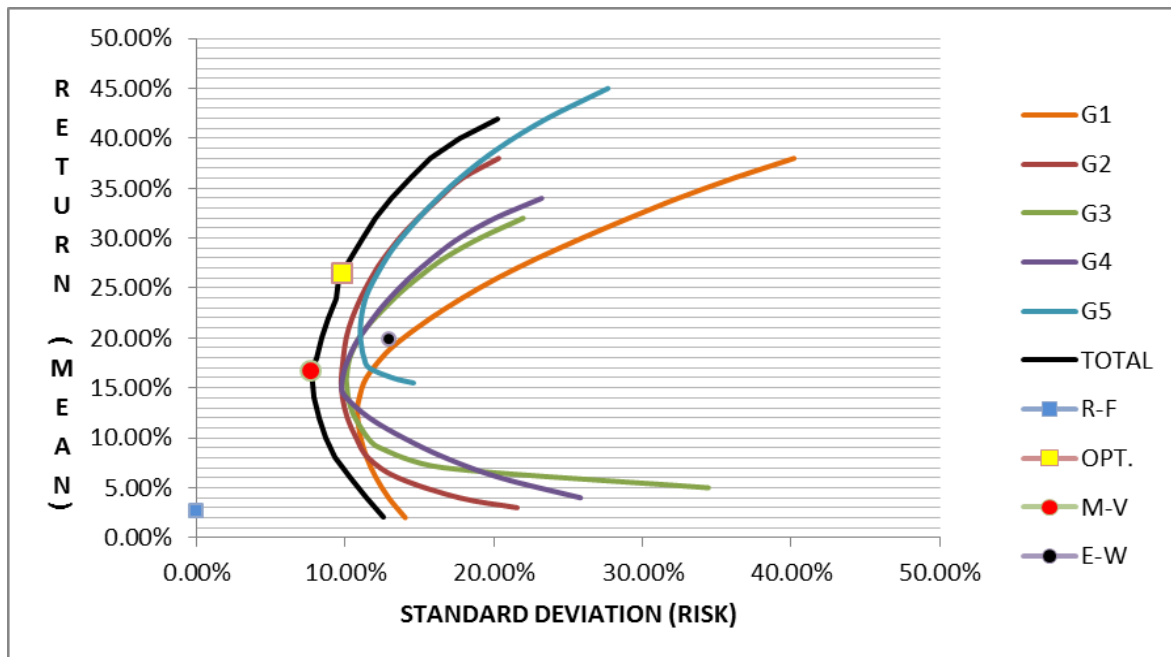
stocks with different weights. The largest contributor to an expected return of 20.76% with a standard deviation of 10.99% and Sharpe ratio of 1.89 is the stock of Astrazeneca (AZN) with 31.91%, closely followed by the stock of Smith and Nephew (SN) with a weight of 31.12%. The optimal risk portfolio is obtained by diversifying between 8 out of 9 stocks, thus obtaining a return of 28.07% and standard deviation of 12.8% and maximum Sharpe ratio of 2.19. The largest stake in the obtained return has the weight of 25.29% of Smith and Nephew (SN) followed by the stock of ARM Holdings (ARM) with a weight of 24.71%. For this group a total of 16 possible scenarios are calculated (appendix E, table 15).

In all of the efficient frontiers the individual stocks are marked on the pictures independently. All of them lay between the borders of the minimum variance frontier. Each stock, if invested in independently, is not efficient as combined with other stocks. In other words, it is riskier for the investor to be exposed only on one stock's movement.

### 3.6 Aggregated efficient frontier

In the following graph, we can see all of the above mentioned efficient frontiers presented. The black line is the efficient frontier created from all of the 50 stocks. For the creation of the frontier 22 possible scenarios are calculated ranging from expected return of 2% up to 42%.

*Figure 13. Efficient frontier from the 50 stocks of the FTSE 100 index*



In appendix E, Table 16 we can see the outcomes of the 23 scenarios and the weights distributed for this efficient frontier. The equally weighted portfolio is obtained when each

of the 50 stocks included has a weight of 2%. It results in expected return of 19.82%, standard deviation of 12.98% and a Sharpe ratio of 1.53. This portfolio is the black dot in Figure 14. The M-V portfolio (red dot in Figure 14) is obtained from 13 out of 50 stocks thus obtaining expected return of 16.75%, standard deviation of 7.68% and a Sharpe ratio of 2.18. The largest weight of 21.51% belongs to the stock of Reckitt Benckiser, followed by the weight of 19.11% belonging to SSE. The optimal risk portfolio (yellow rectangle in Figure 14) is also created from 13 out of 50 stocks. It has an expected return of 26.49% standard deviation of 9.75% and a Sharpe ratio of 2.72 meaning that for each additional unit of risk taken the expected return increases for 2.72 points. The largest weight belongs to the stock of Next with 16.56% closely followed by the stock of Smith and Nephew of 14.39%.

The FTSE 100 index in the used period from 2009-2015 has return of 11.36% with a standard deviation of 14.97% (appendix E, Table 14). The optimal risk portfolio calculated based on 50 randomly selected stocks within the FTSE 100 index has 15% higher expected return and approximately 5% lower standard deviation. Based on this, investing in the diversified optimal portfolio is better decision. Table 13 is a summary of each portfolio's parameters. The "safest" portfolio is the M-V aggregated portfolio created from 13 out of 50 stocks with lowest standard deviation of 7.68%. The riskiest portfolio is the equally weighted one from the first group with a standard deviation of 15.73%. It is already concluded that it is not the optimal one. The best optimal portfolio is from the aggregated group with the highest Sharpe ratio and that is the one created with 13 out of 50 stocks obtaining a Sharpe ratio of 2.72 and a return of 26.49% followed by a standard deviation of 9.75%.

*Table 13. Parameters of the calculated portfolios*

	Group 1				Group 2				Group 3			
	Return	STD	Sharpe	No.Stocks	Return	STD	Sharpe	No.Stocks	Return	STD	Sharpe	No.Stocks
<b>Opt.</b>	18.42%	12.68%	1.45	6	27.56%	12.38%	2.23	7	21.24%	11.52%	1.84	7
<b>M-V</b>	12.42%	10.80%	1.15	5	15.82%	9.72%	1.63	6	15.71%	10.06%	1.56	6
<b>E-W</b>	15.87%	15.73%	1.01	10	21.36%	12.86%	1.66	10	19.32%	14.98%	1.29	10

	Group 4				Group 5				Aggregated			
	Return	STD	Sharpe	No.Stocks	Return	STD	Sharpe	No.Stocks	Return	STD	Sharpe	No.Stocks
<b>Opt.</b>	22.46%	12.10%	1.86	7	28.07%	12.80%	2.19	8	26.49%	9.75%	2.72	13
<b>M-V</b>	15.03%	9.74%	1.54	7	20.76%	10.99%	1.89	9	16.75%	7.68%	2.18	13
<b>E-W</b>	18.01%	13.57%	1.33	11	24.82%	12.96%	1.92	9	19.82%	12.98%	1.53	50

In Figure 15 the portfolios from table 13 are graphically presented. To prove that the effect of diversification is beneficial in Figure 16 we can see the correlation between number of stocks and standard deviation for M-V and optimal risk portfolios. Following the graph, it can be concluded that when the number of stocks in the portfolio increases, the standard deviation tends to decrease thus proving that diversification works. The latter does not prove that the more stocks we add to the portfolio the lower the standard deviation will be. It is impossible to completely eliminate the risk.

Figure 14. Calculated portfolios

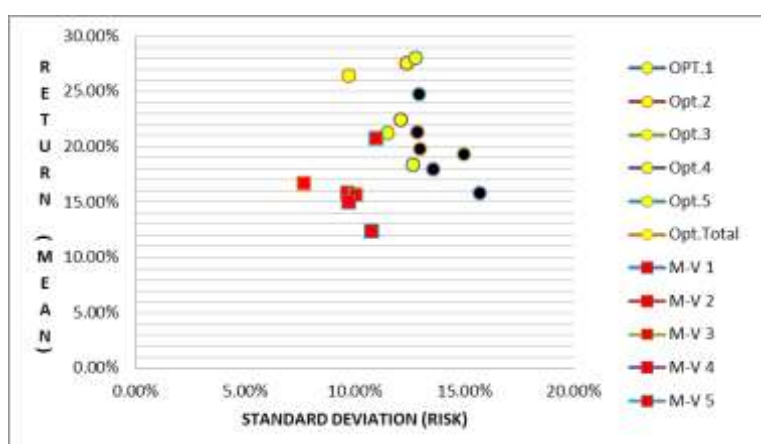
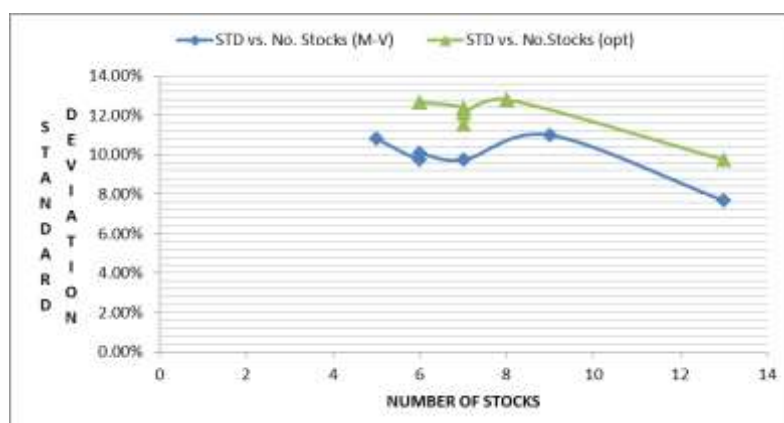


Figure 15. Standard deviation compared to number of stocks in M-V and optimal risk portfolios



## CONCLUSION

When planning an investment, uncertainty follows. Nobody knows what will happen in the future. Investors and analysts have developed numerous models for predicting the future movement of an asset as close as possible. No matter how good the models are, they will never be perfect because in order to create a model you have to base it on some assumptions which are not always correct in the real world. In this research one of the goals is to create efficient portfolio based on 50 randomly selected stocks within the FTSE 100 UK index.

For the whole portfolio creation process to start, one needs to calculate the expected return of each asset. In this research the assumption that stock prices somehow tend to follow a certain pattern and that historical prices are good pointer for the expected return is used. In the past potential investors were only concerned about the return not paying attention to the risk it brings. All that changed with the creation of the Modern Portfolio Theory. This theory puts weight on the risk factor when making an investment decision. Many



modifications by many authors had been made, but the core principle remains the same. The quantitative representation of risk is presented via the standard deviation. The simplest explanation is that it measures the possibility of how much the expected return can differ from the realized one. In order for the standard deviation to work it has to be assumed that the returns are normally distributed (Carol, 1999).

Markowitz's modern portfolio theory proves that by diversification better results regarding the risk return ratio can be achieved. Risk can be significantly reduced but never totally eliminated. The goal is to maximize the return given the risk or the other way around, to minimize the risk given the required return. One basic concept in the portfolio theory is that the investor is not to be concerned with the individual asset's risk/return ratio, but the benefit it contributes that particular asset when added in a portfolio. The latter is connected to the correlation and covariance between the assets in the portfolio. In order to create and correctly make a decision about investing in a well-diversified portfolio first the portfolio expected return and risk should be calculated. This is done via mathematical models and then it can be adjusted for expectations. Also, the correlation and covariance coefficients between each asset have to be calculated and analysed.

This research started by selecting 50 stocks within the UK's FTSE 100 Index for 5-year period (2009–2015) and evaluating them using three different models: Dividend Discount model, Free cash Flow to Company and the Residual income model. Each of the models gave different result, but not too far from the real value of the stock. For each model the valuation errors are calculated using four models: Percentage, Absolute percentage, Squared and Log error. The findings correspond with the earlier researches of certified authors.

Based on the average value of the stock of each of the three models, the absolute percentage error is calculated. In addition, corresponding to the range of the valuation error the stocks were divided in 5 categories. Each category has been separately evaluated and three portfolios were calculated: M-V, Optimal risk and Equal Weights. Those findings are summarized and one aggregated efficient frontier including all stocks is created.

From those findings, I can conclude that every optimal risk portfolio of each group including the aggregated optimal risk portfolio is better than investing in the FTSE 100 index itself. The optimal portfolio for 50 stocks calculated with the help of modern portfolio theory is the best among all portfolios yielding a return of 26.49% with a standard deviation of 9.57 % and a highest Sharpe ratio of 2.72. This portfolio is consisted of 13 stocks. On the other hand, the FTSE 100 index has return of 11.36% with standard deviation of 14.97% and a Sharpe ratio of 0.76. Optimally risky portfolios are not always perfectly diversified so they are in constant need of adjustments.



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## **APPENDIXES**





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## APPENDIX A: Annualized return and standard deviation of the stocks

Table 1. Annualized return and standard deviation

No.	Ticker	Name	Industry	Annualized Average Return	Standard Deviation
1	AAL	ANGLO AMERICAN plc.	Mining	4.69%	37.97%
2	ABF	ASSOCIATED BRITISH FOODS plc.	Food	28.78%	21.01%
3	ANTO	ANTOFAGASTA plc.	Mining	21.52%	37.57%
4	ARM	ARM HOLDINGS plc.	Engineering	46.85%	30.47%
5	AZN	ASTRAZENECA plc.	Pharmaceuticals	15.05%	18.14%
6	BATS	BRITISH AMERICAN TOBACCO plc.	Tobacco	16.97%	16.41%
7	BG	BG GROUP plc.	Oil and Gas	2.51%	23.77%
8	BP	BP GROUP plc.	Oil and Gas	3.92%	26.10%
9	BRBY	BURBERRY GROUP plc.	Fashion	40.87%	31.86%
10	CCL	CARNIVAL plc.	Leisure	17.16%	27.65%
11	CNA	CENTRICA plc.	Energy	6.77%	15.67%
12	CPG	COMPASS GROUP plc.	Food	24.01%	17.12%
13	CRDA	CRODA INTERNATIONAL plc.	Chemicals	33.59%	26.53%
14	DGE	DIAGEO plc.	Beverages	15.14%	14.38%
15	GFS	G4S plc.	Security Services	10.39%	20.15%
16	GKN	GKN plc.	Manufacturing	36.86%	40.30%
17	GSK	GLAXOSMITHKLINE plc.	Pharmaceuticals	7.64%	15.86%
18	HMSO	HAMMERSON plc.	Real Estate	15.99%	24.96%
19	IMI	IMI plc.	Engineering	33.84%	29.66%
20	IMT	IMPERIAL TOBACCO GROUP plc.	Tobacco	12.71%	15.52%
21	ITRK	INTERTEK	Commercial Services	22.19%	21.40%
22	ITV	ITV plc.	Media	38.51%	41.27%
23	JMAT	JOHNSON MATTHEY plc.	Chemicals	24.63%	25.31%
24	KGF	KINGFISHER plc.	Retail Homeware	22.06%	27.11%
25	LAND	LAND SECURITIES GROUP plc.	Property	13.68%	26.66%
26	MGIT	MEGGITT plc.	Engineering	26.96%	28.14%
27	MKS	MARKS & SPENCER plc.	Retail	21.26%	26.90%
28	NG	NATIONAL GRID plc.	Energy	13.74%	14.65%
29	NXT	NEXT plc.	Retail Clothing	36.89%	22.27%
30	PSN	PEARSON plc.	Publishing	16.62%	21.10%
31	RB	RECKITT BENCKISER plc.	Consumer Goods	16.25%	12.65%
32	REL	REED ELSEVIER plc.	Publishing	18.11%	16.63%
33	REX	REXAM plc.	Packaging	11.80%	20.67%
34	RR	ROLLS ROYCE GROUP plc.	Manufacturing	21.11%	22.61%
35	SAB	SABMILLER plc.	Beverages	22.02%	19.89%
36	SBRY	J SAINSBURRY plc.	Supermarkets	1.57%	18.00%
37	SGE	SAGE GROUP plc.	IT	22.16%	20.06%
38	SHPG	SHIRE plc.	Pharmaceuticals	29.32%	26.69%
39	SMIN	SMITHS GRUOP plc.	Engineering	9.36%	20.08%
40	SN	SMITH & NEPHEW plc.	Medical	20.29%	18.47%
41	SSE	SSE plc.	Energy	11.56%	12.91%
42	SVT	SEVERN TRENT plc.	Water	15.76%	17.60%
43	TATE	TATE & LYLE plc.	Food	14.10%	23.36%
44	TSCO	TESCO plc.	Supermarkets	-4.83%	19.83%
45	UU	UNITED UTILITIES plc.	Water	13.39%	18.58%
46	VOD	VODAFONE GROUP plc.	Telecommunications	14.99%	17.06%
47	IHG	INTERCONTINENTAL HOTELS GROUP	Hotels	33.84%	29.66%
48	WOS	WOLSELEY plc.	Building Materials	25.81%	41.38%
49	WPP	WPP plc.	Advertising	25.58%	21.92%
50	WTB	WHITBREAD plc.	Gaming, Lodging and Restaurants	32.97%	22.81%

**APPENDIX B. Stocks divided in groups based on the magnitude of the absolute percentage valuation error**

Table 2. Group 1 (Stocks with an absolute percentage valuation error between 0% - 10%)

Group 1	
Name	Abs.% Error
J SAINSBURRY plc.	0.90
ANTOFAGASTA plc.	1.35
REXAM plc.	3.82
REED ELSEVIER plc.	5.89
NATIONAL GRID plc.	6.69
G4S plc.	6.86
TESCO plc.	8.31
WOLSELEY plc.	8.43
ITV plc.	9.76
KINGFISHER plc.	9.80

Table 3. Group 2 (Stocks with an absolute percentage valuation error between 10% - 17%)

Group 2	
Name	Abs.% Error
LAND SECURITIES GROUP plc.	10.54
NEXT plc.	11.43
BURBERRY GROUP plc.	12.37
BG GROUP plc.	13.86
WEIR	13.91
IMPERIAL TOBACCO GROUP plc.	13.97
CRODA INTERNATIONAL plc.	15.15
BRITISH AMERICAN TOBACCO plc.	16.04
CENTRICA plc.	16.41
SEVERN TRENT plc.	16.54

Table 4. Group 3 (Stocks with an absolute percentage valuation error between 17% - 26%)

Group 3	
Name	Abs.% Error
WPP plc.	17.58
SSE plc.	18.03
HAMMERSON plc.	18.46
IMI plc.	18.99
ANGLO AMERICAN plc.	19.16
MARKS & SPENCER plc.	20.45
INTERTEK	20.48
ROLLS ROYCE GROUP plc.	21.17
GLAXOSMITHKLINE plc.	21.96
SHIRE plc.	25.60

Table 5. Group 4 (Stocks with an absolute percentage valuation error between 26% - 40%)

Group 4	
Name	Abs.% Error
SMITHS GRUOP plc.	26.80
UNITED UTILITIES plc.	28.89
BP GROUP plc.	28.89
CARNIVAL plc.	29.76
WHITBREAD plc.	29.97
VODAFONE GROUP plc.	34.91
GKN plc.	35.68
TATE & LYLE plc.	37.33
RECKITT BENCKISER plc.	38.92
DIAGEO plc.	39.87
COMPASS GROUP plc.	39.89

Table 6. Group 5 (Stocks with an absolute percentage valuation error between 40% - 76%)

Group 5	
Name	Abs.% Error
ASTRAZENECA plc.	40.53
SAGE GROUP plc.	41.19
PEARSON plc.	41.62
JOHNSON MATTHEY plc.	44.26
MEGGITT plc.	45.76
SMITH & NEPHEW plc.	46.66
ASSOCIATED BRITISH FOODS plc.	53.46
SABMILLER plc.	55.59
ARM HOLDINGS plc.	75.61

## APPENDIX C. Calculated valuation errors for each model

Table 7. Valuation errors for the Dividend Discount model

No.	Ticker	Dividend Discount Model			
		Percentage	Abs.Perc.	Squared	Log
1	AAL.LN	2.00	2.00	0.04	1.98
2	ANTO.LN	-50.82	50.82	25.83	-70.97
3	ARM.LN	-77.48	77.48	60.02	-149.05
4	ABF.LN	-64.92	64.92	42.14	-104.74
5	AZN.LN	-61.90	61.90	38.32	-96.50
6	BG.LN	-49.42	49.42	24.43	-68.17
7	BP.LN	-45.33	45.33	20.55	-60.39
8	BTI.LN	-12.27	12.27	1.51	-13.09
9	BRBY.LN	-7.77	7.77	0.60	-8.08
10	CUK.LN	-62.99	62.99	39.67	-99.39
11	CNA.LN	-46.45	46.45	21.57	-62.45
12	CPG.LN	-42.05	42.05	17.68	-54.56
13	CRDA.LN	10.93	10.93	1.19	10.37
14	DGE.LN	-40.68	40.68	16.55	-52.22
15	GFS.LN	-56.24	56.24	31.63	-82.64
16	GKN.LN	7.24	7.24	0.52	6.99
17	GSK.LN	-16.69	16.69	2.79	-18.26
18	HMSO.LN	37.57	37.57	14.12	31.90
19	ITRK.LN	-47.54	47.54	22.60	-64.52
20	IHG.LN	-37.95	37.95	14.40	-47.72
21	IMI.LN	52.01	52.01	27.05	41.88
22	IMT.LN	-23.20	23.20	5.38	-26.39
23	ITV.LN	21.81	21.81	4.76	19.73
24	SBRY.LN	-14.43	14.43	2.08	-15.58
25	JMAT.LN	-41.75	41.75	17.43	-54.05
26	KGF.LN	1.22	1.22	0.01	1.22
27	MKS.LN	-41.65	41.65	17.34	-53.87
28	LAND.LN	-5.10	5.10	0.26	-5.24
29	MGGT.LN	-54.45	54.45	29.65	-78.64
30	NXT.LN	8.51	8.51	0.72	8.17
31	PSON.LN	-52.72	52.72	27.80	-74.92
32	NG.LN	-11.61	11.61	1.35	-12.34
33	RB.LN	-45.56	45.56	20.76	-60.81
34	REL.LN	27.36	27.36	7.49	24.18
35	REX.LN	-40.03	40.03	16.02	-51.13
36	RR.LN	50.82	50.82	25.83	41.09
37	SAB.LN	-57.13	57.13	32.63	-84.69
38	SGE.LN	-58.24	58.24	33.92	-87.32
39	SVT.LN	12.52	12.52	1.57	11.80
40	SHPG.LN	6.94	6.94	0.48	6.71
41	SNN.LN	-60.97	60.97	37.17	-94.07
42	SMIN.LN	-54.72	54.72	29.95	-79.24
43	SSE.LN	-68.22	68.22	46.54	-114.63
44	TATE.LN	41.32	41.32	17.07	34.58
45	TSCO.LN	-74.23	74.23	55.10	-135.59
46	UU.LN	-34.86	34.86	12.15	-42.87
47	VOD.LN	88.46	88.46	78.25	63.37
48	WOS.LN	-33.87	33.87	11.47	-41.36
49	WTB.LN	-37.01	37.01	13.70	-46.21
50	WPP.LN	-35.49	35.49	12.59	-43.83

Table 8. Valuation errors for the Free Cash Flow to the Firm model

No.	Ticker	Free Cash Flow to the Firm model			
		Percentage	Abs.Perc.	Squared	Log
1	AAL.LN	31.10	31.10	9.67	27.08
2	ANTO.LN	72.43	72.43	52.46	54.48
3	ARM.LN	-69.90	69.90	48.86	-120.07
4	ABF.LN	-28.06	28.06	7.87	-32.94
5	AZN.LN	-2.88	2.88	0.08	-2.92
6	BG.LN	34.16	34.16	11.67	29.39
7	BP.LN	-37.67	37.67	14.19	-47.27
8	BTI.LN	-6.11	6.11	0.37	-6.30
9	BRBY.LN	-27.68	27.68	7.66	-32.40
10	CUK.LN	-5.89	5.89	0.35	-6.07
11	CNA.LN	22.08	22.08	4.88	19.95
12	CPG.LN	-31.61	31.61	9.99	-38.00
13	CRDA.LN	-15.13	15.13	2.29	-16.40
14	DGE.LN	-32.69	32.69	10.68	-39.58
15	GFS.LN	80.96	80.96	65.55	59.31
16	GKN.LN	-52.38	52.38	27.44	-74.19
17	GSK.LN	-20.11	20.11	4.05	-22.46
18	HMSO.LN	-94.03	94.03	88.42	-281.86
19	ITRK.LN	36.68	36.68	13.45	31.24
20	IHG.LN	29.71	29.71	8.83	26.01
21	IMI.LN	33.90	33.90	11.49	29.19
22	IMT.LN	92.92	92.92	86.34	65.71
23	ITV.LN	-35.20	35.20	12.39	-43.39
24	SBRY.LN	52.08	52.08	27.12	41.92
25	JMAT.LN	-47.83	47.83	22.88	-65.07
26	KGF.LN	4.69	4.69	0.22	4.59
27	MKS.LN	12.89	12.89	1.66	12.12
28	LAND.LN	-55.63	55.63	30.95	-81.26
29	MGGT.LN	-33.33	33.33	11.11	-40.55
30	NXT.LN	-11.09	11.09	1.23	-11.75
31	PSO.LN	-35.07	35.07	12.30	-43.18
32	NG.LN	11.54	11.54	1.33	10.92
33	RB.LN	-33.07	33.07	10.94	-40.15
34	REL.LN	32.29	32.29	10.43	27.98
35	REX.LN	40.77	40.77	16.63	34.20
36	RR.LN	33.27	33.27	11.07	28.72
37	SAB.LN	-46.47	46.47	21.60	-62.50
38	SGE.LN	-33.24	33.24	11.05	-40.40
39	SVT.LN	-13.00	13.00	1.69	-13.93
40	SHPG.LN	-25.29	25.29	6.40	-29.16
41	SNN.LN	-21.23	21.23	4.51	-23.87
42	SMIN.LN	17.56	17.56	3.08	16.17
43	SSE.LN	22.39	22.39	5.01	20.21
44	TATE.LN	98.35	98.35	96.73	68.49
45	TSCO.LN	117.93	117.93	139.07	77.90
46	UU.LN	-11.93	11.93	1.42	-12.70
47	VOD.LN	51.18	51.18	26.20	41.33
48	WOS.LN	1.77	1.77	0.03	1.75
49	WTB.LN	-5.66	5.66	0.32	-5.83
50	WPP.LN	20.83	20.83	4.34	18.92



Table 9. Valuation errors for the Residual Income Model

No.	Ticker	Residual Income Model			
		Percentage	Abs.Perc.	Squared	Log
1	AAL.LN	24.39	24.39	5.95	21.82
2	ANTO.LN	-25.67	25.67	6.59	-29.67
3	ARM.LN	-79.47	79.47	63.15	-158.32
4	ABF.LN	-67.39	67.39	45.42	-112.06
5	AZN.LN	-56.81	56.81	32.27	-83.96
6	BG.LN	-26.32	26.32	6.93	-30.54
7	BP.LN	-3.67	3.67	0.13	-3.74
8	BTI.LN	-29.75	29.75	8.85	-35.30
9	BRBY.LN	-1.67	1.67	0.03	-1.69
10	CUK.LN	-20.41	20.41	4.17	-22.83
11	CNA.LN	-24.87	24.87	6.19	-28.60
12	CPG.LN	-46.01	46.01	21.17	-61.64
13	CRDA.LN	-41.25	41.25	17.02	-53.19
14	DGE.LN	-46.26	46.26	21.40	-62.09
15	GFS.LN	-4.16	4.16	0.17	-4.25
16	GKN.LN	-61.90	61.90	38.32	-96.51
17	GSK.LN	-29.07	29.07	8.45	-34.34
18	HMSO.LN	1.08	1.08	0.01	1.07
19	ITRK.LN	-50.58	50.58	25.58	-70.48
20	IHG.LN	-33.50	33.50	11.22	-40.80
21	IMI.LN	-28.95	28.95	8.38	-34.18
22	IMT.LN	-27.80	27.80	7.73	-32.58
23	ITV.LN	-15.89	15.89	2.52	-17.30
24	SBRY.LN	-40.34	40.34	16.28	-51.65
25	JMAT.LN	-43.19	43.19	18.65	-56.54
26	KGF.LN	-35.31	35.31	12.47	-43.55
27	MKS.LN	-32.58	32.58	10.61	-39.42
28	LAND.LN	29.12	29.12	8.48	25.56
29	MGGT.LN	-49.49	49.49	24.49	-68.30
30	NXT.LN	-31.72	31.72	10.06	-38.16
31	PSO.LN	-37.06	37.06	13.74	-46.30
32	NG.LN	-20.00	20.00	4.00	-22.31
33	RB.LN	-38.13	38.13	14.54	-48.02
34	REL.LN	-41.97	41.97	17.62	-54.43
35	REX.LN	-12.20	12.20	1.49	-13.01
36	RR.LN	-20.56	20.56	4.23	-23.02
37	SAB.LN	-63.16	63.16	39.89	-99.86
38	SGE.LN	-32.10	32.10	10.31	-38.72
39	SVT.LN	-49.14	49.14	24.14	-67.60
40	SHPG.LN	-58.46	58.46	34.18	-87.86
41	SNN.LN	-57.77	57.77	33.37	-86.19
42	SMIN.LN	-43.22	43.22	18.68	-56.60
43	SSE.LN	-8.26	8.26	0.68	-8.62
44	TATE.LN	-27.69	27.69	7.67	-32.42
45	TSCO.LN	-68.63	68.63	47.10	-115.92
46	UU.LN	-39.87	39.87	15.90	-50.87
47	VOD.LN	-34.91	34.91	12.19	-42.94
48	WOS.LN	6.81	6.81	0.46	6.59
49	WTB.LN	-47.25	47.25	22.33	-63.96
50	WPP.LN	-38.09	38.09	14.51	-47.95

Table 10. Valuation error from the average results of the analysis

No.	Ticker	Analysis Average Results			
		Percentage	Abs.Perc.	Squared	Log
1	AAL.LN	19.16	19.16	3.67	17.53
2	ANTO.LN	-1.35	1.35	0.02	-1.36
3	ARM.LN	-75.61	75.61	57.18	-141.12
4	ABF.LN	-53.46	53.46	28.58	-76.48
5	AZN.LN	-40.53	40.53	16.43	-51.97
6	BG.LN	-13.86	13.86	1.92	-14.92
7	BP.LN	-28.89	28.89	8.35	-34.09
8	BTI.LN	-16.04	16.04	2.57	-17.48
9	BRBY.LN	-12.37	12.37	1.53	-13.21
10	CUK.LN	-29.76	29.76	8.86	-35.33
11	CNA.LN	-16.41	16.41	2.69	-17.93
12	CPG.LN	-39.89	39.89	15.91	-50.90
13	CRDA.LN	-15.15	15.15	2.30	-16.43
14	DGE.LN	-39.87	39.87	15.90	-50.87
15	GFS.LN	6.86	6.86	0.47	6.63
16	GKN.LN	-35.68	35.68	12.73	-44.13
17	GSK.LN	-21.96	21.96	4.82	-24.79
18	HMSO.LN	-18.46	18.46	3.41	-20.41
19	ITRK.LN	-20.48	20.48	4.20	-22.92
20	IHG.LN	-13.91	13.91	1.94	-14.98
21	IMI.LN	18.99	18.99	3.61	17.38
22	IMT.LN	13.97	13.97	1.95	13.08
23	ITV.LN	-9.76	9.76	0.95	-10.27
24	SBRY.LN	-0.90	0.90	0.01	-0.90
25	JMAT.LN	-44.26	44.26	19.59	-58.44
26	KGF.LN	-9.80	9.80	0.96	-10.31
27	MKS.LN	-20.45	20.45	4.18	-22.87
28	LAND.LN	-10.54	10.54	1.11	-11.14
29	MGGT.LN	-45.76	45.76	20.94	-61.17
30	NXT.LN	-11.43	11.43	1.31	-12.14
31	PERSON.LN	-41.62	41.62	17.32	-53.82
32	NG.LN	-6.69	6.69	0.45	-6.92
33	RB.LN	-38.92	38.92	15.15	-49.30
34	REL.LN	5.89	5.89	0.35	5.73
35	REX.LN	-3.82	3.82	0.15	-3.89
36	RR.LN	21.17	21.17	4.48	19.21
37	SAB.LN	-55.59	55.59	30.90	-81.16
38	SGE.LN	-41.19	41.19	16.97	-53.09
39	SVT.LN	-16.54	16.54	2.74	-18.08
40	SHPG.LN	-25.60	25.60	6.56	-29.58
41	SNN.LN	-46.66	46.66	21.77	-62.84
42	SMIN.LN	-26.80	26.80	7.18	-31.19
43	SSE.LN	-18.03	18.03	3.25	-19.88
44	TATE.LN	37.33	37.33	13.93	31.72
45	TSCO.LN	-8.31	8.31	0.69	-8.68
46	UU.LN	-28.89	28.89	8.34	-34.09
47	VOD.LN	34.91	34.91	12.19	29.94
48	WOS.LN	-8.43	8.43	0.71	-8.81
49	WTB.LN	-29.97	29.97	8.98	-35.63
50	WPP.LN	-17.58	17.58	3.09	-19.34

## APPENDIX D. Graphical representation of the calculated valuation errors for each model

Figure 1. Absolute percentage error magnitude for each model

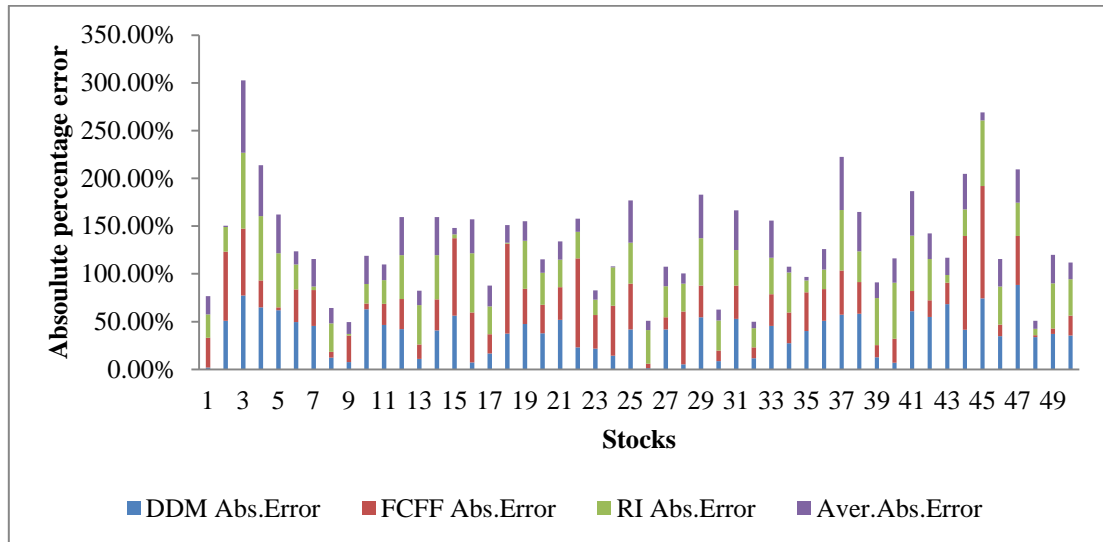


Figure 2. Percentage error magnitude for each model

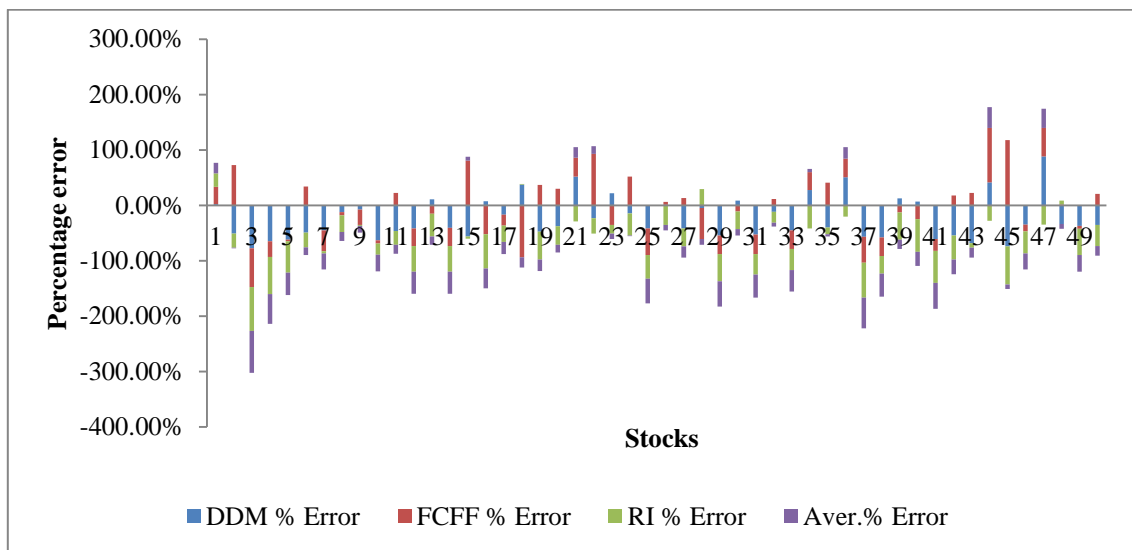


Figure 3. Squared error magnitude for each model

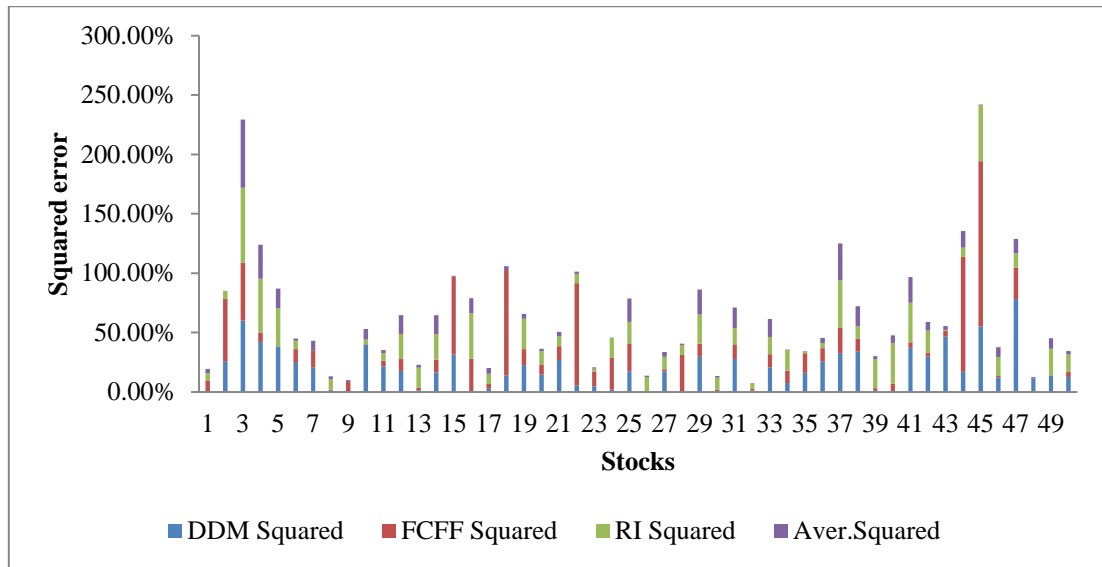
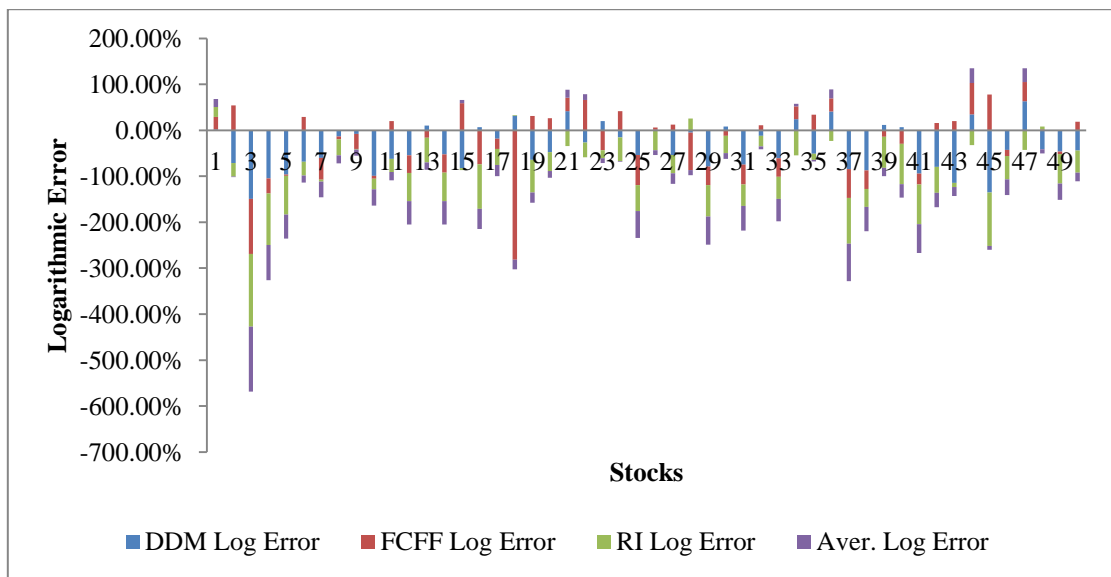


Figure 4. Logarithmic error magnitude for each model



## APPENDIX E. Portfolios which are included on the efficient frontier for each of the five groups of stocks with different magnitude of the absolute valuation error

Table 11. Portfolios which are included in the efficient frontier of group 1 which consists of stocks with an absolute percentage valuation error of 0% - 10%

						M-V		OPT.												E-W
Mean	2.00%	4.00%	6.00%	8.00%	10.00%	12.42%	14.00%	16.00%	18.42%	20.00%	22.00%	24.00%	26.00%	28.00%	30.00%	32.00%	34.00%	36.00%	38.00%	15.87%
STDEV	14.03%	12.90%	12.06%	11.44%	11.02%	10.80%	10.93%	11.40%	12.68%	13.90%	15.75%	17.86%	20.21%	22.92%	25.90%	29.07%	32.38%	36.10%	40.19%	15.73%
Ratio	0.1425843	0.3100158	0.4975614	0.6994748	0.9072279	1.1491647	1.2811206	1.4032736	1.4523298	1.4386506	1.3969837	1.3441157	1.2863813	1.2213747	1.1581138	1.1008787	1.0499507	0.9973542	0.945579	1.0085987
No.Stocks	4	4	5	6	6	5	6	6	6	6	6	5	4	4	4	4	3	3	2	10

W E I G H T S	SBRV	28.24%	28.69%	27.62%	25.77%	24.00%	19.03%	10.69%	0.48%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	ANTO	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	REX	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	REL	0.00%	0.00%	4.59%	11.26%	15.97%	22.51%	28.01%	34.41%	37.89%	39.58%	41.71%	43.73%	38.71%	28.23%	17.74%	7.26%	0.00%	0.00%	10.00%
	NG	16.82%	25.65%	29.76%	31.34%	32.38%	33.52%	34.27%	35.55%	27.86%	22.51%	15.75%	4.90%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	GFS	12.47%	14.65%	16.31%	17.54%	17.84%	17.72%	16.57%	14.82%	8.45%	4.88%	0.36%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	TSCO	42.47%	31.01%	21.72%	13.69%	6.46%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	WOS	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.58%	2.27%	2.97%	3.03%	3.12%	2.49%	1.98%	1.61%	1.24%	0.86%	0.11%	0.00%	10.00%
	ITV	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	6.35%	11.13%	17.16%	24.20%	32.80%	42.53%	52.26%	61.99%	72.58%	84.76%	10.00%
	KGF	0.00%	0.00%	0.00%	0.40%	3.35%	7.23%	9.88%	12.47%	16.48%	18.88%	21.90%	24.68%	26.52%	27.64%	28.76%	29.88%	27.31%	15.24%	10.00%

Table 12. Portfolios which are included in the efficient frontier of group 2 which consists of stocks with an absolute percentage valuation error of 10% - 17%

						M-V		OPT.												E-W
Mean	3.00%	4.00%	6.00%	8.00%	10.00%	12.00%	14.00%	15.82%	18.00%	20.00%	22.00%	24.00%	26.00%	27.56%	30.00%	32.00%	34.00%	36.00%	38.00%	21.36%
STDEV	21.55%	17.66%	13.54%	11.53%	10.72%	10.12%	9.81%	9.72%	9.85%	10.05%	10.45%	11.02%	11.73%	12.38%	13.63%	14.89%	16.31%	17.86%	20.31%	12.86%
Ratio	0.1391816	0.2264566	0.443028	0.6938416	0.9328547	1.1853734	1.4278386	1.62796	1.8269107	1.9902908	2.1046773	2.1775277	2.216328	2.2269181	2.2014159	2.1494734	2.0846353	2.0161193	1.870793	1.6605683
No.Stocks	2	2	3	4	6	6	6	6	9	8	8	8	8	7	6	6	6	5	3	10

W E I G H T S	LAND	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	NXT	0.00%	0.00%	0.00%	0.00%	2.37%	8.88%	14.78%	18.52%	22.27%	23.64%	26.24%	28.81%	31.42%	34.00%	38.78%	42.72%	46.66%	50.60%	10.00%
	BRBY	0.00%	0.00%	0.00%	0.00%	0.19%	0.00%	0.00%	0.00%	0.76%	1.41%	2.46%	3.50%	4.56%	5.96%	9.02%	11.57%	14.12%	16.67%	10.00%
	BG	88.44%	64.99%	34.25%	20.48%	11.64%	11.44%	8.13%	5.38%	1.47%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	IMT	0.00%	0.00%	11.61%	32.08%	35.23%	32.81%	31.54%	30.16%	26.29%	22.81%	17.99%	13.25%	8.34%	3.24%	0.00%	0.00%	0.00%	0.00%	10.00%
	CRDA	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%	0.03%	2.15%	0.03%	4.61%	5.97%	7.32%	8.70%	10.08%	13.35%	16.32%	19.28%	22.24%	10.00%
	BATS	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.98%	6.95%	10.67%	14.29%	18.12%	20.32%	16.65%	11.08%	5.50%	0.00%	10.00%
	CNA	11.56%	35.01%	54.14%	45.23%	41.05%	34.76%	31.67%	28.89%	24.65%	20.49%	14.38%	8.26%	2.16%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	SVT	0.00%	0.00%	0.00%	2.20%	9.52%	12.10%	13.84%	14.90%	16.82%	17.19%	18.27%	19.35%	20.42%	19.88%	15.56%	11.31%	7.07%	2.76%	10.00%
	IHG	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.73%	2.90%	4.03%	5.22%	6.29%	6.52%	6.65%	7.01%	7.37%	7.72%	10.00%

Table 13. Portfolios which are included in the efficient frontier of group 3 which consists of stocks with an absolute percentage valuation error of 17% - 26%

	M-V							OPT.							E-W			
Mean	5.00%	7.00%	9.00%	10.00%	12.00%	13.00%	14.00%	15.71%	16.00%	18.00%	20.00%	21.24%	24.00%	26.00%	28.00%	30.00%	32.00%	19.32%
STDEV	34.43%	16.63%	12.32%	11.50%	10.64%	10.38%	10.19%	10.06%	10.06%	10.29%	10.91%	11.52%	13.29%	14.85%	16.66%	19.04%	21.96%	14.98%
Ratio	0.1452339	0.4208785	0.7304064	0.8694295	1.1274708	1.2529792	1.3743087	1.5614353	1.5898033	1.749875	1.8323776	1.8435865	1.805321	1.7510981	1.6808627	1.5755701	1.4574102	1.2895218
No.Stocks	2	2	3	4	6	6	6	6	6	6	7	7	6	6	5	5	2	10

W E I G H T S	WPP	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.43%	1.32%	0.73%	3.49%	0.68%	0.66%	0.00%	10.00%
	SSE	0.00%	0.00%	40.09%	53.51%	52.78%	51.66%	50.47%	48.44%	48.09%	45.71%	35.65%	29.41%	15.77%	4.75%	0.00%	0.00%	0.00%	10.00%
	HMSO	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	IMI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.61%	4.47%	11.67%	15.54%	26.55%	41.57%	59.35%	10.00%
	AAL	89.39%	21.65%	7.10%	3.56%	0.13%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	MKS	0.00%	0.00%	0.00%	0.00%	3.95%	5.08%	6.15%	7.97%	8.28%	10.41%	12.10%	12.66%	14.14%	14.40%	12.60%	7.56%	0.00%	10.00%
	ITRK	0.00%	0.00%	0.00%	2.54%	8.97%	10.93%	12.79%	15.97%	16.51%	20.23%	24.00%	24.66%	26.33%	26.45%	20.72%	7.88%	0.00%	10.00%
	RR	0.00%	0.00%	0.00%	0.00%	3.37%	3.94%	4.28%	4.87%	4.97%	5.65%	4.32%	2.74%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	GSK	10.61%	78.35%	52.81%	40.39%	30.80%	25.97%	21.19%	13.03%	11.64%	2.09%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	SHPG	0.00%	0.00%	0.00%	0.00%	0.00%	2.41%	5.11%	9.72%	10.51%	15.90%	21.87%	24.73%	31.36%	35.37%	39.45%	42.32%	40.65%	10.00%

Table 14. Portfolios which are included in the efficient frontier of group 4 which consists of stocks with an absolute percentage valuation error of 26% - 40%

	M-V							OPT.							E-W			
Mean	4.00%	6.00%	8.00%	10.00%	12.00%	14.00%	15.03%	17.00%	19.00%	20.00%	22.46%	24.00%	26.00%	28.00%	30.00%	32.00%	34.00%	18.01%
STDEV	25.82%	20.42%	16.72%	13.91%	11.61%	10.07%	9.74%	9.97%	10.53%	10.92%	12.10%	12.98%	14.30%	15.86%	17.65%	20.06%	23.21%	13.57%
Ratio	0.15490171	0.293814	0.4784597	0.7190231	1.0334681	1.390463	1.54	1.7055047	1.8045019	1.831193	1.8560168	1.8484972	1.8187679	1.7652025	1.6997328	1.5953965	1.464953	1.3267912
No. Stocks	2	3	4	6	6	6	7	7	7	7	7	6	4	4	3	3	2	11

WEIGHTS	SMIN	1.56%	34.10%	35.58%	29.27%	20.62%	11.97%	2.67%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	9.09%
	UU	0.00%	2.40%	20.36%	23.36%	23.47%	23.58%	21.67%	17.89%	13.32%	10.92%	5.31%	1.43%	0.00%	0.00%	0.00%	0.00%	0.00%	9.09%
	BP	98.44%	63.51%	42.10%	27.53%	15.32%	3.10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	9.09%
	CCL	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.76%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	9.09%
	WTB	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	5.53%	13.00%	16.82%	23.21%	27.96%	35.01%	42.49%	50.99%	65.72%	73.52%	9.09%
	VOD	0.00%	0.00%	1.96%	11.15%	13.64%	16.13%	15.96%	14.06%	11.01%	9.38%	6.78%	4.07%	0.00%	0.00%	0.00%	0.00%	0.00%	9.09%
	GKN	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.18%	0.18%	3.17%	4.39%	6.13%	8.38%	11.07%	16.36%	26.48%	9.09%
	TATE	0.00%	0.00%	0.00%	1.40%	4.50%	7.60%	7.97%	7.31%	5.10%	3.89%	0.44%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	9.09%
	RB	0.00%	0.00%	0.00%	7.31%	22.46%	37.62%	46.80%	45.68%	42.30%	40.55%	36.33%	32.97%	24.89%	11.50%	0.00%	0.00%	0.00%	9.09%
	DGE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.16%	0.73%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	9.09%
CPG	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	8.80%	15.11%	18.26%	24.76%	29.18%	33.98%	37.63%	37.93%	17.92%	0.00%	9.09%	

Table 15. Portfolios which are included in the efficient frontier of group 5 which consists of stocks with an absolute percentage valuation error of 26% - 40%

						M-V			OPT.							E-W
Mean	15.50%	16.00%	17.00%	18.00%	19.00%	20.76%	23.00%	25.00%	28.07%	30.00%	33.00%	36.00%	39.00%	42.00%	45.00%	24.82%
STDEV	14.58%	13.18%	11.61%	11.26%	11.10%	10.99%	11.18%	11.63%	12.80%	13.76%	15.55%	17.68%	20.29%	23.57%	27.67%	12.96%
Ratio	1.0630807	1.2135153	1.4643714	1.5989425	1.7115569	1.89	2.0576246	2.1489352	2.1937671	2.1807345	2.1228518	2.0360764	1.9223034	1.7820177	1.6264822	1.9155351
No. Stocks	2	3	3	8	9	9	8	8	8	8	7	4	3	2	2	9

WEIGHTS	AZN	71.54%	60.22%	44.97%	37.62%	35.54%	31.91%	26.87%	22.32%	15.20%	10.88%	2.56%	0.00%	0.00%	0.00%	0.00%	11.11%
	SGE	0.00%	0.00%	0.00%	2.40%	1.65%	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	11.11%
	PSON	28.46%	30.99%	25.51%	20.87%	19.18%	16.27%	12.46%	9.07%	3.76%	0.55%	0.00%	0.00%	0.00%	0.00%	0.00%	11.11%
	JMAT	0.00%	0.00%	0.00%	5.00%	5.13%	5.07%	4.80%	4.42%	3.82%	3.53%	1.85%	0.00%	0.00%	0.00%	0.00%	11.11%
	MGGT	0.00%	0.00%	0.00%	0.00%	0.16%	0.81%	1.53%	2.16%	2.88%	3.64%	4.71%	5.90%	4.51%	0.00%	0.00%	11.11%
	SN	0.00%	8.79%	29.52%	32.42%	31.99%	31.12%	29.44%	27.81%	25.29%	23.75%	19.56%	11.59%	0.00%	0.00%	0.00%	11.11%
	ABF	0.00%	0.00%	0.00%	0.46%	2.42%	6.05%	10.29%	14.27%	20.35%	24.19%	31.32%	36.51%	38.47%	26.83%	10.22%	11.11%
	SAB	0.00%	0.00%	0.00%	0.74%	0.96%	1.32%	1.97%	2.56%	3.99%	4.17%	3.36%	0.00%	0.00%	0.00%	0.00%	11.11%
	ARM	0.00%	0.00%	0.00%	0.49%	2.96%	7.23%	12.63%	17.39%	24.71%	29.28%	36.64%	45.99%	57.01%	73.17%	89.78%	11.11%

Table 16. Portfolios which are included in the efficient frontier of all stocks

	MV								OPT																E-W
MEAN	2.00%	4.00%	6.00%	8.00%	10.00%	12.00%	14.00%	16.75%	18.00%	20.00%	22.00%	24.00%	26.00%	26.49%	28.00%	30.00%	32.00%	34.00%	36.00%	38.00%	40.00%	42.00%	19.82%		
STDEV	12.57%	11.40%	10.31%	9.28%	8.65%	8.21%	7.92%	7.68%	8.05%	8.43%	8.88%	9.42%	9.59%	9.75%	10.34%	11.16%	12.07%	13.11%	14.30%	15.75%	17.72%	20.25%	12.98%		
RATIO	0.16	0.35	0.58	0.86	1.16	1.46	1.77	2.18	2.24	2.37	2.48	2.55	2.71	2.72	2.71	2.69	2.65	2.59	2.52	2.41	2.26	2.07	1.53		
No. Stovks	46	8	9	12	15	19	20	13	20	23	19	20	18	13	18	15	14	14	13	11	9	7	50		
AAL	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
ABF	0.63%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.36%	0.65%	1.91%	2.72%	4.02%	6.10%	8.02%	9.19%	9.50%	5.27%	0.00%	0.00%	2.00%		
ANTO	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
ARM	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.88%	7.28%	9.20%	11.49%	10.81%	11.30%	12.91%	14.91%	17.10%	20.10%	23.58%	29.80%	38.70%	48.63%	2.00%		
AZN	0.70%	0.00%	0.00%	0.82%	0.00%	0.00%	0.00%	4.66%	0.00%	0.00%	0.00%	0.00%	0.00%	2.45%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
BATS	0.66%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
BG	7.27%	10.67%	8.97%	6.84%	5.79%	2.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
BP	3.93%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
BRBY	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.59%	1.36%	1.99%	0.00%	0.00%	0.31%	0.82%	1.52%	2.24%	2.98%	4.71%	7.19%	10.04%	2.00%		
CCL	0.48%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
CNA	0.63%	19.64%	14.99%	10.45%	4.19%	0.75%	0.11%	0.00%	0.06%	0.06%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
CPG	0.60%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
CRDA	0.60%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.34%	2.12%	4.40%	4.46%	4.22%	5.88%	6.71%	7.59%	8.57%	9.34%	8.50%	4.74%	0.00%	2.00%		
DGE	0.57%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
GFS	0.59%	4.62%	5.85%	6.91%	7.17%	7.50%	7.61%	7.40%	6.08%	4.72%	4.29%	2.27%	1.95%	1.41%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
GKN	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
GSK	0.52%	10.77%	10.50%	7.86%	6.08%	2.17%	0.31%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
HMSO	0.42%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
IMI	0.53%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
IMT	0.69%	7.59%	9.44%	10.85%	8.51%	7.89%	6.31%	3.40%	3.67%	3.32%	0.12%	0.11%	0.10%	0.00%	0.09%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
ITRK	0.68%	0.00%	0.00%	0.00%	0.00%	2.57%	3.07%	2.65%	1.63%	1.47%	1.09%	0.01%	0.01%	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	2.00%		
ITV	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.30%	1.37%	3.14%	5.68%	8.25%	2.00%		
JMAT	0.49%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
KGF	0.49%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.06%	0.91%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
LAND	0.39%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
MGGT	0.45%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
MKS	0.59%	0.00%	0.00%	0.00%	0.00%	0.00%	0.05%	0.00%	0.06%	0.06%	0.10%	0.09%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
NG	0.69%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.11%	0.18%	0.00%	0.12%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
NXT	0.01%	0.00%	0.00%	0.00%	0.00%	0.03%	0.03%	5.50%	0.04%	0.04%	0.20%	0.97%	17.14%	16.56%	18.03%	19.49%	20.99%	22.37%	23.61%	25.08%	26.92%	28.24%	2.00%		
PSON	0.62%	0.00%	0.00%	0.00%	0.00%	0.92%	1.41%	2.10%	1.76%	1.46%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
RB	0.76%	0.00%	5.41%	9.30%	17.89%	20.01%	22.38%	21.51%	23.91%	22.10%	21.99%	19.40%	13.85%	12.73%	10.55%	6.23%	0.60%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
REL	0.64%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
REX	0.42%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
RR	0.51%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
SAB	0.53%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
SBRY	9.33%	19.23%	20.08%	19.80%	18.58%	13.97%	8.77%	2.13%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
SGE	0.62%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
SHP	0.72%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.30%	1.77%	6.62%	7.82%	7.26%	7.01%	8.73%	9.77%	10.99%	12.46%	13.77%	13.47%	9.08%	2.21%	2.00%		
SMIN	0.55%	0.00%	0.00%	0.00%	0.05%	0.05%	0.05%	0.00%	0.05%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
SN	0.74%	0.00%	0.00%	0.00%	2.77%	6.59%	8.29%	11.58%	12.77%	13.32%	12.72%	12.76%	14.39%	14.39%	14.30%	14.42%	14.22%	10.57%	5.53%	0.00%	0.00%	0.00%	2.00%		
SSE	0.70%	7.08%	13.13%	18.21%	23.37%	25.49%	21.79%	19.11%	17.92%	15.73%	16.54%	13.70%	10.25%	8.73%	6.96%	2.41%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
SVT	0.72%	0.00%	0.00%	2.29%	1.58%	3.56%	9.98%	10.42%	13.22%	13.94%	13.28%	13.15%	12.29%	11.86%	11.96%	11.82%	10.46%	4.60%	0.00%	0.00%	0.00%	0.00%	2.00%		
TATE	0.54%	0.00%	0.00%	0.00%	0.00%	0.11%	0.11%	0.00%	0.11%	0.11%	0.13%	0.08%	0.08%	0.00%	0.06%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
TSCO	56.97%	20.40%	11.62%	4.98%	0.18%	0.16%	0.09%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.00%		
UU	0.60%	0.00%	0																						



## APPENDIX F. List of stocks used in the research, their industry, the ticker, their price and the price obtained from the valuation results

Table 17. List of stocks used in the research, their industry, the ticker, their price and the price obtained from the valuation results

No.	Name	Ticker	Industry	Price	DDM	FCFF	RI	Analysis Average
1	ANGLO AMERICAN plc.	AAL.LN	Mining	\$ 15.50	\$ 15.81	\$ 20.32	\$ 19.28	\$ 18.47
2	ANTOFAGASTA plc.	ANTO.LN	Mining	\$ 11.57	\$ 5.69	\$ 19.95	\$ 8.60	\$ 11.41
3	ARM HOLDINGS plc.	ARML.LN	Engineering	\$ 15.05	\$ 3.39	\$ 4.53	\$ 3.09	\$ 3.67
4	ASSOCIATED BRITISH FOODS plc.	ABF.LN	Food	\$ 46.86	\$ 16.44	\$ 33.71	\$ 15.28	\$ 21.81
5	ASTRAZENECA plc.	AZN.LN	Pharmaceuticals	\$ 70.11	\$ 26.71	\$ 68.09	\$ 30.28	\$ 41.69
6	BG GROUP plc.	BG.LN	Oil and Gas	\$ 12.12	\$ 6.13	\$ 16.26	\$ 8.93	\$ 10.44
7	BP GROUP plc.	BP.LN	Oil and Gas	\$ 6.00	\$ 3.28	\$ 3.74	\$ 5.78	\$ 4.27
8	BRITISH AMERICAN TOBACCO plc.	BTIL.LN	Tobacco	\$ 53.05	\$ 46.54	\$ 49.81	\$ 37.27	\$ 44.54
9	BURBERRY GROUP plc.	BRBY.LN	Fashion	\$ 25.11	\$ 23.16	\$ 18.16	\$ 24.69	\$ 22.00
10	CARNIVAL plc.	CUK.LN	Leisure	\$ 46.20	\$ 17.10	\$ 43.48	\$ 36.77	\$ 32.45
11	CENTRICA plc.	CNA.LN	Energy	\$ 3.94	\$ 2.11	\$ 4.81	\$ 2.96	\$ 3.29
12	COMPASS GROUP plc.	CPG.LN	Food	\$ 16.67	\$ 9.66	\$ 11.40	\$ 9.00	\$ 10.02
13	CRODA INTERNATIONAL plc.	CRDA.LN	Chemicals	\$ 40.00	\$ 44.37	\$ 33.95	\$ 23.50	\$ 33.94
14	DIAGEO plc.	DGE.LN	Beverages	\$ 27.78	\$ 16.48	\$ 18.70	\$ 14.93	\$ 16.70
15	G4S plc.	GFS.LN	Security Services	\$ 4.57	\$ 2.00	\$ 8.27	\$ 4.38	\$ 4.88
16	GKN plc.	GKN.LN	Manufacturing	\$ 5.25	\$ 5.63	\$ 2.50	\$ 2.00	\$ 3.38
17	GLAXOSMITHKLINE plc.	GSK.LN	Pharmaceuticals	\$ 21.33	\$ 17.77	\$ 17.04	\$ 15.13	\$ 16.65
18	HAMMERSON plc.	HMSO.LN	Real Estate	\$ 10.22	\$ 14.06	\$ 0.61	\$ 10.33	\$ 8.33
19	INTERTEK	ITRK.LN	Commercial Services	\$ 40.49	\$ 21.24	\$ 55.34	\$ 20.01	\$ 32.20
20	INTERCONTINENTAL HOTELS GROUP	IHG.LN	Hotels	\$ 39.82	\$ 24.71	\$ 51.65	\$ 26.48	\$ 34.28
21	IMI plc.	IMI.LN	Engineering	\$ 18.17	\$ 27.62	\$ 24.33	\$ 12.91	\$ 21.62
22	IMPERIAL TOBACCO GROUP plc.	IMT.LN	Tobacco	\$ 44.92	\$ 34.50	\$ 86.66	\$ 32.43	\$ 51.20
23	ITV plc.	ITV.LN	Media	\$ 3.21	\$ 3.91	\$ 2.08	\$ 2.70	\$ 2.90
24	J SAINSBURY plc.	SBRY.LN	Supermarkets	\$ 4.09	\$ 3.50	\$ 6.22	\$ 2.44	\$ 4.05
25	JOHNSON MATTHEY plc.	JMAT.LN	Chemicals	\$ 53.00	\$ 30.87	\$ 27.65	\$ 30.11	\$ 29.54
26	KINGFISHER plc.	KGF.LN	Retail Homeware	\$ 4.90	\$ 4.96	\$ 5.13	\$ 3.17	\$ 4.42
27	MARKS & SPENCER plc.	MKS.LN	Retail	\$ 8.38	\$ 4.89	\$ 9.46	\$ 5.65	\$ 6.67
28	LAND SECURITIES GROUP plc.	LAND.LN	Property	\$ 18.03	\$ 17.11	\$ 8.00	\$ 23.28	\$ 16.13
29	MEGGITT plc.	MGGT.LN	Engineering	\$ 7.86	\$ 3.58	\$ 5.24	\$ 3.97	\$ 4.26
30	NEXT plc.	NXT.LN	Retail Clothing	\$104.00	\$112.85	\$ 92.47	\$ 71.01	\$ 92.11
31	PEARSON plc.	PSON.LN	Publishing	\$ 20.56	\$ 9.72	\$ 13.35	\$ 12.94	\$ 12.00
32	NATIONAL GRID plc.	NG.LN	Energy	\$ 13.95	\$ 12.33	\$ 15.56	\$ 11.16	\$ 13.02
33	RECKITT BENCKISER plc.	RB.LN	Consumer Goods	\$ 80.59	\$ 43.87	\$ 53.94	\$ 49.86	\$ 49.22
34	REED ELSEVIER plc.	REL.LN	Publishing	\$ 16.63	\$ 21.18	\$ 22.00	\$ 9.65	\$ 17.61
35	REXAM plc.	REX.LN	Packaging	\$ 6.72	\$ 4.03	\$ 9.46	\$ 5.90	\$ 6.46
36	ROLLS ROYCE GROUP plc.	RR.LN	Manufacturing	\$ 14.64	\$ 22.08	\$ 19.51	\$ 11.63	\$ 17.74
37	SABMILLER plc.	SAB.LN	Beverages	\$ 50.87	\$ 21.81	\$ 27.23	\$ 18.74	\$ 22.59
38	SAGE GROUP plc.	SGE.LN	IT	\$ 7.04	\$ 2.94	\$ 4.70	\$ 4.78	\$ 4.14
39	SEVERN TRENT plc.	SVT.LN	Water	\$ 31.30	\$ 35.22	\$ 27.23	\$ 15.92	\$ 26.12
40	SHIRE plc.	SHPG.LN	Pharmaceuticals	\$ 79.95	\$ 85.50	\$ 59.73	\$ 33.21	\$ 59.48
41	SMITH & NEPHEW plc.	SNN.LN	Medical	\$ 17.19	\$ 6.71	\$ 13.54	\$ 7.26	\$ 9.17
42	SMITHS GRUOP plc.	SMIN.LN	Engineering	\$ 18.00	\$ 8.15	\$ 21.16	\$ 10.22	\$ 13.18
43	SSE plc.	SSE.LN	Energy	\$ 23.00	\$ 7.31	\$ 28.15	\$ 21.10	\$ 18.85
44	TATE & LYLE plc.	TATE.LN	Food	\$ 9.10	\$ 12.86	\$ 18.05	\$ 6.58	\$ 12.50
45	TESCO plc.	TSCO.LN	Supermarkets	\$ 3.57	\$ 0.92	\$ 7.78	\$ 1.12	\$ 3.27
46	UNITED UTILITIES plc.	UU.LN	Water	\$ 14.17	\$ 9.23	\$ 12.48	\$ 8.52	\$ 10.08
47	VODAFONE GROUP plc.	VOD.LN	Telecommunications	\$ 3.38	\$ 6.37	\$ 5.11	\$ 2.20	\$ 4.56
48	WOLSELEY plc.	WOS.LN	Building Materials	\$ 61.11	\$ 40.41	\$ 62.19	\$ 65.27	\$ 55.96
49	WHITBREAD plc.	WTB.LN	Gaming, Lodging and Restaurants	\$ 78.58	\$ 49.50	\$ 74.13	\$ 41.45	\$ 55.03
50	WPP plc.	WPP.LN	Advertising	\$ 23.81	\$ 15.36	\$ 28.77	\$ 14.74	\$ 19.62

Source: Yahoo Finance UK, FTSE 100 Index, 2015.

## APPENDIX G. Annualized covariance matrix for the stocks in each of the five groups and the total covariance matrix.

Table 18. Annualized covariance matrix for the stocks in Group 1 which have an absolute percentage valuation error from 0% - 10%

	<b>SBRY</b>	<b>ANTO</b>	<b>REX</b>	<b>REL</b>	<b>NG</b>	<b>GFS</b>	<b>TSCO</b>	<b>WOS</b>	<b>ITV</b>	<b>KGF</b>
<b>SBRY</b>	0.0323905	0.0201287	0.015169	0.0100273	0.004973	0.0046844	0.0223372	0.0168166	0.0167879	0.01047
<b>ANTO</b>	0.0201287	0.1411591	0.0249637	0.0156316	0.0088374	0.0046229	0.0240728	0.0618666	0.0594746	0.0346879
<b>REX</b>	0.015169	0.0249637	0.0427211	0.0137512	0.0091394	0.0130224	0.0158722	0.0334605	0.0460192	0.0192017
<b>REL</b>	0.0100273	0.0156316	0.0137512	0.0276433	0.0086193	0.0012039	0.0109369	0.0089887	0.0246467	0.0061065
<b>NG</b>	0.004973	0.0088374	0.0091394	0.0086193	0.0214494	0.0076251	0.0078986	0.0004025	0.0242219	0.0034023
<b>GFS</b>	0.0046844	0.0046229	0.0130224	0.0012039	0.0076251	0.0406214	0.0108501	0.0214027	0.0108575	0.0104857
<b>TSCO</b>	0.0223372	0.0240728	0.0158722	0.0109369	0.0078986	0.0108501	0.039338	0.0204554	0.0201722	0.0174896
<b>WOS</b>	0.0168166	0.0618666	0.0334605	0.0089887	0.0004025	0.0214027	0.0204554	0.1712282	0.0619451	0.0497167
<b>ITV</b>	0.0167879	0.0594746	0.0460192	0.0246467	0.0242219	0.0108575	0.0201722	0.0619451	0.1703237	0.0240325
<b>KGF</b>	0.01047	0.0346879	0.0192017	0.0061065	0.0034023	0.0104857	0.0174896	0.0497167	0.0240325	0.0734687

Table 19. Annualized covariance matrix for the stocks in Group 2 which have an absolute percentage valuation error from 10% - 17%

	<b>LAND</b>	<b>NXT</b>	<b>BRBY</b>	<b>BG</b>	<b>IMT</b>	<b>CRDA</b>	<b>BATS</b>	<b>CNA</b>	<b>SVT</b>	<b>IHG</b>
<b>LAND</b>	0.0710581	0.0184407	0.0340276	0.0088155	0.0098588	0.026493	0.010861	0.0097368	0.0130099	0.0378087
<b>NXT</b>	0.0184407	0.0496007	0.0251747	0.007457	0.0012586	0.0121834	-0.000945	-0.003006	0.0005507	0.0145861
<b>BRBY</b>	0.0340276	0.0251747	0.1014945	0.0224615	0.008946	0.024728	0.0100083	0.0028747	0.0010649	0.0472366
<b>BG</b>	0.0088155	0.007457	0.0224615	0.056486	0.0038398	0.0132172	0.012749	0.0095217	0.0055734	0.0269248
<b>IMT</b>	0.0098588	0.0012586	0.008946	0.0038398	0.0240781	0.0059061	0.0179762	0.0028529	0.0053442	0.0001609
<b>CRDA</b>	0.026493	0.0121834	0.024728	0.0132172	0.0059061	0.0703826	0.0121581	0.007585	0.0067375	0.0300306
<b>BATS</b>	0.010861	-0.000945	0.0100083	0.012749	0.0179762	0.0121581	0.026915	0.0087129	0.0065839	0.0087415
<b>CNA</b>	0.0097368	-0.003006	0.0028747	0.0095217	0.0028529	0.007585	0.0087129	0.0245527	0.0092644	0.0154747
<b>SVT</b>	0.0130099	0.0005507	0.0010649	0.0055734	0.0053442	0.0067375	0.0065839	0.0092644	0.0309775	0.0024554
<b>IHG</b>	0.0378087	0.0145861	0.0472366	0.0269248	0.0001609	0.0300306	0.0087415	0.0154747	0.0024554	0.087963

Table 20. Annualized covariance matrix for the stocks in Group 3 which have an absolute percentage valuation error from 17% - 26%

	<b>WPP</b>	<b>SSE</b>	<b>HMSO</b>	<b>IMI</b>	<b>AAL</b>	<b>MKS</b>	<b>ITRK</b>	<b>RR</b>	<b>GSK</b>	<b>SHPG</b>
<b>WPP</b>	0.0480279	0.0063312	0.0311365	0.0418404	0.0477411	0.0187538	0.0206433	0.0268231	0.0121952	0.0138152
<b>SSE</b>	0.0063312	0.0166604	0.0088078	0.0061499	0.0082436	0.0023445	0.0030513	0.0017932	0.0072896	0.0035011
<b>HMSO</b>	0.0311365	0.0088078	0.0622781	0.0366357	0.0518147	0.0259806	0.0200196	0.0218189	0.0116866	0.012183
<b>IMI</b>	0.0418404	0.0061499	0.0366357	0.087963	0.0521082	0.0186989	0.0360547	0.029271	0.0058708	0.0112973
<b>AAL</b>	0.0477411	0.0082436	0.0518147	0.0521082	0.1441514	0.0256058	0.0232193	0.0344946	0.0161099	0.0293881
<b>MKS</b>	0.0187538	0.0023445	0.0259806	0.0186989	0.0256058	0.072344	0.0068274	0.0157534	0.0097095	0.0009867
<b>ITRK</b>	0.0206433	0.0030513	0.0200196	0.0360547	0.0232193	0.0068274	0.0457916	0.0167099	0.0044114	-0.006192
<b>RR</b>	0.0268231	0.0017932	0.0218189	0.029271	0.0344946	0.0157534	0.0167099	0.0511038	0.0112871	0.0140843
<b>GSK</b>	0.0121952	0.0072896	0.0116866	0.0058708	0.0161099	0.0097095	0.0044114	0.0112871	0.025151	0.0132173
<b>SHPG</b>	0.0138152	0.0035011	0.012183	0.0112973	0.0293881	0.0009867	-0.006192	0.0140843	0.0132173	0.0712459

Table 21. Annualized covariance matrix for the stocks in Group 4 which have an absolute percentage valuation error from 26% - 40%

	SMIN	UU	BP	CCL	WTB	VOD	GKN	TATE	RB	DGE	CPG
SMIN	0.0403233	0.0051273	0.0209731	0.010859	0.0140171	0.0071596	0.0395418	0.0146975	0.009565	0.0103722	0.0096428
UU	0.0051273	0.034516	0.0114321	0.0092941	0.0122099	0.0042931	0.0192139	0.0017414	0.0013968	0.0081796	0.0106773
BP	0.0209731	0.0114321	0.0681367	0.0288613	0.0307684	0.0131219	0.0493398	0.0185832	0.0120941	0.0181508	0.0198712
CCL	0.010859	0.0092941	0.0288613	0.076467	0.0323851	0.0135232	0.0341411	0.0112253	0.006652	0.0107724	0.0193693
WTB	0.0140171	0.0122099	0.0307684	0.0323851	0.0520471	0.0131986	0.0368389	0.0153318	0.0079866	0.0124883	0.0150881
VOD	0.0071596	0.0042931	0.0131219	0.0135232	0.0131986	0.0290898	0.0088029	0.0105655	0.0049117	0.0113095	0.0096763
GKN	0.0395418	0.0192139	0.0493398	0.0341411	0.0368389	0.0088029	0.162401	0.029402	0.012158	0.0197747	0.0168271
TATE	0.0146975	0.0017414	0.0185832	0.0112253	0.0153318	0.0105655	0.029402	0.0545855	0.0039582	0.0180215	0.0089286
RB	0.009565	0.0013968	0.0120941	0.006652	0.0079866	0.0049117	0.012158	0.0039582	0.0160103	0.0069647	0.0085231
DGE	0.0103722	0.0081796	0.0181508	0.0107724	0.0124883	0.0113095	0.0197747	0.0180215	0.0069647	0.020674	0.0133355
CPG	0.0096428	0.0106773	0.0198712	0.0193693	0.0150881	0.0096763	0.0168271	0.0089286	0.0085231	0.0133355	0.0292951

Table 22. Annualized covariance matrix for the stocks in Group 5 which have an absolute percentage valuation error from 40% - 76%

	AZN	SGE	PSON	JMAT	MGGT	SN	ABF	SAB	ARM
AZN	0.0329217	0.0083814	0.0019728	0.0099037	0.0032159	-0.00107	0.0134422	0.0114284	0.0012096
SGE	0.0083814	0.0402272	0.0084257	0.0202431	0.022028	0.0143829	0.0121147	0.0095018	0.0193824
PSON	0.0019728	0.0084257	0.0445289	0.0169285	0.0146868	0.0051741	0.0045041	0.0164289	0.0151851
JMAT	0.0099037	0.0202431	0.0169285	0.0640397	0.0395142	-0.000139	0.0204241	0.0247257	0.0141722
MGGT	0.0032159	0.022028	0.0146868	0.0395142	0.0791776	0.0115635	0.0163511	0.0250937	0.014417
SN	-0.00107	0.0143829	0.0051741	-0.000139	0.0115635	0.0341233	0.0083585	0.0067985	0.0034728
ABF	0.0134422	0.0121147	0.0045041	0.0204241	0.0163511	0.0083585	0.044146	0.0138322	0.0067719
SAB	0.0114284	0.0095018	0.0164289	0.0247257	0.0250937	0.0067985	0.0138322	0.0395428	0.0112666
ARM	0.0012096	0.0193824	0.0151851	0.0141722	0.014417	0.0034728	0.0067719	0.0112666	0.092861

Table 23. Annualized covariance matrix for all 50 stocks

	AAL	ABF	ANTO	ARM	AZN	BATS	BG	BP	BRBY	CCL	CNA	CPG	CRDA	DGE	GFS	GKN	GSK	HMSO	IMI	IMT	ITRK	ITV	JMAT	KGF	LAND
AAL	0.1442	0.0194	0.0986	0.0155	0.0182	0.0084	0.0360	0.0528	0.0495	0.0227	0.0057	0.0215	0.0346	0.0216	0.0301	0.0832	0.0161	0.0518	0.0521	0.0036	0.0232	0.0797	0.0523	0.0484	0.0530
ABF	0.0194	0.0441	0.0144	0.0068	0.0134	0.0110	0.0027	0.0169	0.0120	0.0208	0.0054	0.0158	0.0121	0.0122	0.0091	0.0337	0.0122	0.0186	0.0106	0.0092	-0.0005	0.0252	0.0204	0.0112	0.0215
ANTO	0.0986	0.0144	0.1412	0.0189	0.0045	0.0093	0.0342	0.0527	0.0549	0.0362	0.0033	0.0164	0.0434	0.0156	0.0046	0.0922	0.0095	0.0455	0.0598	0.0009	0.0249	0.0595	0.0592	0.0347	0.0423
ARM	0.0155	0.0068	0.0189	0.0929	0.0012	0.0045	0.0164	0.0198	0.0361	0.0206	0.0095	0.0131	0.0213	0.0095	0.0095	0.0306	0.0039	0.0193	0.0196	-0.0022	0.0263	0.0144	0.0142	0.0196	0.0153
AZN	0.0182	0.0134	0.0045	0.0012	0.0329	0.0086	0.0055	0.0107	0.0071	-0.0034	0.0032	0.0071	0.0091	0.0079	0.0047	0.0062	0.0123	0.0112	0.0010	0.0088	0.0003	0.0077	0.0099	0.0043	0.0144
BATS	0.0084	0.0110	0.0093	0.0045	0.0086	0.0269	0.0127	0.0123	0.0100	0.0017	0.0087	0.0119	0.0122	0.0136	0.0044	0.0154	0.0149	0.0077	0.0087	0.0180	0.0105	0.0082	0.0088	0.0025	0.0109
BG	0.0360	0.0027	0.0342	0.0164	0.0055	0.0127	0.0565	0.0314	0.0225	0.0148	0.0095	0.0144	0.0132	0.0083	0.0085	0.0227	0.0070	0.0137	0.0269	0.0038	0.0173	0.0255	0.0240	0.0143	0.0088
BP	0.0528	0.0169	0.0527	0.0198	0.0107	0.0123	0.0314	0.0681	0.0277	0.0289	0.0059	0.0199	0.0289	0.0182	0.0109	0.0493	0.0141	0.0339	0.0311	0.0087	0.0184	0.0446	0.0411	0.0274	0.0338
BRBY	0.0495	0.0120	0.0549	0.0361	0.0071	0.0100	0.0225	0.0277	0.1015	0.0193	0.0029	0.0131	0.0247	0.0114	-0.0004	0.0647	0.0057	0.0312	0.0472	0.0089	0.0304	0.0282	0.0324	0.0388	0.0340
CCL	0.0227	0.0208	0.0362	0.0206	-0.0034	0.0017	0.0148	0.0289	0.0193	0.0765	0.0072	0.0194	0.0207	0.0108	0.0053	0.0341	0.0037	0.0325	0.0246	0.0008	0.0080	0.0297	0.0285	0.0268	0.0250
CNA	0.0057	0.0054	0.0033	0.0095	0.0032	0.0087	0.0095	0.0059	0.0029	0.0072	0.0246	0.0055	0.0076	0.0073	0.0046	0.0134	0.0052	0.0084	0.0155	0.0029	0.0103	0.0216	0.0059	0.0012	0.0097
CPG	0.0215	0.0158	0.0164	0.0131	0.0071	0.0119	0.0144	0.0199	0.0131	0.0194	0.0055	0.0293	0.0173	0.0133	0.0072	0.0168	0.0121	0.0118	0.0148	0.0084	0.0070	0.0121	0.0171	0.0118	0.0141
CRDA	0.0346	0.0121	0.0434	0.0213	0.0091	0.0122	0.0132	0.0289	0.0247	0.0207	0.0076	0.0173	0.0704	0.0174	0.0147	0.0374	0.0108	0.0291	0.0300	0.0059	0.0266	0.0138	0.0372	0.0204	0.0265
DGE	0.0216	0.0122	0.0156	0.0095	0.0079	0.0136	0.0083	0.0182	0.0114	0.0108	0.0073	0.0133	0.0174	0.0207	0.0105	0.0198	0.0119	0.0148	0.0110	0.0103	0.0095	0.0202	0.0152	0.0123	0.0178
GFS	0.0301	0.0091	0.0046	0.0095	0.0047	0.0044	0.0085	0.0109	-0.0004	0.0053	0.0046	0.0072	0.0147	0.0105	0.0406	-0.0001	0.0067	0.0102	0.0013	0.0047	0.0026	0.0109	0.0139	0.0105	0.0062
GKN	0.0832	0.0337	0.0922	0.0306	0.0062	0.0154	0.0227	0.0493	0.0647	0.0341	0.0134	0.0168	0.0374	0.0198	-0.0001	0.1624	0.0165	0.0616	0.0758	0.0050	0.0330	0.1233	0.0630	0.0408	0.0711
GSK	0.0161	0.0122	0.0095	0.0039	0.0123	0.0149	0.0070	0.0141	0.0057	0.0037	0.0052	0.0121	0.0108	0.0119	0.0067	0.0165	0.0252	0.0117	0.0059	0.0105	0.0044	0.0069	0.0112	0.0120	0.0183
HMSO	0.0518	0.0186	0.0455	0.0193	0.0112	0.0077	0.0137	0.0339	0.0312	0.0325	0.0084	0.0118	0.0291	0.0148	0.0102	0.0616	0.0117	0.0623	0.0366	0.0047	0.0200	0.0579	0.0399	0.0269	0.0578
IMI	0.0521	0.0106	0.0598	0.0196	0.0010	0.0087	0.0269	0.0311	0.0472	0.0246	0.0155	0.0148	0.0300	0.0110	0.0013	0.0758	0.0059	0.0366	0.0880	0.0002	0.0361	0.0653	0.0380	0.0284	0.0378
IMT	0.0036	0.0092	0.0009	-0.0022	0.0088	0.0180	0.0038	0.0087	0.0089	0.0008	0.0029	0.0084	0.0059	0.0103	0.0047	0.0050	0.0105	0.0047	0.0002	0.0241	0.0011	0.0061	0.0050	0.0040	0.0099
ITRK	0.0232	-0.0005	0.0249	0.0263	0.0003	0.0105	0.0173	0.0184	0.0304	0.0080	0.0103	0.0070	0.0266	0.0095	0.0026	0.0330	0.0044	0.0200	0.0361	0.0011	0.0458	0.0210	0.0163	0.0159	0.0165
ITV	0.0797	0.0252	0.0595	0.0144	0.0077	0.0082	0.0255	0.0446	0.0282	0.0297	0.0216	0.0121	0.0138	0.0202	0.0109	0.1233	0.0069	0.0579	0.0653	0.0061	0.0210	0.1703	0.0499	0.0240	0.0652
JMAT	0.0523	0.0204	0.0592	0.0142	0.0099	0.0088	0.0240	0.0411	0.0324	0.0285	0.0059	0.0171	0.0372	0.0152	0.0139	0.0630	0.0112	0.0399	0.0380	0.0050	0.0163	0.0499	0.0640	0.0356	0.0379
KGF	0.0484	0.0112	0.0347	0.0196	0.0043	0.0025	0.0143	0.0274	0.0388	0.0268	0.0012	0.0118	0.0204	0.0123	0.0105	0.0408	0.0120	0.0269	0.0284	0.0040	0.0159	0.0240	0.0356	0.0735	0.0262
LAND	0.0530	0.0215	0.0423	0.0153	0.0144	0.0109	0.0088	0.0338	0.0340	0.0250	0.0097	0.0141	0.0265	0.0178	0.0062	0.0711	0.0183	0.0578	0.0378	0.0099	0.0165	0.0652	0.0379	0.0262	0.0711
MGGT	0.0517	0.0164	0.0417	0.0144	0.0032	0.0094	0.0190	0.0266	0.0463	0.0331	0.0049	0.0161	0.0285	0.0181	0.0084	0.0713	0.0099	0.0407	0.0486	0.0074	0.0190	0.0591	0.0395	0.0261	0.0467
MKS	0.0256	0.0100	0.0383	0.0183	0.0029	-0.0010	0.0025	0.0122	0.0391	0.0211	-0.0028	0.0072	0.0173	0.0067	-0.0020	0.0331	0.0097	0.0260	0.0187	0.0034	0.0068	0.0111	0.0288	0.0348	0.0205
NG	0.0129	0.0112	0.0088	0.0000	0.0047	0.0078	0.0021	0.0105	-0.0051	0.0064	0.0126	0.0073	0.0053	0.0093	0.0076	0.0154	0.0091	0.0139	0.0066	0.0060	0.0039	0.0242	0.0093	0.0034	0.0157
NXT	0.0298	0.0088	0.0261	0.0159	0.0053	-0.0009	0.0075	0.0212	0.0252	0.0219	-0.0030	0.0066	0.0122	0.0056	-0.0015	0.0317	0.0066	0.0229	0.0146	0.0013	0.0115	0.0206	0.0206	0.0306	0.0184
PSO	0.0211	0.0045	0.0260	0.0152	0.0020	0.0104	0.0171	0.0237	0.0165	0.0193	0.0102	0.0118	0.0158	0.0117	0.0110	0.0184	0.0094	0.0130	0.0205	0.0029	0.0103	0.0140	0.0169	0.0220	0.0148
RB	0.0054	0.0101	0.0065	0.0063	0.0023	0.0102	0.0070	0.0121	0.0087	0.0067	0.0059	0.0085	0.0054	0.0070	0.0022	0.0122	0.0058	0.0054	0.0090	0.0063	0.0059	0.0069	0.0075	0.0018	0.0051
REL	0.0102	0.0132	0.0156	0.0133	0.0066	0.0092	0.0094	0.0192	0.0160	0.0124	0.0117	0.0124	0.0149	0.0124	0.0012	0.0264	0.0028	0.0123	0.0173	0.0063	0.0079	0.0246	0.0167	0.0061	0.0162
REX	0.0395	0.0120	0.0250	0.0110	0.0074	0.0076	0.0223	0.0245	0.0196	0.0156	0.0114	0.0140	0.0144	0.0157	0.0130	0.0419	0.0090	0.0270	0.0322	0.0064	0.0169	0.0460	0.0200	0.0192	0.0349
RR	0.0345	0.0082	0.0328	0.0184	0.0052	0.0123	0.0182	0.0237	0.0247	0.0219	0.0044	0.0166	0.0162	0.0170	0.0088	0.0411	0.0113	0.0218	0.0293	0.0063	0.0167	0.0340	0.0271	0.0234	0.0239
SAB	0.0451	0.0138	0.0329	0.0113	0.0114	0.0175	0.0234	0.0291	0.0246	0.0128	0.0084	0.0211	0.0220	0.0186	0.0109	0.0379	0.0159	0.0227	0.0308	0.0088	0.0200	0.0291	0.0247	0.0220	0.0242
SBRY	0.0108	0.0085	0.0201	0.0136	0.0076	0.0036	0.0070	0.0124	0.0120	0.0103	0.0081	0.0058	0.0163	0.0084	0.0047	0.0219	0.0051	0.0155	0.0147	0.0013	0.0071	0.0168	0.0124	0.0105	0.0160
SGE	0.0116	0.0121	0.0241	0.0194	0.0084	0.0092	0.0186	0.0233	0.0168	0.0237	0.0058	0.0092	0.0193	0.0095	0.0035	0.0340	0.0053	0.0136	0.0244	0.0074	0.0100	0.0301	0.0202	0.0154	0.0152
SHP	0.0294	0.0153	0.0117	-0.0031	0.0148	0.0125	0.0147	0.0227	0.0106	0.0040	0.0016	0.0154	-0.0037	0.0090	0.0056	0.0114	0.0132	0.0122	0.0113	0.0141	-0.0062	0.0237	0.0051	0.0051	0.0214
SMIN	0.0208	0.0101	0.0238	0.0170	0.0023	0.0079	0.0160	0.0210	0.0158	0.0109	0.0167	0.0096	0.0115	0.0104	0.0000	0.0395	0.0031	0.0200	0.0280	0.0062	0.0135	0.0441	0.0160	0.0051	0.0246
SN	-0.0011	0.0084	0.0100	0.0035	-0.0011	0.0064	0.0043	0.0087	0.0029	0.0100	0.0071	0.0083	0.0026	0.0058	-0.0003	0.0111	0.0008	0.0016	0.0170	0.0048	-0.0008	0.0131	-0.0001	0.0003	0.0048
SSE	0.0082	0.0076	0.0013	0.0018	0.0049	0.0082	0.0052	0.0047	0.0032	0.0085	0.0111	0.0062	0.0049	0.0072	0.0062	0.0061	0.0073	0.0088	0.0061	0.0071	0.0031	0.0079	0.0057	0.0097	0.0100
SVT	0.0177	0.0106	0.0081	-0.0027	0.0054	0.0066	0.0056	0.0055	0.0011	0.0003	0.0093	0.0065	0.0067	0.0060	0.0011	0.0147	0.0057	0.0095	0.0025	0.0053	0.0020	0.0133	0.0062	-0.0007	0.0130
TATE	0.0269	0.0122	0.0081	0.0096	0.0132	0.0114	0.0042	0.0186	0.0037	0.01															

Table 23. Annualized covariance matrix for all 50 stocks (con.)

	MGGT	MKS	NG	NXT	PSON	RB	REL	REX	RR	SAB	SBRY	SGE	SHP	SMIN	SN	SSE	SVT	TATE	TSCO	UU	VOD	IHG	WOS	WPP	WTB
AAL	0.0517	0.0256	0.0129	0.0298	0.0211	0.0054	0.0102	0.0395	0.0345	0.0451	0.0108	0.0116	0.0294	0.0208	-0.0011	0.0082	0.0177	0.0269	0.0234	0.0235	0.0083	0.0570	0.0643	0.0477	0.0325
ABF	0.0164	0.0100	0.0112	0.0088	0.0045	0.0101	0.0132	0.0120	0.0082	0.0138	0.0085	0.0121	0.0153	0.0101	0.0084	0.0076	0.0106	0.0122	0.0182	0.0159	0.0117	0.0002	0.0298	0.0141	0.0168
ANTO	0.0417	0.0383	0.0088	0.0261	0.0260	0.0065	0.0156	0.0250	0.0328	0.0329	0.0201	0.0241	0.0117	0.0238	0.0100	0.0013	0.0081	0.0081	0.0241	0.0098	0.0032	0.0736	0.0619	0.0429	0.0264
ARM	0.0144	0.0183	0.0000	0.0159	0.0152	0.0063	0.0133	0.0110	0.0184	0.0113	0.0136	0.0194	-0.0031	0.0170	0.0035	0.0018	-0.0027	0.0096	0.0142	0.0019	0.0147	0.0305	0.0338	0.0206	0.0084
AZN	0.0032	0.0029	0.0047	0.0053	0.0020	0.0023	0.0066	0.0074	0.0052	0.0114	0.0076	0.0084	0.0148	0.0023	-0.0011	0.0049	0.0054	0.0132	0.0071	0.0063	0.0091	0.0088	0.0106	0.0087	0.0051
BATS	0.0094	-0.0010	0.0078	-0.0009	0.0104	0.0102	0.0092	0.0076	0.0123	0.0175	0.0036	0.0092	0.0125	0.0079	0.0064	0.0082	0.0066	0.0114	0.0112	0.0083	0.0083	0.0108	0.0040	0.0121	0.0066
BG	0.0190	0.0025	0.0021	0.0075	0.0171	0.0070	0.0094	0.0223	0.0182	0.0234	0.0070	0.0186	0.0147	0.0160	0.0043	0.0052	0.0056	0.0042	0.0122	0.0057	0.0090	0.0338	0.0402	0.0254	0.0180
BP	0.0266	0.0122	0.0105	0.0212	0.0237	0.0121	0.0192	0.0245	0.0237	0.0291	0.0124	0.0233	0.0227	0.0210	0.0087	0.0047	0.0055	0.0186	0.0211	0.0114	0.0131	0.0331	0.0459	0.0352	0.0308
BRBY	0.0463	0.0391	-0.0051	0.0252	0.0165	0.0087	0.0160	0.0196	0.0247	0.0246	0.0120	0.0168	0.0106	0.0158	0.0029	0.0032	0.0011	0.0037	0.0148	0.0004	0.0058	0.0480	0.0373	0.0339	0.0154
CCL	0.0331	0.0211	0.0064	0.0219	0.0193	0.0067	0.0124	0.0156	0.0219	0.0128	0.0103	0.0237	0.0040	0.0109	0.0100	0.0085	0.0003	0.0112	0.0179	0.0093	0.0135	0.0164	0.0728	0.0272	0.0324
CNA	0.0049	-0.0028	0.0126	-0.0030	0.0102	0.0059	0.0117	0.0114	0.0044	0.0084	0.0081	0.0058	0.0016	0.0167	0.0071	0.0111	0.0093	0.0082	0.0076	0.0078	0.0055	0.0038	-0.0038	0.0114	0.0059
CPG	0.0161	0.0072	0.0073	0.0066	0.0118	0.0085	0.0124	0.0140	0.0166	0.0211	0.0058	0.0092	0.0154	0.0096	0.0083	0.0062	0.0065	0.0089	0.0164	0.0107	0.0097	0.0124	0.0224	0.0207	0.0151
CRDA	0.0285	0.0173	0.0053	0.0122	0.0158	0.0054	0.0149	0.0144	0.0162	0.0220	0.0163	0.0193	-0.0037	0.0115	0.0026	0.0049	0.0067	0.0151	0.0174	0.0117	0.0115	0.0341	0.0415	0.0288	0.0175
DGE	0.0181	0.0067	0.0093	0.0056	0.0117	0.0070	0.0124	0.0157	0.0170	0.0186	0.0084	0.0095	0.0090	0.0104	0.0058	0.0072	0.0060	0.0180	0.0137	0.0082	0.0113	0.0065	0.0186	0.0167	0.0125
GFS	0.0084	-0.0020	0.0076	-0.0015	0.0110	0.0022	0.0012	0.0130	0.0088	0.0109	0.0047	0.0035	0.0056	0.0000	-0.0003	0.0062	0.0011	0.0165	0.0109	0.0069	0.0065	-0.0005	0.0214	0.0117	0.0029
GKN	0.0713	0.0331	0.0154	0.0317	0.0184	0.0122	0.0264	0.0419	0.0411	0.0379	0.0219	0.0340	0.0114	0.0395	0.0111	0.0061	0.0147	0.0294	0.0262	0.0192	0.0088	0.0627	0.0651	0.0556	0.0368
GSK	0.0099	0.0097	0.0091	0.0066	0.0094	0.0058	0.0028	0.0090	0.0113	0.0159	0.0051	0.0053	0.0132	0.0031	0.0008	0.0073	0.0057	0.0131	0.0150	0.0078	0.0142	0.0075	0.0116	0.0122	0.0130
HMSO	0.0407	0.0260	0.0139	0.0229	0.0130	0.0054	0.0123	0.0270	0.0218	0.0227	0.0155	0.0136	0.0122	0.0200	0.0016	0.0088	0.0095	0.0220	0.0159	0.0159	0.0139	0.0286	0.0642	0.0311	0.0308
IMI	0.0486	0.0187	0.0066	0.0146	0.0205	0.0090	0.0173	0.0322	0.0293	0.0308	0.0147	0.0244	0.0113	0.0280	0.0170	0.0061	0.0025	0.0163	0.0179	0.0053	0.0037	0.0600	0.0388	0.0418	0.0262
IMT	0.0074	0.0034	0.0060	0.0013	0.0029	0.0063	0.0063	0.0064	0.0063	0.0088	0.0013	0.0074	0.0141	0.0062	0.0048	0.0071	0.0053	0.0090	0.0088	0.0090	0.0039	0.0011	0.0053	0.0082	0.0077
ITRK	0.0190	0.0068	0.0039	0.0115	0.0103	0.0059	0.0079	0.0169	0.0167	0.0200	0.0071	0.0100	-0.0062	0.0135	-0.0008	0.0031	0.0020	0.0111	0.0087	0.0015	0.0050	0.0374	0.0191	0.0206	0.0099
ITV	0.0591	0.0111	0.0242	0.0206	0.0140	0.0069	0.0246	0.0460	0.0340	0.0291	0.0168	0.0301	0.0237	0.0441	0.0131	0.0079	0.0133	0.0353	0.0202	0.0220	0.0125	0.0313	0.0619	0.0519	0.0366
JMAT	0.0395	0.0288	0.0093	0.0206	0.0169	0.0075	0.0167	0.0200	0.0271	0.0247	0.0124	0.0202	0.0051	0.0160	-0.0001	0.0057	0.0062	0.0229	0.0149	0.0100	0.0120	0.0297	0.0585	0.0387	0.0259
KGF	0.0361	0.0348	0.0034	0.0306	0.0220	0.0018	0.0061	0.0192	0.0234	0.0220	0.0105	0.0154	0.0051	0.0051	0.0003	0.0097	-0.0007	0.0131	0.0175	0.0020	0.0047	0.0252	0.0497	0.0277	0.0261
LAND	0.0467	0.0205	0.0157	0.0184	0.0148	0.0051	0.0162	0.0349	0.0239	0.0242	0.0160	0.0152	0.0214	0.0246	0.0048	0.0100	0.0130	0.0260	0.0169	0.0184	0.0150	0.0245	0.0544	0.0346	0.0345
MGGT	0.0792	0.0279	0.0020	0.0252	0.0147	0.0064	0.0163	0.0281	0.0416	0.0251	0.0125	0.0220	0.0175	0.0208	0.0116	0.0046	0.0066	0.0280	0.0207	0.0104	0.0153	0.0360	0.0668	0.0401	0.0365
MKS	0.0279	0.0723	0.0023	0.0423	0.0075	0.0078	0.0097	0.0026	0.0158	0.0052	0.0148	0.0096	0.0010	0.0056	-0.0005	0.0023	0.0058	-0.0020	0.0152	0.0053	0.0121	0.0148	0.0364	0.0188	0.0167
NG	0.0020	0.0023	0.0214	0.0028	0.0066	0.0059	0.0086	0.0091	0.0049	0.0091	0.0050	0.0038	0.0057	0.0104	0.0040	0.0097	0.0111	0.0106	0.0079	0.0135	0.0076	-0.0037	0.0004	0.0073	0.0087
NXT	0.0252	0.0423	0.0028	0.0496	0.0086	0.0044	0.0107	0.0079	0.0146	0.0105	0.0080	0.0118	0.0068	0.0050	-0.0018	0.0013	0.0006	0.0043	0.0159	0.0076	0.0106	0.0202	0.0284	0.0179	0.0190
PSON	0.0147	0.0075	0.0066	0.0086	0.0445	0.0047	0.0121	0.0109	0.0169	0.0164	0.0067	0.0084	0.0196	0.0117	0.0052	0.0042	-0.0042	0.0106	0.0094	-0.0020	0.0107	0.0134	0.0195	0.0209	0.0107
RB	0.0064	0.0078	0.0059	0.0044	0.0047	0.0160	0.0088	0.0040	0.0095	0.0109	0.0043	0.0069	0.0054	0.0096	0.0058	0.0012	0.0013	0.0040	0.0046	0.0014	0.0049	0.0018	0.0029	0.0081	0.0080
REL	0.0163	0.0097	0.0086	0.0107	0.0121	0.0088	0.0276	0.0138	0.0085	0.0126	0.0100	0.0133	0.0062	0.0140	0.0108	0.0078	0.0056	0.0061	0.0109	0.0084	0.0080	0.0083	0.0090	0.0200	0.0139
REX	0.0281	0.0026	0.0091	0.0079	0.0109	0.0040	0.0138	0.0427	0.0217	0.0231	0.0152	0.0153	0.0116	0.0204	0.0068	0.0101	0.0117	0.0149	0.0159	0.0142	0.0029	0.0218	0.0335	0.0277	0.0197
RR	0.0416	0.0158	0.0049	0.0146	0.0169	0.0095	0.0085	0.0217	0.0511	0.0208	0.0138	0.0174	0.0141	0.0176	0.0042	0.0018	0.0028	0.0235	0.0161	0.0046	0.0119	0.0212	0.0422	0.0268	0.0148
SAB	0.0251	0.0052	0.0091	0.0105	0.0164	0.0109	0.0126	0.0231	0.0208	0.0395	0.0056	0.0095	0.0163	0.0132	0.0068	0.0069	0.0062	0.0168	0.0152	0.0082	0.0099	0.0289	0.0271	0.0288	0.0182
SBRY	0.0125	0.0148	0.0050	0.0080	0.0067	0.0043	0.0100	0.0152	0.0138	0.0056	0.0324	0.0113	-0.0015	0.0115	0.0099	0.0025	0.0051	0.0075	0.0223	0.0042	0.0051	0.0106	0.0168	0.0117	0.0102
SGE	0.0220	0.0096	0.0038	0.0118	0.0084	0.0069	0.0133	0.0153	0.0174	0.0095	0.0113	0.0402	0.0069	0.0156	0.0144	0.0042	-0.0004	0.0117	0.0128	0.0056	0.0081	0.0223	0.0310	0.0220	0.0184
SHP	0.0175	0.0010	0.0057	0.0068	0.0196	0.0054	0.0062	0.0116	0.0141	0.0163	-0.0015	0.0069	0.0712	0.0147	0.0112	0.0035	0.0054	0.0091	0.0118	0.0118	0.0147	0.0120	0.0116	0.0138	0.0117
SMIN	0.0208	0.0056	0.0104	0.0050	0.0117	0.0096	0.0140	0.0204	0.0176	0.0132	0.0115	0.0156	0.0147	0.0403	0.0114	0.0035	0.0070	0.0147	0.0092	0.0051	0.0072	0.0126	0.0116	0.0221	0.0140
SN	0.0116	-0.0005	0.0040	-0.0018	0.0052	0.0058	0.0108	0.0068	0.0042	0.0068	0.0099	0.0144	0.0112	0.0114	0.0341	0.0010	0.0005	0.0013	0.0095	0.0001	0.0020	0.0014	-0.0006	0.0092	0.0084
SSE	0.0046	0.0023	0.0097	0.0013	0.0042	0.0012	0.0078	0.0101	0.0018	0.0069	0.0025	0.0042	0.0035	0.0035	0.0010	0.0167	0.0084	0.0045	0.0075	0.0127	0.0040	-0.0007	0.0079	0.0063	0.0059
SVT	0.0066	0.0058	0.0111	0.0006	-0.0042	0.0013	0.0056	0.0117	0.0028	0.0062	0.0051	-0.0004	0.0054	0.0070	0.0005	0.0084	0.0310	0.0002	0.0071	0.0260	0.0023	-0.0022	0.0070	0.0038	0.0058
TATE	0.0280	-0.0020	0.0106	0.0043	0.0106	0.0040	0.0061	0.0149	0.0235																

**APPENDIX H. Correlation matrix for the stocks in each of the five groups and the total correlation matrix.**

Table 24. Correlation matrix for the stocks in Group 1 which have an absolute percentage valuation error from 0% - 10%

	<b>SBRY</b>	<b>ANTO</b>	<b>REX</b>	<b>REL</b>	<b>NG</b>	<b>GFS</b>	<b>TSCO</b>	<b>WOS</b>	<b>ITV</b>	<b>KGF</b>
<b>SBRY</b>	1									
<b>ANTO</b>	0.2976821	1								
<b>REX</b>	0.4077812	0.3214644	1							
<b>REL</b>	0.3351063	0.2502388	0.4001527	1						
<b>NG</b>	0.1886697	0.1606056	0.3019177	0.3539733	1					
<b>GFS</b>	0.1291426	0.0610503	0.3126036	0.0359273	0.2583202	1				
<b>TSCO</b>	0.625769	0.3230467	0.3871766	0.3316588	0.2719167	0.2714264	1			
<b>WOS</b>	0.2258097	0.3979371	0.3912228	0.1306514	0.0066422	0.2566275	0.2492384	1		
<b>ITV</b>	0.2260223	0.3835657	0.5394864	0.3591922	0.4007399	0.1305311	0.2464395	0.3627284	1	
<b>KGF</b>	0.2146282	0.3406221	0.3427413	0.1355014	0.0857074	0.1919414	0.3253282	0.443265	0.2148375	1

Table 25. Correlation matrix for the stocks in Group 2 which have an absolute percentage valuation error from 10% - 17%

	<b>LAND</b>	<b>NXT</b>	<b>BRBY</b>	<b>BG</b>	<b>IMT</b>	<b>CRDA</b>	<b>BATS</b>	<b>CNA</b>	<b>SVT</b>	<b>IHG</b>
<b>LAND</b>	1									
<b>NXT</b>	0.3106187	1								
<b>BRBY</b>	0.4006856	0.3548132	1							
<b>BG</b>	0.1391455	0.1408812	0.2966517	1						
<b>IMT</b>	0.2383451	0.0364194	0.1809649	0.1041193	1					
<b>CRDA</b>	0.3746212	0.2062016	0.2925736	0.2096226	0.1434683	1				
<b>BATS</b>	0.248351	-0.025863	0.1914891	0.3269696	0.7061392	0.279341	1			
<b>CNA</b>	0.2331104	-0.086147	0.0575862	0.2556775	0.117336	0.1824634	0.3389338	1		
<b>SVT</b>	0.2772972	0.0140499	0.0189923	0.1332368	0.19568	0.1442927	0.2280133	0.3359282	1	
<b>IHG</b>	0.4782278	0.2208232	0.4999277	0.3819723	0.0034968	0.3816644	0.1796547	0.3329824	0.0470375	1

Table 26. Correlation matrix for the stocks in Group 3 which have an absolute percentage valuation error from 17% - 26%

	<b>WPP</b>	<b>SSE</b>	<b>HMSO</b>	<b>IMI</b>	<b>AAL</b>	<b>MKS</b>	<b>ITRK</b>	<b>RR</b>	<b>GSK</b>	<b>SHPG</b>
<b>WPP</b>	1									
<b>SSE</b>	0.2238178	1								
<b>HMSO</b>	0.5693185	0.2734373	1							
<b>IMI</b>	0.6437232	0.1606489	0.4949797	1						
<b>AAL</b>	0.573768	0.168215	0.5468596	0.4627504	1					
<b>MKS</b>	0.3181572	0.0675327	0.3870616	0.234404	0.2507423	1				
<b>ITRK</b>	0.4401881	0.1104722	0.374883	0.5680928	0.2857897	0.1186205	1			
<b>RR</b>	0.5414218	0.0614538	0.3867566	0.4365775	0.4018971	0.2590876	0.3454245	1		
<b>GSK</b>	0.3508839	0.356109	0.2952873	0.124816	0.26755	0.227625	0.1299901	0.3148317	1	
<b>SHPG</b>	0.2361737	0.1016216	0.1828977	0.1427064	0.289989	0.0137441	-0.108415	0.2334149	0.3122373	1

Table 27. Correlation matrix for the stocks in Group 4 which have an absolute percentage valuation error from 26% - 40%

	SMIN	UU	BP	CCL	WTB	VOD	GKN	TATE	RB	DGE	CPG
SMIN	1										
UU	0.1374373	1									
BP	0.4001234	0.2357345	1								
CCL	0.1955583	0.1809087	0.3998414	1							
WTB	0.305973	0.2880734	0.5166733	0.5133463	1						
VOD	0.2090451	0.1354833	0.2947379	0.2867291	0.3392015	1					
GKN	0.4886346	0.2566314	0.4690424	0.3063703	0.4006951	0.128074	1				
TATE	0.3132765	0.0401189	0.3047131	0.1737488	0.2876442	0.2651425	0.3122797	1			
RB	0.3764486	0.0594181	0.366171	0.1901153	0.2766707	0.2275932	0.2384333	0.1338942	1		
DGE	0.3592367	0.3062039	0.4836068	0.2709331	0.3807093	0.4611701	0.3412733	0.5364638	0.3828148	1	
CPG	0.2805613	0.33578	0.4447706	0.409241	0.386401	0.3314683	0.2439593	0.2232787	0.3935515	0.5418759	1

Table 28. Correlation matrix for the stocks in Group 5 which have an absolute percentage valuation error from 40% - 76%

	AZN	SGE	PSON	JMAT	MGGT	SN	ABF	SAB	ARM
AZN	1								
SGE	0.2303128	1							
PSON	0.0515248	0.199079	1						
JMAT	0.2156917	0.3988347	0.3170107	1					
MGGT	0.0629879	0.3903147	0.2473455	0.5549167	1				
SN	-0.031923	0.3882042	0.1327367	-0.002963	0.2224645	1			
ABF	0.3525998	0.2874788	0.1015876	0.3841247	0.2765672	0.2153561	1		
SAB	0.3167459	0.2382385	0.3915212	0.4913489	0.4484658	0.185078	0.3310648	1	
ARM	0.0218762	0.3171259	0.2361454	0.1837785	0.1681351	0.061693	0.1057664	0.1859275	1

Table 29. Correlation matrix for all 50 stocks

	AAL	ABF	ANTO	ARM	AZN	BATS	BG	BP	BRBY	CCL	CNA	CPG	CRDA	DGE	GFS	GKN	GSK	HMSO	IMI	IMT	ITRK	ITV	JMAT	KGF	LAND	MGGT
AAL	1.0000																									
ABF	0.2426	1.0000																								
ANTO	0.6912	0.1826	1.0000																							
ARM	0.1339	0.1058	0.1649	1.0000																						
AZN	0.2646	0.3526	0.0660	0.0219	1.0000																					
BATS	0.1347	0.3190	0.1511	0.0910	0.2896	1.0000																				
BG	0.3993	0.0546	0.3833	0.2263	0.1282	0.3270	1.0000																			
BP	0.5328	0.3076	0.5373	0.2484	0.2256	0.2869	0.5063	1.0000																		
BRBY	0.4096	0.1794	0.4583	0.3714	0.1233	0.1915	0.2967	0.3327	1.0000																	
CCL	0.2158	0.3587	0.3483	0.2441	-0.0687	0.0375	0.2247	0.3998	0.2189	1.0000																
CNA	0.0966	0.1632	0.0552	0.1990	0.1131	0.3389	0.2557	0.1432	0.0576	0.1667	1.0000															
CPG	0.3311	0.4389	0.2552	0.2514	0.2284	0.4242	0.3539	0.4448	0.2405	0.4092	0.2033	1.0000														
CRDA	0.3437	0.2175	0.4354	0.2633	0.1895	0.2793	0.2096	0.4178	0.2926	0.2823	0.1825	0.3813	1.0000													
DGE	0.3950	0.4036	0.2890	0.2172	0.3025	0.5774	0.2423	0.4836	0.2492	0.2709	0.3236	0.5419	0.4561	1.0000												
GFS	0.3937	0.2158	0.0611	0.1550	0.1294	0.1337	0.1766	0.2067	-0.0070	0.0946	0.1471	0.2084	0.2757	0.3619	1.0000											
GKN	0.5437	0.3977	0.6088	0.2494	0.0854	0.2337	0.2366	0.4690	0.5041	0.3064	0.2125	0.2440	0.3497	0.3413	-0.0018	1.0000										
GSK	0.2675	0.3650	0.1595	0.0812	0.4276	0.5713	0.1865	0.3401	0.1120	0.0833	0.2076	0.4446	0.2569	0.5212	0.2100	0.2584	1.0000									
HMSO	0.5469	0.3549	0.4856	0.2538	0.2468	0.1869	0.2303	0.5203	0.3927	0.4712	0.2136	0.2770	0.4388	0.4136	0.2030	0.6129	0.2953	1.0000								
IMI	0.4628	0.1698	0.5365	0.2169	0.0183	0.1797	0.3820	0.4015	0.4999	0.3002	0.3330	0.2922	0.3817	0.2583	0.0224	0.6340	0.1248	0.4950	1.0000							
IMT	0.0617	0.2825	0.0153	-0.0469	0.3125	0.7061	0.1041	0.2143	0.1810	0.0194	0.1173	0.3156	0.1435	0.4628	0.1511	0.0792	0.4276	0.1226	0.0035	1.0000						
ITRK	0.2858	-0.0111	0.3097	0.4033	0.0066	0.3000	0.3404	0.3290	0.4456	0.1360	0.3084	0.1921	0.4685	0.3101	0.0601	0.3822	0.1300	0.3749	0.5681	0.0335	1.0000					
ITV	0.5085	0.2908	0.3836	0.1147	0.1028	0.1205	0.2598	0.4144	0.2142	0.2606	0.3342	0.1707	0.1263	0.3399	0.1305	0.7412	0.1053	0.5623	0.5335	0.0946	0.2372	1.0000				
JMAT	0.5445	0.3841	0.6227	0.1838	0.2157	0.2117	0.3989	0.6229	0.4019	0.4067	0.1486	0.3954	0.5535	0.4165	0.2729	0.6177	0.2795	0.6316	0.5058	0.1270	0.3018	0.4783	1.0000			
KGF	0.4702	0.1958	0.3406	0.2375	0.0871	0.0565	0.2219	0.3880	0.4492	0.3581	0.0282	0.2534	0.2834	0.3152	0.1919	0.3737	0.2797	0.3982	0.3539	0.0947	0.2750	0.2148	0.5184	1.0000		
LAND	0.5232	0.3846	0.4224	0.1883	0.2980	0.2484	0.1391	0.4864	0.4007	0.3394	0.2331	0.3084	0.3746	0.4639	0.1155	0.6618	0.4338	0.8693	0.4782	0.2383	0.2898	0.5929	0.5620	0.3621	1.0000	
MGGT	0.4841	0.2766	0.3949	0.1681	0.0630	0.2026	0.2844	0.3627	0.5170	0.4252	0.1123	0.3346	0.3824	0.4482	0.1473	0.6288	0.2229	0.5797	0.5828	0.1700	0.3156	0.5093	0.5549	0.4736	0.6220	1.0000
MKS	0.2507	0.1769	0.3790	0.2227	0.0584	-0.0236	0.0386	0.1734	0.4560	0.2843	-0.0653	0.1557	0.2419	0.1731	-0.0360	0.3056	0.2276	0.3871	0.2344	0.0821	0.1186	0.0999	0.4236	0.4768	0.2860	0.3691
NG	0.2319	0.3632	0.1606	0.0006	0.1775	0.3254	0.0611	0.2757	-0.1086	0.1590	0.5482	0.2926	0.1372	0.4427	0.2583	0.2612	0.3926	0.3810	0.1530	0.2655	0.1244	0.4007	0.2517	0.0857	0.4024	0.0489
NXT	0.3525	0.1885	0.3123	0.2345	0.1315	-0.0259	0.1409	0.3650	0.3548	0.3562	-0.0861	0.1724	0.2062	0.1743	-0.0344	0.3534	0.1855	0.4118	0.2208	0.0364	0.2406	0.2236	0.3654	0.5074	0.3106	0.4027
PSON	0.2633	0.1016	0.3284	0.2361	0.0515	0.3008	0.3410	0.4295	0.2454	0.3302	0.3077	0.3258	0.2824	0.3851	0.2584	0.2169	0.2808	0.2460	0.3281	0.0897	0.2271	0.1608	0.3170	0.3839	0.2638	0.2473
RB	0.1114	0.3799	0.1363	0.1643	0.1023	0.4935	0.2331	0.3662	0.2158	0.1901	0.2962	0.3936	0.1618	0.3828	0.0878	0.2384	0.2898	0.1724	0.2398	0.3200	0.2195	0.1320	0.2329	0.0535	0.1503	0.1788
REL	0.1612	0.3773	0.2502	0.2622	0.2201	0.3357	0.2373	0.4416	0.3017	0.2697	0.4490	0.4348	0.3386	0.5193	0.0359	0.3940	0.1079	0.2975	0.3513	0.2433	0.2231	0.3592	0.3961	0.1355	0.3650	0.3490
REX	0.5033	0.2752	0.3215	0.1746	0.1963	0.2249	0.4544	0.4544	0.2978	0.2735	0.3534	0.3968	0.2620	0.5277	0.3126	0.5036	0.2746	0.5226	0.5254	0.1985	0.3826	0.5395	0.3833	0.3427	0.6332	0.4837
RR	0.4019	0.1722	0.3860	0.2669	0.1268	0.3309	0.3388	0.4016	0.3436	0.3506	0.1229	0.4298	0.2707	0.5216	0.1928	0.4515	0.3148	0.3868	0.4366	0.1791	0.3454	0.3647	0.4742	0.3818	0.3969	0.6540
SAB	0.5976	0.3311	0.4398	0.1859	0.3167	0.5353	0.4955	0.5614	0.3877	0.2328	0.2705	0.6202	0.4165	0.6508	0.2724	0.4732	0.5033	0.4565	0.5216	0.2858	0.4705	0.3543	0.4913	0.4076	0.4558	0.4485
SBRY	0.1580	0.2249	0.2977	0.2480	0.2340	0.1230	0.1645	0.2640	0.2089	0.2066	0.2858	0.1897	0.3408	0.3252	0.1291	0.3016	0.1787	0.3454	0.2756	0.0470	0.1853	0.2260	0.2721	0.2146	0.3343	0.2474
SGE	0.1523	0.2875	0.3198	0.3171	0.2303	0.2806	0.3902	0.4443	0.2627	0.4280	0.1833	0.2684	0.3626	0.3306	0.0863	0.4203	0.1677	0.2727	0.4096	0.2392	0.2325	0.3642	0.3988	0.2824	0.2837	0.3903
SHP	0.2900	0.2719	0.1168	-0.0376	0.3055	0.2857	0.2317	0.3263	0.1247	0.0546	0.0387	0.3376	-0.0522	0.2343	0.1034	0.1061	0.3122	0.1829	0.1427	0.3402	-0.1084	0.2151	0.0757	0.0711	0.3001	0.2329
SMIN	0.2730	0.2396	0.3149	0.2784	0.0637	0.2390	0.3362	0.4001	0.2465	0.1956	0.5323	0.2806	0.2166	0.3592	-0.0007	0.4886	0.0967	0.3991	0.4694	0.1992	0.3143	0.5322	0.3140	0.0930	0.4593	0.3679
SN	-0.0155	0.2154	0.1446	0.0617	-0.0319	0.2123	0.0970	0.1795	0.0490	0.1960	0.2457	0.2631	0.0532	0.2166	-0.0072	0.1496	0.0268	0.0357	0.3100	0.1688	-0.0191	0.1714	-0.0030	0.0052	0.0971	0.2225
SSE	0.1682	0.2817	0.0260	0.0470	0.2091	0.3885	0.1685	0.1399	0.0790	0.2377	0.5472	0.2825	0.1432	0.3881	0.2391	0.1173	0.3561	0.2734	0.1606	0.3557	0.1105	0.1483	0.1759	0.2768	0.2915	0.1268
SVT	0.2648	0.2868	0.1228	-0.0500	0.1694	0.2280	0.1332	0.1191	0.0190	0.0061	0.3359	0.2168	0.1443	0.2363	0.0313	0.2077	0.2035	0.2162	0.0470	0.1957	0.0538	0.1826	0.1382	-0.0153	0.2773	0.1333
TATE	0.3033	0.2477	0.0927	0.1344	0.3126	0.2980	0.0750	0.3047	0.0502	0.1737	0.2232	0.2233	0.2443	0.5365	0.3496	0.3123	0.3548	0.3781	0.2351	0.2484	0.2213	0.3664	0.3880	0.2063	0.4169	0.4262
TSCO	0.3109	0.4374	0.3230	0.2351	0.1972	0.3442	0.2578	0.4070	0.2340	0.3255	0.2442	0.4823	0.3298	0.4787	0.2714	0.3282	0.4753	0.3219	0.3047	0.2860	0.2055	0.2464	0.2962	0.3253	0.3190	0.3701
UU	0.3336	0.4073	0.1403	0.0329	0.1882	0.2714	0.1284	0.2357	0.0074	0.1809	0.2684	0.3358	0.2378	0.3062	0.1848	0.2566	0.2651	0.3423	0.0971	0.3123	0.0385	0.2869	0.2131	0.0397	0.3713	0.1987
VOD	0.1287	0.3273	0.0506	0.2837	0.2956	0.2983	0.2217	0.2947	0.1069	0.2867	0.2043	0.3315	0.2548	0.4612	0.1889	0.1281	0.5253	0.3268	0.0737	0.1470	0.1380	0.1770	0.2771	0.1015	0.3303	0.3192
YHG	0.4628	0.1698	0.5365	0.2169	0.0183	0.1797	0.3820	0.4015	0.4999	0.3002	0.3330	0.2922	0.3817	0.2583	0.0224	0.6340	0.1248	0.4950	1.0000	0.0035	0.5681	0.5335	0.5058	0.3539	0.4782	0.5828
WOS	0.4094	0.3423	0.3979	0.2678	0.1412	0.0595	0.4084	0.4248	0.2828	0.6366	-0.0590	0.3162	0.3780	0.3133	0.2566	0.3906	0.1774	0.6221	0.3162	0.0830	0.2160	0.3627	0.5582	0.4433	0.4931	0.5741
WPP	0.5738	0.3064	0.5212	0.3078	0.2176	0.3358	0.4874	0.6153	0.4862	0.4484	0.3320	0.552														



Table 29. Correlation matrix for all 50 stocks (con.)

	MKS	NG	NXT	PERSON	RB	REL	REX	RR	SAB	SBRY	SGE	SHP	SMIN	SN	SSE	SVT	TATE	TSCO	UU	VOD	IHG	WOS	WPP	WTB
AAL																								
ABF																								
ANTO																								
ARM																								
AZN																								
BATS																								
BG																								
BP																								
BRBY																								
CCL																								
CNA																								
CPG																								
CRDA																								
DGE																								
GFS																								
GKN																								
GSK																								
HMSO																								
IMI																								
IMT																								
ITRK																								
ITV																								
JMAT																								
KGF																								
LAND																								
MGGT																								
MKS	1.0000																							
NG	0.0594	1.0000																						
NXT	0.7062	0.0844	1.0000																					
PERSON	0.1322	0.2139	0.1838	1.0000																				
RB	0.2295	0.3182	0.1566	0.1763	1.0000																			
REL	0.2178	0.3540	0.2892	0.3443	0.4161	1.0000																		
REX	0.0466	0.3019	0.1710	0.2497	0.1524	0.4002	1.0000																	
RR	0.2591	0.1487	0.2898	0.3543	0.3338	0.2259	0.4638	1.0000																
SAB	0.0980	0.3109	0.2379	0.3915	0.4338	0.3802	0.5630	0.4621	1.0000															
SBRY	0.3053	0.1887	0.1992	0.1769	0.1868	0.3351	0.4078	0.3391	0.1560	1.0000														
SGE	0.1786	0.1292	0.2638	0.1991	0.2724	0.3985	0.3690	0.3848	0.2382	0.3142	1.0000													
SHP	0.0137	0.1462	0.1137	0.3479	0.1585	0.1406	0.2106	0.2334	0.3070	-0.0310	0.1288	1.0000												
SMIN	0.1029	0.3531	0.1117	0.2752	0.3764	0.4202	0.4904	0.3872	0.3302	0.3173	0.3876	0.2749	1.0000											
SN	-0.0095	0.1474	-0.0432	0.1327	0.2484	0.3503	0.1774	0.1006	0.1851	0.2968	0.3882	0.2279	0.3068	1.0000										
SSE	0.0675	0.5154	0.0443	0.1543	0.0753	0.3645	0.3798	0.0615	0.2685	0.1068	0.1609	0.1016	0.1349	0.0401	1.0000									
SVT	0.1235	0.4302	0.0140	-0.1122	0.0567	0.1921	0.3226	0.0695	0.1774	0.1598	-0.0108	0.1155	0.1980	0.0165	0.3679	1.0000								
TATE	-0.0314	0.3092	0.0831	0.2151	0.1339	0.1574	0.3081	0.4457	0.3624	0.1779	0.2501	0.1452	0.3133	0.0299	0.1484	0.0057	1.0000							
TSCO	0.2845	0.2719	0.3598	0.2247	0.1826	0.3317	0.3872	0.3601	0.3851	0.6258	0.3223	0.2228	0.2319	0.2595	0.2936	0.2042	0.3171	1.0000						
UU	0.1053	0.4967	0.1836	-0.0502	0.0594	0.2717	0.3702	0.1106	0.2220	0.1257	0.1503	0.2379	0.1374	0.0026	0.5302	0.7948	0.0401	0.3331	1.0000					
VOD	0.2644	0.3058	0.2800	0.2960	0.2276	0.2834	0.0827	0.3078	0.2912	0.1648	0.2368	0.3231	0.2090	0.0643	0.1835	0.0773	0.2651	0.3012	0.1355	1.0000				
IHG	0.2344	0.1530	0.2208	0.3281	0.2398	0.3513	0.5254	0.4366	0.5216	0.2756	0.4096	0.1427	0.4694	0.3100	0.1606	0.0470	0.2351	0.3047	0.0971	0.0737	1.0000			
WOS	0.3267	0.0066	0.3082	0.2234	0.0563	0.1307	0.3912	0.4516	0.3291	0.2258	0.3735	0.1053	0.1396	-0.0083	0.1486	0.0961	0.2815	0.2492	0.2123	0.2957	0.3162	1.0000		
WPP	0.3182	0.2266	0.3674	0.4509	0.2914	0.5495	0.6119	0.5414	0.6605	0.2973	0.5005	0.2362	0.5015	0.2267	0.2238	0.0981	0.3492	0.3541	0.2137	0.3492	0.6437	0.4950	1.0000	
WTB	0.2720	0.2592	0.3736	0.2216	0.2767	0.3665	0.4179	0.2874	0.4003	0.2480	0.4022	0.1925	0.3060	0.1983	0.1995	0.1451	0.2876	0.3283	0.2881	0.3392	0.3875	0.4797	0.6115	1.0000