FACULTY OF ECONOMICS
UNIVERSITY OF LJUBLJANA

MASTER'S THESIS

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RISK ATTITUDES AND LEARNING: AN EMPIRICAL ANALYSIS OF THE CENTIPEDE GAME

Ljubljana, June 2014

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The undersigned Nejc Lukač, a student at the University of Ljubljana, Faculty of Economics, (hereafter: FELU), declare that I am the author of the master's thesis entitled Risk Attitudes and Learning: an Empirical Analysis of the Centipede Game, written under supervision of doc. dr. Aljoša Feldin.

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Table of Contents

INTRODUCTION 1
1 THE CENTIPEDE 2
1.1 Solving the centipede game 2
1.1.1 Instrumental rationality 5
1.1.2 Common knowledge of rationality 8
1.1.2 Players know the rules of the game 11
1.2 Experiments with the centipede game 11
2 EXPERIMENTAL DESIGN 14
3 RESULTS 17
4 LEARNING 20
4.1 Evolutionary dynamics and belief-based learning 20
4.2 Reinforcement learning and other models 21
4.3 Simulation of fictitious play in the centipede game 21
4.4 Modeling learning in the centipede game 23
4.4.1 Fixed effects 24
4.4.2 Mixed effects 26
4.4.3 Ordered logit model 30
4.5 Convergence to equilibrium 36
5 ATTITUDES TOWARD RISK 37
5.1 Introduction to attitudes toward risk 37
5.2 Attitude toward risk experimental results 40
5.3 Risk aversion and the centipede game 42
5.4 Modeling risk aversion 50
CONCLUSION 54
REFERENCE LIST 56
APPENDIXES

List of Tables

Table 1: Experimental sessions 15
Table 2: Relative frequencies of end-of-game observations 17
Table 3: Relative frequencies of observed take/pass strategies 19
Table 4: Cumulative relative frequencies of end-of-game observations 19
Table 5: Hausman test performed in Stata 24
Table 6: Fixed-effects model 25
Table 7: Mixed effect (multi-level) model 27
Table 8: Expanded mixed effect (multi-level) model 29
Table 9: Frequencies of strategic choices of player 1 32
Table 10: Frequencies of strategic choices of player 2 32
Table 11: Cross-tabulated results 32
Table 12: Ordinal logit model 33
Table 13: Estimated and actual relative frequencies of strategic choices 34
Table 14: Odds ratios 34
Table 15: Brant test 35
Table 16: Likelihood ratio test 35
Table 17: Expanded ordinal logit model 35
Table 18: Exponential function 41
Table 19: Conditional relative frequencies of accepting a bet 41
Table 20: Indufference points 49
Table 21: Linear model 53

List of Figures

Figure 1: Extensive form of the centipede game 2
Figure 2: Backward induction 4
Figure 3: A's knowledge and his knowledge about C's knowledge 9
Figure 4: C's knowledge and his knowledge about B's knowledge when A has green eyes 10
Figure 5: Three versions of the centipede game used by McKelvey & Palfrey (1992) 12
Figure 6: Constant-sum centipede game 13
Figure 7: The trust building game 14
Figure 8: Three-person centipede game 14
Figure 9: Game interface for player 1 16
Figure 10: Centipede game with geometrically increasing payoffs 17
Figure 11: Distribution of relative "end-of-game" frequencies in individual sessions 18
Figure 12: Simulation of fictitious play 22

Figure 13: Average number of times players passed per game, comparison between simulated fictitious play results and experimental data 23
Figure 14: Decomposition to fixed and random effects 27
Figure 15: Decomposition to fixed and random effects (second model) 30
Figure 16: Results of the simulation.. 37
Figure 17: Concave utility function 39
Figure 18: Convex utility function 39
Figure 19: Linear utility function 40
Figure 20: Curve fit in SPSS.. 42
Figure 21: Second half of the centipede game 43
Figure 22: Structure of the game from player 1's perspective 44
Figure 23: Player 1’s most profitable strategies as a function of probabilities x and y of player 2 passing the pot... 45
Figure 24: Certainty equivalent of a risk-averse player 46
Figure 25: Distribution of utility-maximizing strategies of a risk averse individual............ 47
Figure 26: Certainty equivalent of a risk-loving player 48
Figure 27: Distribution of utility-maximizing strategies of a risk loving individual............ 49
Figure 28: Structure of the game from player 2's perspective 49
Figure 29: Point of indifference... 52

INTRODUCTION

The thesis reports on the results of a behavioral experiment conducted and funded by myself under the supervision of my mentor, dr. Aljoša Feldin. In particular, it focuses on the difficult problem of squaring the predictions of game theory with the experimental results which show that subjects rarely use the expected equilibrium strategies (McKelvey & Palfrey, 1992; Mailath, 1998). In my thesis, I attempt to use various models of learning, including fictitious play (Camerer, 2003) and reinforcement learning (Fudenberg & Lavine, 1998) as well as utilizing concepts of attitudes toward risk to better explain the behavior exhibited by subjects in the study. In the experiment, pairs of subjects played a version of the Centipede game, first introduced by Robert W. Rosenthal in his 1981 article *Games of Perfect Information, Predatory Pricing and the Chain-Store Paradox*. It is a finite, two-player game in which players take turns deciding either to take the larger share of an increasing pot of money or to pass the pot on to the other player. A detailed explanation of the game is provided in Chapter 1.

The main purpose of the experiment and the master's thesis is to test the main finding of similar experiments – that subjects, on average, do not behave as standard game theory predictions dictate. In addition, I test the following specific hypotheses:

1. Players learn from perceived errors in their past plays based on the difference between players' expected payoffs consistent with their chosen strategies and the actual payoffs they received. In other words, the bigger the difference between the payoffs players expected by playing their strategies and the actual payoff received, the less likely players are to pass the pot in subsequent games (on average).

2. Attitudes toward risk are not constant, but are a function of money amounts. There is a negative correlation between payoff amounts (the amounts being gambled) and the relative frequency of the probability of accepting a fair gamble.

3. More risk-averse individuals are, on average, less likely to *pass* the pot than risk-loving individuals

Chapter 1 introduces the game used in the experiment and provides the theoretical background required to understand it. It concludes with a discussion of notable centipede experiments conducted in the past. Chapters 2 and 3 describe the design of the experiment and provide summary results, respectively.

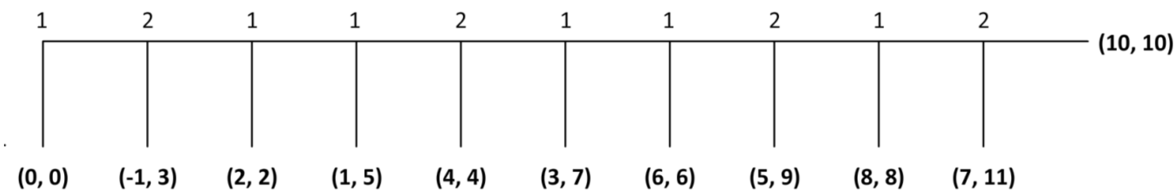
In Chapter 4, I discuss the various concepts of learning in behavioral game theory. I present a simulation of a basic, belief-based learning algorithm, *fictitious play*, using data from the experiment. To test the hypothesis that players learn from perceived errors in their past play, the idea which is drawn from the concepts of reinforcement learning, I estimate a series of models, including a fixed-effects panel-data model, a mixed-effects panel-data model and an ordered logit model.

Chapter 5 focuses on attitudes toward risk and provides a detailed analysis of the centipede game for risk-neutral, risk-averse and risk-seeking players. I attempt to estimate players' attitudes toward risk and test the hypothesis that players' risk aversion is correlated with their strategies – specifically, that players displaying greater risk aversion are more likely to end the game sooner.

1 THE CENTIPEDE

The Centipede game was first introduced by Robert W. Rosenthal in his 1981 article *Games of Perfect Information, Predatory Pricing and the Chain-Store Paradox*. It is a finite two-player game in which players take turns deciding either to take the larger share of an increasing pot or to pass the pot to the other player. Figure 1 shows the extensive form representation of a 10 round centipede game.

Figure 1. Extensive form of the centipede game



Source: R. W. Rosenthal*, Games of Perfect Information, Predatory Pricing and the Chain-Store Paradox,* 1981, p. 96

1.1 Solving the centipede game

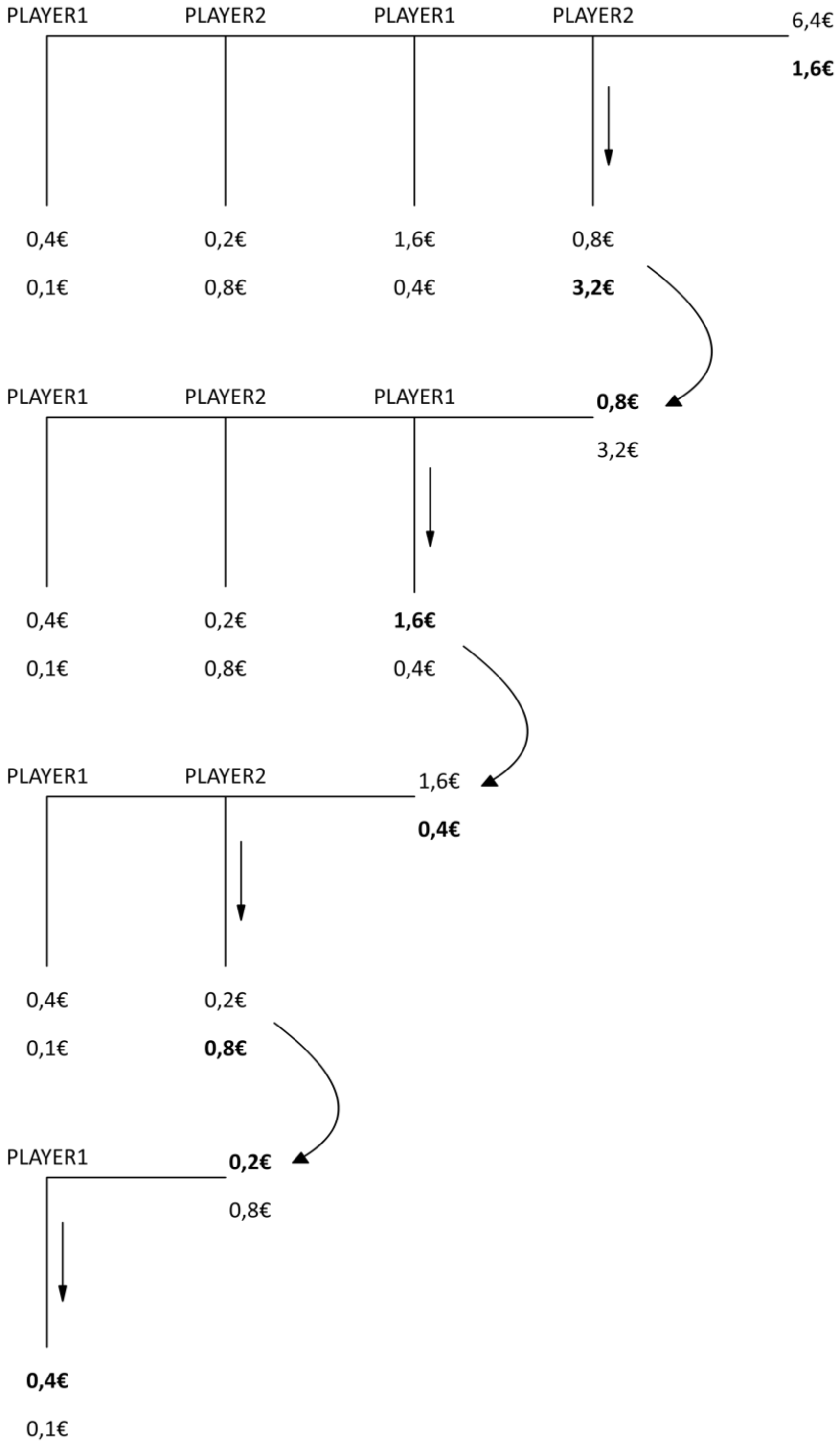
The standard method of solving finite sequential games is a process called backward induction. Starting at the last move of the game, we determine the player's optimal strategy. The reasoning is repeated in the next-to-last move of the game in which the optimal strategy is determined by taking the last player's optimal action as given. This iterative process is continued until all players' actions have been determined (Fudenberg & Tirole, 1991, pp. 92-93; Broome & Rabinowicz, 1999, pp. 237-242).

Figure 2 shows individual iterations in the process of backward induction for a four-move centipede game that was used in our experiment. In the last move of the game, player 2 has a choice between taking the pot and gaining 3,2€ or passing the pot for a payoff of 1,6€. If we assume player 2 is rational and maximizes his utility by maximizing his payoffs, we see that given the choice between 3,2€ and 1,6€, player 2 will choose 3,2€ - taking the pot. Moving one node to the left, player one is now faced with a decision between taking the pot (1,6€) or passing, in which case player 2 will take 3,2€, leaving player 1 with only 0,8€. Therefore, if player 1 is also rational and maximizes his utility, he will choose to take the pot. Player 2 in node 2 has a choice between taking the pot (0,8€) or passing, in which case player 1 will take the pot, leaving player two with only 0,4€, therefore player 2 takes the pot in node 2. Finally, player 1 in the first move of the game faces a decision between taking the pot (0,4€) or passing the pot to player 2 who would take the pot, leaving player 1 with only 0,1€, therefore player 1 takes the pot in the first move of the game.

This type of solution is referred to as a *subgame perfect Nash equilibrium* (hereinafter: SPNE). A *Nash equilibrium* is "an action profile a^* with the property that no player i can do better by choosing an action different from a_i^*, given that every other player j adheres to a_j^* (Osborne, 2003, p.32)." A solution is said to be a Nash equilibrium solution if each player is making the best decision they can, taking into account the other players' decisions. In this sense, the Nash equilibrium represents a steady state solution in which none of the players in the game can (or is therefore willing to) change their strategy to benefit themselves. A *subgame* is a part of an extensive game "beginning with a decision point and including everything that branches out below it (Nicholson & Snyder, 2010, p. 197)." Strategies are said to be in SPNE, when they constitute a Nash equilibrium in *each* subgame (Heap & Varoufakis, 1995, p. 84).

The proposed solution might seem quite counterintuitive when we consider that practically every other outcome has each of the players receiving higher payoffs. One could coin a version of the centipede game with much higher stakes in which players would be millionaires after only a few moves, yet as long as each player were even a fraction of a cent better-off if they took the pot, the backward induction solution would be exactly the same.

Figure 2. Backward induction



This is the consequence of some very rigid assumptions game theory imposes on players. According to Heap and Varoufakis (1995, p. 2) there are three key assumptions in game theory:

- Players are instrumentally rational
- They have common knowledge of this rationality
- They know the rules of the game

In addition to these three assumptions we must also assume either that players are self-interested (maximize payoff) or that the monetary amounts in Figure 2 are actually the utilities of the two players in order to come to the proposed solution.

1.1.1 Instrumental rationality

Individuals who are *instrumentally rational* are those who maximize their (expected) utility consistent with their preferences. Individuals have preferences over various things but regardless of the ends they pursue, they are instrumentally rational as long as they select actions/means that best achieve those ends. A selfish individual will choose actions that maximize their own gains while altruistic preferences might result in a very different set of actions. So the assumption of rationality does not put any restrictions on how we choose our preferences or rather what our preferences are, only on how we go about satisfying those preferences (Heap & Varoufakis, 1995).

Frequently, there are additional conditions put on individual preferences, so that they are well-ordered (first three axioms) and useful mathematically (continuity), stated by the following axioms:

a) Completeness: $A \gtrsim B$ or $B \gtrsim A$ for all A and B
b) Reflexivity: $A \gtrsim A$ for all A
c) Transitivity: for any A, B, C, if $A \gtrsim B$ and $B \gtrsim C$, then $A \gtrsim C$
d) Continuity: there are no jumps in people's preferences. In other words, if we prefer *A* to *B*, something sufficiently close to *A* should also be preferred to *B*.

The statement $A \gtrsim B$ is read as: "*A* is at least as good as *B*". For the preferences over prospects to be consistent, two additional conditions are required (Heap & Varoufakis, 1995):

e) Monotonicity (preference increasing with probability): the prospect improves if the probability of a preferred outcome within a prospect increases, while the probability of the poorer outcome falls. If outcome *A* is preferred to outcome *B* and p_1 and $1-p_1$ are the respective probabilities of those two outcomes within a lottery, $L_1 = (A, B; p_1, 1 - p_1)$, then a different lottery, $L_2 = (A, B; p_2, 1 - p_2)$, is preferred to L_1 only if $p_2 > \ p_1$.

f) Independence (substitution): assumes that "the choice between two lotteries, *X* and *Y*, is independent of the possible existence of a common (and hence "irrelevant") prospect Z (Holt, 1986)." In other words, if outcome *A* is preferred to outcome *B*, then $L_1 = (A, Z; p, 1 - p) \succ L_2 = (B, Z; p, 1 - p)$ for all *Z* and $p \in (0,1)$.

Expected utility theory has been criticized extensively, the independence axiom being particularly controversial. The Allais paradox (originally designed by Maurice Allais (Allais, 1953)) and its many variants show that the predictions of expected utility theory are inconsistent with observed behavior. Allais proposed two hypothetical experiments where participants would choose between two lotteries. In the first experiment, participants could choose between a certain payoff of $1 million and a gamble with 89% chance of winning $1 million, 10% chance of winning $5 million and a 1% chance of winning nothing. In the second experiment, participants could choose between a gamble with 89% chance of winning nothing and 11% chance of winning $ 1 million, and a gamble with 90% chance of winning nothing and 10% chance of winning $5 million. Most people would choose the first option (certain payoff of $1 million) in the first experiment but the second gamble in the second experiment (90% chance of winning nothing and 10% chance of winning $5 million), yet this violates the independence axiom. Kahneman and Tversky (1979) argued that people overvalue certain outcomes relative to merely probable ones (certainty effect) and discard components common to all prospects they are considering (isolation effect).

The above example demonstrates what is commonly referred to as the framing effect which exposes the fact that peoples' preferences are often not robust to different presentations of the same choices – in particular, we are sensitive to whether a proposition is phrased in terms of losses or gains. This can be interpreted either as an error in our decision-making or perhaps even more challenging - as "frames" being a determinant of our preferences (Rabin, 1998).

1.1.2 Common knowledge of rationality

A regularly employed assumption in game theory is *common knowledge of rationality* (hereinafter: CKR). In simple terms, each player knows all players are rational and know all players are rational and know all players know all players are rational and so on ad infinitum (Aumann, 1995).

This is different from the assumption of mutual knowledge of rationality by which all players simply know the other players are rational but don't necessarily know whether or not the other players know all other players are rational. To explain the implications of this difference a variant of the following logical puzzle is often employed (Gintis, 2009, p. 53):

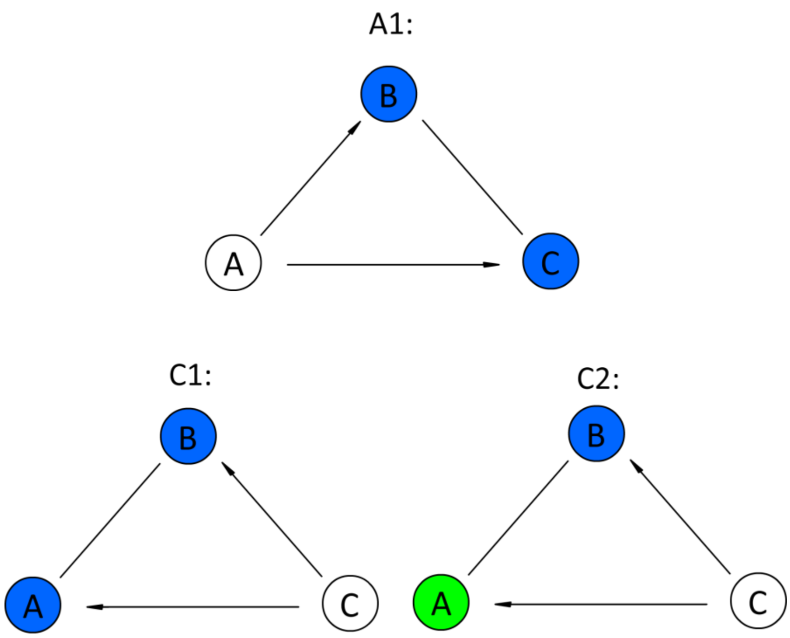
Imagine an isolated island with a tribe of 100 blue-eyed people. Each person in the tribe knows the eye color of every other member of the tribe except their own, and they cannot directly establish the color of their eyes (discussion with other members about this topic is forbidden, there are no mirrors or other reflective surfaces, etc.). They all desperately wish to leave the island, but tradition dictates that only blue-eyed people may venture into the world. Every morning, a ferry lands on the island, expecting to carry passengers to the mainland and every night it returns empty. One night, as the ferry is leaving, the blue-eyed captain, whom they trust implicitly, addresses the entire tribe in their own language, saying: "It's nice to see another blue-eyed person; there are hardly any in my hometown!" Exactly 100 days later, all members of the tribe board the ferry and travel to the mainland.

Before the captain's announcement, it was *mutually* known that there was at least one blue-eyed person on the island (in fact, each of them knew that there were at least 99 blue-eyed people on the island), but only after the announcement did this fact become common-knowledge. That is, each person knew there was at least one blue-eyed person on the island, but did not know whether or not other people knew that until the captain's announcement. To solve this riddle, the problem can be reduced to an island with only two people. Each of them can see the other person's eye color but doesn't know their own; therefore they cannot leave the island. After the captain's announcement, they also know that each of them knows that at least one person has blue eyes and therefore each of them knows that if the other person saw that their friend's eyes weren't blue, they would conclude that they must have been the blue-eyed person the captain was talking about and would leave the island the next morning. If nobody leaves the next morning, both of them know neither of them observed a non-blue-eyed person, therefore both of them must have blue eyes and can leave on the next ferry (on the second morning after captain's announcement).

The same logic applies for a three-person island. If one of them saw that both of his friends had green eyes, for instance, he could conclude that he must have blue eyes and would leave on the first morning. If nobody leaves on the first morning, at least two of them must have blue eyes – therefore anyone who saw *one* of their friends having green eyes would leave the island on the next morning. If nobody does, each of them concludes they must all have blue eyes, and they all leave the island on the third morning. Therefore, on an island with *k* blue-eyed people, nobody leaves for *k-1* days and everybody leaves together on the *k*-th day.

This reasoning might not seem sound at first glance if we apply it past *k*=2, where one can clearly see that in the case of there being only one person with blue eyes on the island, the captain's announcement would truly be new information for one of them as there would have been no other blue-eyed people for them to observe and therefore the fact that there were blue-eyed people on the island could not have been common knowledge. But what new information did the captain convey on the 100-person island? Each of them knew there were at least 99 blue-eyed people on the island and each of them must also have been able to conclude that those 99 blue-eyed people knew that there were at least 98 blue-eyed people on the island and so on. It is not immediately clear that "at least one person has blue eyes" wasn't common knowledge all along. But examining the easier 3-person island example demonstrates that the assumption of common knowledge does break down even for *k*>2. We can denote the three people on the island as *A*, *B* and *C*. Figure 3 shows the *C'*s beliefs from *A'*s perspective.

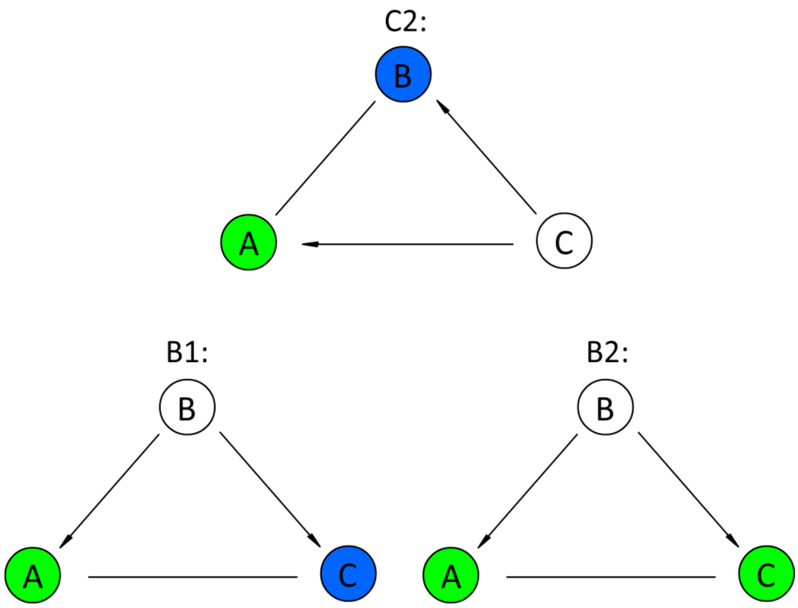
Figure 3. A's knowledge and his knowledge about C's knowledge



Clearly, *A* knows that there is at least one blue-eyed person on the island, since she sees both *B* and *C*, who both have blue eyes (A1 in figure 3). *A* also knows that *B* and *C* each know that there is at least one blue-eyed person on the island as they are able to observe each other. And *A* knows that *C* (*B*) knows that *A* knows that there is at least one blue-eyed person on the island, since they should both be able to observe *B* (*C*). But *A* doesn't know whether or not *C* (*B*) knows that *B* (*C*) knows that there is at least one blue-eyed person on the island.

To simplify, let's assume *A* believed he had green eyes, while he could see that *B* and *C* had blue eyes. From *A*'s perspective, *C* could see that *B* had blue eyes and *A* had green eyes (C2 in figure 4), but since *C* could not know the color of his own eyes, he could not know whether *B* was seeing two pairs of green eyes or one pair of green eyes and one pair of blue eyes (B1 and B2 in Figure 4).

Figure 4. C's knowledge and his knowledge about B's knowledge when A has green eyes



Obviously, not everyone knows that everyone knows that everyone knows that there is at least one person with blue eyes on the island. There is no common-knowledge of this fact and it is only made common-knowledge by the captain's announcement.

In the centipede game, CKR ensures that all players believe other players are rational and think all other players are rational (and so on for as many levels as it takes for common knowledge not to break down in that particular game), which allows us to use backward induction to come to a specific solution. If player 1 believes player 2 is rational but believes that player 1 is not rational – the same reasoning cannot be applied. Even if we assume CKR there are problems with this approach. Backward induction begins at the end of the game (player 2’s second turn in Figure 2), but if CKR and the other assumptions hold, there is no way for the game to actually reach that point – or any *out-of-equilibrium* point. Should player 2 in node 2 (see: Figure 2) not take the fact that player 1 played *pass* on his first turn into consideration when trying to maximize his utility? Selten (1975) proposed the idea of *trembles* – a small chance of a player making an error on their move (a similar concept, the sequential equilibrium, was proposed by Kreps and Wilson in 1982. Out-of-equilibrium points could then be reached without necessarily violating CKR. However, considering how many out-of-equilibrium beliefs would have to be considered in, for example, a 100-move centipede game, the idea of using random trembles as an explanation becomes much less plausible. In the unlikely event of a game reaching nodes close to the end of the game by random trembles alone, players could interpret this as a systematic feature of other players’ strategies rather than random perturbations (Heap & Varoufakis, 1995, pp. 87-90).

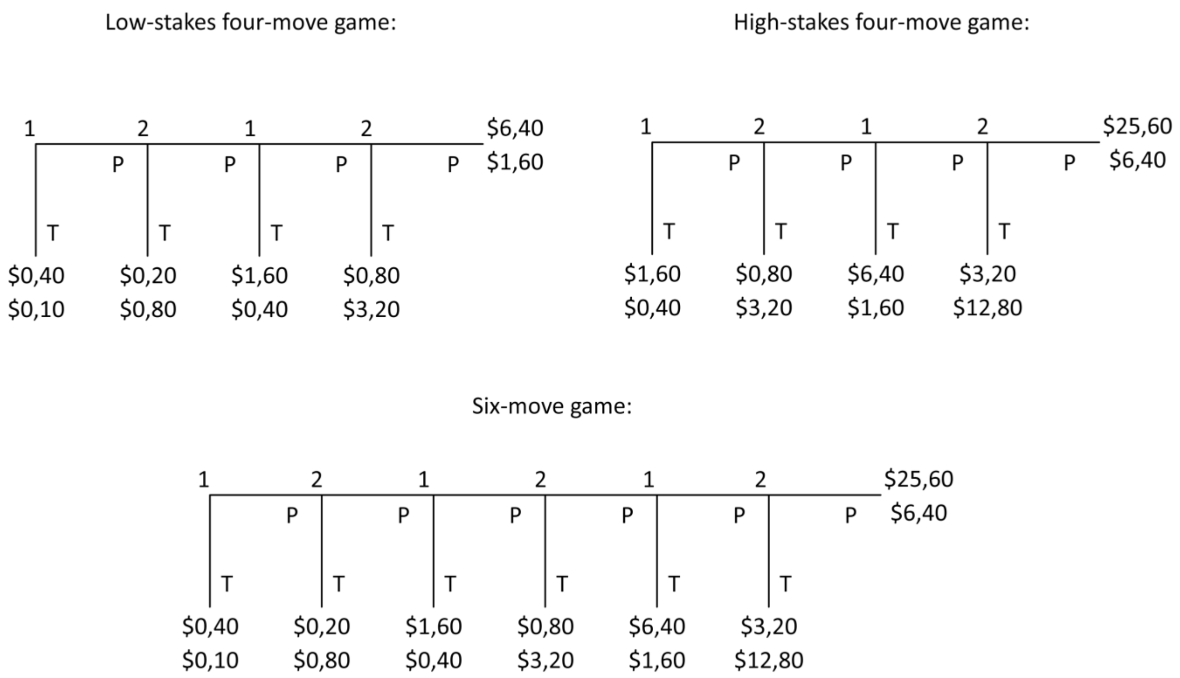
1.1.2 Players know the rules of the game

The final assumption is that all players know the rules of the game – all possible actions and all the utility payoffs (or expected payoffs) a particular action or combination of actions is going to bring them and their opponents. This assumption might be reasonable in simple, well-structured games, where the set of all possible moves and players’ payoffs are reduced to a manageable size. But in complex or loosely structured interactions players often “invent” moves and do not necessarily know exactly how their decisions will affect each player’s (expected) payoffs (Heap & Varoufakis, 1995, p. 28).

1.2 Experiments with the centipede game

One of the earlier laboratory experiments utilizing the centipede game was conducted by Richard D. McKelvey and Thomas R. Palfrey in 1992. Their study involved seven sessions of 18-20 subjects each playing 9-10 games for a total of 662 games. The first three sessions subjects played a low-stakes, four-move game with the pot doubling each move, three sessions were conducted using a six-move version of the game and one session was conducted using a high-stakes version of the four-move game used in the first three sessions, with all the payoffs quadrupled. Figure 5 shows the three versions of the centipede game used in the experiment. The results of the study confirmed that players rarely take the pot in early nodes as subgame perfect Nash equilibrium predicts, however the probability of “take” increases as the game gets closer to the last move. McKelvey and Palfrey suggested that this result could be explained by a small proportion of altruistic players (they estimated the level of altruism of the order of 5%) with a utility function that was monotonically increasing in the sum of both players’ payoffs (passing the pot on each turn would maximize such a player’s utility) and a large proportion of selfish players who could use a wide variety of strategies, including mimicking altruistic behavior, to maximize their own payoffs. Neither the higher number of moves in a game nor higher payoffs seemed to have any significant effect (except, the authors noted, a lower estimated initial error rate in the six-move game).

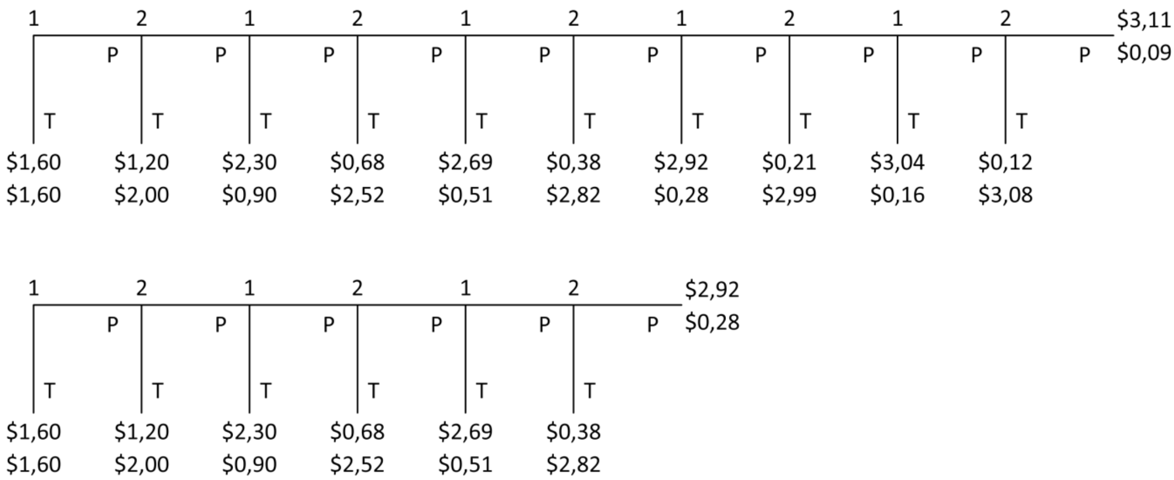
Figure 5. Three versions of the centipede game used by McKelvey & Palfrey (1992)



Source: R. D. McKelvey & T. R. Palfrey, *An Experimental Study of the Centipede Game*, 1992, p. 806.

In 1996, Fey, McKelvey and Palfrey conducted an experiment utilizing a constant-sum centipede game. The game started with $3,2 divided evenly among the two players. If player one passed, a quarter of the money from their pile would be subtracted from their payoff in node 2 and added to player 2’s pile. This process would continue every time a player passed the pot and the distribution of payoffs would get ever more uneven as the game progressed. Unlike the McKelvey and Palfrey (1992) study, deviations from Nash equilibrium solution could not be explained by a percentage of altruistic players (with utilities that increase monotonically in the sum of both players’ payoffs), since there are no social gains in passing the pot. Despite this, a surprisingly high percentage of games (23% in the ten-move version and 8% in the six move game) continued past node 2. The authors proposed a *quantal response equilibrium* (*QRE*) model to explain the data.

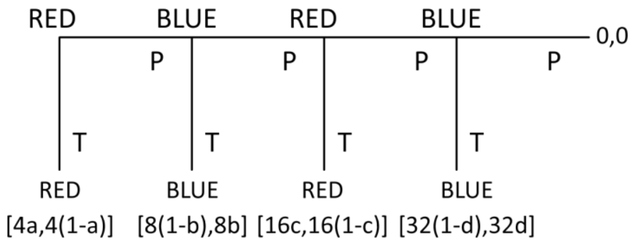
Figure 6. Constant-sum centipede game



Source: M. Fey, R. D. McKelvey & T. R. Palfrey, *An Experimental Study of Constant-sum Centipede Games,* 1996, p. 271.

In 2005, Ho and Weigelt conducted a trust building experiment utilizing a four-move centipede game where the payoffs doubled each move. Unlike the standard centipede game, where payoff amounts of each player are determined, only the value of the pot (the sum of both players' payoffs) was determined. On their turns, players could decide to *take* and divide the pot between themselves and the other player as they saw fit, ending the game, or to "trust" their opponent and pass the pot to them. If they decided to pass, the pot would double and their opponent would have to decide whether to pass the pot back or to take as much as they wanted. The unique design of the experiment allowed individuals to display both trusting (passing the pot to their opponent) and trustworthy (depending on how players chose to divide the pot between themselves and their opponents) behavior. Results showed that subjects exhibited some trusting behavior, but also exhibited a high level of self-interest, with 30% of player 1s taking at the first node and 50% of player 2s taking at the second node. Players were also not trustworthy, taking, on average, 95% of the social gains. Trusting behavior decreased monotonically across decision stages, while trustworthiness monotonically increased. Subjects were found to be more trusting as well as more trustworthy when playing for higher stakes. Figure 7 shows the design of the game.

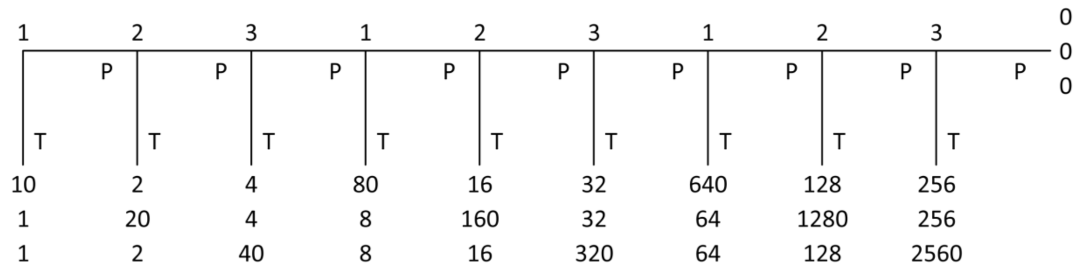
Figure 7. The trust building game



Source: T. Ho & K. Weigelt, *Trust Building Among Strangers*, 2005, pp. 519-530.

In 2003, Rapoport, Stein, Parco & Nicholas conducted a four-turn, three-person centipede game with very high stakes (see: Figure 8) and compared it to the same game with much lower stakes. In both cases, a total of sixty students participated in four session of 15 subjects each. As in the Ho and Weigelt study, results showed a much greater proportion of games ending in the first two nodes (around a third in the high-stakes version) compared to the results in McKelvey and Palfrey experiment. Camerer (2003, p. 95) suggests the difference might be due to the presence of a (0,0) terminal node in the Ho and Weigelt and the Rapoport et al. studies. It is also quite difficult to directly compare the results of a two-person and a three-person centipede game as the different structure of the game might affect players' strategies in various ways.

Figure 8. Three-person centipede game



Source: A. Rapoport, W. E. Stein, J. E. Parco & T. E. Nicholas, *Equilibrium play and adaptive learning in a three-person centipede game*, 2003, p. 240.

2 EXPERIMENTAL DESIGN

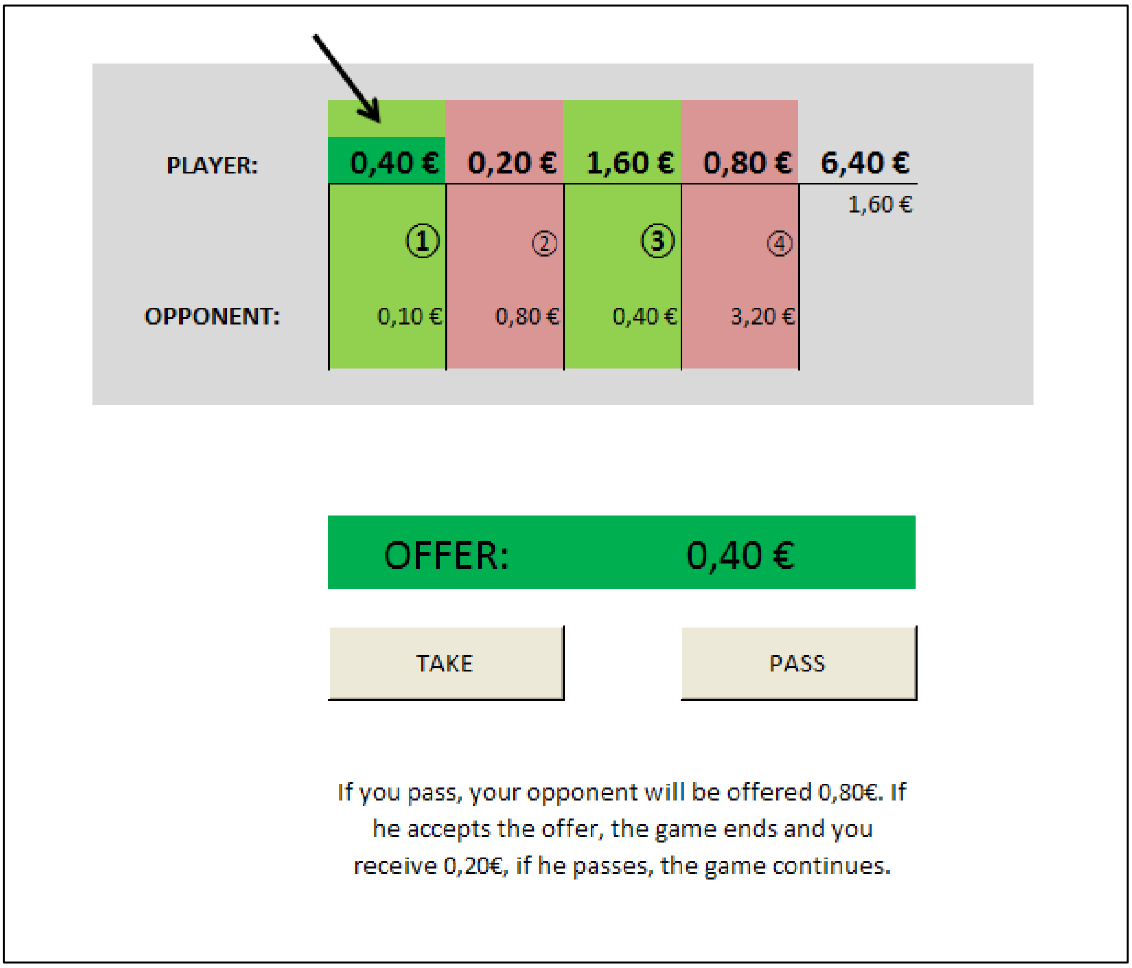
The experiment was conducted in three sessions in which 38 subjects played a total of 123 games. Subjects were graduate students from the Faculty of Economics in Ljubljana; the third session was conducted exclusively with students from the faculty's MBA programme. All games were played on computers using a simple interface designed specifically for this experiment. Figure 9 shows the game interface for player 1. The game structure, including payoffs and a short verbose description were available throughout the game. The players' current position (the node at which they were making the decision) was clearly marked by an arrow and their payoff for *taking* on that turn was prominently displayed above the *take* and *pass* buttons. The interface was designed in such a way in an effort to ensure all players were as informed as possible about the choices they had, the consequences of each choice and about the structure of the game as a whole. Due to capacity constraints and other factors (not all subjects showed up) the numbers of subjects participating in the sessions 1-3 were 10, 14, and 14 respectively. In each session subjects would draw numbered cards corresponding to numbers assigned to the computers to randomize seating order and to divide them into two groups, determining whether or not players would get to act first in games. For each game, a player from the first group (player 1) was randomly paired with a new player from the second group (player 2), ensuring that nobody played with the same person more than once. This fact was thoroughly explained to the subjects. As the number of subjects in each session determined the number of unique pairs (and consequently, the number of games played in the session), students in the second and the third session played 7 games each (a total of 49 games per session) while students in the first session played only 5 games each (for a total of 25 games). Table 1 shows a breakdown of the sessions.

Table 1. Experimental sessions

Session number	Subject pool	Number of subjects	Games per subject	Total games
1	Graduate students	10	5	25
2	Graduate students	14	7	49
3	MBA programme	14	7	49

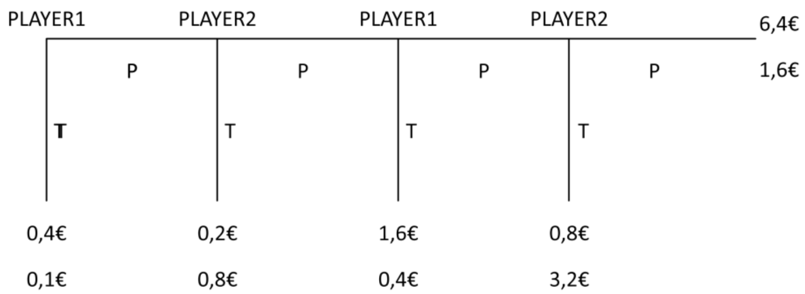
In each session, the subjects were given a series of instructions by the experimenter, explaining the nature of the experiment and providing a detailed explanation of the game about to be played. Subjects were not allowed to communicate with each other except via the actions selected while playing the game. For a complete transcription of the instructions, see Appendix A.

Figure 9. Game interface for player 1



The game chosen for the study is a variation on the game introduced in chapter 1, a four-move (two-round) centipede game with geometrically increasing payoffs. The basic payoff structure is similar to the one used in Mckelvey & Palfrey (1992) study (as well as some other studies) as it seems to offers a good trade-off between suitable payoff amounts and the strictly constrained budget of a student's self-funded endeavor. The extensive-form structure of the game used in the experiment is shown in Figure 10. After each game, subjects were given a choice to either collect that game's winnings or to gamble them on a flip of a coin to either double their payoff or lose what they had won in that game. The addition of a fair gamble at the end of each game allowed me to expand the study to include attitudes towards risk in reference to individuals as well as varying amounts of winnings being gambled.

Figure 10. Centipede game with geometrically increasing payoffs



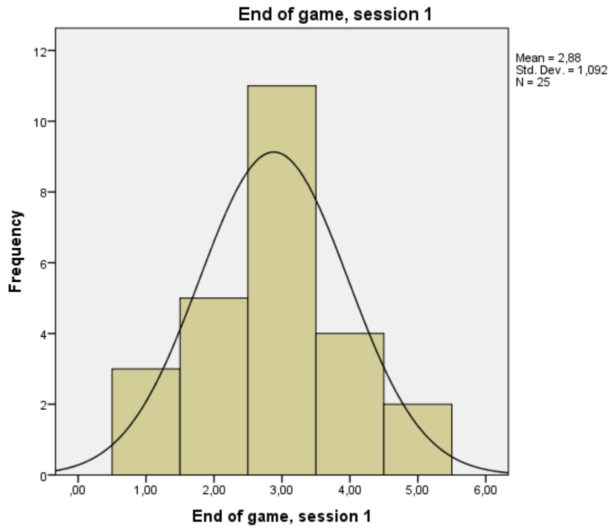
3 RESULTS

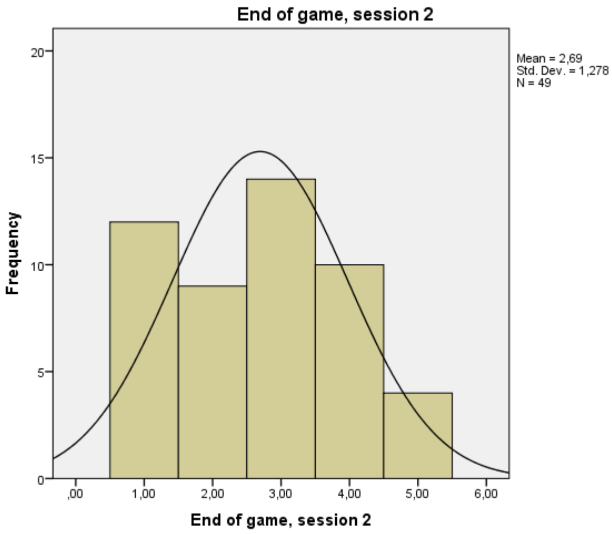
Table 2. Relative frequencies of end-of-game observations

Session	Subjects	N	f_1	f_2	f_3	f_4	f_5
1	Graduate students	25	0,12	0,20	0,44	0,16	0,08
2	Graduate students	49	0,24	0,18	0,29	0,20	0,08
3	MBA students	49	0,02	0,27	0,45	0,16	0,10
Total		123	0,13	0,22	0,38	0,18	0,09

Table 2 shows the relative frequencies of the game-terminating nodes reached in 123 games. f_5 corresponds to the frequency of games ending with both players having passed on each of their turns. Because of the different number of games in each session, the totals are weighted averages of the corresponding relative frequencies in individual sessions ($f_i^{TOTAL} = \sum_{j=1}^{3} N_j f_{ij}$). Despite the standard game-theory predictions ($f_1 = 1$ and $f_2 = f_3 = f_4 = f_5 = 0$), only 13% of all games ended in the first decision node and 9% of all games ended with none of the players taking the pot. 38% of all games ended in decision node three, which was also the mode of all three sessions. The somewhat different frequency distribution in session 2 (see: Figure 11) can in large part be attributed to a single player's strategy of consistently taking the pot in the first node in all 7 games.

Figure 11. Distribution of relative “end-of-game” frequencies in individual sessions





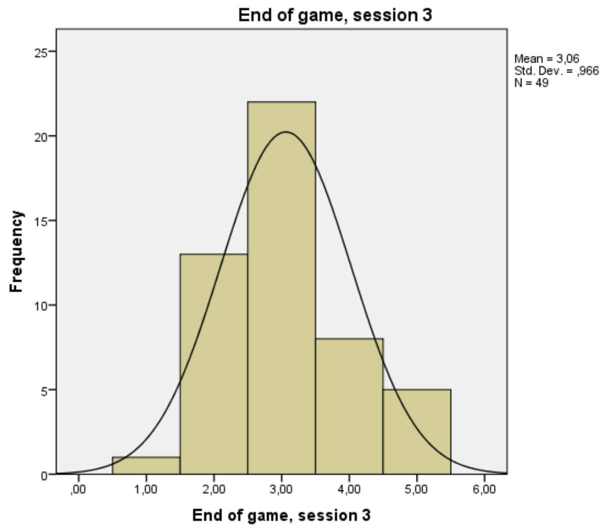


Table 3. Relative frequencies of observed take/pass strategies

Session	Subjects	N	Player 1 f^1_{TT}	Player 1 f^1_{PT}	Player 1 f^1_{PP}	Player 2 f^2_{TT}	Player 2 f^2_{PT}	Player 2 f^2_{PP}
1	graduate students	25	0,12	0,52	0,36	0,20	0,44	0,36
2	graduate students	49	0,24	0,39	0,37	0,20	0,57	0,22
3	MBA students	49	0,02	0,63	0,35	0,27	0,57	0,16
Total		123	0,13	0,51	0,36	0,23	0,54	0,23

Table 3 shows the relative frequencies of observed take/pass strategies adopted by the two player types. There are three distinct strategies players can adopt for any particular game (*TT*, *PT* and *PP*). *TT* represents the backward-induction predicted strategy of players taking the pot in their first turn, *PT* represents the strategy of passing on the player's first turn and taking the pot in their next turn and *PP* represents the strategy of players passing on both of their turns. Due to the specific way in which data was collected in this experiment – allowing each player to play their game regardless of their opponent's decisions - the frequencies presented in Table 2 are the actual frequencies of players of a particular player-type having chosen a particular strategy. Since player 1's strategy of *TT* directly determines the node at which the game ends, frequencies in column f^1_{TT} are identical to the corresponding frequencies in column f_1 in Table 1. For both player types *PT* was the most frequently selected strategy. While $f^2_{PP} \leq f^1_{PP}$ for each session, the strictly dominated strategy of *PP* was still chosen by player 2 a surprisingly high percentage of the time. This means that faced with the decision to collect their 3,2€ (leaving player 1 with 0,8€), player 2, on average, decided to pass the pot back to player 1 a final time (halving their own payoff but increasing player 1's payoff to 6,4€) 23% of the time. If the standard assumption of players being habitual payoff-maximizers holds, this would imply very high errors in action – experimenting with different strategies, failing to correctly ascertain their player-type or which round of the game they are in, pressing the wrong button etc. (McKelvey & Palfrey (1992), p. 815). Alternatively, some players' utility functions could include other players' payoffs or they could even gain utility from experimenters' losses. These "altruists" would not necessarily choose the highest payoff when maximizing their utility. Table 4 shows the cumulative relative frequencies of end-of-game observations.

Table 4. Cumulative relative frequencies of end-of-game observations

Session	Subjects	N	F_1	F_2	F_3	F_4	F_5
1	Graduate students	25	0,12	0,32	0,76	0,92	1,00
2	Graduate students	49	0,24	0,43	0,71	0,92	1,00
3	MBA students	49	0,02	0,29	0,73	0,90	1,00

Total		123	0,13	0,35	0,73	0,91	1,00

4 LEARNING

In an attempt to explain how equilibrium actually arises in a game, a number of distinct theories of learning have been proposed. This chapter provides an overview of these different approaches.

4.1 Evolutionary dynamics and belief-based learning

Models of evolutionary learning attempt to study the evolution of strategies in a population (or multiple populations) of agents who repeatedly interact to play a game. Strategies are subject to selection pressures and payoffs can be interpreted as a measure of the relative success (fitness) of a strategy. The basic idea is that successful strategies tend to spread more (are more frequent) than less successful ones. The main mechanisms at work in evolutionary models mirror those of biological evolution: selection, replication and mutation. Selection favors players who have obtained higher payoffs over those with lower payoffs and replication ensures that strategies of selected players are transmitted across consecutive generations. Mutation describes the process of experimentation or innovation by which new strategies or new patterns of behavior appear (Izquierdo, Izquierdo & Vega-Redondo, 2012).

Three common models of evolutionary learning are fictitious play, partial best response and the replicator model. All three models can also be categorized as belief-based. In fictitious play, which is only vaguely connected with evolutionary ideas, each player assumes their opponents are playing stationary strategies, but does not know the distribution of those strategies. Players then simply play their best replies against their opponents' past play. This approach is interesting in terms of explaining the convergence to equilibrium, because if at any point all players play Nash equilibrium, they will continue to do so for all subsequent turns (Battigalli, Gilli & Molinari, 1992). The partial best response model is very similar to the process of fictitious play, but here only a fixed portion of the players adjust their play. In the replicator model the share of the population using a particular strategy increases at a rate proportional with the relative payoff advantage of that strategy (Fudenberg & Tirole, 1991).

4.2 Reinforcement learning and other models

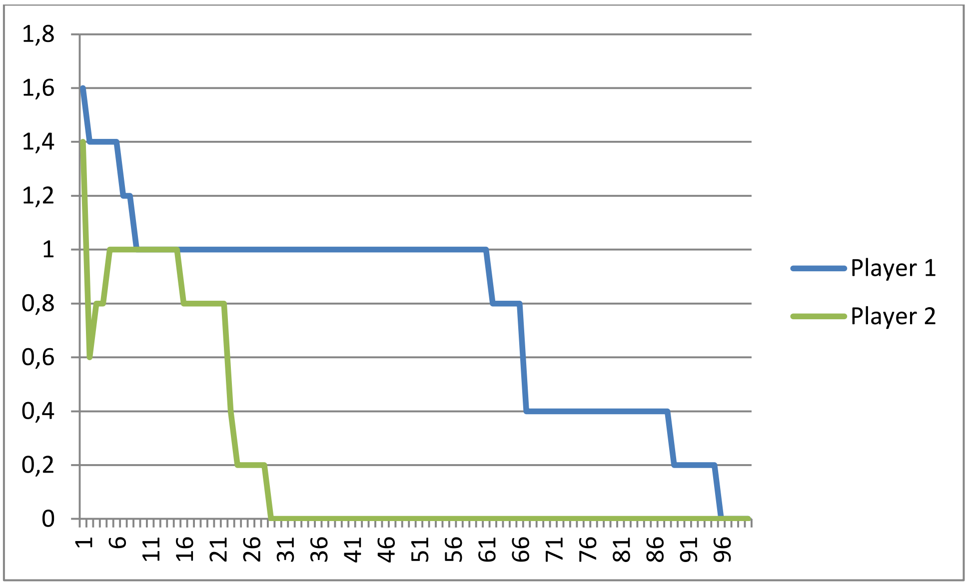
Reinforcement learning models were largely inspired by developments and experimentation in behavioral psychology. The basic premise of reinforcement learning is that players' strategies are reinforced by their previous payoffs. Players learn which actions to perform based on the rewards they receive, strategies with satisfactory outcomes tend to be repeated and strategies with unsatisfactory results tend to be avoided. Payoffs received for a specific strategy may also act as reinforcement for similar strategies (reinforcement may "spill-over"). A draw-back of simple reinforcement learning models is that they assume players only care about their past payoffs and disregard any other information they might have, such as the payoff structure, other players' payoffs, etc (Camerer, 2003, p. 273).

More sophisticated models such as experience-weighted (EWA) learning (Camerer & Ho, 1999; Camerer, Ho & Chong, 2002), which combines features of reinforcement and the weighted fictitious play model, have also been proposed. Models with anticipatory learning assume players can use information about other players' payoffs to reason about their future actions. Players might also learn by imitating the strategies of other, more successful players. Finally, rule learning allows players to learn which rules, rather than specific strategies, to use.

4.3 Simulation of fictitious play in the centipede game

The most rudimentary form of learning players in the centipede game could adopt would be a form of fictitious play - assuming players could be made aware of their opponents' strategies after each game. To examine fictitious play as a viable explanation of experimental data, I designed a simulation of this model of learning. Every game after the initial round, players would simply play the best response to their opponents' strategies (best response to the frequency of the opponents' past strategies) and the game should eventually unravel to the equilibrium with both players taking on their first turns which would repeat in every subsequent game. Figure 12 shows a simulation of fictitious play using data from the first game in session 1 of the experiment. The y-axis is the average number of times player's passed the pot in the game and the x-axis shows the number of iterations of the game. For the purposes of the simulation, players were assumed to be fully aware of their opponents' choices. Players were paired using a rotating matching scheme rather than truly randomized.

Figure 12. Simulation of fictitious play



The simulation confirms that the game reaches NSPE, but only after 96 iterations of the game. Players 1 lagged behind players 2 who all reached the predicted equilibrium strategy within 29 iterations. Expectedly, the fictitious play model is not adequate to explain the experimental data, but it does demonstrate the concept of convergence to equilibrium. While the centipede game does eventually reach NSPE in this case, a high number of iterations was required even in such a simple game which was further simplified by assuming players knew their opponents strategies. These facts undermine the plausibility of this approach in explaining how we arrive at equilibrium. The last player to reach equilibrium strategy in this simulation would have had to keep track of 95 games and be able to derive the best response based on the frequency distribution of 95 opponents' strategies and at the same time be completely myopic with regards to the fact that (at least) the last 67 games his opponents all played the same strategy.

Figure 13. Average number of times players passed per game, comparison between simulated fictitious play results and experimental data

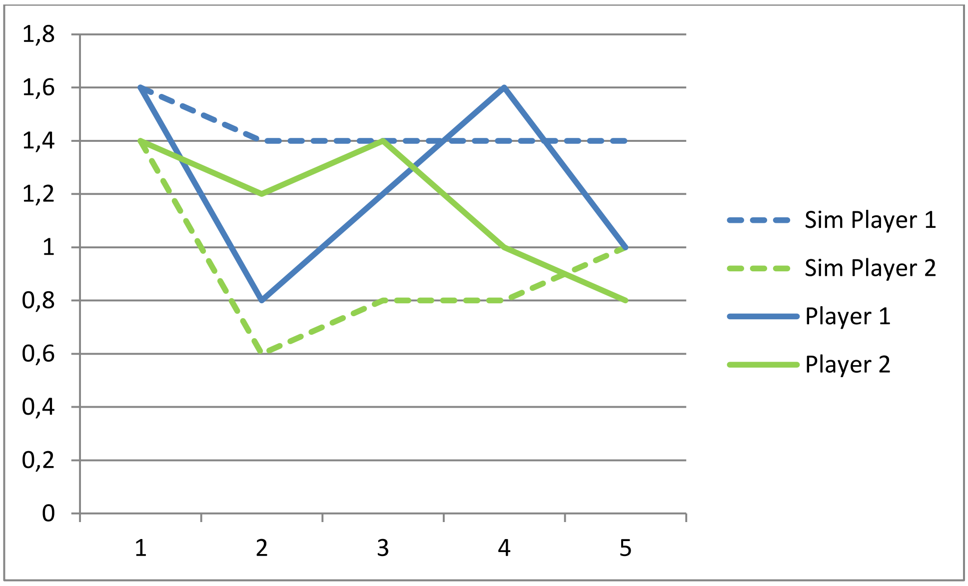


Figure 13 shows the comparison between the simulated results of the first five iterations of the game and the experimental data. Except for the starting point, which was used as input for the simulation, there is little similarity with the observed data.

4.4 Modeling learning in the centipede game

The motivations of players are complex and diverse and it is unlikely that simplifications such as treating all individuals as habitual payoff-maximizers would prove effective in explaining the data collected in this or similar experiments sufficiently well (Beard & Beil, 1994). However, it is my contention that while the *tendencies* of individual players, the beliefs and motivations that inform their actions, may be hard to explain adequately, the *evolution* of players' strategies throughout a session are based on a relatively simple approach. Each player has preexisting beliefs that inform their strategy in the first game and presumably players choose strategies they believe maximize their utilities – whether that's purely a function of their payoffs or some combination of their payoffs and other peoples' payoffs. Whatever strategy they choose to play, players are immediately confronted with their opponent's strategy in the first game, which brings about two possible outcomes; either the game ended in the node in which they chose to end the game (they were able to effectively play the strategy they had chosen to maximize their utility) or the game ended before they were able to take the pot and end the game.

Players observe the error – the difference between the payoff they expected by playing their strategy and the actual payoff received and react to it by adjusting their strategies in the subsequent games. The bigger the difference between the expected payoff consistent with players' strategies (the payoff associated with the node reached playing a certain strategy) and the actual payoff received, the less likely players are to pass the pot.

4.4.1 Fixed effects

To test this hypothesis, the following panel-data fixed effects regression model was estimated:

$$Y_{it} = \alpha_i + \beta_1 X_{i,t-1} + u_{it} \tag{1}$$

where Y_{it} is the number of times subject *i* passed in game *t*, $X_{i,t-1}$ is the error (the difference between the expected payoff consistent with the player's chosen strategy and the payoff actually received) of subject *i* in *t-1*, α_i is the unknown intercept for each subject, β_1 is the slope coefficient and u_{it} is the error term. With the subject-specific intercepts we can control for any unobserved heterogeneity (effects particular to each subject). The fixed effects model was selected as the use of the random effects model (generally preferred due to higher efficiency) was counter-indicated by the Hausman test, shown in Table 5 (Hausman, 1978).

Table 5. Hausman test performed in Stata

```
                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |     fixed        random       Difference          S.E.
-------------+----------------------------------------------------------------
  error_lag2 |   -.0623079    -.0396503        -.0226576        .0048083
------------------------------------------------------------------------------
                           b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)*
                          =       22.20
                Prob>chi2 =      0.0000
```

The null hypothesis of the Hausman test in this case is that both the random-effects and the fixed-effects estimators are consistent. We reject the null hypothesis at Prob>chi2 = 0,0000 and accept the alternative hypothesis that the random-effects estimator is inconsistent and therefore the fixed-effects model is preferred. The model was estimated in Stata, the results are shown in Table 6

Table 6. Fixed-effects model

```
Fixed-effects (within) regression               Number of obs      =       208
Group variable: subject                         Number of groups   =        38

R-sq:  within  = 0.0408                         Obs per group: min =         4
       between = 0.1503                                        avg =       5.5
       overall = 0.0039                                        max =         6

                                                F(1,169)           =      7.19
corr(u_i, Xb)  = -0.2026                        Prob > F           =    0.0081

------------------------------------------------------------------------------
        pass |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  error_lag2 |  -.0623079   .0232399    -2.68   0.008    -.1081859     -.01643
       _cons |   1.181543   .0544271    21.71   0.000     1.074098    1.288987
-------------+----------------------------------------------------------------
     sigma_u |  .40952337
     sigma_e |  .61704803
         rho |  .30578365   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(37, 169) =     2.31             Prob > F = 0.0002
```

Since the independent variable (the lagged difference between actual end-of-game and the strategy end-of-game) is not applicable to individuals' first games of a session, 10 games from the first session and 14 games from each of the two consequent sessions are omitted from this regression, resulting in the lower number of observations (n=208).

The number of times players passed the pot in a game varied, but on average, they passed 1,1815 times per game (with players 1 passing, on average, 1,3043 times per game and players 2 passing, on average, 1,0647 times per game). On the basis of the t-value (and its corresponding P-value of 0,008) we can conclude that our slope coefficient is different from zero. It implies that the as the error (the difference between the expected payoff consistent with the player's chosen strategy and the payoff actually received) in t-1 increases by 1€, the number of time players pass the pot in a game decreases, on average, by 0,0623.

The value of »rho« ($\rho = 0{,}30578365$), also known as the intraclass correlation, shows that 30,6% of the variance is due to differences across panels.

4.4.2 Mixed effects

The fixed effects model cannot be used to estimate time-invariant variables, but it does control for them with subject-specific intercepts. The slope coefficient is fit on the population level and does not vary from subject to subject. However, a random slope term could be introduced to the model, which would likely improve the fit on the subject level. To do this, we can estimate the following mixed effect (multi-level) model:

$$Y_{it} = \underbrace{\beta_0 + \beta_1 X_{i,t-1}}_{\text{FIXED EFFECT}} + \overbrace{b_{i0} + b_{i1} X_{i,t-1} + \varepsilon_{it}}^{\text{RANDOM EFFECT}} \tag{2}$$

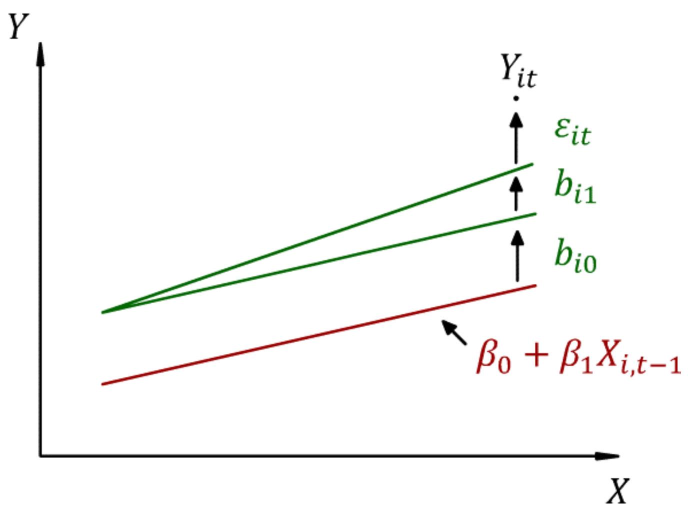
where Y_{it} is the number of times subject *i* passed in game *t*, $X_{i,t-1}$ is the error (the difference between the expected payoff consistent with the player's chosen strategy and the payoff actually received) of subject *i* in *t-1*, β_0 is the fixed effect intercept (the *grand mean*) and $\beta_1$is the fixed effect slope coefficient for $X_{i,t-1}$. b_{i0} is the random effect intercept, b_{i1} is the random slope coefficient for $X_{i,t-1}$ and ε_{it} is the error term. The model was estimated in Stata, using the maximum-likelihood approach, the results are shown in Table 7.

The number of times players passed in a game varied across subjects and games, but on average they passed 1,158 times per game; this is the grand mean (population-level mean). *sd(_cons)* is a measure of the between-subject variability for the intercept, the standard deviation of subject-specific intercepts from the grand mean was 0,289 passes per game. The fixed effects (population level) slope coefficient is statistically significantly different from zero (P=0,044), which means that all other things held constant, when the error in the previous game (*t-1*) increases by 1€, the number of times players pass in the game (*t*) decreases, on average, by 0,0486. *Sd(*error_~g*)*= 0.0460559 can be interpreted as the standard deviation of the random slope for subjects. Figure 14 shows the decomposition to fixed and random effects. Instead of retrieving individual random effects coefficients, the estimated random effects parameters represent the standard deviations of those coefficients assumed to be distributed normally with a mean of zero ($N \sim (0, \sigma^2)$). All random-effects parameters are statistically significantly different from zero at the *0.05* alpha level. The LR test's null hypothesis is that the random-effects equal zero, we reject that hypothesis at *Prob > chi2* = 0.0024.

Table 7. Mixed effect (multi-level) model

```
Mixed-effects ML regression                     Number of obs      =       208
Group variable: subject                         Number of groups   =        38

                                                Obs per group: min =         4
                                                               avg =       5.5
                                                               max =         6


                                                Wald chi2(1)       =      4.04
Log likelihood =  -209.6558                     Prob > chi2        =    0.0443

------------------------------------------------------------------------------
        pass |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   error_lag |  -.0485552    .024147    -2.01   0.044    -.0958825   -.001228
       _cons |   1.158009   .0711315    16.28   0.000     1.018594   1.297424
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
subject: Independent         |
                sd(error_~g) |   .0460559   .0414247      .0079009    .2684682
                   sd(_cons) |   .2890718   .0640319      .1872653    .4462252
-----------------------------+------------------------------------------------
                sd(Residual) |   .6091059   .0348761      .5444461     .681445
------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(2) =    12.06   Prob > chi2 = 0.0024
```

Figure 14. Decomposition to fixed and random effects



From the random effects parameters and the standard deviation of the residual, we can calculate the intraclass correlation (IC):

$$IC = \frac{\sigma^2_{\_cons} + \sigma^2_{error\_\sim g}}{\sigma^2_{\_cons} + \sigma^2_{error\_\sim g} + \sigma^2_{residual}} = \frac{0{,}0460559^2 + 0{,}2890718^2}{0{,}0460559^2 + 0{,}2890718^2 + 0{,}6091059^2} = 0{,}186 \qquad (3)$$

IC is a measure of the correlation of cases within a cluster (observations within a group – in our case, a subject). An IC close to zero would imply that observations are independent therefore there would be no need for a panel data treatment and a standard regression would suffice. A high IC (close to 1) means there is little variation of observations within each subject and differences between subjects are of more use for statistical inference (Snijders & Bosker, 2011, pp. 16-22).

Finally, we can control for the differences between players 1 and players 2 by nesting subjects within the two subject-types (the two subject-types can be seen as two different treatments, since each type is faced with different decisions and different payoffs in the game):

$$Y_{jit} = \beta_0 + \beta_1 X_{ji,t-1} + b_{j0} + b_{ji0} + b_{ji1} X_{ji,t-1} + \varepsilon_{jit} \qquad (4)$$

where Y_{it} is the number of times subject *i* of type *j* passed in game t, $X_{ji,t-1}$ is the error (the difference between the expected payoff consistent with the player's chosen strategy and the payoff actually received) of subject *i* of type *j* in *t-1*, β_0 is the fixed effect intercept and $\beta_1$is the fixed effect slope coefficient for $X_{ji,t-1}$. b_{ji0} is the subject random intercept, b_{ji1} is the subject random slope coefficient for $X_{i,t-1}$, b_{j0} is the type-of-subject random intercept and ε_{jit} is the error term. Type-of-subject random slope coefficient was omitted from the model as if was not statistically significantly different from zero. The model was estimated in Stata, using the maximum-likelihood approach, the results are shown in Table 8.

Table 8. Expanded mixed effect (multi-level) model

Mixed-effects ML regression | **Number of obs = 208**

Group Variable	No. of Groups	Observations per Group: Minimum	Average	Maximum
type	2	104	104.0	104
subject	38	4	5.5	6

	Wald chi2(1)	=	4.10
Log likelihood = -209.51719	Prob > chi2	=	0.0428

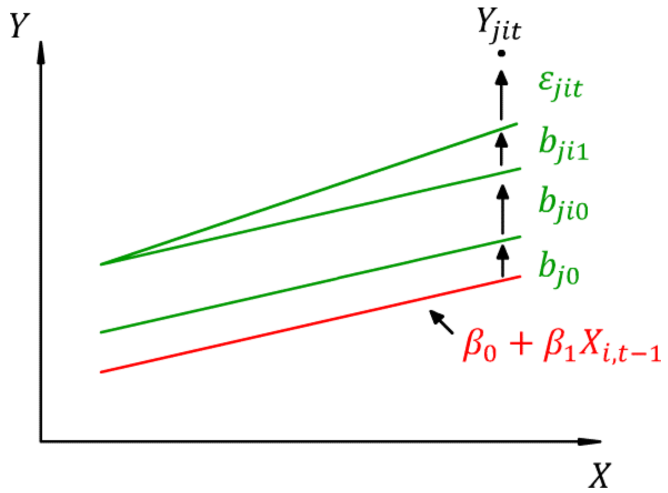
pass	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
error_lag	-.0485338	.0239562	-2.03	0.043	-.0954872	-.0015805
_cons	1.15864	.0869836	13.32	0.000	.9881552	1.329125

Random-effects Parameters	Estimate	Std. Err.	[95% Conf.	Interval]
type: Identity				
sd(_cons)	.0725073	.0929892	.0058713	.8954284
subject: Independent				
sd(error_~g)	.0434392	.0429416	.0062579	.3015313
sd(_cons)	.2807612	.0651892	.1781144	.4425631
sd(Residual)	.6097262	.034954	.5449262	.6822319

LR test vs. linear regression: chi2(3) = 12.33 Prob > chi2 = 0.0063

The inclusion of type-of-subject level seems to have very little overall effect on the model. The number of times players passed in a game varied across subjects and games, but on average they passed 1,159 times per game. The standard deviation of subject-specific intercepts from the fixed effect intercept, *sd(_cons),* was 0,281 passes per game. The standard deviation of type-specific intercepts from the fixed effect intercept, *sd(_cons),* was 0,0725 passes per game. The fixed effects (population level) slope coefficient is statistically significantly different from zero (P=0,043), which means that all other things held constant, when the error in the previous game (*t-1*) increases by 1€, the number of times players pass in the game (*t*) decreases, on average, by 0,0485. The standard deviation of the random slope for subjects, sd(error_g), was 0,0434. All random-effects parameters are statistically significantly different from zero at the *0.05* alpha level. We reject the LR test's null hypothesis that random-effects equal zero at *Prob > chi2* = 0.0063. Figure 15 shows the decomposition of the second mixed model to fixed and random effects.

Figure 15. Decomposition to fixed and random effects (second model)



4.4.3 Ordered logit model

A slightly different approach in analyzing the collected data would be to directly examine the adjustments in strategies players made after each game. As each of the players was made aware of the actual amount of money they had won in the game compared to what they were expecting to make based on their own choices, each player could make one of three strategic choices:

- decrease cooperation by taking the pot sooner in the game
- continue with the current strategy
- increase cooperation by taking the pot later in the game (or not at all)

Based on the results of the panel data models we would expect to see a correlation between the errors players made in the previous game and the change in cooperation in the current game. Table 9 and Table 10 show the cross-tabulated results for each type of player as well as the relative frequencies of particular changes of strategy for each possible value of error. The numbers in the *change in cooperation* columns correspond to the difference between the number of times the player passed the pot in the current game and the number of times a player passed the pot in the previous game. For instance, a player changing their strategy from passing the pot twice to passing the pot only once corresponds to a change in cooperation of -1.

Table 9. Frequencies of strategic choices of player 1

	Change in cooperation					
Error (€)	-2	-1	0	1	2	Total
6,2	2	6	1	0	0	9
5,6	2	10	6	0	0	18
1,4	0	2	6	7	0	15
0,0	0	9	31	20	2	62
Total	4	27	44	27	2	104

	Change in cooperation				
Error (€)	-2	-1	0	1	2
6,2	22,2%	66,7%	11,1%	0,0%	0,0%
5,6	11,1%	55,6%	33,3%	0,0%	0,0%
1,4	0,0%	13,3%	40,0%	46,7%	0,0%
0,0	0,0%	14,5%	50,0%	32,3%	3,2%
Overall	3,8%	26,0%	42,3%	26,0%	1,9%

Table 10. Frequencies of strategic choices of player 2

	Change in cooperation					
Error (€)	-2	-1	0	1	2	Total
3,1	0	4	2	1	0	7
2,8	0	10	12	8	0	30
1,5	0	4	1	0	0	5
1,2	3	2	3	0	0	8
0,7	0	0	0	1	0	1
0,0	0	9	25	14	5	53
Total	3	29	43	24	5	104

	Change in cooperation				
Error (€)	-2	-1	0	1	2
3,1	0,0%	57,1%	28,6%	14,3%	0,0%
2,8	0,0%	33,3%	40,0%	26,7%	0,0%
1,5	0,0%	80,0%	20,0%	0,0%	0,0%
1,2	37,5%	25,0%	37,5%	0,0%	0,0%
0,7	0,0%	0,0%	0,0%	100,0%	0,0%
0,0	0,0%	17,0%	47,2%	26,4%	9,4%
Overall	2,9%	27,9%	41,3%	23,1%	4,8%

Both types of players seem to exhibit the same basic pattern of being more likely to increase cooperation after making a smaller error than after making a large error. For further analysis, the data for both types of players was combined and the variable »change in cooperation« was reduced to only three categories (increase, same, decrease):

Table 11. Cross-tabulated results

	cooperation			
error_lag	decrease	same	increase	Total
6.2	8	1	0	9
5.6	12	6	0	18
3.1	4	2	1	7
2.8	10	12	8	30
1.5	4	1	0	5
1.4	2	6	7	15
1.2	5	3	0	8
.7	0	0	1	1
0	18	56	41	115
Total	63	87	58	208

```
Pearson chi2(16) =  59.3835   Pr = 0.000
```

Pearson's chi-squared test conducted in Stata confirms the association between the magnitude of the error in the previous game and the change in strategy in the current game, however, since the test is non-directional, it gives no indication as to any correlation between the two. To estimate the relationship between the ordered dependent variable, change in cooperation, and the independent variable, error in the previous game, an ordered logit model (a generalization of the binary logit model) can be utilized using Stata's *ologit* command, the results are shown in Table 12.

Table 12. Ordinal logit model

```
Ordered logistic regression                       Number of obs   =        208
                                                  LR chi2(1)      =      37.69
                                                  Prob > chi2     =     0.0000
Log likelihood = -206.30498                       Pseudo R2       =     0.0837

------------------------------------------------------------------------------
cooperation2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  error_lag2 |  -.4259064   .0744753    -5.72   0.000    -.5718752   -.2799375
-------------+----------------------------------------------------------------
       /cut1 |  -1.538469   .2056002                     -1.941437     -1.1355
       /cut2 |   .5032041    .173134                      .1638678    .8425405
------------------------------------------------------------------------------
```

The lagged error variable is expressed in absolute terms. The coefficient is the ordered log-odds estimate for a 1€ increase in the error in the previous game on the expected change in cooperation in the current game. A 1€ increase in the error in the previous game would result in a 0.4259 unit decrease in the ordered log-odds of being in a »higher category« of cooperation (choosing to stick to the same strategy instead of decreasing cooperation, for instance). *Cut1* and *Cut2* are the estimated cutpoints on the latent variable used to differentiate between the categories of the dependent variable. This can be expressed as:

$$S_j = -0{,}4259064 \cdot error\_lag2_j + u_j \quad (5)$$

For instance, for an error of 1€ in the previous game, the probability of choosing to decrease cooperation can be calculated as follows:

$$-0{,}4259064 + u_j \leq -1{,}538469 \quad (6)$$

$$u_j \leq -1{,}1125626 \quad (7)$$

Since the error term is assumed to be logistically distributed, the probability of $u_j \leq -1{,}1125626$ is:

$$Pr(u_j \leq -1{,}1125626) = \frac{1}{1 + e^{1{,}113}} = 0{,}2473 \tag{8}$$

Table 13. Estimated and actual relative frequencies of strategic choices for particular values of errors

	Change in cooperation (model)			**Change in cooperation (data)**			**Number of observations**
Error (€)	Decrease	Same	Increase	Decrease	Same	Increase	
6,2	75%	21%	4%	89%	11%	0%	9
5,6	70%	25%	5%	67%	33%	0%	18
3,1	45%	42%	14%	57%	29%	14%	7
2,8	41%	43%	16%	33%	40%	27%	30
1,5	29%	47%	24%	80%	20%	0%	5
1,4	28%	47%	25%	13%	40%	47%	15
1,2	26%	47%	27%	63%	38%	0%	8
0,7	22%	47%	31%	0%	0%	100%	1
0	18%	45%	38%	16%	49%	36%	115

Corresponding odds ratios can also be derived by using the *or* option after the *ologit* command, the results are shown in Table 14.

Table 14. Odds ratios

```
Ordered logistic regression                       Number of obs   =        208
                                                  LR chi2(1)      =      37.69
                                                  Prob > chi2     =     0.0000
Log likelihood = -206.30498                       Pseudo R2       =     0.0837

------------------------------------------------------------------------------
cooperation2 | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  error_lag2 |   .6531775   .0486456    -5.72   0.000     .5644659     .755831
-------------+----------------------------------------------------------------
       /cut1 |  -1.538469   .2056002                     -1.941437     -1.1355
       /cut2 |   .5032041    .173134                      .1638678    .8425405
------------------------------------------------------------------------------
```

For a 1€ increase of the error in the previous game, the odds of increased cooperation versus the two other strategic choices are 0,653 times lower. Similarly, for a 1€ increase of the error in the previous game, the odds of decreased cooperation versus the combined odds of the two other strategic choices are also 0,653 lower. This is the consequence of the proportional odds assumption used in the model, which assumes that the relationship between each pair of outcome groups is the same. To test the proportional odds assumption, two additional methods can be used, the Brant test (using the *brant* command) and a likelihood ratio test (using the *omodel* command), the results of the tests are shown in Tables 15 and 16.

Table 15. Brant test

Brant Test of Parallel Regression Assumption:

Variable	chi2	p>chi2	df
All	0.30	0.585	1
error_lag2	0.30	0.585	1

Table 16. Likelihood ratio test

```
Approximate likelihood-ratio test of proportionality of odds
across response categories:
         chi2(1) =      0.18
       Prob > chi2 =    0.6701
```

Both results are non-significant, indicating that the proportional odds assumption has not been violated. Finally, to test whether or not players' responses were constant throughout the game, an expanded model including a variable *game* (representing the iteration of the game having been played) is estimated, the results are shown in Table 17.

Table 17. Expanded ordinal logit model

```
Ordered logistic regression                       Number of obs   =        208
                                                  LR chi2(2)      =      38.58
                                                  Prob > chi2     =     0.0000
Log likelihood = -205.85998                       Pseudo R2       =     0.0857
```

cooperation2	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
error_lag2	-.4280986	.0744624	-5.75	0.000	-.5740423	-.2821549
game	.0767	.0813897	0.94	0.346	-.082821	.2362209
/cut1	-1.215998	.3967525			-1.993619	-.4383775
/cut2	.8333136	.391074			.0668226	1.599805

The coefficient corresponding to the *game* variable is not statistically significant, indicating that the relative frequencies of strategic choices for particular values of errors did not change with the number of games players played. In other words, players were just as likely to increase/decrease cooperation after observing a particular result in their first game as they were in their last game.

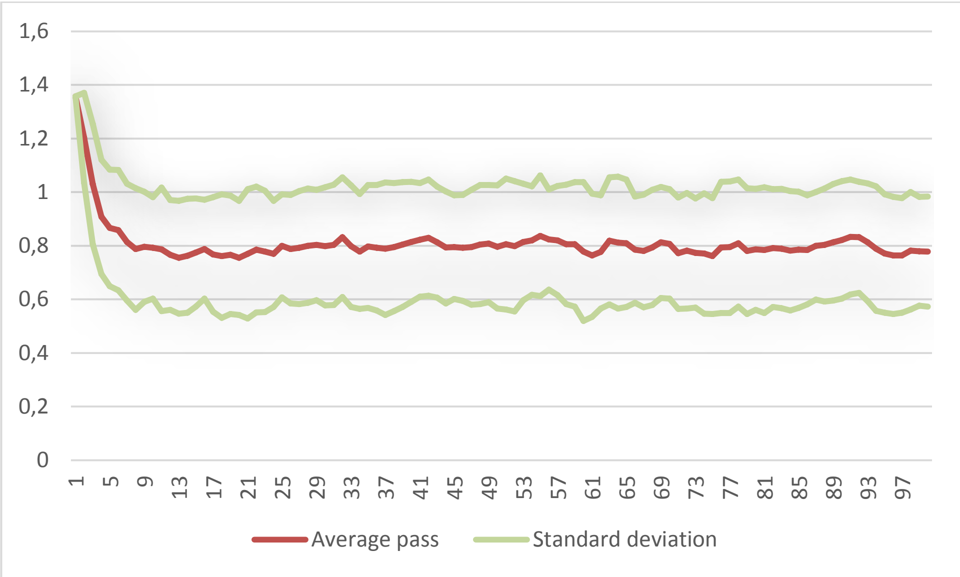
The main advantage of the ordered logit model over the panel data models used earlier is the proper treatment of the ordinal variable – the exact values the ordinal variable takes are irrelevant, but higher values are assumed to correspond to higher outcomes (in this case, increasing levels of cooperation). The downside of this model is that, unlike the panel data models, it treats observations from the same individual as independent.

4.5 Convergence to equilibrium

Using the figures in Table 9 and Table 10 it is possible to at least approximately simulate the game over a large number of iterations, which would otherwise not be feasible due to numerous real-world constraints. In every subsequent iteration of the game players know the magnitude of their errors in the previous iteration and attempt a change in strategy consistent with the relative frequencies in Table 9 and Table 10. If the attempted change in strategy is not possible (for instance, if a player attempts to decrease cooperation when his previous strategy was already to never pass the pot), no change in strategy is made (this is why the simulated relative frequencies are not completely identical to the relative frequencies that have been empirically established). In each iteration players of one type are randomly paired with players of the other type and the output of the previous iteration (error) is used to determine players' actions in the current iteration. The entire code used to run the simulation can be viewed in Appendix B, the results of the simulation are shown in Figure 16.

After each iteration, the average number of times players passed the pot is calculated and all 100 iterations of the game are repeated 100 times. The red line represents the mean value of the 100 simulations at each iteration, while the green lines show the standard deviation of those 100 simulations. The jump in the beginning of the graph is due to the fact that the initial (first iteration) results of the game are taken from the empirical results of the first game of session 2. The starting values are therefore the same for each of the 100 simulations and there is no variability. Other than that, the variability is fairly constant throughout all 100 iterations, meaning that, based on the simulation, the results of a game are just as (un)predictable after 100 iterations as they are after 7 iterations and there seems to be no convergence to an equilibrium, let alone to an equilibrium based on the predictions of game theory.

Figure 16. Results of the simulation

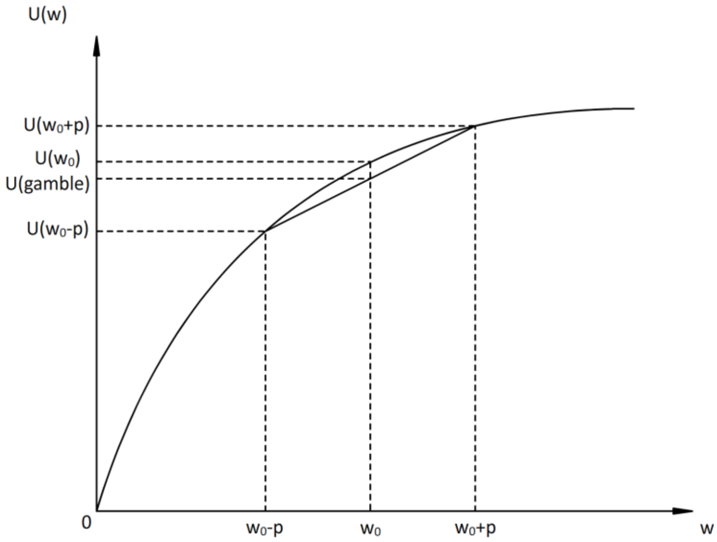


5 ATTITUDES TOWARD RISK

5.1 Introduction to attitudes toward risk

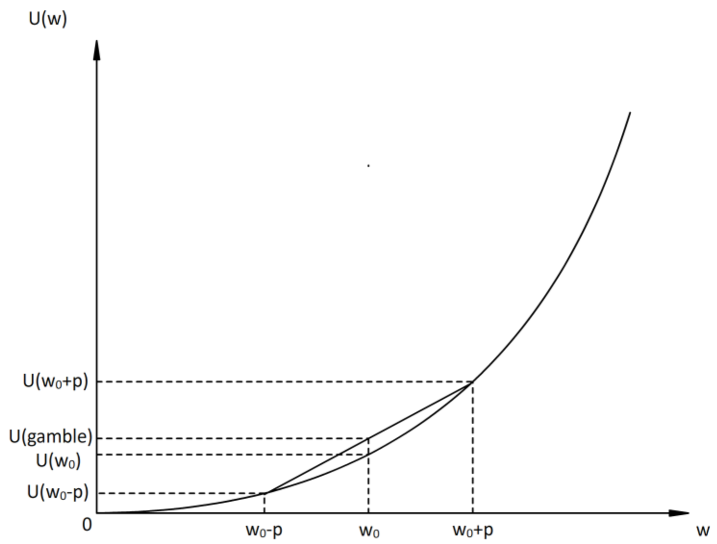
After each game, subjects were individually offered the choice to gamble their winnings (in that game) on a flip of a coin. Those who accepted would call out either "heads" or "tails" while the experimenter flipped the coin. If they won, their winnings in that game would double, if their guess was incorrect, they lost their winnings in that game. The frequency of a fair coin landing on either side is 0,5 so the expected value of this gamble, irrespective of players' strategies, is zero. When dealing with attitudes toward risk, economists generally put individuals in one of three categories: risk loving, risk averse and risk neutral (McCarty & Meirowitz, 2006, pp. 28-40). Risks averse are those individuals who, when given a choice between a gamble and the expected value of that gamble, choose the latter. This attitude can be explained by assuming a concave utility function shown in Figure 17.

Figure 17. Concave utility function



Conversely, a risk-taking individual would prefer the gamble over the expected value of the gamble. This does not take into account any utility one might gain from the act of gambling itself, it is simply a consequence of a convex utility function – higher money amounts offer this individual disproportionately higher utility, therefore the expected value of the gamble undervalues higher money amounts in terms of this individual's utility.

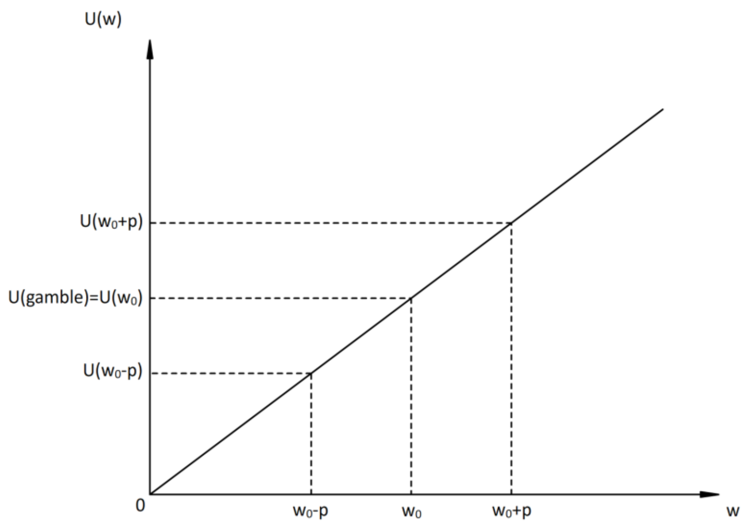
Figure 18. Convex utility function



A risk neutral individual is completely indifferent between a gamble and a payoff equivalent to the expected value of a gamble, his utility as a function of money can therefore be represented as a straight line (Figure 19).

However, these three simple categories are insufficient to fully explain an individual's attitude toward risk as this will likely vary as money amounts increase (an individual might be risk-taking when smaller amounts of money are involved and might exhibit risk-aversion when stakes increase), when the probabilities of different outcomes in a gamble are distributed, etc.

Figure 19. Linear utility function



5.2 Attitudes toward risk experimental results

Table 19 shows the conditional relative frequencies of accepting a bet at a given payoff amount. Clearly there is a negative correlation between payoff amounts and the relative frequency of accepting a bet. Furthermore, the scatterplot diagram in Figure 20 reveals a non-linear relationship between the two variables, an exponential curve fitted in SPSS confirms there is an exponential increase in conditional relative frequencies of accepted bets as money amounts decline.

The results of fitting the log-linearized exponential function $\ln \hat{y}_i = \ln a + x_i \ln b$ in Stata are shown in Table 18.

Table 18. Exponential function

```
      Source |       SS       df       MS              Number of obs =       7
-------------+------------------------------           F(  1,     5) =   72.19
       Model |  4.00268896     1  4.00268896           Prob > F      =  0.0004
    Residual |  .277233617     5  .055446723           R-squared     =  0.9352
-------------+------------------------------           Adj R-squared =  0.9223
       Total |  4.27992258     6   .71332043           Root MSE      =  .23547

------------------------------------------------------------------------------
 lnfrequency |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      payoff |  -.3560807   .0419093    -8.50   0.000     -.463812   -.2483494
       _cons |   -.315195    .117057    -2.69   0.043    -.6160997   -.0142902
```

The slope coefficient is significantly different from zero (at P=0,000). Through anti-logaritmazation, we can obtain the regression coefficients of the original exponential function:

$$\widehat{frequency}_i = \hat{\alpha}\hat{\beta}^{payoff_i} = 0{,}730 \cdot (0{,}700)^{payoff_i} \tag{9}$$

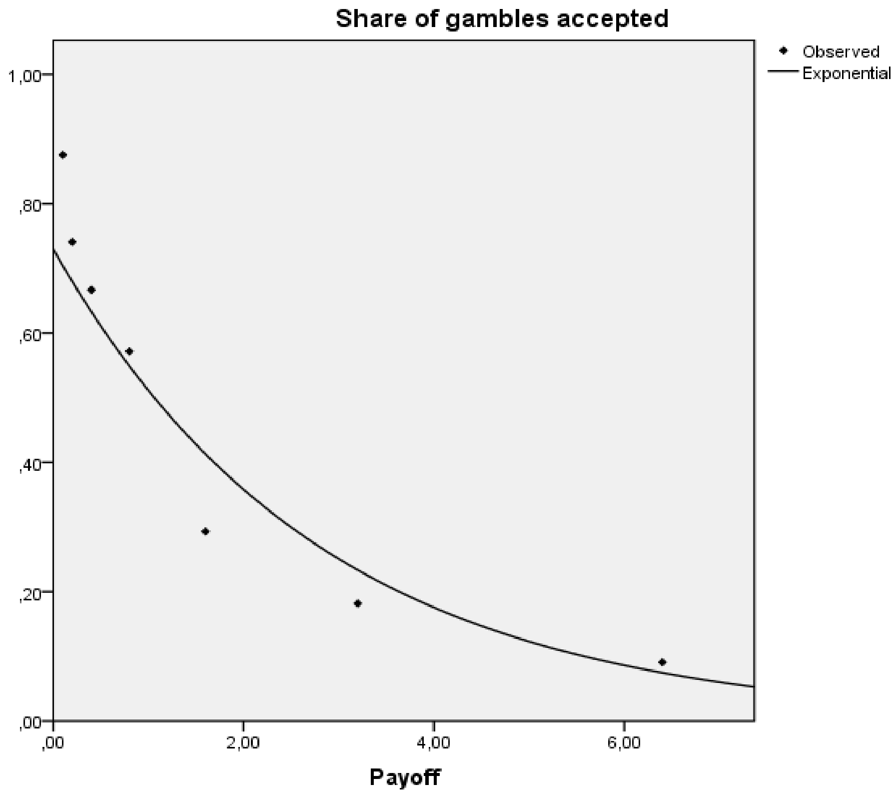
Based on our sample, the conditional relative frequency of accepting a gamble, on average, decreases by 30%, when payoffs increase by \$1.

Table 19. Conditional relative frequencies of accepting a bet

Payoff	**6,40 €**	**3,20 €**	**1,60 €**	**0,80 €**	**0,40 €**	**0,20 €**	**0,10 €**
Times payoff wagered	1	4	17	28	42	20	14
Times payoff reached	11	22	58	49	63	27	16
Relative frequency	0,09091	0,18182	0,2931	0,57143	0,66667	0,74074	0,875

Putting individuals in one of the three basic categories according to their attitudes toward risk (risk-loving, risk-averse and risk-neutral) clearly does a poor job of explaining these results. A risk-averse person would never accept a fair gamble and a risk-loving individual would never pass up on one, while the risk-neutral player would simply *flip a coin* as to whether or not to flip the coin. If this were the case, the distribution of conditional relative frequencies of accepting the bet would, to a large extent, mirror the relative probability of a risk-loving player reaching node *i* in a particular game (although these would not necessarily be identical due to the presence of risk-neutral players). This would imply that the vast majority of players to reach the last node in a game were risk-averse while the majority of players deciding to *take* in early stages were risk-loving, the exact reverse of what one might expect. It seems much more likely that attitudes toward risk are not constant, but rather a function of monetary amounts.

Figure 20. Curve fit in SPSS



These results are consistent with similar experiments dealing with attitudes toward risk when stakes increase. Holt and Laury (2002) found that there was a sharp increase in risk aversion when stakes were scaled up by factors 20, 50 and 90 from the initial low payoff of a couple of dollars and that a large proportion of subjects exhibited risk aversion even with low stake. Similar conclusions can be drawn from Kachelmeier and Shehata (1992) and Binswanger (1981).

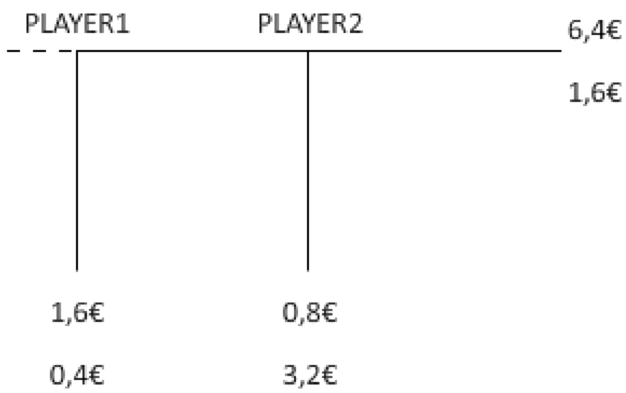
5.3 Risk aversion and the centipede game

The lagged differences between expected payoffs and actual payoffs go far in explaining the evolution of players' strategies throughout a session, but provide no insight into players' average tendencies. Based on the results of the experiment it is clear that following standard game theory assumptions, in particular common knowledge of instrumental rationality, which leads to the backward induction solution discussed in Chapter 1, would be of little use in explaining players' motivations. While a portion of the subjects might have altruistic tendencies, it is probably not reasonable to assume an overwhelming majority of them would pass the pot to their opponents and forgo a higher payoff if they believed with certainty that their opponents would always *take*. On the other hand, passing the pot to one's opponent could easily be justified if one believed there was a high-enough probability of their opponent making a "mistake", passing the pot back to them. For example, player 1 making a decision in node 3 of the game could perform a simple odds calculation to determine (at least) how often his opponent would have to *pass* in his next move to make player 1's strategy of passing profitable. If player 1 *takes*, he is guaranteed 1,6€, if he *passes*, he gets 6,4€ with probability *x* and 0,8€ with probability *1-x*, where *x* is the probability of player 2 *passing*.

$$1{,}6 \leq 6{,}4x + 0{,}8(1-x) \quad \Rightarrow \quad x \geq 0{,}143 \tag{10}$$

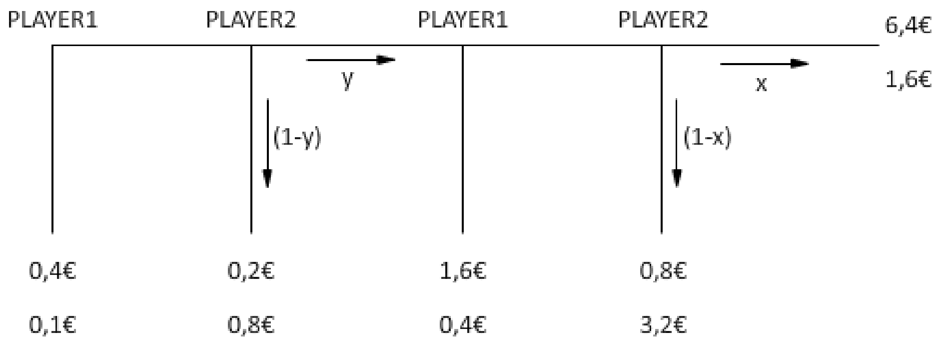
Player 2 would have to pass at least 14,3% of the time to make player 1's strategy of passing in decision node 3 profitable on average. This is assuming the risk neutrality of player 1.

Figure 21. Second half of the centipede game



In the above example the player only had to consider a single action from his opponent after which the game would end, but the same reasoning could be applied for the entire game. Based on their beliefs regarding the probabilities of their opponents *passing* on specific decision nodes, players could decide on the appropriate strategy to maximize their payoffs. The calculation in (10) has shown that player 1's decision to pass in node 3 is profitable only if player 2 decides to pass in node 4 more than 14,3% of the time. However, this explains little about player 1's overall strategy, as it is not clear whether or not node 3 will ever be reached at all. Figure 22 shows the structure of the entire game from the viewpoint of player 1, y and x are the probabilities that player 2 decides to pass in node 2 and node 4 respectively.

Figure 22. Structure of the game from player 1's perspective



To simplify the analysis, we can first assume that the probability of player 2 making a mistake is constant, such that $x = y$. Because the ratio between player 1's payoffs if player 2 decides to take or to pass is the same in decision nodes 2 and 4 (1:3), this implies that the only feasible strategies for player 1 are *PP* (if $y = x < 14{,}3\%$) or *TT* if ($y = x \geq 14{,}3\%$). If $x < 14{,}3\%$, then there is no $x = y$, such that:

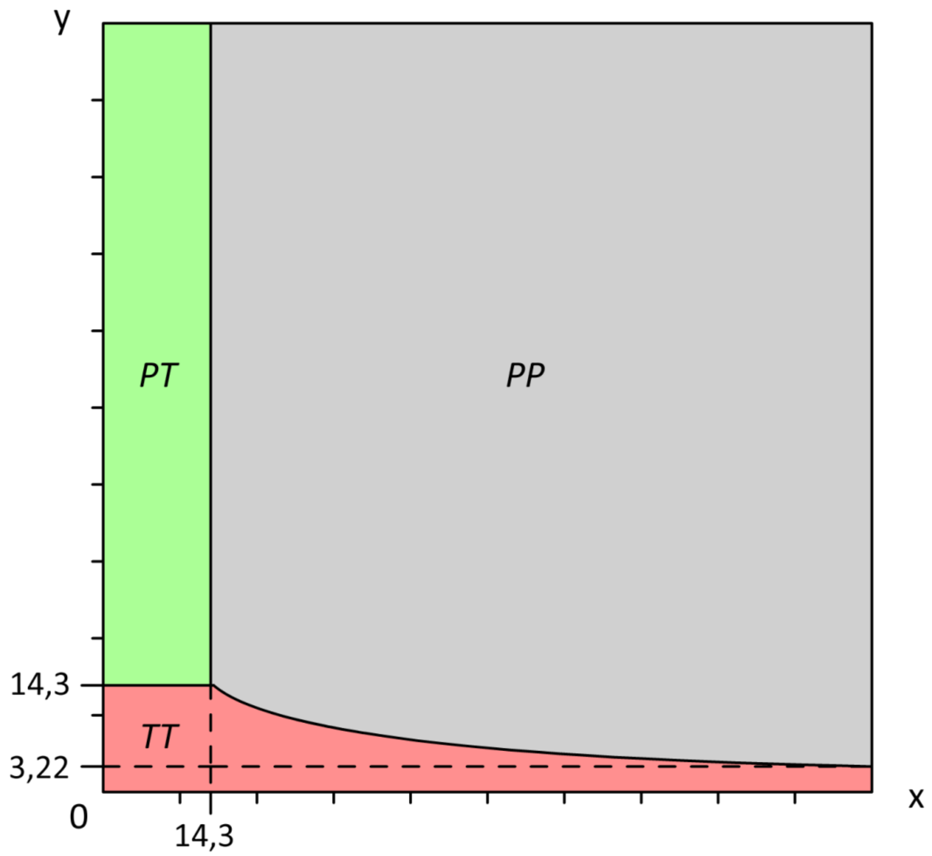
$$0{,}4 \leq (1 - y)0{,}2 + 1{,}6y \;\Rightarrow y \geq 14{,}3\% \qquad (11)$$

There is no reason to assume player 2 would be just as likely to pass in decision node 4, where he is directly giving up half his payoff to quadruple player 1's payoff, as he is to pass in decision node 2, where he has a real prospect of gaining greater payoff later in the game. Figure 23 shows player 1's most profitable strategies as a function of probabilities x and y of player 2 passing the pot. Clearly, if both probabilities are greater than 14,3%, passing on both turns (*PP*) is most profitable, whereas taking on player 1's first turn (*TT*) is most profitable if both x and y are lower than 14,3%. Passing once and taking on the next turn (*PT*) is the most profitable strategy when $x < 14{,}3\%$ and $y > 14{,}3\%$. If the probability of player 2 passing on his first turn is lower than 14,3%, while the probability of him passing on his next turn is higher than 14,3%, either *PP* or *TT* are the most profitable strategies, depending on whether or not the lower probability of player 1 reaching node 3 is offset by the higher probability of him subsequently reaching node 5.

The curve separating *PP* and *TT* (for $x > 14{,}3\%$) can be described by the following equation:

$$y = \frac{0{,}2}{x \cdot 6{,}4 + (1 - x) \cdot 0{,}8 - 0{,}2} \approx 0{,}032 \cdot x^{-0{,}78} \tag{12}$$

Figure 23. Player 1's most profitable strategies as a function of probabilities x and y of player 2 passing the pot

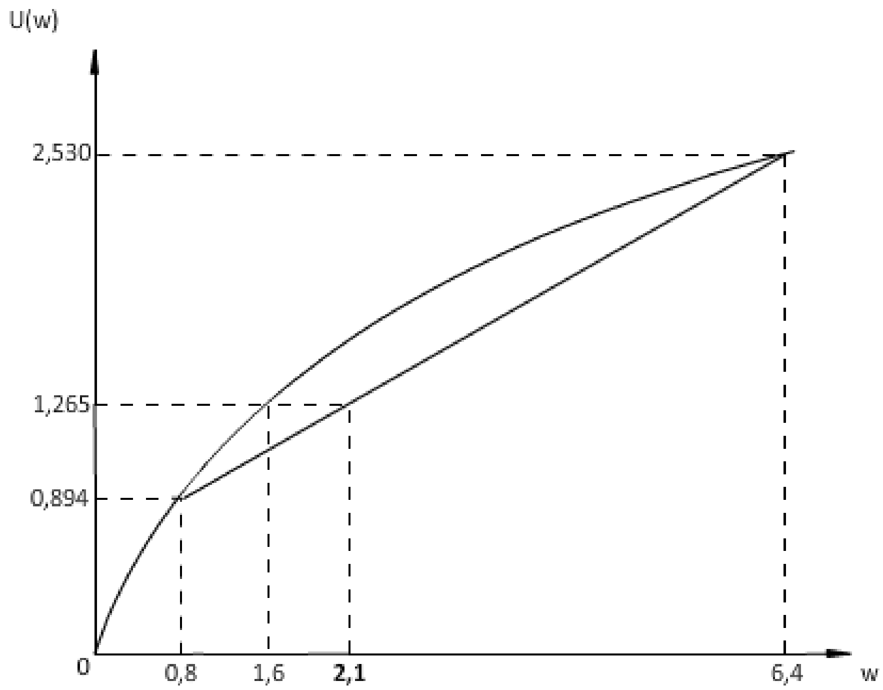


However, calculating whether or not a play is profitable in terms of expected payoffs is only part of the problem. In the first example, player 1 was faced with a choice between a certain payoff of 1,6€ and a lottery between 6,4€ with probability *x* and 0,8€ with probability *(1-x)*. Depending on the player's attitude toward risk, their certainty equivalent value – a payoff such that the player would be indifferent between it and the lottery – could be lower of higher than 1,6€. A more risk-averse individual will prefer a certain payoff of 1,6€ to a lottery between 6,4€ and 0,8€ with the expected value of 1,6€, therefore he will require a higher probability *x* of player 2 passing (higher that the calculation in (10) suggests) to forgo *taking* the pot immediately. Conversely, a risk-loving individual will prefer the lottery to a certain payoff in the amount of the expected value of the lottery. For example, we could consider a risk-averse individual with a VNM utility function of;

$$U(w) = \sqrt{w}, \tag{13}$$

where w are the individual's winnings in a game.

Figure 24. Certainty equivalent of a risk-averse player



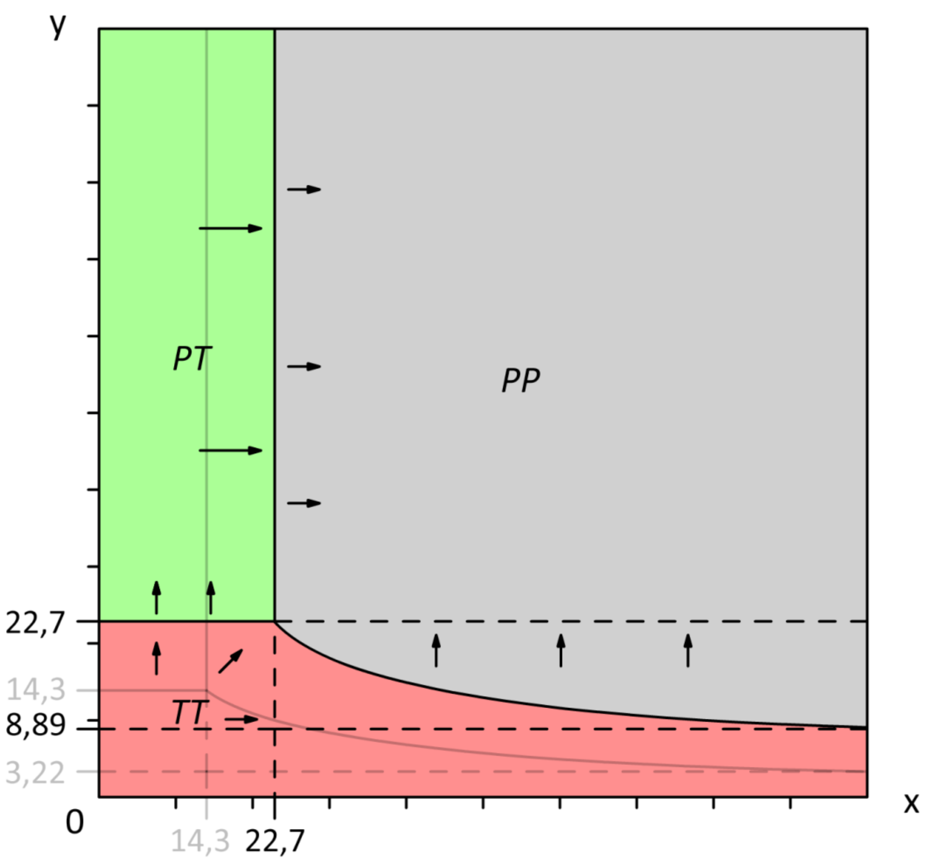
When considering *passing* the pot, this individual might calculate the probability *x* of player 2 passing where he would be indifferent between a certain payoff of 1,6€ and a lottery between 6,4€ with probability *x* and 0,8€ with probability *(1-x),* such that:

$$\sqrt{1{,}6} = x\sqrt{6{,}4} + (1-x)\sqrt{0{,}8} \quad \Longrightarrow \quad x = 0{,}227 \tag{14}$$

In this individual's case, the probability of player 2 passing the pot would have to be higher than 22,7% in order for them to consider the strategy of passing. Given this value of *x*, the expected value of the lottery is 2,07€ and 1,6€ is the certainty equivalent of the lottery.

Figure 25 shows the effect this calculation has on the distribution of chosen strategies by player 1. The area representing strategy PP shrinks, while the areas representing strategies TT and PT grow. This result shows that, on average, a risk averse player should be less likely to pass the pot to their opponent.

Figure 25. Distribution of utility-maximizing strategies of a risk averse individual



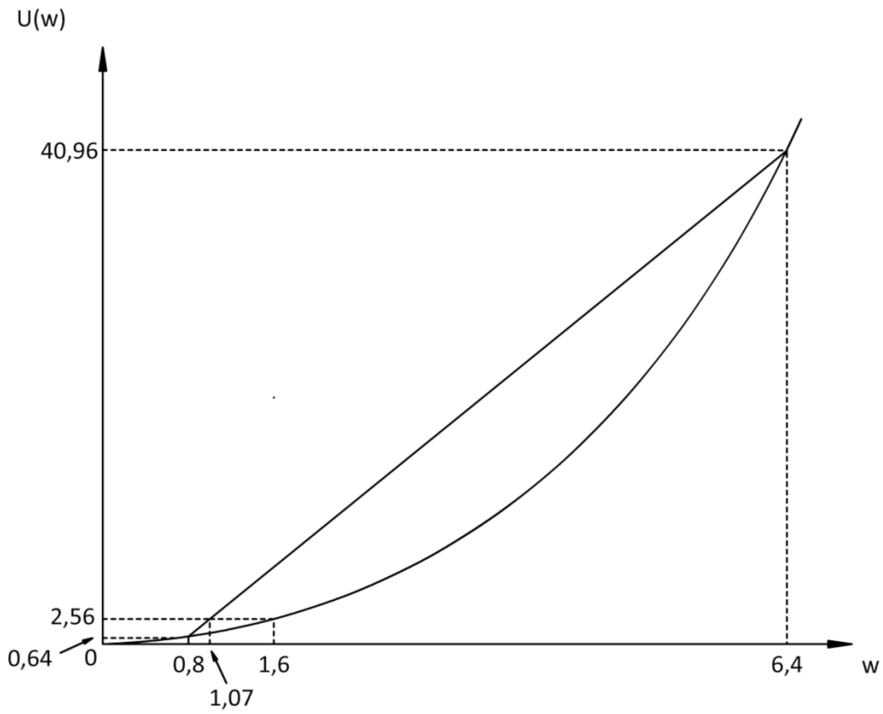
A risk-loving individual will prefer the lottery to a certain payoff in the amount of the expected value of the lottery, so a risk-loving player's utility could be represented by a quadratic utility function:

$$U(w) = w^2 \tag{15}$$

Again, when considering *passing* the pot, this individual might calculate the probability *x* of player 2 passing where he would be indifferent between a certain payoff of 1,6€ and a lottery between 6,4€ with probability *x* and 0,8€ with probability (1−*x*), such that:

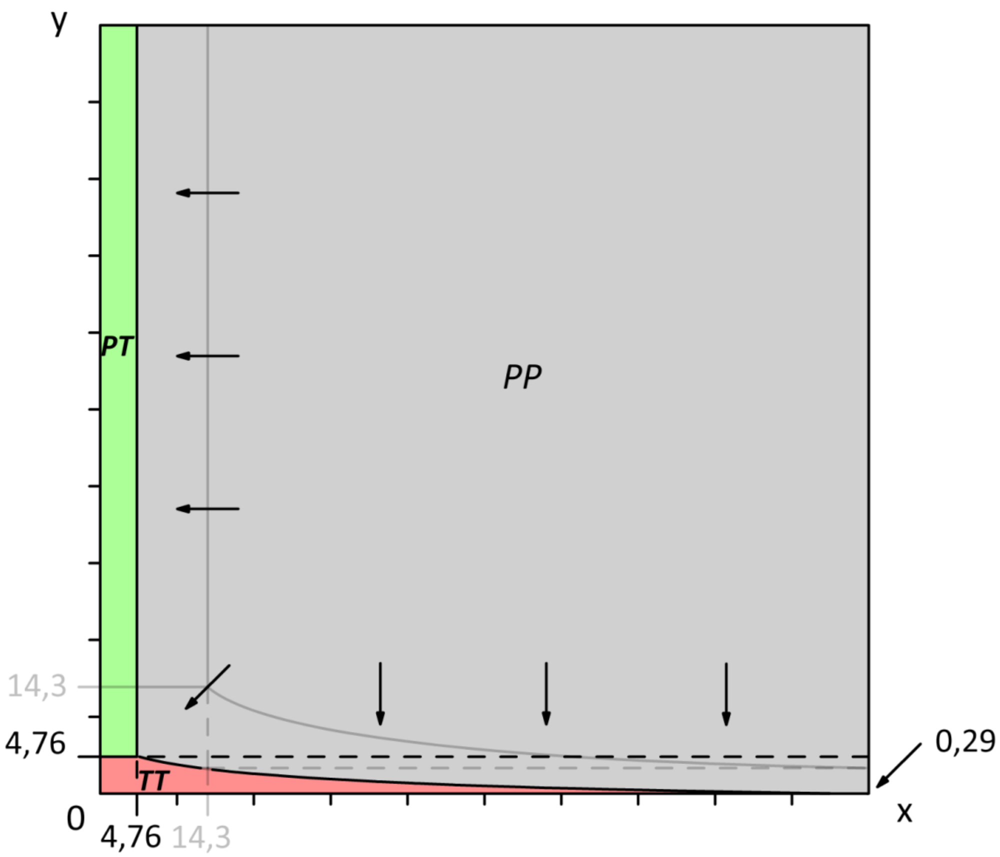
$$1{,}6^2 = x \cdot 6{,}4^2 + (1 - x) \cdot 0{,}8^2 \quad \Rightarrow \quad x = 0{,}047619 \tag{16}$$

Figure 26. Certainty equivalent of a risk-loving player



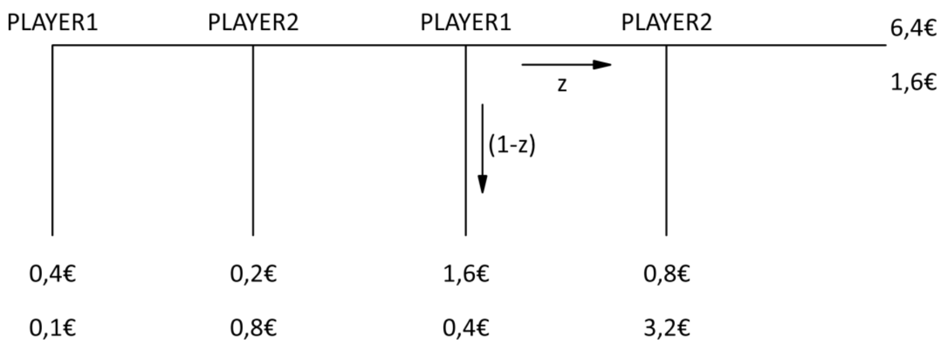
In this individual's case, the probability of player 2 passing the pot would have to be higher than 4,76% in order for him to consider the strategy of passing. Given this value of *x*, the expected value of the lottery is 1,066€ and 1,6€ is the certainty equivalent of the lottery. Figure 26 shows the effect this calculation has on the distribution of chosen strategies by player 1. The area representing strategy PP increases, while the areas representing strategies TT and PT decrease. This result shows that, on average, a risk-loving player should be more likely to pass the pot to their opponent.

Figure 27. Distribution of utility-maximizing strategies of a risk loving individual



Player 2 has a similar decision on his first turn – what is the optimal strategy if he believes player 1 will pass the pot on his next turn a certain percentage (*z*) of the time? On player 2's second turn, however, his strategic choice is clear – assuming he's trying to maximize his payoff, he'll simply take the pot and collect 3,2€.

Figure 28. Structure of the game from player 2's perspective



A risk neutral player 2 can choose to pass in node 2 if the following equation holds:

$$0{,}8 \leq (1 - z) \cdot 0{,}4 + z \cdot 3{,}2 \qquad \Longrightarrow \quad z \geq 0{,}143 \tag{17}$$

This is exactly the same probability as probability *x* of player 2 passing in node 4 at which a risk-neutral player 1 switches his optimal strategy in node 3 from taking to passing. And the same considerations regarding attitudes toward risk apply to player 2. A risk averse player with a utility function $U(w) = \sqrt{w}$ will require higher probability *z* to pass on his first turn:

$$\sqrt{0{,}8} \leq (1 - z) \cdot \sqrt{0{,}4} + z \cdot \sqrt{3{,}2} \qquad \Longrightarrow \quad z \geq 0{,}227 \tag{18}$$

A risk-loving player 2 with a utility function $U(w) = w^2$ will require lower probability *z* to pass in node 2:

$$0{,}8^2 \leq (1 - z) \cdot 0{,}4^2 + z \cdot 3{,}2^2 \qquad \Longrightarrow \qquad z \geq 0{,}0476 \tag{19}$$

The only feasible strategies for a payoff-maximizing player 2 who believes that player 1 will make a “mistake” (not selecting take in node 3) with a certain probability *z* are *TT* and *PT*. There is no *z* at which player 2, regardless of his attitude toward risk, should choose *PP* in order to maximize payoff. In fact, player 2 acts as if his first turn were effectively his last turn, giving him only one viable opportunity to pass the pot. Therefore, there should be a clear distinction between how many times per game player 1 and player 2, on average, pass the pot.

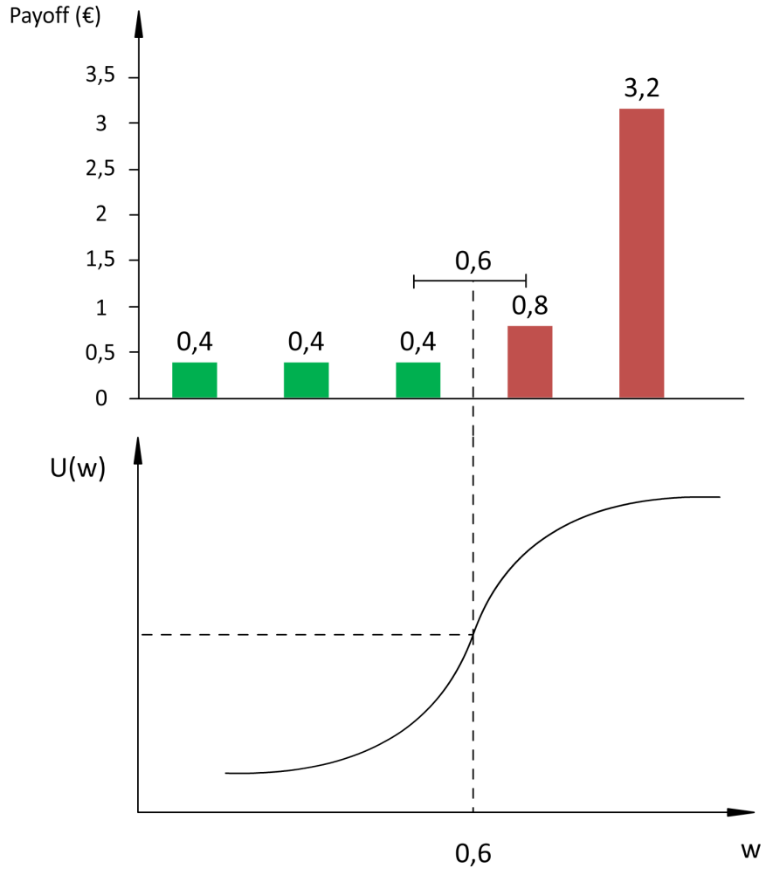
5.4 Modeling risk aversion

Given the analysis in [5.3] it might not be unreasonable to assume that (more) risk-averse individuals are, on average, less likely to *pass* the pot. However, testing this hypothesis is made less straightforward by the fact that directly quantifying a measure of an individual’s risk-aversion is difficult. Furthermore, if risk-aversion is a function of monetary amounts as discussed in [5.2], this non-linear relationship has to be taken into account.

The data collected in our experiment shows that only 2 out of 38 subjects (5%) never accepted the lottery, 4 out of 38 subjects (10%) always accepted the lottery and the vast majority of subjects sometimes accepted the lottery (85%). Considering the nature of the three standard categories of attitudes toward risk, we could conclude that *at least* 85% of the subjects in our experiment were risk-neutral, *at most* 5% were risk-averse and *at most* 10% were risk-loving. The reason these values cannot be established exactly is that a risk-neutral person could theoretically always accept the lottery or always reject it. But if 85% of our subjects were truly risk-neutral and their accepting or rejecting the lottery were unaffected by payoff amounts, we would expect a fairly equal distribution of relative frequencies of accepting the lottery, which is not consistent with the data.

Appendix C shows individual players' payoffs, ordered from lowest to highest. Red columns denote instances in which the lottery was rejected by the subject and green columns show instances where the lottery was accepted. By examining individuals' decisions we can approximately determine the payoff at which a subject changes from risk-seeking behavior to risk-averse behavior, *point of indifference* – if one exists at all. We can assume that the lower the value at which a subject is still indifferent between a fair gamble and a certain payoff of the expected value of that gamble, the more risk averse an individual is on average. Figure 29 shows an example of determining a player's point of indifference as the arithmetic mean between the highest payoff wagered and the lowest payoff at which the lottery was rejected.

Figure 29. Point of indifference



While it is not possible to derive a comprehensive measure of risk aversion in this way, it is still possible to establish a rough outline of players' attitudes toward risk. The majority of subjects display quite a clear switch from risk-taking to risk-averse behavior even on such a narrow interval of relatively low payoffs. Table 20 shows the proposed indifference points for all 38 subjects.

Table 20. Indifference points

Subject	POI	Subject	POI	Subject	POI	Subject	POI
1	0,5	**11**	6,4	**21**	4,8	**31**	2,4
2	0,15	**12**	0,2	**22**	0,6	**32**	0,6
3	0,3	**13**	1,2	**23**	0,6	**33**	0,6
4	0,5	**14**	1,2	**24**	0,6	**34**	4,8
5	1,2	**15**	1,2	**25**	1,2	**35**	0,6
6	0,6	**16**	0,4	**26**	1,2	**36**	0,6
7	0,6	**17**	0,3	**27**	1,2	**37**	0,4
8	1,6	**18**	0,25	**28**	0,6	**38**	1,2
9	3,2	**19**	2,4	**29**	1,2		

10	3,2	**20**	2,4	**30**	0,6

To test the hypothesis that players who are more risk-averse are less likely, on average, to pass the pot, we estimate the following linear model:

$$avgpass_i = \beta_0 + \beta_1 lnIP_i + \beta_2 D_i \quad (20)$$

where $avgpass_i$ is the mean of the number of times individual *i* passed the pot in a game, $lnIP_i$ are natural logarithms of subjects' measures of risk attitudes represented by the indifference points proposed in Table 20, D_i is a dummy variable to control for the two different types of players (0 for player 1 and 1 for player 2). The results are shown in Table 21.

Table 21. Linear model

```
      Source |       SS       df       MS              Number of obs =      38
-------------+------------------------------           F(  2,    35) =    4.71
       Model |  1.14784533     2  .573922667           Prob > F      =  0.0154
    Residual |  4.26139187    35  .121754053           R-squared     =  0.2122
-------------+------------------------------           Adj R-squared =  0.1672
       Total |   5.4092372    37    .1461956           Root MSE      =  .34893

------------------------------------------------------------------------------
     avgpass |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnIP |   .1555688   .0647702     2.40   0.022     .0240783    .2870594
       type2 |  -.2657926   .1150508    -2.31   0.027    -.4993582   -.032227
       _cons |   1.269907   .0818797    15.51   0.000     1.103682    1.436132
------------------------------------------------------------------------------
```

The mean value of the average number of times players 1 passed the pot per game was 1,269907 and the mean value of the average number of times players 2 passed the pot per game was 1,004177. The slope coefficient is different from zero (P=0,022), which implies that as the value at which a subject is indifferent between a fair gamble and a certain payoff of the expected value of that gamble increases by 100% (all other things being held constant), the average number of times a player passes per game increases, on average, by 0,156*ln(2)= 0,1078.

CONCLUSION

Results of the experiments confirmed that standard game theory predictions are not adequate to explain players' behavior, which is consistent with the results of similar experiments. There is also no evidence of convergence to equilibrium, neither by testing the statistical significance of the number of games having been played in the ordered logit model or by simulating further iterations of the game using empirically derived relative frequencies of strategic choices. Both the panel data models and the ordered logit model show a strong correlation between the perceived errors of players' past plays, based on the difference between players' expected payoffs consistent with their chosen strategies and the actual payoffs they received, and the players' strategic choices in the subsequent game. On, average, players are less willing to pass the pot (or less willing to increase cooperation) as the magnitude of the errors increase. Even on a relatively narrow interval of low payoffs, ranging from 0,1€ to 6,4€, players, on average, exhibited a very clear exponential increase in conditional relative frequencies of accepted bets as money amounts declined. These results are consistent with similar experiments dealing with attitudes toward risk when stakes increase. Finally, the analysis in [5.4] suggests that more risk-averse players are, on average, less likely to *pass* the pot and that overall, player 2s are less likely to pass the pot than player 1s. This result is consistent with the calculations and conclusions derived in [5.3].

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APPENDIXES

List of Appendixes

Appendix A: Instructions 1
Appendix B: Code 3
Appendix C: Lotteries 9
Appendix D: Povzetek 14

Appendix A: Instructions

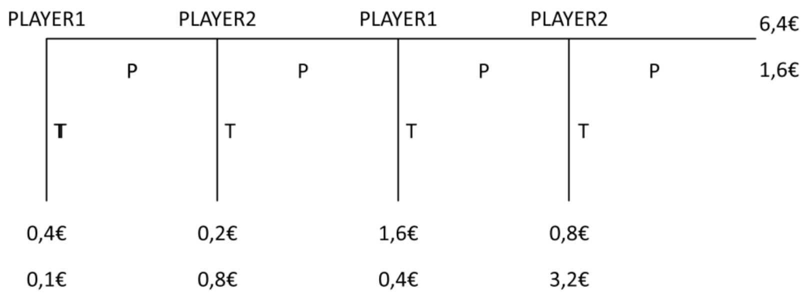
Thank you for your participation in this experiment. Before we get started, we're going to draw cards to determine the seating arrangement. The number on the card corresponds to the number of your computer. Your number will also determine whether it's you or your opponent who gets to act first in the game – an odd number means you get to act first, an even number means your opponent acts first.

[each player draws a card]

Please take your seats and do not log on to the computer at this time. The game we are going to play is a variant of the centipede game. You will be playing for real money and will be paid in cash at the end of the session. Please do not communicate to each other during the experiment.

The game begins with a pot of 50 cents divided unevenly among two players – you can see the payoffs on this diagram:

Figure 1. Centipede game with geometrically increasing payoffs



[experimenter uses diagram on whiteboard to aid in explaining the game].

The player to act first is given a choice to either accept the larger share (40 cents) of the pot or to pass the pot to the other player. If he chooses to take the pot, he gets 40 cents, the second player gets 10 cents and the game ends. If he passes the pot to the second player, the pot doubles to 1€, but in this (second) round, the second player gets to choose to either take the larger share of the pot - 80 cents (in which case the first player only gets 20 cents) or to pass the pot back to the first player. If he takes the 80 cents, the first player receives 20 cents and the game ends. If he passes, the pot gets doubled again to 2€ and the first player is offered 1,6€. The payoff for the second player in this round, if the first player accepts the offer, is 40 cents. If the first player decides to pass, the pot is doubled again to 4€. The second player is offered 3,2€. If he takes the offer, he gets 3,2€, while the first player gets 80 cents and the game ends. If the second player decides to pass, the pot doubles one last time to 8€. However, the game now ends and the second player receives the smaller share of the pot, 1,6€, while the first player gets 6,4€.

We're going to repeat this game several times during this session. You will not play with the same opponent more than once. At the end of each game you'll be given the option to gamble your winnings in that particular game on a flip of a coin. Call the right side of the coin, and your winnings (for the game) double, get it wrong, and you're left with nothing.

[experimenter takes questions]

The game will be played on your computers, so please log on and open file []. Before we start the actual experiment, we're going to do a practice run.

[experimenter explains interface]

There is one important point I have to make clear. Because we want to gather as much information as possible about your decision making process, the program is designed to let you keep playing even if your opponent has already taken the pot and effectively ended the game. For example, if the first player, in round 1, passed the pot to their opponent, who decided to take the pot thereby ending the game in round 2, the first player would still be made to decide on their action in round 3 as if their opponent had decided to pass. The first player's decision in round 3 would have no bearing on the outcome of the game in this case. So you won't necessarily know the actual results of the game until we post them on the whiteboard at the end of each game.

[experimenter takes questions]

[practice run]

Appendix B: Code

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <time.h>

int takeat [14];
double exppayoff [14];
int gameend [14];
double actualpayoff [14];
double error [14];
int pass [14];

int pairs [7]= {8, 8, 8, 8, 8, 8, 8};
int i,n,j;

int main()
{
    srand(time(NULL));

    for(j=0; j<100; j++)
    {

        FILE *datoteka;
        datoteka=fopen("magisterij.txt", "a+");

        takeat[0]=5;
        takeat[1]=1;
        takeat[2]=5;
        takeat[3]=5;
        takeat[4]=5;
        takeat[5]=5;
        takeat[6]=3;

        takeat[7]=5;
        takeat[8]=4;
        takeat[9]=4;
        takeat[10]=5;
        takeat[11]=2;
        takeat[12]=4;
        takeat[13]=4;

        for(i=0; i<7; i++)
        {
            if(takeat[i]==5)
                pass[i]=2;
            if(takeat[i]==3)
                pass[i]=1;
            if(takeat[i]==1)
                pass[i]=0;
        }

        for(i=7; i<14; i++)
        {
            if(takeat[i]==5)
            {
                pass[i]=2;
            }
            if(takeat[i]==4)
            {
                pass[i]=1;
            }
            if(takeat[i]==2)
            {
                pass[i]=0;
            }

        }

        for(n=0; n<100; n++)
        {
```

```
            fprintf(datoteka,"%d\t", j+1);
            fprintf(datoteka,"%d\t", n+1);
            for(i=0; i<14; i++)
            {
                fprintf(datoteka,"%d\t", pass[i]);
            }
            fprintf(datoteka,"\n");


            //calculate expected payoffs
            for(i=0; i<7; i++)
            {
                if(takeat[i]==5)
                    exppayoff[i]=6.4;
                if(takeat[i]==3)
                    exppayoff[i]=1.6;
                if(takeat[i]==1)
                    exppayoff[i]=0.4;
            }

            for(i=7; i<14; i++)
            {
                if(takeat[i]==5)
                {
                    exppayoff[i]=1.6;
                }
                if(takeat[i]==4)
                {
                    exppayoff[i]=3.2;
                }
                if(takeat[i]==2)
                {
                    exppayoff[i]=0.8;
                }
            }


//pairing, int pair is a random order of player 1s to be paired with player 2s.

            pairs[0]=rand()%7;

            for(i=1; i<7; i++)
            {
                do
                {
                    pairs[i]=rand()%7;
                }
                while((pairs[i]==pairs[i-1])
                        ||(pairs[i]==pairs[i-2])
                        ||(pairs[i]==pairs[i-3])
                        ||(pairs[i]==pairs[i-4])
                        ||(pairs[i]==pairs[i-5])
                        ||(pairs[i]==pairs[i-6])
                     );
            }


            for(i=0; i<7; i++)
            {
                if(takeat[i+7]>=takeat[pairs[i]])
                {
                    gameend[i+7]=takeat[pairs[i]];
                    gameend[pairs[i]]=takeat[pairs[i]];
                }

                if(takeat[i+7]<takeat[pairs[i]])
                {
                    gameend[i+7]=takeat[i+7];
                    gameend[pairs[i]]=takeat[i+7];
                }

            }


            //calculate actual payoffs
            for(i=0; i<7; i++)
```

```
{
    if(gameend[i]==5)
        actualpayoff[i]=6.4;
    if(gameend[i]==4)
        actualpayoff[i]=0.8;
    if(gameend[i]==3)
        actualpayoff[i]=1.6;
    if(gameend[i]==2)
        actualpayoff[i]=0.2;
    if(gameend[i]==1)
        actualpayoff[i]=0.4;
}

for(i=7; i<14; i++)
{
    if(gameend[i]==5)
    {
        actualpayoff[i]=1.6;
    }
    if(gameend[i]==4)
    {
        actualpayoff[i]=3.2;
    }
    if(gameend[i]==3)
    {
        actualpayoff[i]=0.4;
    }
    if(gameend[i]==2)
    {
        actualpayoff[i]=0.8;
    }
    if(gameend[i]==1)
    {
        actualpayoff[i]=0.1;
    }
}

for(i=0; i<14; i++)
{
    error[i]=exppayoff[i]-actualpayoff[i];
}


for(i=0; i<7; i++)
{
    if(takeat[i]==5)
        pass[i]=2;
    if(takeat[i]==3)
        pass[i]=1;
    if(takeat[i]==1)
        pass[i]=0;
}

for(i=7; i<14; i++)
{
    if(takeat[i]==5)
    {
        pass[i]=2;
    }
    if(takeat[i]==4)
    {
        pass[i]=1;
    }
    if(takeat[i]==2)
    {
        pass[i]=0;
    }

    for(i=0; i<14; i++)
    {
        double roll=(double)rand()/RAND_MAX;

        if(error[i]==6.2)
        {
            if(roll<=0.22222)
                pass[i]=0;
```

```
    if(roll>0.22222&&roll<=0.88888)
    {

        if (pass[i]==0)
        {
            pass[i]=0;
        }
        else
        {
            pass[i]--;
        }
    }
    if(roll>0.88888)
        pass[i]=pass[i];
}

if(error[i]==5.6)
{
    if(roll<=0.11111)
        pass[i]=0;
    if(roll>0.11111&&roll<=0.66666)
    {
        if (pass[i]==0)
            pass[i]=0;
        else pass[i]--;
    }
    if(roll>0.66666)
        pass[i]=pass[i];
}

if(error[i]==3.1)
{
    if(roll<=0.57142)
    {
        if (pass[i]==0)
            pass[i]=0;
        else pass[i]--;
    }
    if(roll>0.57142&&roll<=0.85714)
        pass[i]=pass[i];
    if(roll>0.85714)
    {
        if (pass[i]==2)
            pass[i]=2;
        else pass[i]++;
    }
}

if(error[i]==2.8)
{
    if(roll<=0.33333)
    {
        if (pass[i]==0)
            pass[i]=0;
        else pass[i]--;
    }
    if(roll>0.33333&&roll<=0.73333)
        pass[i]=pass[i];
    if(roll>0.73333)
    {
        if (pass[i]==2)
            pass[i]=2;
        else pass[i]++;
    }
}

if(error[i]==1.5)
{
    if(roll<=0.8)
    {
        if (pass[i]==0)
            pass[i]=0;
        else pass[i]--;
    }
    if(roll>0.8)
        pass[i]=pass[i];
```

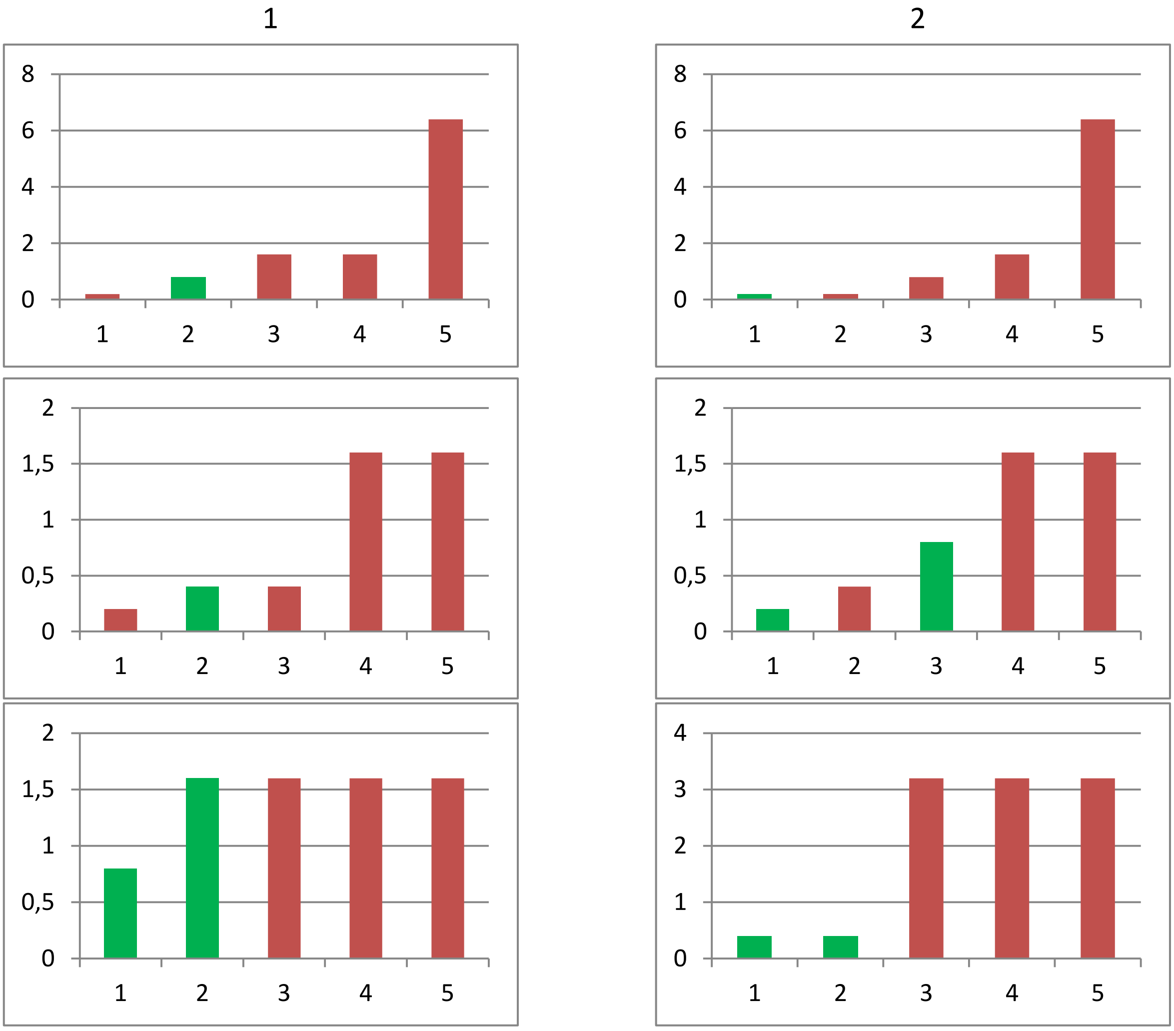
```
                }

                if(error[i]==1.4)
                {
                    if(roll<=0.33333)
                    {
                        if (pass[i]==0)
                            pass[i]=0;
                        else pass[i]--;
                    }
                    if(roll>0.33333&&roll<=0.53333)
                        pass[i]=pass[i];
                    if(roll>0.53333)
                    {
                        if (pass[i]==2)
                            pass[i]=2;
                        else pass[i]++;
                    }
                }

                if(error[i]==1.2)
                {
                    if(roll<=0.375)
                        pass[i]=0;
                    if(roll>0.375&&roll<=0.625)
                    {
                        if (pass[i]==0)
                            pass[i]=0;
                        else pass[i]--;
                    }
                    if(roll>0.625)
                        pass[i]=pass[i];
                }

                if(error[i]==0.7)
                {
                    if (pass[i]==2)
                        pass[i]=2;
                    else pass[i]++;
                }

                if(error[i]==0)
                {
                    if(roll<=0.45652)
                    {
                        if (pass[i]==0)
                            pass[i]=0;
                        else pass[i]--;
                    }

                    if(roll>0.45652&&roll<=0.64347)
                        pass[i]=pass[i];

                    if(roll>0.64347&&roll<=0.93913)
                    {
                        if (pass[i]==2)
                            pass[i]=2;
                        else pass[i]++;
                    }
                    if(roll>0.93913)
                        pass[i]=2;
                }
            }

        }

        for(i=0; i<7; i++)
        {
            if(pass[i]==2)
                takeat[i]=5;
            if(pass[i]==1)
                takeat[i]=3;
            if(pass[i]==0)
                takeat[i]=1;
        }

```

```
            for(i=7; i<14; i++)
            {
                if(pass[i]==2)
                {
                    takeat[i]=5;
                }
                if(pass[i]==1)
                {
                    takeat[i]=4;
                }
                if(pass[i]==0)
                {
                    takeat[i]=2;
                }

            }

        }

    }

    return 0;
}
```

Appendix C: Lotteries

Figure 2 shows individual players' payoffs ordered from lowest to highest, red columns denote instances in which the lottery was rejected by the subject and green columns show instances where the lottery was accepted.

Figure 2. Individual players' payoffs



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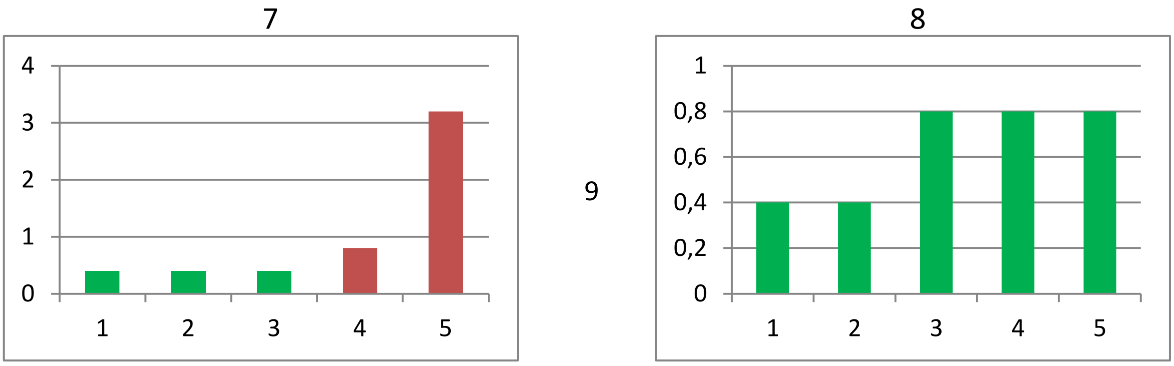

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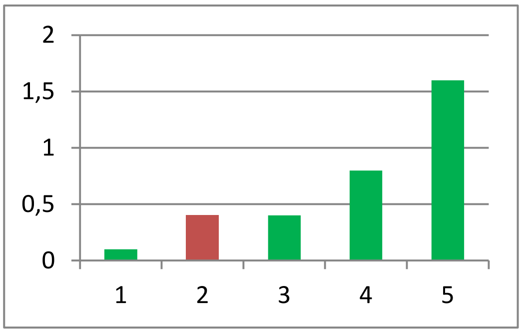

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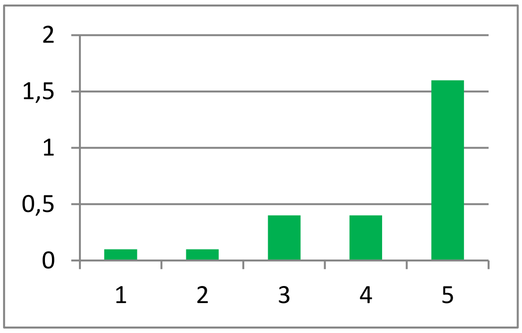

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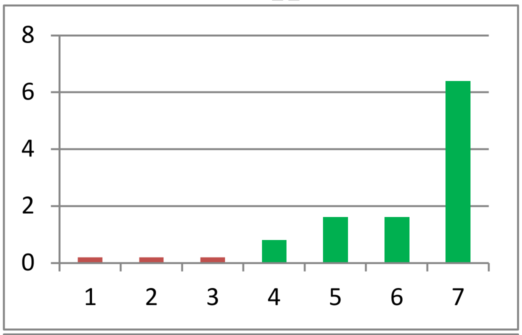

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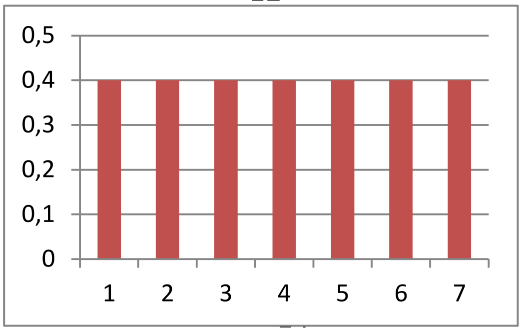

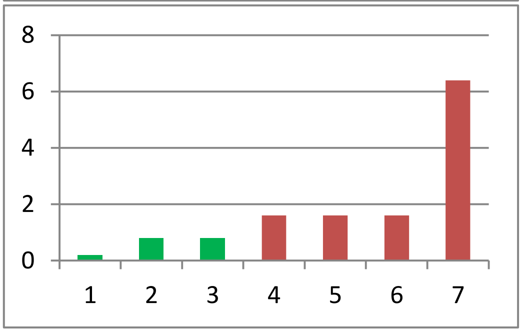

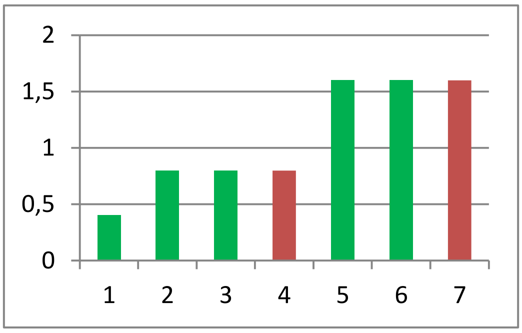

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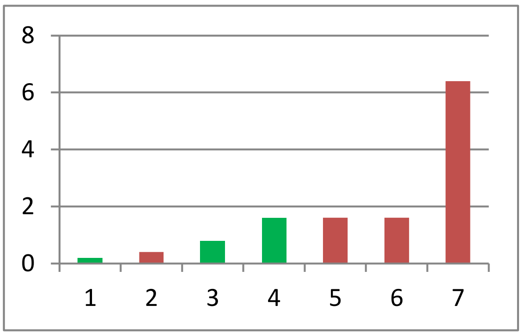

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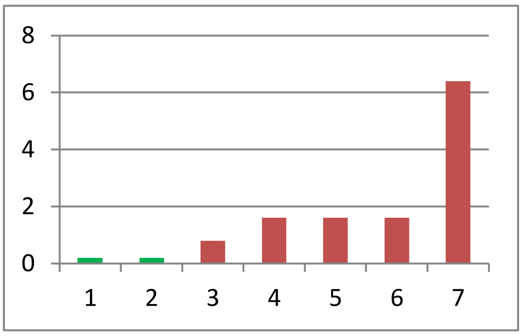

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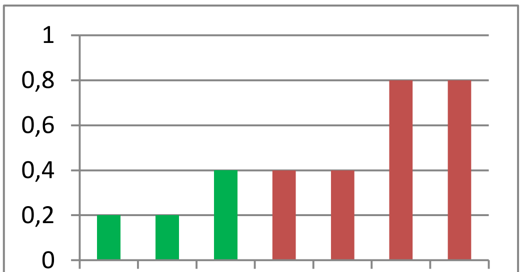

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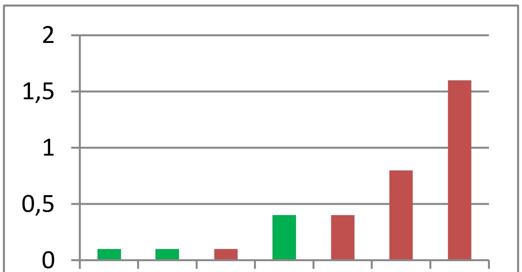

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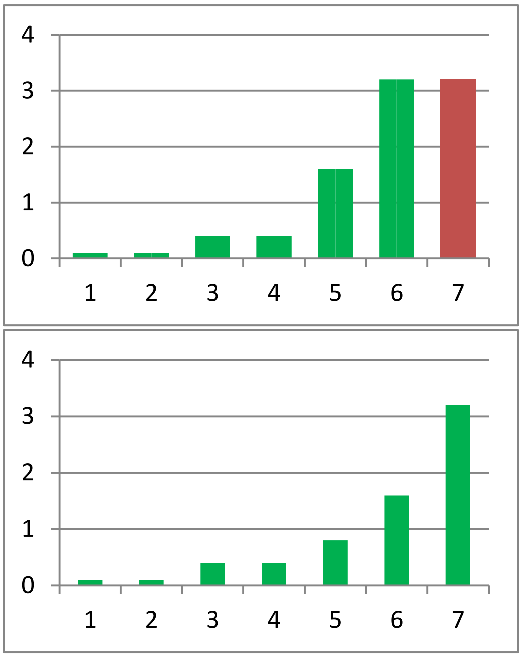

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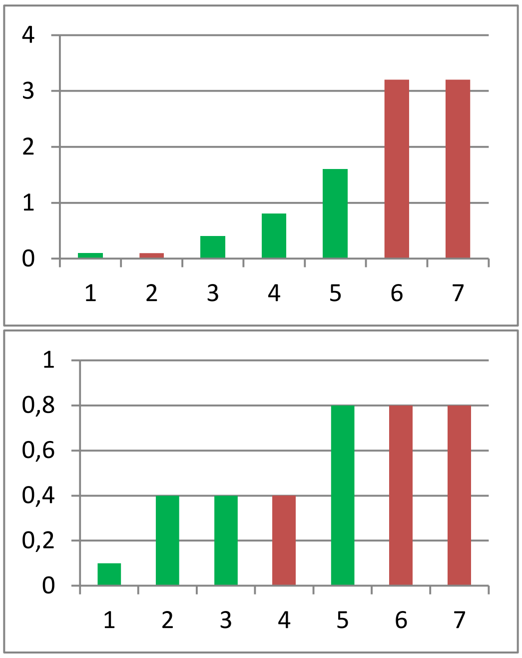

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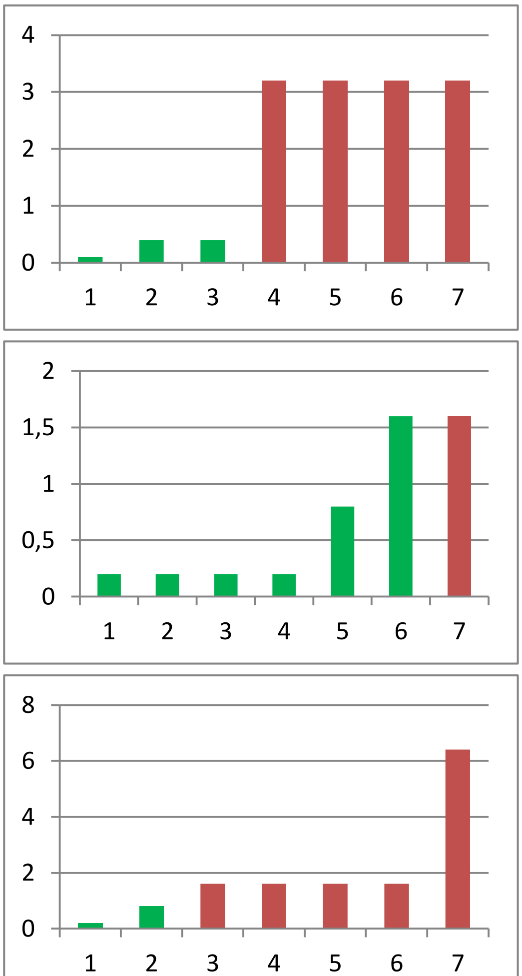

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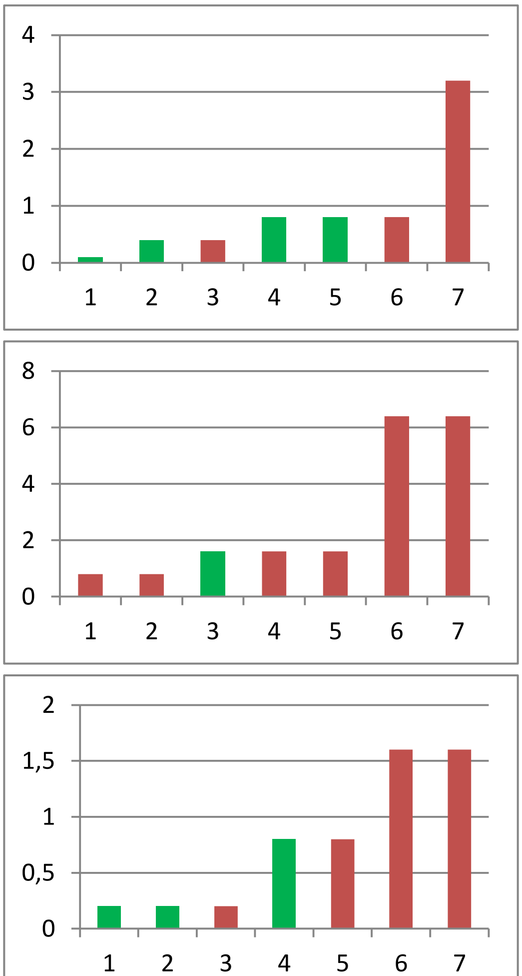

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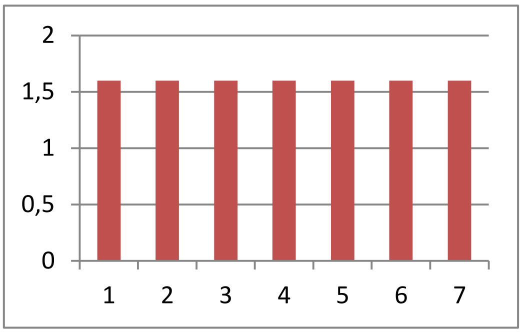

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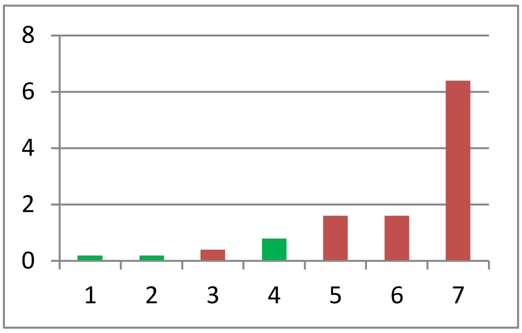

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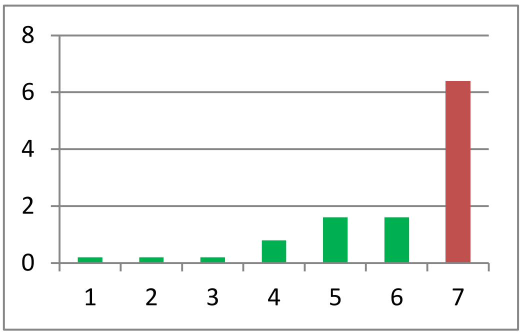

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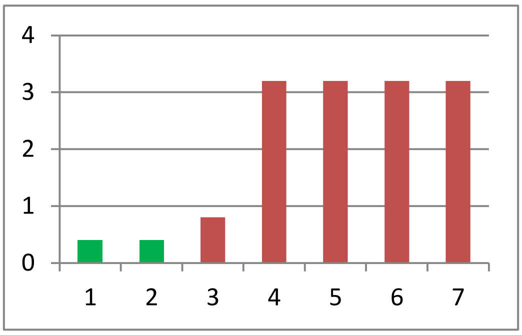

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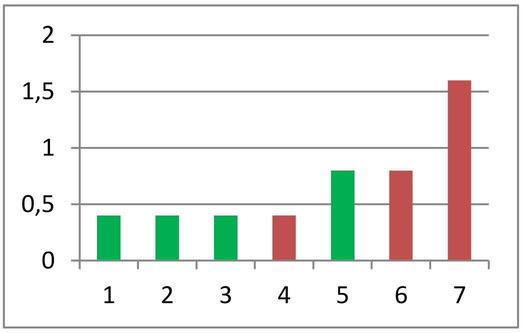

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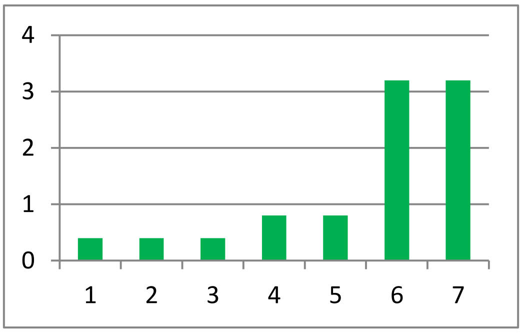
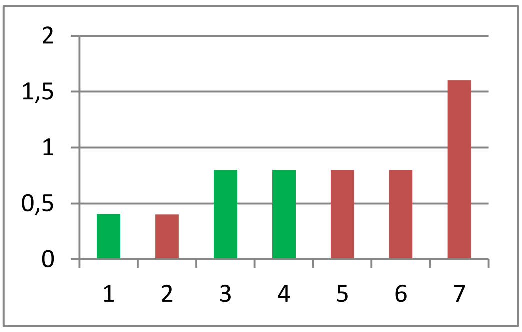
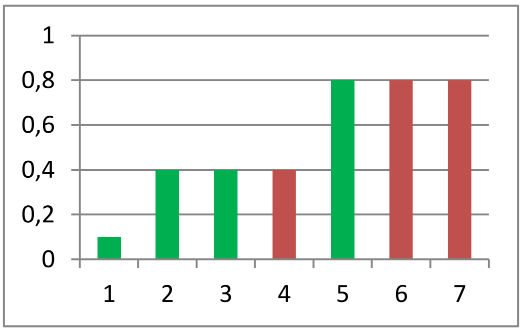
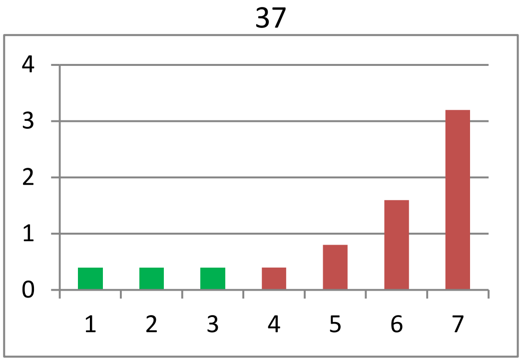
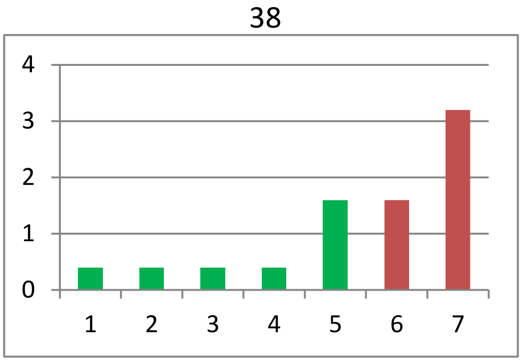







37
4
3
2
1
0
1 2 3 4 5 6 7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4
3
2
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1 2 3 4 5 6 7

Appendix D: Povzetek

ODNOSI DO TVEGANJA IN UČENJE: EMPIRIČNA ANALIZA IGRE STONOGA (POVZETEK)

UVOD

V magistrskem delu povzemam rezultate vedenjskega eksperimenta, ki sem ga izvedel s pomočjo mentorja, dr. Aljoše Feldina. Pri analizi se upiram na različne modele učenja, ki vključujejo model fiktivne igre (angl. *fictitious play* (Camerer, 2003)) in model učenja z ojačitvijo (angl. *reinforcement learning* (Fudenberg & Lavine, 1998)), ter na ustaljene koncepte odnosov to tveganja. Namen magistrske naloge je pojasniti razlike med rešitvami, ki jih predlaga teorija iger, in empiričnimi rezultati, ki se praviloma z njimi ne ujemajo. V eksperimentu so študenti Ekonomske fakultete igrali igro Stonoga (angl. *Centipede* (Rosenthal, 1981)), ki je podrobneje opisana v prvem poglavju. Poleg glavne ugotovitve podobnih študij, da se empirični rezultati razlikujejo od rezultatov, ki temeljijo na analizi teorije iger, v magistrski nalogi preverim naslednje domneve:

1. Igralci se učijo iz napak v prejšnjih igrah, ki temeljijo na razliki med dobitki, ki jih igralci pričakujejo z igranjem izbrane strategije, in dejanskimi dobitki, ki jih prejmejo. Večja kot je razlika med pričakovanim in dejanskim dobitkom v prejšnji igri, manj verjetno je, v povprečju, da igralci zavrnejo ponujeno vsoto denarja v naslednji igri.

2. Odnosi do tveganja niso konstantni, temveč odvisni od različnih denarnih vsot. Obstaja negativna korelacija med višino dobitka (višino stave) in verjetnostjo, da je igralec svoj dobitek pripravljen staviti na met kovanca.

3. Igralci, ki so manj nagnjeni k tveganju, v povprečju tudi manjkrat zavrnejo ponujeno vsoto denarja v igri.

V prvem poglavju predstavim igro, ki je uporabljena v eksperimentu, in podam osnove teorije iger, ki so potrebni za njeno razumevanje. Poglavje se zaključi s pregledom literature na temo sorodnih študij. V drugem poglavju sledi podroben opis zasnove in poteka eksperimenta, v tretjem poglavju pa so predstavljeni rezultati.

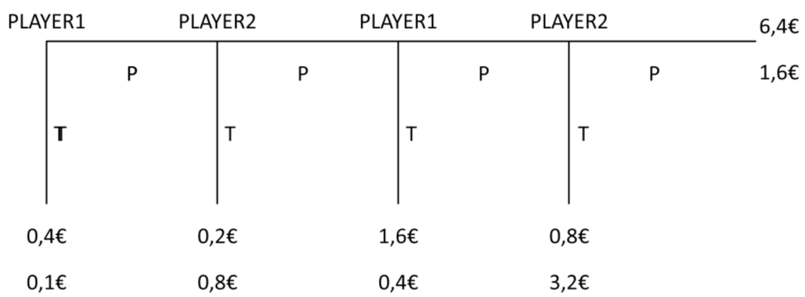
Četrto poglavje uvede različne modele učenja v teoriji iger. Za testiranje domneve, da se igralci učijo iz napak v prejšnjih igrah (ideja, ki je sorodna konceptom v modelu učenja z ojačanjem), uporabim več regresijskih modelov (model s konstantnimi efekti, model z mešanimi efekti in ordinalni logit model). Za boljšo predstavo rezultatov uporabim tudi simulacijo, s katerimi s pomočjo vhodnih podatkov (rezultati prve igre eksperimenta) modeliram učenje po principu modela fiktivne igre in rezultate simulacije primerjam z empiričnimi podatki.

V petem poglavju podrobneje analizira koncepte povezane z odnosi do tveganja in kako vplivajo na odločitve posameznikov v igri. V sklepnem delu poglavja preverim domnevo, da obstaja korelacija med posameznikovim odnosom do tveganja in njihovo strategijo v igri, oziroma, da igralci, ki so manj nagnjeni k tveganju, v povprečju tudi manjkrat zavrnejo ponujeno vsoto denarja v igri.

1 IGRA STONOGA

Stonoga (angl. *Centipede*) je igra, v kateri se dva igralca izmenjujeta pri odločanju o tem, ali bosta vzela večji delež kupa denarja, ki se v vsakem krogu igre poveča. Igro je kot primer prvi uporabil Robert W. Rosenthal (1981). Od prve uporabe je igra in njene mnoge različice postala priljubljen način razlage določenih konceptov teorije iger, uporabljena pa je bila tudi v številnih eksperimentalnih študijah. Za lastno delo sem izbral Stonogo s štirimi krogi in geometrijsko naraščajočimi vsotami denarja, igra je prikazana v sliki 1.

Slika 1. Stonoga s štirimi krogi in geometrijsko naraščajočim kupom denarja



Na začetku igre se manjša vsota denarja (0,5€) neenakomerno razdeli med oba igralca, 0,4€ prvemu igralcu in 0,1€ drugemu igralcu. V začetnem krogu odloča prvi igralec, če ponudbo sprejme, se igra konča, igralca pa dobita vsak svoj delež denarja, če ponudbo zavrne, se igra nadaljuje. V drugem krogu se vsota v skupnem kupu denarja podvoji (1€), vendar sedaj drugi igralec odloča o tem, ali bo sprejel večji delež denarja (0,8€). Če ponudbo sprejme, vzame ponujeno vsoto denarja, prvi igralec prejme le 0,2€ in igra se konča. Če ponudbo zavrne, se igra nadaljuje v naslednji krog, vsota v skupnem kupu denarja se ponovno podvoji (2€), denar pa se razdeli med oba igralca (1,6€ prvemu in 0,4€ drugemu igralcu). V tretjem krogu o ponudi odloča prvi igralec. Če ponudbo sprejme, se igra konča, igralca pa prejmeta ponujeni vsoti denarja. Če ponudbo zavrne, se igra nadaljuje v naslednji krog, vsota v skupnem kupu denarja se ponovno podvoji (4€), denar pa se razdeli med oba igralca (0,8€ prvemu in 3,2€ drugemu igralcu). V četrtem krogu o ponudi odloča drugi igralec. Če ponudbo sprejme, se igra konča, igralca pa prejmeta ponujeni vsoti denarja. Če ponudbo zavrne, se skupni kup denarja še zadnjič podvoji (8€), prvi igralec prejme 6,4€, drugi igralec pa le 1,6€ in igra se konča.

2 ZASNOVA EKSPERIMENTA

Eksperiment je potekal v treh skupinah, v katerih je 38 študentov podiplomskega študija Ekonomske fakultete odigralo 123 iger. V tretji skupini so sodelovali le študenti MBA programa Ekonomske fakultete. Zaradi zasedenosti prostorov in manjkajočih udeležencev število igralcev v vseh treh skupinah ni bilo enako, v prvi je sodelovalo deset igralcev (pet parov), v drugi in tretji pa po štirinajst igralcev (sedem parov).

Vse igre so bile igrane preko preprostega vmesnika na računalniku, ki je poleg gumbov za sprejem ali zavrnitev ponudbe vseboval tudi kratek opis trenutnega kroga s ponujenimi vsotami za oba igralca in grafično predstavitev poteka igre. Slika 2 predstavlja grafični vmesnik drugega igralca.

Pred začetkom igre so igralci izžrebali številko računalnika, na katerem so igrali. S tem je bila poleg sedežnega reda zagotovljena tudi naključna porazdelitev udeležencev na dva tipa igralcev, tiste, ki odločajo v prvem in tretjem krogu igre (prvi igralec) in tiste, ki odločajo v drugem in zadnjem krogu igre (drugi igralec). Na začetku vsake igre so bili naključno izbrani pari igralcev obeh tipov na tak način, da je vsak igralec prvega tipa igral točno eno igro z vsakim igralcem drugega tipa. Udeleženci so bili jasno seznanjeni s tem, da bodo vsako igro igrali z novim igralcem. Zaradi te omejitve je bilo v prvi skupini odigranih le 25 iger, v preostalih dveh pa 49 iger. Podrobni podatki o vsaki skupini so zbrani v tabeli 1.

Slika 2. Grafični vmesnik drugega igralca

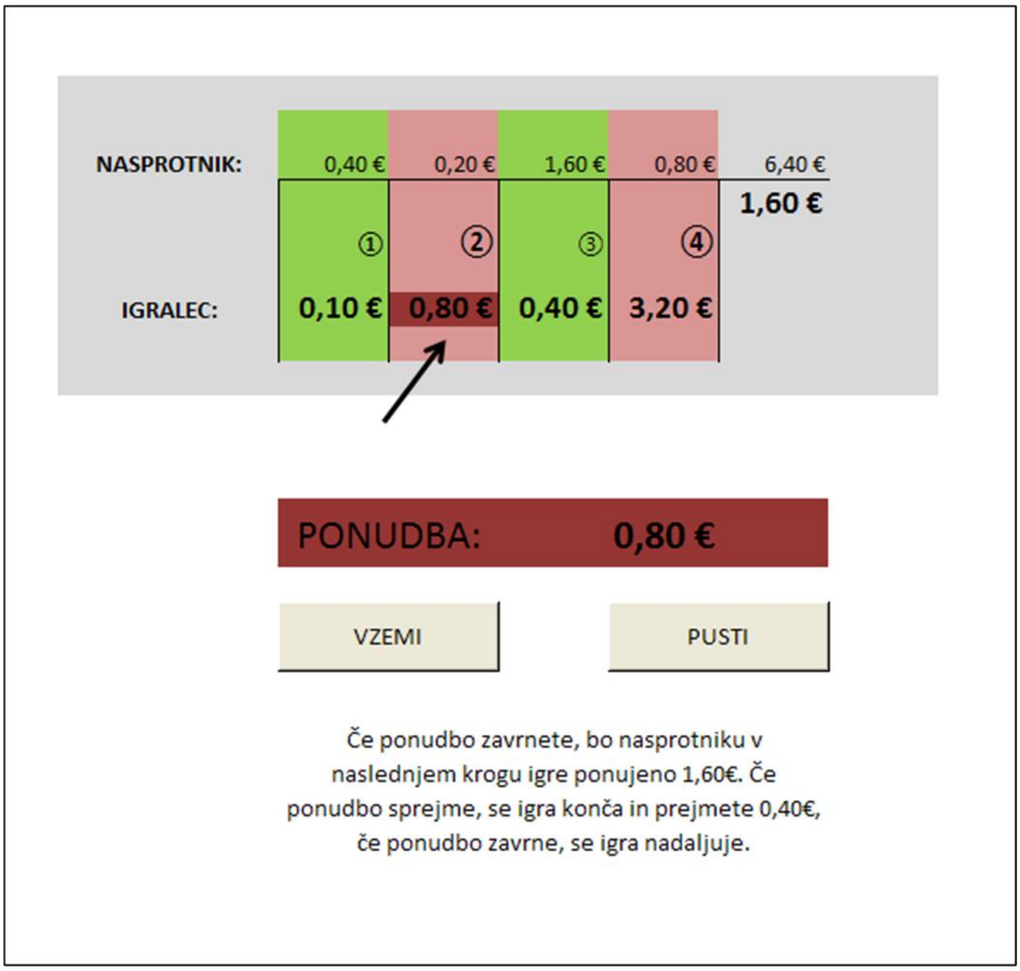


Tabela 1. Podatki o skupinah

Skupina	Število udeležencev	Število iger na udeleženca	Skupno število iger
1	10	5	25
2	14	7	49
3	14	7	49

Vsaki skupini so bila podrobno razložena pravila igre in podana navodila glede poteka eksperimenta. Komunikacija med udeleženci ni bila dovoljena, da bi se omejili kakršnikoli poskusi sodelovanja.

Po koncu vsake igre je bila vsakemu udeležencu ponujena dodatna *možnost*, da svoj dobitek stavi na met kovanca. Če zmaga, se njegov dobitek podvoji, če izgubi, ostane brez denarja, ki ga je dobil v igri. Namen ponujene stave ob koncu igre je vključitev analize odnosov do tveganja udeležencev.

3 REZULTATI EKSPERIMENTA

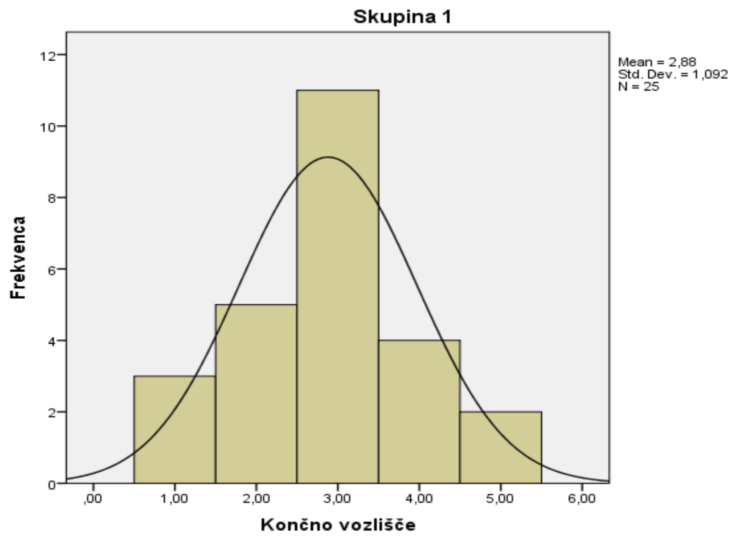
Tabela 2 prikazuje relativne frekvence doseženih končnih vozlišč v vseh 123 igrah. f_5 je relativna frekvenca iger, ki so se končale v petem vozlišču (igre, v katerih nihče od igralcev ni sprejel ponujenih vsot denarja).

Tabela 2. Relativne frekvence doseženih končnih vozlišč

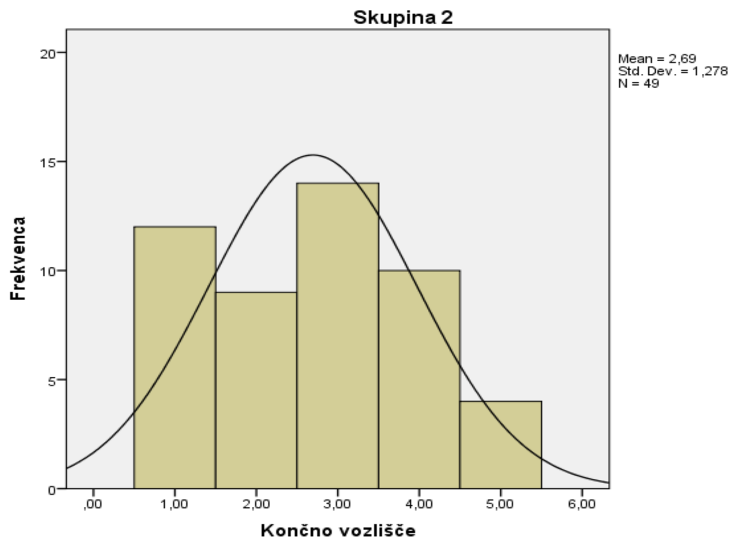
Skupina	Udeleženci	N	f_1	f_2	f_3	f_4	f_5
1	Podiplomski študij	25	0,12	0,20	0,44	0,16	0,08
2	Podiplomski študij	49	0,24	0,18	0,29	0,20	0,08
3	MBA	49	0,02	0,27	0,45	0,16	0,10
Skupaj		123	0,13	0,22	0,38	0,18	0,09

Ker je bilo odigrano število iger po posameznih skupinah različno, relativne frekvence v vrstici *skupno* predstavljajo uteženo povprečje relativnih frekvenc doseženih vozliščih. Kljub temu, da standardna rešitev v teoriji iger predvideva, da bo prvi igralec vedno takoj sprejel prvo ponudbo in se bodo zato vse igre končale v prvem vozlišču (f_1=1; $f_2 = f_3 = f_4 = f_5 = 0$), se je v eksperimentu le 13% iger končalo pri prvi ponudbi. Kar 9% iger se je končalo v petem vozlišču, kar pomeni, da prvi in drugi igralec nikoli nista sprejela ponujenih vsot. 38% iger se je končalo v tretjem vozlišču, f_3 je tudi modus pri vseh treh skupinah. Slike 3-5 prikazujejo frekvenčno porazdelitev končnih vozlišč doseženih v igrah posameznih skupin.

Slika 3. Frekvenčno porazdelitev doseženih končnih vozlišč v 1. skupini



Slika 4. Frekvenčno porazdelitev doseženih končnih vozlišč v 2. skupini



Slika 5. Frekvenčno porazdelitev doseženih končnih vozlišč v 3. skupini

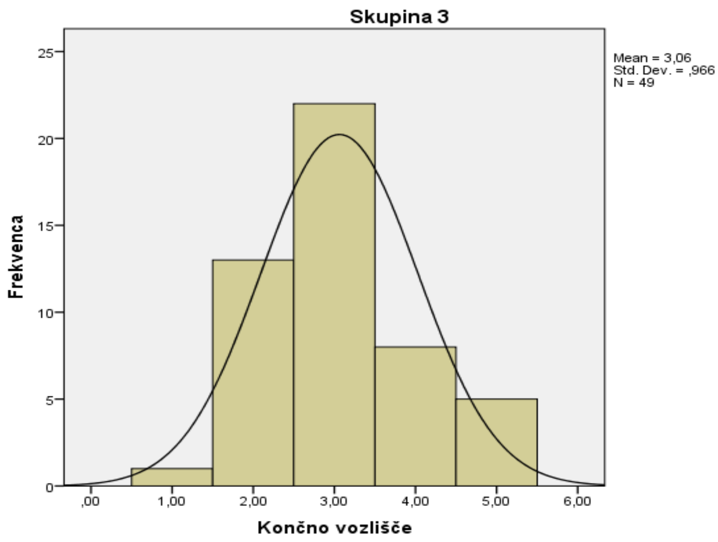


Tabela 3 prikazuje relativne frekvence strategij obeh tipov igralcev. f_{VV}^1 je relativna frekvenca strategije, pri kateri je prvi igralec vedno vzel ponujeno vsoto denarja (*VV*). f_{PV}^1 je relativna frekvenca strategije, pri kateri je prvi igralec zavrnil (pustil) prvo ponudbo, a sprejel (vzel) drugo (*PV*). f_{PP}^1 je relativna frekvenca strategije, pri kateri je prvi igralec zavrnil obe ponudbi (*PP*).

Tabela 3. Relativne frekvence strategij obeh tipov igralcev

			Igralec 1			Igralec 2		
Skupina	**Udeleženci**	**N**	f_{VV}^1	f_{PV}^1	f_{PP}^1	f_{VV}^2	f_{PV}^2	f_{PP}^2
1	Podiplomski št.	25	0,12	0,52	0,36	0,20	0,44	0,36
2	Podiplomski št.	49	0,24	0,39	0,37	0,20	0,57	0,22
3	MBA	49	0,02	0,63	0,35	0,27	0,57	0,16
Skupaj		123	0,13	0,51	0,36	0,23	0,54	0,23

Posebnost tega eksperimenta je zasnova igre in računalniškega vmesnika, ki je vsakemu igralcu omogočila, da svojo strategijo izpelje do konca, ne glede na igro nasprotnika. Če, na primer, prvi igralec že v prvem krogu igre sprejme ponudbo (in igro s tem dejansko konča), drugi igralec kljub temu lahko igra naprej, v skladu s svojo strategijo. Z rezultatom in odločitvami nasprotnika je seznanjen šele po koncu vsake igre, zato prilagojen postopek igre ne vpliva na odločitve igralcev v naslednjih igrah, rezultat tekoče igre pa je določen s prvo sprejeto ponudbo (ali pa se konča v petem vozlišču, če igralca zavrneta vse ponudbe). Prilagojen postopek tako ne spremeni igre, kljub temu pa omogoči, da je zbranih več podatkov o odločitvah igralcev. Relativne frekvence v tabeli 3 tako predstavljajo prave frekvence strategij igralcev, ne le ocene na podlagi rezultatov igre, ki so običajno uporabljene v podobnih študijah. Ker strategija *VV* prvega igralca direktno določi rezultat igre (konec v prvem vozlišču), se relativne frekvence strategije *VV* prvega igralca v tabeli 3 popolnoma ujemajo z relativnimi frekvencami v stolpcu f_1 (relativna frekvenca iger, ki se konča v prvem vozlišču) v tabeli 2.

4 UČENJE V STONOGI

V četrtem poglavju preizkušam domnevo, da se igralci učijo iz napak v prejšnjih igrah, ki temeljijo na razliki med dobitki, ki jih igralci pričakujejo z igranjem izbrane strategije, in dejanskimi dobitki, ki jih prejmejo. Večja kot je razlika med pričakovanim in dejanskim dobitkom v prejšnji igri, manj verjetno je, v povprečju, da igralci zavrnejo ponujeno vsoto denarja v naslednji igri. V ta namen je ocenjen regresijski model s konstantnimi efekti;

$$Y_{it} = \alpha_i + \beta_1 X_{i,t-1} + u_{it}, \tag{1}$$

kjer je α_i konstanta igralca *i*, β_1 pa je skupni regresijski koeficient. $X_{i,t-1}$ je napaka igralca *i* v prejšnji igri (razlika med dobitkom, ki ga igralec pričakuje na podlagi lastne strategije, in dejanskim dobitkom v igri). Y_{it} je število zavrnjenih ponudb igralca *i* v trenutni igri in u_{it} je napaka. Rezutat ocenjenega regresijskega modela v program Stata je prikazan v tabeli 4.

Tabela 4. Ocenjen model s konstantnimi učinki

```
Fixed-effects (within) regression               Number of obs      =       208
Group variable: subject                         Number of groups   =        38

R-sq:  within  = 0.0408                         Obs per group: min =         4
       between = 0.1503                                        avg =       5.5
       overall = 0.0039                                        max =         6

                                                F(1,169)           =      7.19
corr(u_i, Xb)  = -0.2026                        Prob > F           =    0.0081

------------------------------------------------------------------------------
        pass |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  error_lag2 |  -.0623079   .0232399    -2.68   0.008    -.1081859     -.01643
       _cons |   1.181543   .0544271    21.71   0.000     1.074098    1.288987
-------------+----------------------------------------------------------------
     sigma_u |  .40952337
     sigma_e |  .61704803
         rho |  .30578365   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(37, 169) =     2.31             Prob > F = 0.0002
```

Na podlagi vzorčnih podatkov ocenjeni regresijski koeficient je enak -0,0623, kar pomeni, da se v povprečju število zavrnjenih ponudb v igri zmanjša za 0,0623, če se napaka v prejšnji igri poveča za 1€. Na podlagi vzorčnih podatkov zavrnemo ničelno domnevo, da je regresijski koeficient enak 0, pri stopnji značilnosti P=0,008. V povprečju so igralci zavrnili 1,1815 ponudb na igro (prvi igralec v povprečju 1,3043 ponudbe na igro in drugi igralec v povprečju 1,0647 ponudbe na igro). Negativno korelacijo med napakami v prejšnjih igrah (ki temeljijo na razliki med dobitki, ki jih igralci pričakujejo z igranjem izbrane strategije, in dejanskimi dobitki, ki jih prejmejo) in številom zavrnjenih ponudb, sta potrdila tudi model z mešanimi učinki in ordinalni logistični model.

5 ODNOSI DO TVEGANJA

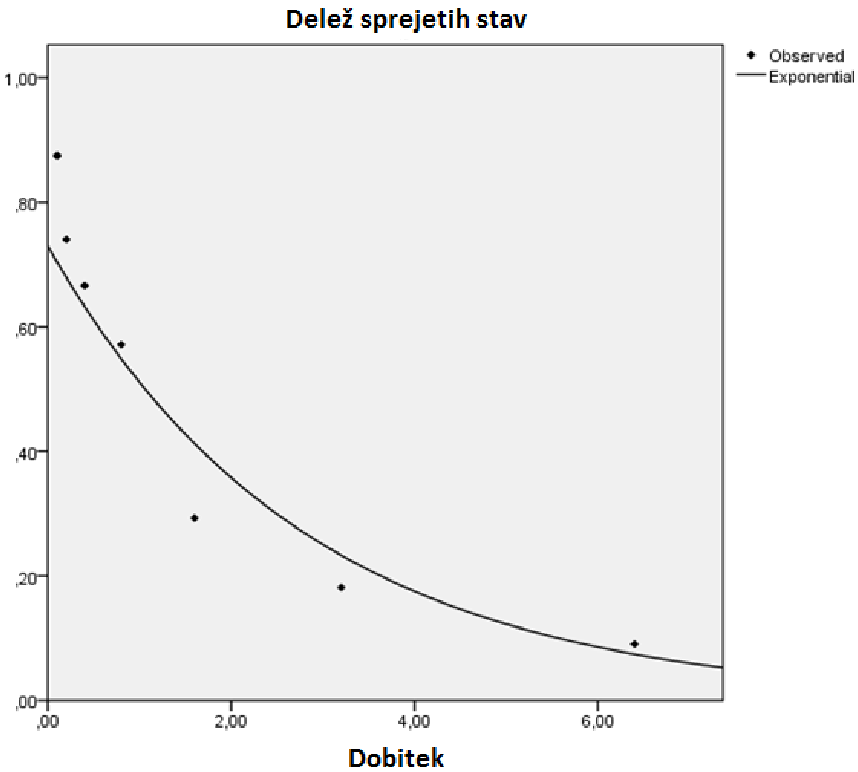
Tabela 5 prikazuje relativne frekvence sprejetja stave pri posameznih dobitkih. Takoj je razvidno, da relativna frekvenca monotono pada z naraščanjem dobitka, kar pomeni, da odnosi igralcev do tveganja niso konstantni.

Tabela 5. Pogojna relativna frekvenca sprejetja stave

Dobitek (stava)	**6,40 €**	**3,20 €**	**1,60 €**	**0,80 €**	**0,40 €**	**0,20 €**	**0,10 €**
Število sprejetih stav	1	4	17	28	42	20	14
Število doseženih dobitkov	11	22	58	49	63	27	16
Relativna frekvenca	0,09091	0,18182	0,2931	0,57143	0,66667	0,74074	0,875

Rezultati so skladni z ugotovitvami podobnih eksperimentov (Holt & Laury, 2002; Kachelmeier & Shehata, 1992; Binswanger, 1981). Slika 6 prikazuje razsevni grafikon in ocenjeno eksponentno regresijsko funkcijo.

Slika 6. Delež sprejetih stav pri posameznih dobitkih



$$\widehat{frekvenca_i} = \hat{\alpha}\hat{\beta}^{dobitek_i} = 0{,}730 \cdot (0{,}700)^{dobitek_i} \quad (2)$$

Na podlagi vzorčnih podatkov lahko trdim, da se relativna frekvenca sprejetja stave v povprečju zmanjša za 30%, če se dobitek poveča za 1 €.

ZAKLJUČEK

Rezultati eksperimenta so potrdili, da se vedenje igralcev v veliki meri razlikuje od rezultatov, ki temeljijo na analizi teorije iger. Poleg tega empirični rezultati ne kažejo na kakršnokoli težnjo k ravnovesju, ki bi lahko upravičila uporabnost teoretičnih napovedi. Ocenjeni modeli konstantnih učinkov, mešanih učinkov in ordinalni logistični model potrjujejo močno pozitivno korelacijo med napakami igralcev v prejšnji igri (ki temeljijo na razliki med dobitki, ki jih igralci pričakujejo z igranjem izbrane strategije, in dejanskimi dobitki, ki jih prejmejo) in hitrejšim sprejemanjem ponudb v tekoči igri.

Kljub relativno nizkim denarnim zneskom (in njihovemu ozkemu razponu), je iz podatkov jasno razvidno, da odnosi do tveganja posameznih igralcev niso konstantni, temveč med višino dobitka in relativno frekvenco sprejetja stave obstaja negative eksponentna odvisnost. Podati kažejo na to, da igralci, ki so bolj nagnjeni k tveganju, v povprečju tudi bolj pogosto zavrnejo ponujeno vsoto denarja.