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INFLATION FORECASTING DRIVEN BY INFLATION EXPECTATIONS: A DYNAMIC MULTI-FACTOR MODEL

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LIST OF ABBREVIATIONS

3PRF	Three-Pass Regression Filter
AIC	Akaike Information Criterion
ADF	Augmented Dickey-Fuller
APP	Asset Purchasing Programme
AR	Autoregressive Model
ATS	Affine Term Structure
BIC	Bayes Information Criterion
BMA	Bayesian Model Averaging
BVAR	Bayesian Vector Autoregression
BEIR	Break-Even Inflation Rate

CPPI	Commercial Property Price Index
DFM	Dynamic Factor Model
EA	Euro Area
EBM	Empirical Bayes Method
ECB	European Central Bank
FAVAR	Factor-Augmented Vector Autoregression
FPG	Forecasting Precision Gain
GATS	Gaussian Affine Term Structure
HICP	Harmonized Index of Consumer Prices
ILS	Inflation Linked Swap
IRP	Inflation Risk Premium
LASSO	Least Absolute Shrinkage and Selection Operator
MLE	Maximum Likelihood Estimator
MSFE	Mean Square Forecast Error
OLS	Ordinary Least Squares
OOS	Out Of Sample
PCA	Principal Component Analysis
PCR	Principal Component Regression
PLS	Partial Least Squares
PPI	Producer Price Index
RMSE	Root Mean Square Error
RMSFE	Root Mean Square Forecasting Error
RPPI	Residential Property Price Index
rMSFE	relative Mean Square Forecasting Error
VAR	Vector Autoregression

INTRODUCTION

Inflation expectations play an important role in the effective implementation and transmission of monetary policy. They provide valuable evidence about the credibility of a central bank and its capability to achieve its inflationary target and deliver on its price stability mandate. Besides the headline inflation over the medium-term central banks use measures of inflation expectations as one of the indicators to monitor the effect of their monetary policy decisions and as a cross-check on the macroeconomic projections of the inflation outlook.

The key interest of policy makers is well-anchored expected inflation in the medium- to long-term period, which is the precondition for effective monetary policy actions. Inflation expectations are said to be anchored when the mean forecasts of inflation are stable and close to the inflation target set by the central bank. The view on the future inflation should be, on average, close to the inflation target set by the central bank. Definition of the euro area price stability, a year-on-year harmonized index of consumer prices (HICP hereinafter) inflation rate of below, but close to 2 % over the medium-term, reflects the focus for inflation expectations and provides a guide for markets.

Expected inflation can be measured by using first, model-free or survey-based measures, or second, model-based measures. Survey based measures use either household or firms views about the future level of inflation or professional forecasters opinion. The inflation expectations of the former tend to differ from those of either professional forecasters or the market. The expected inflation of household or firms tend to be also more dispersed and misaligned with either current inflation dynamics or the central bank target (Coibion, Gorodnichenko, Kumar & Pedemonte, 2020). Using the model-based approach inflation expectations can be derived from the prices of instruments that are linked to the future inflation outcomes and are traded in financial markets, for example Inflation-Linked Swap (ILS hereinafter) rates and Break-Even Inflation rates (BEIR). Central banks use both of the approaches when measuring the inflation expectations, however, survey-based measures act more as an additional indicator than the primary source of information on the future inflation (Camba-Mendez & Werner, 2017; Grothe & Meyler, 2018; Ang, Bekaert & Wei, 2007; Trehan, 2015).

An important input in policy-making are inflation forecasts, usually measured by the HICP. For the short-term inflation forecasting various tools and models have been developed. However, on the medium- to long-term, inflation forecasting is not trivial, nevertheless important as of its relevance for implementation of monetary policy and for the assessment of long run inflation objectives.

Ang, Bekaert and Wei (2007) compared the Out-Of-Sample (OOS hereinafter) inflation forecasts using four different alternatives, among them survey-based measures which use information about the future inflation from agents (consumers or professionals), model-based measures using term structure models, using time series ARIMA models and running forecasting regressions with real activity measures motivated from the Phillips curve. Comparing the alternatives their finding was that the forecasts using the survey-based inflation expectations measures outperformed the other measures. Stock and Watson (1999) studied the inflation forecasts based on the real economic activity (the Phillips curve) and used estimated common factors as proxies for the unobserved state of the economy. Their findings were that in comparison with other asset prices and macro series their OOS forecasts of U.S. inflation turned out to be the most accurate, however performance of the forecasts based on the Phillips curve strongly depends on the sample period. Forecasting power is additionally affected by the instable output gap coefficients of the Phillips curve (Fisher, Liu, & Zhou, 2002; Atkeson & Ohanian, 2001; Clark & McCracken, 2006).

Inflation forecasting can also be performed by combining different alternative models, which usually outperform inflation predictions using autoregressive model as a benchmark. Combination of forecasts can be based on Bayesian Model Averaging (BMA hereinafter), simple equal weighted averaging, or linear combinations of forecasts using weights based on prior information and past performance, among others (Ang, Bekaert, & Wei, 2007; Stock & Watson, 1999; Brave & Fisher, 2004; Wright, 2009).

Some authors use the indirect approach to forecast inflation, modelling the HICP components separately instead of modelling overall HICP (direct approach), however in this way the model for inflation forecasting lacks some of the vital properties (Benalal, Dial del Hoyo, Landau, Roma, & Skudelny, 2004). The literature offers methods to model structural changes, such as change in persistence of inflation or a sudden shift in expected inflation due to a supply shock. In order to account for a regime-switching behavior in inflation a Markov switching model can be used (Evans & Lewis, 1995; Evans & Wachtel, 1993). Giannone, Lenza, Momferatou and Onorante (2014) proposed Bayesian Vector Autoregressive (BVAR) framework, which can be used to construct conditional and un-conditional forecasts of inflation in the short run. The model can capture the dynamic relationships between determinants of inflation and its main components, and can be also used for the scenario analysis, where alternative paths for inflation determinants and components can be used to construct conditional forecasts of inflation.

The key idea of my thesis is to evaluate whether by using dynamic factor model (DFM), which uses targeted predictors extracted from inflation expectations as underlying factors, good predictions for the euro area inflation can be obtained. I focus on medium- to long-term euro area inflation forecasting using a set of different model specifications. Inflation is reported to be a challenging series to forecast due to its volatile features which can be hardly captured. Forecasts of inflation on the medium- to long-term are subject to more pronounced forecasting errors than the ones on the short-term. However, inflation forecasts on the medium- to long-term horizons are of vital policy relevance.

The purpose of the thesis is to propose an alternative modelling strategy, where the constructed model incorporates relevant factors behind the inflation expectations. The constructed dynamic multi-factor model considers the maximum amount of information available at any given point in time. This can include, inter alia, information about recent and expected developments in the main drivers of inflation expectations. The purpose of the thesis is also the assessment of the forecasting power of the constructed model for different forecast horizons.

The main contributions of my thesis to the existing literature are the following. First, the construction of dynamic multi-factor model for inflation forecasting for the euro area, driven by inflation expectations and second, the comparison of the inflation expectations-based inflation forecasts and forecasts of inflation using the target variable itself. In the master's thesis I investigate and provide an answer to the following key empirical questions:

- Do inflation expectations embed information that has a predictive power for the realized inflation?
- Is the forecasting performance of the inflation expectations driven factor forecasting model better than the one using the autoregressive model for inflation forecasting?
- Does the forecasting accuracy of inflation improve when using the inflation forecasts through inflation expectations compared to the forecasting model driven by the variable of interest itself?

In addition, one of the important sub-questions is the identification of possible different groups of macroeconomic variables that are the most informative about the expected inflation and macroeconomic segments that could be most informative about the future HICP inflation.

In this thesis I model the euro area market-based inflation expectations extracted from the zero-coupon ILS curve and study whether this information can be used in euro area inflation forecasting in particular on the medium- to long-term. The fixed swap rate over the relevant horizon is indirectly disclosed in the quoted ILS rate, indicating the expected inflation on the market. Nevertheless, the ILS rate is composed of two main components. First, inflation expectations, and second, the inflation risk premium. The latter represents the compensation for risk related to inflation uncertainty and makes the extraction of long-term inflation expectations from financial instruments further complicated as it is unobservable, however it can be estimated using the affine term structure model.

To decompose the forward ILS-implied inflation rate into inflation expectations and inflation risk premium I estimate the Gaussian Affine Term Structure (GATS hereinafter) model. Affine Term Structure (ATS hereinafter) models are broadly used in the literature, as the expectations about the future events in the market are embedded in the term structure. ATS models are, for instance, used to investigate what is the effect of monetary policy on the term structure of interest rates (Ang & Piazzesi, 2003; Bauer, 2011; Beechey & Wright, 2009), to infer inflation expectations of the market using break-even inflation rates

(Christensen, Lopez, & Rudebusch, 2010) and for the assessment of the non-conventional central bank interference effects during the financial crisis (Christensen, Lopez, & Rudebusch, 2014).

However, in the estimation of the ATS model parameters numerical challenges might occur (Ang & Piazzesi, 2003; Kim, 2008; Christensen, Diebold, & Rudebusch, 2011). To overcome these issues, different methods were constructed (Hamilton & Wu, 2012; Joslin, Singleton, & Zhu, 2011). In this thesis I adopt the Joslin, Singleton and Zhu (2011) canonical representation of the GATS model and I use their algorithm for maximum likelihood estimation, which allows for a computationally efficient estimation of the model. In their model observable pricing factors in the form of the collection of yields can be used instead of latent factors, which are, on a standalone basis or in a combination with the observable factors, used in the standard formulations of the ATS models (Diebold, Rudebusch, & Aruoba, 2006; Duffee, 2011b; Kim & Wright, 2005).

Using Joslin, Singleton and Zhu (2011) approach I decompose the ILS yield curve to the expected inflation under probability measure $\mathbb{P}$ and to the inflation risk premium component. The decomposition is performed at 1-, 2- and 3-year horizons for the period from July 2004 to December 2019. I assume that the pricing factors are the first three principle components of the euro area zero-coupon ILS curve and that underlying factors included in the model are governed by unrestricted $\text{VAR}(1)$ dynamics. After GATS model estimation I use the obtained maximum likelihood parameters to calculate first, the expected value of future inflation under $\mathbb{Q}$, which is represented as model-implied forward rate, and second, inflation expectations under $\mathbb{P}$, which are calculated using dynamic forecasting of the corresponding factors h-periods ahead, with h being the expected future inflation horizon of interest. Inflation risk premium for a particular horizon is then calculated as the spread between the two.

In the next step of my research I identify the underlying factors that drive inflation expectations at different expectation horizons and investigate whether this information can be useful in inflation forecasting using the dynamic factor model. From a large dataset of possible predictor variables that cover the most important macroeconomic segments, I extract the most relevant factors using the Three-Pass Regression Filter (3PRF hereinafter) procedure developed by Kelly and Pruitt (2015), which is an extension of partial least squares. Using the 3PRF procedure, I estimate the relevant factors that drive the movement in the expected inflation obtained from the decomposition of the ILS curve and I use them in inflation forecasting. The number of factors selected in the forecasting regression is based on the relative mean-square-forecasting-error comparing the forecasting accuracy of the model using the estimated factors only versus the benchmark autoregressive model. Using the underlying factors that drive the variable of interest instead of predictor variables themselves reduces the so-called 'curse of dymensionality' that occurs when using the large dataset approach in time series forecasting. Many methods were developed to address and reduce these difficulties (Kelly & Pruitt, 2015; Bulligan, Marcellino, & Venditti, 2015; Zou & Hastie, 2005; Tibshirani, 1996; Bai & Ng, 2008).

In my initial model specification, the inflation forecasts are obtained using the 3PRF factors estimated using expected inflation as a proxy variable. For each of the horizons considered, three different models are estimated. One with an autoregressive term only (the benchmark model), one with an autoregressive term and factors, and the last model with factors only. I compare the resulting pseudo-real-time forecasts from different model specifications to the euro area HICP inflation. In the forecasting evaluation section, I report the outcomes of the OOS forecasting evaluation of competing models. The OOS forecast performance of various models are compared based on the root-mean-square-error value, which is the widely used measure for the model forecasts comparison in the literature (Stock & Watson, 1999; Duffee, 2011a; Ciccarelli & Osbat, 2017).

My findings are that comparing forecast accuracy using different model specifications, ILS rates without inflation risk premium obtained from the ILS curve decomposition dominate for all of the horizons considered. Inflation expectations themselves exhibit the best forecasting performance, as they provide the lowest OOS prediction root-mean-square-forecasting-error (RMSFE hereinafter). It turns out, that expected inflation is the best predictor of inflation on the medium-term.

In addition, from my initial analysis I conclude that using the factors constructed from the relevant predictors of the expected inflation only improves inflation forecasts relative to the AR(1) benchmark model. Even though the forecasts obtained using this model specification offer no improvement over ILS rates, the results suggest there is a predictive content for the inflation in the factors obtained from the 3PRF estimation using inflation expectations as a proxy variable. Therefore, I extend the analysis and investigate the role of inflation expectations for the purpose of forecasting inflation further.

The key idea is to evaluate the performance in forecasting inflation indirectly through inflation expectations. I construct and estimate two additional models which are the modifications of the initial model that I use. In the first model modification I produce inflation expectations forecasts using the same proxies from the 3PRF procedure as in the initial approach. The difference from the primary approach is in the dependent variable, which is no longer realized yearly inflation, but expected inflation for different horizons. In the second model modification I construct the inflation forecasts based on the factors estimated with the 3PRF procedure as well, however as a proxy I use the HICP inflation instead of inflation expectations as in the initial modelling approach and in the first model modification.

The key at this point is to infer which of the modified models results in a better medium-term pseudo-real-time OOS forecasts of the realized yearly euro area seasonally-adjusted HICP inflation. I compare the accuracy of inflation forecasts calculated through inflation expectations forecasts and inflation forecasts themselves using the RMSFE as a metric of comparison. In addition, I investigate what is the macroeconomic information embedded in expected inflation. I identify the macroeconomic segments that carry most of the medium- to long-term information about inflation expectations and compare them to the segments most informative above the HICP inflation.

My findings are that pseudo-real-time OOS forecasts of inflation through forecasting expectations are more stable than inflation forecasts using the factors obtained from the target variable itself, resulting in a better forecasting precision for all of the horizons considered. I support my findings with different starting points to initiate OOS forecasts. I argue that forecasts of inflation through inflation expectations obtained from the standard Kelly and Pruitt (2015) regression could be used as an important additional information for policy makers, for the policy implications estimation and as a robustness check to central bank's own inflation projections.

To summarize, the main result and the contribution of my thesis is that on the medium- to long-term inflation forecasts through forecasting expectations are on average more accurate and reliable compared to the inflation forecasts using the estimated 3PRF factors from the inflation itself. According to my knowledge these results are new and have not been discussed in the literature before.

The thesis is structured as follows. First section describes inflation expectations, inflation risk premium and presents the procedures that can be used to measure the expected inflation. Section 2 provides theoretical overview of ATS models and extends this framework to the case of inflation-linked swap curve. In this section the GATS model is implemented and inflation expectations are derived from the ILS curve. Section 3 gives an overview of time series forecasting using many predictor variables and in detail describes the 3PRF estimation procedure. Section 4 describes the inflation forecasting framework, followed by data description and identification of the relevant factors that drive the inflation expectations. Additionally, this section consists of the following. First, the resulting inflation forecasts based on inflation expectations factors. Second, the intermediate results of inflation expectations forecasting using expected inflation as a proxy variable when estimating the factors using the 3PRF procedure. Next, the inflation forecasts constructed applying the factors estimated with the 3PRF procedure using inflation itself as a proxy variable. Finally, a comparison of pseudo-real-time OOS forecasts of inflation through forecasting expectations and OOS forecasts obtained from the variable of interest itself. The results of the comparison are supported with the robustness check in this section as well. Summary of the most important results and concluding remarks are provided in the conclusion.

1 INFLATION EXPECTATIONS AND INFLATION RISK PREMIUM

Inflation expectations play an important part of modern central banking practice. Their role in the effective implementation and transmission of monetary policy is critical, as they indicate the confidence of the public in the central bank's capability to deliver on its price stability mandate. Central banks use measures of inflation expectations as one of the key indicators (besides headline inflation over a medium-term) to monitor the effectiveness of their policy changes in achieving inflationary target. There exists a close link between monetary policy effectiveness and inflation expectations, therefore, the latter provide valuable evidence about the credibility of a central bank in terms of how well it is meeting its inflation objective. In addition, inflation expectations enter the New Keynesian Philips curve, which represents one of the key relations in the modern monetary policy toolkit. Most importantly, inflation expectations can be used as a cross-check on central bank's own macroeconomic projections of the inflation outlook, which influences its monetary policy decisions.

An important question related to formation of inflation expectations is that of expectations anchoring. There is no widely agreed-upon definition regarding the concept of anchored inflation expectations. Expected inflation is said to be anchored, if the mean forecasts of inflation across agents remain stable and close to the central bank's inflation target, especially in the long run. The literature suggests that inflation expectations should be centered around the target over a sufficiently long horizon and therefore should not react to transitory fluctuations in short-term inflation expectations or in actual inflation. When expectations are poorly anchored, particularly shocks in economic activity, monetary policy and food price inflation move the expectations, with more sharp response on the short-term than on the long-term (Clark & Davig, 2008).

On the other hand, when expectations are strongly anchored, inflation returns quickly to its pre-shock level. Thus, well-anchored inflation expectations in the medium- to long-term perspective are the key interest of policymakers as this is the precondition for effective monetary policy actions. To ensure well anchored expected inflation price stability needs to be delivered by implementing systemic and consistent policy action. Better anchoring of long-term inflation expectations is associated with the rise in central bank transparency and the presence of an inflation targeting regime (ECB, 2018, pp. 73-86; ECB, 2011, pp. 73-86; Bems, Caselli, Grigoli, Gruss, & Lian, 2018, pp. 5-12).

Inflation dynamics is affected by inflation persistence, the slope of the Phillips curve and its responsiveness to other shocks. Inflation persistence is defined by the length of the effects of a shock to the inflation, whether the effects of the shock persists, or if reversion back to inflation trend level is quick (Mishkin, 2007). Inflation should exhibit low persistence and inflationary shocks should have only temporary effects. Temporary shocks to volatile components of inflation tend to affect the expected inflation particularly on the short period. Expectations need to be monitored, as temporary shocks boost the inflation, which might lead to the longer-lasting effects on inflation through their impact on domestic price and wage setting. With respect to that, the medium- to long-term inflation expectations present a more relevant measure for the purpose of monetary policy, as they have turned out to be broadly insensitive to the temporary shocks.

The longer the horizon of the expectations, the more they reflect the level of credibility, accorded to monetary policy by economic agents, regarding central bank's commitment to achieving price stability. Benati's (2008) research results on a sample of advanced economies suggested the existence of the relation between monetary regime and inflation persistence. If inflation expectations are well anchored, the view on the future inflation should be, on average, close to the inflation target pursued by the monetary authority. Price stability of the euro area is defined as a year-on-year HICP inflation rate of below, but close to 2 % over the medium-term, which sets the goal of European Central Bank (ECB hereinafter) to maintain the euro area inflation rates on this level. This definition presents a prime focus for inflation expectations in the euro area and provides a guide for markets (ECB, 2018; García & Werner, 2018).

In the period after the global financial crisis of 2008-09, inflation has remained persistently low such that the question of de-anchoring of inflation expectations has attracted attention among policymakers and academics in evaluation of the effectiveness of unconventional monetary policy measures (Ciccarelli & Osbat, 2017; Coibion & Gorodnichenko, 2015; García & Werner, 2018; Kose, Matsuoka, Panizza, & Vorisek, 2019). In their survey, Ciccarelli and Osbat (2017) provide evidence of weak anchoring of inflation expectations in the euro area in the period after 2014 and report that the euro area has experienced the most pronounced de-anchoring of inflation expectations from the central bank target. This de-anchoring followed a protracted slowdown of economic activity after the onset of the global financial crisis of 2008 and stabilized only after the implementation of the ECB large-scale Asset Purchasing Programme (APP) in 2015. Despite significant monetary expansion, however, inflation expectations had not anchored to the ECB target of inflation below but close to 2 %, and had remained persistently below that level.

García and Werner (2018) report that inflation expectations anchoring has weakened due to the persisting period of low and below-target inflation in the euro area since 2013. Their results show, that there is a significant impact of inflation news on inflation expectations. As reported in Kose, Matsuoka, Panizza and Vorisek (2019) there exists a positive relationship between anchoring and fiscal sustainability proxies and institutions that strengthen sustainability.

1.1 Inflation risk premium

One way of measuring the inflation expectations is the model based approach, in more detail explained in next section, where the expected inflation is derived from prices of instruments that are traded in financial markets. However, the measures derived from financial instruments based on the market trades are affected by two important unobserved components, namely expected inflation and the time-varying Inflation Risk Premium (IRP hereinafter). All market-based indicators of inflation expectations incorporate an IRP component in order to compensate investors for the risks surrounding inflation expectations over the forecast horizon. It can be interpreted as a correction that reveals central expectations and, additionally, informs about which inflation outcomes investors care about most.

Investors bear the risks surrounding their internal estimates of inflation over the forecast horizon and IRP compensates them for those risks that surround their baseline inflation expectations. If investors believe that the future inflation will be stable, inflation expectations remail well anchored and consequently a lower IRP is requested (ECB, 2011; Camba-Mendez & Werner, 2017; ECB, 2018). Due to the IRP component, the signals about expected inflation might be misinterpreted. From this perspective, survey-based measures of inflation expectations explained below might be more backward-looking and not necessarily linked to actual economic behavior.

A substantial research literature covers the analysis of IRP using the arbitrage-free affine term structure (ATS hereinafter) models. Among others, Chernov and Mueller (2008) estimated a model using survey-based inflation forecasts, Chen, Liu and Cheng (2005) used a two-factor model to represent real and nominal yields and Ang, Bekaert and Wei (2008) estimated the inflation data using a regime-switching arbitrage-free ATS model. In addition, Joyce, Lildholdt, and Sorensen (2010) used the ATS model to decompose nominal forward rates into inflation expectations and IRP, and Camba-Mendez and Werner (2017) constructed model-free and model-based indicators for IRP in the euro area and in the US.

1.2 Inflation expectations measures

There is no uniform approach to measuring inflation expectations. Measures of inflation expectations can be classified into two groups. First, survey-based measures and second, model-based measures. Central banks use both, model-based, as well as survey-based measures of expected inflation, where the latter act more as a complementary source of information on the future inflation.

1.2.1 Survey-based measures of inflation expectations

In the first group of measures survey-based evidence is used, whereby either households or firms report their (subjective) views about the level of inflation in some future period, or surveys of professional forecasters who report their internal projections of inflation. Survey-based forecasts of expected inflation cover both the short- and medium- to long-term period and incorporate predictions of future inflation as seen by consumers or professionals (academia or financial institutions), reported in surveys such as ECB Survey of Professional Forecasters or Consensus Economics.

The advantage of survey-based measures is the capability to capture opinion of different types of agents, as participating agents in the survey are financial and non-financial institutions. However, with inflation movement and absolute value of the change in inflation there tends to be a considerable disagreement about the future outlook between them (Mankiw, Reis, & Wolfers, 2003). Additionally, the inconvenience of survey-based measures is that the frequency of surveys is fairly low, at quarterly or at best, monthly frequency, which makes them of limited use for policymakers. Nevertheless, survey-based measures of inflation expectations help with identification of expected inflation and are usually used in modelling in addition to the second group of inflation expectations measures as a robustness check.

As shown in Camba-Mendez and Werner (2017), using the model-free (survey-based) approach, IRP can be calculated as:

$$IRP = \pi^{ILS} - E(\pi), \tag{1}$$

where π^{ILS} denotes the ILS rate as quoted in the market and $E(\pi)$ inflation forecasts as reported in the surveys. The IRP can be computed as the spread between the two.

1.2.2 Model-based measures of inflation expectations

The second group of measures consists of inflation expectations extracted from financial markets using the model-based approach, covering the short- and medium- to long-term horizons. Inflation expectations are embodied in financial asset prices and can be obtained from the prices of instruments that are traded in financial markets and linked to future inflation outcomes such as ILS rates, break-even inflation rates, inflation-linked bonds and inflation options. All of the mentioned instruments are inflation-linked and can thus help monitor developments in short-term inflation expectations as well as provide meaningful information on longer-term inflation expectations (ECB, 2018, pp. 73-86). Model based measures are available at higher frequency and for a more extensive range of horizons than survey-based measures. As the trades happen continuously, market or model based measures provide a quicker indication of the potential shift in the inflation outlook compared to the survey-based measures (Kose, Matsuoka, Panizza, & Vorisek, 2019; Grothe & Meyler, 2018).

In this thesis I use the model-based approach to obtain the inflation expectations, derived from the ILS rates. Zero-coupon ILS rates are the most commonly traded inflation derivatives in the euro area. They incorporate information about private sector inflation expectations. The swap agreement is such that one of the counterparties pays a fixed rate (the ILS rate) π^{ILS}, which is agreed in advance, and the other pays a floating rate, the realisation of π, which is linked to the inflation index over the period of the swap. At the maturity of the swap contract only the net cash flows are exchanged, which are calculated as the difference between the fixed-leg rate and the realized inflation rate applied on the notional value of the contract (Grothe & Meyler, 2018; Camba-Mendez & Werner, 2017).

The construction of the contract is such that over the relevant horizon the fixed swap rate is indirectly disclosed, indicating inflation expectations on the market. ILS rate for maturity n, denoted as $\pi_t^{ILS}(n)$, includes the compensation for the expected changes in the price level. Following derivation and notation of Camba-Mendez and Werner (2017) the ILS rate is set to equalise the cash flows of the inflation beneficiary and inflation payer:

$$(1+\pi_t^{ILS}(n))^n - 1 = E_t^{\mathbb{Q}}\left[\frac{P_{t+n}}{P_t} - 1\right], \tag{2}$$

with $\pi_t^{ILS}(n)$ being the market-quoted discretely compounded inflation swap rate and $E_t^{\mathbb{Q}}\left[\frac{P_{t+n}}{P_t} - 1\right]$ expected changes in the price level. Under the assumption of continuously compounded inflation $\pi_t = \ln(\frac{P_t}{P_{t-1}})$ and ILS rate $y^{\pi}_{t,n} = \ln(1+\pi^{ILS}_t(n))$, equation from above can be re-arranged to:

$$y^{\pi}_{t,n} = \frac{1}{n}\log\left(E^{\mathbb{Q}}_t\left[\exp\left(\sum_{j=1}^{n}\pi_{t+j}\right)\right]\right). \tag{3}$$

The ILS rates for maturity n embed the future inflation expectations over the period t to $t+n$. However, ILS rates are not a perfect measure of inflation expectations as they embody first, the expected inflation, and second, the inflation risk premium related to inflation uncertainty as a compensation for risk.

In the model-based approach, the IRP can be calculated as the difference between the future inflation expectations under $\mathbb{Q}$-probability measure and future inflation expectations under $\mathbb{P}$-probability measure. The price of future payoffs of the ILS contract under the risk-neutral measure should be the same, hence:

$$E^{\mathbb{Q}}\left(\frac{\pi^{ILS}}{1+r}\right) = E^{\mathbb{Q}}\left(\frac{\pi}{1+r}\right), \tag{4}$$

with the realized inflation π, the ILS rate π^{ILS} and risk-free rate r. From the fact that first, ILS rate is set in advance and stays the same after the realisation of π (implying $E^{\mathbb{Q}}(\pi^{ILS}) = \pi^{ILS}$), and second, because risk-free rate is deterministic, it follows that $\pi^{ILS} = E^{\mathbb{Q}}(\pi)$.

Inflation expectations embodied in ILS under $\mathbb{Q}$ and under $\mathbb{P}$ can be extracted using the affine term structure model. With the assumption of no arbitrage opportunities remaining in trading ILS rates at different maturities, the IRP using the ATS model approach can be calculated as the difference between the inflation expectation under both of the probability measures:

$$IRP \equiv E^{\mathbb{Q}}(\pi) - E^{\mathbb{P}}(\pi). \tag{5}$$

Grothe and Meyler (2018) showed that survey-based and market-based inflation expectations are informative sources for forecasting future inflation developments in the short run, having significant predictive power compared to statistical benchmark models. Ang, Bekaert and Wei (2007) also reported that survey-based measures of inflation expectations produce non-negligibly accurate inflation forecasts, however Trehan (2015) documents that the forecasting performance using professional inflation survey has deteriorated.

2 AFFINE TERM STRUCTURE MODELS OF THE YIELD CURVE

In this thesis the model based approach of measuring inflation expectations is adopted, using the Gaussian Affine Term Structure model (GATS hereinafter). One of the founding papers covering ATS models are papers from Vasicek (1977) and Cox, Ingersoll and Ross (1985). Among other they proposed the tractable pricing formulas of instantaneous short rate $r_t = \rho_0 + \rho_{1,X}' X_t$ as an affine function of unobserved N-dimensional state vector $X_t = X(t) = (X_1(t), X_2(t), ..., X_N(t))$. ATS models still remain one of the most popular specifications in the literature. They have been used to address a broad range of questions, as the term structure embodies the expectations about the future events in the market. This information can be extracted and used to predict the effects on the yield curve caused by the changes in the underlying variables, however, usually only a few factors drive the term structure in affine models (Duffee, 2011b).

In ATS models estimation is usually performed using the maximum likelihood estimator (MLE hereinafter). MLE is asymptotically efficient. However, the disadvantage of this method is the numerical challenge in the estimation of the parameters of the model that can occur, especially when the number of parameters to be estimated is large and if the relationship between yields and parameters is non-linear. Due to their latent nature, the underlying factors might rotate during the estimation of a canonical model. This results in a potentially badly behaved likelihood surface with multiple likelihood maxima. Despite being obtained from the same data, model and estimation method, the results from the modelling can turn out to have identical fit to the data with different yields decompositions and consequently different implications for economic behavior. Therefore, affine models estimation can be problematic from the convergence point of view (Ang & Piazzesi, 2003; Kim, 2008; Christensen, Diebold, & Rudebusch, 2011).

Various methods were constructed in order to overcome these issues. Hamilton and Wu (2012) proposed the minimum-chi-square estimation method, Christensen, Diebold and Rudebusch (2011) introduced the model which offered better predictive performance than previous specifications with convenient representation of level, slope and curvature factors, i.e. Nelson-Siegel interest rates model, which is an arbitrage-free ATS model. Furthermore, better behaviour of likelihood functions and hence improvement in maximum likelihood estimation was also achieved by Joslin, Singleton and Zhu (2011), who used the canonical representation of the ATS models, which is a method maximally flexible subject only to constraints necessary for econometric identification. In addition, Joslin, Singleton and Zhu (2011) method and the one from Hamilton and Wu (2012) can be applied to GATS models only.

GATS models are a special type of ATS models, where yields are presented as affine functions of common factors with Gaussian dynamics. The underlying common factors should, however, embody all joint variation in yields. In discrete time framework the joint distribution of factors and yields in GATS models is assumed to be multivariate normal with constant conditional variances. These models represent the fundamental tools for empirical research in macroeconomics and finance (Hamilton & Wu, 2012).

GATS models are of large use when analyzing the relations between yields on assets of different maturities for the purposes of using a no-arbitrage framework (Christensen, Diebold, & Rudebusch, 2011; Ang & Piazzesi, 2003). Nevertheless, in canonical Gaussian dynamic term structure models the conditional forecasts of the pricing factors are invariant to the imposition of no-arbitrage restrictions (Duffee, 2011a). The efficiency of the model estimation does not increase and forecast accuracy of the model is not affected, as the dynamics of the yield curve factors under physical (empirical) probability measure $\mathbb{P}$ is not affected by the constraints imposed under the so-called risk-neutral probability measure (also called an equivalent martingale measure or equillibrium measure) $\mathbb{Q}$. The concept of the two is in more detail described in the section that follows.

2.1 Probability measures $\mathbb{Q}$ and $\mathbb{P}$

The risk-neutral measure is a fundamental concept of modern financial mathematics. The $\mathbb{Q}$-distribution is essentially a skewed $\mathbb{P}$-distribution, with more probability mass allocated to the negative outcomes, as risk averse agents pay more attention to the undesirable states of the world. On the other hand, the price of the underlying asset itself evolves under the physical probability measure. If at a given point in time the financial market participants are more risk averse, then more weight will be given to unfavorable states of the world. There is no exact answer on whether the mean of the $\mathbb{Q}$-distribution is higher or lower than $\mathbb{P}$-distribution and which situation is more unpleasant for the risk averse financial market participants.

The risk-neutral probability measure proved to be necessary and sufficient for the concept of absence of arbitrage in the theory of market models with finite number of assets. The assumption of the no-arbitrage assures the existence of the equivalent martingale measure $\mathbb{Q}$ (Harrison & Kreps, 1979; Nyholm, 2019; Rásonyi, 2004). Joslin, Singleton and Zhu (2011) model is used in this thesis due to the possibility to decompose the yield curve under $\mathbb{Q}$ and under $\mathbb{P}$. The implementation and details of the model are discussed below.

2.2 JSZ decomposition of GATS models

In this thesis the Joslin, Singleton and Zhu (2011) approach is used, which enables the decomposition of the underlying yield curve to time series under the risk-neutral probability measure and physical probability measure. Their algorithm for maximum likelihood estimation follows the step-wise estimation approach and converges to the global optimum almost instantaneously, allowing for a computationally efficient estimation of GATS models.

Standard formulations of the affine term structure models use unobservable risk factors or combination of latent and observable ones (Diebold, Rudebusch, & Aruoba, 2006; Duffee, 2011b; Kim & Wright, 2005). However, in the canonical form model representation by Joslin, Singleton and Zhu (2011, pp. 926-939) observable factors can be used. Invariant transformations are applied to the pricing factors, which allows for the replacement of latent factors by the observable pricing factors in the form of the collection of yields. Observable portfolios of yields can be formed with different sets of pricing factors, with each spanning different spaces, as all can be rotated into a proposed canonical form GATS model.

Linear combination of yields can be formed by applying a weighting matrix W to the yield curve data. Yield curve factors under $\mathbb{Q}$-measure can be thus represented as:

$$X = Wy, \tag{6}$$

with observable portfolios of yields X and yield curve data y. The underlying assumption is that N linear combinations of observed yields are priced exactly, with N being the number of unobserved pricing factors. As long as the pricing factors matrix X is measured without error, unconstrained Ordinary Least Squares (OLS hereinafter) within canonical GATS model gives the maximum likelihood estimates of conditional $\mathbb{P}$ expectations of the pricing factors X.

The assumed factor dynamics under $\mathbb{Q}$ and $\mathbb{P}$ is the following. The vector of the modelled yield curve factors at time t is denoted as X_t. However, following Joslin, Singleton and Zhu (2011), the Vector Autoregressive (VAR hereinafter) models are specified in difference form. Under both, the empirical measure $\mathbb{P}$ and the pricing measure $\mathbb{Q}$, the dynamics of X_t is governed by VAR processes of order one:

$$\Delta X_t = \mu_{0,X}^{\mathbb{Q}} + K_{1,X}^{\mathbb{Q}} X_{t-1} + \Sigma_X e_t^{\mathbb{Q}}, \quad e_t^{\mathbb{Q}} \sim N(0, \mathbb{1}_N), \tag{7}$$

$$\Delta X_t = \mu_{0,X}^{\mathbb{P}} + K_{1,X}^{\mathbb{P}} X_{t-1} + \Sigma_X e_t^{\mathbb{P}}, \quad e_t^{\mathbb{P}} \sim N(0, \mathbb{1}_N). \tag{8}$$

Excluding the idiosyncratic components, the relation between the risk-free short rate r_t and the state vector X_t can be written as:

$$r_t = \rho_{0,X} + \rho_{1,X}' X_t. \tag{9}$$

Given N yield factors, the construction of the entire time-t yield curve can be constructed by setting three parameters only. First, the long run mean of the short rate under the $\mathbb{Q}$-measure ($r_\infty^{\mathbb{Q}}$), second, $\lambda^{\mathbb{Q}}$, which denotes the mean reversion speed of the yield factors under $\mathbb{Q}$ (eigenvalues of $K_{1,X}^{\mathbb{Q}}$), and third, the conditional covariance matrix of yield factors from the VAR model Σ_X. The underlying assumption here is that Σ_X is the same under both of the probability measures, $\mathbb{Q}$ and $\mathbb{P}$. Parameters $\mu_{0,X}^{\mathbb{Q}}$, $K_{1,X}^{\mathbb{Q}}$, $\rho_{0,X}$ and $\rho_{1,X}$ are explicit functions of ($\lambda^{\mathbb{Q}}$, $r_\infty^{\mathbb{Q}}$, Σ_X). Under the assumption of stationarity of the VAR model, eigenvalues of $K_{1,X}^{\mathbb{Q}}$ specify how fast is the convergence to a steady-state, hence expressing the degree of persistence of the process.

The continuously-compounded yields on a m-maturity zero-coupon bond, denoted by $y_{t,m}^o$ can be represented as a linear function of the underlying factors in a form of an affine function of the state X_t:

$$y_{t,m}^o = A_m(\Theta_X^{\mathbb{Q}}) + B_m(\Theta_X^{\mathbb{Q}}) X_t + \eta_{m,t}, \quad \eta_{m,t} \sim N(0, \sigma_\eta^2). \tag{10}$$

The model-implied component of yields $y_{t,m}$ can be thus represented as:

$$y_{t,m} = A_m(\Theta_X^{\mathbb{Q}}) + B_m(\Theta_X^{\mathbb{Q}})X_t, \tag{11}$$

with coefficients (A_m, B_m) that satisfy standard Ricatti equations.

Using this approach, the likelihood function can be partitioned in a convenient way. Since X are observed factors, the parameters of the $\mathbb{P}$-conditional likelihood function of the observed yields can be separated to the ones that govern the conditional distribution of observed yields ($\mathbb{Q}$-measure parameters) and the ones that influence the conditional $\mathbb{P}$ density of the pricing factors X_t ($\mathbb{P}$-measure parameters):

$$f(y_t^o|y_{t-1}^o;\Theta) = \underbrace{f(y_t^o|X_t;\lambda^{\mathbb{Q}},r_\infty^{\mathbb{Q}},X^{\theta_m},\Sigma_X,)}_{\text{Conversion of factors into yields}} \times \underbrace{f(X_t|X_{t-1};\mu_{0,X}^{\mathbb{P}},K_{1,X}^{\mathbb{P}},\Sigma_X)}_{\text{Time-series evolution of the factors}}, \tag{12}$$

with observed yields y_t^o, observed factors $X_t = X_t^o \in \mathbb{R}^N$, and X^{θ_m} being the conditional distribution of measurement errors $y_t^o - y_t$, for some $\theta_m \in \Theta_m$. In such parametrization, the covariance of innovations is the only link between the two parts of the conditional density (Joslin, Singleton, & Zhu, 2011). The implementation of the GATS model on ILS curve is discussed below.

2.3 Affine term structure of ILS rates

Inflation expectations in the euro area can be, as already mentioned, derived using the survey-based measures or can be extracted from financial markets information using the model-based approach. In this thesis the market-based inflation expectations are obtained from the market prices of euro area zero-coupon ILS rates. To model the ILS curve, I use the arbitrage-free GATS model. Using this model, I decompose ILS rates to obtain the inflation expectations under $\mathbb{P}$ and compare them to the model implied ILS forward rate curve under probability measure $\mathbb{Q}$, as well as to gauge the size of the inflation risk premium.

For the decomposition of the ILS yield curve to inflation expectations and inflation risk premium I adopt the Joslin, Singleton and Zhu (2011) approach. This approach is appropriate as affine models in general only characterize the $\mathbb{Q}$-dynamics, but for the term premium decomposition the market prices of risk are needed ($\mathbb{P}$-dynamics). Instead of using the latent factors, the model is rotated to obtain the model that depends on principal components $X_t \in \mathbb{R}^N$ of the ILS rates as underlying factors.

As in Camba-Mendez and Werner (2017) I assume that the pricing factors X_t are the first three principle components of the euro area zero-coupon inflation swap curve. The underlying factors included in the model are governed by unrestricted VAR(1) dynamics, as in (7) and (8), and represent linear combinations of the collection of yields y_t. Hence, for any full-rank matrix $W \in \mathbb{R}^{N\times J}$, with N being the number of principle components and J number of maturities of ILS rates that constitute the ILS curve, $X_t = Wy_t$ denotes N-dimensional set of portfolios of observed yields, where W contains the weights obtained from the Principal Components Analysis (PCA hereinafter) of the observed ILS rates. The model-implied yields on a zero-coupon ILS rate $y^{\pi}_{t,m}$ of maturity m and inflation π_t are assumed to be linear functions of the pricing factors X_t:

$$\pi_t = \rho_{0,X} + \rho'_{1,X} X_t, \tag{13}$$

$$y^{\pi}_{t,m} = A_m(\Theta^{\mathbb{Q}}_X) + B_m(\Theta^{\mathbb{Q}}_X) X_t. \tag{14}$$

After the estimation of the GATS model, the obtained maximum likelihood parameters allow for the calculation of expected value of future inflation under $\mathbb{Q}$ and under $\mathbb{P}$. In order to use the Joslin, Singleton and Zhu (2011) approach, the underlying assumptions are that pricing errors are normally distributed and that pricing factors are observed without measurement errors, which allows to estimate the model without using the Kalman filter, which is efficient, but can be computationally very demanding.

Inflation expectations under $\mathbb{Q}$ can be represented as model-implied forward rates, calculated using the estimated parameters from the GATS model. On the other hand, inflation expectations under the physical measure can be obtained by dynamic forecasting of the corresponding factors (following a VAR(1) process) h-periods ahead, where h denotes the expected future inflation horizon of interest. Inflation risk premium at a given horizon can be then constructed as the spread between the expected value of the future inflation under the risk-neutral measure (fitted forward rate) and expected value of future inflation under the physical measure (expected inflation).

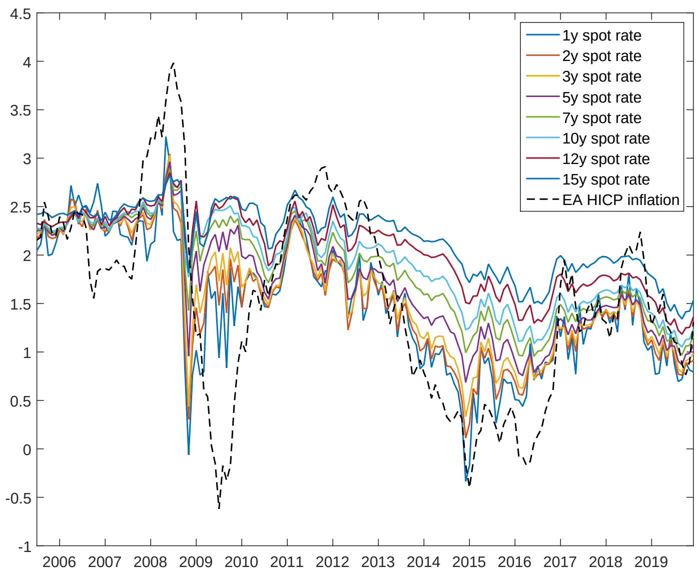
2.4 JSZ model implementation

I perform the decomposition of euro area zero-coupon ILS curve to inflation expectations under $\mathbb{P}$ and IRP for the period from July 2004 to December 2019 for three different horizons: one, two and three years, $h \in \{1,2,3\}$. The estimation of the model is based on a set of ILS rates obtained from Bloomberg on the intra-day frequency over the period considered, for different maturities $m \in \{1,2,3,4,5,6,7,8,9,10,12,15,20,25\}$. Pre-decomposition I aggregate daily time series of ILS rates to a monthly level, which results in the sample covering data from 2014:M7 to 2019:M12. Additionally, as affine models do not capture the seasonal patterns embedded in ILS rates quoted in the market, I adjust the ILS rates prior to modelling to replicate the ILS curve construction approach following Camba-Mendez and Werner (2017).

I perform the PCA on the euro area zero-coupon ILS curve and store the first three principle components of the curve and their corresponding weights W and use them in the further processing. On the calculated principle components I use the unrestricted VAR(1) to obtain the eigenvalues. By sorting the latter in descending order I obtain the $\lambda^{\mathbb{Q}}$ parameter, which enters the numerical optimization constructed as in Wu and Xia (2016). With grid search I calculate the optimal $K^{\mathbb{Q}}_{1,X}$. By running the Joslin, Singleton and Zhu (2011) procedure with the required inputs I compute the $\mathbb{Q}$-likelihood and $\mathbb{P}$-likelihood and estimate all the corresponding $\mathbb{Q}$- and $\mathbb{P}$-parameters of the GATS model.

In Figure 1 the yields for a selected subset $m_s \in \{1,2,3,5,7,10,12,15\}$ of all of the maturities m from the lowest to the highest line (with some exceptions of cross-overs when the yield curves are inverted) are plotted. As observed the dynamics of ILS rates for various maturities is similar to the realized seasonally-adjusted HICP inflation, however their level is different. HICP inflation in the euro area was quite volatile throughout the period under consideration. Two declines particularly stand out, the steaper one from the middle of 2008 until the middle of 2009, where the inflation rate decreased from levels nearly 4 % to around -0.62 %, and the other from approximately 2.9 % in the beginning of 2012 to -0.4 % in the first quarter of 2015. In 2016:Q2 inflation rates were strongly increasing, growing from levels around -0.17 % to 1.95 % in just two quarters. The last prominent peak occurred in the last quarter of 2018, reaching roughly 1.9 %. Since then the inflation rate series trend movement is mostly downward, not fulfilling the ECB target of being close to, but below the price stability target of 2 %.

Figure 1: Time series of swap yields



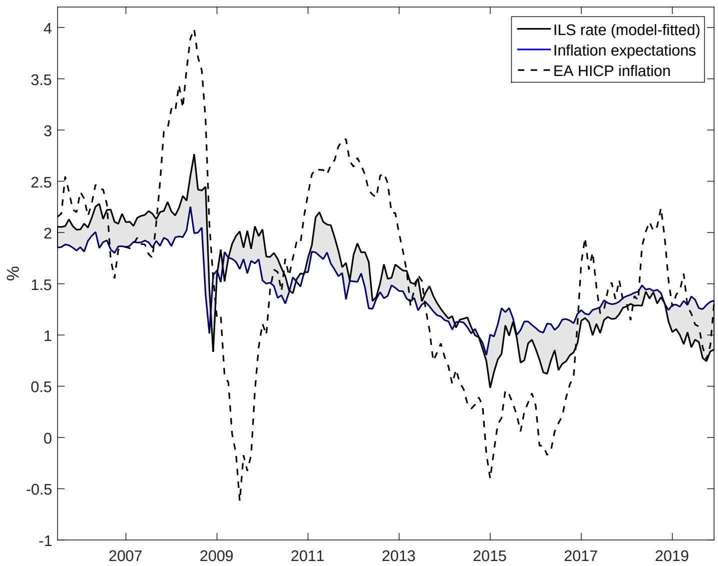
Source: Own work.

In Figures 2-4, the decomposition of one-year forward ILS rate for horizons between one and three years (denoted 1y1y, 1y2y and 1y3y, respectively) is presented. For a more clear comparison the derived inflation expectations for all of the considered horizons are presented in Figure 5 together with the realized seasonally-adjusted HICP inflation. The graphs display some of the key features of inflation expectations and the related inflation risk premium in the euro area. The euro area 1y1y ILS rate reached a trough in the first quarter of 2015 (at around 0.63 %), and both, 1y2y and 1y3y ILS rate, reached the bottom in the third quarter of 2016 (approximately 0.82 % and 1.03 %, respectively). Since then, the ILS rates had an increasing trend until the third quarter of 2018, when they started to decline again, approaching their deepest levels in the last period of the considered sample.

In the time period considered, the expected inflation had been consistently below the ECB target. At a longer expectation horizon of three years, the expected inflation declined significantly less, but also remained persistently below 2% and stabilized at below 1.5% after 2012. Since mid-2016, the measures of inflation expectations for all of the horizons considered experienced a recovery and had an upward trend up until the second half of 2018, which could have been driven by the global economic cycle. Since then, inflation expectations were declining again. Nevertheless, in the last quarter of 2019 a slight increase in inflation expectations can be observed again. The overall movement of inflation expectations over all of the horizons considered is similar in its direction, but different in its volatility. The shorter the horizon, the higher the volatility of inflation expectations.

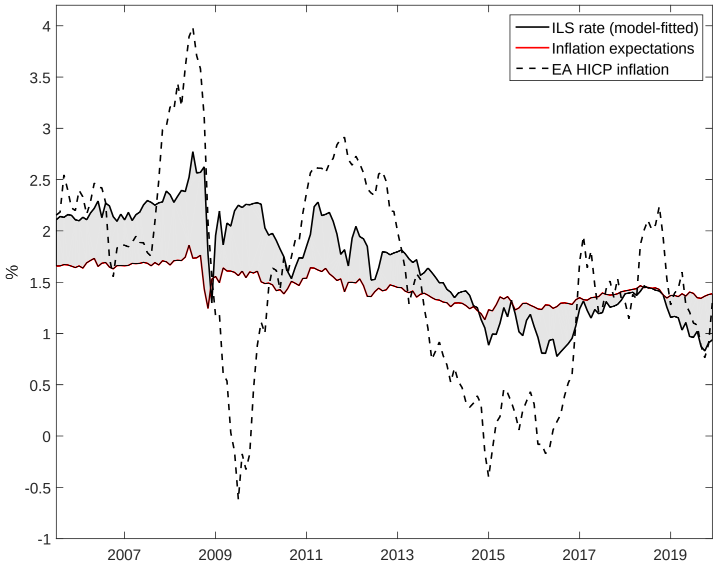
The surface between model-fitted ILS rates and inflation expectations shows the inflation risk premium which represents the inflation protection. Inflation risk exposure increases with maturity, which implies an increasing IRP over the increasing forecast horizon. On average, 1y1y IRP has been around 0.07 %, 1y2y IRP about 0.23 % and 1y3y IRP approximately 0.44 %.

Figure 2: Expected inflation and the inflation risk premium at 1-year horizon



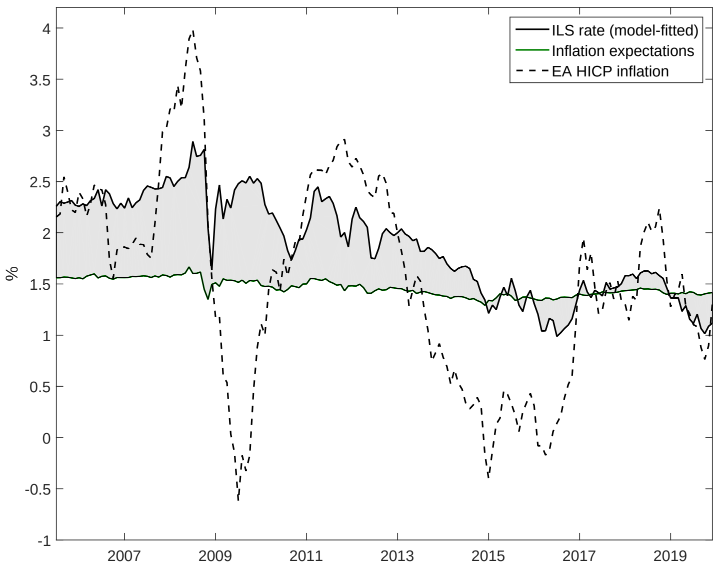
Source: Own work.

Figure 3: Expected inflation and the inflation risk premium at 2-year horizon



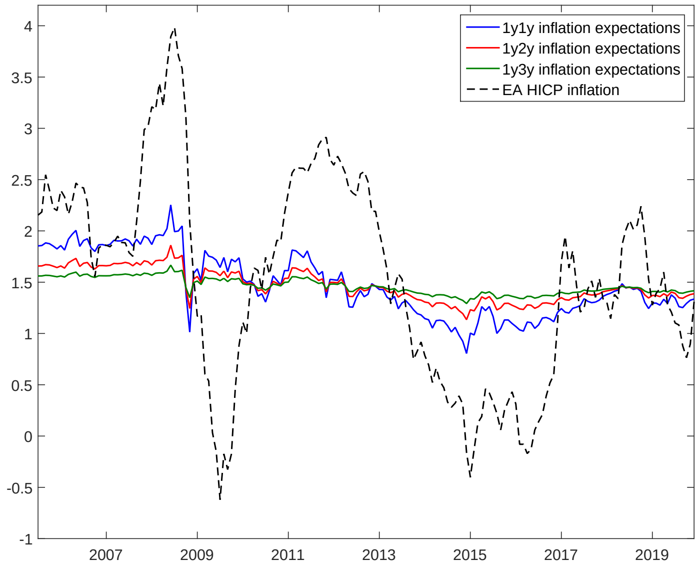
Source: Own work.

Figure 4: Expected inflation and the inflation risk premium at 3-year horizon



Source: Own work.

Figure 5: Euro area expected inflation at 1- to 3-year horizon



Source: Own work.

Note: 1y1y—1y3y denote expectations of yearly inflation 1- to 3-years ahead, respectively.

IRP component of the ILS curve is far more volatile than inflation expectations, accounting for a significant portion of the volatility in inflation-linked swap rates. It can be observed that the ILS rates are prone to move with IRP, as inflation expectations do not fluctuate as much. From the resulting decomposition to inflation expectations and IRP it is evident that most of the fall in ILS rates happened due to the latter component. Similarly, in the last period of decreasing ILS rates, most of the decline occurred because of the IRP component. In addition, as can be observed, the volatility of inflation compensation at longer horizon is almost fully driven by the IRP.

The IRP can be positive or negative depending on economic circumstances. If the inflation is positively skewed, this implies that the risk of high inflation is perceived to be higher than that of lower inflation. These asymmetries in inflations risks contain important information for premia developments interpretation (Garciá & Werner, 2010).

Results for all of the horizons considered suggest that one-year ahead and two-year ahead IRP turned negative around the last quarter of 2014 and have been persisting in negative numbers thereafter. Similar movement can be observed in three-year ahead decomposition of ILS rates, except for the short period from the beginning of 2017 until the last quarter of 2018 when IRP turned out to be positive again. However, for all of the horizons considered IRP remains in negative numbers at the end of the observation period, implying a downside skewness of inflation risk as observed by market participants.

Negative risk premium from the mentioned period forward also supports recent findings in the literature, which emphasize the possibility that the sign of IRP has changed. The cause of a decline in IRP to lower levels or even negative ones could reflect growing fears of the global economy slowdown, weakly perceived inflation risks. inflation uncertainty, or an expectation of a deflationary recession among investors. Following Camba-Mendez and Werner (2017), negative IRP can be the result of deflation fears, which impact the inflation expectations specifically in the short run.

3 TIME SERIES FORECASTING USING MANY PREDICTOR VARIABLES

Expected development of the state of the economy and its real-time assessment is of substantial value for economic agents and policy-makers, which even emphasized in the recent crisis. Many methods were developed to reduce the difficulties which come forth when dealing with large panels. Even though a large dataset of predictors offers a much richer base from where information can be extracted, several predictor variables imply many parameters to estimate. Having many possible regressors in a dataset, when N is large, asymptotic difficulties of OLS emerge. Structure of the model should be such, that estimation error is asymptotically negligible but still allowing for extraction of as much relevant information as possible, as the information from a large number of predictors should be embedded into the forecast. In this section different methods to tackle the problems that come with the large panel approach are discussed.

In general, possible solutions to reduce the problem of dimensionality are first, using the models with no more than a few variables, and second, estimation of the underlying factors that drive the variable of interest. For the purpose of the former a lot of econometric work was done on shrinking. These methods have the shrinkage representation, such as BMA, empirical Bayes methods (EBM) and bagging. In shrinking methods the weights attached to the possible predictor variables from a large dataset are calculated as a product between the OLS estimator and the value of shrinkage factor (Bulligan, Marcellino, & Venditti, 2015).

Some of the most popular regularization techniques used when fitting a linear regression model to a high-dimensional data, or if variables from the data sample tend to be correlated are elastic net regularization, the Least Absolute Shrinkage and Selection Operator (LASSO) method and the ridge regression, which use penalized regression to drop the uninformative variables (Zou & Hastie, 2005; Tibshirani, 1996). Bai and Ng (2008) label these methods as soft thresholding rules for variable selection in the forecasting context. Possible predictors are ordered considering some soft-thresholding rule, elements corresponding to weak predictors are set to zero and only predictors on top of the rank are kept. This feature is important especially when dealing with the set of possible predictors which includes correlated variables, as information in other predictors needs to be taken into account as well. Additional to methods with soft thresholding rules there exist methods with hard thresholding rules. Both can be used to identify the variables from which the factors should be extracted.

3.1 Dynamic factor models

Second solution to reduce the dimensionality of the problem is extraction of the underlying factors from the initial dataset to forecast the variable of interest using the dynamic factor model. This kind of approach is called the factor-based forecasting, which has been in the foreground of developments in the macroeconomic forecasting literature in the recent past. The underlying assumption in the dynamic factor models is that the comovements among a large number of variables are coming from a handful of unobserved factors. The underlying factors embed the covariation amid possible predictor time series and can be used to forecast individual target variable (Giannone, Reichlin, & Small, 2008; Bernanke & Boivin, 2003). The underlying factors in large dimensional panels can be obtained using different methods, where the additional structure on the possible predictors coefficients is imposed and exploited. Using only the most informative factors was found to bring improvements at all forecast horizons (Stock & Watson, 2012a; Bai & Ng, 2008).

When the forecasting problem contains of many predictors, the literature mainly relies on Principal Component Regression (PCR hereinafter). PCR focuses on the cross section according to the covariance within the predictors, meaning that identification of factors driving the panel of predictors in PCR is such, that some of the factors might not be relevant for the target variable dynamics. In order to achieve consistency in PCR method, all common factors should be estimated, including relevant as well as irrelevant ones. When the number of predictive factors is equal to the number of factors driving the target, the PCR forecast is asymptotically efficient. However, if factors are weak and sparsely contribute to the total variability in the predictors the identification of the factors behind the objective can turn out to be a difficult task. Even further, it is not necessary true that using the principal components, predictive factors relevant to the target variable are extracted first (Forni & Reichlin, 1998; Bai & Ng, 2006; Bai & Ng, 2008; Stock & Watson, 2012b).

The link between factors and predictors can be non-linear as well, using the quadratic principal components method, where the matrix of predictors is expanded by including non-linear functions of the observed variables. Bai and Ng (2008) show that allowing for non-linearity can lead to additional gains in forecasts. Another possible method is the diffusion index forecasting method, where common factors are first estimated from a large dataset using the principal components, which are then augmented to a linear forecasting equation that consists of lags of the target variable and other predictors (Stock & Watson, 1999; Stock & Watson, 2002; Stock & Watson, 2012b; Boivin & Ng, 2005). Also, Stock and Watson (1998) constructed an approximate dynamic factor model for balanced and unbalanced panels using weighted averages as predictors.

Forecasting using the dynamic factor model can be performed in two steps. First, using the selected method factors should be estimated from the matrix of predictors $X \in \mathbb{R}^{T \times N}$, and second, linear regression should be used to regress the variable of interest on estimated factors together with (or without) lags of the variable of interest. Matrix of candidate predictors can be represented as:

$$X_t = \Lambda(L)F_t + e_t, \tag{15}$$

with $F_t \in \mathbb{R}^{N\times 1}$ denoting useful factors for target variable forecasting, $\Lambda(L)$ indicating lag polynomial allowing lags of the factors to enter the equations (15) and (16), and $e_t \in \mathbb{R}^{N\times 1}$ vector of idiosyncratic disturbances. Following the notation from McCracken and Ng (2016, pp. 577-583), the dynamic factor model can be in general presented as

$$y^h_{t+h} = \alpha_h + \beta_h(L)\hat{F}_t + \gamma_h(L)y_t + \varepsilon^h_{t+h}, \tag{16}$$

where y_t represents the selected lag of the target variable and y^h_{t+h} the time series variable forecast of interest for the horizon h. $\hat{F}_t$ denotes the estimated common latent factors useful for target variable forecasting from step one of the two-step forecasting process, $\beta_h(L)$ and $\gamma_h(L)$ are lag polynomials with finite order and ε^h_{t+h} stands for the forecasting error. The forecast can be then constructed as

$$\hat{y}^h_{t+h} = \hat{\beta}_h(L)\hat{F}_t + \hat{\gamma}_h(L)y_t, \tag{17}$$

with estimated coefficients $\hat{\beta}_h(L)$ and $\hat{\gamma}_h(L)$.

3.2 Time series forecasting using targeted predictors

The key part of the thesis is the identification of the underlying factors that drive inflation expectations at different expectation horizons. For the empirical estimation I use the 3PRF estimation procedure developed by Kelly and Pruitt (2015). The 3PRF procedure is reviewed in this section, closely following the notation of the authors.

3.2.1 The Three-Pass Regression Filter (3PRF)

Let Y be the $T\times 1$ target variable vector and X the $T\times N$ matrix of predictors, with x_t being a large set of N variables, driven by both, relevant and irrelevant factors. Predictors $(x_1', x_2\prime, \ldots, x_T\prime)' = (x_1, x_2, \ldots, x_N)$ from the matrix of predictors X need to be standardized in order to have unit time series variance. The target variable Y can be presented as a linear function of a subset of the latent factors and some unforecastable noise. The systematic variations of both, the forecast target Y and matrix of predictors X is driven by the latent factors, which are, however, unobservable, causing the infeasibility of the best forecast, as the optimal forecast would come from a regression on the true underlying relevant factors. To forecast Y the factors which are significant drivers of variation in X can be extracted and used. Generally holds that the factor extraction can consist of relevant as well as irrelevant information for the forecast target (Kelly and Pruitt, 2015, pp. 4-10).

The advantage of the 3PRF procedure is that it allows selective identification of the subset of relevant factors only. Hence, the factors that influence the target variable Y are selected and factors which are irrelevant for the target itself are discarded. The subset of factors is denoted by F_t, which is constructed as $F_t = (f_t\prime, g_t\prime)'$, where f_t represents the subset of relevant factors and g_t the subset of irrelevant factors. Irrelevant factors are those, which might drive the cross section of predictive information of x_t, but do not influence the forecast target. The model can be presented in the following way:

$$Y = \beta_0 + F\beta + \eta, \tag{18}$$

$$Z = \lambda_0' + F\Lambda' + \omega, \tag{19}$$

$$X = \phi_0' + F\Phi' + \varepsilon, \tag{20}$$

or in a cross section at time-t as:

$$y_{t+1} = \beta_0 + \beta' F_t + \eta_{t+1}, \tag{21}$$

$$z_t = \lambda_0 + \Lambda F_t + \omega_t, \tag{22}$$

$$x_t = \phi_0 + \Phi F_t + \varepsilon_t, \tag{23}$$

where Z denotes the $T \times L$ matrix of proxies driven by factors and proxy noise ω, with L being significantly smaller than the number of predictors N and the amount of the available time series observations T. Matrix $Z = (z_1\prime, z_2\prime, \ldots, z_T\prime)'$ contains of period-by-period proxy data. The dimension of vectors f_t and g_t is $K_f > 0$ and $K_g > 0$, respectively, where $K = K_f + K_g$. For dimension of a proxy z_t holds that $0 \ < L \ll min(N,T)$. $\Phi = \left(\Phi_f, \Phi_g\right)$ represents the factor loadings of the model and $\Lambda = \left(\Lambda_f, \Lambda_g\right)$ the proxy factor loadings.

As mentioned, Z includes a small set of L proxies, that are driven by the same underlying factors as the target variable, such that $\Lambda = (\Lambda_f, 0)$ with nonsingular Λ_f. This implies three properties. First, loadings of proxies on irrelevant factors are equal to zero, second, proxies' loadings on the required factors are linearly independent and third, the number of proxies is equal to the number of relevant factors. The factor loadings of the target are $\beta = \left(\beta_f\prime, 0'\right)'$, with $\left|\beta_f\right| > 0$, allowing the target to depend only on a strict subset of the factors that drive the predictors.

In finite samples, or if the irrelevant factors g_t are strong while the required factors f_t are weak, it is convenient to estimate and use only the target-relevant factors f_t in (18) or (21). The target-relevant factors are characterized by the so-called proxy variables, which are used for the target variable forecasting. Proxy variables can be created from the target variable and predictor variables themselves. Using the 3PRF procedure, the relevant factors behind the variable of interest can be obtained in an intuitive and simple manner and the computation is almost instantaneous, as it is expressed in closed form.

This problem was also tackled by Boivin and Ng (2006), who performed variable pre-selection before the factor extraction. In addition, the target variable Y can be in principle driven by less factors than the matrix of predictors X, however the estimated loadings of additional factors to those strictly needed in (18) or (21) will converge to zero. The best time-t forecast given by $\beta_0 + \beta_f\prime f_t$ as η_{t+1} is a martingale sequence with respect to all information known at time t. Nevertheless, as the relevant factors f_t are unobserved, the forecast is infeasible.

3.2.2 Properties of 3PRF

The 3PRF is an extension of Partial Least Squares (PLS) and it is constructed as the sequence of OLS regressions. Compared to the linear principal components framework, the 3PRF procedure allows for a more flexible factors structure. It first concentrates the cross section corresponding to the covariance with the forecast target and, second, uses only predictors informative for the objective in estimation of the factors (target-relevant factors). The latter property of the 3PRF is of particular importance especially when the number of relevant factors is strictly less than the number of total factors in the matrix of predictor variables and when other components from the data matrix dominate the target-relevant principal components.

In order to achieve identification, two assumptions must hold. First, factors should be orthogonal to one another and second, the covariance of predictor loadings should be the identity matrix when N and T diverge. In other words, in order to be cross-sectionally regular, factors and loadings are required to have behaved covariance matrices for large T and N. Together with the additional assumption of nonsingular matrix Λ_f this means that none of the proxy variation is due to the factors that are irrelevant. Therefore, the common component of proxies spans the relevant factors space. Furthermore, some cross section correlation among ε_{it} is allowed, as well as serial dependence among ε_{it} and proxy noise dependence with factors and idiosyncratic shocks.

Kelly and Pruitt (2015) proposed a general and simple solution to estimate only f_t in the model (21)-(22). Their procedure can be represented with the three main steps:

- First, predictors from matrix X are regressed on the proxies. In the first step N time series regressions of x_i on Z are ran, one for each variable of X:

$$x_{i,t} = \phi_{0,i} + z_t\prime\phi_i + e_{i,t}, \tag{24}$$

  for $i = 1, \ldots, N$, with $\widehat{\phi_i}$ being the first pass time series OLS regression coefficient estimates. The estimated coefficients inform about the sensitivity of a particular predictor to factors represented by the proxies.

- Second, coefficients estimated in the first pass are used in T separate cross section regressions. Similarly as in the first step of the procedure, predictors act as a dependent variable, however are regressed on the first pass coefficients $\hat{\phi}_i$. Second pass cross section factor estimates $\widehat{F_t}$ are acquired running the cross section regression of x_t on $\widehat{\phi_i}$:

$$x_{i,t} = \phi_{0,t} + \hat{\phi}_i^{\prime} F_t + \varepsilon_{i,t}, \tag{25}$$

  for $t = 1, \ldots, T$.

- Third pass consists of time series regression of the target variable y_{t+1} on the second pass estimated predictive factors $\widehat{F}_t$:

$$y_{t+1} = \beta_0 + \beta' \widehat{F}_t + \eta_{t+1}. \tag{26}$$

  From the OLS regression estimates $\hat{\beta}_0$ and $\hat{\beta}'$ are retained used with the second pass estimated predictive factors $\widehat{F}_t$ to construct the forecast $\widehat{y}_{t+1} = \hat{\beta}_0 + \hat{\beta}' \widehat{F}_t$.

3PRF factor estimator is consistent if both, first and second pass regressions are consistent, which implies that there is no omitted variable bias in regressions. In the simultaneous limit as cross section size N and time series dimension T increase to infinity, the difference between the feasible forecast and the infeasible best forecast decays, implying that the 3PRF based forecast $\widehat{y}_{t+1} = \hat{\beta}_0 + \hat{\beta}' \widehat{F}_t$ converges to the unfeasible best forecast $\beta_0 + \beta' F_t$:

$$\widehat{y}_{t+1} \to \beta_0 + \beta' F_t. \tag{27}$$

The 3PRF forecast is asymptotically normal. Consistency remains even in the case when target-irrelevant factors dominate the variation in predictors.

3.2.3 Automatic proxy selection approach

Kelly and Pruitt (2015, pp. 15-18) proposed the automatic proxy selection algorithm, which comes handy particularly in the panels with many predictor variables, unbalanced panels, and panels with missing data. The presented selection of the proxies is the following. In the case of only one relevant factor f_t ($K_f = 1$), the target variable itself can be directly used as a sole proxy Z for one factor. When the number of relevant factors is more than one ($K_f > 1$) not enough factors are extracted by the target-proxy 3PRF to asymptotically achieve the infeasible best forecast. The proposed approach is to either use the theory suggested proxies or use the automatic proxy selection algorithm, which can be implemented in the following way.

Let r_k indicate the k^{th} automatic proxy, with the initial automatic proxy being the target-proxy 3PRF, $r_1 = y$. As the second proxy residuals from the initial proxy can be used. These are linearly independent of the first proxy and have non-zero loading on relevant factors and zero-loading on irrelevant factors. As a third proxy the residuals from the two-proxy 3PRF are used, up until the L-automatic-proxy 3PRF. When $L = K_f$ the L-automatic-proxy 3PRF forecast of target variable y satisfies the required assumptions automatically, implying the consistency and asymptotic normal properties of the L-automatic-proxy. Following the notation used in Hepenstrick and Marcellino (2016, pp. 4-7), the iterative procedure of proxy construction to construct L predictive factors can be presented as follows.

- Step 1: initial proxy should be set to $r_1 = y$, followed by the calculation of the 3PRF forecast $\hat{y}_{t,1}$ and the residuals $e_{t,1} = y_t - \hat{y}_{t,1}$.

For $j = 2, \ldots, L$:

- Step j: set $r_j = e_{j-1}$ and compute the 3PRF forecast $\hat{y}_{t,j}$ using $r_1, \ldots, r_j$ as proxies. Similarly as in Step 1 the corresponding residuals can be obtained as $e_{t,j} = y_t - \hat{y}_{t,j}$.

According to Kelly and Pruitt (2015, pp. 21-30) the 3PRF exhibits a strong forecasting performance across a variety of simulation specifications and it produces good nowcasts and short-term forecasts for a variety of financial and macroeconomic variables. Compared to more complex alternatives it performs well in finite samples. However, the disadvantage of the procedure is that difficulties may occur in small samples, as only target-relevant factors need to be estimated, whereas in large samples this does not cause any troubles. Hepenstrick and Marcellino (2016) even extended the 3PRF procedure into the mixed-frequency 3PRF to make the 3PRF applicable in a forecasting context also in large mixed-frequency datasets with possible ragged edges.

4 INFLATION FORECASTING FRAMEWORK

Inflation forecasts represent an important input in monetary and fiscal policy-making and are of crucial importance in decision making. Central banks aim to conduct forecast-based monetary policy, therefore, forecasts should be as accurate and reliable as possible. In addition, inflation forecasts bear important information also for investment decisions and settlement of prices for firms and for investors hedging the nominal assets' risk.

Academic literature covers different methods for inflation forecasting, among which four of them are the main methods. First, forecasts from surveys, second, forecasts from the yield curve, third, time series forecasting and forth, forecasts based on the Phillips curve. Model appropriate and flexible enough for inflation forecasting should have the following two properties at minimum. First, it should enable the timely use of the maximum amount of available information about the main variables that drive inflation, and secondly, it should explain the short run inflation dynamics. The model should capture interactions between the variables that drive the inflation, as well as interactions among those determinants to take into account the potential spill-over between the drivers of the inflation.

Forecasts h-period ahead can be performed using the iterative or direct forecast method. The former uses one-period ahead model which is iterated forward for the selected number of periods and, if properly specified, produces more efficient parameter estimates than the latter. In the direct forecast method, the h-period ahead value of the target variable acts as the dependent variable in the horizon-specific estimate model. Compared to the iterative forecasting method, the direct method of forecasting resulted to be more robust to model misspecification, hence less prone to bias. However, comparing the forecast performance, the iterated forecasts accuracy tends to dominate and even improve with the forecast horizons (Marcellino, Stock, & Watson, 2006, pp. 502-515).

The aim of the thesis is forecasting euro area inflation using large number of predictors, from which common factors that are most strongly correlated with inflation expectations are extracted. Let y_{t+h}^{h} denote the seasonally-adjusted HICP for euro area which is the variable of interest in a h-period ahead forecast. The idea is to compress information embedded in the large dataset of variables into a handful of factors from the initial dataset and then use factor estimates as predictors. The 3PRF regression filter is applied to obtain the factor estimates using inflation expectations derived in Section 2.3 as a proxy. The 3PRF procedure is selected mainly due to the fact that first, only target-relevant factors can be extracted from a matrix of possible predictor variables and second, factors obtained are linked to a group of variables, which is from the policy implication point of view more robust than using specific variables.

4.1 Data description

In the analysis I use 47 variables for the euro area that cover the most important macroeconomic segments from real economic activity and prices, to labour market variables, monetary aggregates, stock market indices, exchange rates and confidence survey indicators. I exclude the yield curve rates and market interest rates because such variables embed inflation expectations and could thus lead to spurious correlation. Analysis is conducted on a quarterly time series available from 2004:Q4 to 2019:Q4 for a total of 61 time series observations. The dataset of possible predictor variables is constructed using the data from different sources, including the FRED Database, Eurostat Database, ECB Warehouse and Yahoo Finance.

Because many of those time series are non-stationary, the data is beforehand appropriately transformed to achieve stationarity. Table 1 lists the (short) name of each series included in the matrix of possible predictors, to which macroeconomic group of variables it belongs and the transformation applied to each specific series. Where transformation is not needed, the variables enter into the model in levels. For other possible drivers of inflation expectations the transformations applied are the first difference of the series (Δ), the first difference of the logarithm of the series (Δln), or the second difference of the logarithm of the series (Δ^2ln). The Augmented Dickey-Fuller (ADF) test is used to infer whether time series are stationary or not. The null hypothesis when using this test is that a unit root exists in the time series versus the alternative hypothesis that the data is stationary. The only time series which remains non-stationary after the transformation is the variable which denotes the Total Employment. With additional differencing of the mentioned time series stationarity could be achieved. Nevertheless, I leave the transformation of the Total Employment as is in order to have consistent transformations across the variables.

4.2 Identification of relevant factors of inflation expectations

To take into account various groups of macroeconomic variables I use the quarterly data, as on a monthly frequency less variables are available. Therefore, I aggregate monthly expected inflation obtained using the Joslin, Singleton and Zhu (2011) decomposition in Section 2.3 on a quarterly level as well. Hence, I estimate the underlying relevant factors on a quarterly level and use them for forecasting euro area HICP on a quarterly basis, presented in Sections 4.3.1, 4.3.2 and 4.3.3.

Table 1: Data description and transformations

Short description	Group	Transformation
DE DAX	Stock market	$\Delta \ln$
VIX volatility index	Stock Market	$\Delta \ln$
USD/EUR	Exchange rates	$\Delta \ln$
CHF/EUR	Exchange rates	$\Delta \ln$
GBP/EUR	Exchange rates	$\Delta \ln$
Real Effective Exchange Rate, CPI deflated	Exchange rates	$\Delta \ln$
M1, in million EUR, WDA and SA	Money aggregates	$\Delta^2 \ln$
M2, in million EUR, WDA and SA	Money aggregates	$\Delta^2 \ln$
M3, in million EUR, WDA and SA	Money aggregates	$\Delta^2 \ln$
HICP: Overall Index, WDA and SA	Prices	$\Delta^2 \ln$
HICP: Unprocessed Food,WDA and SA	Prices	$\Delta^2 \ln$
HICP: Industrial Goods Excluding Energy, WDA and SA	Prices	$\Delta^2 \ln$
HICP: Services, WDA and SA	Prices	$\Delta^2 \ln$
HICP: Processed Food including Alcohol and Tobacco, WDA and SA	Prices	$\Delta^2 \ln$
Producer Price Index: Total Consumer Goods	Prices	$\Delta^2 \ln$
Producer Price Index: Energy	Prices	$\Delta^2 \ln$
Producer Price Index: Industry	Prices	$\Delta^2 \ln$
Residential Property Price Index	Prices	$\Delta^2 \ln$
Commercial Property Price Index	Prices	$\Delta^2 \ln$
Deposits from Corporations, in million EUR	Deposits	$\Delta \ln$
Deposits from Households, in million EUR	Deposits	$\Delta \ln$
Unemployment Rate, % of labour force, SA	Labour market	level
Employment: Total, in thousands of persons, WDA and SA	Labour market	$\Delta \ln$
Unit Labour Costs: Total, WDA and SA	Labour market	$\Delta \ln$
Labour Productivity: Total, per hours worked, WDA and SA	Labour market	$\Delta \ln$
Economic Sentiment Indicator, SA	Survey data	Δ
Industrial Confidence Indicator, SA	Survey data	Δ
Consumer Confidence Indicator, SA	Survey data	Δ
Euro area Current account, as % of GDP	Output data	Δ
GDP: Total, WDA and SA	Output data	$\Delta \ln$
GDP: Gross Fixed Capital Formation, WDA and SA	Output data	$\Delta \ln$
GDP: Exports of Goods, WDA and SA	Output data	$\Delta \ln$
GDP: Exports of Services, WDA and SA	Output data	$\Delta \ln$
GDP: Imports of Goods, WDA and SA	Output data	$\Delta \ln$
GDP: Imports of Services, WDA and SA	Output data	$\Delta \ln$
GDP: Final Consumption Expenditure, WDA and SA	Output data	$\Delta \ln$
GDP: Final Consumption Expenditure of General Government, WDA and SA	Output data	$\Delta \ln$
Industrial Production Index: Total Industry, Excluding Construction, WDA and SA	Output data	$\Delta^2 \ln$
Industrial Production Index: Consumer Goods, WDA and SA	Output data	$\Delta^2 \ln$
Industrial Production Index: Energy, WDA and SA	Output data	$\Delta^2 \ln$
Industrial Production Index: Manufacturing, WDA and SA	Output data	$\Delta^2 \ln$
Final Consumption Expenditure: Households, WDA and SA	Output data	$\Delta \ln$
Final Consumption Expenditure: General Government, WDA and SA	Output data	$\Delta \ln$
Government Total Revenue, as % of GDP	Output data	Δ
Change in Government Debt, as % of GDP	Output data	Δ
Exports of Goods and Services, in million EUR, WDA and SA	Output data	$\Delta \ln$
Imports of Goods and Services, in million EUR, WDA and SA	Output data	$\Delta \ln$

Source: ECB Datawarehouse, (n.d.); Yahoo Finance, (n.d.); Eurostat Database, (n.d.); FRED Database, (n.d.)

The empirical procedure to extract the relevant factors that drive the dynamics of inflation expectations is the following. Before forecasting the target, the data is transformed as presented in Table 1 and standardized, to have sample mean zero and sample variance equal to one. To construct a time $t+h$ out-of-sample forecast, the data at time t, $\Psi_t \equiv \{y_t, x_t, y_{t-1}, x_{t-1}, \ldots\}$, must be known. Then, the 3PRF's three steps are calculated in order to decompose x_t.

From the first pass regressions, which are separately run for each $i = 1, \ldots, N$, I obtain slope coefficient estimators $\hat{\phi}_i$. Also, the second pass regressions are run separately for each $\tau = 1, \ldots, T$, yielding $\hat{F}_t$. The final step differs between the modelling approaches presented in the following sections. I regress the variable of interest y_t on the constant and estimated factors $\hat{F}_{t-h}$ to obtain $\hat{\beta}_0$ and $\hat{\beta}$, which are used to construct the OOS forecast as $\hat{\beta}_0 + \hat{F}_{t-h}\hat{\beta}$, not necessarily explicitly following the Kelly and Pruitt (2015) notation from the final pass of the 3PRF procedure, described in Section 3.2.2.

4.2.1 Measuring forecast accuracy and number of factors selection

I assess the forecast accuracy of each model with the Root-Mean-Square-Error (hereinafter RMSE) of the forecasts, which measures the average magnitude of the forecasting error and is one of the most used measures for the comparison of the accuracy of the forecasts obtained from various models (Stock & Watson, 1999; Duffee, 2011a; Ciccarelli & Osbat, 2017). Giannone, Lenza, Momferatou and Onorante (2014) used similar metric to measure the forecast performance, the Mean-Square-Forecasting-Error (MSFE), which can be split to two components, the bias and variance of the forecast errors.

I use the RMSFE which is computed for the forecast horizon h-quarters ahead as:

$$RMSFE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\pi_{t+h}^{h} - \hat{\pi}_{t+h}^{h})^2}, \tag{28}$$

where $\pi_{t+h}^{h} - \hat{\pi}_{t+h}^{h}$ denotes the difference between the realized inflation and the predicted inflation over the period under consideration. Parameter T indicates the number of forecasts made over the period for which the forecasts are constructed.

The number of factors in the multifactor model can be assumed and set by expert judgement or determined by data. However, the number of factors determination is a complex issue. Only k informative factors that best capture the variation in the dataset of the predictors X should be chosen. When N and T are large, various selection procedures can be used to set the number of factors used in the model. In order to select the number of factors, different criteria are available. It is not, however, as important which assumptions are imposed on the factor model, as it is that the same criteria is used for all of the models considered.

Various selection procedures can be adopted. For instance, Bai and Ng (2002) proposed the panel criteria to consistently estimate the number of factors from observed data, while also information criteria such as Bayes Information Criterion (BIC) and Akaike Information

Criterion (AIC) could be used to select the appropriate number of factors (Brezigar Masten, Glažar, Kušar & Masten, 2008; McCracken & Ng, 2016). For the determination of the suitable number of factors selection in the forecasting regression I use the following approach.

For $k \in \{1, \ldots, 10\}$, I extract k factors from the dataset of possible predictor variables and use them in two forecasting model specifications. First, the model that depends on estimated factors only and second, the benchmark AR(1) model, which contains the autoregressive term only. The number of factors selection depends on the relative Mean-Square-Forecasting-Error (rMSFE hereinafter) comparing the forecasting accuracy of the two based on the squared ratio of the RMSFE values. rMSFE ratio is calculated as:

$$\mathrm{rMSFE}_{\mathrm{F_k}} = \left(\frac{RMSFE_{F_k}}{RMSFE_{\mathrm{AR(1)}}} \right)^2, \tag{29}$$

where AR(1) denotes the benchmark forecast and F_k stands for the candidate model forecast with k estimated 3PRF factors. The final number of factors is set as the k that minimizes the rMSFE:

$$k \equiv \underset{k}{argmin}\{\mathrm{rMSFE}_{\mathrm{F_k}}\}. \tag{30}$$

4.3 Forecasting models and methodology

The main goal of the thesis is to forecast inflation on the medium-term. Initially, I obtain the inflation forecasts applying the factors estimated using inflation expectations as a proxy. I compare the resulting forecasts to the inflation realization using different competing models − one with an autoregressive term only (denoted as AR), one with autoregressive term and factors (denoted as FAR), and the last model with estimated factors only (denoted as F). Forecasts vary for different underlying inflation expectations horizons. The goal of using this approach is to evaluate the usefulness of the factors extracted from expected inflation and to see if the factors have more predictive content compared to the simple autoregressive model, which is used as a baseline model.

The resulting output of OOS forecasting exercises are forecasts of the annual inflation. In all of the model specifications considered and in more detail described below, model forecasts are compared to the yearly seasonally-adjusted euro area HICP inflation rate:

$$\pi_t = \ln \left(\frac{P_t}{P_{t-4}} \right), \tag{31}$$

where π_t denotes the inflation rate for the period from $t-4$ to t with inflation index level P_t at the end of quarter t.

OOS forecasts are computed using the direct forecasting method where the variable of interest is forecast multiperiod ahead (Marcellino, Stock, & Watson, 2006). Additionally, OOS forecasts are recursive, as all available data at time t is used to forecast annual future inflation from t to $t+h$. Therefore, the window length used for the estimation increases through time. In each step h-quarter ahead forecasts are computed, then the sample is augmented by 1 quarter, followed by the corresponding computation of h-step ahead forecast. In this way OOS forecasting procedure produces the inflation forecasts as would have been constructed if the models would be historically used to generate them. The models result in quarterly forecast of the target variable considered and are calculated at different horizons. In Section 4.4 the forecast evaluation sample is extended, allowing for computation of forecasts for different initial dates and displayed as a robustness check.

In the initial modelling approach, realized inflation is compared to inflation forecasts obtained using the factors estimated with the 3PRF procedure where inflation expectations $E_t(\pi_{t+h})$ for different horizons are used as a proxy. My findings are that on the medium-term inflation expectations $E_t(\pi_{t+h})$ are the finest predictor for inflation dynamics for all of the horizons under consideration, as their forecasting accuracy turns out to be best. In other words, forecasts using the primary modelling approach turn out to be weaker than directly using the expected inflation derived from the ATS model. The stated holds for all of the considered competing models.

That said, I try to forecast inflation through inflation expectations $E_t(\pi_{t+h})$ forecasts. In order to do that, I estimate two additional models, denoted as model modifications of the initial model. First model modification (hereinafter Case 1) produces expected inflation forecasts using the same proxies from the 3PRF procedure as in the first approach. Second model modification, indicated as Case 2, constructs inflation forecasts based on the factors estimated with the 3PRF procedure as well, however as a proxy HICP inflation is used instead of inflation expectations as in the previous two modelling approaches.

The competing models used for forecasting in each of the three approaches (namely, initial, Case 1 and Case 2) have the following specification:

$$\text{AR}: \quad y_{t+h}^{h} = \alpha_h + \gamma_h y_t + \varepsilon_{t+h}^{h}, \tag{32}$$

$$\text{FAR}: \quad y_{t+h}^{h} = \alpha_h + \beta_h \hat{f}_t + \gamma_h y_t + \varepsilon_{t+h}^{h}, \tag{33}$$

$$\text{F}: \quad y_{t+h}^{h} = \alpha_h + \beta_h \hat{f}_t + \varepsilon_{t+h}^{h}, \tag{34}$$

with coefficients β_h and γ_h, while definitions for dependent variable y_{t+h}^{h} and independent variable y_t, as well as factors f_t differ in the modelling approaches considered.

- In the initial modelling approach, dependent variable is yearly seasonally-adjusted euro area inflation, defined as $y_{t+h}^{h} = \ln\left(\frac{P_{t+h}}{P_{t+h-4}}\right)$. Inflation observed in time t is denoted as $y_t := \pi_t$. Factors f_t are estimated using appropriately transformed expected inflation $\Delta E_t(\pi_{t+h})$ as a proxy observed in time t for different horizons $h \in \{8, 12, 16\}$.
- In the first modification of the modelling approach, dependent variable is defined as expected inflation $y_{t+h}^{h} = E_t(\pi_{t+h})$, and y_t stands for the lag of inflation expectations in

time t. Factors f_t are estimated using the differenced expected inflation as a proxy for all of the horizons covered, which is the same as in the initial modelling approach. Forecasts of expected inflation for all of the horizons considered (1y1y, 1y2y and 1y3y) are performed from t to $t+h$ with $h = 1$, implying the 1-quarter ahead forecasts.

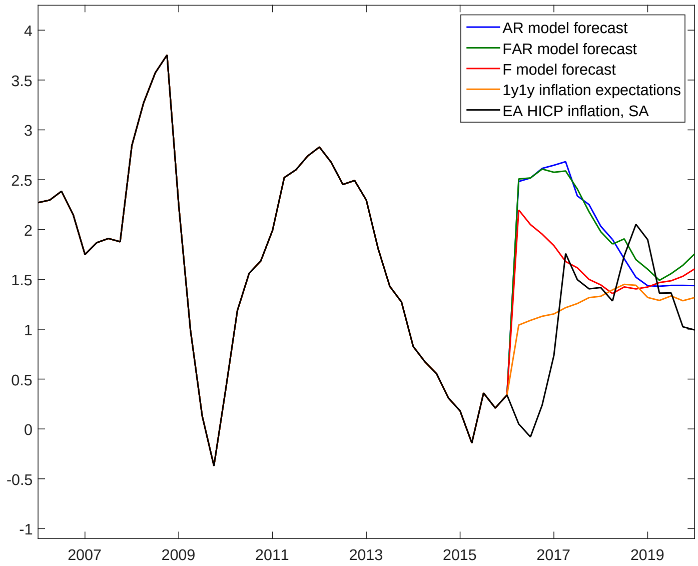
- In the second model modification the dependent variable y^h_{t+h} and lag of the dependent variable y_t are the same as in the initial modelling approach. The difference between the latter and the second model modification is in the proxy variable used in the 3PRF estimation of factors. Factor estimation is performed with the 3PRF procedure using the transformed HICP inflation as a proxy. All of the three competing models construct h-quarters ahead inflation forecasts for $h \in \{9, 13, 17\}$. Step h is set in such a way that the allignment with the inflation expectations forecasts produced from the first model modification is achieved.

4.3.1 Inflation forecasts based on inflation expectations

In this section I construct the inflation forecasts adopting the factors obtained in the 3PRF procedure using inflation expectations as a proxy. For each of the horizons considered, I estimate three competing models as referred above. When using 1y1y expected inflation as a proxy I set parameter h to 8 and for 1y2y and 1y3y inflation expectations h is set to 12 and 16, respectively.

For all of the horizons considered the performance of the model with factors only compared to the benchmark turns out to be best when the selected number of factors is one. Therefore, only the first factor is used in all of the competing models $-$ AR, FAR and F. In Figures 6-8 pseudo-real-time OOS forecasts for the yearly euro area seasonally-adjusted HICP inflation are presented. Additional to model forecasts also inflation expectations for the appropriate horizon are presented. From the presented results it is evident, that the best forecast performance is achieved using the inflation expectations derived from the ATS model decomposition. This result is the consequence of the volatile features of the HICP inflation, which can be hardly captured at the particularly long forecasting horizons considered ($h \in \{8, 12, 16\}$). On the other hand, inflation expectations series are a lot more stable through time. They do not capture the peaks and troughs of the underlying realized inflation, but overall their forecasting performance turns out to be best, supported with forecast accuracy calculations in Table 2.

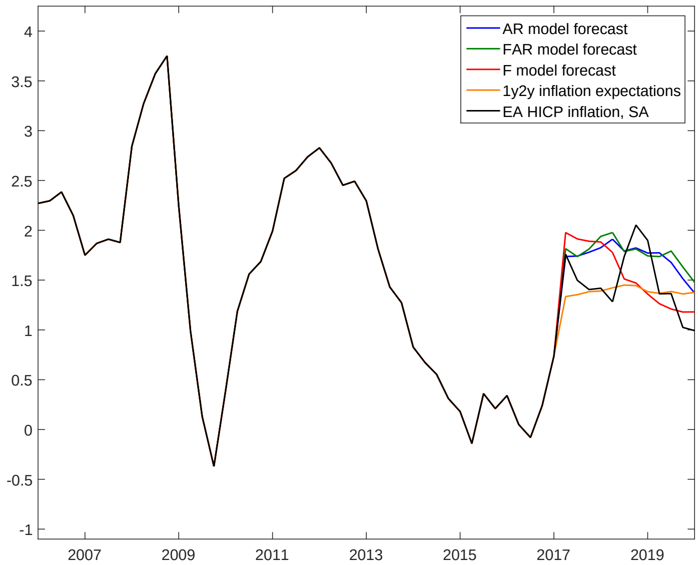
Figure 6: OOS forecasts of inflation: 1y1y inflation expectations factors



Source: Own work.

Note: 1y1y denote the expectation of yearly inflation at 1-year horizon.

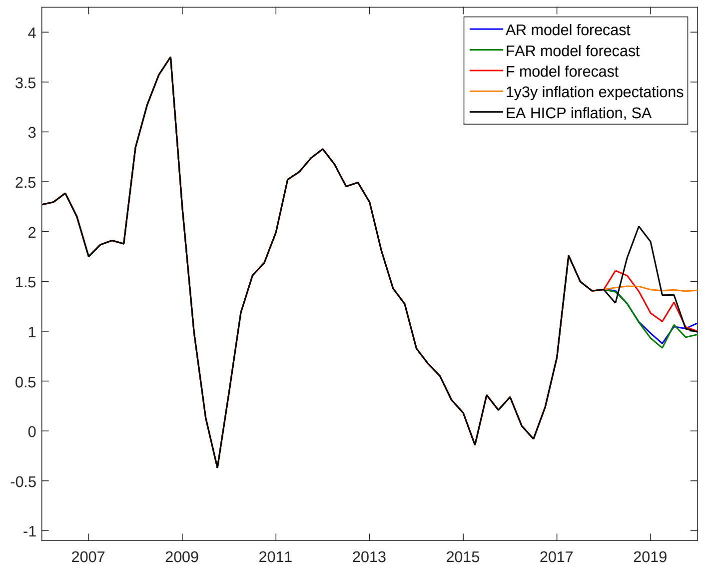
Figure 7: OOS forecasts of inflation: 1y2y inflation expectations factors



Source: Own work.

Note: 1y2y denote the expectation of yearly inflation at 2-year horizon.

Figure 8: OOS forecasts of inflation: 1y3y inflation expectations factors



Source: Own work.

Note: 1y3y denote the expectation of yearly inflation at 3-year horizon.

The RMSFE results are summarized in Table 2, which reports eight-, twelve-, and sixteen-quarter ahead pseudo-real-time OOS RMSFEs calculated relative to the euro area inflation realization for the period considered. Additionally to models AR, FAR and F, I also include the expected inflation $E_t(\pi_{t+h})$ obtained using the Joslin, Singleton and Zhu model for different horizons (denoted as E) in the analysis. It is straightforward that the estimation of the factors affects the RMSFE. Across various factor estimation methods and formations of the forecasts there are differences particularly when having a higher T (lower h). RMSFEs for 1y2y and 1y3y inflation expectations all lie in the range from 0.316 to 0.555, while this range is broader for the shortest horizon considered, namely $h = 8$, with RMSFE ranging from 0.540 to 1.271. Comparing only inflation expectations performance (E) for different horizons implies the worst performance for the shortest horizon $h = 8$ and the best for the 12-quarters ahead horizon.

First finding is, that ILS rates without inflation risk premium for all of the horizons considered dominate as they have the lowest RMSFE. The second finding is, that model F outperforms the benchmark autoregressive model AR when using 1y1y and 1y3y inflation expectations as proxy variables. It should be noted that when using 1y2y inflation expectation the forecast performance of model F is worse, yet similar compared to the AR forecast accuracy. To summarize the results, for all h-steps ahead forecasts ILS rates without the IRP exhibit the best forecasting performance, as they provide the lowest OOS prediction RMSFE.

Table 2: RMSFE: inflation expectations factors

Model	Horizon		
	1y1y*, *h = 8*	1y2y*, *h = 12*	1y3y*, *h = 16*
AR	1.271	0.351	0.540
FAR	1.269	0.407	0.555
F	0.960	0.375	0.380
E	0.540	0.316	0.357

Note: *1y1y−1y3y denote expectations of yearly inflation 1- to 3-years ahead, respectively.

Source: Own work.

I compare the forecast performance of the candidate models using the rMSFE ratio. For all of the horizons considered, forecast accuracy of the model consisting of factors only (F) is compared to the model having an autoregressive term only, which is a standard approach in the literature. Due to the forecast performance of the models presented in the Table 2, I additionally compare the RMSFE of the inflation expectations (E) to the RMSFE of models AR, FAR, and F. The rMSFE ratio is calculated as the ratio between the RMSFE of model i versus the one of model j:

$$\text{rMSFE}_{i/j} = \left(\frac{RMSFE_i}{RMSFE_j}\right)^2, \tag{35}$$

where $i, j \in \{\text{AR}, \text{FAR}, \text{F}, \text{E}\}$. Model i's forecasting performance is superior to the one using the model j if the resulting rMSFE ratio indicates less than 1, and vice versa. The gain or loss in forecasting precision of model specification (FPG hereinafter) i relative to j is then calculated as:

$$\text{FPG}_{i/j} = (1 - \text{rMSFE}_{i,j}) * 100. \tag{36}$$

The results of the analysis are presented in Table 3. The results show that the model using the factors obtained from the 3PRF using inflation expectations as a proxy (F) outperforms the autoregressive model (AR) in 8-steps ahead forecast and 16-steps ahead forecast. The average gain in the former is around 43.0 % and in the latter 50.6 %. On the other hand, in 3-years ahead forecast, the autoregressive model outperforms the one using the factors only, as the forecasting loss using the AR model is approximately 14.0 %. The results imply, that there are generally efficiency gains in inflation forecasts using the estimated factors compared to the benchmark autoregressive model.

Table 3: Relative MSFE

	Relative MSFE	FPG
	1y1y*, h = *8*	
F - AR	0.570	43.0 %
E - AR	0.181	81.9 %
E - FAR	0.181	81.9 %
E - F	0.317	68.3 %
	1y2y*, h = *12*	
F - AR	1.140	-14.0 %
E - AR	0.810	19.0 %
E - FAR	0.603	39.7 %
E - F	0.711	29.0 %
	1y3y*, h = *16*	
F - AR	0.494	50.6 %
E - AR	0.438	56.2 %
E - FAR	0.415	58.5 %
E - F	0.887	11.3 %

Note: *1y1y−1y3y denote expectations of yearly inflation 1- to 3-years ahead, respectively.

Source: Own work.

For all of the forecast horizons considered the pseudo-real-time OOS results using inflation expectations obtained with the Joslin, Singleton and Zhu (2011) decomposition result in substantially better performance at all horizons, relative to all of the other model specifications considered. None of the models generally improve upon the inflation expectations forecasts. For 8-quarters ahead forecasts, the model specification using the inflation expectations provides the highest model forecasting improvements over the models AR, FAR and F. Forecasting precision gain of model E versus the model AR and model FAR is almost 82 %, while compared to the model forecast performance with estimated factors only (model F), gain in forecasting precision is around 68.3 %. For longer horizons considered, the forecasting gain of using expected inflation for inflation forecasting reduces, however still dominates forecasts produced using all the other model specifications. For instance, gain in forecasting performance using 1y3y inflation expectations compared to the benchmark AR and model FAR is around 56.2 % and 58.5 %, respectively. It is interesting, that the forecasting gain is the lowest for the $h = 12$ quarters horizon. Looking at this horizon only, the forecasting gain of 1y2y inflation expectations is the highest when compared to the model including the estimated factors and AR(1) term (model FAR), being approximately 39.7 %.

The presented analysis suggests that using the factors constructed from the relevant predictors of the target variables only, improves inflation forecasts relative to the AR(1) benchmark model, however, offers no improvement over ILS rates without inflation risk premium. Nevertheless, there is a predictive content in the factors obtained from the 3PRF estimation using inflation expectations as a proxy variable. From this investigation follows, that for the purpose of forecasting inflation the role of inflation expectations should be studied further. Therefore, I extend the analysis in the next subsections.

4.3.2 Inflation expectations forecasting

In the previous section we saw that expected inflation $E_t(\pi_{t+h})$ turns out to be the best predictor of inflation in the medium-term. For this reason I investigate in this section the performance in forecasting inflation indirectly through inflation expectations. I construct additional two models, presented in this and the following subsection 4.3.3. First model modification, denoted as Case 1 and presented in this section, is constructed as follows. I estimate the factors with the 3PRF procedure using inflation expectations for different horizons as a proxy variable. This part is the same as in the starting model specification. The difference is in the dependent variable, which is no longer realized yearly inflation, but expected inflation for different horizons.

For all, 1y1y, 1y2y and 1y3y inflation expectations I calculate 1-quarter ahead forecasts, using the direct forecasting method. Such horizon is set in order to be in line with pseudo-real-time OOS inflation forecasts from the subsection 4.3.3 that follows. I construct the 1-quarter ahead pseudo-real-time OOS expected inflation forecast and obtain the inflation forecast for $h+1$ steps ahead. For $h \in \{8, 12, 16\}$ this implies the 9-quarter, 12-quarter, and 17-quarter ahead pseudo-real-time OOS forecasts of inflation.

Specification of each pass in the first model modification is such, that the 3PRF procedure from Kelly and Pruitt (2015) is replicated. Hence, I construct time t forecasts of the realization in time $t+1$. For all of the inflation expectations horizons considered the selected number of factors using the approach from Section 4.2.1 is eight. Therefore, in all of the three model specifications I use the first eight factors. I present the in-sample fit curves to the expected inflation (for the aligned horizon), using the models AR, FAR and F in Figures 9−11.

In Figures 12−14 I present the pseudo-real-time OOS forecasts using the factors estimated with the 3PRF procedure for the expected yearly inflation at 1-, 2-, and 3-year horizon. It is evident, that the performance of the model with factors only is not the best. However, the purpose of model modification is not assessing the forecasting precision of expected inflation forecast compared to the inflation expectations as obtained from the Joslin, Singleton and Zhu (2011) model, but comparison of inflation forecasts through inflation expectations and inflation forecasts as obtained in subsection 4.3.3.

Figure 9: Model specification in-sample fit: 1y1y inflation expectations

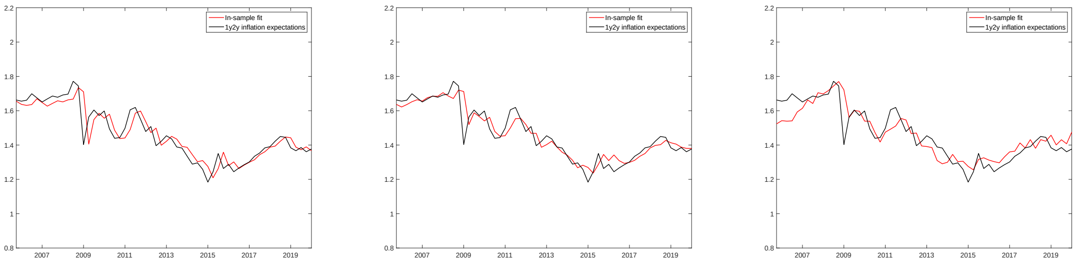
(a) Model AR *(b) Model FAR* *(c) Model F*

Source: Own work.

Note: 1y1y denote the expectation of yearly inflation at 1-year horizon.

Figure 10: Model specification in-sample fit: 1y2y inflation expectations

(a) Model AR *(b) Model FAR* *(c) Model F*

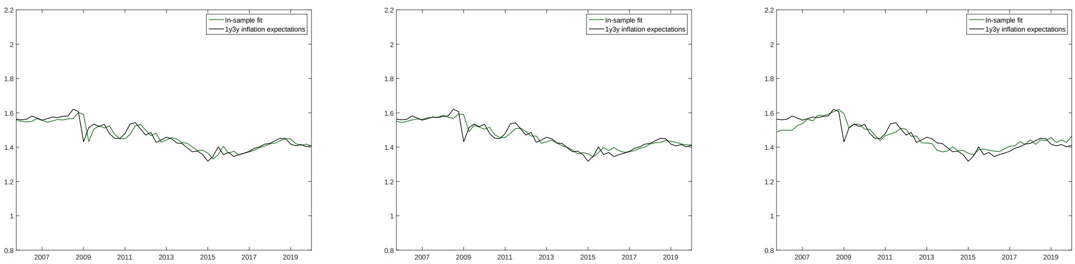


Source: Own work.

Note: 1y2y denote the expectation of yearly inflation at 2-year horizon.

Figure 11: Model specification in-sample fit: 1y3y inflation expectations

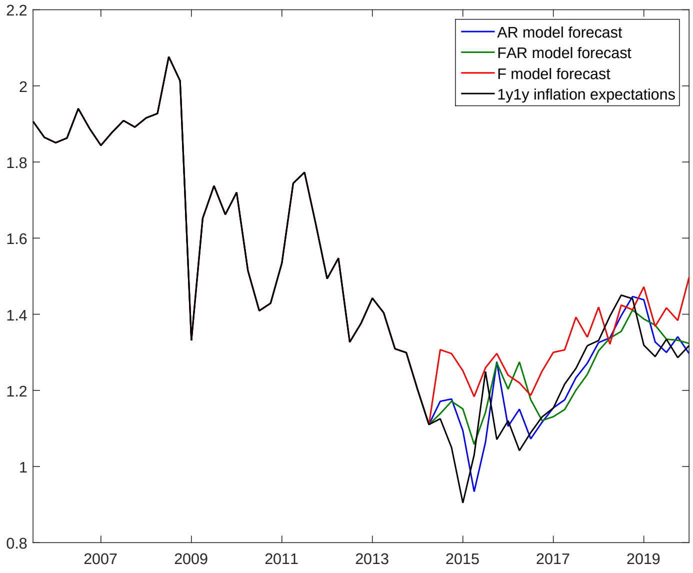
(a) Model AR *(b) Model FAR* *(c) Model F*



Source: Own work.

Note: 1y3y denote the expectation of yearly inflation at 3-year horizon.

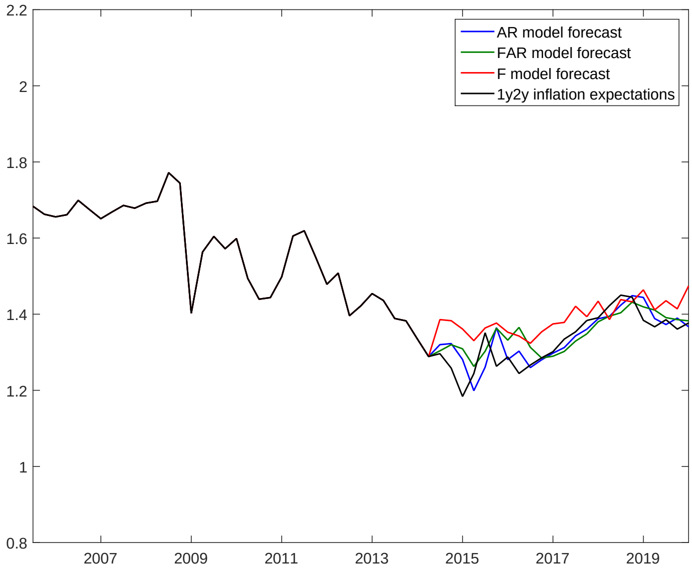
Figure 12: OOS forecasts of 1y1y inflation expectations



Source: Own work.

Note: 1y1y denote the expectation of yearly inflation at 1-year horizon.

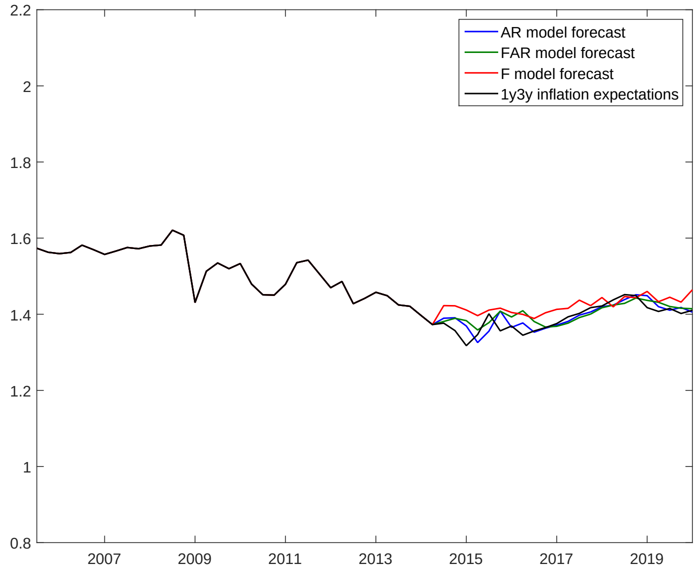
Figure 13: OOS forecasts of 1y2y inflation expectations



Source: Own work.

Note: 1y2y denote the expectation of yearly inflation at 2-year horizon.

Figure 14: OOS forecasts of 1y3y inflation expectations



Source: Own work.

Note: 1y3y denote the expectation of yearly inflation at 3-year horizon.

4.3.3 Inflation forecasting through forecasting expectations

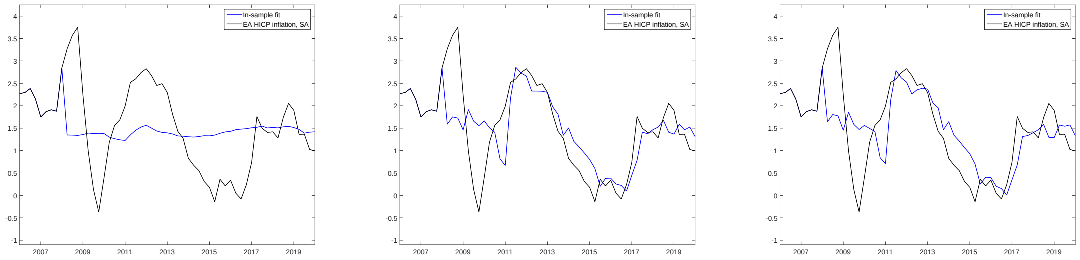
In this subsection I present the results from the second modification of the initial model. The crucial difference between the initial model and second model modification, denoted as Case 2, is in the proxy variable in the 3PRF estimation of the factors. Instead of using transformed expected inflation as in Case 1, as a proxy variable transformed HICP inflation is used. In contrast to Case 1, the dependent variable remains the same as in the initial model, being the realized yearly euro area seasonally-adjusted HICP inflation.

I estimate the same specifications of the models (AR, FAR and F) and construct the forecasts for $h \in \{9, 13, 17\}$ quarters ahead. Such horizons are chosen in order to achieve allignment with the $h = 1$ quarter ahead forecasts of 1y1y, 1y2y and 1y3y inflation expectations obtained in the previous subsection. For $h = 9$ quarter horizon the optimal number of factor results to be three. For the other two horizons considered, namely $h = 13$ and $h = 17$, the model with the first factor only turns out to be best when compared to the benchmark model.

I present in-sample fit curves of the competing models to the euro area seasonally-adjusted HICP inflation in Figures 15$-$17. As evident from Figure 15, the best in-sample fit is achieved when I use the estimated factors in the modelling, as in-sample fit using the benchmark AR(1) model is poor. In-sample fit of the baseline model is meagre also for the other two horizons considered, $h = 13$ and $h = 17$. Figures 16 and 17 suggest that in-sample fit is best when using the model specification that includes AR(1) term as well as estimated factors using the 3PRF procedure.

Figure 15: Model specification in-sample fit: inflation ($h = 9$)

(a) Model AR *(b) Model FAR* *(c) Model F*



Source: Own work.

Figure 16: Model specification in-sample fit: inflation ($h = 13$)

(a) Model AR *(b) Model FAR* *(c) Model F*

Source: Own work.

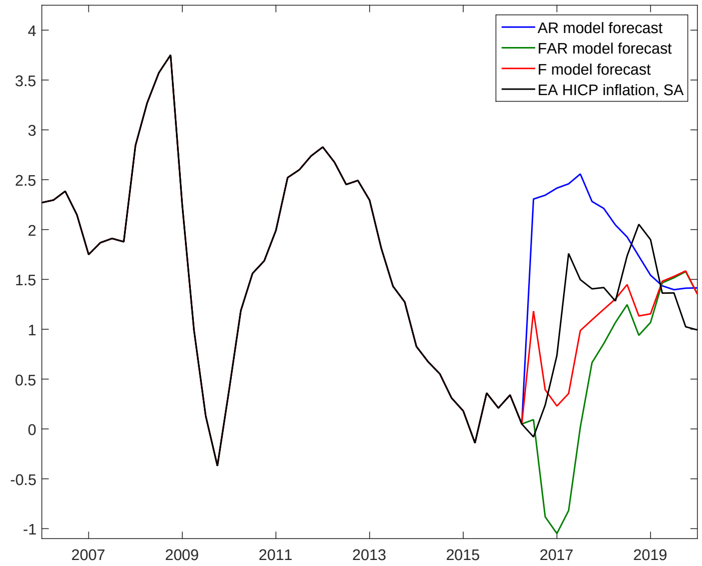
Figure 17: Model specification in-sample fit: inflation ($h = 17$)

(a) Model AR *(b) Model FAR* *(c) Model F*

Source: Own work.

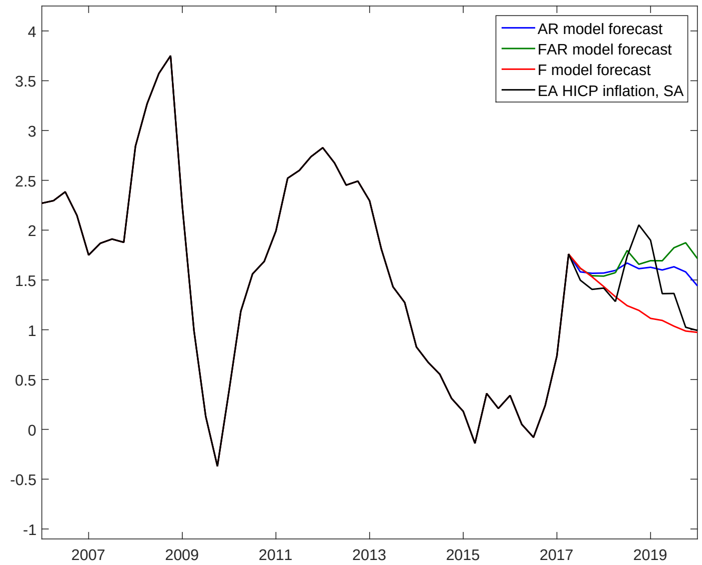
In Figures 18−20 I present the pseudo-real-time OOS forecasts exploiting the factors estimated with the 3PRF procedure using the inflation time series as a proxy variable. Again, more important than comparison of the model specification for inflation forecasting among each other is the comparison of RMSFEs of inflation forecasts using each of the competing models to the RMSFEs of expected inflation forecasts resulting from the subsection 4.3.2. The comparative analysis is presented further below.

Figure 18: OOS forecasts of inflation: inflation factors ($h = 9$)



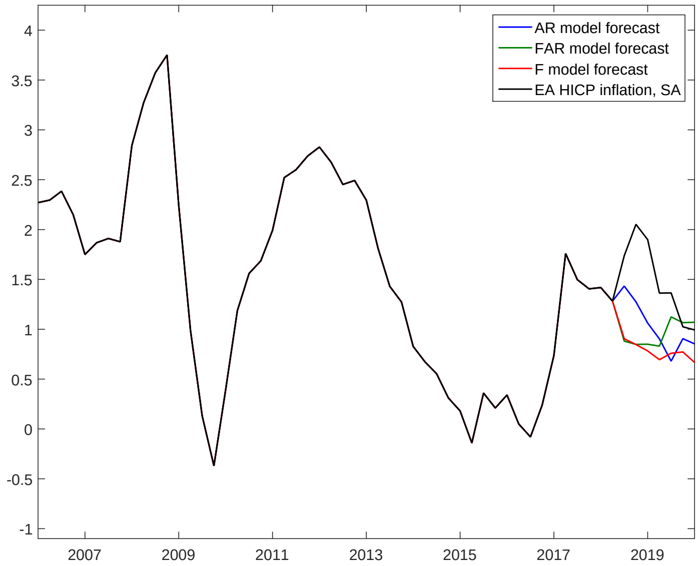
Source: Own work.

Figure 19: OOS forecasts of inflation: inflation factors ($h = 13$)



Source: Own work.

Figure 20: OOS forecasts of inflation: inflation factors ($h = 17$)



Source: Own work.

The goal is to infer which of the two approaches presented above results in a better medium-term pseudo-real-time OOS forecast of the realized yearly euro area seasonally-adjusted HICP inflation. As a metric of comparison I use the RMSFEs of each of the model specifications comparing the forecasting performance from Case 1 and Case 2. The better of the approaches is the one with the lower inflation forecasting error. The resulting RMSFEs for each model specification, estimated by rolling forecasts h-quarters ahead, are presented in Table 4. As already mentioned, inflation expectations OOS forecasts for all of the horizons considered were estimated by rolling forecasts $h = 1$ quarter ahead, while inflation OOS forecasts were estimated for $h \in \{9, 13, 17\}$ quarters ahead, to achieve alignment of the forecasts.

Table 4: RMSFE comparison

	Model specification		
	AR	FAR	F
1y1y*, *h = 1*	0.615	0.589	0.596
1y2y*, *h = 1*	0.346	0.330	0.326
1y3y*, *h = 1*	0.401	0.403	0.389
HICP**, *h = 9*	1.069	1.060	0.644
HICP**, *h = 13*	0.311	0.415	0.406
HICP**, *h = 17*	0.549	0.720	0.791

Notes: *1y1y−1y3y denote expectations of yearly inflation 1- to 3-years ahead, respectively. **HICP denotes euro area seasonally-adjusted yearly inflation.

Source: Own work.

The results of the comparison analysis of the inflation forecasts through inflation expectations and the inflation forecasts using inflation as a proxy in the 3PRF estimation of the factors are the following. Comparing the results obtained from Case 1 for 1y1y expected inflation forecasts and inflation forecasts 9-quarters ahead from Case 2, I can conclude that for all of the model specifications and horizons considered, forecasts of inflation through inflation expectations forecasts result to achieve better forecast accuracy.

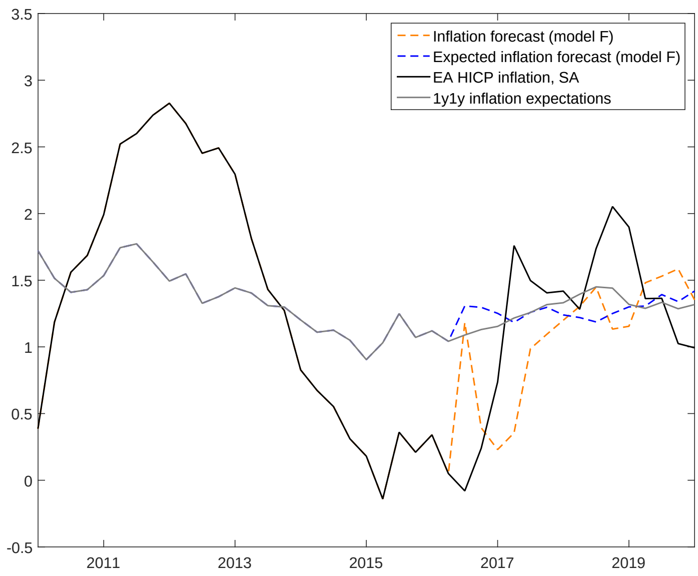
The model that provided the most precise forecasts is the model using estimated factors from the 3PRF with inflation expectations as a proxy and lagged values of expected inflation as well. For the second horizon considered, namely forecast of 1y2y expected inflation and inflation forecasts $h = 13$ quarters ahead, RMSFEs of each of the model specifications result to be more similar than in the previous example. If I use the FAR or F model specification for the forecasts of the expected inflation on the medium-term, I obtain more accurate inflation forecasts. The only exception is the AR(1) model, where the forecast accuracy is better in Case 2 (inflation forecast directly) for $h = 13$ quarters ahead horizon.

Finally, comparing the resulting forecast precision using 1y3y inflation expectations forecast 1-quarter ahead and inflation forecast 17-quarters ahead, model performance of the former is, for all of the model specifications, better than the latter. The best forecasting accuracy is achived using inflation expectations forecasts with the model containing only factors estimated from the 3PRF procedure. To summarise, comparing the inflation forecasts through inflation expectations it turns out that for 1y2y and 1y3y inflation expectations the model consisting of estimated 3PRF factors only results to have the highest forecast accuracy. For 1y1y inflation expectations FAR model is the one which performs the best.

I present pseudo-real-time OOS forecasts from the two considered model modifications in Figures $21-23$. As the main question is whether the factors obtained from the inflation expectations improve the forecasting precision of inflation forecasts in comparison to using the ones obtained from the variable of interest itself, I compare the forecasts obtained with model specification of 3PRF estimated factors only (model F). The RMSFEs of inflation forecasts $h = 9$ quarters ahead using just the estimated factors is around 0.644 and the one using inflation expectations forecasts amounts to 0.596, implying the RMSFE of the former is around 1.1-times larger than the latter. From Figure 21 can be observed that OOS forecast of inflation through inflation expectations is far more stable than the inflation OOS forecast, resulting in a better forecasting precision.

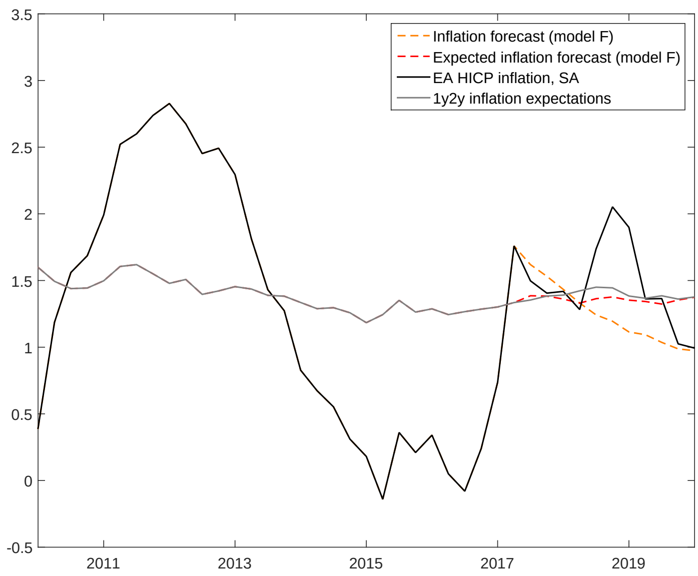
In Figure 22 can be observed, that inflation forecast $h = 13$ quarters ahead in the last part of the forecast tightly approaches the realized inflation. Nevertheless, overall RMSFE of the inflation forecast covering this period results to be 0.406 compared to RMSFE of 0.326 using the 1y2y inflation expectations forecasts. The highest forecasting gain of inflation expectations forecasts results to be when comparing 1y3y expected inflation forecasts and $h = 17$ quarters ahead inflation forecasts. RMSFE of the latter is about 0.791 and of the former 0.389, resulting in approximately half smaller RMSFE if expected inflation forecasts using the factors estimated from the inflation expectations are used for inflation forecasting.

Figure 21: OOS forecasts comparison ($h = 9$)



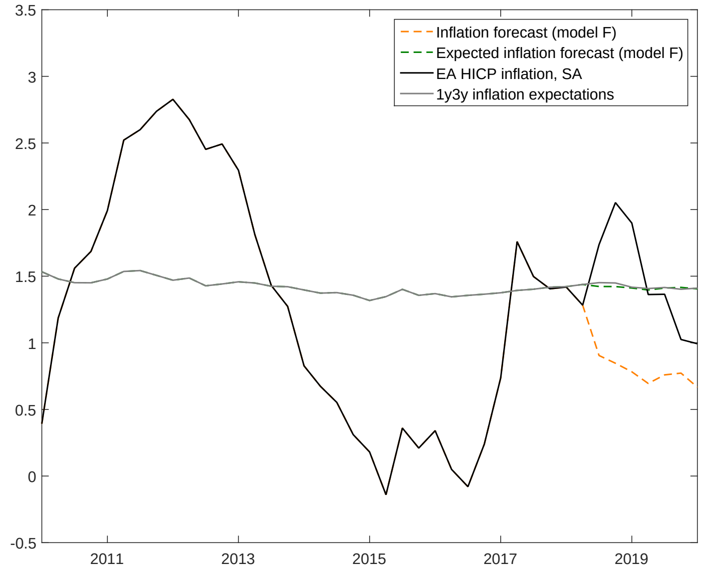
Source: Own work.

Figure 22: OOS forecasts comparison ($h = 13$)



Source: Own work.

Figure 23: OOS forecasts comparison ($h = 17$)



Source: Own work.

To summarize, for all of the horizons considered euro area yearly inflation forecasts result to be less accurate than inflation forecasts through 1y1y, 1y2y, and 1y3y inflation expectations forecasts. Inflation expectations forecasts obtained from the standard Kelly and Pruitt (2015) regression can be, therefore, used as an additional information for the policy makers at minimum. On the medium- to long-term inflation expectations forecasts of inflation produce results that are more accurate than inflation forecasts using the factors obtained from the target variable itself.

4.3.4 Macroeconomic indicators of inflation expectations

Factors obtained from the 3PRF procedure embed the relevant information about the variable of interest. In this section I identify groups of macroeconomic variables that carry medium- to long-term information about inflation expectations and inflation. In other words, I identify macroeconomic segments that are most correlated with 3PRF estimated factors using inflation expectations or inflation as a proxy.

I assess the importance of each particular group of variables in the following way. For each estimated factor, variables from the dataset of possible predictor variables act as dependent variables and are separately regressed on the current factor in the regression. For each I report the R^2 statistics. Next, I obtain the macroeconomic segment R^2_{group} as the mean of R^2 statistics of the respective variables.

Table 5 shows the mean proportion of the variance explained for each group of macroeconomic variables by each of the estimated factors from the 3PRF. The resulting R^2_{group} statistics are presented using expected yearly inflation 1-year ahead as a proxy only, as the resulting most informative groups of variables are similar also when using expected yearly inflation 2-years and 3-years ahead as a proxy. I present the results of the latter two in Appendix 2.

As presented in Table 5, the first factor explains approximately 11.4 % of the variation in the group of prices. Variation of exchange rates, survey data and deposits groups explained by the first factor is around 4.0 %, 3.8 %, and 3.5 %, respectively. Output data and money aggregates groups variation explained is even less, while variation explained in the labour market and stock market group is negligible. All of the factors from the second to the last, eight factor, explain the highest amount of the variation in the stock market group, ranging from about 6.5 % to 9.2 %. Among the deposits, prices and output data groups the variation explained by those factors is similar, while the variation explained for all of the other groups of variables is immaterial.

Table 5: R^2 values: 1y1y inflation expectations

Group	f1	f2	f3	f4	f5	f6	f7	f8
Stock market	0.005	0.077	0.075	0.065	0.080	0.084	0.092	0.092
Exchange rates	0.040	0.001	0.005	0.006	0.007	0.004	0.009	0.007
Money aggregates	0.023	0.005	0.015	0.012	0.011	0.013	0.013	0.013
Deposits	0.035	0.042	0.056	0.044	0.047	0.046	0.052	0.051
Prices	0.114	0.041	0.036	0.036	0.039	0.039	0.039	0.044
Output data	0.028	0.037	0.036	0.041	0.037	0.038	0.035	0.040
Labour market	0.004	0.007	0.001	0.003	0.001	0.002	0.002	0002
Survey data	0.038	0.059	0.041	0.030	0.034	0.036	0.035	0.036

Source: Own work.

I repeat the similar analysis using the HICP inflation as a proxy and present the results in Tables 6 and 7. Table 6 contains the information about the variation explained by the three factors, as this is the optimal number of factors that enter the model. Table 7, however, contains of the R^2 values when regressing the variables on the first factor only, as just this one enters into the model after performing the factor selection procedure.

The results presented in Table 6 suggest, that the first factor explains a similar amount of variation for survey data group of variables, labour marker, exchange rate groups and money aggregates. The variation explained is around 16 % in the survey data group, 15.3 % and 15.0 % in the labour market and exchange rates group, and 14.9 % when looking at money aggregates group. Non-negligible amount of variation explained is also in the stock market and output data segments, while variation explained in the remaining groups of prices and deposits results to be much less. The second factor explains most of the variation in money aggregates and exchange rates groups, while the proportion of the variation explained in the other macroeconomic groups of variables is minor. Similarly to the second factor, the third factor explains most of the variation in money aggregates and exchange rates groups.

R^2 statistics from Table 7 imply, that the first factor from the 3PRF procedure with transformed HICP inflation as a proxy variable explains around 55.8 % of variation in the labour market group, 40.7 % in the survey data and 35.5 % of variation in the output data group. Additionally, proportion of variation explained in the stock market group of variables is roughly 18.5 % and 10.9 % in the money aggregates group. Results for horizon $h = 17$ are essentially the same as for the horizon $h = 13$ and are, therefore, presented in Appendix 2.

Table 6: R^2 values: inflation ($h = 9$)

Group	f1	f2	f3
Stock market	0.110	0.033	0.124
Exchange rates	0.150	0.197	0.189
Money aggregates	0.149	0.251	0.289
Deposits	0.034	0.003	0.028
Prices	0.052	0.026	0.078
Output data	0.105	0.032	0.040
Labour market	0.153	0.010	0.011
Survey data	0.160	0.019	0.034

Source: Own work.

Table 7: R^2 values: inflation ($h = 13$)

Group	f1
Stock market	0.185
Exchange rates	0.063
Money aggregates	0.109
Deposits	0.067
Prices	0.060
Output data	0.355
Labour market	0.558
Survey data	0.407

Source: Own work.

The findings of this section are the following. The macroeconomic segment that carries the most medium- to long-term information about expected inflation is the group of prices, followed by the stock markets. However, except from 11.4 % of variation explained in the prices group by the first factor, the overall percentage of variation explained in each of the groups in Case 1 is quite low, especially compared to the proportion of the variation explained by the factors estimated from 3PRF procedure using inflation as the proxy variable in Case 2. For the $h = 9$ quarter horizon, most informative groups of variables about inflation are survey data, labour market and exchange rates variables. Finally, for the longer horizons of $h = 13$ and $h = 17$, labour market, survey data and output data groups of variables undoubtedly dominate in terms of the correlation with the inflation used as a target-proxy.

4.4 Robustness check

In this subsection I consider various initial estimation sample lengths for calculation of the pseudo-real-time OOS forecasts in order to ensure the robustness of the results presented above. The forecasts presented in the previous subsection are constructed using the parameters estimated on the sample covering the period from 2006 to 2014. I sequentially extend the OOS period, re-estimate the models and compute the forecasts using the model with estimated factors only (model F). However, due to the short sample size I present only the pseudo-real-time OOS forecasts which could be constructed.

I present the resulting RMSFEs for each of the model specifications in Tables 8−10. For all of the tested initial estimation samples the OOS forecasting performance of inflation forecasts through inflation expectations forecasts dominates the forecasts obtained with estimated factors from the 3PRF procedure using inflation as a proxy. Hence, confirming the results presented above.

I perform the first robustness check using the initial estimation sample covering the period from 2006 to 2013 for the parameter estimation. The results are presented in Table 8. RMSFE of inflation forecast $h = 9$ quarters ahead compared to 1y1y inflation expectations forecast results to be nearby 2.2-times higher, RMSFE of inflation forecast $h = 13$ quarters ahead compared to 1y2y inflation expectations forecast about 1.4-times higher and RMSFE of inflation forecast $h = 17$ quarters ahead compared to 1y3y inflation expectations forecast almost 2.6-times higher. I present the resulting comparisons in Appendix 3 in Figures 24−26.

Table 8: Initial estimation sample: 2006−2013

	Model specification		
Expected inflation	AR	FAR	F
1y1y	0.755	0.693	0.697
1y2y	0.619	0.591	0.584
1y3y	0.323	0.329	0.327
Realized inflation	AR	FAR	F
HICP**, h = 9	1.294	1.704	1.503
HICP**, h = 13	0.713	0.972	0.828
HICP**, h = 17	0.544	0.808	0.843

Source: Own work.

Next, I shorten the initial estimation sample for one year, implying the sample from 2006 to 2012. Table 9 shows that RMSFE of inflation forecast $h = 9$ quarters ahead compared to 1y1y inflation expectations forecast results to be approximately 2.2-times higher (1.780 compared to 0.806) and RMSFE of inflation forecast $h = 13$ quarters ahead compared to 1y2y inflation expectations forecast around 1.3-times higher (RMFSE of 0.988 versus 0.758). The comparison of the OOS forecasts constructed using the parameters estimated in this period can be seen in Appendix 3 in Figures 27 and 28.

Table 9: Initial estimation sample: 2006−2012

	Model specification		
Expected inflation	AR	FAR	F
1y1y	0.875	0.799	0.806
1y2y	0.796	0.758	0.758
Realized inflation	AR	FAR	F
HICP, h = *9*	1.374	2.287	1.780
HICP, h = *13*	0.942	1.24	0.988

Source: Own work.

Finally, I set the initial estimation sample to cover the period from 2006 to 2011. Table 10 shows that RMSFE of inflation forecast $h = 9$ quarters ahead compared to 1y1y inflation expectations forecast results to be around 2.4-times higher (1.876 compared to 0.778) for the initial estimation sample considered. This is evident from Figure 29 presented in Appendix 3 as well. Inflation forecast performance is much worse due to the realized peaks and troughs in the initial estimation sample period which has a high impact on the produced forecasts. Forecasts for inflation expectations, on the other hand, remain quite stable throughout the forecasting period resulting in a lower RMSFE.

Table 10: Initial estimation sample: 2006−2011

	Model specification		
Expected inflation	AR	FAR	F
1y1y	0.850	0.768	0.778
Realized inflation	AR	FAR	F
HICP, h = *9*	1.308	2.449	1.876

Source: Own work.

CONCLUSION

In this master's thesis I present a new approach to inflation forecasting on the medium- to long-term. I propose the alternative modelling strategy and construct the model which produces the forecasts of inflation through market-based inflation expectations using the factors estimated with the 3PRF procedure. I assess the accuracy of inflation forecasts based on inflation expectations and study whether the information embedded in the inflation expectations can be used in inflation forecasting.

I analyse the formation of expected inflation in the euro area, focusing on the market-based measure of expectations extracted from the yield curve of inflation-linked swap contracts using the affine term structure model. I model the expected inflation using the Gaussian affine term structure model to obtain the parameters needed to decompose forward inflation quotes into the expected inflation and inflation risk premium.

The results obtained from the decomposition suggest that the expected inflation had been consistently below the ECB target. This needs to be contrasted, however, with the fact that the average level of expected inflation is well below the ECB inflation target, which is ultimately an indication of de-anchoring. Over all of the horizons considered, the movement of expected inflation is similar in its direction, but different in its volatility. The shorter the horizon, the higher the volatility of inflation expectations. Nevertheless, inflation expectations do not fluctuate as much as does the inflation risk premium component of the ILS curve, implying that ILS rates are prone to move with inflation risk premium.

After decomposition of ILS curve to the inflation expectations and inflation risk premium, I use the three-pass regression filter of Kelly and Pruitt (2015) to extract the most relevant factors driving first, the inflation expectations, and then additionally also the factors driving the inflation itself. Using the estimated factors three model specifications are evaluated, namely the AR(1) model, the model with estimated factors only and the model that includes both, the estimated factors and the autoregressive term.

Initially I construct the inflation forecasts using the common factors obtained with the 3PRF procedure using inflation expectations as factor proxy. In this way I assess whether inflation expectations disclose any additional information about the inflation. My findings are that adding the relevant factors that drive inflation expectations into a forecasting equation for inflation improves the forecast accuracy in the case of using 1y1y and 1y3y inflation expectations compared to the forecast accuracy when using the baseline AR(1) model. Therefore, using the dynamic factor model with targeted predictors extracted from inflation expectations as underlying factors provide better predictions for the euro area HICP inflation than the benchmark. The stated indicates, that inflation expectations embed some useful information and have some predictive power for realized inflation forecasting.

However, expected inflation obtained from the GATS model decomposition turns out to be the best predictor of inflation at all horizons considered. Comparing the forecast accuracy of the factor model to the expected inflation from the ILS curve decomposition I discover that for all dynamic h-steps ahead forecasts ILS rates without the IRP have systematic predictive power for inflation. The latter exhibit the best forecasting performance, as they provide the lowest pseudo-real-time OOS prediction RMSFE.

Nevertheless, the factors obtained from the 3PRF estimation using market-based inflation expectations as a proxy variable contain some useful information about the inflation. Compared to the benchmark model I report the forecasts accuracy gains using the 3PRF estimated factors when forecasting inflation. Hence, I extend my analysis and study the usefulness of inflation expectations for inflation forecasting on the medium- to long-term further.

I construct two additional models. The idea is to compare the forecasting performance of inflation forecasts calculated through inflation expectations and inflation forecasts obtained using the 3PRF estimated factors using inflation as factor proxy. Forecasting inflation indirectly using the measure of inflation expectations is the novel approach in the literature. I assess which of the two modified models results to be more accurate in terms of OOS forecasts of the realized euro area seasonally-adjusted HICP inflation. I modify the initial model and estimate two model modifications.

First, I estimate the model based on the factors estimated with the 3PRF procedure using inflation expectations for different horizons as a proxy variable to forecast the expected inflation. The difference from the starting model in my analysis is in the dependent variable, which is expected inflation for different horizons and no longer realized yearly inflation. Second, I construct and estimate the forecasting model for inflation based on the 3PRF estimated factors using the target variable − inflation itself as a proxy. For both of the model modifications rolling forecasts h-steps ahead are constructed in such a way that the allignment of the forecast periods is achieved.

I assess whether inflation expectations disclose any additional information about the inflation and my findings can be summarized as follows. In-sample fit and out-of-sample forecasting exercises suggest that market-based inflation expectations contain useful information about inflation. Inflation expectations have systematic predictive power for inflation at all horizons considered. Pseudo-real-time OOS forecasts of inflation through inflation expectations result in a better forecasting precision and are more stable than the inflation OOS forecasts using the factors estimated with the 3PRF procedure using the target variable itself. This claim is supported by robustness checks with various starting points as well.

Additionally, I find that the macroeconomic segment that is most important in explaining the inflation expectations in the medium- to long-term is the group of prices, which consists of various HICP and PPI indices, as well as RPPI and CPPI. On the other hand, when analyzing the HICP inflation, most informative macroeconomic segments for the longer horizons are labour market, survey data and output data segments, while on the shortest period considered survey data and labour market as before, as well as exchange rate group of macroeconomic variables.

On the medium- to long-term horizons forecasting inflation is not trivial due to the underlying features of the HICP inflation dynamics. Inflation expectations bear the informational value for the realized inflation and contain a predictive power for the forecasts of the latter. The results produced using the modelling strategy for inflation forecasting that I propose suggest that inflation expectations-based inflation forecasts are more accurate than inflation forecasts using the factors obtained from the target variable directly. The constructed inflation forecasts are more robust which implies that policy makers could improve their efficiency when forecasting inflation. Forecasting inflation indirectly using the measure of inflation expectations as approached in this master's thesis is, according to my knowledge, a novel approach in the literature and could be of great interest in policy implications. Inflation expectations forecasts obtained from the standard Kelly and Pruitt (2015) regression can be used to monitor the effect of the monetary policy decisions, to assess the effectiveness in achieving the inflationary target on the medium- to long-term and to check the robustness of central bank's own inflation outlook.

The thesis could be improved and extended even further by additionally evaluating the forecast accuracy for some sub-aggregates of the overall HICP inflation. For instance, inflation based on overall HICP index excluding energy and unprocessed food prices could be used, which excludes the components of consumer prices which are the most volatile, as volatility component, particularly when pronounced, makes it harder to forecast the inflation on the medium- to long-term. Moreover, different approaches could be employed to obtain the underlying factors or individual variables that drive the dynamics of inflation expectations. Finally, the model specification and its estimation could be further extended using, for instance, the Factor-Augmented Vector Autoregressive (FAVAR) models.

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APPENDICES

Appendix 1: Povzetek v slovenskem jeziku

NAPOVEDOVANJE INFLACIJE NA OSNOVI INFLACIJSKIH PRIČAKOVANJ: DINAMIČNI VEČFAKTORSKI MODEL

Ključni ideji moje magistrske naloge sta napovedovanje inflacije evrskega območja z dinamičnim faktorskim modelom s faktorji, ki ženejo inflacijska pričakovanja in ocena natančnosti teh napovedi za različne horizonte. Gre za alternativno strategijo napovedovanja inflacije, kjer faktorje ocenim s tro-stopenjskim regresijskim filtrom (angl. *Three-Pass Regression Filter*, v nadaljevanju: 3PRF metoda). Ta omogoča ekstrakcijo faktorjev, ki so najbolj relevantni za napovedno spremenljivko.

Glavna prispevka moje magistrske naloge k že obstoječi literaturi sta uporaba dinamičnega večfaktorskega modela za napovedovanje inflacije evrskega območja s faktorji, ki ženejo inflacijska pričakovanja in primerjava posrednih napovedi inflacije preko inflacijskih pričakovanj in neposrednih napovedi inflacije z uporabo faktorjev, ki ženejo inflacijo samo. Primarna raziskovalna vprašanja magistrskega dela so:

- Ali ima informacija, ki jo vsebujejo inflacijska pričakovanja, napovedno moč za napovedovanje inflacije?
- Ali je napovedna moč modela s faktorji, ki ženejo inflacijska pričakovanja, boljša od napovedne moči običajnega avtoregresijskega modela za napovedovanje inflacije?
- Ali se natančnost napovedovanja inflacije izboljša ob uporabi modela s faktorji, ki ženejo inflacijska pričakovanja, v primerjavi z napovedno močjo modela s faktorji, ki ženejo ciljno spremenljivko − inflacijo samo?

Poleg tega v magistrskem delu identificiram in analiziram razlike v skupinah makroekonomskih spremenljivk, ki so najbolj informativne za inflacijska pričakovanja in tistih makroekonomskih segmentov, ki so najbolj informativni za inflacijo.

Za napovedovanje inflacije najprej uporabim dinamični večfaktorski model s faktorji, ki ženejo inflacijska pričakovanja. Ta imajo pomembno vlogo pri učinkovitem izvajanju denarne politike, saj predstavljajo enega izmed osrednjih pokazateljev verodostojnosti centralne banke. Inflacijska pričakovanja odražajo zmožnosti centralne banke pri doseganju inflacijskih ciljev in zagotavljanju stabilnosti cen. Centralne banke poleg osnovne mere inflacije na srednji rok uporabljajo inflacijska pričakovanja kot enega od kazalnikov za spremljanje učinka sprememb in odločitev, ki se vežejo na izvajanje denarne politike ter za navzkrižno preverjanje makroekonomskih projekcij glede obetov inflacije.

Predpogoji za učinkovitost ukrepov denarne politike so dobro zasidrana inflacijska pričakovanja na srednji do dolgi rok, zato vprašanje zasidranosti inflacijskih pričakovanj predstavlja enega izmed ključnih interesov oblikovalcev politike. Inflacijska pričakovanja so zasidrana, ko so povprečne napovedi inflacije stabilne in blizu inflacijskega cilja, določenega s strani centralne banke. Stabilnost cen evrskega območja je definirana kot medletna stopnja inflacije, ki je nižja od 2 %, vendar blizu tej ravni na srednji rok, kar predstavlja vodilo za trge in fokus za inflacijska pričakovanja.

Inflacijska pričakovanja se lahko meri na podlagi anket, lahko pa se jih oceni na podlagi modela. Anketna inflacijska pričakovanja zajemajo (subjektivni) pogled gospodinjstev, podjetij ali profesionalnih napovedovalcev o prihodnji stopnji inflacije. Slabost pričakovane inflacije, merjene na podlagi anket, je v frekvenci merjenja le-teh. Izvajanje anket poteka na četrtletni ravni (oziroma v najboljšem primeru na mesečni ravni) kar omejuje njihovo uporabnost za oblikovalce politike. Iz tega razloga pričakovano inflacijo običajno modeliramo, se pa kljub temu inflacijska pričakovanja na osnovi anket uporabljajo za preverjanje robustnosti inflacijskih pričakovanj pridobljenih iz modela.

V magistrski nalogi ocenim inflacijska pričakovanja z modelom z afino časovno strukturo (angl. *Affine Term Structure model*, v nadaljevanju: ATS model), ki omogoča dekompozicijo krivulje brezkuponskih inflacijskih zamenjav (angl. *zero-coupon Inflation-Linked Swap rates*, v nadaljevanju: ILS) na pričakovano inflacijo in premijo za inflacijsko tveganje. Ta komponenta krivulje predstavlja nadomestilo investitorjem za tveganje, povezano z negotovostjo inflacije v prihodnosti. Uporabila sem pristop avtorjev Joslina, Singletona in Zhu-ja (2011), ki uporabijo Gaussov ATS model (angl. *Gaussian Affine Term Structure model*, v nadaljevanju: GATS model). Ti modeli predstavljajo eno izmed temeljnih orodij za empirične raziskave na področju makroekonomije in financ. V GATS modelih so donosi predstavljeni kot afine funkcije faktorjev z Gaussovo dinamiko. Poleg tega je v diskretnem času skupna porazdelitev faktorjev in donosov multivariatna normalna s konstantnimi pogojnimi variancami. Prednost metode Joslina, Singletona in Zhu-ja (2011) je v tem, da omogoča dekompozicijo krivulje donosnosti na dva dela in sicer na časovno vrsto v do tveganja nevtralnem verjetnostnem prostoru $\mathbb{Q}$ in časovno vrsto v empiričnem verjetnostnem prostoru $\mathbb{P}$. Metoda omenjenih avtorjev omogoča konvergenco h globalnemu optimumu praktično v trenutku, kar omogoča računsko učinkovito ocenjevanje GATS modela.

Dekompozicijo krivulje ILS naredim za različne dospelosti in sicer za 1-, 2- in 3-letni horizont za obdobje od julija 2004 do decembra 2019. S krivulje brezkuponskih inflacijskih zamenjav ocenim glavne komponente in uporabim prve tri kot faktorje, na katerih se tekom ocenjevanja izvedejo invariantne transformacije. Uporaba metode Joslina, Singletona in Zhu-ja (2011) omogoča izračun parametrov, ki so ocenjeni ločeno – izračun $\mathbb{Q}$ in $\mathbb{P}$ parametrov.

Po oceni GATS modela uporabim ocenjene parametre največjega verjetja za izračun pričakovane prihodnje inflacije pod verjetnostno mero $\mathbb{Q}$ in $\mathbb{P}$. Inflacijska pričakovanja v prostoru $\mathbb{Q}$ izračunam kot terminske stopnje po načinu, kot izhajajo iz modela (angl. model-implied forward rates), inflacijska pričakovanja v verjetnostnem prostoru $\mathbb{P}$ pa z uporabo dinamičnega napovedovanja za h-prihodnjih obdobij (VAR(1) model), pri čemer h sovpada z dospelostjo inflacijskih pričakovanj. Premijo za inflacijsko tveganje izračunam kot razliko med pričakovano inflacijo v verjetnostnem prostoru $\mathbb{Q}$ in pričakovano inflacijo pod verjetnostno mero $\mathbb{P}$.

Za inflacijska pričakovanja vseh dospelosti – letna inflacijska pričakovanja čez eno leto, čez dve leti in čez tri leta ugotovim, da je njihova dinamika podobna v smeri, a različna v volatilnosti. Za krajše horizonte so inflacijska pričakovanja bolj volatilna. Druga komponenta ILS krivulje je bolj volatilna kot inflacijska pričakovanja, tj. premija za inflacijsko tveganje. Dinamika ILS krivulje je precej podobna inflacijski premiji. Nihanje inflacijskih pričakovanj je precej manj izrazito. Iz tega sledi, da na gibanje in volatilnost ILS krivulje bolj vpliva inflacijska premija, kar je najbolj razvidno na daljših horizontih.

Za nadaljnjo analizo iz velikega nabora makroekonomskih spremenljivk izračunam faktorje, ki so za pričakovano inflacijo, za vsako obravnavano dospelost posebej, najbolj relevantni. Problem velikega števila možnih napovednih spremeljivk je namreč v tem, da je v praksi tako veliko količino informacij težko uporabiti (angl. *curse of dimensionality*). V splošnem se ta problem rešuje z uporabo nekaj spremenljivk, ki so najbolj pomembne za napovedno spremenljivko, ali pa z uporabo ocenjenih faktorjev.

Uporabljeno podatkovno bazo sestavlja 47 četrtletnih serij evrskega območja, ki pokrivajo obdobje od zadnjega kvartala leta 2004 do konca leta 2019. Makroekonomske skupine, zajete z uporabljenimi spremenljivkami, so kapitalski trgi (delniški indeksi), cenovni indeksi, monetarni agregati, menjalni tečaji, trg dela, narodno-gospodarski podatki ter anketni kazalniki sentimenta in zaupanja. Iz analize izločim kratkoročne in dolgoročne obrestne mere, saj že vsebujejo inflacijska pričakovanja, kar bi lahko vodilo do napačne korelacije (angl. *spurious correlation*). Za dosego stacionarnosti časovnih vrst sem podatke predhodno ustrezno transformirala.

Faktorje, ki ženejo inflacijska pričakovanja izračunam s pomočjo 3PRF metode avtorjev Kelly-ja in Pruitta (2015). V literaturi je sicer najbolj uporabljena PCA metoda. 3PRF, ki predstavlja zaporedje treh regresij po metodi najmanjših kvadratov (angl. *Ordinary Least Squares*, v nadaljevanju: OLS), pa je njena alternativa. V 3PRF metodi so faktorji, pomembni za napovedovanje ciljne spremenljivke, selektivno izbrani.

V prvem koraku 3PRF metode se izvede toliko regresij časovnih serij (angl. *time series regressions*), kolikor je različnih možnih prediktorjev v podatkovni bazi. Prediktorji vstopajo v enačbo kot odvisna spremenljivka, kot regresorji pa *proxy* spremenljivke. Kot *proxy* v prvi iteraciji uporabim inflacijska pričakovanja, v naslednjih iteracijah pa napake (angl. *residuals*) iz prejšnjega *proxy*-ja kot nov *proxy*, kar avtorja Kelly in Pruitt (2015) imenujeta avtomatična izbira *proxy*-jev (angl. *Automatic Proxy Selection*). V tem koraku z OLS metodo ocenim koeficiente ϕ_i za vsak $i = 1, \ldots, N$ iz enačbe (24) iz poglavja 3.2, kjer N predstavlja število različnih makroekonomskih spremenljivk, ki v regresijo vstopajo kot prediktorji. V drugem koraku 3PRF postopka ponovno uporabim prediktorje v vlogi odvisne spremenljivke, kot regresorje pa ocenjene koeficiente $\hat{\phi}_i$ iz prvega koraka. V tem koraku s t *cross section* regresijami ocenim koeficient F_t za vsak $t = 1, \ldots, T$ iz enačbe (25).

V napovedni regresiji časovne serije inflacije v nadaljevanju uporabim ocenjene koeficiente $\hat{F}_t$ iz prejšnjega koraka, kar predstavlja tretji in zadnji korak 3PRF metode.

Z uporabo 3PRF postopka za napovedovanje inflacije uporabim zgolj faktorje relevantne za inflacijska pričakovanja, ki sem jih pridobila iz dekompozicije ILS krivulje z GATS modelom. Končno število faktorjev uporabljenih v dinamičnem faktorskem modelu določim glede na relativno povprečno kvadratno napako napovedi (angl. *relative Mean-Square-Forecasting-Error*) v primerjavi natančnosti napovedi iz modela, ki vsebuje zgolj faktorje, in avtoregresijskega modela AR(1), ki igra vlogo primerjalnega modela (angl. *benchmark model*).

Za napovedovanje izven vzorca (angl. *Out-Of-Sample forecasting*, v nadaljevanju: OOS) uporabim dinamični faktorski model s faktorji, ki ženejo inflacijska pričakovanja in raziščem, ali so informacije vsebovane v inflacijskih pričakovanjih koristne za napovedovanje inflacije. V analizi ocenim napovedno moč več različnih specifikacij modela. Ocenim osnovni AR(1) model, model s faktorji in avtoregresijskim členom inflacije, ter model, v katerega vstopajo zgolj faktorji. OOS napovedi, ki izhajajo iz različnih specifikacij modela primerjam z realizirano časovno serijo HICP inflacije evro območja. Napovedno moč različnih modelskih specifikacij primerjam na podlagi vrednosti korena povprečne kvadratne napake napovedi (angl. *Root-Mean-Square-Forecasting-Error*, v nadaljevanju: RMSFE).

Rezultati so, izhajajoč iz mojih ugotovitev analize primerjave napovedne natančnosti različnih specifikacij modela sledeči. Inflacijska pričakovanja neposredno pridobljena iz GATS modela so najbolj indikativna za realizirano inflacijo za vse obravnavane dospelosti. Primerjalno z drugimi modelskimi specifikacijami je njihova RMSFE vrednost OOS napovedi najnižja. Izkaže pa se, da je napovedna natančnost modela zgolj s faktorji, ki ženejo inflacijska pričakovanja, vseeno boljša od natančnosti primerjalnega AR(1) modela. To nam pove, da pričakovana inflacija vsebuje informacijo, ki je koristna za napovedovanje inflacije.

Zaradi omenjene ugotovitve sem prvotno analizo razširila z dvema dodatnima modeloma, ocenjenima za vse tri različne modelske specifikacije. Prva modifikacija modela se s prvotno ocenjenim modelom razlikuje v tem, da v model kot napovedna spremenljivka namesto inflacije vstopa pričakovana inflacija dobljena iz GATS modela. Kot faktorji v dinamični faktorski model pa še vedno vstopajo tisti faktorji, ki ženejo inflacijska pričakovanja pridobljeni s 3PRF metodo. V drugi modifikaciji modela je napovedna spremenljivka, tako kot v prvotno ocenjenem modelu, inflacija. Relevantni faktorji, ocenjeni s 3PRF metodo, ki vstopajo v dinamični faktorski model, pa so, namesto iz inflacijskih pričakovanj, izločeni iz inflacije same.

Moj cilj na tej točki je ugotoviti, kateri od dveh novih pristopov ima pri napovedovanju inflacije evrskega območja izven vzorca večjo napovedno moč. V primerjalni analizi so tako napovedi inflacijskih pričakovanj kot inflacije izračunane za soležne horizonte. OOS napovedi obeh dodatnih modelov za vsako od treh specifikacij ponovno primerjam z RMSFE. Ugotovim, da so napovedi inflacije, pridobljene z napovedmi inflacijskih pričakovanj, za vse obravnavane dospelosti bolj natančne, kot napovedi dobljene z inflacijo samo. Ugotovitve podprem tudi s testi robustnosti, kjer za začetno ocenjevanje parametrov uporabim različne dolžine vzorca. V vseh obravnavanih primerih je napovedna moč inflacijskih pričakovanj pri napovedovanju inflacije boljša kot tista z uporabo napovedi inflacije.

Na srednji do dolgi rok je inflacijo zaradi njene volatilnosti težko natančno napovedovati, so pa napovedi inflacije na teh horizontih ključnega pomena pri izvajanju monetarne politike, katere glavni cilj je zagotavljanje stabilnosti cen. Prav zaradi njihovega pomena morajo biti napovedi inflacije čim bolj natančne in zanesljive. Rezultati kažejo, da so srednje- do dolgoročne napovedi inflacije za obravnavane dospelosti z alternativno metodo, predstavljeno v tej magistrski nalogi, bolj natančne kot neposredne napovedi inflacije same. To pomeni, da bi bilo napovedovanje inflacije lahko bolj učinkovito. V literaturi napovedovanje inflacije posredno prek inflacijskih pričakovanj po mojem vedenju še ni bilo uporabljeno, zato je predstavljen pristop v magistrski nalogi na tem področju nov. Napovedi inflacije preko inflacijskih pričakovanj bi se lahko uporabile za spremljanje učinka odločitev o spremembah denarne politike centralne banke, za ocenjevanje učinkovitosti srednje- do dolgoročnega doseganja inflacijskega cilja in za preverjanje robustnosti inflacijskih napovedi.

Appendix 2: Macroeconomic indicators of inflation expectations

Table 11: R^2 values: 1y2y inflation expectations

Group	f1	f2	f3	f4	f5	f6	f7	f8
Stock market	0.006	0.075	0.073	0.063	0.077	0.082	0.089	0.089
Exchange rates	0.038	0.013	0.004	0.006	0.007	0.004	0.009	0.007
Money aggregates	0.022	0.005	0.014	0.011	0.011	0.012	0.013	0.012
Deposits	0.036	0.044	0.057	0.044	0.047	0.046	0.052	0.052
Prices	0.114	0.043	0.039	0.038	0.042	0.041	0.042	0.046
Output data	0.029	0.038	0.037	0.042	0.038	0.039	0.037	0.041
Labour market	0.004	0.007	0.001	0.004	0.001	0.003	0.002	0.002
Survey data	0.037	0.055	0.038	0.029	0.032	0.034	0.033	0.034

Source: Own work.

Table 12: R^2 values: 1y3y inflation expectations

Group	f1	f2	f3	f4	f5	f6	f7	f8
Stock market	0.006	0.073	0.071	0.061	0.075	0.080	0.086	0.086
Exchange rates	0.038	0.013	0.004	0.005	0.007	0.004	0.008	0.007
Money aggregates	0.023	0.005	0.014	0.011	0.010	0.012	0.012	0.012
Deposits	0.036	0.044	0.057	0.045	0.047	0.046	0.052	0.051
Prices	0.114	0.045	0.040	0.039	0.043	0.043	0.043	0.048
Output data	0.030	0.040	0.038	0.042	0.040	0.040	0.038	0.042
Labour market	0.004	0.007	0.001	0.004	0.001	0.003	0.002	0.002
Survey data	0.037	0.052	0.036	0.027	0.030	0.032	0.031	0.032

Source: Own work.

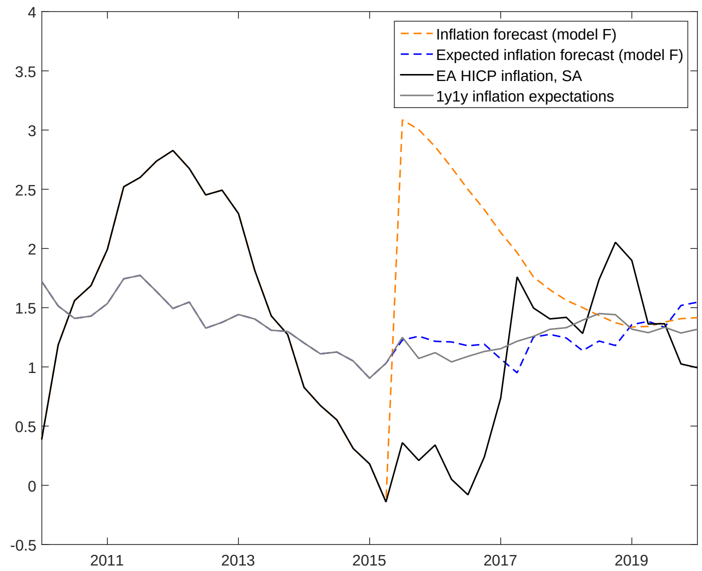
Table 13: R^2 values: inflation ($h = 17$)

Group	f1
Stock market	0.185
Exchange rates	0.063
Money aggregates	0.109
Deposits	0.067
Prices	0.060
Output data	0.355
Labour market	0.558
Survey data	0.407

Source: Own work.

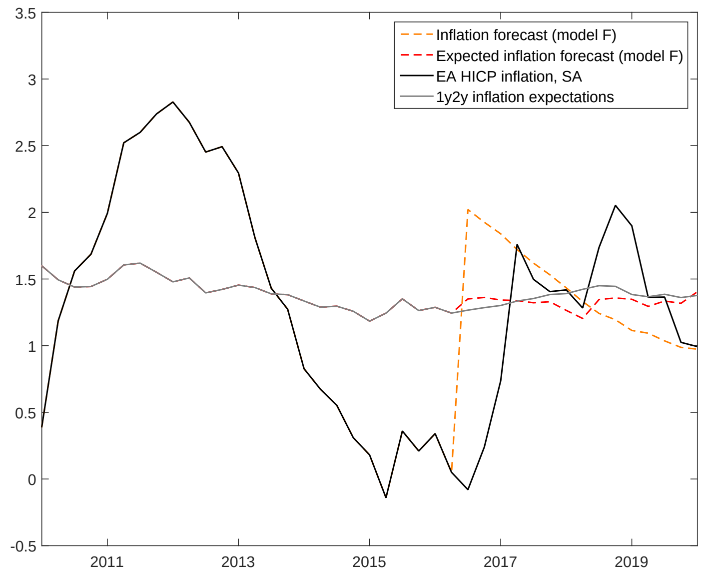
Appendix 3: Robustness check

Figure 24: Initial estimation sample: 2006–2013 ($h = 9$)



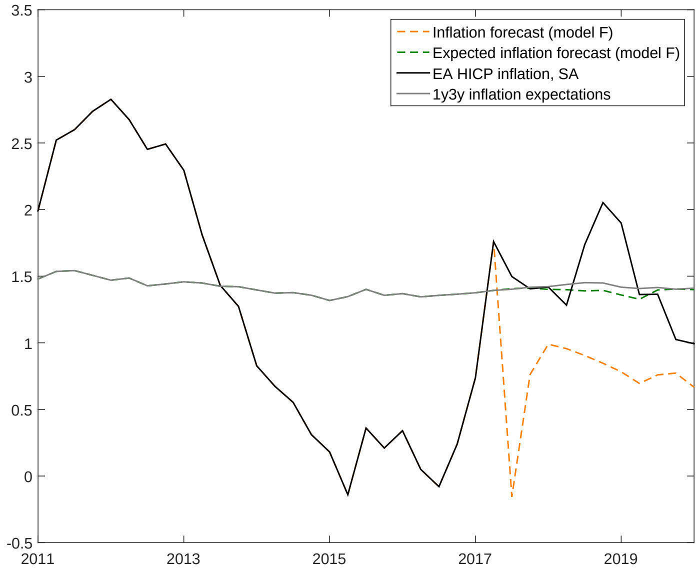
Source: Own work.

Figure 25: Initial estimation sample: 2006–2013 ($h = 13$)



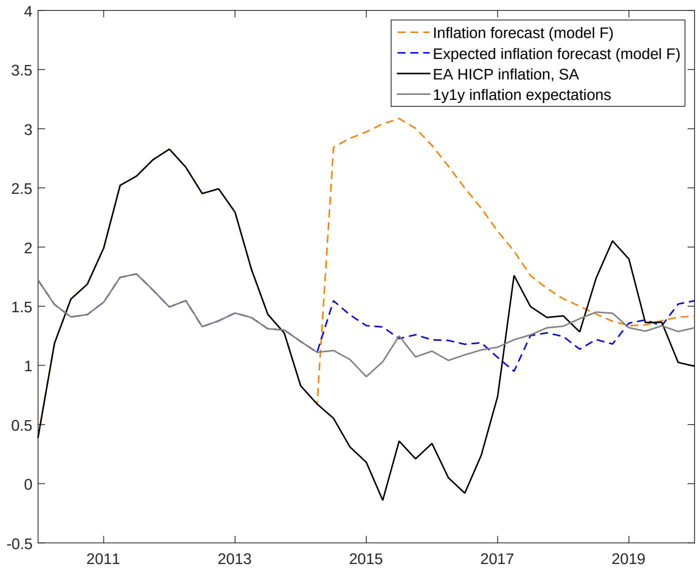
Source: Own work.

Figure 26: Initial estimation sample: 2006–2013 ($h = 17$)



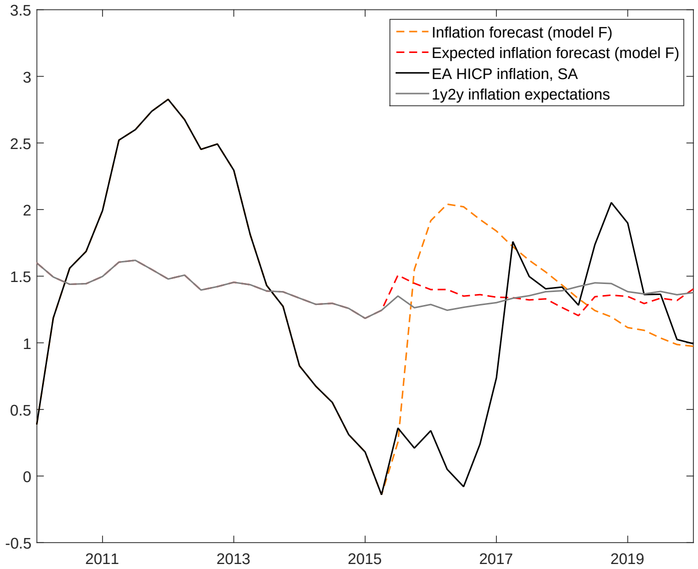
Source: Own work.

Figure 27: Initial estimation sample: 2006–2012 ($h = 9$)



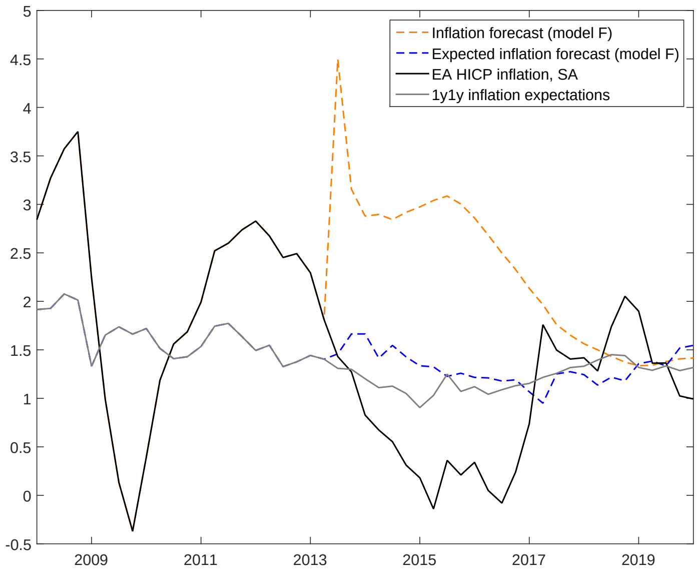
Source: Own work.

Figure 28: Initial estimation sample: 2006−2012 ($h = 13$)



Source: Own work.

Figure 29: Initial estimation sample: 2006−2011 ($h = 9$)



Source: Own work.