UNIVERSITY OF LJUBLJANA
FACULTY OF ECONOMICS

MASTER'S THESIS

GLOBAL VARIANCE RISK PREMIUM

Ljubljana, August 2017 KATJA NOVAK

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INTRODUCTION

Stock prices fluctuate widely due to investor's emotions. In addition, studies on equity premium puzzle generate extra proof that emotions play a major role. Mehra and Prescott (1985) concluded that the risk premium from the year 1870 on was on average 7%, which they believe is too high for the factor two or three. Furthermore, equity premium puzzle has been studied for many years and researchers tried to provide many explanations why stocks outperform bonds for quite a large margin. For example, Benartzi and Thaler (1995) reported in their paper that for equity premium puzzle there is an explanation based on behavioral reasons - they assume that investors are loss averse and believe that investors evaluate their portfolios often, even long term ones. Thus, the equity premium can be explained by short-term loss aversion as well as by investors evaluating their portfolios frequently. Consequently, it can be concluded that emotions are a reason for the stock market volatility as well.

In general, stock predictability is usually linked with dividend yields, price-to-earnings ratios and other more traditional valuation measures. However, in my thesis I decide to use variance risk premium (described in detail in the next paragraph) as an approximation for the short-term loss aversion in order to see whether it has any predictive power and if it is able to explain excess stock returns of eight countries that I analyze (United States, United Kingdom, Germany, Switzerland, Japan, Korea, Canada and France).

Variance risk premium is connected to variance swap strike being greater than realized variance (on average). Variance swap is an over-the-counter derivative that pays the difference between realized variance and a fixed swap rate in a specific time frame. Naturally, it costs zero at the inception. Variance swap rate represents the risk neutral expected value of the realized variance and thus I use the difference between synthetic swap rate and realized variance in my variance risk premium construction. My examined time period is between November 2010 and November 2015 and I check market risk aversion changes in times of financial turmoil and how it is influenced by other events in the market. In addition, I am also interested to see the sign of the variance risk premium for each country - in case it is positive it means that investors are willing to suffer losses just to hedge against unfavorable volatility movements.

In my thesis I construct country-specific variance risk premium as a difference between implied and realized volatility. Implied volatilities are calculated by the approach adopted by Whaley in 1993, when he introduced the new methodology of indices based on volatility implied by option prices. In the same year Chicago Board Options Exchange (hereinafter: CBOE) was the first exchange that officially introduced an implied volatility index called VIX index and was based on the previously mentioned methodology. However, this index was updated in the year 2003 to model-free calculation and it is based on estimating the expected volatility by averaging the weighted prices of S&P 500 puts and calls over a wide range of strike prices.

In my empirical part, implied volatility in time t is measured as the end-of-month implied volatility-squared and it is de-annualized ($IV_t^2/12$). Realized variance is the sum of squared daily returns of corresponding equity index over one month (these are 22 trading days), in the interval between t and t+1.

According to researchers, variance risk premium is considered as a measure of economic uncertainty and risk aversion. Financial crisis has amplified the need for indicators of risk aversion of market participants, because variations in risk appetite lead to variations in asset prices. Thus, variance risk premium can be seen as compensation for a risk-averse investor that is exposed to the variance of risky assets as well as jumps in their prices. Numerous papers investigate whether variance risk premium can be predicted (especially for the S&P 500 index). Analyzing this research question is important for variance traders. It turns out that variance-trading strategies are profitable in cases of short volatility positions e.g. Coval and Shumway (2001); Bakshi and Kapadia (2003); Driessen and Maenhout (2007), etc. In contrast, these strategies are very sensitive if market volatilities increase. An example of such consequence can be found in the period of financial crisis. If market participants are able to correctly predict variance risk premium, it could help them to build profitable trading strategies without having excessive risk.

Furthermore, variance risk premium is also used when explaining asset predictability puzzle. For example, Bollerslev and Zhou (2006) show that variance risk premium explains more than 15% of the variation in quarterly stock returns in the period from 1990 up to 2005. According to Bakshi and Madan (2006) it is (non-linearly) connected to the coefficient of relative risk aversion. This means, that when variance risk premium is low (high), it generally implies low (high) degree of risk aversion. As a result, investors tend to increase (decrease) their spending (investment, consumption) and switch assets from less (more) to more (less) risky assets in portfolio. Thus, on the one hand expected returns decrease (rise) but on the other hand, economic growth increases (decreases).

So far, research papers have mainly focused on the American market and used CBOE calculated VIX index. Thus, the purpose of my thesis is to expand the analysis also to other countries that have available implied volatility indices, which are calculated according to the new model-free VIX methodology. The main contribution of my research is to check whether findings in connection with American variance risk premium also hold in an international setting. Besides United States I analyze also Germany, United Kingdom, Japan, Canada, Switzerland, Korea and France, because they have available data that is calculated in the same way as VIX index.

In my thesis I construct variance risk premium for different countries around the world. The advantage of using the proxy is that it is model-free and is directly observable. So far, the research papers have mainly focused on the American market. This thesis thus differs by exploring and analyzing variance risk premium and the global risk aversion in the international setting and in the time period after financial crisis. I investigate whether the market is pricing global variance risk premium in contrast to country-specific variance risk premium and check the relationship between implied and realized volatility for each of the eight countries. The focus of my thesis is on constructing variance risk premium for each country separately first and then to obtain a global one weighted by market capitalization.

An implied volatility index conveys the market expectations regarding the future volatility of the underlying equity index and thus I perform some tests in order to see whether there is any information content regarding realized volatility of publically available (in addition there are also academic implied volatility indices) implied volatility indices around the world. In addition, I perform country-specific regressions in order to see whether there are any patterns that can be observed across different countries.

After constructing and examining global variance risk premium, I focus only on the American market and check whether there are any macroeconomic variables that would be able to explain time-varying nature of variance risk premium. I use univariate and multivariate regressions to check possible benefits of adding a new potential explanatory variable.

My thesis is organized as follows:
The first chapter starts with definition of variance risk premium and how it is built. Short review of existing literature is added in order to present relevant findings about the topic. I describe in details the calculation of old and new VIX index, since all subsequent indices used in my thesis are calculated in the same model-free method. First, I present the implied volatility index for the American market and then I add short description of implied volatility indices of other countries with highly liquid option markets: Germany (VDAX-New), Switzerland (VSMI), France (VCAC), United Kingdom (VFTSE), Japan (VXJ), Korea (VKOSPI) and Canada (VICX). These are all officially available indices that are created by an organized exchange. There exist also model-free indices that were created for academic purposes, but I do not consider those, as they are not publically available.

In the second chapter I analyze relationship between implied and realized volatility of all eight previously mentioned countries. Implied volatility is an important indicator of the market expectation regarding future volatility of the corresponding underlying market index. Recent articles imply that implied volatility is a superior estimator of future volatility in comparison to Black-Scholes implied volatility and historical volatility. I check whether this assumption holds also for my chosen countries.

In the third chapter I construct local variance risk premiums for the same countries I analyzed in the previous chapters. I follow the procedure of Bollerslev, Marrone, Xu and Zhou (2014). My approach takes into account also Canada and Korea when calculating global variance risk premium, because they introduced model-free calculated implied volatilities later on. Due to the fact that there is not a long common time frame of historical time series data, I limit my time horizon to five years.

Later on, I check regression return predictabilities using more recent data. In particular, I analyze if the time-varying variance risk premium and its ability to predict returns holds internationally. Time-series plots of variance risk premium for each individual country are presented and analyzed. Then I continue with country specific regressions. In the first step I regress monthly excess returns against local variance risk premium for each individual country and then I replace it by global variance risk premium. Proxy for the global variance risk premium is based on the capitalization-weighted average of previously calculated country specific variance risk premiums.

In order to check sensitivity of my results, I build global and country individual forward-looking global variance risk premium and investigate its return-predictability pattern. The use of proxy for the variance risk premium considers the assumption that volatility follows a random walk. Thus, the forward-looking variance risk premium is used in order to explore the sensitivity of the international empirical findings to this simplified assumption.

After constructing and examining global variance risk premium, I focus only on the American market in the sixth chapter and check whether there are any macroeconomic variables that would be able to explain time-varying nature of variance risk premium. I use univariate and multivariate regressions to check the possible benefits of adding a new possible explanatory variable.

In the seventh chapter I discuss open issues and possible questions for further research.

1 VARIANCE RISK PREMIUM DEFINITION AND CALCULATION

In my thesis I construct model-free variance risk premium. In order to define what does model-free calculation mean, let $\mathrm{C_t(T,K)}$ denote the price of a European call option with maturity time T and strike price K. Price of a zero-coupon bond (in time t) with maturity in T is denoted by $\mathrm{B(t,T)}$. The equation below shows the model-free calculation of the implied variance, which is market's risk-neutral expectation of the return variance in the time interval between t and t+1:

$$IV_{t,t+1} \equiv E_t^Q\left(Var_{t,t+1}\right) = 2\int_0^{\infty} \frac{C_t\left(t+1, \frac{K}{B(t,t+1)}\right) - C_t(t,K)}{K^2}\, dK \qquad (1)$$

Implied variance can be seen as portfolio of European calls with strike prices spanning from zero to infinity (see Jiang & Tian (2005) for more details). In practice, implied variance is constructed of calls with finite number of strike prices. Also limited number of options tends to provide quite good approximation of risk-neutral expectation of the future market variance, which is still better than the one following Black-Sholes formula.

The variance risk premium is defined as the difference between the risk-neutral and objective expectations of realized variance. In my empirical part, implied volatility in time t is measured as the end-of-month implied volatility-squared and it is de-annualized ($\mathrm{IV}_t^2/12$). The realized variance is the sum of squared daily returns of corresponding equity index over one month (these are 22 trading days), in the interval between t and t+1.

In order to define realized variance calculation in details, let p_t denote the logarithmic price of the asset. Now the model-free realized variance in the time period between t and t+1, $\mathrm{RV}_{t,t+1}$ can be measured in discrete time in the following way:

$$RV_{t,t+1} \equiv \sum_{j=1}^{n} \left(p_{t+\frac{j}{n}} - p_{t+\frac{j-1}{n}}\right)^2 \tag{2}$$

where $n \to \infty$. Furthermore, because of the theory of quadratic variation (see e.g. Anderson and Benzoni (2008)),

$$\lim_{n\to\infty} RV_{t,T} = \int_t^T RV_s \, ds. \tag{3}$$

Therefore, when sampling frequency n increases, the measurement error becomes smaller ($\mathrm{RV}_{t,T} \approx \int_t^T \mathrm{RV}_s \, \mathrm{ds}$). Researchers use different sampling procedures when computing realized variance. Some researchers use daily and others use intraday observations, because they argue that daily returns lead to poor estimation of the actual realized variance. For example, Bollerslev et al. (2014) use in their variance risk premium five minutes intraday return observations for the realized variance computation. However, there are two ways to select the time window regarding the calculation of the realized volatility. First, such computation can be based on overlapping daily data, which means that every two subsequent computed realized volatilities have in common 21 daily returns, because time frame shits just one day at the time. Second, some papers e.g. Christensen and Prabhala (1998) take non-overlapping monthly sample in order to avoid autocorrelation in regressions. In my thesis I follow the second approach using no overlapping monthly data.

Now variance risk premium denoted by $\mathrm{VRP_t}$ can be defined as the difference between ex-ante risk-neutral expectation of the future return variation in the time interval [t, t + 1] and the ex-post realized return variation over the time interval [t-1, t]:

$$VRP_t \equiv E_t^Q\left(Var_{t,t+1}\right) - E_t^P\left(Var_{t,t+1}\right). \tag{4}$$

In reality, this is not observable. Therefore, $\mathrm{E_t^Q}\left(\mathrm{Var_{t,t+1}}\right)$ is replaced by implied volatility index, such as CBOE implied variance index (VIX -squared) in case of American market and $\mathrm{Var_{t,t+1}}$ is replaced by its discretized realization $\mathrm{RV_{t,t+1}}$.

In this section a general overview of the existing implied volatility indices is presented as well as detailed description of Vix Index calculation. I start with the description and calculation method of the VIX Index, since all other indices follow the same methodology. I shortly summarize CBOE White Paper called The CBOE Volatility Index - VIX in order to introduce model-free calculation of an index and compare it with previous method of calculation.

Seminal work of Whaley (1993) introduced the new methodology of indices that are based on the volatility implied by option prices. He was the first to take into account index options rather than individual stock options and used both call and put options in implied volatility index calculations. Previously it was common to use only call options in the calculations.

CBOE was the first organized exchange that officially introduced an implied volatility index called VIX in the year 1993. It was called CBOE OEX Volatility Index and its calculation method was based on previously mentioned Whaley approach. At the beginning it was used to measure the market's expectation of 30-day volatility, which was implied by just eight at-the money S&P 100 Index option prices. In the year 2003 CBOE introduced the new VIX Index, whose calculation was based on wider basket of S&P 500 out-of-the-money put and call options. The new VIX Index provided better measurement and quickly became the benchmark for the American stock market volatility. Nowadays it is also known as "fear index". It reaches higher levels in crises periods and several spikes when market crashes.

The calculation of the original VIX is based on the Black-Sholes/Merton option valuation formula. Volatility is derived from four pairs of call and put options, which are based on the S&P 100 Index. Black-Sholes formula for the calculation of the theoretical price of an option is a function of the strike price of an option, its spot underlying price, time to maturity, interest rate as well as volatility of the underlying asset. However, option prices are also influenced by the demand and supply of the market. Therefore, by using Black-

Sholes formula the expected volatility for the time frame until the expiration of the option can be implied. Below is the formula of the original VIX Index, denoted as VXO:

$$\text{VXO} = \sigma_1\left(\frac{N_{t2}-22}{N_{t2}-N_{t1}}\right) + \sigma_2\left(\frac{22-N_{t1}}{N_{t2}-N_{t1}}\right), \tag{5}$$

where $\sigma_1 = \sigma_1^{X_l}\left(\frac{X_u-S}{X_u-X_l}\right) + \sigma_1^{X_u}\left(\frac{S-X_l}{X_u-X_l}\right)$ and $\sigma_2 = \sigma_2^{X_l}\left(\frac{X_u-S}{X_u-X_l}\right) + \sigma_2^{X_u}\left(\frac{S-X_l}{X_u-X_l}\right)$.

With S is denoted the spot underlying price, X_l means the lower exercise price, X_u the upper exercise price, N_{t1} and N_{t2} are numbers of trading days to expiration of the first and the second contract respectively.

Ten years later, the measurement of expected volatility was modified. The new VIX Index is now based on the S&P 500 Index, because it is the benchmark of the American stock market return. Now it is calculated by averaging the weighted prices of puts and calls of S&P 500 Index over a wide range of strike prices. The new VIX Index is no longer based on any model, it is said that it is model-free. Calculation of the "fear index" is based on the idea of fair values of future variances, which are directly observable from the market prices of interest rates and put and call option prices. It is an approximation of the one-month variance swap rate that has zero value at the inception date. In order to better understand how VIX Index and all other indices that follow CBOE methodology are calculated, the detailed description of the method is presented following CBOE White Paper: The CBOE Volatility Index - VIX. The generalized formula used in VIX Index calculation is as follows:

$$\sigma = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2}\, e^{RT} Q(K_i) - \frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2, \tag{6}$$

where:

σ: VIX/100 ⇒ VIX = σ* 100,

T: time to expiration,

F: forward index level derived from index option prices,

K_0: first strike below the forward index level (F),

K_i: strike price of the ith out-of-the-money option; it is a put if $K_\sigma > K_i$ and vice versa for call; in case $K_i = K_\sigma$ we consider both,

ΔK_i interval between strike prices, it is calculated as follows:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}, \tag{7}$$

$Q(K_i)$:the midpoint of the bid-ask spread for each option with strike K_{i},

R: risk-free interest rate to expiration.

Time to expiration (T) is retrieved according to the following equation:

$T = (M_{\text{current day}} + M_{\text{settlemetn day}} + M_{\text{other days})} /$ Minutes in a year,

where:

M_{today}: minutes remaining until midnight today,

M_{today}: minutes from midnight until 0:30 (for "standard" SPX expirations) or 15:00 (for "weekly" SPX expirations).

Time to expiration is measured in calendar days and each day is further divided into minutes due to better precision of calculations of volatility.

In order to better understand formula, each component of the equation has to be specified and explained more into details. First, the VIX-components are near-term and next-term put and call options, meaning that they have more than 23 but less than 37 days to expiry. In order to avoid errors in computation when considering close to expiration (near-term) options, they must have at least one week to expiration.

After the selection of near- and next- term options with the corresponding maturities (T_1 for near-term and T_2 for next term options) the computation of the VIX Index can be divided into three different steps.

FIRST STEP: Selection of options. In the first step out-of-the-money SPX calls and puts options are selected (with non-zero bid prices), gathered around at-the-money strike price denoted with K_0. Price range of strikes varies due to volatility changes. Consequently, number of options considered in the VIX calculation can fluctuate even minute-to-minute.

In the formula below, F denotes the level of the forward SPX level. It is obtained by determining the strike price at which the absolute difference between call and put prices (denoted by C_t and P_t respectively) is the smallest. The following example from the CBOE

White Paper: The CBOE Volatility Index - VIX provides the forward prices for near- and next-term options:

$$F_1 = 1965 + e^{(0.000305 \times 0.0683486)} \times (21{,}05 - 23{,}15) = 1962{,}89996$$

$$F_1 = 1960 + e^{(0.000286 \times 0.0882686)} \times (27{,}30 - 24{,}90) = 1962{,}40006$$

The general formula:

$$F \ = Strike\ Price + e^{RT} \times (Call\ Price - Put\ Price) \tag{8}$$

Table 1. Near Term Options

Near Term Options				**Next Term Options**			
Strike Price	**Call**	**Put**	**Difference**	**Strike Price**	**Call**	**Put**	**Difference**
.	.	.	.	.	.	.	.
1940	38,45	15,25	23,20	1940	41,05	18,80	22,25
1945	34,70	16,55	18,15	1945	37,45	20,20	17,25
1950	31,10	18,25	12,85	1950	34,05	21,60	12,45
1955	27,60	19,75	7,85	1955	30,60	23,20	7,40
1960	24,25	21,30	2,95	1960	27,30	24,90	2,40
1965	21,05	23,15	2,10	1965	24,15	26,90	2,75
1970	18,10	25,05	6,95	1970	21,10	28,95	7,85
1975	15,25	27,30	12,05	1975	18,30	31,05	12,75
1980	12,75	29,75	17,00	1980	15,70	33,50	17,80

Source: *The CBOE Volatility Index - VIX*, 2015.*

Once the forward prices are computed, the strike price (K_0) immediately below the forward index level is determined. In this example there are two: $K_{0,1} = 1960$ and $K_{0,2} = 1960$. Then, out-of-the-money put options with strike prices immediately lower than K_0 are selected (puts without bid prices are excluded from the calculation). Furthermore, when two consecutive strike prices have zero bid prices, all puts with lower strikes are no longer considered for calculation. The same procedure is then applied for out-of-the-money calls with strike higher than K_0.

Table 2. Out-of-the-money put options selection

Put Strike	Bid	Ask	Include?
1345	0,00	0,15	Not considered following two zero bids
1350	0,05	0,15	
1355	0,05	0,35	
1360	0,00	0,35	No
1365	0,00	0,35	No
1370	0,05	0,35	Yes
1375	0,10	0,15	Yes
1380	0,10	0,20	Yes

Source: *The CBOE Volatility Index - VIX*, 2015.*

The same procedure (Table 3 below) is then applied for out-of-the-money calls with strike higher than K_0.

Table 3. Out-of-the-money call option selection

Call Strike	Bid	Ask	Include?
.	.	.	.
2095	0,05	0,35	Yes
2100	0,05	0,15	Yes
2120	0,00	0,15	No
2125	0,05	0,15	Yes
2150	0,00	0,10	No
2175	0,00	0,05	No
2200	0,00	0,05	Not considered following two zero bids
2225	0,05	0,10	
2250	0,00	0,05	
.	.	.	

Source: *The CBOE Volatility Index - VIX*, 2015.*

In the final part of the first step, puts and calls with strike K_0 are averaged.

In the table below are represented options that are used in the VIX Index calculation with their average quoted bid and ask prices (mid quote). For example, the corresponding price for the 1960 near-the-term strike is calculated as (24,25+21,30)/2 = 22,775.

Table 4. Put and call option selection

Near term Strike	**Option Type**	**Mid-quote Price**	**Next term Strike**	**Option Type**	**Mid-quote Price**
1370	Put	0,2	1275	Put	0,075
1375	Put	0,125	1325	Put	0,15
1380	Put	0,15	1350	Put	0,15
.	.	.	.	.	.
1950	Put	18,25	1950	Put	21,6
1955	Put	19,75	1955	Put	23,2
1960	Put/Call Average	22,775	1960	Put/Call Average	26,1
1965	Call	21,05	1965	Call	24,15
1970	Call	18,1	1970	Call	21,1
.	.	.	.	.	.
2095	Call	0,2	2125	Call	0,1
2100	Call	0,1	2150	Call	0,1
2125	Call	0,1	2200	Call	0,08

Source: *The CBOE Volatility Index - VIX*, 2015.*

SECOND STEP: Volatility calculation for near-term and next-term options. In the second step VIX formula is applied to the near-term and next-term options as follows:

$$\sigma_1^2 = \frac{2}{T_1}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_1,T_1} Q(K_i) - \frac{1}{T_1}\left(\frac{F_1}{K_0} - 1\right)^2 \tag{9}$$

and

$$\sigma_2^2 = \frac{2}{T_2}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_2,T_2} Q(K_i) - \frac{1}{T_2}\left(\frac{F_2}{K_0} - 1\right)^2. \tag{10}$$

In the formula, ΔK_i denotes half the difference between the strike prices on either side of K_i. An example of calculation for the next-term 1325 put can be: $\Delta K = (1350 - 1275)/2$. In addition, ΔK_i for the upper and lower edges for a strip of options is simply computed as the difference between K_i and the adjacent strike price. More specifically, for the 1370 put that is the lowest strike in the strip of near-term options, 1375 is the strike that is adjacent

$(\Delta K = 5(1375 - 1370))$. The contribution to the final VIX value is proportional to the ΔK. In this case, the near-term1370 contribution is given by:

$$\frac{\Delta K_{1370Put}}{K_{1370Put}^2} e^{R_{1,}T_1} Q(1370Put) = \frac{5}{1370^2} e^{0,000305(0,0683486)}(0,20) = 0,0000005328.$$

Below (Table 5) are summarized the results of contribution calculations.

Table 5. Options used in VIX calculation

Near term Strike	Option Type	Mid-quote Price	Contribution by Strike	Next term Strike	Option Type	Mid-quote Price	Contribution by Strike
1370	Put	0,2	5,328E-07	1275	Put	0,075	2,3069E-06
1375	Put	0,125	3,306E-07	1325	Put	0,15	3,2041E-06
1380	Put	0,15	3,938E-07	1350	Put	0,15	2,0577E-06
.	.	.	.	.	.	.	.
1950	Put	18,25	2,39979E-05	1950	Put	21,6	2,84031E-05
1955	Put	19,75	2,58376E-05	1955	Put	23,2	3,03512E-05
1960	Put/Call Average	22,775	2,96432E-05	1960	Put/Call Average	26,1	3,39711E-05
1965	Call	21,05	2,72588E-05	1965	Call	24,15	3,12732E-05
1970	Call	18,1	0,000233198	1970	Call	21,1	2,71851E-05
.	.	.	.	.	.	.	.
2095	Call	0,2	2,278E-07	2125	Call	0,1	5,536E-07
2100	Call	0,1	3,401E-07	2150	Call	0,1	8,113E-07
2125	Call	0,1	5,536E-07	2200	Call	0,075	7,748E-07
	$\frac{2}{T_1}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_1 T_1} Q(K_i)$		0,018495		$\frac{2}{T_2}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_2 T_2} Q(K_i)$		0,018838

Source: *The CBOE Volatility Index - VIX*, 2015.*

The final part of the VIX formula that needs to be calculated is $\frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2$ for both T_1, T_2 near-terms:

$$\frac{1}{T_1}\left(\frac{F_1}{K_0} - 1\right)^2 = \frac{1}{0,0683486}\left(\frac{1962,89996}{1960} - 1\right)^2 = 0,00003203,$$

and

$$\frac{1}{T_2}\left(\frac{F_2}{K_0}-1\right)^2 = \frac{1}{0{,}0882686}\left(\frac{1962{,}40006}{1960}-1\right)^2 = 0{,}00001699.$$

Consequently, the implied variances number one and two are:

$$\sigma_1^2 = \frac{2}{T_1}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_1,T_1}\, Q(K_i) - \frac{1}{T_1}\left(\frac{F_1}{K_0}-1\right)^2 = 0{,}018495 - 0{,}00003203 = 0{,}01846292$$

and

$$\sigma_2^2 = \frac{2}{T_2}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_2,T_2}\, Q(K_i) - \frac{1}{T_2}\left(\frac{F_2}{K_0}-1\right)^2 = 0{,}018838 - 0{,}00001699$$
$$= 0{,}01882101.$$

THIRD STEP: Final calculations. In the last step, the 30-day weighted average of σ_1^2 and σ_2^2 is computed by taking the square root of that value and multiplying it by 100 in order to get VIX.

$$VIX = 100\times\sqrt{T_1\sigma_1^2\left(\frac{N_{T_2}-N_{30}}{N_{T_2}-N_{T_1}}\right) + T_2\sigma_2^2\left(\frac{N_{T_{30}}-N_{T_1}}{N_{T_2}-N_{T_1}}\right)\times\frac{N_{365}}{N_{30}}} \qquad (11)$$

where:

N_{T_1}= number of minutes to settlement of the near-term options (e.g. 35,924),

N_{T_2}= number of minutes to settlement of the near-term options (e.g. 46,394),

N_{30}= number of minutes in 30 days (30 × 1,440 =43,200),

N_{365} = number of minutes in a 365-day year (e.g. 365 × 1,440 = 525,600).

Finally, VIX is calculated by the formula:

$$VIX = 100\times\sqrt{\left\{0{,}0683486\times0{,}0184629\times\left[\frac{46{,}394-43{,}200}{46{,}394-35{,}942}\right] + 0{,}0882686\times0{,}018821\times\frac{43{,}200-35{,}942}{46{,}394-35{,}942}\right]\right\}\times\frac{526{,}600}{43{,}200}}$$

$$VIX = 100\times0{,}13685821 = 13{,}69.$$

In the summary, VIX is calculated as the square root of the risk neutral expectation of the next 30 calendar days of S&P 500 variance. For example, if today the VIX level is at 20%, denoted the expected annualized volatility of the S&P 500 over the next 30 calender days, taking into consideration of course risk-neutral measure. When we want to have a volatility calcultation for the shorter time period, VIX has to be divided by the square root of the time.

The VIX Futures. On 23.03.2004 CBOE Futures Exchange started with trading of futures contracts that were based on VIX index. This step was certainly one of the most talked-about financial innovations in that time, because it allowed to transform an index into a tradable asset. VIX Futures (VX) are contracts that are based on 30-day forward implied volatilities and with maturities up to nine months. Their main thecnical charactheristics are represented in the following rows below.

1. The contract multiplier is $1000 for each VIX futures contract.

2. The minimum price movement (value per tick) is 0.05, which is equal to 50.00 per contract.

3. Trading terminates on the business day immediately preceding the final settlement date of the VIX futures contract for the relevant spot month. When the last trading day is moved because of a certain holiday, the last trading day for an expiring VIX futures contract will be the day immediately preceding the last regularly-scheduled trading day.

4. Final Settlement Date: the Wednesday that is thirty days prior to the third Friday of the calendar month immediately following the month in which the contract expires.

5. Final settlement value: it shall be a Special Opening Quotation (hereinafter: SOQ) of VIX calculated from the sequence of opening prices of the options used to calculate the index on the settlement date.

6. Delivery: settlement of VIX futures contracts will result in the delivery of a cash settlement amount on the business day immediately following the Final Settlement Date. The cash settlement amount on the Final Settlement Date shall be the final mark to market amount against the final settlement value of the VIX futures multiplied by $1000.

2 RELATIONSHIP BETWEEN IMPLIED AND REALIZED VOLATILITY ACROSS COUNTRIES

Estimates of future volatility of assets are one of the most important factors when deciding about investment strategies or calculating risk exposure. Usually, there are two ways to obtain such estimates. The first one is to use an econometric model in order to forecast volatility in the future. The second one is to estimate future volatility with implied volatility, which is the market's risk neutral expectation of the future volatility of the underlying.

Researchers use both methods for predicting and have not come up to the agreement which method is better. For example, Poon and Granger (2003) conducted an extensive study about implied volatility forecasts with different approaches (using historical, stochastic and implied volatility estimates). Their findings suggest that implied volatility based forecasts often provide more accurate estimates than other methods. In addition, Jiang and Tian (2005) test informational efficiency of the option market using model-free implied volatility, Black-Sholes implied volatility and past realized volatility. Their findings also show that implied volatility (reflected by the new VIX index) provides the best forecast. On the other hand, some other studies e.g. Becker and Clements (2007) suggest that combinations of model-based forecasts of realized volatility are superior to implied volatility estimates.

Outside of the American research, there are Siriopoulos and Fassas (2009) that analyzed information content of all publically available implied volatility indices across the world, using realized volatility and equity index returns of corresponding countries. Their results conclude that implied volatility proxied by the corresponding index contain information about future volatility beyond that included in past volatility, although they are biased estimates.

Thus, this chapter examines the information content of publically available implied volatility indices across the world (United States, United Kingdom, France, Switzerland, Germany, Korea, Japan and Canada) regarding returns as well as realized volatility of the corresponding underlying equity markets.

Academic researches have conducted many studies regarding implied volatility, specifically various issues regarding the estimation of it. If markets were efficient and the option-pricing model was correct, all implied volatilities that would be calculated from options on the same underlying and expiry but with different strike prices should be identical. In reality, this does not hold, because deep-in-the-money or out-of-the-money options are linked with higher implied volatility in comparison to at-the-money options (see follow Siriopoulos & Fassas, 2009). Hence, it can be questionable how to best measure market's volatility expectation.

One class of finance literature studies implied volatility's information content that is linked to future realized volatility and how it can be predicted. I follow Siriopoulos and Fassas (2009) approach and test and document information content of eight countries publically available indices. In my research I change examination period after crisis and add certain countries (for example Canada and Japan) to the analysis, as they introduced their model-free volatility indices later on. The importance of having implied volatilities in the study rather than actual price fluctuations is to check what is the market participants' expectation regarding future uncertainty.

2.1 DATA

An implied volatility index conveys what is the market expectation regarding the future variation of the underlying equity index. Table 6 below shows eight implied volatility indices that I use in my variance risk premium calculation together with their underlying assets and short method calculation description retrieved from Siriopoulos and Fassas (2009). It can be seen that all indices are calculated in a model-free manner, which means that they are no longer based on any model (for example the old VIX Index calculation was based on the Black-Sholes/Merton model). All indices follow the new VIX Index calculation methodology described in the first chapter and use out-of-the-money put and call options with 30 days to expiration and consider wide range of strike prices.

My empirical research excludes the original VIX (United States), VDAX (Germany) and MVX (Canada) form the econometric analysis, because there are already available "updated" versions of those indices. Among the previously mentioned countries Canada has introduced the new implied volatility index the latest, at the end of the year 2009. The new VICX measures market expectation of the 30-day volatility of the Canadian stock market and is implied by the near-term and next-term options on the S&P/TSX 60 index (SXO).

Table 6. List of analyzed implied volatility indices

Volatility Index	Exchange	Underlying Asset	Short Summary
VIX	CBOE	S&P 500	Out-of-the money put and call options in the two nearest-term to 30 days expiration of a wide range of strike prices
VDAX-New	Deutsche Börse	DAX 30	Based on 8 DAX option series from 2-24 months expiration. The VDAX-NEW index is calculated via an interpolation of the two sub-indices closest to the 30 days expiration.
VSMI	SWX Swiss Exchange	SMI 20	Based on 8 SMI option series from 2-24 months expiration. The VDAX-NEW index is calculated via an interpolation of the two sub-indices closest to the 30 days expiration.
S&P/TSX60 VIX (VICX)	Montreal exchange	S&P/TSX60	Near-term and next-term options on the S&P/TSX60 index. VICX indicates implied volatility of the fixed 30-day period.
CAC 40 Volatility Index	Euronext (Paris)	CAC 40	Out-of-the money put and call options in the two nearest-term to 30 days expiration of a wide range of strike prices
VFTSE	Euronext	FTSE 100	Out-of-the money put and call options in the two nearest-term to 30 days expiration of a wide range of strike prices
VKOSPI	Korea Stock Exchange	KOSPI 200 index	Out-of-the money put and call options in the two nearest-term to 30 days expiration of a wide range of strike prices
VXJ	Japanese Exchange group	Nikkei 225	Out-of-the money put and call options in the two nearest-term to 30 days expiration of a wide range of strike prices

Source: C. Siriopoulos & A. Fassas, *Implied Volatility Indices - A review*, 2009, pp. 25-28.

Table 7. Summary statistics of implied and realized volatility

USA	Mean	Median	Min	Max	Std	Skew	Ex. Kurtosis	Jarque-Bera	Prob	#NA
IV_m	29,062	22,963	10,830	153,800	22,475	3,360	14,307	635,078	0,000	0
RV_m	20,926	11,667	0,000	195,970	29,298	4,169	20,627	196,200	0,075	3
lnIV	32,016	31,339	2,382	5,036	0,529	1,105	1,385	172,947	0,000	0
lnRV	26,359	24,613	1,048	5,278	0,822	0,898	0,960	985,381	0,000	4
Switzerland	**Mean**	**Median**	**Min**	**Max**	**Std**	**Skew**	**Ex. Kurtosis**	**Jarque-Bera**	**Prob**	**#NA**
IV_m	24,417	19,686	9,577	89,872	14,916	2,477	7,125	191,433	0,000	0
RV_m	20,545	13,675	2,922	154,270	24,666	3,764	15,872	694,278	0,000	7
lnIV	3,070	2,980	2,259	4,498	0,471	0,877	0,816	950,379	0,000	0
lnRV	2,675	2,615	1,072	5,039	0,769	0,589	1,192	6,322	0,040	7
Canada	**Mean**	**Median**	**Min**	**Max**	**Std**	**Skew**	**Ex. Kurtosis**	**Jarque-Bera**	**Prob**	**#NA**
IV_m	23,794	19,127	8,979	89,489	14,151	2,255	6,598	162,336	0,000	0
RV_m	15,013	9,997	2,714	80,748	14,993	2,324	6,005	139,343	0,000	3
lnIV	3,042	2,951	2,195	4,494	0,485	0,631	0,310	42,996	0,117	0
lnRV	2,361	2,302	0,998	4,391	0,804	0,500	-0,404	280,909	0,245	3
Korea	**Mean**	**Median**	**Min**	**Max**	**Std**	**Skew**	**Ex. Kurtosis**	**Jarque-Bera**	**Prob**	**#NA**
IV_m	26,622	19,712	9,828	137,090	20,620	3,132	12,485	495,291	0,000	0
RV_m	27,444	16,875	4,106	181,270	31,660	3,402	13,113	345,573	0,000	23
lnIV	3,103	2,981	2,285	4,921	0,554	0,898	0,845	100,081	0,007	0
lnRV	2,963	2,826	1,413	5,200	0,775	0,705	0,591	37,001	0,157	23
Japan	**Mean**	**Median**	**Min**	**Max**	**Std**	**Skew**	**Ex. Kurtosis**	**Jarque-Bera**	**Prob**	**#NA**
IV_m	47,723	42,979	21,094	111,810	21,407	1,374	1,424	24,336	0,000	0
RV_m	42,273	29,033	7,392	253,290	42,349	3,091	10,978	350,537	0,000	8
lnIV	3,781	3,761	3,049	4,717	0,403	0,461	0,213	2,274	0,321	0
lnRV	3,461	3,368	2,000	5,535	0,698	0,738	0,654	57,555	0,056	8
UK	**Mean**	**Median**	**Min**	**Max**	**Std**	**Skew**	**Ex. Kurtosis**	**Jarque-Bera**	**Prob**	**#NA**
IV_m	26,475	22,468	9,894	118,790	17,787	2,775	10,500	358,514	0,000	0
RV_m	20,837	15,080	2,647	106,900	20,953	2,705	7,535	200,747	0,000	5
lnIV	3,126	3,112	2,292	4,777	0,521	0,680	0,437	51,852	0,075	0
lnRV	2,719	2,713	0,974	4,672	0,769	0,347	0,371	1,442	0,486	5
France	**Mean**	**Median**	**Min**	**Max**	**Std**	**Skew**	**Ex. kurtosis**	**Jarque-Bera**	**Prob**	**#NA**
IV_m	41,148	34,071	14,901	143,270	24,394	1,823	3,861	716,721	0,000	0
RV_m	39,366	26,512	6,562	187,020	36,419	2,258	5,325	117,830	0,000	3
lnIV	3,581	3,528	2,701	4,965	0,506	0,529	-0,168	2,915	0,233	0
lnRV	3,365	3,277	1,881	5,231	0,769	0,305	-0,218	1,015	0,602	3
Germany	**Mean**	**Median**	**Min**	**Max**	**Std**	**Skew**	**Ex. kurtosis**	**Jarque-Bera**	**Prob**	**#NA**
IV_m	41,898	32,538	16,124	184,480	27,757	2,723	9,946	326,812	0,000	0
RV_m	39,371	27,740	3,333	197,910	38,953	2,460	6,216	141,392	0,000	7
lnIV	3,592	3,482	2,780	5,218	0,503	0,880	0,568	8,695	0,013	0
lnRV	3,333	3,323	1,204	5,288	0,824	0,020	0,502	0,570	0,752	7

Before testing the information content of implied volatility regarding realized volatility, I provide summary statistics of those series and their logs first. Monthly non-overlapping observations are used in order to avoid over estimation of past volatility. Table 7 represents descriptive statistics of monthly implied (IV_m) and realized volatilities (RV_m) and log-volatility series (lnRV for realized and lnIV for implied volatility) for each country separately. In total there are 61 observations, spanning from November 2010 until November 2015. Data were retrieved from Bloomberg and Reuters.

Mean values of implied volatilities range from 23,794 to 47,723, while the median prices are to some extent lower (19,127 to 42,979). These differences can be contributed to certain market break-downs/events and each country reacted of course differently to it. However, the other statistics give quite consistent results: for example all countries have positive skewness, which implies longer right tails and mainly positive kurtosis implies fatter tails.

From the table above can be seen that averages of both implied and log-implied volatilities are higher than average realized and log-realized volatilities respectively. In addition, Jarque-Bera test rejects the null hypothesis of normal distribution of analyzed time series (also for the log ones). Because the distributions of log-volatility time series seem to approximate better the normal distribution, this data set is used in the following section rather then standard volatility data set.

In order to have consistent measurements of implied and realized volatility, I aligned them also in respect to their time frame. Thus, some of them have missing values, as on that specific day due to time zone differences it was not a trading day. However, later on in calculations I replace missing values with the latest available observation.

Figures 1-8 below show monthly implied and realized volatiles in the five-year span for all eight examined countries. Also here can be seen that implied volatilities are on average higher than realized volatilities.

*Figure 1.*VIX and S&P 500 implied and realized variance

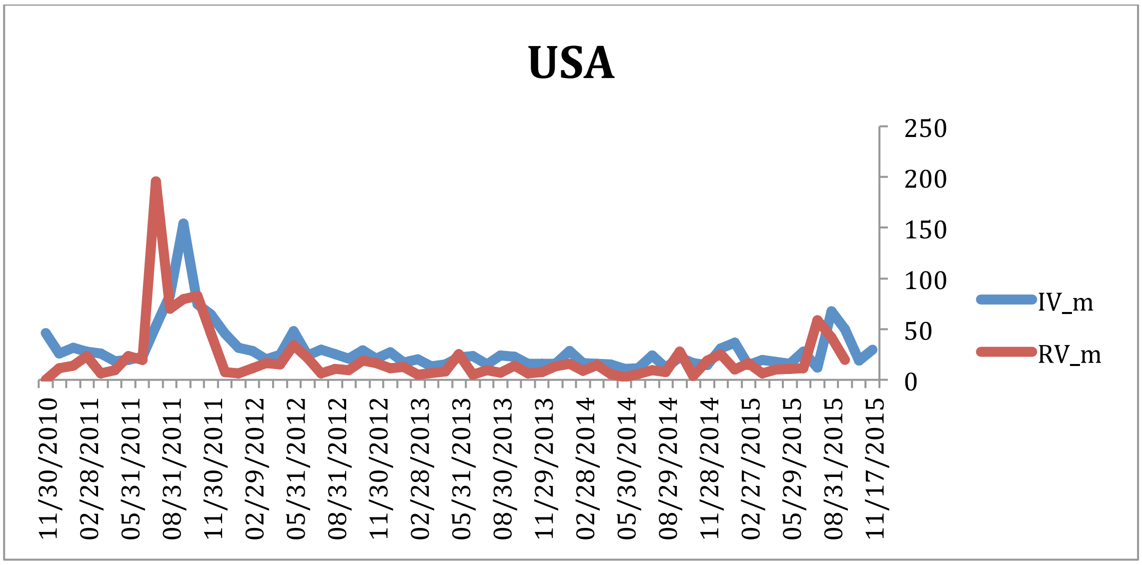


Figure 2. VSMI and SMI implied and realized variance

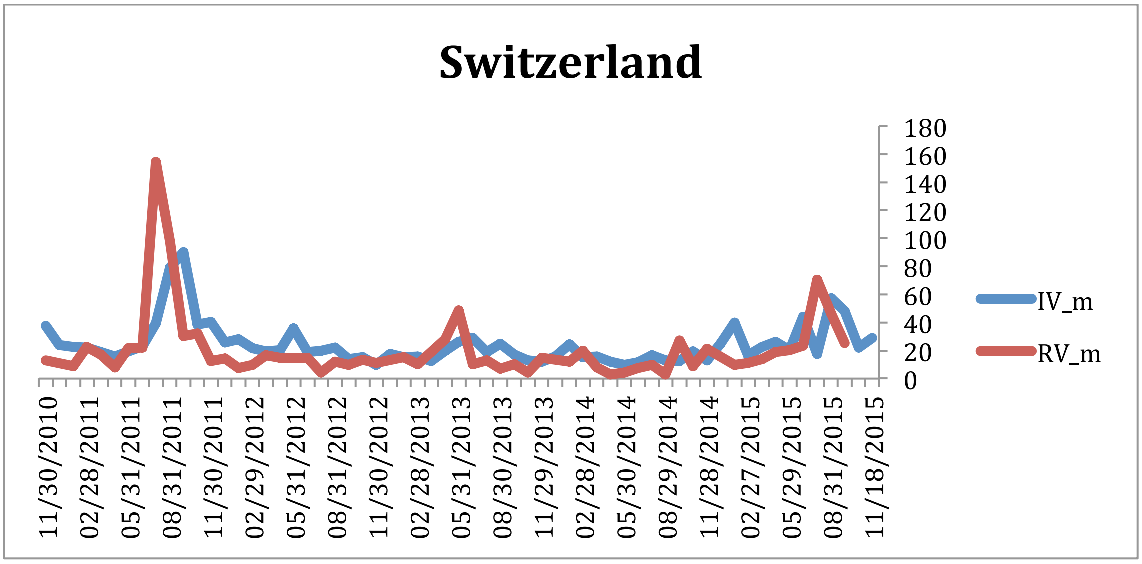


Figure 3. VICX implied and SXO realized volatility index

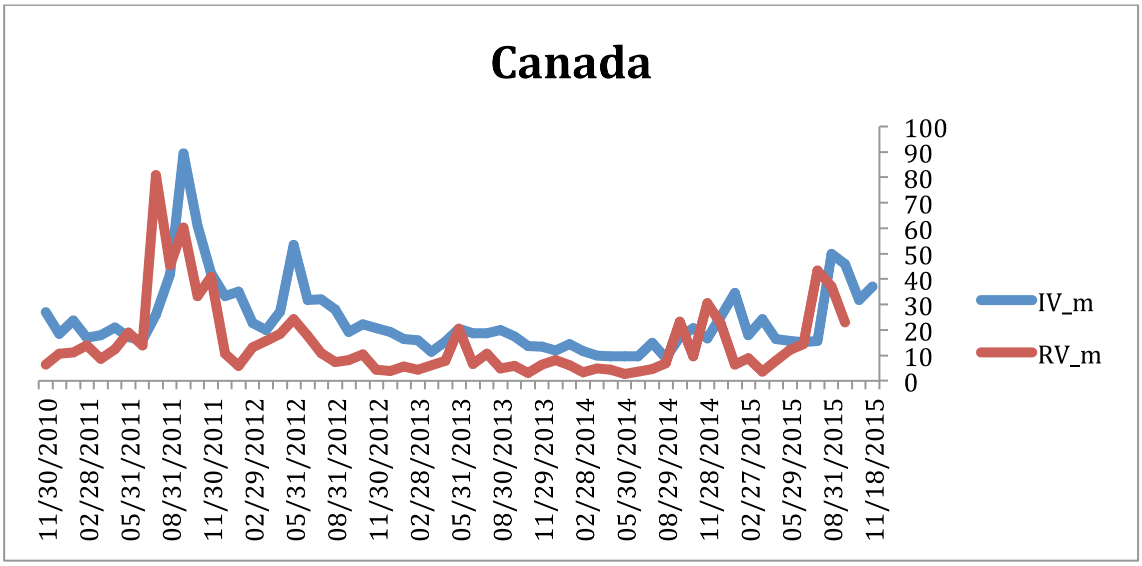


Figure 4. VKOSPI implied volatility and KOSPI 200

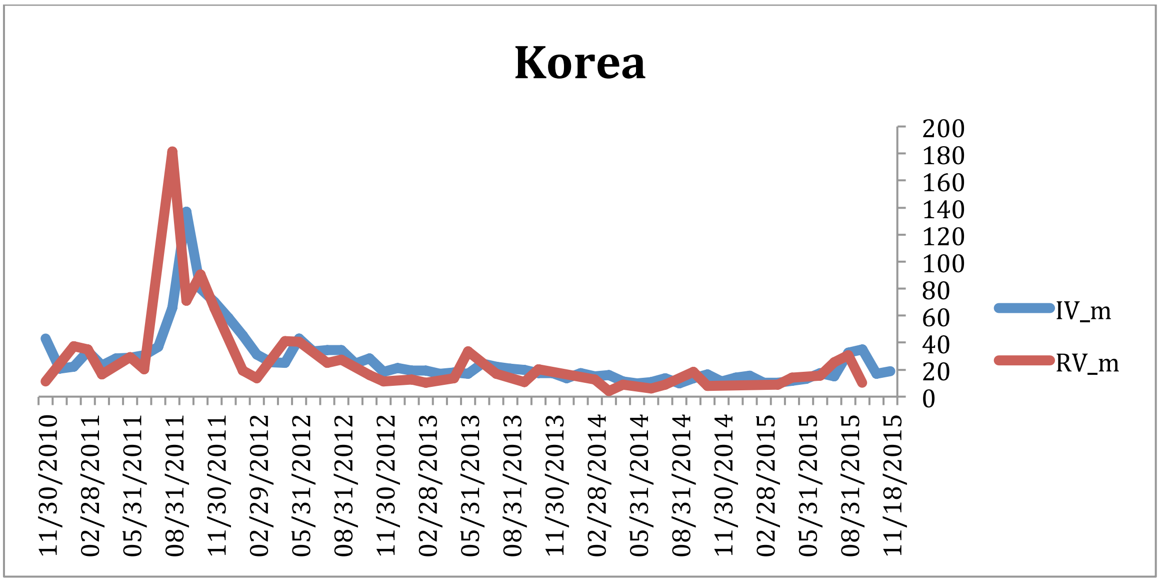


Figure 5. VXJ implied volatility and Nikkei 225

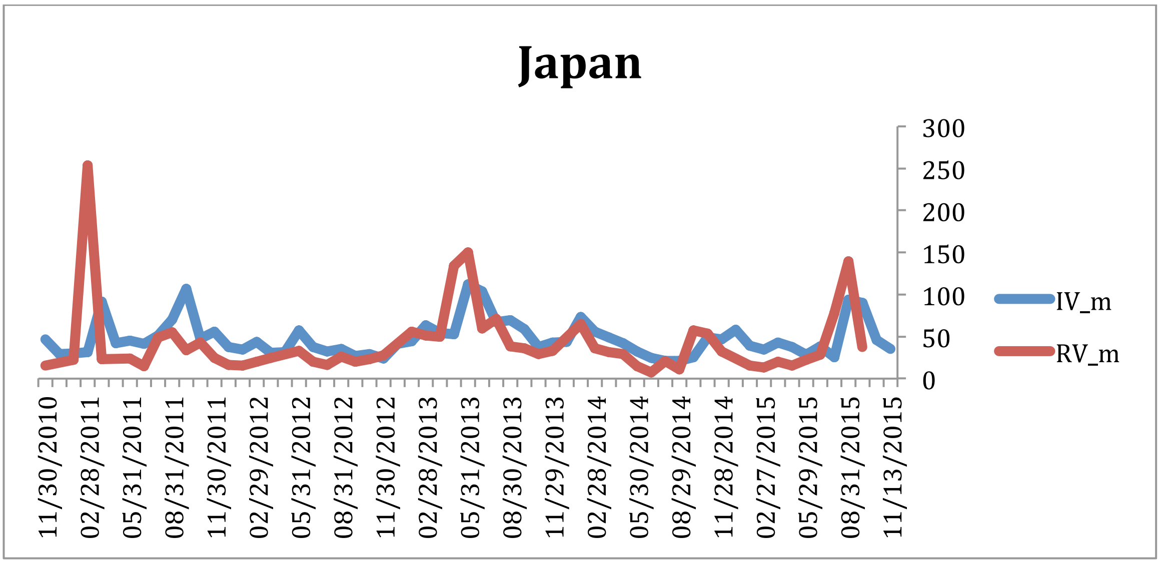


Figure 6. VFTSE implied volatility and FTSE 100 realized volatility

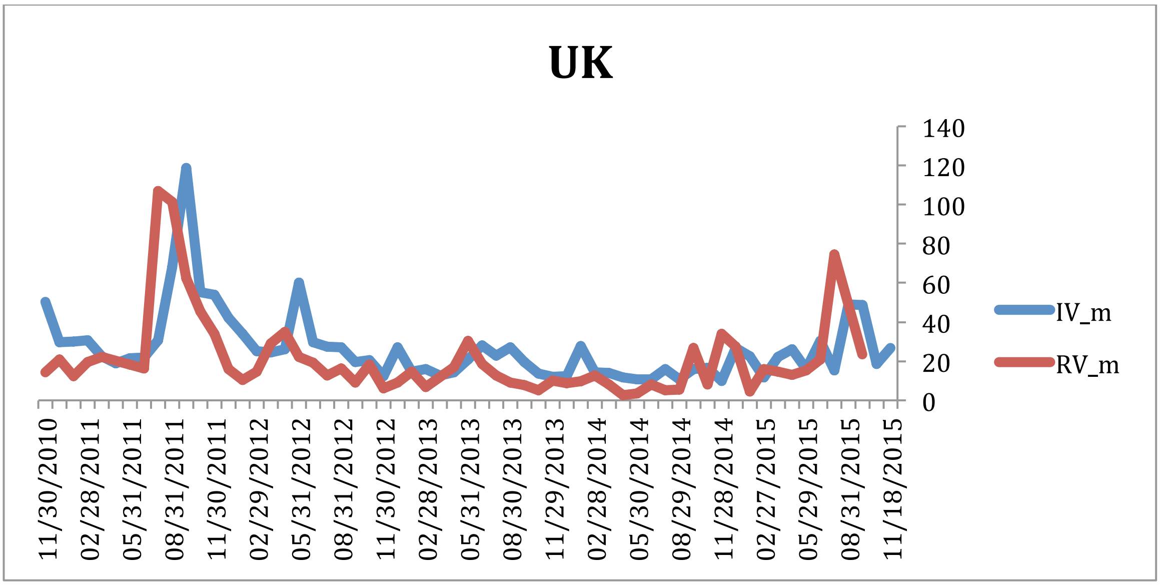


Figure 7. VCAC implied volatility and CAC 40 realized volatility

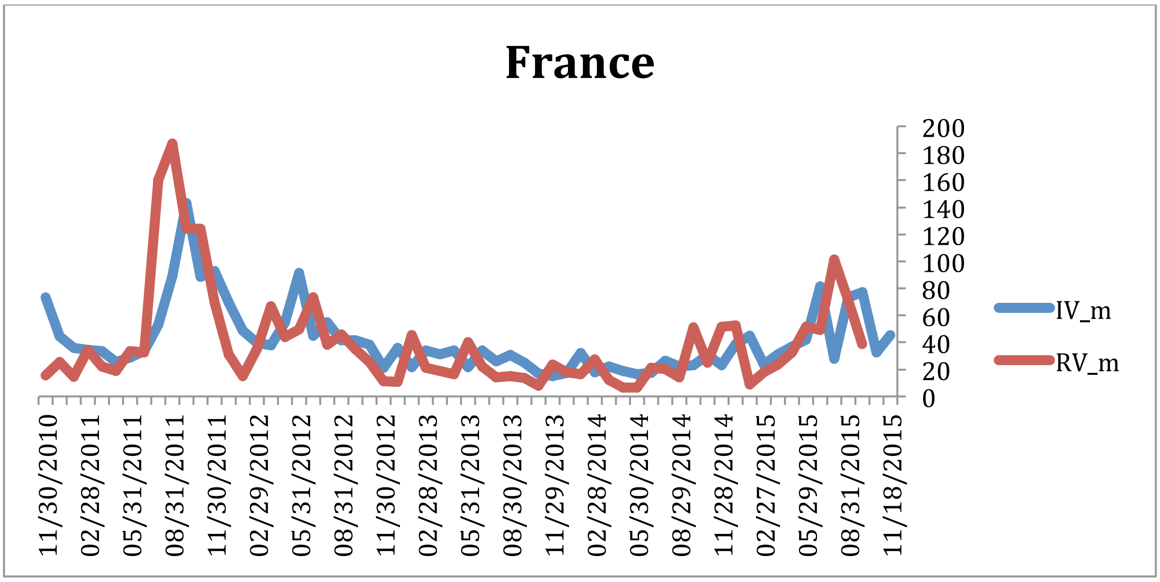
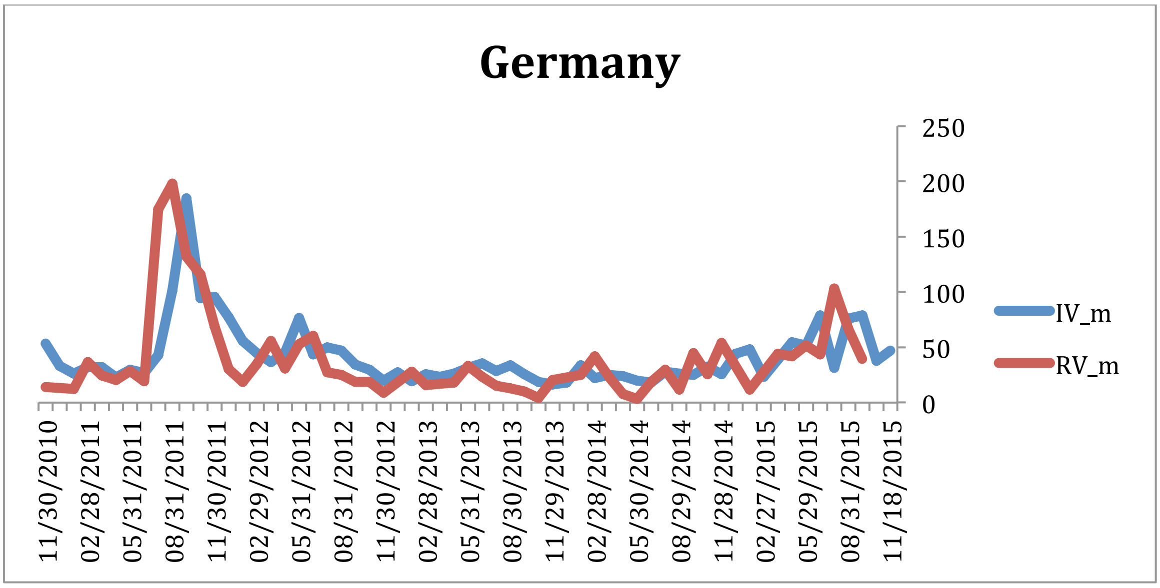


Figure 8. VDAX-New implied volatility and DAX 30 realized volatility



Graphs above show a great correlation between the two series in all eight countries, because they tend to co-move throughout the examined five-year time frame. In times of financial turmoil (for example in the year 2011) spikes in the implied variance graph can be seen. In addition, the same “behavior” is also shown by realized variance time series that experienced at the same time huge spikes in market crash events. On average, realized variance is lower than implied variance, but in periods of financial turmoil there is a reversion in majority of countries.

2.2 EMPIRICAL ANALYSIS

In order to examine the relationship between implied (IV) and realized volatility (RV), the following OLS regression is used:

$$RV_m = a_0 + a_i IV_m + e_m. \quad (12)$$

Implied and realized volatilities are aligned, meaning that realized volatility in the month m has the corresponding implied volatility observed in the time m -1.

If implied volatility holds some information regarding future realized volatility then coefficient a_i should be statistically significant and the null hypothesis $a_i = 0$ should be thus rejected. In addition, if implied volatility is an unbiased estimate of realized volatility, then a_0=0 and $a_i = 1$. Furthermore, if implied volatility is an efficient estimate then residuals should be uncorrelated with any other variable (and should be pure white noise).

Regression results are represented in the Table 8 below. It can be seen that implied volatility contains some information regarding future realized volatility and it is a biased estimate of realized volatility, because previously mentioned joint hypothesis are rejected in all cases. Moreover, implied volatility is in all eight cases efficient predictor of future volatility due to Durbin-Watson statistics not being significantly different from two - no autocorrelation of residuals.

In the second step, the following multiple regression was examined in order to compare the efficiency of implied volatility to the past one:

$$RV_m = a_0 + a_i IV_m + a_v RV_{m-1} + e_m. \quad (13)$$

Table 8 reports multiple regression results. Past realized volatility explains future realized volatility, however when the corresponding implied volatility index is added as an additional explanatory variable, only implied volatility remains statistically significant. Again, F-statistics rejects joint hypothesis ($a_0 = a_v = 0$ and $a_i = 1$), implying that implied volatility is a biased estimator.

Table 8. Regression results

	Intercept	IV_m	RV_{m-1}	Adj R-squared	F-statistics	Missing Obs.
USA	- 0,503	0,980***		39,330%	37,310	4
	(0,338)	(0,000)			(0,000)	
	-17,398		14,7667***	15,010%	10,537	6
	(0,173)		- 0,002		0,002	
	- 0,543	1,032***	- 0,045	37,876%	17,455	6
	(0,344)	(0,000)	(0,786)		(0,000)	
Switzerland	0,088	0,840***		27,354%	20,957	7
	(0,879)	(0,000)			(0,000)	
	1,374***		0,491***	22,697%	14,799	13
	(0,0004)		(0,0004)		(0,000)	
	- 0,075	0,834**	0,078	33,440%	11,777	13
	(0,907)	(0,012)	(0,695)		(0,000)	
Canada	- 0,737	1,021***		36,338%	33,535	3
	(0,179)	(0,000)			(0,000)	
	0,750***		0,668	45,063%	46,030	5
	(0,005)		(0,000)		(0,000)	
	0,098	0,519***	0,347	45,532%	23,988	5
	(0,867)	(0,004)	(0,244)		(0,000)	
Korea	- 0,092	0,953***		51,891%	40,909	23
	(0,852)	(0,000)			(0,000)	
	0,908*		0,672***	53,360%	23,877	40
	(0,054)		(0,0001)		(0,000)	
	0,108	0,730	0,159	55,055%	13,249	40
	(0,886)	(0,206)	(0,705)		(0,000)	
Japan	0,462	0,790***		21,396%	15,155	8
	(0,554)	(0,0003)			(0,000)	
	1,968***		0,433***	17,992%	10,783	15
	(0,0001)		(0,0019)		(0,0019)	
	0,841	0,553	0,156	19,710%	6,522	15
	(0,372)	(0,171)	(0,516)		(0,003)	

table continues

continued

	Intercept	IV_m	RV_{m-1}	Adj R-squared	F-statistics	Missing Obs.
UK	0,186	0,810***		29,578%	24,101	5
	(0,732)	(0,000)			(0,000)	
	(0,0026)		(0,000)		(0,000)	9
	0,277	0,500*	0,326	33,873%	14,062	9
	(0,634)	(0,0921)	(0,1123)		(0,000)	
France	0,217	0,879***		33,285%	30,136	3
	(0,709)	(0,000)			(0,000)	
	1,309***		0,617	37,601%	34,142	5
	(0,0007)		(0,000)		(0,000)	
	0,398	0,532*	0,323*	40,451%	19,681	5
	(0,509)	(0,064)	(0,089)		(0,000)	
Germany	- 0,262	0,996***		38,189%	33,745	7
	(0,676)	(0,000)			(0,000)	
	1,336***		0,610***	35,094%	26,412	13
	(0,002)		(0,000)		(0,000)	
	- 0,238	0,857***	0,155	42,896%	18,653	13
	(0,735)	(0,0098)	(0,448)		(0,000)	

Empirical results from this section suggest that implied volatilities of all countries are biased estimates of future realized volatility and they have better predictive power in comparison to past realized volatility when comparing either adjusted R-squared or statistical significance of the respective regression coefficients.

3 VARIANCE RISK PREMIUM: LOCAL MARKET

The purpose of this section is to construct local variance risk premiums for all the countries in the world that have calculated implied volatility index according to the new VIX Index model-free methodology and have highly liquid options market (see Siriopoulos and Fassas (2009).

As previously mentioned, variance risk premium is the difference between the risk-neutral and statistical expectation of the future return variation (some researchers calculate it also the other way around, but this does not influence results). I follow Bollerslev, Tauchen and Zhou (2009) approach and use the directly observable proxy of variance risk premium, retrieved as the difference between 1-month forward looking model-free option-implied variance and the actual 1-month realized variance. More detailed approach is described in the first chapter of thesis.

Daily realized variance calculations are based on data for French CAC 40, German DAX 30, Japanese Nikkei 225, Swiss SMI 20, British FTSE 100, Korean Kospi 200, Canadian S&P/TSX and the American S&P 500 (all obtained from Thomson Reuters and Bloomberg). The corresponding monthly model-fee implied volatilities for the CAC (VCAC), DAX (VDAX), Nikkei 225 (VXJ), SMI (VSMI), FTSE (VFTSE), Kospi 200 (VKOSPI), S&P/TSX (VXC) and S&P500 (VIX) are also obtained from Bloomberg and Thomson Reuters. The sample period for each of the time series is from the November 2010 until November 2015. My time period spans only five years, because Canadian model-free implied volatility index was introduced at the end of the year 2009. Finally, the risk-free rates used in the excess return calculations were retrieved from Bloomberg.

I use proxy for individual country variance risk premium, defined as follows:

$$VRP_t^i \equiv IV_t^2 - E(RV_{t+1}^{22}). \tag{14}$$

IV_t^2 denotes implied option volatility of the corresponding country's equity index in time t with maturity of one month and $\mathrm{RV}_{t+1}^{22}$is the realized variance measured over the next month (22 trading days). Note that $\mathrm{RV}_{t+1}^{22} - \mathrm{IV}_t^2$ is the return one gets when buying variance in a variance swap contract. In my approach I use ex-post realized variance over a time period of 22 trading days (one month) and compute logarithmic RV_{t+1}^{22}as a proxy for the true realized variance. It is computed using daily returns, but at the end I consider only monthly returns when constructing monthly variance risk premium. There are numerous studies that attempt to answer question about sampling frequency of realized variance, which hinges on different factors. For example, Ait-Sahalia, Mykland and Zhan (2005) show that for longer time horizon (e.g. one month), realized volatility should not be sampled with high frequency as in the case of shorter time period (e.g. one day).

Time-series graphs of country-specific variance risk premiums are shown in the following eight graphs. They all clearly show spike in August 2011 and exceptionally negative variance risk premium due to sharp drop in stock market prices. Stock exchanges in United States, Europe, Asia and Middle East were all negatively affected due to fears of European sovereign bond crisis contagion to Italy and France, slow economic growth and United States credit rating downgrade. The so-called Black Monday 2011 refers to 08.08.2011 when global stock market crashed due to United States credit rate downgrade by Standard and Poor's from AAA to AA+, which was the first downgrade in the history of United States. Extremely high market volatility continued also after August 2011 for the rest of the year.

Another negative spike (but not as drastic as the previous one) occurred in August 2015, when the Dow Jones Industrial Average fell. World stock markets suffered huge losses connected also with price drops in oil, copper, majority of Asian currencies lost value against United States Dollar.

Figure 9. American variance risk premium

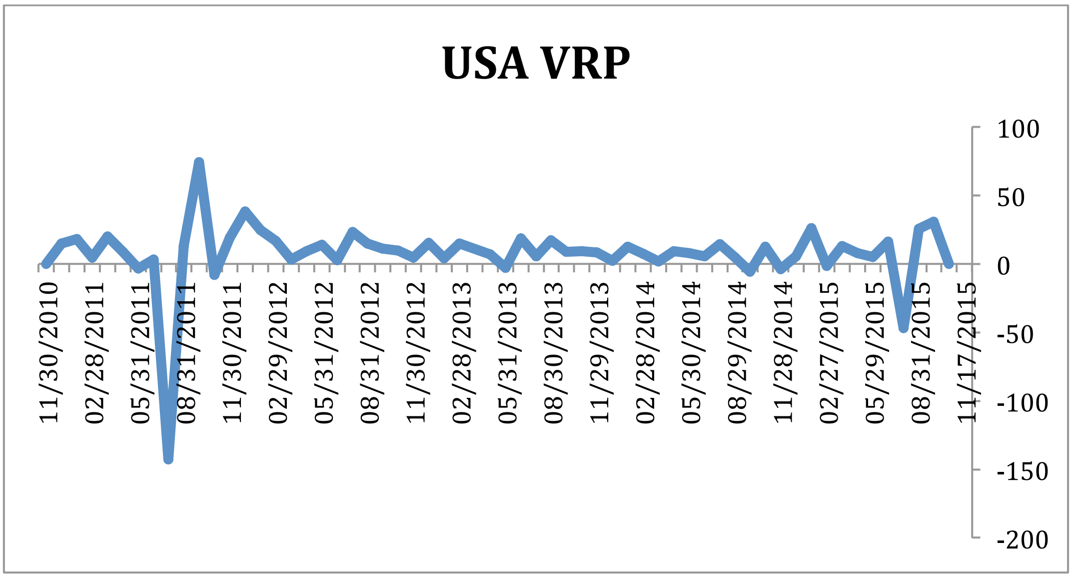


Figure 10. Swiss variance risk premium

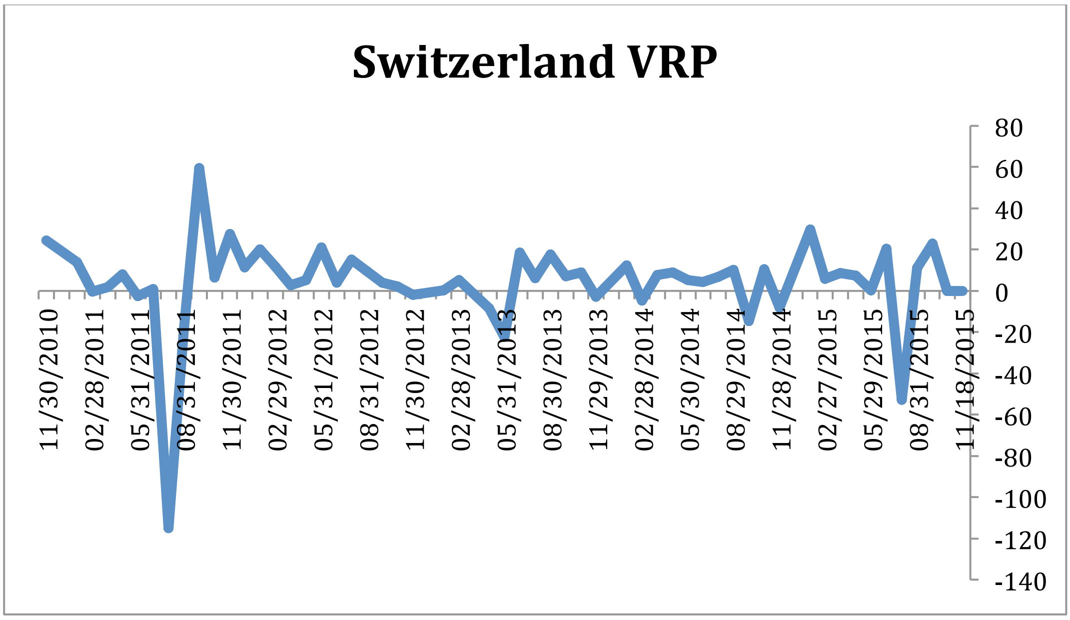


Figure 11. Canadian variance risk premium

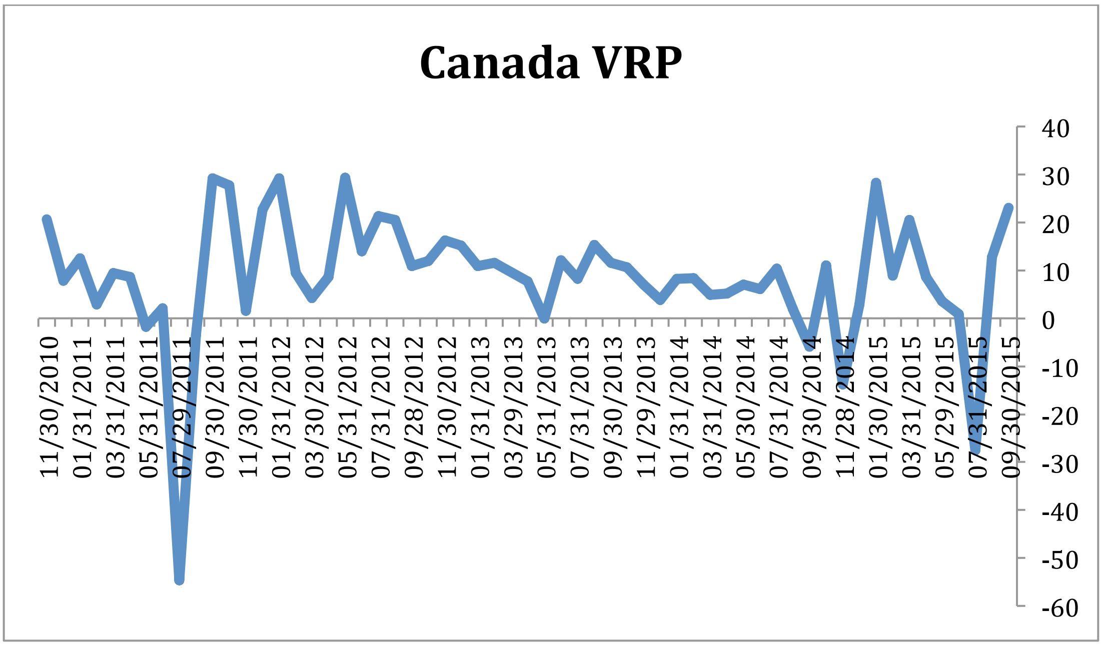


Figure 12. Korean variance risk premium

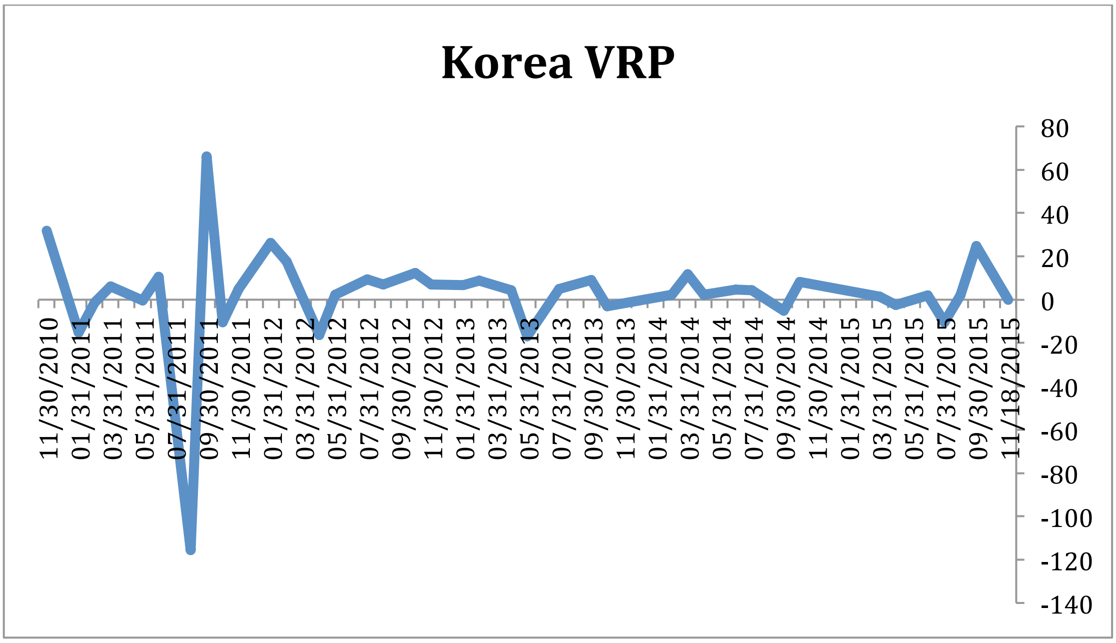


Figure 13. Japanese variance risk premium

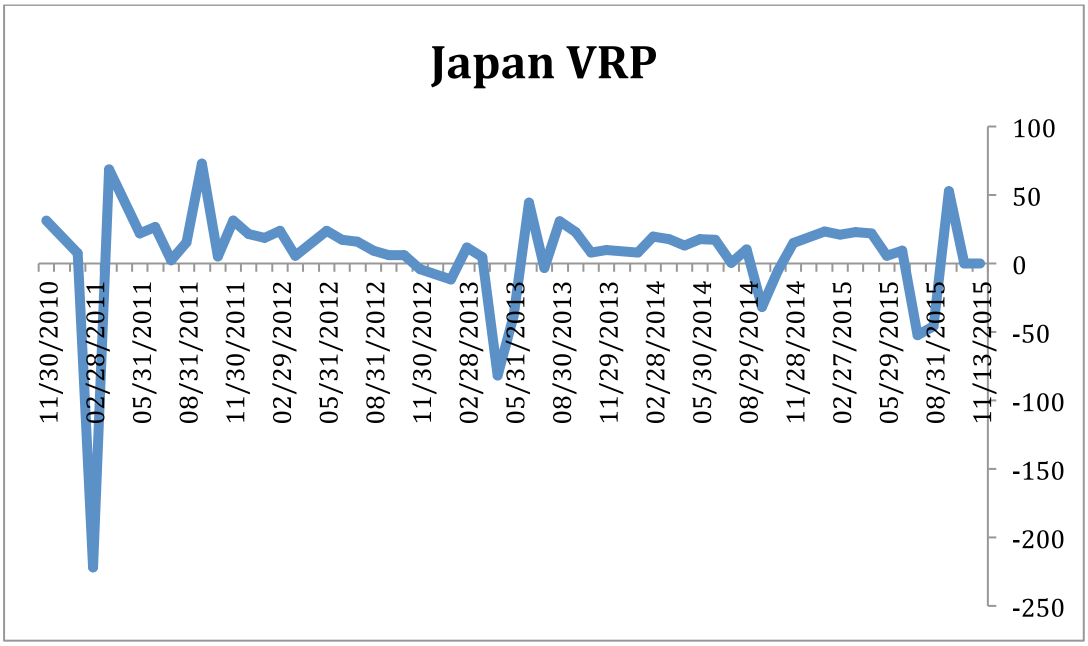


Figure 14. British variance risk premium

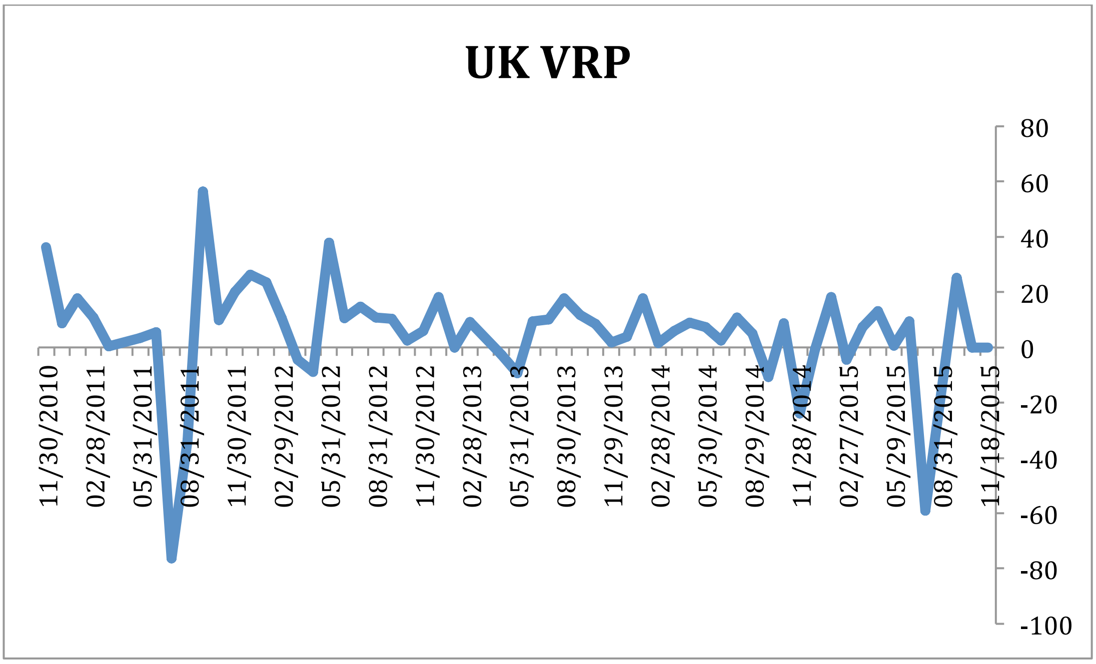
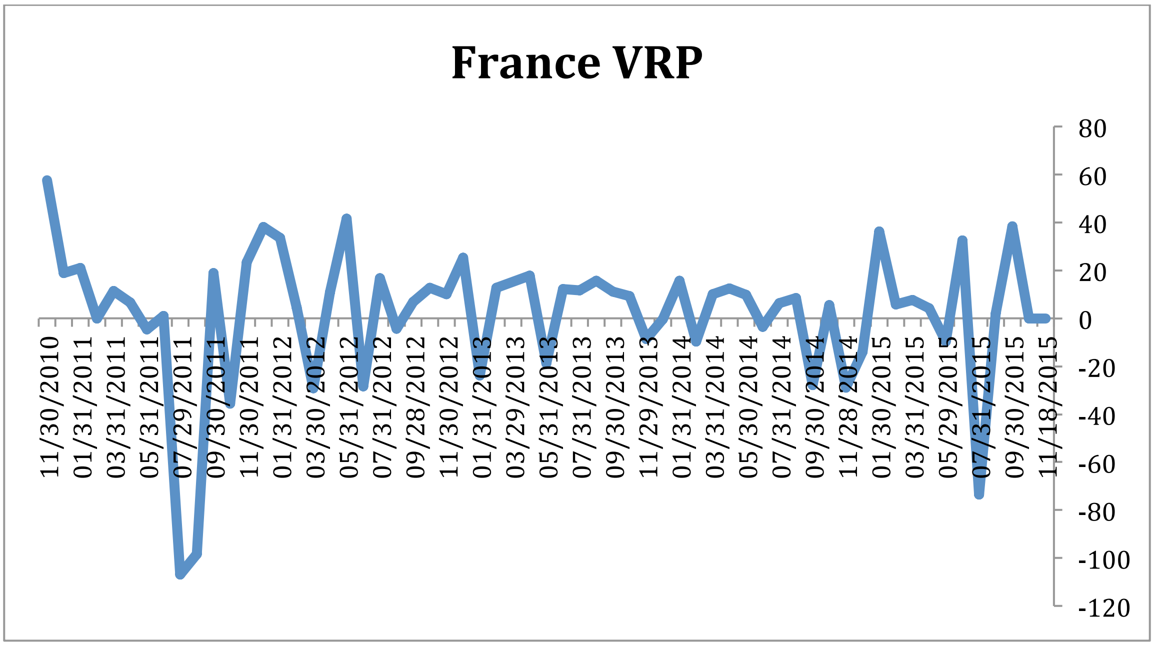


Figure 15. French variance risk premium



4 VARIANCE RISK PREMIUM: INTERNATIONAL SETTING

In the fourth chapter I expand local variance risk premium calculation into international setting and construct a global variance risk premium based on the capitalization weighted average of all eight countries.

Numerous studies found out that American stock market return is predictable up to two quarters when using variance risk premium in the regression. Moreover, Bollerslev et al. (2014) expand their research to other countries with available volatility indices and show that predictive relationship between future returns and current variance risk premium holds as well, although significance is not so high in case of United States.

In this chapter I examine regression return predictabilities. In particular, I analyze if the time-varying variance risk premium and its ability to predict returns holds internationally in one-month horizon. I start with country-specific regressions and regress monthly excess returns against local variance risk premium for each individual country and then I replace it by global variance risk premium. Proxy for the global variance risk premium is based on the capitalization-weighted average of the previously calculated country specific variance risk premiums.

My monthly aggregate market returns are based on the French CAC 40, the German DAX 30, the Japanese Nikkei 225, the Swiss SMI 20, the British FTSE 100, the American S&P 500, the Korean KOSPI 200 and the Canadian S&P/TSX60 index. The corresponding implied volatilities needed in the following analysis are VCAC (France), VDAX-New (Germany), VIX (United States), VFTSE (United Kingdom), VSMI (Switzerland), VXJ (Japan), VICX (Canada) and VKOSPI (Korea). They were all retrieved either from Reuters or Bloomberg.

In addition, the risk-free rates used in the excess return calculation were obtained from Bloomberg and I used the average risk free rate of five-year period. Sample period expands from November 2010 until November 2015 and is based on monthly frequency.

Table 9. Summary statistics

Country	**Mean**	**Median**	**Minimum**	**Maximum**	**Std. Deav.**	**Skewness**	**Ex. kurtosis**
USA Excess Ret.	0,008	0,010	- 0,074	0,106	0,034	- 0,041	0,456
USA VRP	7,766	8,908	- 142,840	74,066	24,393	- 3,8	23,499
Canada Excess Ret.	0,001	0,004	- 0,091	0,052	0,028	- 0,613	0,641
Canada VRP	8,876	9,166	- 54,700	29,381	13,137	- 2,050	8,251
France Excess Ret.	0,004	0,003	- 0,115	0,098	0,046	- 0,340	- 0,241
France VRP	1,209	7,201	- 106,930	41,582	27,907	- 1,971	5,031
Germany Excess Ret.	0,008	0,011	- 0,193	0,122	0,052	- 0,761	2,596
Germany VRP	3,127	9,076	- 131,640	52,303	29,857	- 2,253	7,390
UK Excess Ret.	0,001	0,004	- 0,075	0,079	0,034	- 0,177	- 0,051
UK VRP	3,940	7,962	- 76,321	56,346	20,468	- 1,722	5,378
Switzerland Excess Ret.	0,006	0,006	- 0,067	0,083	0,034	- 0,212	- 0,004
Switzerland VRP	3,774	5,543	- 114,990	59,378	21,303	- 2,923	15,503
Japan Excess Ret.	0,012	0,013	- 0,103	0,117	0,051	- 0,265	- 0,143
Japan VRP	7,307	10,994	- 221,800	73,001	39,486	- 3,410	17,562
Korea Excess Ret.	-0,002	0,000	- 0,129	0,085	0,041	- 0,183	0,555
Korea VRP	3,117	4,917	- 115,710	66,079	19,795	- 3,107	21,354

The standard set of summary statistics reported in Table 9 above implies coherence in the distribution of monthly country specific variance risk premium and corresponding monthly excess returns. Mean excess returns are on average positive and very small, which can be a result of the time frame of my analysis. In contrast, researchers that focused their analysis on the time period around financial crisis, report negative excess returns and much higher variations in variance risk premium. This can be confirmed also with the standard deviation results from my table.

Moreover, averages of variance risk premium for all countries are positive and are in the interval between 1,209 (for France) up to 8,876 (for Canada). It can be concluded that "volatility selling" has been profitable for the observed five-year period. Looking at skewness, it is negative in all cases and variance risk premiums exhibit large excess kurtosis.

Table 10. Correlation matrix for excess returns

	USA	France	Canada	Germany	UK	Switzerland	Japan	Korea
USA	1,0000	0,7649	0,7294	0,7401	0,8362	0,677	0,5985	0,5855
Fra		1,0000	0,6283	0,8717	0,8292	0,6538	0,5882	0,5441
Canada			1,0000	0,5439	0,7218	0,4226	0,3152	0,5347
Germany				1,0000	0,7472	0,5559	0,6095	0,6426
UK					1,0000	0,6467	0,5103	0,6020
Switzerland						1,0000	0,5173	0,2545
Japan							1,0000	0,3294
Korea								1,0000

Table 11. Correlation matrix for country specific variance risk premium

	USA	Switzerland	Canada	Korea	Japan	UK	France	Germany	GlobalVRP
USA	1,0000	0,9051	0,3546	0,4445	0,3586	0,7396	0,7912	0,7586	0,9236
Switzerland		1,0000	0,3387	0,3094	0,4219	0,8782	0,7982	0,8276	0,9983
Canada			1,0000	0,3476	0,3492	0,3981	0,3984	0,3512	0,2583
Korea				1,0000	0,3067	0,5692	0,4247	0,4912	0,2357
Japan					1,0000	0,4962	0,3918	0,3685	0,4846
UK						1,0000	0,7592	0,7235	0,7823
France							1,0000	0,9385	0,7582
Germany								1,0000	0,8535
Global VRP									1,0000

4.1 COUNTRY-SPECIFIC REGRESSIONS

In this section I perform standard OLS regressions, using monthly-calculated variance risk premiums and excess returns for each country separately. I want to check whether there is any power predictability for one-month horizon. The null hypothesis is that the true coeffieicnt is zero.

In the first step I regress monthly excess returns against local variance risk premium for each of the eight individual countries:

$$h^{-1} r_{t,t+h}^{i} = a^{i}(h) + b^{i}(h) VRP_{t}^{i} + u_{t,t+h}^{i}; \;\; h = 1 \tag{15}$$

where VRP_t^i denotes the variance risk premium for the corresponding country i and $r_{t,t+h}^i$ refers to the monthly excess return of the country's equity index. With h is denoted month excess return and in my case it is set to one month.

Looking at the results reported in Table 12, estimated coefficients are relatively low and insignificant, except for Canada and Korea. These two countries are also the ones with relatively newly calculated model-free implied volatilities. These findings are in line with other researcher's articles such as Bollerslev et al. (2014) who also found out that for the after-crisis period one-month horizon does not show statistically significant predictability. Previously mentioned researchers found out that horizon ranging between three and five months has the highest significance of estimated coefficients. However, I decided to analyze just one-month returns, as horizons up to twelve months result in lower amount of data for regression. Because some indices launched model-free implied volatility indices just recently (Canada for example), it makes sense to wait with longer-horizon analysis and collect more data if we want to include those countries into regressions as well.

Certain patterns that are common for analyzed countries suggest that perhaps better regression results might be achieved by adding global variance risk premium into the regression and that is the reason why I construct global variance premium weighted by market capitalization in the next section.

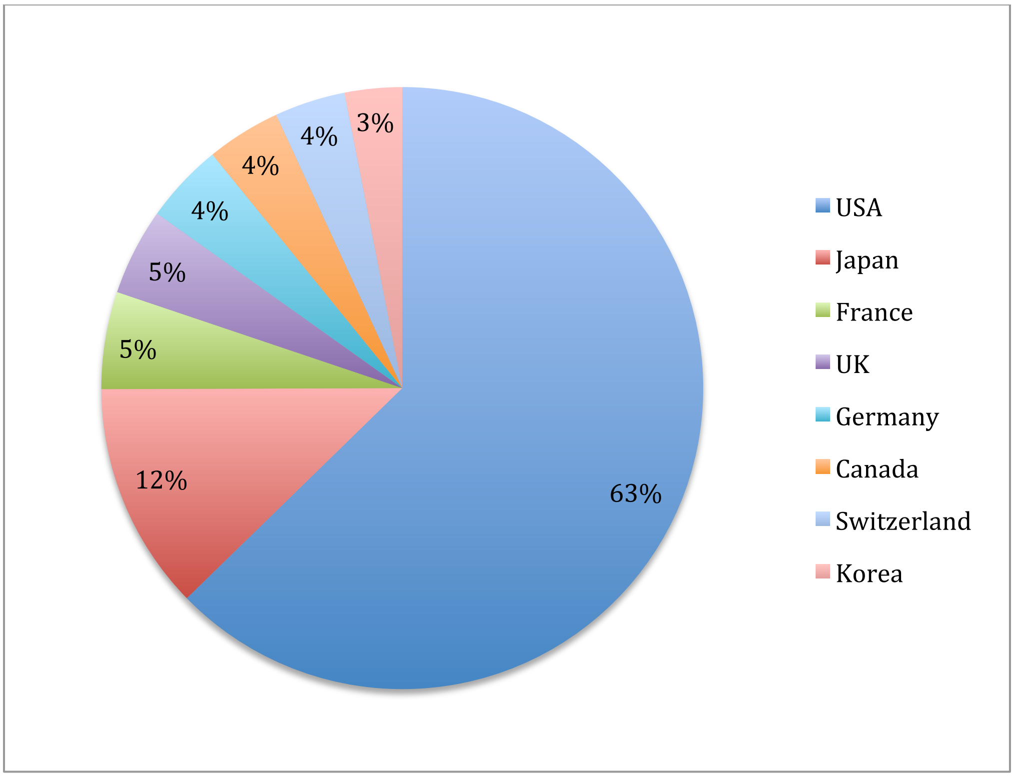
Table 12. Country-specific regressions

COUNTRY:	FRANCE	USA	CANADA	GERMANY	SWITZERLAND	UK	JAPAN	KOREA
Constant	0,0042	0,0096**	-0,0068	0,0075	0,0058	0,0004	0,0144**	-0,0049
	(0,4828)	(0,0046)	(0,1117)	(0,2662)	(0,1974)	(0,9273)	(0,0305)	(0,3303)
VRP	0,0002	-0,002	0,0007**	0,0003	0,0067	0,0001	-0,0003*	0,0008**
	(0,3816)	(0,2491)	(0,0170)	(0,2429)	(0,9821)	(0,6745)	(0,0546)	(0,0026)
Adj. R-squared	0,0132	0,0228	(0,0943)	0,0234	0,0009	0,0031	0,0622	0,1457
White test	10,5126	5,3401	4,6444	6,3091	8,3450	3,4193	0,4046	8,7403
	(0,0052)	(0,0692)	(0,0980)	(0,0427)	(0,0154)	(0,1809)	(0,8169)	(0,0126)
Breusch-Godfrey	0,6548	0,6039	1,0543	1,0991	0,7544	1,7832	0,5275	3,0013
	(0,7840)	(0,8281)	(0,4192)	(0,3842)	(0,6854)	(0,0798)	(0,886)	(0,0036)
Jarque-Bera	0,7550	1,1284	0,1710	6,3939	0,2376	0,5236	1,0742	0,9773
	(0,6855)	(0,5688)	(0,9180)	(0,0409)	(0,9950)	(0,7697)	(0,5845)	(0,6134)

4.2 GLOBAL VARIANCE RISK PREMIUM

In this section I construct a proxy for the global variance risk premium, which is based on the capitalization-weighted average of the previously calculated country specific variance risk premiums. Source for the market capitalization was World federation of Exchanged database. In my sample of eight countries by far the largest market capitalization has USA, accounting for more than 60% of the total number, followed by Japan. Figure below shows market capitalization for France (CAC 40), Germany (DAX 30), the United Kingdom (FTSE 100), Japan (Nikkei 225), Switzerland (SMI 20), United States (S&P 500), Canada (S&P/TSX60) and Korea (KOSPI 200).

Figure 16. Market Capitalization



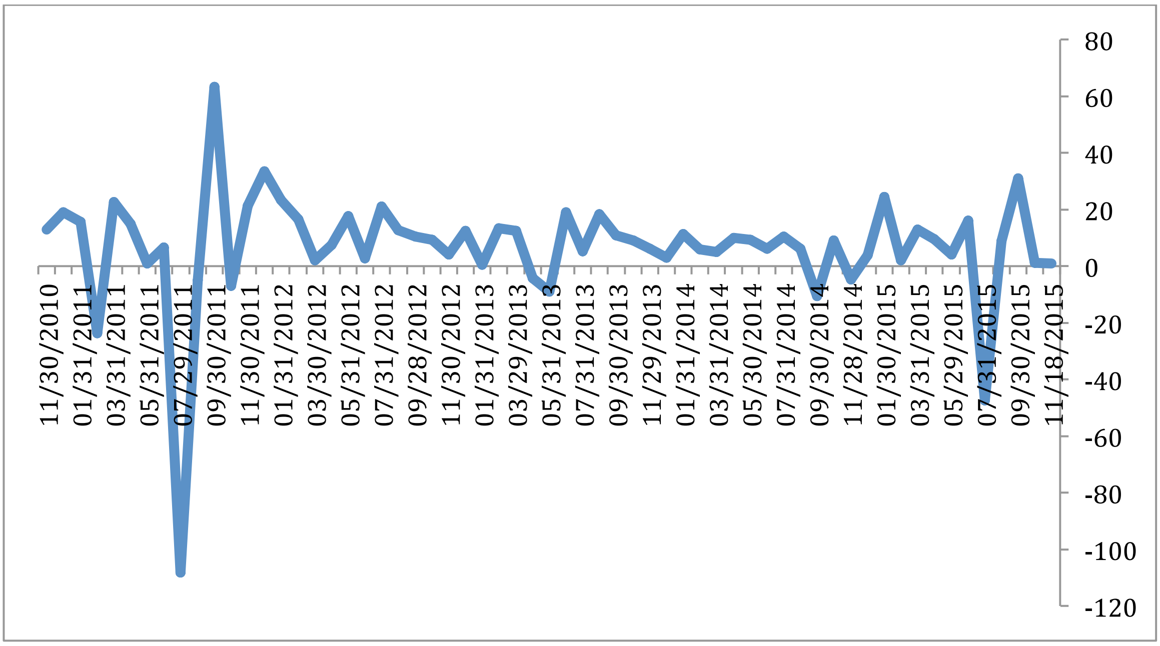
Global variance risk premium $\mathrm{VRP}_t^{\mathrm{GLOBAL}}$ is then calculated with the following formula:

$$VRP_t^{GLOBAL} = \sum_{i=1}^{8} w_t^i VRP_t^i \,, \tag{16}$$

where i = 1, 2,… 8 denotes the corresponding country and w_t^i the corresponding weight.

From the graph below can be seen that in comparison to country individual variance risk premium also global variance risk premium shows huge spikes in times of financial turmoil, like August 2015 and August 2011.

Figure 17. Global variance risk premium



In order to see whether global variance risk premium has any power predictability over one-month global excess return weighted by all eight countries market capitalization, I re-estimate the equation number 15. Results in the table below show no statistically significant parameters with low R-squared.

Table 13. Global excess returns regressed against global variance risk premium

Horizon	1-month
Constant	0,0086* (0,0611)
Global VRP	-0,0022 (0,2999)
R-squared	0,0185

4.3 INDIVIDUAL COUNTRY REGRESSIONS

The newly obtained global proxy from the previous section substitutes the local variance risk premium in the country regression equation, which gives us:

$$h^{-1} r^{i}_{t,t+h} = a^{i}(h) + b^{i}(h) VRP^{GLOBAL}_{t} + u^{i}_{t,t+h}; h = 1. \tag{17}$$

The null hypothesis is that the true coefficient equals zero.

Regression results are reported in the Table 14. Comparing these results with the one obtained in country-specific regressions, for one-month horizon global variance risk premium does not perform any better than local variance risk premiums, as it is not statistically significant. This can appear due to the observed horizon. As previously mentioned when there will be more data available, it would make sense to calculate up to one year returns and see whether power predictability increases with the time-horizon.

Table 14. Global variance risk premium regressions

COUNTRY:	FRANCE	USA	CANADA	GERMANY	SWITZERLAND	UK	JAPAN	KOREA
Constant	0,0048	0,0095**	0,0002	-5,1180**	0,0059	0,0020	0,0149**	-0,0037
	(0,4452)	(0,0466)	(0,9674)	(0,0212)	(0,2059)	(0,6724)	0,0330	(0,5146)
GLOBAL VRP	-0,0001	-0,0203	-0,0002	1,2226***	-0,0000	-0,0002	-0,0004	0,0002
	(0,8364)	(0,2711)	(0,3456)	(0,0000)	(0,9578)	(0,4112)	(0,1844)	(0,4839)
Adj. R-squared	0,0007	0,0203	0,0153	0,7215	0,0000	0,0117	0,0302	0,0085
White test	7,5491	3,1557	18,2880	0,8474	8,9717	0,2375	0,0032	0,3569
	(0,0229)	(0,2064)	(0,0001)	(0,6546)	(0,0112)	0,8880	(0,9984)	(0,8366)
Breusch-Godfrey	0,7467	0,5828	1,4254	0,6574	0,2378	0,8941	0,6414	3,3069
	(0,6821)	(0,8446)	(0,1891)	(0,7817)	(0,9951)	(0,5592)	(0,7958)	(0,0017)
Jarque-Bera	1,5750	1,3646	2,9904	40,5145	0,8167	0,5133	0,6027	2,5092
	(0,4550)	(0,5055)	(0,2242)	(0,0000)	(0,6647)	(0,7737)	(0,7398)	(0,2852)

5 (GLOBAL) FORWARD-LOOKING VARIANCE RISK PREMIUM

The use of proxy for the variance risk premium considers the assumption that volatility follows a random walk. Thus, the forward-looking variance risk premium is used in order to explore the sensitivity of the international empirical findings to this simplified assumption and for robustness check.

In previous chapters I estimated realized variance and used the assumption that conditional variance of each country's stock returns follows a martingale. In contrast, data does not always satisfy the martingale assumption, which can lead to biased results of variance risk premium. Consequently, the physical expectation of the future realized return variance is substituted with a forward-looking model-based expectation in the forward-looking variance risk premium calculation for the individual country at time t:

$$FVRP_t^i = IV_t^i - E\left(RV_{t,t+22}^i\right), \tag{19}$$

where: : IV_t^i = implied variance in time t for country i and

$E\left(RV_{t,t+22}^i\right)$ = forward expectation of realized variance for country i.

In order to calculate the forward expectations of realized variance $E\left(RV_{t,t+22}^i\right)$, the following model is used:

$$RV_t{}^i = \alpha_t + \beta_t RV_{t-22}^i + \varepsilon_{t.} \tag{20}$$

This generates one-step ahead static forecasts.

Next, I re-estimate the equation 15 from the previous chapter and replace variance risk premium with forward-looking variance risk premium.

Table 15. Country specific excess returns regressed against global forward VRP

	FRANCE	USA	CANADA	GERMANY	SWISS	UK	JAPAN	KOREA
Constant	0,008	0,014***	0,008	0,016**	0,013***	0,004	0,020***	0,000
	(0,183)	(0,003)	(0,109)	(0,023)	(0,006)	(0,439)	(0,003)	(0,946)
FVRP	-0,002***	-0,001***	-0,001***	-0,001**	-0,002***	-0,001**	-0,001***	-0,001*
	(0,009)	(0,002)	(0,008)	(0,007)	(0,001)	(0,037)	(0,003)	(0,024)
R-squared	0,113	0,154	0,115	0,125	0,166	0,073	0,147	0,060

It can be seen that forward-looking variance risk premium has better power predictability in comparison to global variance risk premium for all eight countries. Moreover, also R-squared is much higher.

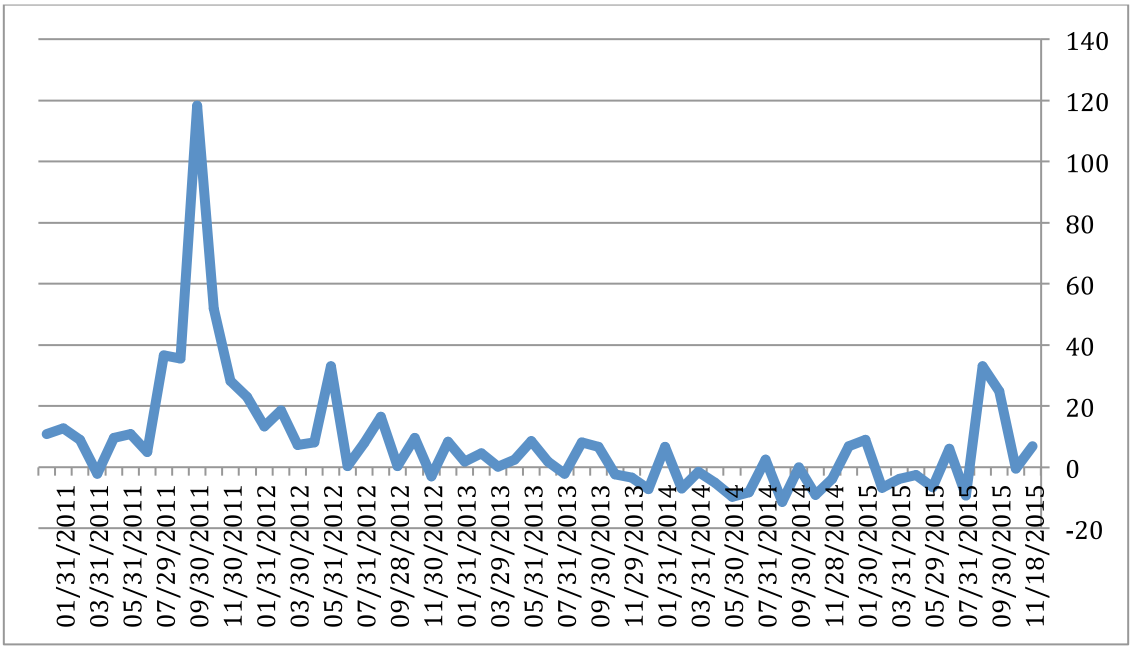
After examining country-specific forward-looking variance risk premium $FVRP_t^i$ I build global forward-looking global variance risk premium $FVRP_t^{GLOBAL}$ and investigate its return-predictability pattern. It is constructed according to the following formula:

$$FVRP_t^{GLOBAL} = \sum_{i=1}^{8} w_t^i \, FVRP_t^i \,, \tag{18}$$

where FVRP_t^i is calculated with a model that produces the forward expectations. In addition, $\mathrm{FVRP}_t^i = \mathrm{IV}_t^i - \mathrm{E}_t(\mathrm{RV}_{t,t+1}^i)$ is the equation for the individual corresponding country's forward-looking variance risk premium.

The resulting global forward-looking variance risk premium, weighted by each country's market capitalization is displayed in the following figure.

Figure 18. Global forward variance risk premium



Graphs with global variance risk premium and global forward variance risk premium noticeably differ. On one hand, global variance risk premium shows negative spikes in times of market turmoil (August 2011 and August 2015 for instance). On the other hand, the global forward variance risk premium exhibits positive spikes in that period. Consequently, negative pattern present in global variance risk premium is diminished when looking at global forward variance risk premium computed with the forecasted values of countries' local realized variances and then weighted by market capitalizations.

In the next step I estimate the regression equation 15 and switch global variance risk premium for global forward-looking variance risk premium and compare both results. The following table shows results where global excess returns weighted by market capitalization of all eight countries were regressed against the global forward variance risk premium:

Table 16. Global excess returns regressed against global forward VRP

Horizon	1-month
Constant	0,0129*** (0,0042)
FVRP	-0,001*** (0,0015)
R-squared	0,1614

It turns out that power predictability by global forward-looking variance risk premium is very high for one-month horizon and parameters are statistically significant. Moreover, also R-squared is here much higher.

In addition, global forward variance risk premium appears to have some power predictability also over country-specific excess returns for all eight countries. They all have also much higher R-squared in comparison to results from previous table, except for Germany.

Table 17. Country specific excess returns regressed against global forward VRP

	FRANCE	**USA**	**CANADA**	**GERMANY**	**SWISS**	**UK**	**JAPAN**	**KOREA**
Constant	0,012*	0,014***	0,004	0,015**	0,010**	0,005	0,02***	0,001
	(0,053)	(0,004)	(0,237)	(0,043)	(0,042)	(0,285)	(0,007)	(0,895)
Global FVRP	-0,001***	-0,001***	-0,001***	-0,001**	-0,001**	-0,001**	-0,001**	-0,000**
	(0,003)	(0,002)	(0,005)	(0,030)	(0,043)	(0,025)	(0,013)	(0,044)
R-squared	0,144	0,150	0,190	0,079	0,069	0,083	0,103	0,060

Overall, these results suggest that the global forward-looking variance risk premium has higher power predictability in explaining global excess returns in comparison to global variance risk premium for the one-month horizon. Moreover, individual country forward-looking variance risk premium perform even better in comparison to global forward-looking variance risk premium. Although coefficients still remain quite low, R-squared are much higher.

However, these results are in line with findings from Bollerslev et al. (2014) who also concluded that for the after-crisis period one-month horizon does not show statistically significant predictability when using country- or global-variance risk premium. Previously mentioned researchers found out that horizon ranging between three and five months has the highest significance of estimated coefficients. However, I decided to analyze just one-month returns, as horizons up to twelve months result in lower amount of data for regression. Because some indices launched model-free implied volatility indices just recently (Canada for example), it makes sense to wait with longer-horizon analysis and collect more data if we want to include those countries into regressions as well.

6 TIME-VARYING VARIANCE RISK PREMIUM IN USA

Variance risk premium changes over time. Bollerslev and Zhou (2006) found out that it is able to explain more than 15% of the ex-post time series variation in quarterly excess returns for the period from 1990 until 2005, which is more in comparison to other predictor variables, such as P/E ratio, the dividend yield, the default spread, etc. Their results show the importance of temporal variation in risk as well as risk-aversion when determining the independent variables in regression that explain stock market returns.

Overall, variance risk premium plays an important role in explaining variation in stock market returns. On the other hand, there is also a question which variables explain variance risk premium? Variance risk premium is one of the indicators of the risk-aversion coefficients of market participants and if volatility premium were constant, it would imply a constant risk aversion coefficient, which would be too restrictive when explaining stock return dynamics.

There are many papers written in that regard. For example, Brandt and Wang (2003) obtain statistically significant results that risk aversion changes in response to news and inflation. In another study, other researcers constructed different models in order to explain time-varying variance risk premium (risk aversion). Their findings suggest that some macro-economic variables can be linked to temporal variation in the volatility risk premium. In addition Carr and Wu (2007) examine variance risk premium in relation to asset allocation and check whether excess returns of selling or buying variance swaps can be explained by CAPM model and Fama-French factors. However, their findings suggest that none of previously mentioned models are able to strongly explain excess returns on variance swaps and obtain a strongly negative variance risk premium due to systematic variance risk factor. There exists a negative correlation between index returns and volatility. Consequently, also beta is negative and can explain a small portion of the negative variance risk premium.

Following the Fama-French approach, also additional factors SMB (factor related to the firm size) and HML (factor related to the book-to-market-value) are unable to explain negative variance risk premium. The negative sign shows that investors are willing to pay a large premium in order to protect themselves against increasing market volatility. Another approach adopted by Ait-Sahalia, Karaman and Mancini (2013) is to regress variance risk premium against various maturities of S&P500 Index, certain economic variables and the VIX Index in order to check time-varying variance risk premium. Their analysis shows that previously mentioned variables are able to explain a large amount of the variation of variance risk premium.

Factor model. In this subsection I conduct an empirical analysis and check whether there are any macroeconomic variables that can explain time-varying variance risk premium for the American market. I focus only on the United States and not on other seven countries from the analysis before, because it has longer historical data availability.

Time period is from January 1990 until December 2015, using monthly variance risk premium. In order to have more accurate measurements, I decided to use logarithmic variance risk premium, consistent with the above-mentioned research. The following equation is estimated:

$$VRP_t = \alpha + \sum_{i}^{N} \beta_i \, M_{t,i}, \tag{21}$$

where $\mathrm{VRP_t}$ denotes the logarithm of the variance risk premium, α is the constant and $\mathrm{M_{t,i}}$ is the i-th macroeconomic variable at time t.

I collected monthly series data of seven different macroeconomic variables: Industrial Production, Default Spread (difference between Moody's BAA and AAA bond yield indices), Consumer Price Index for All Urban Consumers (CPI), Producer Price Index for All Commodities (PPI), Civilian Unemployment Rate, Chicago Fed National Activity Index (CFNAI) and Dividend Yield on S&P500. Data was retrieved from Federal Reserve Economic Data for the time period from February 1990 until November 2015.

In the first step individual regressions of all macroeconomic variables were performed (using a threshold of 5% p-value) in order to see which combination of independent variables could be used in a multivariate regression model. Therefore, the following standardized macroeconomic variables were used in my model:

1. Logarithm of Producer Price Index (PPI)
2. Logarithm of Unemployment Rate (UNRATE)
3. Logarithm of Industrial Production Index (INDPROD)

Figure 19. Producer Price Index (PPI)

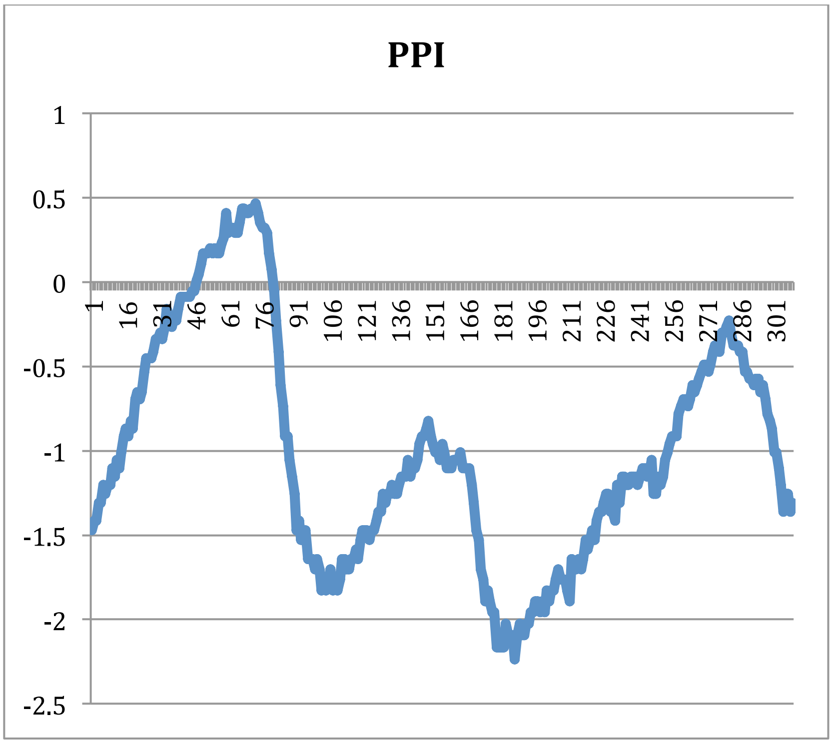


Figure 20. Unemployment Rate (UNRATE)

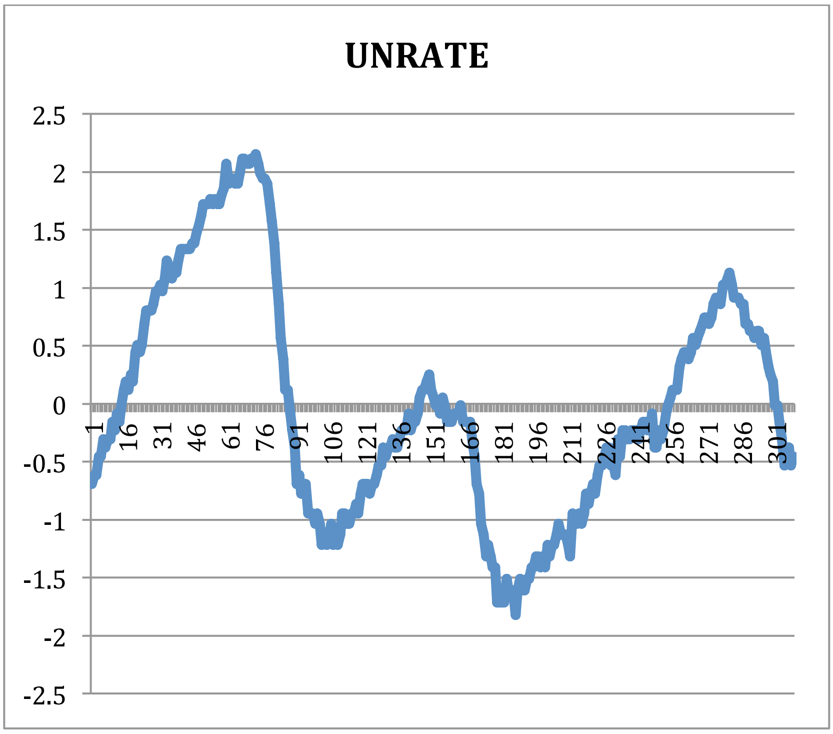
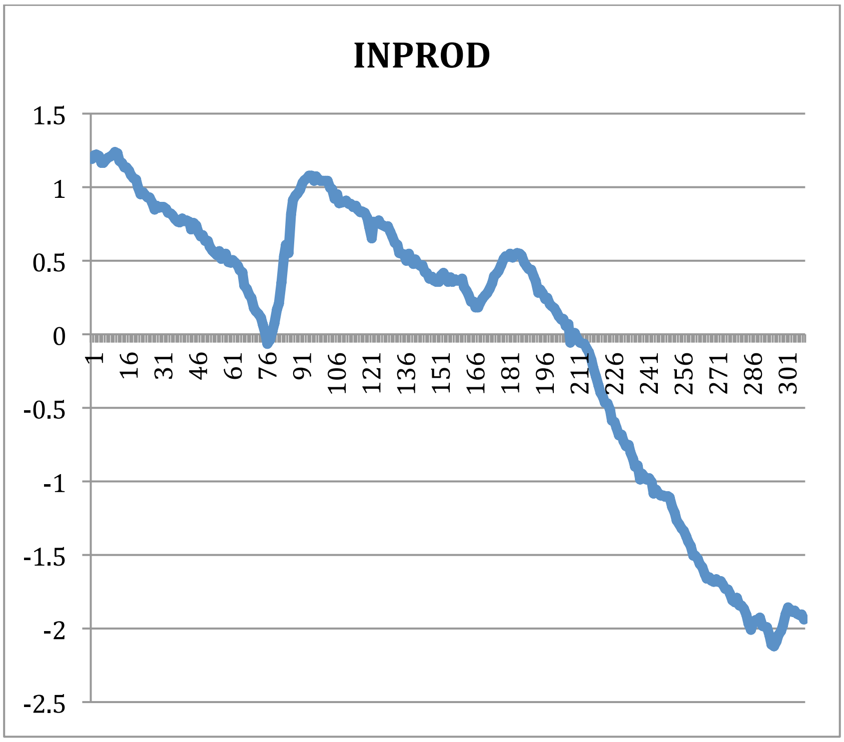


Figure 21. Industrial Production Index (INDPROD)



In regressions I standardized all variables in order to obtain zero mean and standard variation of one, so that their marginal contributions are easier to observe. Finally, the following regression equation is estimated:

$$lnVRP_t = \alpha + \beta_1 INDPROD + \beta_2 PPI + \beta_3 UNRATE, \qquad (22)$$

where: lnVRP = logarithm of variance risk premium in time t
INDPROD = Industrial Production Index
PPI = Producer Price Index
UNRATE = Unemployment Rate.

Table 18. Summary statistics

	Mean	Std. Dev.	Minimum	Maximum	Skewness	Ex. Kurtosis
UNRATE	0	1	- 1,823	2,1523	0,4093	- 0,6558
PPI	0	1	2,2362	0,4645	0,4093	- 0,6558
INDPROD	0	1	- 2,210	1,2374	- 0,8484	- 0,6525

Table 19. Regression results

	Coefficient	Std. error	t-ratio	p-value
Const.	8,067***	2,836	2,844	0,005
INDPROD	- 0,006***	0,001	- 3,832	0,000
PPI	- 2,908***	1,023	- 2,842	0,005
UNRATE	0,272***	0,062	4,411	0,000

Table 20. Additional tests

Mean dependent var	0,000
Sum squared resid	276,731
R-squared	0,102
Log-likelihood	- 421,412
Schwarz criterion	865,756
Rho	0,098
S.D. dependent var	1,000
S.E. of regression	0,953
Adjusted R-squared	0,093
Akaike criterion	850,823
Hannan-Quinn	856,793
Durbin-Watson	1,789

From the above tables it can be seen that all macroeconomic variables used in multivariate regression are statistically significant when using 5% as the threshold for the p-value. Consequently, I can reject all null hypotheses that all explanatory variables are jointly equal to zero.

In addition, signs of the independent variables are also important to consider when explaining what drives temporal variation in the American variance risk premium. For example, PPI Index has the greatest negative contribution of approximately -2,91. As previously mentioned, variance risk premium is commonly used as one of the indicators for risk aversion of market participants and according to my regression results this means that an increase in PPI Index causes smaller risk aversion. The next variables are Unemployment Rate with the positive coefficient of 0,27 and Industrial Production Index with the magnitude of -0,01. All these variables explain approximately 10,15% of the total variation in the variance risk premium when looking at the Adjusted R-squared.

However, results might be misleading if OLS assumptions do not hold. In order to examine that, there are three main assumptions that must hold:

1. Homoscedasticity in the error terms
2. Error terms must be uncorrelated with one another
3. Normally distributed disturbances

If the first assumption does not hold, it means that OLS estimators no longer have the minimum variance among the class of unbiased estimators, although they are still consistent as well as unbiased. Thus, if OLS method is used despite heteroscedasticity being present, the standard error could be incorrect and consequently the inference is wrong. If the second assumption is violated, errors are heteroscedastic, which implies that obtained OLS estimators are no longer Best Linear Unbiased Estimator (hereinafter: BLUE). Finally, third assumption must hold in order to conduct single or joint hypothesis about the model parameters.

In the previous table are results from the analysis of White test and Breusch-Godfrey test with 12-lags. White test is a statistical test that checks whether the residual variance of a variable is constant - its null hypothesis assumes that residuals are not heteroscedastic. On the other hand, the null hypothesis of the Breusch-Godfrey test is that the residuals have no autocorrelation.

From the two tables below can be seen that Breusch-Godfrey test has p-value grater than 5%, which means that null hypothesis cannot be rejected. On the other hand, null hypothesis about homoscedasticity is rejected with the White test.

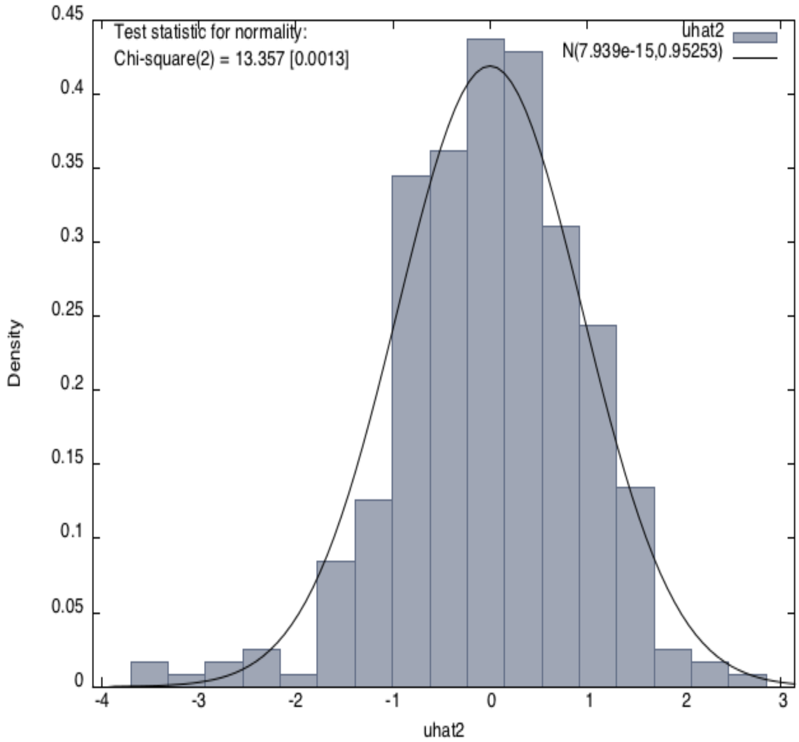
Table 21. White test and Breusch-Godfrey test

TEST	**Null Hypothesis**	**Test Statistic**	**P-Value**
White test	Homoscedasticity	18,244	0,032
Breusch-Godfrey	No-autocorrelation	1,436	0,149

In the next step of the residual analysis I check the distribution of the error terms by plotting their distribution against normal one. The graph is displayed below together with

Jarque-Bera test results. The obtained p-value is 0,0013, which rejects the null hypothesis of normality.

Figure 22. Distribution frequency of residuals



7 OPEN ISSUES AND FURTHER RESEARCH QUESTIONS

1. Global variance risk premium can be seen as a proxy for a worldwide economic uncertainty and some researchers have shown that it provides better predictions of future returns rather than local variance risk premium. Although my regression results report statistically insignificant coefficients for one-month returns, there are research papers that report statistically significant results from three to up to five-month returns. I did not calculate those due to lack of data availability.

My analysis focuses on five-year time frame from 2010 until 2015 because of Canada (they launched model-free calculated implied volatility index at the end of 2009). When there will be longer historical time series, it would make more sense to conduct again regressions for different horizons and check whether those findings hold also for Canada and other countries that recently introduced such indices.

2. Derivatives products are becoming more and more popular also due to changing regulation. For example, IFRS 9 moves cost of hedging to equity and thus reduces P&L

volatility. One of the most popular derivative products is variance swap and as mentioned in the first chapter, long side of the variance swap contract receives at the end of specified period the difference between realized variance and fixed variance swap rate. Thus, another further development of this thesis could be research of possible trading strategies that are based on variance swaps and underlying local index returns (depending on each country separately). New ways of asset allocation could be analyzed in order to see if they provide better investment performances.

3. In the past few years there has been a growing interest of using various derivative products related to variance. The most actively traded product among them has been variance swap, where a long-side in a variance swap contract at the end of holding period gets the difference between realized and fixed variance swap rate. Consequently, another way in which this thesis could be extended is to implement some trading strategies based on variance swaps and country-individual returns (here we assume that variance risk premium is a good predictor for country-individual excess returns - see previous chapters). This analysis could show that with the use of variance swap rate in certain trading strategies there could be some significant changes in asset allocation as well as higher Sharpe ratios in order to increase the performance of investment portfolios.

CONCLUSION

In my thesis I decided to use variance risk premium as an approximation for the short-term loss aversion in order to see whether it has any predictive power and if it is able to explain country-specific excess stock returns for one-month horizon and later on global exess returns regressed against global variance risk premium.

I use model-free implied variance (for example squared VIX index in case of United States) and realized variance in order to build country-specific variance risk premium and later on global variance risk premium. I use monthly non-overlapping data for the time period from November 2010 to November 2015.

The variance risk premium is defined as the difference between the risk-neutral and objective expectations of realized variance. In my empirical part, implied volatility in time t is measured as the end-of-month implied volatility-squared and it is de-annualized ($\mathrm{IV}_t^2/12$). The realized variance is the sum of squared daily returns of corresponding equity index over one month (these are 22 trading days), in the interval between t and t+1.

I conduct econometric analysis for eight countries (Unites States, United Kingdom, France, Japan, Korea, Switzerland, Germany and Canada) that have available model-free calculated implied volatility indices. I check the information content of implied volatility regarding realized volatility and empirical findings suggest that that implied volatilities of all analyzed countries are biased estimates of future realized volatility and they have better predictive power in comparison to past realized volatility when comparing either adjusted R-squared or statistical significance of the respective regression coefficients.

In addition, I check whether local variance risk premiums and global variance risk premiums have any predictive power for the local excess returns or global excess returns and my findings for one-horizon show mainly statistically insignificant results. It would make sense to conduct analysis for longer time horizons, but this would result in less data, if I continue using monthly-calculated variance risk premium. Because some countries introduced model-free calculated implied volatility indices just few years ago, they do not have long time series of historical data and this can negatively affect the quality of regressions. However, these results are in line with findings from Bollerslev et al. (2014) who also concluded that for the after-crisis period one-month horizon does not show statistically significant predictability. Previously mentioned researchers found out that horizon ranging between three and five months has the highest significance of estimated coefficients. However, I decided to analyze just one-month returns, as horizons up to twelve months result in lower amount of data for regression. Because some indices launched model-free implied volatility indices just recently, it makes sense to wait with longer-horizon analysis and collect more data if we want to include those countries into regressions as well.

In contrast, when regressing global and country individual forward-looking variance risk premium against global excess returns weighted by countries market capitalization, results show statistically significant parameters for all eight countries, although with marginal contribution. Individual country forward-looking variance risk premium show better power predictability in comparison to the global one.

Furthermore, statistical summary of variance risk premiums shows that for the observed period between November 2010 and November 2015 local variance risk premiums have been on average positive for all countries and this implies that “volatility selling” has been profitable in the observed time frame.

After constructing and examining global variance risk premium, I focus only on the American market (due to longer historical time-series data availability) and check whether there are any macroeconomic variables that would be able to explain time-varying nature of variance risk premium. I use univariate and multivariate regressions to examine the possible benefits of adding a new potential explanatory variable. Results regarding the factor model suggest that some macroeconomic variables such as Producer Price Index, Unemployment Rate and Industrial Production Index can explain approximately 10,15% of the total variation in the variance risk premium when looking at the Adjusted R-squared. Other tests and models that follow are used for examinging wheter errors are homoscedastic, normally distributed and not autocorrelated - suggesting that the factor model is well specified.

REFERENCE LIST

1. Ahoniemi, K. (2008). Modeling and Forecasting the VIX Index. Retrieved June 21, 2015, from http://epub.lib.aalto.fi/pdf/diss/a340.pdf
2. Ait-Sahalia, Y., Karaman, M., & Mancini, L. (2013). The term structure of variance swaps, risk premia and the expectation hypothesis. Retrieved June 21, 2015, from https://workspace.imperial.ac.uk/businessschool/Public/RiskLab/8th%20HF%20conf/8_Mancini.pdf
3. Ait-Sahalia, Y., Mykland, A., & Zhang, L. (2005). A tale of two time scales: determining integrated volatility with noisy high-frequency data. Retrieved June 21, 2015, from https://www.princeton.edu/~yacine/twoscales.pdf
4. Amengual, D. (2008). The Term Structure of Variance Risk Premia. Retrieved June 21, 2015, from http://www.cemfi.es/~amengual/docs/termS.pdf
5. Andersen, T. G., & Benzoni, L. (2008). Realized Volatility. Retrieved June 21, 2015, from https://link.springer.com/chapter/10.1007%2F978-3-540-71297-8_24?LI=true
6. Ang, A., & Bekaert, G. (2007). Stock Return Predictability: Is It There. *Review of Financial Studies, 20*, 651–707.
7. Angelidis, T., Benos, A., & Degiannakis, S. (2003). The use of GARCH models in VaR estimation. Retrieved June 21, 2015, from http://macro.soc.uoc.gr/8conf/docs/The%20Use%20of%20GARCH%20Models%20in%20VaR%20Estimation.pdf
8. Bakshi, G., Kapadia, N., & Madan, D. (2003). Stock return characteristics, skew laws and the differential pricing of individual equity options. *Review of Financial Studies 16*, 101-143.
9. Becker, R., Clements, A.C., & White, S. (2007). Does implied volatility provide any information beyond that captured in model-based volatility forecasts?. *Journal of Banking and Finance 31*, 2535–2549.
10. Bekaert, G., Engstrom, E., & Xing, Y. (2009). Risk, Uncertainty and Asset Prices. *Journal of Financial Economics, 91*, 59–82.
11. Bekaert, G., & Hoervoa, M. (2014). The VIX, the variance premium and stock market volatility. Retrieved June 21, 2015, from http://www.nber.org/papers/w18995
12. Benartzi, S., & Thaler, R. (1995). Myopic Loss Aversion and the Equity Premium Puzzle. Quarterly *Journal of Economics, 110*(1), 73-92.
13. Bollerslev, T., & Zhou, H. (2006). Volatility puzzles: a simple framework for gauging return-volatility regression. *Journal of Econometrics, 131*, 123–150.
14. Bollerslev, T., Gibson, M., & Zhou, H. (2011). Dynamic estimation of volatility risk premia and investor risk aversion from option-implied and realized volatilities. *Journal of Econometrics, 160*, 235-245.
15. Bollerslev, T., Marrone, J., Xu, L., & Zhou, H. (2014). Stock return predictability and variance risk premia: statistical inference and international evidence. Retrieved June 21, 2015, from http://laixu.me/sites/default/files/general/intvrp_bmxz.pdf

16. Bollerslev, T., Tauchen, G., & Zhou, H. (2009). Expected stock return and variance risk premia. Retrieved June 21, 2015, from http://public.econ.duke.edu/~boller/Published_Papers/rfs_09.pdf
17. Boudoukh, J., Richardson, M., & Whitelaw, R. (1998). *The best of both worlds: A hybrid approach to calculating value at risk, 11*, 64-67.
18. Brandt, M. W., & Wang, K. Q. (2003). Time-varying risk aversion and unexpected inflation. *Journal of Monetary Economics, 50*, 1457-1498.
19. Chang, C.-L., Jiménez-M., J.-A., McAleer, M., & Amaral, T.P. (2011*). Risk Management of Risk Under the Basel Accord: Forecasting Value-at-Risk of VIX Futures.* Retrieved June 21, 2015, from https://www.ucm.es/data/cont/docs/518-2013-11-05-1102.pdf
20. Chicago Board of Options Exchange. (n.d.). *Additional Features Of VIX Futures.* Retrieved June 21, 2015, from http://cfe.cboe.com/education/vixprimer/-Features.aspx.
21. Chicago Board of Options Exchange. (n.d.). *The CBOE Volatility Index - VIX.* Retrieved June 21, 2015, from http://www.cboe.com/-Strategies/VIX/IndexIntro/Part1.aspx.
22. Chicago Board of Options Exchange. (n.d.). *The Relationship of the SPX and the VIX Index.* Retrieved June 21, 2015, from http://www.cboe.com/Strategies/-VIX/IndexIntro/part3.aspx.
23. Chicago Board of Options Exchange. (n.d*.). VIX Futures: The Basics.* Retrieved June 21, 2015, from http://cfe.cboe.com/education/vixprimer/Basics.aspx. 67
24. Christensen, B. J., & Prabhala, N. R. (1998). The relation between implied and realized volatility. *Journal of Financial Economis*, vol. 50, issue 2, 125-150.
25. Clements, A.C., & Fuller, J. (2012*).* Forecasting Increases in the VIX: A Time-Varying Long Volatility Hedge for Equities. Retrieved June 21, 2015, from http://www.ncer.edu.au/papers/documents/WP88.pdf
26. Coudert, V., & Gex, M. (2008). Does Risk Aversion Drive Financial Crises? Testing the Predictive Power of Empirical Indicators. *Journal of Empirical Finance, 15*, 167-184.
27. Coval, J. D., & Shumway, T. (2001). Expected option returns. *Journal of Finance, 56*, 983-1009.
28. Day, T. E., & Lewis, C. M. (1992). Stock market volatility and the information content of the stock index options. *Journal of Econometrics, 52*, 267-287.
29. Driessen, J., & Maenhout, P. (2007). An empirical portfolio perspective on option pricing anomalies, *Review of Finance, 11*, 561-603.
30. Fernandes, M., Medeiros, M. C., & Scharth, M. (2013). Modeling and predicting the CBOE market volatility index. Retrieved June 21, 2015, from http://bibliotecadigital.fgv.br/dspace/bitstream/handle/10438/11333/TD%20342%20%20CEQEF%2010%20-%20Marcelo%20Fernandes%20-%20Marcelo%20C.%20Medeiros%20-%20Marcel%20Scharth.pdf
31. Ferson, W. E., & Harvey, R. (1993). The Risk and Predictability of International Equity Returns. *Review of Financial Studies, 6*, 527–566.
32. Fleming, J. (1998).The Quality of Market Volatility Forecasts Implied by S&P 100

Index Option Prices. *Journal of Empirical Finance, 5,* 317-345.
33. Ghysels, E., Santa-Clara, P., & Valkanov, R. (2005). There is a risk-return trade-off afer all. *Journal of Financial Economics,* 76(3), 509-548.
34. Giot, P. (2005). Relationships Between Implied Volatility Indexes and Stock Index Returns. *Journal of Portfolio Management,* 31, 92-100.
35. Goyal, A., & Welch, I. (2008). A Comprehensive Look at the Empirical Performance of Equity Premium Prediction. *Review of Financial Studies, 21*, 1455–1508.
36. Han, B., & Zhou, Y. (2010). Variance Risk Premium and Cross-Section of Stock Returns. Retrieved June 21, 2015, from http://cn.ckgsb.com/Userfiles/doc/VRP_Dec2011a.pdf
37. Hjalmarsson, E. (2010). Predicting Global Stock Returns. *Journal of Financial and Quantitative Analysis, 45*, 49–80.
38. Jiang, G. J., & Tian, Y. S. (2005). The Model-Free Implied Volatility and Its Information Content. *Review of Financial studies,* 18(4), 1305-1342, 2005.
39. Johnson, T. (2015). Risk premia and the VIX term structure. Retrieved June 21, 2015, from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2548050
40. Londono, J. (2011). The variance risk premium around the world. Retrieved June 21, 2015, from http://www.federalreserve.gov/pubs/ifdp/2011/1035/ifdp1035.pdf
41. Lu, Z., & Zhu, Y. (2010). Volatility components: the term structure dynamics of VIX futures. *The Journal of Futures Markets*, 30(3), 230–256.
42. Martin, I. (2013). Simple Variance Swaps. Retrieved June 21, 2015, from http://personal.lse.ac.uk/martiniw/SVS%20slides%20latest.pdf
43. Mehra, R., & Prescott, E. (1985). The Equity Premium: A Puzzle. *Journal of Monetary Economics, 15*(2), 145–161.
44. Menzly, L., Santos, T., & Veronesi, P. (2004). Understanding Predictability. *Journal of Political Economy, 112*, 1-47.
45. Nossman, M., & Wilhelmsson, A. (2011). Non-Parametric Future Looking Value-at-RisK. Retrieved June 21, 2015, from https://www.researchgate.net/profile/Anders_Wilhelmsson/publication/228204707_Non-Parametric_Future_Looking_ValueatRisk/links/546df5530cf2bc99c21504f7.pdf?origin=publication_detail
46. Poon S., & Granger C. (2003). Forecasting Volatility in Financial Markets: A Review. *Journal of Economic Literature*, 41(2), 478-539.
47. Sahalia, Y., Karaman, M., & Mancini, L. (2015). The term structure of variance swaps and risk premia. Retrieved June 21, 2015, from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2136820
48. Siriopoulos C., & Fassas, A.(2009). Implied Volatility Indices - A Review. Retrieved June 21, 2015, from http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1421202 .
49. Whaley, R. E. (2000). The Investor Fear Gauge. *Journal of Portfolio Management*, 26(3), 12-17.
50. Whaley, R.E. (2008). Understanding VIX. Vanderbilt University – Finance. *Journal of Portfolio Management*, 35(3), 98-105.

51. Zhou, G., & Zhu, Y. (2009). A Long-Run Risks Model with Long- and Short-Run Volatilities: Explaining Predictability and Volatility Risk Premium. Retrieved June 21, 2015, from http://cn.ckgsb.com/frsc2009/papers/d53f2d23-7cbe-4588-890c-bc64b1bb8be4.pdf
52. Zhou, H. (2010). Variance risk premia, asset predictability puzzles, and macroeconomic uncertainty. Retrieved June 21, 2015, from http://www.federalreserve.gov/pubs/feds/2010/201014/201014pap.pdf