UNIVERSITY OF LJUBLJANA
SCHOOL OF ECONOMICS AND BUSINESS

MASTER'S THESIS

# IMPACT OF LOCAL AND GLOBAL STRUCUTRAL BREAKS IN VOLATILITY ON ITS PERSISTENCE

Ljubljana, August 2020 ROK POŽUN

# AUTHORSHIP STATEMENT

The undersigned Rok Požun, a student at the University of Ljubljana, School of Economics and Business, (hereafter: SEB LU), author of this written final work of studies with the title Impact of local and global structural breaks in volatility on its persistence prepared under supervision of dr. Igor Lončarski,

D E C L A R E

1. this written final work of studies to be based on the results of my own research;
2. the printed form of this written final work of studies to be identical to its electronic form;
3. the text of this written final work of studies to be language-edited and technically in adherence with the SEB LU's Technical Guidelines for Written Works, which means that I cited and/or quoted works and opinions of other authors in this written final work of studies in accordance with the SEB LU's Technical Guidelines for Written Works;
4. to be aware of the fact that plagiarism (in written or graphical form) is a criminal offense and can be prosecuted in accordance with the Criminal Code of the Republic of Slovenia;
5. to be aware of the consequences a proven plagiarism charge based on the this written final work could have for my status at the SEB LU in accordance with the relevant SEB LU Rules;
6. to have obtained all the necessary permits to use the data and works of other authors which are (in written or graphical form) referred to in this written final work of studies and to have clearly marked them;
7. to have acted in accordance with ethical principles during the preparation of this written final work of studies and to have, where necessary, obtained the permission of the Ethics Committee;
8. my consent to use the electronic form of this written final work of studies for the detection of content similarity with other written works, using similarity detection software that is connected with the SEB LU Study Information System;
9. to transfer to the University of Ljubljana free of charge, non-exclusively, geographically and time-wise unlimited the right of saving this written final work of studies in the electronic form, the right of its reproduction, as well as the right of making this written final work of studies available to the public on the World Wide Web via the Repository of the University of Ljubljana;
10. my consent to the publication of my personal data that are included in this written final work of studies and in this declaration, when this written final work of studies is published.

Ljubljana, ________________________ Author's signature: _________________________

## TABLE OF CONTENTS

## LIST OF FIGURES



## LIST OF TABLES



## LIST OF APPENDICES

# INTRODUCTION

Volatility in asset returns plays an important role for investors and researchers. It is commonly associated with the second moments of asset returns, namely variance and standard deviation and is one of the key indicators of the risk associated with an asset. The combination of practical importance and academic research potential has fuelled an ongoing interest in volatility.

One of the patterns that has been discovered in volatility is that shocks in volatility appear to have long lasting effects. If volatility increases in a certain period, it is likely that it will be also elevated in the following period. This was first described by Mandelbrot (1963), when he noted that large stock returns of either sign are followed by large returns of either sign. The direction of the returns does not matter, only the magnitude matters. Since then, persistency in volatility has been linked with various asset classes not just stocks, for example exchange rates, see Cheung (1993), and macroeconomic variables such as real interest rates, see Neely and Rapach (2008). Poterba and Summers (1984) also formally showed that if the expected shocks to volatility exhibit persistence it should increase the expected price of a security. Among others, Lamoureux and Lastrapes (1990) and Granger and Hyung (2004) also studied the effect of structural breaks in variance on the degree of persistency in volatility and their main conclusion was that if the structural breaks are not included in the analysis, the estimated degree of persistence might be too high. If the persistence really is a feature of volatility, this has several significant implications. To begin with, this has a large practical consequence, because that would mean that volatility today can be used to forecast volatility in the future, which reduces the uncertainty and riskiness stemming from it. This offers the possibility of using current and historical realizations of volatility to form trading strategies and portfolio allocations for the future. Secondly, this also uncovers that there are some dependencies over time in the second moments of the underlying processes in the financial markets, which cause the long memory in volatility. Understanding the reasons why these dependencies exist would expand our knowledge about the fundamentals of the financial world. From the following, it becomes clear that persistence in volatility plays an important role for practitioners, academics and researchers.

Additionally, persistence in volatility has been studied for several decades and researchers are yet to establish a uniformed conclusion about the statistical and economic significance of persistence in volatility of different asset classes, which indicates that it will remain among the studied features of volatility until a credible consensus is reached. A clear sign that persistency in volatility has sparked academic interest is that the observance of it lead to introduction of new econometric concepts such as fractional integration and new econometric model, which are specifically meant for capturing persistence in volatility such as the fractionally integrated GARCH model.

The purpose of the master's thesis is to check whether persistence in volatility can be associated with the volatility of the S&P 500 index and 50 individual stocks that were included in the index. The observed period is from beginning of January 2000 to beginning of May 2020. This would help to understand if the current volatility is correlated with the volatility in the future. If this is the case, it has important implications on the pricing of the stocks and on forecasting future volatilities. I also want to examine if the statistical significance and degree of persistence change when structural breaks in the variance of returns are considered. This would enhance the understanding of the persistency in volatility as it would show how persistency changes over time and in shorter subperiods. The main aim of the master's thesis is to estimate the memory parameter, which is the measure of the persistence in volatility, using various semi-parametric and parametric methods for the S&P 500 index and the 50 individual stocks and portfolio of these individual stocks. The next aim of the master's thesis is the decomposition of the portfolio' variance on a systemic component, which is related to the variability of the S&P 500 and the non-systemic component. This will be done by rolling regressions, where the portfolio returns are the dependent variable and the returns of the S&P 500 are the explanatory variable. Another important aim is to determine the locations of the structural breaks in the variance of the S&P 500 returns and the residual variances obtained by the previously mentioned rolling regressions. After identification of the structural breaks by using the loglikelihood approach and division of the analysed period into subperiods, the procedures for estimating the memory parameter are applied to the portfolio and S&P 500 return series to check if the memory parameter changes through time.

The structure of the master's thesis can be roughly divided into two parts: theoretical and empirical. To begin with the theoretical part, some basic information about volatility is provided. This is followed by chapters, which focus on persistency in volatility: definition of persistency, asset pricing implications, economic explanations, econometric background and methods for measuring and detecting. The theoretical part is concluded with the literature review. In the empirical part I analysed the daily data for the S&P500 and 50 individual stocks that are included in this index from January 2000 to May 2020. In the beginning, some descriptive statistics are provided for the 51 individual returns series of logarithmic daily returns. The main part of the empirical analysis is focused on estimating the degree of persistence using various semi-parametric and parametric methods for the entire period and then for the different subperiods, which were determined by identifying structural breaks in the volatility of the S&P500 and the portfolio consisting from the chosen stocks. The thesis ends with a conclusion, where I recap the may findings.

# 1 THEORETICAL BACKGROUND OF VOLATILITY AND ITS PERSISTENCE

In the following section the theoretical framework of volatility and its persistence will be described. The first subsection will focus on volatility in general and the rest will deal with persistence in more detail.

## 1.1 Volatility fundamentals

The volatility of an asset is a measure of variation of its price in a certain period (De Silva, McMurran & Miller, 2017, p. 1). Intuitively, volatility of an asset helps us to determine its riskiness. Generally, an asset with a more volatile return is riskier in the eyes of the investors. Standard asset pricing theory teaches that higher risks needs to be compensated with higher returns. Statistically speaking, volatility tells us about dispersion of the asset's returns (Volatility definition, 2019). Due to this, one of the first volatility measures was standard deviation of asset's returns in a certain period. This is often called the historical volatility of an asset.

In general, the measures of volatility can be categorized in two groups: ex-ante measures and ex-post measures depending on their horizons (Linton, 2019, p. 359). Standard deviation of asset's returns would fall into the latter category. Another commonly used ex-post measure of volatility is the beta coefficient, which is a relative measure of volatility and it tells how volatile a stock is compared to its respective market. Nevertheless, the question of how to determine the suitable respective market arises. Implied volatility is one of the most important and commonly used measures of the ex-ante category. It is most commonly associated with the pricing of options as it is a component of the Black-Scholes equation. Implied volatility can also be used as a metric of the projection of future volatility and expresses the likelihood of a change of an asset's price in the future based on the demand and supply forces on the market (Implied Volatility-IV, 2020). It is important to point out that the implied volatility offers no indication on the direction of the price change, just the size of it. For example, high implied volatility would indicate that there might be a large price swing in the future but would not tell us if the price will increase or decrease by a lot. One of the most used and followed volatility metrics by the practitioners in the financial field is the Volatility Index (VIX). It tells us the expected volatility of the U.S. stock market for the next 30 days based on the S&P500 call and put options (CBOE, 2020). It was designed by the Chicago Board Options Exchange and was one of the first real time indices of future market volatility. The use of this index has become widespread. It can be used as an investor sentiment tracker on the stock market, measure of risk and volatility and a potential indicator of incoming recessions or bear markets. In addition, it can also serve as an underlying asset for financial derivatives such as future and options. VIX options and futures are frequently used as hedging or speculative instruments.

Volatility has been one of the hottest research topics in the past couple of decades. In the period between 1996 and 2015, the term "volatility" has been the third most used keyword in publications in the financial and economic journals, only behind the terms "risk" and "corporate" (Brooks & Schopohl, 2018, p. 38). Techniques for modelling, estimating and measuring volatility have made big steps forward. This can be attributed to advances in technology, which lead to better computers with more computational power and better data collection, which allows researchers to work with more precise and recent data. Volatility is also becoming more and more important for practitioners such as investors, traders, asset managers etc… Just one of the indicators of volatility's importance is that there are many financial instruments, which treat volatility as a security (for example options on implied volatility).

According to Linton (2019, p. 358), volatility also plays important role in risk management, portfolio allocation and determining the market quality. When it comes to risk management, standard deviation is a necessity when calculating Value at Risk (VaR) and its derivatives, such as Conditional Value at Risk (CVaR). This family of risk measures is one of the most commonly used in risk management. When deciding on your portfolio allocation a crucial input for calculating weights assigned to each individual asset is the variance-covariance matrix, which requires the estimation of second moments. An example of this kind decision making would be the mean-variance framework developed by Markowitz in 1952. Finally, market volatility is often considered as one of the most important indicators of market quality, because there is a common agreement among the investors that a more volatile market is considered less good. This probably relates to the fact that more volatility means more risk and less predictability in the eyes of the investors, which makes their tasks harder. It is also important to point out, that the volatility of a market is by no means the only or the most important factor when it comes to determination of the market quality. If a stock market in a certain country has only a few illiquid stocks listed, the volatility of such stock market would be lower, but on the other hand it is highly unlikely that investors would classify it as a quality market.

When modelling volatility, the main goal is that your model can forecast volatility as best as possible. Classification of the volatility models used in practice can be done on how the volatility is formulated. There are two main classes of volatility models: the first case is when the conditional variance as a measure of volatility is modelled as a function of directly observed variables and the second class of models model volatility as a function of variables that are not observable (Engle & Patton, 2001. p. 49). GARCH family of models belongs to the first class. The models belonging in the second class are often referred to as latent volatility models, because volatility is modelled as a function of latent variables. Such models are exceedingly difficult to estimate. Their main working assumption is that all the shocks affecting the volatility are not observable and thus can not be modelled as a function of the current or past information sets. The shocks can be divided in two categories: observable and unobservable. These models are sometimes categorized as stochastic volatility models, although this name might be misleading because their main feature is that the volatility function includes latent variables. For example, the GARCH equation also includes a stochastic component of the volatility and it belongs to the first class of the models.

Researchers have come up with some of the features of volatility that need to be incorporated in a volatility model. The features are the following: volatility exhibits persistence (already pointed out), volatility has mean-reversion, volatility shocks are asymmetric (also known as the leverage effect or risk premium effect) and volatility can be influenced by external factors (Engle & Patton, 2001, p. 52).

A variety of external factors can impact a volatility of an asset. An increase in volatility of a certain asset class can transfer a surge of volatility into another asset class. This is a common sight in times of economic recessions when investors are reducing their exposures, closing their positions and fleeing into save havens. Such transmission of volatility happened during the economic recession in 2008-09, when the shock from the mortgaged backed securities gradually spread through the entire financial industry. Changes in volatility of an asset can also be caused by different exogenous events such as macroeconomic announcements (for example, monthly unemployment data and PMI manufacturing data are closely watched by stock market investors), political events and other one-time events (COVID-19 pandemic). Mean reversion is a common feature of financial time series and it can be best described as fluctuation around a mean of level. Mean reversion is not associated only with volatility as some also believe that mean-reversion is a feature of other key macroeconomic variables, for example real interest rates (Kim & Ji, 2011). The first one to observe the so-called "leverage effect" was Black (1976) when he observed a negative correlation between returns of an asset and the asset's volatility. In addition to that, the effect appears to be asymmetric, because negative returns are accompanied with larger increases in volatility than the decreases in volatility that accompany the positive returns (Aït-Sahalia, Fan & Li, 2013, p. 224). The economic rationale behind this proposed by Black was that because of the decline in stock price the company becomes relatively more indebted (leveraged, hence the leverage effect) as the proportion of debt relative to the equity rises. In eyes of the investors, this stock becomes riskier and consequently more volatile because the investors are adjusting their exposure to the stock. However, no conclusive proof was ever provided for this hypothesis. An alternative explanation suggests that investors require higher rate of returns to compensate for the anticipated surge in volatility. This can only be achieved if the asset price decreases (Aït-Sahalia, Fan & Li, p. 225). Nevertheless, no hypothesis has been uniformly accepted and this topic remains in the focus of researchers and it is often referred to as the equity risk premium puzzle. Persistence of volatility will be examined in more detail in the following sections.

Due to the fact, that volatility has been extensively studied in the past, researchers have discovered a few stylized facts about it. One of the most discussed among those features is volatility persistence (or long memory, volatility clustering and long-range dependencies) in the volatility process. However, persistence has been encountered in some natural sciences such as meteorology, hydrology and physics before economics (Lo, 1989, p. 3014). In the field of economics, the first one to mention that persistence might be associated with volatility was Mandelbrot (1963), who observed that large (small) movements in returns are usually followed by large (small) movements in the near future and that volatility usually appears in clusters. To this day, this is the simplest description that captures the essence of volatility persistence. In practical terms, this means that if we observe high volatility of an asset in a certain period, we can expect that the next period will be similarly volatile. Since then, there have been many different definitions proposed, but no generally accepted definition has emerged yet.

## 1.2 Formal definition of persistence

Volatility persistence can be generally defined in two different ways: in frequency domain using spectral densities or in the time domain (can also be referred to as in covariance sense) using autocorrelation functions. I will focus on persistence in the time domain. At this point is also important to note, that a process can be persistent in one sense but not in the other or it can also be persistent in both domains (Guegan, 2005, p. 117). Autocorrelation function is a measurement tool, which can tell how correlated the observed data points at different lags in a time series are. When it comes to volatility, we would expect that the autocorrelation function for short time lags would be positive but at larger lags they would start reducing quickly and become equal to zero fast. This means that the change in volatility would be relevant for only a shorter amount of time. Suppose that we would be able to isolate the effect that an external shock has on a random stock's volatility. The day that this event occurs, the impact on the increase (or decrease) of volatility would be the biggest. Then, each period the effect of this would get smaller and smaller, until it would quickly die out. The longer the period before the effect dies out, the higher is the persistence of volatility. On the other hand, if this event would have only a one-time effect on the day it would occur, this stock would have non-persistent volatility (or short memory volatility process. When dealing with a logarithmic return series from a variety of financial assets, the observed autocorrelations functions of their absolute or power transformations (absolute returns, squared returns etc…), are all positive, decay relatively fast at the first lags and stabilize at a positive value for larger lags and do not reach zero (Mikosch & Starica, 2005, p. 378).

A financial asset's volatility is said to be persistent if it is stationary and its observed autocorrelation functions can be expressed in the following way (Teyssière & Kirman, 2002, p. 283):

$$\rho(k) \sim L(k) k^{2d-1} \text{ as } k \rightarrow \infty \tag{1}$$

$\rho(k)$ denotes the autocorrelation function at lag k and L(k) is a slowly varying function. The main feature of these autocorrelations is the hyperbolic decay with respect to time. The parameter that determines the speed of the decay and is often referred to as the memory (or long memory) parameter and is denoted as "d" in Equation 1. The higher the value of this parameter, the more persistent the process and slower is the decay in the autocorrelation functions. This means that there will be a stronger dependence between distant observations and also the distance itself will increase. Depending on the value of the memory parameter, we can distinguish three cases (Nguyen, Prokopczuk & Sibbertsen, 2020, p. 5). If $d$ is equal to zero, than the process has a short memory, if $d$ is smaller than 0, the process is said to be anti-persistent and if the value of $d$ is between 0 and 1 then the process is persistent.

However, in the latter case we distinguish two different scenarios. If $d$ is between 0 and ½ the process is persistent and stationary and if $d$ is between ½ and 1 the process is again persistent, but in this case it is nonstationary. If the process has a short memory, then the decay associated with the autocorrelation functions is exponential. The main differences between hyperbolic and exponential decay are, that the exponential decay has a constant decrease rate and that hyperbolic decay asymptotically stabilizes around a certain positive value as the number of time lags reaches infinity. In more general terms, the exponential decay is faster at reaching zero from the same starting point compared to the hyperbolic decay.

A more general definition of persistent process can be written as (Banerjee & Urga, 2005, p. 13):

$$\lim_{n\to\infty}\left(\sum_{j=-n}^{n}\left|\rho_j\right|\right)=\infty \tag{2}$$

An explanation of this would be that when the lags approach infinity the values of the autocorrelations ($\rho_j$ in Equation 2) become smaller and smaller and close to zero but never actually reach it and consequently their sum is not finite.

Daily asset prices possess the martingale property and are usually found to be integrated of order one, denoted as I(1) (Teyssière & Kirman, 2002, p. 284). This feature implies the non-stationarity of daily prices and the presence of a unit root. The reason for this comes from the assumption of the efficient market hypothesis, which assumes that $E(P_{T+1}/I_t)=P_t$ , where $I_T$ is the information set available today). The martingale property ensures that the log returns of these daily prices are uncorrelated. The log return series are also commonly found to be I(0), because the differencing in $\ln(\frac{Pt}{Pt-1})$ when obtaining returns eliminates the unit root. When studying persistence many authors have chosen to take transformation of the returns, such as absolute returns or squared returns as a measure of volatility, and the evidence indicate the that the transformed returns are correlated (detected with autocorrelation functions), hence volatility is found to be persistent. Although many power transformations were used in these studies, the strongest evidence came from the studies, where absolute returns were used as a transformation, although majority of studies uses squared returns (Cont, 2004, p. 161).

Two important conclusions can be derived from this. Firstly, asset returns and consequently asset prices seem to be dependent across time and secondly, because correlation is detected after using transformations of returns, these dependencies seem to be non-linear. If asset returns are indeed dependent across time, this may lead to important practical implications for investors especially regarding their investment decision and portfolio allocation. They can include the knowledge about those nonlinearities in their decision-making process in order to achieve higher yields. However, detection and modelling of these dependencies could turn out to be a difficult task, because they appear to be nonlinear. Nonlinear models are far more complex than their linear counterparts.This raises the questions if exploiting this is a worthwhile challenge for investors as additional gains, if there are any, might not be worth the additional resources spent.

When talking about persistence in volatility the main question is, for how long volatility stays high when a shock occurs. The longer the periods of high volatility, higher is the persistence. Consequently, autocorrelation functions need to also be estimated at larger lags, which has proven to be difficult to measure precisely in reality.

## 1.3 Asset pricing implications

Why should investors care if stocks (or other financial assets) exhibit persistence in their volatility? The importance for investor stems from the dependence of distant observations on today's observation in the transformed return series. This means that the returns realized in the past can convey important information about returns today. Returns realized today can have valuable insights and clues about future returns, which can be used in forecasting and forming a future trading strategy. The question of how persistence in volatility should be priced arises. Two important aspects are important for this issue. To begin with, an asset with persistent volatility would have higher levels of overall volatility, because when a shock that would increase volatility, would occur, the volatility would stay higher and/or longer compared to a case without persistence. This would make an asset riskier in the eyes of the investor and that would lead to a compensation in the form of a positive equity premium. On the contrary, persistence in volatility implies higher predictability, because there is some information in today's returns, that are relevant for future returns, due to long range dependencies in the autocorrelation function. This means that persistence can lead to better forecasting performance and thus, higher predictability, which is negatively priced with a negative equity premium. These two aspects are working in opposite directions, so this is a classic trade-off from investor's standpoint: predictability versus higher volatility.

### 1.3.1 Persistency in volatility and the arbitrage principle

Additionally, persistence may play an important role for investors through arbitrage. A necessity for existence of arbitrage is that a statistical dependence of observations in a price series is short lived (Sadique & Silvapulle, 2001, p. 59). Put it in another way, there must not be any long-term dependencies present in the return series of a financial assets, otherwise this allows the investors to adapt their trading strategy accordingly, if they observe some arbitrage opportunities coming from these dependencies and collect the capital gains. Persistence can thus be connected to the efficient market hypothesis via the principle of arbitrage, which ensures that any mispricing of financial instruments goes away quickly as investors take advantage of it and buy (sell) the undervalued (overvalued) asset and sustain the asset's fundamental price. If due to the presence of persistence, these price deviations would last longer, this would raise questions if the efficient market hypothesis really holds.

Another aspect, which indirectly relates to efficient markets, is that arbitrage increases competition and quality among the investors, because only the first ones will gain benefits of arbitrage before the pricing mismatch is arbitraged away. Consequently, only the best among investors have positive gains.

### 1.3.2 Persistency and mean reversion in volatility

As already mentioned, another stylized fact about volatility that is based on empirical findings is the mean reversion of volatility. At this point, I would like to focus on the link between persistence and mean reversion. Mean reversion means that volatility tends to converge to a certain mean level in the long run despite developments that may raise or reduce volatility in the short run. This also suggest that current new information does not have any (or only limited) effect on long term volatility forecasts. This is the fundamental difference between persistence and mean reversion. If an asset shows persistence in its volatility, then the current information will impact our predictions in the long run, whilst for an asset with mean-reverting volatility this is not the case. Nevertheless, it also has to be pointed out that these two phenomena are not mutually exclusive and volatility can in fact be both, persistent and mean-reverting. In reality, most assets usually possess both of them, except for the cases, where shocks to volatility last forever (no time invariant mean of volatility), which is also referred to as the "integrated-GARCH" effect. The stronger the persistence of volatility, the longer an effect of a shock will be visible and will push the volatility level away from its historical mean and more time will be needed for volatility to converge back to normal. With this in mind, it becomes clear that the size of persistence is negatively correlated with the speed of mean reversion. A conclusion based on this would be, that mean reversion and persistence try to capture and describe similar patterns in volatility behaviour but from opposite angles and can be regarded to as different sides of the same coin. In terms of asset pricing, the mean reversion also implies higher predictability of the volatility an asset and consequently the asset returns. As it is in the case of persistence in volatility, higher predictability should result in the negative risk premium.

The main takeaway of this subsection is that persistency in volatility has economic significance as it impacts the risk premium of an asset, which is an important determinant of the asset price. If the persistence is a statistically significant feature of the asset's volatility, then investors should pay attention to the degree of it. Long memory has also been linked to the arbitrage principle, which is among the cornerstones of the asset pricing theories and most important concepts in finance.

## 1.4 Econometric view of persistence

Up to now, econometric terms such as (non)-stationarity, order of integration etc… have been used. In this section, I would like to concentrate on persistence from a more econometric point of view. To begin with, the definition of stationarity is needed.

Econometric literature distinguishes between two forms of stationarity: strong and weak. Weak stationarity, also called covariance stationarity, is more generally used and suffices at this point. A weak stationary time series has time invariant first two moments (mean and variance) and autocovariance structure (Nau, 2020). If we have a time series process $X_t$, which is covariance stationary, then it will possess the following properties:

$$E(Xt)=\mu \text{ for every } t \tag{3}$$

$$cov(Xt,Xt\text{-}i)=E((Xt\text{-}\mu)(Xt\text{-}i\text{-}\mu))=\delta i \quad \text{for all } t \text{ and any } i \tag{4}$$

From the second property, we can derive the *j*-th autocorrelation lag just by dividing $\delta_j$ with $\delta_0$. Another conclusion coming from the second property is also that the value of autocorrelation function depends only on the distance between the lags. The variance of this process is also a finite number. Practically speaking, stationarity means that no sustained increase or decrease can be observed in the time series, i.e. no deterministic trend is present. The realized observations will fluctuate around (above and below) a certain level without any visible long-term trend that would lead away from the mean. Another way of saying this is that a stationary time series shows signs of aforementioned mean reversion.

When dealing with a time series of (transformed) returns, persistence will be less difficult to estimate, if this series is stationary. Because the autocovariance, which is used to calculate the autocorrelations, is time invariant. As will be discussed later, non-stationarity is also one of the leading causes that empirical findings on the presence of persistence might be spurious.

A related term to stationarity is order of integration. Order of integration tells us, how many times we need to differentiate a non-stationary time series to transform it to a stationary one. Burke and Hunter (2005) offer another explanation on how to interpret order of integration, which can be regarded to as the number of unit roots that studied time series has. A more formal definition of order integration can be expressed in the following manner (Pierse, 2020, p. 1):

$$(1\text{-}L)^d X_t=u_t \tag{5}$$

$X_t$ is the time series of observations, $u_t$ are disturbances, which are independent and identically distributed with a mean equal to zero and a constant variance and *L* is the lag operator. The order of integration equals to *d*. Most commonly estimated orders of integration from real life data are 0, 1 and 2, with the last one being the rarest. In the context of the volatility persistence, the important cases are the first two. When the series is I(0),we are dealing with a stationary time series, which was discussed briefly in the previous paragraph. The case when order of integration equals to 1 is referred to as a random walk with a unit root. If a time series possesses a unit root, this implies that variance becomes infinite when the number of observations approaches infinity (Mills, 2019, p. 72).

Nonetheless, a common finding among the researchers dealing with stationary and integrated time series was that many time series in financial economics do not show signs of being integrated of order one (possession of a unit root) or order zero i.e., being stationary (Banerjee & Urga, 2005, p. 14). Such series are not stationary and do not have the features of stationarity that were discussed at the beginning of this section. On the other hand, they seem "overdiferenced" when first differencing to eliminate the unit roots is applied. This implies that order of integration for such time series is between 0 and 1. This is called fractional integration (Baillie, 1996, p. 6). Fractional integration can be viewed as a generalization of integration mentioned before. The mathematical equation for it is the same as Equation 3, as it allows the values of order of integration to be fractions and thus adding more flexibility in time series. Therefore, a fractionally integrated process can be seen as a link between integration of order 0 and 1, because these two cases represent the most extreme (border) cases. Another description of fractionally integrated time series is that they seem to satisfy the assumption of stationarity, but they show some small long-range dependencies, which can not be neglected (Mills, 2019, p. 90).

From this definition the link to long memory in volatility is immediately clear. A possible way to determine the exact value of the fractional differencing parameter is by identifying the speed of the decay of the autocorrelation functions of the time series. As discussed before, integration of order zero is synonymous with exponential decay in the autocorrelations, whilst integration of order one is associated with linear decline in the autocorrelations. Nevertheless, if the order integration is between those two cases, the decay in the autocorrelation functions can be described as hyperbolic, is which one of the most common ways to describe the persistence found in volatility process. I have already mentioned that literature distinguishes between two cases of long memory in volatility: stationary and non-stationary case with the deciding factor being the value of the long memory parameter. If the value of this parameter exceeds ½, the volatility is persistent and non-stationary and if the value is below ½ the volatility exhibits persistence and stationarity. By considering, the previous discussion about fractional integration it is clear why ½ is set as a borderline value between those two cases. If the value of $d$ is below ½ the process is closer to being stationary, because zero is closer and also the decay is closer to exponential decay. The opposite is true for the other case. Mills (2019, p. 91) also argues that even if the value of fractional integration order is on the interval between ½ and 1 the process is less non-stationary than the case when integration order equals to one. Expanding this analogy, one might conclude that higher fractional integrator, which also means stronger persistence, means stronger non-stationarity, especially if the order of integration is higher than ½.

We may also find cases where the value of the parameter $d$ is negative. Such processes are called non-persistent or anti-persistent (Mills, 2019, p. 91). Their main features are negative short-term and long-term dependencies, which are non-smooth. As opposed to the case with long memory, anti-persistent processes have negative autocorrelations functions. For such cases we would expect that large moves are followed by small moves and conversely. But persistence has been studied far more extensively because it is a more common finding in empirical studies and is considered as one of the most important features of volatility.

## 1.5 Economic explanations of persistence

At the beginning of this section, it is important to note that persistence in volatility has been first encountered in empirical studies. Its identification as a feature of volatility behaviour stems from the analysis of empirical data and not from theoretical models. The economic explanations can be classified into two broader categories: exogenous and endogenous (McQueen & Vorkink, 2004, p. 916). The clustering of information belongs to the exogenous group and all others mentioned belong to the endogenous group.

### 1.5.1 Exogeneous explanations

The first hypothesis established in the literature attributes the persistence in volatility to the rate of arrival of information that affects the asset price. This rate of arrival of new information itself is said to be persistent (Berger, Chaboud & Hjalmarsson, 2009, p. 1). This would mean that the volatility clustering is a consequence of information clustering. This holds some merit if we look at, for example, earnings announcement. It is common practice that a lot of companies announce their quarterly or annual results during the same period. This means that investors get a lot of new information (from competitors, suppliers, customers) in a short span, which may lead to the accommodation of their investing strategies. As a result of investors changing their portfolios, the stock prices change rapidly, which leads to higher volatility.

An interesting question arises within this explanation. What is the role of information technology and information age within this? If we think about it, the rise of internet, social media and smart gadgets causes that investors are exposed to a vast quantity of information. A couple of decades ago, investors had considerably less sources of news. Nevertheless, volatility persistence is not a new phenomenon since it was first mentioned by Mandelbrot in 1963. This means that persistence did not arise together with the information age, however it would be interesting to study its effect on persistence, because a potential consequence could be that volatility persistence increased in the last couple of decades since investors are exposed to more information, which also comes with a higher frequency. A possible extension of this would be adding economic integration to this equation. Because economies are more connected (more international trade, foreign investments, global value chains etc…), economic and financial shocks in one country impact financial markets in other countries as well. In case a shock happens in one country, it is fair to assume, that the information about it will reach the others countries that are relevant, faster, with more details and also in higher frequency (more news, reports, analysis). In addition, foreign investors might attach more weight to it if their home country and the country in which shock occurred share tighter economic links. In a sense, both globalization and information age, might exacerbate the persistence found in volatility.

### 1.5.2 Endogenous explanations

Müller et al. (1997) propose an explanation based on different types of market investors. Their idea focuses on foreign exchange rates, but the idea can be generalized also to other assets. Their main claim is, that investor horizon is one of the most important determinants of investor behaviour and can also explain volatility behaviour. Investors can be classified as short-term or long-term investor depending on their horizon and they behave differently in each of these categories. Short-term investors try to exploit even the smallest price movements and thus give more weight to every new information arrival. Their trading frequency is high, which leads to higher price movements. On the other hand, long-term investors will not try to exploit every change in price and will only react if the fundamentals change. A prime example of this kind of investing strategy is Warren Buffet's philosophy. Accordingly, long-term investors will not react to every news and will only care about only more fundamental news, which impact the long-term outlook of an asset. Their trading frequency will be lower and consequently volatility and. The strength of persistence then hinges on the proportion of each category in the investor structure, which can change through time. The changing structure of investors, then causes time-varying volatility behaviour.

Another possible explanation are information costs and information asymmetry stemming from them as proposed by De Fontnouvelle (2000). Obtaining private information is costly and thus causes information asymmetry. If information costs are high, there is on average less private information gathered by investors, which also means that information endowment among majority of investors is low. This also means that investors will probably have the same information, which normally generates low volatility due to lower trading frequency and similar patterns in the trading regime. This could cause lower persistency. To the contrary, low information costs means higher variation in information endowment, which also leads to higher frequency of trades and also different strategies. The varying cost of information leads to the exchange of periods with low and high volatility.

McQueen and Vorkink (2004) attribute persistence and time-varying volatility to time-varying risk aversion. Investor responses to information change through time due to changes in risk aversion, which is tied to past performance through level of wealth. Past performance changes the level of wealth that investor possesses. If his past performance was good (bad), he will have higher (lower) level of wealth. Because of the change in the level of wealth, investor temporarily displays different risk aversion and sensitivity to news. When his past performance is good (bad), he has more (less) wealth and is therefore temporarily less (more) risk averse. Nevertheless, after a certain period their risk aversion starts to reverse to the previous level. If his sensitivity to news is greater and more persistent as a response to higher risk aversion, this will cause volatility clustering. Additionally, this explanation can also be extended to include prospect theory and loss aversion, which are important pillars of behavioural economics.

Persistence can also be explained by investor attention (Andrei & Hasser, 2015). Empirical studies have proven that investor attention changes over time and also that is positively correlated with volatility. The main framework is that lower investor attention causes slower learning process, i.e. investors slowly react to the news, and that produces lower volatility as a result. When investor attention is high, the situation is reversed. The switching between these two regimes of either high or low attention causes periods of low or high volatility, which depends on the size of investors' attention. A potential consequence of the switching is that volatility might exhibit persistent behaviour.

Finally, switching between different trading strategies can also result in volatility clustering as proposed by Cont (2004, p. 172–173). Trading strategies can be defined as trading rules that investors follow based on their behavioural patterns. Switching between them then causes fluctuations in asset prices, which also results in higher volatility. An example of this would be if investors are divided between fundamentalist and chartist based on their trading strategy and strategy switching is allowed. In this setting, fundamentalists are similar to long-term investors, as their strategy is based on a principle that an asset price follows the fundamental value in the long run. The chartists are similar to noise trades and/or short-term investors because they want to take advantage of every mispricing opportunity and they base their strategy on realized gains. This setting become unstable, when the proportion of the latter category reaches a certain threshold and jump in volatility occurs, which can also lead to heavy tails in distribution and long-range dependencies.

All of the explanations in the endogenous group have in common that they are characterized by two different states of strategies, behaviours and patterns. The majority of them also includes switching regimes between these states and this repeating switching is crucial because it results in periods with low and periods with high volatility and additionally autocorrelation in volatility (Cont, 2004, p. 175). It is also worth noting, that the news clustering is labelled as the most problematic because it seems to be overly simplistic to capture the complexity of real world (McQueen & Vorkink, 2004, p. 917). It also has another downside compared to the majority of other explanations, because it isn't able to explain other stylized facts about volatility such as leverage effect, while others can do that or at least have the potential to do that if suitable extensions are applied. Nevertheless, one common thing stands out within the group of endogenous explanations, since they all incorporate some sort of heterogeneity of investors. A possible and generalized conclusion might be, that persistence in volatility stems from the fact that financial markets consist of different types of investor groups, where switching from on group to another is possible.

### 1.6 Past empirical research

In this subsection, some previously obtained empirical results will be presented. The studies included in this section are chosen based on variety of criteria. The first and most important criterion is their relevance, which is judged by the number of their citations and also the journals they were published in. I tried to include articles in which author(s) use different modelling techniques, different sets of data, different frequencies and time periods for the analysed data etc… I also tried to include articles that were amongst the first and most influential from this field but also some of the most recent ones.

Poterba and Summers (1984) were among the first to formally prove that if volatility persistence in fact exists, then it affects security prices and therefore should be priced in. Higher expected persistence in volatility will lead to higher security prices since the persistence affects the discount factor to the future cash flows. However, using daily and monthly data of market returns from the periods 1926–1983 (monthly data) and 1968-1983 (daily data) they show that volatility is only weakly serially correlated and that impacts of shocks to volatility are almost negligible in the two years after the shock. In their estimation, they used autocorrelelograms and the autoregressive models to estimate the autoregressive coefficient. Another finding in their article is that persistence is stronger with the daily data.

Chou (1988) was among the first to try to determine the presence of persistence in volatility using the GARCH model, which was introduced just a couple of years earlier. His main modelling assumption for the market returns was that their variance follows a GARCH specification, where today's variance is influenced by past realization of the variances and past forecasting errors made by investors. He decides to use an adaptation of the classical GARCH (1,1), the GARCH (1,1)-M, also referred to as the GARCH in the mean, which is sometimes used in financial studies, if there is suspicion of serial correlation in the returns. The weekly data for NYSE value weighted index was obtained from the CRSP, with the analysed period being from July 1962 to December 1985. The justification for the usage of weekly data was that weekly data exhibits less serial correlation than daily data. The weekly returns were calculated as a difference between logarithmic closing prices of two consecutive Tuesdays. Tuesday was chosen to avoid the problems that might be caused by the weekend effects. With MLE estimation, which at least theoretically provides consistency and efficiency, the estimates of the GARCH parameters were obtained. For the entire period, the sum of the parameters was equal to 0,986, with both being statistically significant. Then the same procedure was repeated with the only difference being that the period was divided in two subperiods, where the boundary is at the end of the year 1973. The results remained similar. Chou also decided to compare the AR (1) model with GARCH (1,1) for modelling persistence. His main conclusion was that the GARCH(1,1) is better in capturing long memory dynamics, produces more stable estimates and is generally more suitable for modelling this topic. In the last part, he tested if the sum of GARCH parameters is equal to 1, which is a test for the presence of the so-called "I-GARCH effects" and a unit in root in the variance. He found out that I-GARCH is supported for almost every frequency of the measured data, which he included as a robustness check.

Lo (1989) applied the modified R/S statistic to daily and monthly data in the time period from 1962 to 1987 and 1926 to 1987, respectively. The data used was provided by the Centre for Security Prices. In addition, he used weekly data from Lo and MacKinlay (1988) and annual data from 1872 to 1986 for S&P Composite Index. Using this statistic, he did not find any evidence of long memory, as the statistic was not statistically significant for any of the returns.

Lamoureux and Lastrapes (1990) were among the first to study the effects of structural change in variance and how it impacts the estimation of persistence. They characterized persistence as momentum in the conditional variance. Their main hypothesis was that in cases where structural changes, i.e. level shifts in variance, are present the size of the persistence is overstated. They applied a GARCH (1,1) specification estimated by the maximum likelihood to the daily returns of 30 randomly selected companies from the CRSP from the period from January 1963 to November 1973. The average of the sum of the GARCH parameters was equal to 0.987, which implies that strong persistence is present. However, when they added shift dummies, the sum of the parameters decreased for all 30 stocks and the average decreased to 0.817. Using the Monte Carlo simulation they also tested their hypothesis on a hypothetical GARCH process with low persistence and small number of shifts in the unconditional variance and they concluded that level shifts can lead to the spurious findings of persistence in stock returns. Another conclusion of their study was that the direction of the level shift is not relevant.

Ding, Granger and Engle (1993) used daily closing price of the S&P500 to calculate daily log returns. Using this data, they estimated the autocorrelations for returns, absolute and squared returns and they found that autocorrelations for the transformed returns (squared and absolute) are statistically significant for at least 100 lags and in some cases for 2500 lags. This means that the shocks to these returns were significant for at least 100 trading days after the occurrence of the shock. Based on these sample autocorrelations functions they estimated the autocorrelations curve using ordinary least squares and concluded that it displayed hyperbolic decay, which is consistent with the presence of the persistence. They also established a connection between the high volatility and high persistence giving the example of the pre-war period in the late twenties and early thirties of the previous century, when there was high volatility because of the Great Depression and their estimates for persistence were higher.

Andersen and Bollerslev (1997) studied the five-minute returns of S&P500 Composite Index Futures from January 1986 to December 1989. They focused on intraday volatility, how to model it and if there are any visible patterns. Among those patterns they also focused on long memory in volatility, which they studied using autocorrelations and also an extension of the GARCH, the Moving Average(1)-GARCH (1,1) estimated by quasi maximum likelihood. They included the moving average term to take care for potential first order autocorrelations in returns. They found that the autocorrelations for the first ten lags are all statistically significant, but there is no visible pattern as the size and sign of the coefficients varied. However, they did not study the behaviour of any transformed returns at longer lags, with which the long-range dependencies are most commonly associated. Using the MA(1)-GARCH (1,1), they found significant evidence for persistence in the five minute-returns. They also checked their results using standardized and absolute standardized results and got similar results even in those cases. The returns exhibited strong persistence with smooth and stable decline over longer horizons.

Lobato and Savin (1998) tested for the presence of persistence with an adaptation of a Langrange multiplier test, where the null hypothesis is that returns feature only week dependence (short memory) and alternative is strong dependence (long memory). They derived the critical values and the limiting distribution for this test and backed them up with Monte Carlo simulations. They used the daily returns of S&P500 and 30 individual companies that are included in the Dow Jones Industrial Average from February 1962 to December 1994. They found no evidence of long memory in daily returns of the index but found strong evidence in squared returns and absolute returns. The absolute returns exhibit strongest persistence. For further analysis they focused only on squared returns. Their next step was to check if this evidence is influenced by non-stationarity. They divided the period in two, with the break point being in January 1973, when the oil shocks happened and there was a jump in volatility. Even after this, the squared returns still exhibited persistence in both divided periods. To check if the results for the S&P500 are caused by aggregation, they analysed 30 individual stocks from the Dow Jones. Tests were made for the same three periods as before for the squared returns. For the period July 1962 to December 1972, persistence is found in 24 stocks out of 30 stock. For the next period, the data suggests that persistence is even stronger than in the previous. Based on all of this, their main conclusion was that long memory is probably real feature of the volatility process and that is unlikely that this would be a spurious finding because of non-stationarity or aggregation.

A theoretical derivation of the fact that persistence or breaks can lead to similar conclusions about the properties of stock returns was first shown by Granger and Hyung (2004). They defined breaks in a time series as level shifts in the mean. Their main claim was that it is very troublesome to distinguish between real persistence or a spurious one caused by level shifts. By analysing S&P500 absolute stock returns from January 1928 to October 2002 the results obtained show that at least partially long memory could be caused by breaks in the series. They estimated the GPH estimator for the entire period, which equalled to 0.45 and was significant. But after dividing the entire period into shorter sub-periods the value of the GPH estimator ranges between 0.15 and 0.715 with the mean value being lower than for the entire period. Comparison of the long memory model and occasional break model based on the data from this time period showed, that these two models have very similar explanatory power and also very similar BIC and AIC criteria. The main guidance from their article is that it is not enough to consider only linear behaviour (autocorrelations) but you also have to check for other nonlinearities, whether they are present and how big they are.

Malik, Ewing and Payne (2005) used weekly data from Canadian Stock Exchange to check whether long memory is a feature of the Canadian stock returns. They used weekly data from June 1992 to October 1999. To account for the possible shifts in the variance, they decided to use the Iterated cumulated sum of squares algorithm (ICSS), but they also modified it to account for conditional heteroscedasticity, which is found in stock returns. The algorithm identified two different shifts, which also means that there were three variance regimes altogether in that period. After that they used the GARCH (1,1) model, which included the control variables for regime changes mentioned earlier, to check if they can detect any persistence. The sum of GARCH parameters were equal to 0.967 and 0.941 for Vancouver and Toronto Stock Exchange, respectively, when they did not include the control variables for regime changes. In the model that included the control variables, the sum of parameters decreased to 0.672 and 0.846, respectively. They conclude that every regime shift has to be taken into the account, when estimating long memory in order to get the best accuracy of the results.

McMillan and Ruiz (2009) checked for the presence of long memory in volatility in 10 international stock indices from the following countries: Canada, France, Germany, Hong Kong, Italy, Japan Singapore, Spain, UK and the US. They inspected daily stock index absolute returns from the time period between January 1990 and December 2005. They used three different tools: autocorrelations, GARCH (1,1) and GPH estimator. Autocorrelations were all significant for 9 countries for all of their 100 tested lags. The exception is Japan, where the correlations are significant up the 55th lag. The sum of the GARCH parameters ranged from 0.981 to 0.995 and all parameters were significant, which indicates strong persistence. The fractional integration parameter estimated by the GPH method was statistically different from 0 for every country and ranged from 0.21 to 0.78. This can be interpreted as another sign for the presence of persistence. After that, they wanted to see if the variance of these returns is time invariant and if there are any breaks present. They discovered several breaks and decided to calculate moving averages of returns with 130 days in a single window in order to eliminate the effect of long-term trends and check again if the persistence is present. The GPH in this case did not support the presence of long memory. They also used an adaptation of the GARCH model, which was adjusted by the time-varying mean, where they replaced the constant term with a moving average mentioned before. The range of the sum of parameters for this model decreased evidently. They ranged from 0.899 to 0.979, which indicates lower persistence than before.

Huang, Liu and Wang (2016) came up with another extension of the classic GARCH model called the Heteroscedastic Autoregressive Realized GARCH or the HAR Realized GARCH. A realized GARCH model builds on the initial GARCH model by adding a measurement equation of the realized volatility such as realized variance or realized kernel. This adds another channel alongside the returns through which expectations about future volatility are formed. The measurement equation is basically introduced to connect the realized measure to the conditional volatility. The authors also incorporated the heterogeneous autoregressive structure of the realized variance into their model with a cascade structure for the lags. This would enable them to capture the long memory property in the volatility more adequately. Their dataset included daily returns of the Dow Jones Industrial Average, SP500 Composite Index and 27 individual liquidly traded stock from January 2002 to December 2013. As a measure of the realized volatility they used the realized kernel and for the estimation of the fractional integration parameter they used the local Whittle estimator. The HAR Realized GARCH was estimated by the maximum likelihood. The purpose of their article was twofold. First, they wanted to confirm the presence of volatility persistence in the studied indices and stock and then they wanted to see if their HAR Realized GARCH model is better at capturing it then the Realized GARCH. For most of the analysed time series of squared returns they obtained the estimation of the fractional integration parameter above 0.5, which means that these time series exhibit non-stationarity and long memory. The sum of the parameters, which indicated persistence, in their HAR Realized GARCH model equalled to 0.974. That suggests strong persistence as the sum of 1 would mean infinite persistence. They also concluded that their model captures the autocovariance structure at longer lags horizons then the Realized GARCH based on the comparison of the sample autocorrelation functions.

Schmitt and Westerhoff (2017) tried to build a model based on behavioural finance, which would be able to capture all the important features of the stock markets: bubbles, crashes, excess volatility, fat-tails of returns, uncorrelated returns and volatility clustering. Their model focused on the herding behaviour of the investors and how herding can explain their aforementioned phenomena. Their main premise was that herding behaviour of investors becomes stronger in times of elevated uncertainty, because the investors "see safety in numbers" and start observing more what others are doing. Because this leads to repetition of the same behaviour this means that certain assets experience capital inflows and the remaining assets experience outflows. The first group can be regarded as safe havens for example gold, or some currencies like the Japanese yen and certain type of stocks (counter-cyclical stocks). Because there is a significant amount of resource reallocation happening in such times, the volatility also increases. The volatility clustering is produced by the switching between periods with low uncertainty (less herding) and high uncertainty (more herding). Their hypothetical stock market is populated by a single market maker, who mediates transactions and determines the stock prices in response to the excess demand and a number of heterogeneous investors. They follow a mix of technical and fundamental trading rules. Each investor has an individual demand function, which consists of three components. Because of the technical component the investors follow a current price trend, which they determine through technical analysis. According to the fundamental components, they follow the stock's fundamental value, which also implies the presence of mean reversion. The final component is a multivariate normally distributed random variable with a time-varying variance-covariance structure. Through this they introduced the herding into the investors' behaviour because the random component is positively correlated with past market volatility. To check the performance of their model they used daily returns of the S&P500 from January 1964 to December 2014 with 12 797 daily observations. To capture the five facts associated with the stock markets they use 12 summary statistics (tail index, three autocorrelations functions for raw returns…). To capture the volatility clustering feature they use autocorrelation functions for absolute returns with lags 3, 6, 12, 25, 50 and 100. The statistics are estimated by the method of moments. The measure for the general goodness of fit of the model, the average moment matching score (AMMS) equals to 0.855, which means that 85.5% of the estimates produced by the model fall into the 95% confidence interval of the actual values for the statistics. The AMMS for the autocorrelations of the absolute returns is the highest for the shortest lags (3) and equals to 0.974 and then gradually declines for longer lags. For the autocorrelations with 50 lags the AMMS equals to 0.754 and for the ones with 100 lags it equals to 0.404. This means that the model produces the persistence of volatility similar to the actual one with the shorter lags but has some trouble with that when it comes to longer lags. All in all, the results of this study provide an indication that herding behaviour might be at least partially responsible for the observance of volatility clustering in the stock markets.

Although all the studies presented up to this point have dealt with long memory in stock markets, this is not the only area with which persistence in volatility is associated with. Extensive research has also been done about persistence in exchange rates, interest rates and other assets such as metals etc… I will present additional studies, which studied persistence in exchange rates and interest rates.

Cheung (1993) used the GPH estimator and an ARIFMA model on the data of five major dollar spot rates: British pounds, Deutsche mark, Swiss franc, French franc and Japanese yen using weekly data from January 1974 to December 1987. By plotting them, he determined that the exchange rates seem stationary. Additionally, he conducted an Augmented Dicky Fuller test to determine the presence of a unit root, but the test rejected the presence of it. He then applied the GPH estimator to estimate the fractional integration parameter and modelled the exchange rates using the ARIFMA model. The order of parameters in this model was determined by the Aikake's information criteria, which is a standard procedure. The GPH estimation indicated the presence of long memory for all of the exchange rates except for the British pound and that the strength of long memory is of similar size in the exchange rates with Deutsche mark, Swiss franc and Japanese yen. The fractional integration parameter in the ARIFMA models was not statistically significant larger than 0 only for the Swiss franc. The value of this parameter ranged from 0.045 to 0.36. Based on these results he concluded that there is a substantial amount of evidence, which indicate towards persistent volatility. As a possible source of long memory in exchange rates he pointed towards the variability of the purchasing power parity and fluctuations in other national macroeconomic variables.

Neely and Rapach (2008) studied the persistence in the real interest rate of the US economy. They use quarterly data from the first quarter in 1953 until second quarter of 2007. They measured real interest rate as nominal interest rate reduced for actual inflation. First, they performed the Augmented Dickey Fuller test to test for non-stationarity and their results indicate the presence of unit root(s) in the US real interest rate. After that, they used an adaption of the Local Whittle estimator, the two-step feasible Local Whittle estimator to determine the value of the fractional parameter, which captures the effects of long memory. Their estimate of the parameter was 0.71 with a 95% confidence interval between 0.51 and 0.9, which implies that this parameter is statistically different from 0 (stationarity) and 1 (unit root or non-stationarity). Based on this they concluded that the real interest rates in the US possess low mean-reversion with persistence and no unit roots. After employing the Bai and Perron methodology for structural breaks, they found 3 of them and consequently 4 different regimes. Nevertheless, their hypothesis was that such breaks minimize only the local persistence (persistence found within an individual regime) and do not affect the global persistence. As a most probable cause for persistence found in real interest rates, they point out shocks in the monetary policy. Other listed possible reasons are changes in consumer preferences, technology growth and fiscal shocks.

Discussion from this section can bring to light some important takeaways. The first one is, that long memory or persistence in stock market volatility is not just a phenomenon associated with the US stock market, although it has been the most extensively studied there. Evidence supporting persistence has been found worldwide from Japan to Europe and the emerging markets. Another important finding that relates to the methodological approach is that the measured persistence decreases substantially after the structural breaks in the data are accounted for. It does not disappear, but its relative importance in asset pricing is smaller than it was originally. Additionally, from the relevance and applicability of the data, it is better if the break points in the regimes are not assumed a priori. It is better to use such methodological frameworks that identify the break points based on the actual data at hand. Third, long memory in stock market volatility is usually found more frequently with that data that is in higher frequency such as daily returns or intra-day returns. Fourth, semi parametric models of estimation usually find stronger persistence in the same datasets compared to the parametric models. Probably this has to the fact, that semi parametric models focus only on the long memory parameter (an example of it is the GPH estimator) and they do not try to account for others features of volatility. This means that they are a more simplistic approach and less time consuming, but on the other hand their estimates might be less price. This is a classic trade off when it comes to modelling decisions. Based on the review of the literature, an interesting trend can be observed in the last couple of years. More and more researchers are trying to explain volatility clustering in the financial markets with the help of the behavioural finance (for example herding behaviour). Last but not least, the majority of the studies described used a model from the GARCH family of models, which is not surprising, because the GARCH models are one of the most popular, widely used and regarded as one of the best choices when it comes to modelling volatility.

# 2 DETECTING AND MEASURING PERSISTENCE IN VOLATILITY

Since long memory has been extensively studied in the field of economics since the 1980s, various methods and techniques have been developed for detecting and measuring the presence of long memory. Presence of persistence in the volatility of a financial time series can be confirmed by parametric and non-parametric tests. For measuring persistence, semi-parametric and parametric models can be applied. The difference between the two approaches is that in the semi-parametric methods, the whole model does not have to be estimated and one can focus only on the estimation of the parameter of interest. When it comes to persistence in volatility the parameter of interest is the memory parameter. In the following section, the rescaled range statistic and the modified rescaled range statistic from the group of non-parametric tests will be presented. In addition, Geweke Porter-Hudak (GPH) and local Whittle estimators, which belong to semi-parametric methods, will be described. From the parametric methods, the GARCH model will be presented alongside its extension the fractionally integrated GARCH (FIGARCH) model. Another possible distinction of the long memory models is between discrete and continuous long memory models (Banerjee & Urga, 2005, p. 15).

The main aim of the semi-parametric and parametric methods, which can measure the degree of persistency, is to estimate the value of the memory parameter. One way of identifying the value of the memory parameter is to estimate the order of fractional integration, which is denoted as $d$ in Equation 5. By identifying the order of fractional integration, the degree of persistence is also identified. The fractionally integrated GARCH model, that will be studied in more detail is closely related to the notion of fractional integration and was developed mainly by the introduction of the concept of fractional integration.

## 2.1 Non-parametric tests

The rescaled range statistic was first used in hydrology and was developed by H.E Hurst in 1951 (Voss, 2013), but has found its way in the financial world since then. The rescaled range statistic (R/S) is calculated as the range of partial sums of deviations of a time series from its mean divided by the standard deviation of the time series (Lo, 1991, p. 1287). Suppose a sample of returns consists of $X_1, X_2, \ldots, X_t$. The mean of the sample is calculated as usual: $\frac{\sum_j^t xj}{t}$.

The R/S statistic for the corresponding time series would be:

$$Q = \frac{1}{s_t}(Max_{1\leq k\leq t}\sum_{j=1}^{k}(X_j - \bar{X}) - Min_{1\leq k\leq t}\sum_{j=1}^{k}(X_j - \bar{X})) \quad (6),$$

where $s_t$ and $\bar{X}$ are the standard deviation of the returns and the mean, respectively (Lo, 1991, p. 1287). After obtaining the value of the R/S statistic, the value of the so-called Hurst coefficient is calculated as, where T is the number of observations (Equation 7) (Baillie, 1996, p. 27):

$$H = \frac{\log(\frac{Q}{s_t})}{\log T} \qquad (7)$$

Alternatively, Hurst coefficient can also be obtained in the regression where the nominator is regressed on the denominator in the Equation 7. If the value of the Hurst coefficients is equal to ½ or lower, then the process can be characterized as a short memory process. If the value of the Hurst coefficient exceeds ½ then the process possesses long memory. The higher the difference between the Hurst coefficient and ½ the stronger is the persistence. However, the biggest shortcoming of the R/S statistic is the sensitivity to short-range dependencies (Lo, 1991, p. 1288). The reason for this is that the R/S statistic often exhibits similar type of asymptotic behaviour when it is used on a stationary time series that exhibits persistence or a non-stationary short memory time series (Guegan, 2005, p. 141). Moreover, the performance of this statistic is questionable in all the cases, in which data exhibits any type of heteroscedastic behaviour (Banerjee & Urga, 2005, p. 21).

Because of the stated reasons, Lo (1991) decided to introduce the modified R/S statistic, which is robust to short-range dependence and hence performs better than the original R/S statistic. To achieve this, Lo (1991, p. 1290) modified the denominator in the R/S statistic (the nominator stays the same as in Equation 6).

$$s_t^2 = \sigma_x^2 + 2\sum_{j=1}^{q} \omega_j(q)\gamma_j \,, \ \omega_j(q) = 1 - \frac{j}{q+1}, \ q<t \qquad (8)$$

The $\sigma_x^2$ is the variance of the sample and $\gamma_j$ is the sample autocovariance. Lo justifies this modification with the fact that if analysed time series exhibits short-range dependencies, the variance of the partial sum can not be obtained by adding up the variances of individual terms, but also the autocovariances must be accounted for. This is done by the inclusion of weighted autocovariances. The original R/S statistic corresponds to the case where q=0. Nevertheless, in practice when using the modified R/S statistic an important question arises. As Baillie (1996, p. 28) points out, it is exceedingly difficult to choose the value of $q$, as it hinges on the presence of short-range dependence. The problem is that when $q$ is too small, some of the autocovariances after the $q$-th lag that are important may be left out. If $q$ is too large, the sample distribution of the estimator could become distinct from its asymptotic limit. The bottom line is, that the value of $q$ will depend on the data at hand and that there is no indubitable correct choice.

Lo's version is not the only test for detecting the presence of long memory based on the test developed by Hurst. Another example of such a test would be the rescaled variance test presented by Giraitis, Kokoszka, Leipus and Teyssière (2003). The common reason for the development of all these modifications is the improvement of the robustness of the test to the presence of short memory. The use of such test is so widespread because of the easy application. On the other hand, this ease of application also leads to some downfalls. All of the R/S based tests involve non-standard asymptotic behaviour (Banerjee & Urga, 2005, p. 21).

Another testing approach for discovering if a financial time series possess long-range dependencies can be employed. Studies have shown that tests for stationarity can also be modified to test the presence of long memory. Such example is the KPSS test statistic originally meant for testing for stationarity against the alternative of a unit root (Giraitis, Kokoszka, Leipus & Teyssière, 2003, p. 269). Studies have also been conducted, that show that in some cases the Dickey-Fuller and the augmented Dickey-Fuller (initially developed to test for the presence of unit roots), can also be used to test for the presence of long memory (Banerjee & Urga, 2005, p. 22).

## 2.2 Semi-parametric estimators

Among the semi-parametric methods for estimating the long memory parameter, the local Whittle estimator and the GPH estimator are most commonly used and studied. Similarly, to the non-parametric tests, both of these two estimators have been modified to enhance their performance by increasing their robustness to the short-range dependencies. Compared to the tests described in the previous section, semi-parametric approach is more complex, but on the other hand offers more insight into the memory characteristics of the analysed process.

To define the GPH estimator, definitions of a spectral density and periodogram are needed. In the time domain, the long memory is defined through the autocovariance structure. On the other hand, the GPH estimates the memory parameter in the frequency domain, where the spectral density has the same role as the autocovariance function in the time domain. If $\gamma(h)$ is the autocovariance function and $f(\omega)$ is the spectral density of the same process, where h represents the time lag and $\omega$ the frequency then the relationship can be described by (PennState, 2020):

$$\gamma(h) = \int_{-1/2}^{1/2} e^{2\pi i\omega h} f(\omega) d\omega \tag{9}$$

To put it in other words, spectral density and autocovariance provide the same information about a process, but in a different manner. Periodogram is then just sample estimate of a population's spectral density. It also contains information about the periodic components of a process (PennState, 2020). The periodogram of a stationary process $X_t$ s denoted by $I_x(\omega_j)$ and calculated in the following manner (Nguyen, Prokopczuk & Sibbertsen, 2020, p. 6):

$$I_x(\omega_j) = \frac{1}{2\pi} \left|\sum_{t=1}^{T} X_t e^{-it\omega}\right|^2 \qquad (10).$$

Consider the process given by Equation 3. It can be also expressed with spectral densities

$$f(\omega)_x = \left|1 - e^{-i\omega}\right|^{-2d} f(\omega)_u \tag{11}$$

To obtain the GPH estimator, the periodogram is estimated from the data. The logarithm of the periodogram is then regressed on a trigonometric function (Banerjee & Urga, 2005, p. 19). as shown in Equation 10. The GPH estimator equals to the negative slope of this regression denoted by parameter b (Baillie, 1996, p. 33).

$$\log\left(I_X(\omega_J)\right) = a + b\log(4\sin^2\left(\frac{\omega_j}{2}\right) + \nu_j,\ \nu_j = \log\left(\frac{f_u(\omega_j)}{f_u(0)}\right), j = 1, \ldots, m \qquad (12)$$

The asymptotic distribution equals $\sqrt{m}(d_{GPH} - d) \xrightarrow{d} N(0, \frac{\pi^2}{24})$, if $d$ is between – ½ and ½ (Busch & Sibbertsen, 2018, p. 4). Similarly, to the modified rescaled range statistic, a practical question of choosing the value of $m$, which is the bandwidth parameters, arises when using the GPH estimator. There is a trade-off when choosing the value of $m$. If the $m$ is too high the GPH estimator might be biased, but if it is too low, this will lead to a rise in the variance of the estimator (Nguyen, Prokopczuk & Sibbertsen, 2020, p. 7). One possible choice for the value of $m$, is that $m$ equals to the square root of T, which is the number of observations (Baillie, 1996, p. 33). Practically, the value of $m$ tells us how many estimates from the periodogram will be used in calculating the GPH estimator. Of course, this decision also depends on the data at hand. Consequentially, different choices for the value of m can produce a wide range of estimates for the memory parameter in the same dataset. The GPH estimator is relatively robust to non-normality in $u_t$, but autocorrelation in the $u_t$ can lead to significant bias in the estimation of the memory parameter (Baillie, 1996, p. 35).

The local Whittle estimator also estimates the memory parameter in the frequency domain. It was proposed by Kuensch in 1987, just a couple of years after the introduction of the GPH estimator (Busch & Sibbertsen, 2018, p. 4). It is also referred to as the Gaussian semiparametric estimator. It is obtained by minimizing an objective function, which is in a discrete approximate frequency domain Gaussian likelihood averaged over frequencies near zero (Banerjee & Urga, 2005, p. 20). The objective function is defined in the following way (Shimotsu & Philips, 2004, p. 658):

$$Q_m(G, d) = \frac{1}{m}\sum_{j=1}^{m}(\log\left(G\,\omega_j^{-2d}\right) + \frac{\omega_j^{2d}}{G} I_x(\omega_j) \qquad (13)$$

The objective function is a Gaussian objective function and for this reason the local Whittle estimator is for that reason also called the Gaussian semi parametric estimator. Minimization of this function provides the estimates of the parameters G and d (Shimotsu & Philips, 2004, p. 659).

$$d_{LW} = argmin\ (\log\left(\frac{1}{m}\sum_{1}^{m}\omega_j^{2d}\, I_x(\omega_J)\right) - 2d\,\frac{1}{m}\sum_{1}^{m} log\omega_j) \qquad (14)$$

Due to the widespread use of this estimator, its properties have been extensively studied and compared to the GPH estimator. The local Whittle estimator has an asymptotically normal distribution if –½ < $d$ < ¾ and it is also consistent for –½ < $d$ < 1 (Shimotsu & Phillips, 2004, p.656). The asymptotic distribution of the estimator equals to $\sqrt{m}(d_{LW} - d) \xrightarrow{d} N(0, \frac{1}{4})$ (Busch & Sibbertsen, 2018, p.5). It is evident that the local Whittle performs particularly good when applied to stationary time series. His performance starts lagging after the memory parameter exceeds ½, which is consistent with a non-stationary time series. Nevertheless, if the non-stationarity is not to strong its performance remains acceptable. The performance of the estimator becomes problematic when $d$ >1 (Shimotsu & Phillips, 2005, p.1890). Because this is a common feature of financial time series, a series of modifications of the local Whittle estimator have been proposed. The main purpose of all these modifications is improving the consistency of the estimator when it is applied to time series with severe non-stationarity. An example would be the exact local Whittle estimator proposed by Shimotsu and Phillips (2005). Their main additions are that the exact Local Whittle estimator uses an assumption that the initial value of the data is known and a modified objective function, which is computationally heavier. They provided evidence that indicate that their modifications increase consistency of the estimator. It has also been shown that the local Whittle estimator is more efficient than the group of estimators based on the approach of the regression of the logarithmic periodogram (Shimotsu & Phillips, 2005, p. 1890). The GPH estimator belongs to this group. Nevertheless, the GPH estimator and the LW estimator share the same practical dilemma of choosing the bandwidth parameter m, which substantially affect the obtained estimates of the memory parameter.

## 2.3 Parametric methods

The GARCH (p,q) model was first developed in 1986 by Bollerslev and Taylor as the generalization of the ARCH model. If the $\upsilon_t$ is the information set available and if the following conditions hold:

$$E(\in_t |\upsilon_{t-1}) = 0 \; and \; var(\in_t |\upsilon_{t-1}) = h_t,$$

then the GARCH (p,q) defines the volatility equation as (Tayefi & Ramanthan, 2012, p. 177):

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{15}$$

The GARCH model is used to describe the conditional variance with heteroscedasticity. It is widely used in financial econometrics for modelling various financial time series. As will be shown in the literature review, it has been commonly used to measure persistence in volatility. The indicator of the degree of persistence is the sum of $\alpha_i$ and $\beta_j$. The higher the sum, the stronger the persistence. If the sum is equal to 1, then this is the case of integrated GARCH model (Tayefi & Ramanthan, 2012, p. 178). In such instances the persistence is infinite, meaning that the shocks to volatility have permanent effect and the unconditional variance is not finite and thus the underlying process is non-stationary.

The GARCH (p,q) can be rewritten as:

$$\left(1-\alpha(L)-\beta(L)\right)\epsilon_t^2=\alpha_0+\left(1-\beta(L)\right)\nu_t, \nu_t=\epsilon_t^2-h_t \tag{16}$$

The lag operators are denoted as L and $\nu_t$ are the innovations of the conditional variance. According to Tayefi and Ramanthan (2012, p. 179), the aforementioned integrated GARCH model can be written as:

$$\left(1-\alpha(L)-\beta(L)\right)(1-L)\epsilon_t^2=\alpha_0+\left(1-\beta(L)\right)\nu_t \tag{17}$$

A lot of derivations have been proposed since the introduction of the GARCH model in order to incorporate the features that volatility exhibits. For example, to incorporate the leverage effect into the GARCH model Nelson (1991) proposes the exponential GARCH model. Building on the concept of fractional integration, Baillie, Bollerslev and Mikkelsen (1996) propose the fractionally integrated GARCH as an adaptation for capturing the long memory property in volatility. To obtain the fractionally integrated GARCH model the *(1-L)* term in Equation 15 is replaced by $(1-L)^d$, where 0<*d*<1.

$$\left(1-\alpha(L)-\beta(L)\right)(1-L)^d\epsilon_t^2=\alpha_0+\left(1-\beta(L)\right)\nu_t \tag{18}$$

This offers more flexibility in modelling the conditional variances and also enables measuring the degree of persistency, through the memory parameter *d*. Higher value of the memory parameter implies stronger persistence. If the value of *d* would be set to 0, this would correspond with the classic GARCH model and for value of *d* equal to 1, this would be the I-GARCH model.

# 3 SPURIOUS PERSISTENCE, NON-STATIONARITY AND STRUCTURAL BREAKS

Since the work of Granger and Newbold (1974), spurious regression has become a serious issue in time series econometrics. Their main thesis was that serial correlation in the error term, which is a common feature in time series data, could cause the significance tests to be inaccurate and support wrong conclusions. This also means that the value of the determination coefficient ($R^2$) may be misleading and that the high value of that coefficient does not imply a good fit as it is usually suggested by the data. To simplify, due to the features of the data in time series econometrics, one might mistakenly deduce that there exists a causal relationship between two (or more) variables when conducting a classical regression. This is also often referred to as a spurious regression problem. A common observation is also that regression using integrated series are more prone to produce spurious results and inference is especially hard when trying to regress series of the same integration order (Mills, 2019, pp. 240–242). The reason for this is this that time series with order of integration that is higher than zero usually possess deterministic trends. Suppose that we have two unrelated time series that both possess an upward deterministic trend (non-stationary time series). That means that the data points from this series will be increasing with time. Consequently, if we regress one of them on the other, the regression could show some correlation, because of the upward movement, but that might not be completely justified.

When dealing with persistency, time series data is employed, thus one must be on alert for spurious regression stemming from non-stationarity. Either, actual long memory in the time series or non-stationarity in the time series, can cause the findings in the autocorrelations functions and other metrics, which point to persistent behaviour in volatility (Guegan, 2005, p. 114). Non-stationarity, which is a very plausible option, actually causes the statistical tools intended to identify persistence to behave the same way as if they were used on stationary long-range dependent sequence. This means that the parameters and that statistics we obtained might tell us that data exhibits persistence when it actually does not. The field of long memory behaviour is especially known for statistical tools that behave similarly when under the assumption of stationarity and long-range dependence or weak dependence and non-stationarity (Guegan, 2005, p. 133). This is also the reason why there are so many different and inconclusive empirical results when researching persistence and why the researchers have not yet come to a general consensus about it.

### 3.1 Definition of structural breaks

A special branch of research has developed within the field of non-stationarity in time series and it focuses on structural breaks. A simple definition of a structural break would be a sudden an unexpected change in the time series (Stata, 2020). The change is most commonly associated with a change in the mean, but there can also be a change in any other parameter that defines the time series, for example in the second moment. When a structural break occurs the parameter vector that defines a model permanently changes (Clements & Hendry, 2006, p. 607). The permanence of the structural break comes from the commonly used assumption that these breaks are exogenous. A wide range of economic developments can cause a structural break in a financial time series. For example, economic crises are often thought as the events that have the ability to structurally change the stock market volatility because of their effect on the fundamentals of economies. Another assumed feature of the structural breaks is that they are unanticipated (Clements & Hendry, 2006, p. 607). This poses a problem when it comes to modelling. This is also one of the reasons, why test for structural breaks are essential diagnostic tools. When dealing with time-varying volatility they should be used in a preliminary and precautionary manner. Usually, tests meant to detect structural breaks are formed as a hypothesis tests, where the test statistics can be viewed as two-sample test adjusted for a presence of a break (Aue & Horváth, 2012, p. 1). The null hypothesis usually assumes structural stability and the alternative suggest presence of the break(s). Structural stability has a couple of different definitions. A time series can be labelled as structurally stable if its unconditional mean does not change over time. Another meaning of the structural stability is that a time series has a time invariant conditional mean.

This also referred to as constancy of regression coefficients. The last type of structural stability is the one regarding the unconditional second moments of the time series, which are expressed through the auto-covariance function (Aue & Horváth, 2012, p. 2). The most examined are the first two types. In regression context, parameter instability is often associated with the misspecification of the model. A misspecification in the model usually means that the conclusion coming from the model might not be in line with reality and thus compromises the obtained results. When model misspecification is present, the standard econometric theory and inference do not apply. Additionally, the parameters that are the subject of estimation need to be stable through time when estimating time-varying volatility (Smith, 2008, p. 845).

After the confirmation of the presence of the break, the next step is to determine the time of the break, because this tells when the analysed time series has changed. This procedure is more straightforward, if there are some potential candidates when the break might have occurred. The problem with cases when the date of the break is not known is that the standard tests used for detecting breaks do not have the standard limiting distributions and this often leads to invalid inference of such tests (Smith, 2008, p. 846). In general, it is difficult to the determine the date breaks in finite samples. It has also been shown that the testing works best if the break date is in the middle of the timeframe of the sample (Aue & Horváth, 2012, p. 5). Probability of correctly identifying the presence and the location of a structural break increases with the sample size (Gil-Alana, 2007, p. 171).

One can show that breaks seriously impact important features of persistence tests (and estimators) such as their size and power, which means that inference based on them is misleading and the results can be severely affected (Banerjee & Urga, 2005, p. 2). An elementary explanation for this would be that non-stationary series or time series with breaks do not possess time invariant first two moments and autocovariance structure and that could give an impression that a shock to the volatility of this time series (asset returns) persists with the moments being unchanged. In reality however, the moments itself are changing and there is no mean reversion just a deterministic trend, which falsely indicates persistence. Long memory and structural breaks exhibit similarly in the second order properties of a time series (Aue & Horváth, 2012, p. 1). They both share slow decay in the autocorrelation for larger lags. This means that at the first glance, there is no contextual difference between the two. Earlier, the prevailing mindset was that short-memory models with structural breaks can be interchangeable with long-memory models (Wenger, Leschinski & Sibbertsen, 2018, p. 90). Recently, this was refuted by empirical studies. For example, Perron and Qu (2010) were among those, who showed that the behaviour of autocorrelations of a short memory model with structural change is similar but not identical to the behaviour of autocorrelations in a long memory model. They showed that autocorrelation functions of these two models can be differentiated by observing their behaviour at longer langs. The autocorrelation function of a short memory model with a structural break has a specific structure, because the value of the autocorrelations at distant lags depends only on the sample size. Meanwhile, the long memory model has an autocorrelation function that also depends on the underlying process and the long memory parameter, which defines it. In a short memory model with a structural break the autocorrelation function will reach zero at larger lags compared to the long memory model, where this is defined by sample size and the long memory parameter.

This similarity in behaviour also exuberates the problem in testing. Inference about structural breaks is adversely impacted in the presence of long memory and vice versa (Wenger, Leschinski & Sibbertsen, 2018, p. 91). The conclusion following from this is that when testing for the presence of either of them, the presence of the other must also be accounted and checked for. For modelling persistence GARCH models are usually used. The problem is that the GARCH family of models is more susceptible to structural changes than linear models and this problem is independent of the estimation method used to obtain the GARCH equation (Andreou & Ghysels, 2006, p. 6). Part of the reason lies within the assumption that the GARCH model normally assumes the stationarity of the data if not adjusted for. The other reason is the formulation of the GARCH itself, because, the stochasticity of the conditional volatility comes only from the lagged squared residual and there is no contemporaneous error orthogonal to the to the regressors (Hillebrand & Medeiros, 2008, p. 308).

To put structural breaks in a broader context, they also play an important role in macroeconomics. A common approach in macroeconomics is to tie policy changes or key economic developments to structural change in key macroeconomic variables. For example, Garcia and Perron (1996) link structural breaks in US real interest rate to the spike of oil prices in 1973 and the US federal budget deficit expansion in 1981. The analysis of previous policy induced structural breaks in macroeconomics can be a valuable source of information for key officials and decision-makers, whether they are involved in fiscal or monetary policy decision. Structural breaks are also important for investors because they can affect the financial indicators, on which they base their investment allocations.

### 3.2 Detecting structural breaks

One of the methods for detecting structural breaks, which will be used in my master thesis, is based on the loglikelihood approach as proposed by Killick & Eckley (2014). For a single changepoint the approach is formed on as a statistical test, where the null hypothesis is that there is no changepoint. The test statistic is calculated from the loglikelihood (Killick & Eckley, 2014, p. 3):

$$ML(\tau_1) = logp\left(y_{1:\tau_1}|\theta_1\right) + logp\left(y_{(\tau_1+1):n}|\theta_2\right) \tag{19}$$

The probability density function is denoted by p(.) and the changepoint occurs at time $\tau_1$. The corresponding test statistic is then:

$$\lambda = 2(maxML(\tau_1) - logp\left(y_{(y1:n)}|\theta\right)) \tag{20}$$

The approach can be extended for multiple changepoints by summing the likelihood for each segment (Killick & Eckley, 2014, p. 3).

$$\sum_{i=1}^{m+1}(C(y_{\tau_{i-1}+1):\tau_1})) + \beta f(m) \tag{21}$$

By minimizing the objective function in Equation 4 one can find multiple structural breaks in a time series. The first term in the bracket is the negative loglikelihood (the cost function) and the second term is the penalty function to prevent overfitting (Killick & Eckley, 2014, p. 4). In order to minimize the objective function, the segment neighbourhood algorithm was used. It is a dynamic programming technique, which gives exact solutions but is computationally heavy, but in my case  the datasets are not too big, so this is not a concern.

Long memory in volatility has been detected in different asset classes from stock and precious metals to foreign exchange rates and has been tested for many different time periods. This makes the notion that persistence is a spurious finding in empirical research less likely than one might think. Perhaps the importance, frequency and strength of persistence are overestimated due to non-stationarity, but after all the research it is highly unlikely that the entire concept of long memory in asset returns is spurious.Although that non-stationarity is the most studied potential candidate for spurious long memory, there are also other reasons that may cause it.

### 3.3 Other causes for spurious persistence

The second most mentioned reason for spuriousness is aggregation (Lobato & Savin, 1998, p. 264). In some studies, persistence has been found in stock indices such as S&P 500. The core principle involved here is that stocks individually do not exhibit any significant long-range dependencies, but when you aggregate them together in a form of an index they do. To put it in more general terms, a combination of weakly dependent series of observations may produce a strongly dependent one. Lobato and Savin (1998, p. 265) also mention two other reasons: non-existence of higher order moments and seasonal long memory component. The existence of the fourth moment of a time series is a necessary condition to perform some of the tests for long memory. That means that if the fourth moment does not exist or is not finite, we can not or at least should not preform inference test, because the results may be spurious (Lobato & Savin, 1998, p. 266). The last reason is a seasonal long memory component (Lobato & Savin, 1998, p. 266). This means that a seasonal movement in the time series has such strong impact, that even though persistence is present only seasonally, the results can spuriously show year-round persistence for the entire period. However, one might argue that this might not technically qualify as a spurious finding, because persistence is present but only seasonally. Nevertheless, it can still lead to wrong inference and deduction regarding the presence of persistence.

The main takeaway from this part is that we need to do our due diligence when it comes to the data even before we start modelling. We need to know its features in order to apply the right statistical tools and methods and to get viable results. We need to gather as much information about the analysed time series as possible: main parameters that define it, its features and especially all the things that make this time series unique, if they exist. The reason behind this is, because only when we are equipped with this information we are able to make the right decision on the use of statistical tools and modelling techniques. Additionally, the information about the time series will affect how to interpret the results. Most importantly, we are also able to identify where the potential issues lies and can be prepared for them.

# 4 DESCRIPTIVES

In the following section, some descriptive statistics of the used data will be presented. The return series for the S&P 500 and the fifty individual stocks starts on 4$^{th}$ of January 2000 and ends on 1$^{st}$ of May 2020. Based on the daily closing prices logarithmic daily returns were calculated.

*Table 1: List of chosen companies*

| | **Company** | **Ticker** | **Sector** |
|---|---|---|---|
| 1 | 3M Company | MMM | Industrials |
| 2 | Abbott Laboratories | ABT | Healthcare |
| 3 | Altria Group Inc | MO | Consumer staples |
| 4 | American Express Co | AXP | Financials |
| 5 | Apple Inc | AAPL | Technology |
| 6 | Archer Daniels Midland Co | ADM | Consumer staples |
| 7 | Barrick Gold Corp | GOLD | Basic materials |
| 8 | The Boeing Co | BA | Industrials |
| 9 | Bank of America Corp | BAC | Financials |
| 10 | Bristol Myers Squibb Co | BMY | Healthcare |
| 11 | Caterpillar Inc | CAT | Industrials |
| 12 | Chevron Corp | CVX | Energy |
| 13 | Cigna Corp | CI | Healthcare |
| 14 | Cisco Systems Inc | CSCO | Technology |
| 15 | Citigroup Inc | C | Financials |
| 16 | Coca-Cola | KO | Consumer staples |
| 17 | Colgate Palmolive Co | CL | Consumer staples |
| 18 | Eastman Chemical Co | EMN | Basic materials |
| 19 | Eaton Corp | ETN | Industrials |
| 20 | Exxon Mobil Corp | XOM | Energy |
| 21 | Foot Locker Inc | FL | Consumer discretionary |
| 22 | Ford Motor Co | F | Consumer discretionary |
| 23 | The Goodyear Tire & Rubber Co | GT | Consumer discretionary |

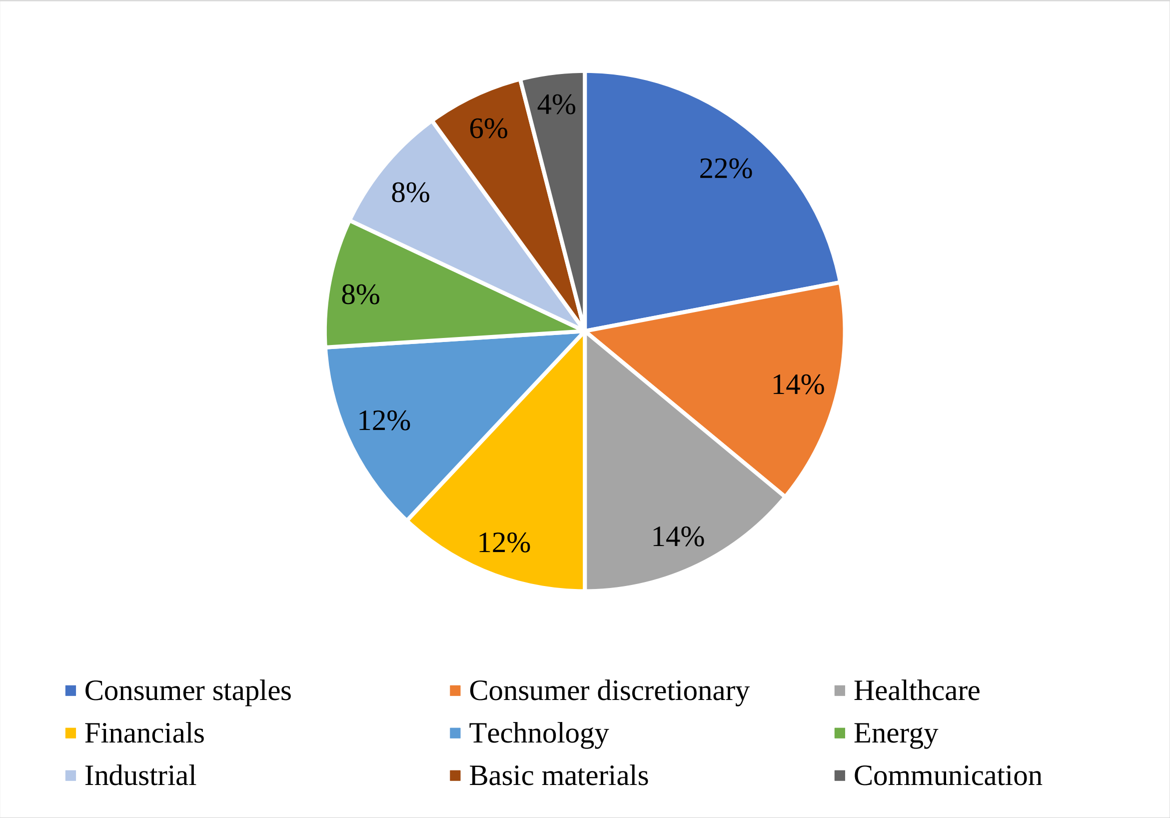
*Tables continues*

*Table 1: List of chosen companies (cont.)*

| | Company | Ticker | Sector |
|---|---|---|---|
| 24 | Home Depot Inc | HD | Consumer discretionary |
| 25 | IBM Corp. | IBM | Technology |
| 26 | Intel Corp | INTC | Technology |
| 27 | Johnson&Johnson | JNJ | Healthcare |
| 28 | JPMorgan & Chase Co | JPM | Financials |
| 29 | Kellogg Co | K | Consumer staples |
| 30 | Kimberly Clark Corp | KMB | Consumer staples |
| 31 | McDonald's Corp | MCD | Consumer discretionary |
| 32 | Merck & Co Inc | M | Healthcare |
| 33 | Microsoft Corp | MSFT | Technology |
| 34 | Morgan Stanley | MS | Financials |
| 35 | Newmont Corp | NEM | Basic materials |
| 36 | Nike Inc | NKE | Consumer discretionary |
| 37 | Occidental Petroleum Corp | OXY | Energy |
| 38 | Oracle | ORCL | Technology |
| 39 | PepsiCo Inc | PEP | Consumer staples |
| 40 | Pfizer Inc | PFE | Healthcare |
| 41 | The Procter & Gamble Co | PG | Consumer staples |
| 42 | Schlumberger Ltd | SLB | Energy |
| 43 | Target Corp | TGT | Consumer staples |
| 44 | The Unilever Group | UN | Consumer staples |
| 45 | Verizon Communications Inc | VZ | Communication |
| 46 | Walgreens Boots Alliance Inc | WBA | Healthcare |
| 47 | Walmart Inc | WMT | Consumer staples |
| 48 | The Walt Disney Co | DIS | Communication |
| 49 | Wells Fargo & Co | WFC | Financials |
| 50 | Whirlpool Corp | WHR | Consumer discretionary |

*Source: Own work.*

*Figure 1: Structure of chosen companies by sector (in %)*




*Source: Own work.*

Table 1 and Figure 1 show the list of the companies and their structure by sectors. Out of 50 randomly selected stocks, 22% of them come from the consumer staples sector, 14% from the consumer discretionary sector and 14% from the healthcare sector. In the chosen group of stocks there are none from the real estate and utilities sector. Companies are fairly diversified among sectors with cyclical and non-cyclical sectors included (roughly half and half in each group). However, two out of the three largest sectors in the portfolio are considered non-cyclical (consumer staples and healthcare), which may put a downward pressure on the returns and also volatility. When it comes to the selection of the stocks from the S&P500, one must bear in mind that the structure of the index is constantly changing and there are not so many stocks that have been included in the index for the entire 20 years that were examined. The majority of the companies that were selected were founded quite some time before the start of observation period. Another thing also needs to be pointed out. In empirical finance the survivorship bias is a common concern. Here this concern becomes even more severe, because the firms that are included in the S&P500 must not only survive, but they must also maintain sufficient market capitalization and through good performance to be included in the index. Consequently, findings from this specific part of the stock market may deviate from the overall stock markets or other segments such as small and mid-caps.

*Table 2: Descriptive statistics of the logarithmic returns from January 2000 to May 2020*

| Descriptives | Returns of the selected stocks | S&P500 returns |
|---|---|---|
| Mean | 0.01% | 0.01% |
| Median | 0.03% | 0.05% |
| Min | -23.17% | -12.7% |
| Max | 18.45% | 10.96% |
| SD | 2.01% | 1.26% |
| Skewness | -0.50 | -0.37 |
| Kurtosis | 20.99 | 11.16 |
| 1st quartile | -0.84% | -0.48% |
| 3rd quartile | 0.90% | 0.57% |

*Source: Own work.*

In Table 2, the descriptive statistics are presented for the 51 log return series of the individual stocks and the S&P500 index. The number of observations for each of the return series equals to 5,114. First thing that becomes clear is that the magnitude of returns and volatility is much lower for the S&P500 than for the individual stocks. Both means and medians are positive implying that investing in these 50 stocks or the S&P500 would be beneficial for an investor. The buy and hold return for the S&P500 for the analysed period equals to 66,54%. The positive means from both columns also offer confirm the view that investing in the stock market is beneficial on the long run. The analysed period has seen two of the biggest bear markets (Dotcom crash and after the financial crisis of 2007 and 2008) in the history of the US stock market and yet the returns of the S&P500 and the majority of chosen stocks are positive. Out of the 50 analysed stock 8 of them have negative average daily returns: BMY, BAC, CSCO, C, F, GT, MS and SLB. Three of them are banks, which can be explained by the fact that the financial sector was in the epicentre of the financial crisis in the 2007-2008 period. After the crisis, the banks were also exposed to significant legal and regulatory scrutiny. Additionally, the unprecedented low interest rate environment is also putting downward pressures on the banks' margins and challenging their business models. In both columns, the means are smaller than the medians. Combined with the negative skewness coefficients, this implies that the distributions for the S&P500 and 50 stocks are skewed to the left, which means that the left tails are longer. The minimums are also higher in absolute terms than the maximus, which confirms that the left tails are longer than the right. However, the magnitude of the skewness coefficients does not indicate that the asymmetry would be extreme, only moderate. On the other hand, the coefficients for kurtosis imply that in both cases returns exhibit fat tails, which means that the extreme returns are more likely than in a normal distribution. This is line with the previous research. The coefficients for kurtosis and skewness are higher for the 50 selected stocks compared to the coefficients for the S&P500. This also implies that the returns of the S&P500 are more symmetric, less fat tailed and in general more like the normal distribution. Both range and the standard deviation indicate that the volatility is higher for the series of returns for the selected stocks than for the

S&P500 returns, which is expected because the S&P500 consists of 10 times more stocks and diversification effects are stronger. The first and third quartiles for the individual stocks are higher in absolute terms than for the S&P500. Additionally, in both cases the first quartile is closer to the median than the third quartile.

*Table 3: Highest and lowest daily returns of the S&P500 (in %)*

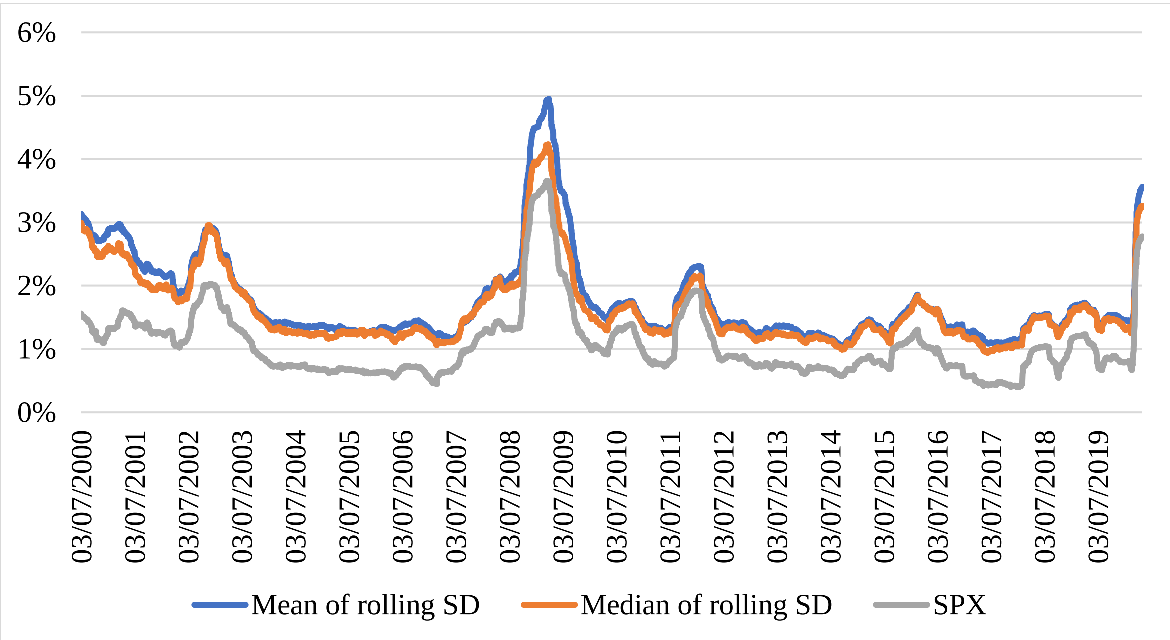
| Rank | Lowest daily return | Date | Highest daily return | Date |
|---|---|---|---|---|
| 1 | -12.77 | 16/03/2020 | 10.96 | 13/10/2008 |
| 2 | -9.99 | 12/03/2020 | 10.25 | 28/10/2008 |
| 3 | -9.47 | 15/10/2008 | 8.97 | 24/03/2020 |
| 4 | -9.35 | 01/12/2008 | 8.88 | 13/03/2020 |
| 5 | -9.20 | 29/09/2008 | 6.84 | 23/03/2009 |
| 6 | -7.92 | 09/10/2008 | 6.80 | 06/04/2020 |
| 7 | -7.90 | 09/03/2020 | 6.69 | 13/11/2008 |
| 8 | -6.95 | 20/11/2008 | 6.27 | 24/11/2008 |
| 9 | -6.90 | 08/08/2011 | 6.17 | 10/03/2009 |
| 10 | -6.31 | 19/11/2008 | 6.13 | 21/11/2008 |
| 11 | -6.30 | 22/10/2008 | 6.05 | 26/03/2020 |
| 12 | -6.00 | 14/04/2000 | 5.82 | 17/03/2020 |
| 13 | -5.91 | 07/10/2008 | 5.57 | 24/07/2002 |
| 14 | -5.43 | 20/01/2009 | 5.28 | 30/09/2008 |
| 15 | -5.41 | 05/11/2008 | 5.27 | 29/07/2002 |

*Source: Own work.*

Table 3 shows 15 highest and lowest daily returns of the S&P500 with the corresponding dates in the last 20 years. This can be an indicator of days or periods of time, when some market shifting events happened. To begin with, all the corresponding dates can be linked to periods of heightened uncertainty in the markets or real economy. Most of the extreme moves happened in two periods: during the financial crisis in 2007-2008 and the COVID-19 induced recession in 2020. Among the lowest daily returns of the S&P500 there are only two days that are not from that period. The first is from 2011, which was the height of the sovereign debt crisis and the other one is from 2000, which was the build-up period for the Dotcom bubble burst. Among the highest returns in the last 20 years only two do not belong to the two aforementioned periods and they happened in April and June 2002. This was in the aftermath of the Dotcom bubble, 9/11 terrorist attack and Enron accounting scandal, which induced the markets with elevated volatility. Another interesting thing is that extreme returns in either direction happen close to each other. For example, the lowest returns in the examined period happened just in the span of 5 days. Similar story is for the highest returns, where the two highest returns happened in the span of two weeks in the October of 2008. More importantly, the period from 9$^{th}$ of March 2020 to 26$^{th}$ of March 2020 has had 3 out of 15 lowest returns and 4 out of 15 highest returns in the last 20 years. This can be explained by the unique situation in this period when the majority of the developed economies shut down, which is unprecedented and has caused panic among the investors. Similarly, the S&P500 rose for almost 11% on 13th of October 2008 but then declined for almost 10% two trading sessions later. This simple exercise shows the essence of Mandelbrot's volatility clustering. It is clearly visible that large changes in a stock (index) price are usually followed by large changes and that the sign does not particularly matter. A large or negative change can lead to another large positive or negative change. In both cases the volatility remains high. The data from this table can also offer some insights about when a regime changes in the volatility (second moments of the return series) of the S&P500 might have occurred. Two potential candidates immediately stand out based on the findings in the Table 2. The first one is the period from the end of September 2008 to mid-November in 2008. The other is the second half of March of 2020, because of the large changes and increased volatility in those periods. This table also indicates another stylized fact about the volatility. On average, the lowest daily returns from this table are greater in absolute terms than the highest daily returns, which is consistent with the asymmetric shocks associated with volatility, where negative shocks have a stronger effect than positive.

Before the emergence of the ARCH and GARCH models, the rolling standard deviation was one of the first measure of time-varying volatility. Due to the moving window it has at least some dynamic component included and can give at least some insight into how volatility is changing with time.

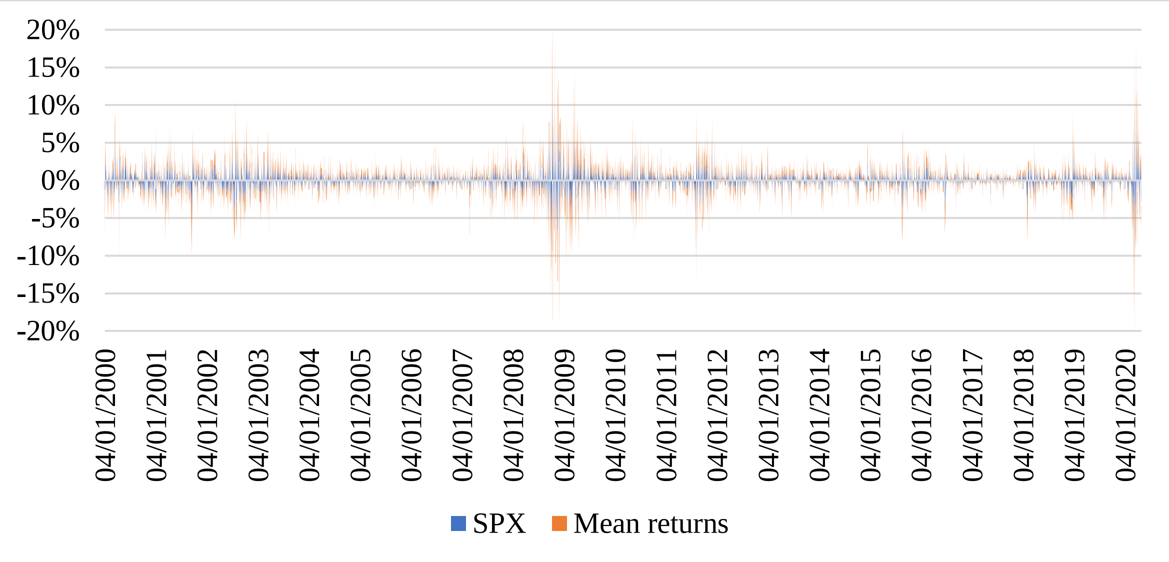
*Figure 2: Rolling standard deviation of the S&P500 and mean and median returns of the selected stocks*




*Source: Own work.*

In Figure 2 the three lines represent the rolling standard deviation of the S&P500 and the mean and median of the rolling standard deviation for each rolling window for the 50 analysed stocks. The window length equals to 126 trading days, which equals to a half a year with the assumption that there is approximately 252 trading days in one year. When choosing the length of the rolling window there is always a trade-of present. Choosing a larger window means that there is a possibility that the sensitivity of standard deviation will be too low to capture the dynamics. If the window is too narrow the metric will be sensitive to outliers. Since World War II the average length of the stock market corrections is estimated to be around 4 months (Franck, 2020). Additionally, in macroeconomics, recession is usually announced when the GDP growth is negative for two consecutive quarters. Based on the rolling standard deviations, volatility has spiked a couple of times during the last 20 years. The entire period from mid-2000 to mid-2002 was marked with elevated volatility. The largest spike happened during the financial recession in 2008 and 2009, when volatility was almost twice as high in the first two years of the examined period. Volatility has also spiked substantially at the end of the examined period in March 2020. If we compare the COVID-19 induced market turmoil with the financial crisis, we notice a couple of differences. Firstly, the volatility measured by rolling standard deviation was higher in the financial crisis, but this is at least partially explained with the fact that the analysed period doesn't capture the entire COVID-19 bear market, since the chosen periods ends in the middle of the COVID-19 induced decline of the stock markets. Secondly, in the months, prior to the financial crisis in 2008 volatility was already gradually climbing upwards, whilst before March 2020 the volatility was actually declining. Finally, the rate of increase of volatility in March 2020 appears to be larger (the lines almost become vertical in that period). If we compare the three lines, we notice that the rolling standard deviation of the S&P500 is constantly below the mean and median of the rolling standard deviations of the 50 chosen stocks. But the movements in the rolling standard deviations tend to follow each other. In fact, the calculated correlation coefficients between the mean of the rolling standard deviations and the rolling standard deviation of the S&P500 and the median of the rolling standard deviations and the rolling standard deviation of the S&P500 are 0.948 and 0.95, respectively. This figure can again serve as an indicator of the periods when structural changes in the volatility of returns stemming from systemic factors might have occurred in the last 20 years. Based on this figure, the periods with potential structural changes remain the same as those that were identified with the table 2, with the possible addition of the first half of 2016.

*Figure 3: Returns off the S&P500 and mean returns of the selected stocks*




*Source: Own work.*

Figure 3 shows the daily returns of the S&P500 and the mean returns of the 50 stocks at each date (equivalent way of saying would be the returns of an equally weighted portfolio constructed from these 50 stocks). The movement is similar, although the graph of the mean returns has a larger magnitude. The correlation coefficient for these two-time series equals to 0.97, which implies a strong correlation. Both return series seem to fluctuate around the mean level, which means that they might both be stationary as is often found in empirical studies.

*Table 4: Statistical tests for serial correlation, stationarity and conditional heteroscedasticity*

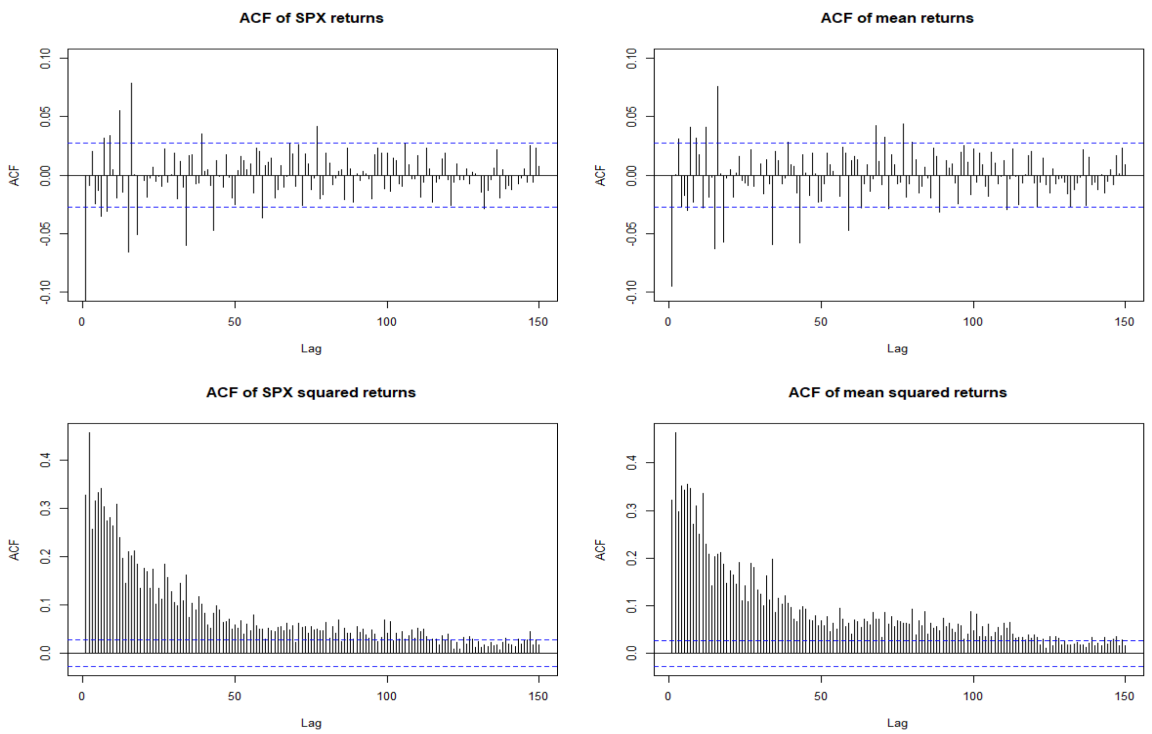
| **Time series** | | | **S&P 500 returns** | | **Mean returns** | |
|---|---|---|---|---|---|---|
| **Test** | **Purpose** | **Null hypothesis** | **Test statistic** | **P-value** | **Test statistic** | **P-value** |
| Ljung-Box test | Autocorrelation | No serial correlation | 95.15 | <0.001 | 78.86 | <0.001 |
| KPSS test | Stationarity | Stationarity | 0.23 | 0.1 | 0.057 | 0.1 |
| ADF test | Stationarity | Presence of a unit root | -17.09 | 0.01 | -17.21 | 0.01 |
| ARCH LM-test | Conditional heteroscedasticity | No ARCH effects | 1536.30 | <0.001 | 1576.50 | <0.001 |

*Source: Own work.*

For both return series I performed statistical tests to check for autocorrelation, stationarity and conditional heteroscedasticity. The results are presented in Table 4. For testing for the presence of autocorrelation, I used the Ljung-Box test with the number of lags equal to 10.

Based on the *p*-values of the test we can reject the null hypothesis that there is no serial correlation in the first 10 lags for both series. This means that the autocorrelations are significant at least one lag in the first ten lags, which implies that today's return is correlated to at least one of the returns in the next 10 trading days. To check whether the series are stationary I used the Augmented Dickey-Fuller test and Kwiatkowski–Phillips–Schmidt–Shin test. The ADF test examined whether the return series possesses a unit root. However, the *p*-value suggest that we can reject the null hypothesis of a unit root, which implies stationarity. The KPSS test offers a similar conclusion, because based on the value of the test statistic, we can not reject the null hypothesis of trend stationarity. Finally, Engle's ARCH test revealed that there is conditional heteroscedasticity present in both series. The evidence to support this hypothesis are pretty convincing as p-value is smaller than 0.01. Practical implications of these testing procedures are the following. Today's realized return of these two series will affect at least one of the returns realized in the next 10 trading sessions. Both return series do not have a unit root, which means that the means are finite and that the unconditional means are time invariant. The variances also vary with time, which means that the GARCH family of models is needed to capture the time-varying volatility. Conditional heteroscedasticity also means that in some periods have higher variance and lower variance in other periods.

*Figure 4: Autocorrelation functions of returns and squared returns*




*Source: Own work.*

To conclude this section, Figure 4 shows plotted autocorrelation functions for returns and squared returns of the S&P500 and equally weighted portfolio are presented. The first thing that is noticeable is their pairwise similarity. In case of ACF of the returns, we can see that there is significant autocorrelation in the first lag. This means that today's return has strong implications for tomorrow's return. This is also consistent with the Ljung-Box test, which found statistically significant autocorrelations in the first 10 lags. For the next lags there is no clear pattern. Some of the autocorrelations are significant at individual lags and there positive and negative autocorrelations present. In the autocorrelations of squared returns the distinct pattern that is linked to long memory emerges immediately. The autocorrelations are positive and significant and remain such until distant lags. The autocorrelations also display the hyperbolic decay. This means that today's volatility of a return series has significant positive effect on the volatility many trading days ahead. There are autocorrelations that are positive even lags close to 150, which means that today's volatility is positively correlated with volatility half a year in the future (assumption that a year has around 252 trading days).

The results from this section are consistent with financial theory and previous empirical research. Additionally, the autocorrelations provide indication of behaviour, which is consistent with long memory associated with volatility. However, procedures conducted so far offer no concrete evidence if the long memory in volatility is a statistically significant feature of the volatility based on the chosen data.

# 5 MODELLING PERSISTENCE IN VOLATILITY

In this section, the results from different methods of estimation of the memory parameter will be presented. To begin with, I estimated the memory parameter for the S&P500 and each individual stock for the entire period with the GPH and ELW estimators. Both estimators were applied to squared returns. The value of the bandwidth parameter is set at 0.6. Based on the survey of empirical studies the bandwidth parameter is usually set between 0.5 and 0.7.

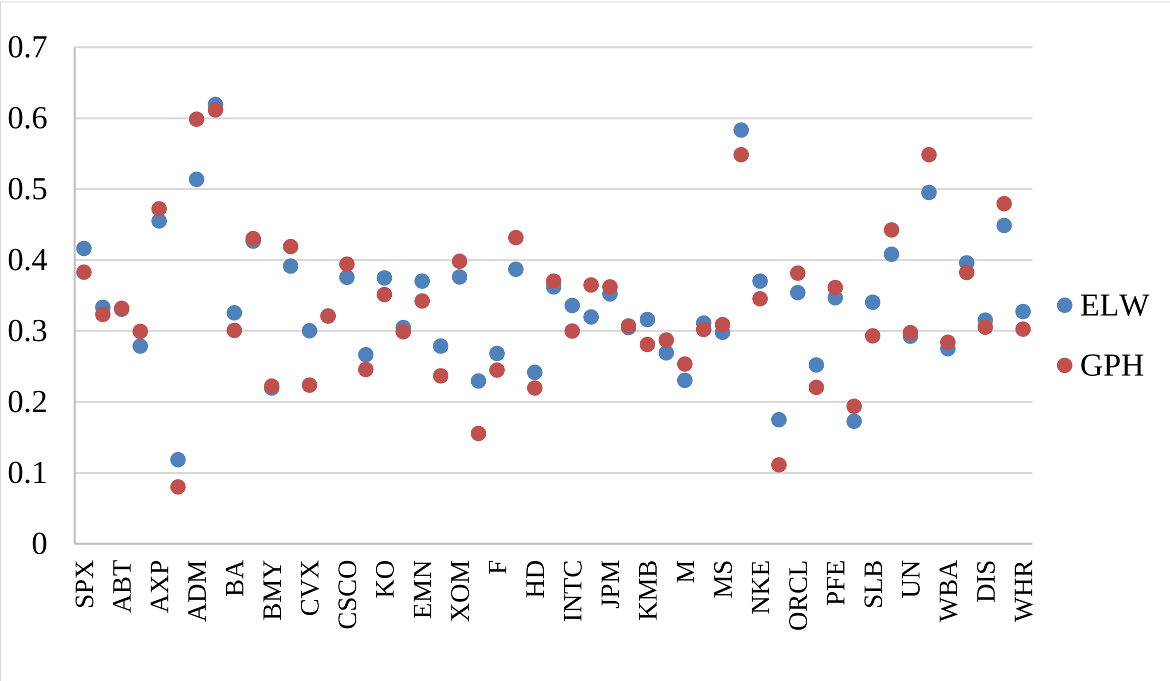
*Table 5: Semi-parametric estimators of the memory parameter*

| **Estimator** | **GPH** | **ELW** |
|---|---|---|
| Mean | 0.333 | 0.337 |
| Median | 0.309 | 0.327 |
| Min | 0.080 | 0.119 |
| Max | 0.611 | 0.619 |
| 1st quartile | 0.282 | 0.279 |
| 3rd quartile | 0.382 | 0.376 |
| Number of significant results (p=0.05) | 50 | 51 |
| Number of significant results (p=0.01) | 49 | 51 |
| Number of d>0.5 | 4 | 3 |

*Source: Own work.*

The estimates obtained with both estimators are very similar as shown in Table 5. The means are almost identical, while other statistics are close to each other. This means that the method itself does not have a huge impact on the values obtained. With the ELW estimator, all of the computed memory parameters are significant at 99% confidence interval. The GPH estimation produced 49 and 50 significant estimates at 99% confidence interval and 95% confidence interval, respectively. The number of memory parameters that exceed ½, which marks the boundary between a stationary series with long memory and non-stationary series with long memory, is 4 based on GPH estimator and 3 based on the ELW estimator. This is also consistent with the findings from ADF and KPSS tests in the previous chapter, which suggest that the returns of the equally weighted portfolio consisting of the picked 50 stocks is stationary. The semi-parametric methods also point towards the fact that the hyperbolic behaviour in the ACFs of the squared returns are caused by long memory.

*Figure 5: Scatterplot of the memory parameters obtained with the GPH and ELW estimator*




*Source: Own work.*

Figure 5 plots the estimates summarized in Table 4. For the S&P500 the estimates are around 0.4, which is below the 0.5 boundary and consistent with the KPSS and ADF test performed on the returns of S&P500. The scatterplot also shows that the pairs of estimates are close together and that neither of the two estimators consistently produces higher values.

The memory parameter for the equally weighted portfolio obtained with the GPH estimator equals to 0.384 and with the exact local Whittle estimator to 0.399, which is in line with the individual estimates. The results from the semi-parametric methods support the thesis that the persistence in volatility of stock returns is not just a feature of isolated cases but can be found in majority of the stocks from different sectors.

For the returns of the S&P500 and mean returns I estimated the GARCH (1,1) model with t-distribution as conditional distribution for innovations. Distributions of stock returns often have fat tails, which was also confirmed by descriptive statistics, henceforth the Student distribution seems an appropriate choice.

*Table 6: Coefficients of the GARCH (1,1) models for the entire period*

| **S&P 500 returns** | | | | |
|---|---|---|---|---|
| Coefficients | Estimate | Standard error | T-statistic | P-value |
| Alpha | 0.122 | 0.059 | 2.05 | 0.0400 |
| Beta | 0.876 | 0.053 | 16.63 | 0.0000 |
| **Returns of the equally weighted portfolio** | | | | |
| Coefficients | Estimate | Standard error | T-statistic | P-value |
| Alpha | 0.124 | 0.218 | 0.57 | 0.5709 |
| Beta | 0.871 | 0.202 | 4.32 | 0.0000 |

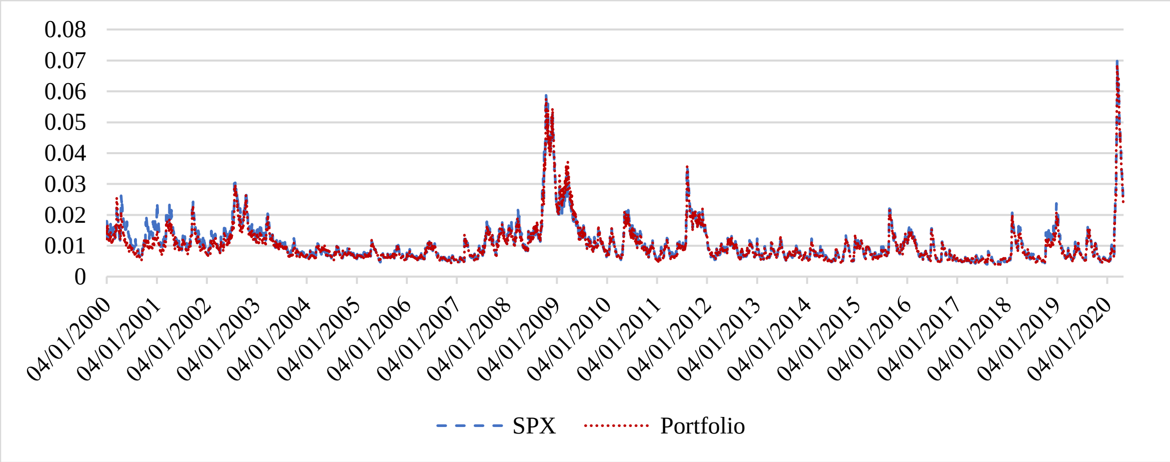
*Source: Own work.*

Table 6 shows the coefficients of the GARCH (1,1) model of the S&P 500 and equally weighted portfolio returns. For the S&P500 returns both coefficients are statistically significant and their sum equals to 0.997, which implies strong persistence. For the equally weighted portfolio’s returns the alpha coefficient is not statistically significant and the sum equals to 0.995. This is an indication that strong persistence in volatility is present in both cases. From the GARCH coefficients, the half-life of volatility can be calculated in the following manner:

$$\tau = \frac{-ln2}{ln\,(\alpha+\beta)} \tag{22}$$

For the S&P500, the half-life of volatility equals to 504.9 days, and for the equally weighted portfolio the half-life equals to 138.5 days. If a spike in volatility of the S&P500 occurs, it takes roughly 505 trading days for volatility to move halfway to the unconditional mean of the volatility. This means that based on this measure, the volatility of the S&P500 is far more persistent than the volatility of the portfolio. From Equation 10 it becomes clear, that if the sum of the GARCH equals to 1, the I-GARCH case, the half-life of the volatility can not be calculated because the denominator is not defined and the shocks last forever. But closer the sum is to 1, the half-life is higher and process is more persistent.

*Figure 6: Conditional volatilities based on the GARCH (1,1) model*




*Source: Own work.*

In Figure 6, the conditional volatilities for the portfolio and S&P500 based on the two GARCH models are plotted. Except for brief period around the year 2001, the conditional volatility of the portfolio is higher than the conditional volatility of the S&P500. A possible explanation for this is that in the portfolio there are not many stocks from the tech sector, which was the most volatile in the Dotcom bubble. If we compare Figure 6 to Figure 2 with the rolling standard deviations, a difference can be spotted. Based on Figure 2 the volatility was higher during the financial crisis 2007-08 than the Covid-19 induced crisis. Figure 6 tells the opposite story. The discrepancy could be explained by the fact that the stock market sell-off that and the subsequent rebound were among the fastest and the rolling standard deviation approach is not capable of capturing such fast dynamics.

The next step was the estimation of the FIGARCH (1,d,1) model for the same two return series. Here the measure of persistence is the parameter *d*, which is the memory parameter. The results are shown in Table 7.

*Table 7: FIGARCH (1,d,1) coefficients for the entire period*

| **S&P 500 returns** | | | | |
|---|---|---|---|---|
| Coefficients | Estimate | Standard error | T-statistic | P-value |
| Alpha | 0.031 | 0.053 | 0.60 | 0.5496 |
| Beta | 0.566 | 0.189 | 2.99 | 0.0027 |
| Memory parameter | 0.595 | 0.128 | 4.64 | 0.0000 |
| **Returns of the equally weighted portfolio** | | | | |
| Coefficients | Estimate | Standard error | T-statistic | P-value |
| Alpha | 0.055 | 0.018 | 3.04 | 0.0024 |
| Beta | 0.576 | 0.086 | 6.67 | 0.0000 |
| Memory parameter | 0.598 | 0.052 | 11.29 | 0.0000 |

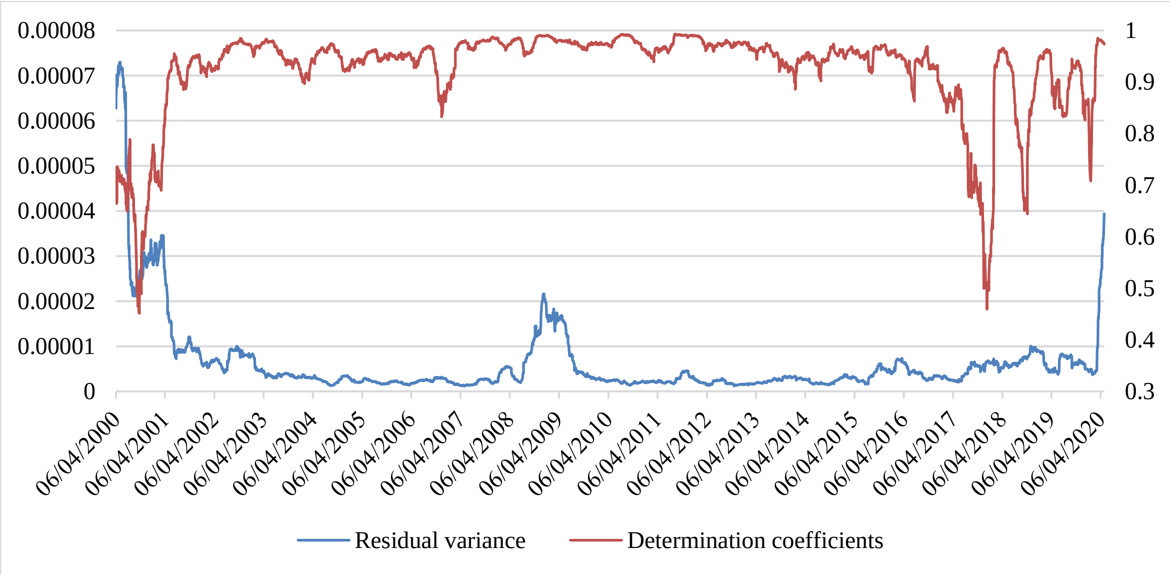
*Source: Own work.*

The memory parameters obtained with the FIGARCH model are similar to each other and both of them have the p-value lower than 0.001. The semi-parametric methods also yielded similar values of the memory parameter for the equally weighted portfolio and the S&P500. The only difference here is that the memory parameters from the FIGARCH exceeds ½, which would imply that both series are non-stationary with long memory properties, despite the fact that the KPSS and ADF tests offered the opposite evidence regarding stationarity.

For detecting the global structural breaks in the volatility, I decided to check for structural breaks in the variance of the returns in the S&P500. Because I am interested in structural breaks in the volatility, which is associated with the second moment of returns, structural breaks in variance must be examined. The S&P500 is among the three most followed U.S. stock indices, together with the Dow Jones Industrial Average and Nasdaq Composite index. However, due to the fact that the S&P500 has the most members and also members from the different sectors (Nasdaq is a tech heavy index and Dow Jones consists mostly from industrial companies) it is often thought as one of the best representation of the overall U.S. stock market. Consequently, I have decided that structural breaks in the variance of the S&P500 returns would be a good approximation of global structural breaks in the volatility. Global structure breaks are perceived as market-wide.

To determine the local structural breaks, I decided to estimate the residual variances from the rolling regressions with the explanatory variable being the return series for the S&P500 and the dependent variable being the returns of the equally weighted portfolio. The length of the rolling window is 66 trading days or approximately one quarter. The residual variances are the squared residual standard errors from these regressions (in residual standard error the correction term for the degrees of freedom is included but the effect of this is minimal, because the main interest lies in the structural breaks and not in the absolute size of the variances). The chosen window is based on the fact that each window should have enough observations in order that regressions will yield consistent and efficient estimates, but it also should not be too long so that not too many observations are lost. The logic behind this step is to eliminate the variability in the returns of the portfolio that can be explained by the variability of the S&P500 returns, which can be viewed as a systemic factor. The residual variance would than contain the portfolio specific variability, which is not associated with the variability in the S&P500. It is also important to point out that the stocks included in the portfolio are also included in the index and there is going to be some overlap.

*Figure 7: Residual variance from the rolling regressions and determination coefficients*




*Source: Own work.*

Figure 7 shows the residual variances and determination coefficients from the rolling regressions. The residual variances are the highest at the beginning of the period (this period starts in April 2000, because 66 observations are "lost" because of the rolling window approach). They then gradually decline and are low until the financial crisis of 2008-2009, when they increase again. The next spike does not happen until the beginning of 2020, when the Covid-19 epidemic erupted. The residual variances in the last couple of months are higher than the residual variance during the financial crisis in 2008-2009. It appears that every time a turmoil in the stock market occurs, the residual variances rise. In Figure 7, the determination coefficients from the rolling regressions are also plotted (on the right-hand side scale). For the most part of the last two decades, the variability of the S&P500 returns explains almost all of the variability in the returns of the portfolio. High determination coefficient is a common occurrence when regressing one time series on another. However, there are some years when this relationship seems to break down (the first two years of the period and the last two years of the period). Especially in the last two years the determination coefficients vary a lot, ranging from 0.48 to mid-0.9 with a couple of up and down moves. A potential reason for this could lie in the fact that in the last couple of years the relative significance of the largest stocks (measured in market capitalization) included in the index is becoming bigger. Just in 2020, the share of the 5 largest companies (Microsoft, Apple, Google, Amazon and Facebook) in the S&P500 increased from 17.5% in January (Levy & Konish, 2020) to over 20% at the end of April (Fox, 2020). This also indicates that the movement in stock prices of these 5 companies can swing the entire S&P500. Similar trend was happening during the Dotcom bubble build up, when coincidentally the determination coefficients were also far lower than during most of the period.

The described approach for detecting structural breaks in section 3 identified four breakpoints in the variance of the S&P500 and four breakpoints in the residual variances from the rolling regressions, which would imply 5 different regimes for each as shown in Table 8. But because the final breaks in both cases occurred almost at the end of the period, the fifth regime is not included in the estimations of the memory parameter for the subperiods as it would be too short to obtain efficient and consistent estimates.

*Table 8: Structural breaks in the variance of S&P 500 returns and in the residual variances*

| Variance of S&P 500 returns | Residual variances |
|---|---|
| 25.7.2003 | 23.5.2001 |
| 23.7.2007 | 7.5.2004 |
| 20.12.2011 | 7.8.2015 |
| 21.2.2020 | 13.3.2020 |

*Source: Own work.*

The first structural break in the variance of S&P500 returns was identified at the end of July in 2003 as volatility gradually decreased after the Dotcom bubble period with elevated volatility. The second structural break occurred in the summer of 2007, several months before the recession in 2008. However, by mid-2007 it became apparent that the U.S housing market was in a bubble as default rates on mortgage payments skyrocketed. The next structural break was in December 2011, which was during the European sovereign debt crisis, which also had spill-over effects on the U.S stock markets. The final structural break in the S&P500 return variance was in February 2020 as the spread of the Covid-19 was becoming more imminent, which reduced the confidence of stock market investors. The structural breaks in the residual variance do not occur in similar periods than the structural in the S&P500, apart from the last structural break. Based on this, one might argue that the Covid-19 pandemic caused a such strong disruption in the stock market that even after accounting for the systemic factor a structural break in the residual variances was detected. The first structural break in the residual variances was dated at the end of May 2001 during the Dotcom bubble. During the years 2015 and 2016 with the beginning in June 2015 there was a decline in Chinese stock markets, which lead to Chinese devaluation of yuan, which rattled the investors worldwide and also had an impact on the U.S. stock market, which may help explaining the structural break. It is also interesting that there are 3 structural breaks in the variance of the S&P500 in the first 11 years of the period and the last one is only a couple of months before the end of the period.

This is another confirmation that the 2010s were one of the most favourable decades in history, during which the longest bull market in the U.S. occurred. The common link between all of these identified structural breaks is that there was an event (or series of events) that caused that the parameters that defined both variances changed. In most cases, these events can be associated with macroeconomic developments such as the Great financial crisis. Because the parameters that defined the variances changed, this also implies that the feature of these variances changed. Therefore, the trading strategies and modelling approaches have to be accommodated for such changes in order to still reflect the reality.

After obtaining the dates of structural breaks, I then estimated the memory parameter in the subperiods using both previously used semi-parametric methods and the FIGARCH model. The memory parameter was estimated for 4 different subperiods for each of the two return series. The S&P500 return series is divided according to the structural breaks in the variance of the S&P500 and the return series of the equally weighted portfolio is divided according to the breaks in the residual variances. This means that the first regime starts with the first observation in the whole period and ends at the first structural break, the second one starts the day after the first structural break and ends with the second structural break and so on… As I have already mention, the periods after the last structural break is disregarded as they would contain only a couple dozen of observations.

*Table 9: Exact local Whittle estimator for the subperiods in the S&P 500 returns*

| **Estimate** | **Regime 1** | **Regime 2** | **Regime 3** | **Regime 4** |
|---|---|---|---|---|
| Memory parameter | 0.329 | 0.218 | 0.694 | 0.274 |
| Standard error | 0.065 | 0.062 | 0.060 | 0.051 |
| T-statistic | 5.05 | 3.49 | 11.45 | 5.42 |
| P-value | <0.001 | 0.005 | <0.001 | <0.001 |

*Source: Own work.*

*Table 10: GPH estimator for the subperiods in the S&P 500 returns*

| **Estimate** | **Regime 1** | **Regime 2** | **Regime 3** | **Regime 4** |
|---|---|---|---|---|
| Memory parameter | 0.340 | 0.192 | 0.712 | 0.188 |
| Standard error | 0.129 | 0.079 | 0.066 | 0.064 |
| T-statistic | 2.62 | 2.42 | 10.79 | 2.95 |
| P-value | 0.009 | 0.0157 | <0.001 | 0.0032 |

*Source: Own work.*

All of the estimated memory parameters from Tables 9 and 10 for the squared returns of the S&P 500 are statistically significant and again the values obtained with the exact local Whittle estimator are similar to those obtained with the GPH estimator. In the first regime the memory parameter is similar to the one from the whole period. In the second and last sub period, the memory parameters are lower, which implies that between 2003 and 2007 and 2011 and 2020 the degree of persistency in volatility was lower than in the overall period. In the third regime, the memory parameter is substantially higher and even implies non-stationarity. This coincides with the period between 2007 and 2011 with the financial crisis and sovereign debt crisis, which was an extremely volatile period with frequent jumps and falls in the stock market.

*Table 11: Exact local Whittle estimator for the subperiods in the portfolio returns*

| **Estimate** | **Regime 1** | **Regime 2** | **Regime 3** | **Regime 4** |
|---|---|---|---|---|
| Memory parameter | 0.185 | 0.421 | 0.552 | -0.289 |
| Standard error | 0.086 | 0.069 | 0.046 | 0.060 |
| T-statistic | 2.156 | 6.12 | 11.99 | -4.79 |
| P-value | 0.0316 | <0.001 | <0.001 | <0.001 |

*Source: Own work.*

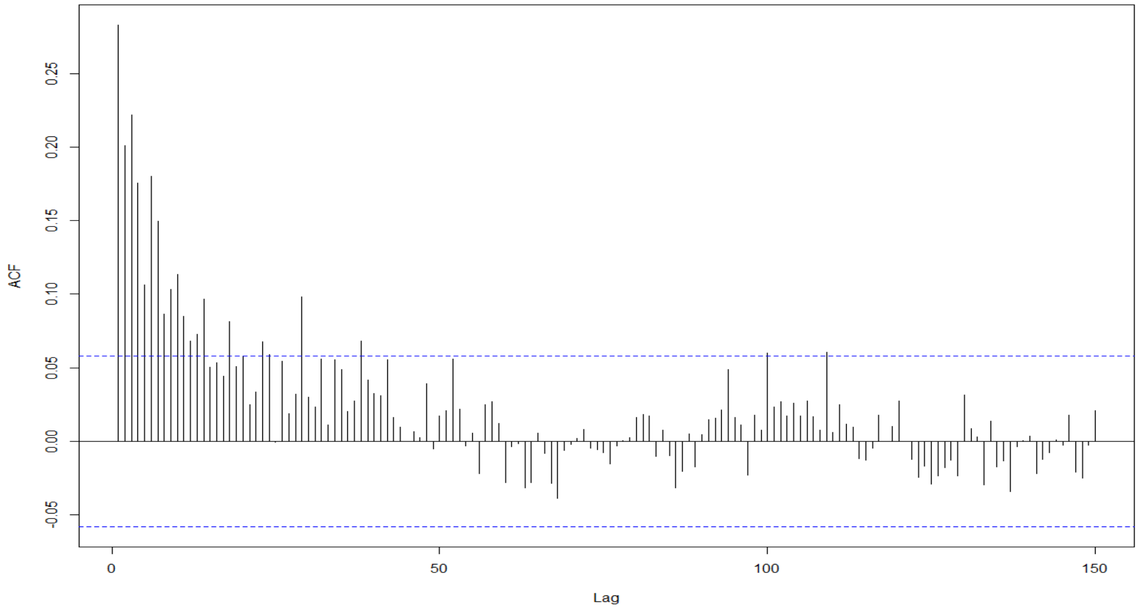
*Table 12: GPH estimator for the subperiods in the portfolio returns*

| **Estimate** | **Regime 1** | **Regime 2** | **Regime 3** | **Regime 4** |
|---|---|---|---|---|
| Memory parameter | 0.262 | 0.542 | 0.598 | 0.242 |
| Standard error | 0.111 | 0.094 | 0.054 | 0.021 |
| T-statistic | 2.36 | 5.78 | 11.08 | 11.60 |
| P-value | 0.0186 | <0.001 | <0.001 | <0.001 |

*Source: Own work.*

The squared portfolio returns produced statistically significant memory parameters for all of the returns as presented in Tables 11 and 12. However, the exact local Whittle estimator gives a negative memory for the last regime, from July 2015 to March 2020, which implies that the volatility of the portfolio exhibits anti-persistence, which is not in line with any of the previous results. With the GPH estimator the obtained memory parameter equals to 0.242 within the same subperiod. The difference is even more interesting, because for the entire period and all of the other subperiods the methods yield similar results and that this is the only instance where the memory parameter is negative. Additionally, the bandwidth parameter is equal as in other cases. A potential explanation for this discrepancy would be that, the GPH estimator estimates the memory parameter in the frequency domain and the local Whittle estimator is based in the time domain and that this leads to the opposite implications.

*Figure 8: ACF of squared returns of the equally weighted portfolio for the last regime from March 2015 to August 2020*




*Source: Own work.*

The autocorrelations of the squared portfolio returns for the last regime from Figure 8 exhibit no distinct hyperbolic decay as observed in Figure 4. In fact, most autocorrelations at larger lags appear to be negative, although not significant. This implies that increase in volatility in that period would actually lead to reduction in volatility in the future.

For the other three regimes the memory parameters are positive, statistically significant and similar for both estimators. For the second and third regime, the latter one includes the period of the financial crisis, the estimated of values of the memory exceed the one for the entire period, which means that the portfolio's volatility exhibited stronger persistency.

For the last step in the empirical part, I estimated the memory parameter using the FIGARCH (1,d,1) model in the return series of the S&P500 and equally weighted portfolio. The results are in Tables 13 and 14.

*Table 13: Estimates of the memory parameter from the FIGARCH model for the subperiods in the S&P 500 returns*

| **Estimate** | **Regime 1** | **Regime 2** | **Regime 3** | **Regime 4** |
|---|---|---|---|---|
| Memory parameter | 0.377 | 0.2715 | 0.716 | 0.551 |
| Standard error | 0.093 | 0.044 | 0.112 | 0.155 |
| T-statistic | 4.04 | 6.21 | 6.38 | 3.56 |
| P-value | <0.001 | <0.001 | <0.001 | <0.001 |

*Source: Own work.*

The memory parameters from the FIGARCH model fitted to the subperiod returns of the S&P500 are all highly statistically significant. Identically to the semi-parametric estimators the highest memory parameter is found in Regime 3, which is also higher than the value obtained with the FIGARCH for the entire return series of S&P500. The memory parameter for the fourth regime in the S&P500 is a lot higher than suggested by the semi parametric estimators and it exceeds ½. In the first two regimes the persistence in volatility appears to be lower than in the in the last two. This implies that in the latter two regimes the realized volatility on a certain day had stronger effect on the future volatility meaning that a spike in volatility lasted for a longer period of time.

*Table 14: Estimates of the memory parameter from the FIGARCH model for the subperiods in the returns of the portfolio*

| **Estimate** | **Regime 1** | **Regime 2** | **Regime 3** | **Regime 4** |
|---|---|---|---|---|
| Memory parameter | 0.999 | 0.477 | 0.679 | 0.598 |
| Standard error | 0.226 | 0.127 | 0.245 | 0.848 |
| T-statistic | 4.42 | 3.76 | 2.77 | 0.71 |
| P-value | <0.001 | <0.001 | 0.006 | 0.4807 |

*Source: Own work.*

The FIGARCH results for the subperiods of the equally weighted portfolio are much more erratic. To begin with, the memory parameter for the first regime is equal to 0.999, which basically implies infinite persistence and that shocks in volatility have permanent effect on the level of volatility in that period. The value of the memory parameter also implies the presence of a unit root and stationarity of the first order in the returns of the portfolio from January 2000 to May 2001. Secondly, the memory parameter for the last regime is statistically insignificant, with *p*-value equal to 0.48, which implies that persistence is not a statistically significant feature of volatility of the portfolio returns from August 2015 to March 2020. This is consistent with the mixed results obtained with the semi-parametric estimators for the last regime of the portfolio. Combined with the autocorrelations plotted in Figure 8, the evidence suggest that the long memory properties of the portfolio's volatility may not play an important role in from August 2015 to March 2020. The memory parameters for the second and third regime are statistically significant and similar to those obtained with the semi-parametric methods. Similar to the case with S&P500, the regime 3, which includes the financial crisis of 2007-08 and the sovereign debt crisis, has the memory parameter above the one estimated for the whole period, which means that portfolio volatility exhibited stronger persistence in that period.

Estimating long memory in volatility after accounting for structural breaks, showed that the structural breaks do affect the degree persistency in volatility. For the volatility of the S&P500, persistence in volatility remained a statistically significant feature for all the subperiods, however the degree of it differed from subperiod to subperiod. This means that shocks to the S&P500 volatility in the analysed period had effects on the future realizations of volatility and that an increase of volatility in a certain period lead to higher volatility in the following periods. For the volatility of the equally weighted portfolio the results are less clear, especially for the period from August 2015 to March 2020, where the estimates of the degree of persistency in volatility based on different methods differ and cast doubt on the statistical significance. Another interesting finding from this part is that the regime (in both cases this was the third one), which included the period of the financial crisis of 07-08 and its aftermath, had higher persistence in volatility than the whole period independent from the method of estimation that indicates that the shocks to volatility during that time had longer-lasting effects compared to the entire period. Additionally, the value of the memory parameters is generally higher when obtained with the FIGARCH model then with the semi-parametric estimators. Based on this, it appears that application of the parametric methods leads to higher values of the memory parameter and consequently persistence in volatility. The last finding of my master thesis would be, that indexation does not lead to spurious persistence. Based on my analysis, persistence in volatility is a statistically significant feature of individual stocks and S&P 500.

## CONCLUSION

Over the year, several patterns and features of volatility has been documented. Persistence in volatility was studied since Mandelbrot (1963) first mentioned the observed clustering in volatility. Researchers have studied persistence from a more theoretical perspective and also from a more empirical standpoint. For example, the observation of long memory in volatility prompted Baillie (1996) to extend the notion of integration to allow the order of stationarity to be a fraction, which offers theoretical foundation for the presence of persistence in volatility. Researchers also tried to provide economic explanations and models, which would provide economic rationale for its presence. The two main groups of explanations can be divided in exogeneous and endogenous theories. The information clustering theory belongs to the first, which explains volatility clustering with rate of arrival of information, which itself exhibits clustering. The endogenous group of theories explains persistence through heterogeneity of investors, which have different features that affect their portfolio allocation. The time-varying structure of investors leads to regime switching, which then causes long memory in volatility. Among the most significant empirical findings is the fact that structural breaks affect the degree of persistence and that structural breaks need to be taken into account when estimating the memory parameter as shown (among others) by Granger and Hyung (2004). This topic was also the main focus in my master thesis.

Based on various methods for estimating the memory parameter, which is a measure of persistency, my analysis showed that persistence is a statistically significant feature of the volatility of the S&P500, 50 chosen stocks and an equally weighted portfolio constructed from those stocks. The applied method for identifying structural breaks showed that there are 4 structural breaks present in the variance of the S&P500 returns and in the residual variances that were obtained with regressing the returns of the portfolio on the S&P500

returns. Most of the structural breaks is linked to the periods with increased uncertainty and economic turmoil such as the Dotcom bubble, financial recession in 2007-08, sovereign debt crisis and the Covid-19 pandemic. Accounting for structural breaks does not affect the statistical significance of persistency in volatility for the S&P500, while for the portfolio there is a period from August 2015 to March 2020, when the analysis does not offer evidence that the persistence in the portfolio's volatility is important. Despite the fact, that the memory parameters remain statistically significant in 7 out of the 8 sub periods for the two time series, the degree of persistence varies in a wide range between the subperiods, which indicates that it is important to account for structural breaks when estimating persistency in volatility. The obtained memory parameters for the whole analysed period indicate that persistence in volatility is higher for the S&P500, but when it comes to the divided periods there is no clear pattern. There are some periods when persistence appears to be higher in the S&P500 and in others in the portfolio's volatility. If the structural breaks in the volatility of the S&P500 represent global structural breaks and the breaks in residual variances are considered as local structural breaks, then when it comes to the degree of persistency no clear pattern can be established. However, after the global structural breaks all the memory parameters remained highly statistically significant, while this was not the case with local structural breaks. This would indicate that there exists some difference on how global and local structural breaks affect the memory parameters, which are the measure of persistency in volatility. Nevertheless, in order to offer concrete evidence and meaningful interpretations larger datasets would need to be studied.

Practical findings of my master thesis indicate that there exist long range dependencies in the volatility meaning that today's volatility is positively correlated with volatility in the future. This means that past and current realizations of volatility can be used in predicting future volatility and that periods of high volatility are followed by periods with high volatility and vice versa. This also uncovers that there are some underlying mechanisms in the asset markets that allow the volatilities from different times to be connected. Because the returns are generally not serially correlated at longer time horizon, these dependencies are not linear. However, dependencies are found in the volatilities, which are linked to the second moments, this also means that these dependencies are non-linear. The question what exactly these mechanisms are and how do they operate arises. It also means that there are some aspects of the financial markets that we do not fully understand yet.

## REFERENCE LIST

1. Aït-Sahalia, Y., Fan, J. & Li, Y. (2013). The leverage effect puzzle: Disentangling sources of bias at high frequency. *Journal of Financial Economics*, *109*(1), 224–249.
2. Andersen, T. G. & Bollerslev, T. (1997). Intraday periodicity and volatility persistence in financial markets. *Journal of Empirical Finance*, *4*(2–3), 115–158.
3. Andrei, D. & Hasler, M. (2015). Investor Attention and Stock Market Volatility. Review of Financial Studies. *Society for Financial Studies*, *28*(1), 33–72.

4. Andreou, E. & Ghysels, E. (2006). Structural Breaks in Financial Time Series. *SSRN Electronic Journal, 23*, 90–94.
5. Aue, A. & Horváth, L. (2012). Structural breaks in time series. *Journal of Time Series Analysis*, *34*(1), 1–16.
6. Baillie, R. (1996). Long memory processes and fractional integration in econometrics. *Journal of Econometrics, 73*(1), 5–59.
7. Baillie, R., Bollerslev, T. & Mikkelsen, H. O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, 74*(1), 3–30.
8. Banerjee, A. & Urga, G. (2005). Modelling structural breaks, long memory and stock market volatility: an overview. *Journal of Econometrics, 129*(1–2), 1–34.
9. Berger, D., Chaboud, A. & Hjalmarsson, E. (2009). What drives volatility persistence in the foreign exchange market? *Journal of Financial Economics*, *94*(2), 192–213.
10. Black, F. (1976). Studies of stock price volatility changes, proceedings of the 1976 meetings of the business and economic statistics section. In *American Statistical association* (pp. 177–191). Washington: The Association.
11. Brooks, C. & Schopohl, L. (2018). Topics and trends in finance research: What is published, who publishes it and what gets cited?. *The British Accounting Review, 50*(6), 615–637.
12. Burke, S. P. & Hunter, J. (2005). *Modelling Non-Stationary Economic Time Series: A Multivariate Approach*. London: Palgrave Macmillan.
13. Busch, M. & Sibbertsen, P. (2018). An Overview of Modified Semiparametric Memory Estimation Methods. *Econometrics, 6*(1), 1–21.
14. CBOE. (2020). VIX. Retrieved on 8 April 2020 from http://www.cboe.com/vix
15. Cheung, Y. W. (1993). Long Memory in Foreign-Exchange Rates. *Journal of Business & Economic Statistics. 11*(1), 93–101.
16. Chou, R. (1988). Volatility Persistence and Stock Valuations: Some Empirical Evidence Using GARCH. *Journal of Applied Econometrics*, *3*(4), 279–294.
17. Clements, M. P. & Hendry, D. F. (2006). Chapter 12 Forecasting with Breaks. *Handbook of Economic Forecasting, 1,* 605–657).
18. Cont, R. (2004). *Long range dependence in financial markets. Fractals in Engineering.* London: Springer.
19. De Fontnouvelle, P. (2000). Information dynamics in financial markets. *Macroeconomic Dynamics, 4*(2), 139–169.
20. De Silva, H., McMurran, G. M. & Miller, M. N. (2017). Diversification and the Volatility Risk Premium. In *Factor Investing* (pp. 365–387). London: Springer.
21. Ding, Z., Granger, C. & Engle, R. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance, 1*(1), 83–106.
22. Engle, R. F. & Patton, A. J. (2001). What Good Is a Volatility Model? *Quantitative Finance*, *1*(2), 237–245.
23. Fox, M. (2020). 5 companies now make up 20% of the S&P 500. Here's why Goldman Sachs says that's a bad signal for future market returns. *Markets Insider*. Retrieved on 21

June 2020 from https://markets.businessinsider.com/news/stocks/sp500-concentration-large-cap-bad-sign-future-returns-effect-market-2020-4-1029133505#
24. Franck, T. (2020, 27 February). Here's how long stock market corrections last and how bad they can get. *CNBC*. Retrieved on 21 June 2020 from https://www.cnbc.com/2020/02/27/heres-how-long-stock-market-corrections-last-and-how-bad-they-can-get.html
25. Garcia, R. & Perron, P. (1996). An Analysis of the Real Interest Rate Under Regime Shifts. *The Review of Economics and Statistics*, *78*(1), 111–123.
26. Gil-Alana, L. A. (2007). Fractional integration and structural breaks at unknown periods of time. *Journal of Time Series Analysis*, *29*(1), 163–185.
27. Giraitis, L., Kokoszka, P., Leipus, R. & Teyssière, G. (2003). Rescaled variance and related tests for long memory in volatility and levels. *Journal of econometrics*, *112*(2), 265–294.
28. Granger, C. W. J. & Hyung, N. (2004). Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns. *Journal of Empirical Finance*, *11*(3), 399–421.
29. Granger, C. W. J. & Newbold, P. (1974). Spurious regressions in econometrics. *Journal of Econometrics*, *2*(2), 111–120.
30. Guegan, D. (2005). How can we define the concept of long memory? An econometric survey. *Econometric Reviews*, *24*(2), 113–149.
31. Hillebrand, E. & Medeiros, M. C. (2008). Estimating and forecasting GARCH models in the presence of structural breaks and regime switches. In *Forecasting in the Presence of Structural Breaks and Model Uncertainty* (pp. 303–327). Bingley: Emerald Group Publishing Limited.
32. Huang, Z., Liu, H. & Wang, T. (2016). Modeling long memory volatility using realized measures of volatility: A realized HAR GARCH model. *Economic Modelling*, *52*, 812–821.
33. Implied Volatility-IV. (2020). In *Investopedia*. Retrieved on 17 July 2020 from https://www.investopedia.com/terms/i/iv.asp
34. Killick, R. & Eckley, I. (2014). changepoint: An R Package for Changepoint Analysis. *Journal of Statistical Software*, *58*(3), 1–19.
35. Kim, J. H. & Ji, P. I. (2011). Mean-reversion in international real interest rates. *Economic Modelling*, *28*(4), 1959–1966.
36. Lamoureux, C. G & Lastrapes, W., D. (1990). Persistence in Variance, Structural Change, and the GARCH Model. *Journal of Business & Economic Statistics*, *8*(2), 225–234.
37. Levy, A. & Konish, L. (2020, 28 January). The five biggest tech companies now make up 17.5% of the S&P 500 — here's how to protect yourself. *CNBC*. Retrieved on 21.06.2020 from: https://www.cnbc.com/2020/01/28/sp-500-dominated-by-apple-microsoft-alphabet-amazon-facebook.html
38. Linton, O. (2019). *Financial Econometrics: Models and Methods*. Cambridge: Cambridge University Press.

39. Lo, A. W. (1989). *Long-term memory in stock market prices, Working papers*. Cambridge: Massachusetts Institute of Technology (MIT), Sloan School of Management.
40. Lo, A. W. (1991). Long-Term Memory in Stock Market Prices. *Econometrica, 59*(5), 1279–1313.
41. Lo, A. W., & MacKinlay, A. C. (1988). Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test. *Review of Financial Studies*, 1(1), 41–66.
42. Lobato, I. N. & Savin, N. E. (1998). Real and Spurious Long-Memory Properties of Stock-Market Data. *Journal of Business & Economic Statistics, 16*(3), 261–268.
43. Malik, F., Ewing, B. & Payne, J. (2005). Measuring volatility persistence in the presence of sudden changes in the variance of Canadian stock returns. *Canadian Journal of Economics. Canadian Economics Association, 38*(3), 1037–1056.
44. Mandelbrot, B. B. (1963). The variation of certain speculative prices. *Journal of Business, 36*, 392–417.
45. McMillan, D. G. & Ruiz, I. (2009). Volatility persistence, long memory and time-varying unconditional mean: Evidence from 10 equity indices. *The Quarterly Review of Economics and Finance*, *49*(2), 578–595.
46. McQueen, G. & Vorkink, K. (2004). Whence GARCH? A Preference-Based Explanation for Conditional Volatility. *Review of Financial Studies, 17*(4), 915–949.
47. Mikosch, T. & Starica, C. (2005). Nonstationarities in financial time series, the long range dependence and the IGARCH effect. *Review of Economics and Statistics, 86*(1), 378–390.
48. Mills, T. C. (2019). *Applied Time Series Analysis - A Practical Guide to Modeling and Forecasting.* London: Academic Press.
49. Müller, U. A., Dacorogna, M. M., Davé, R. D., Olsen, R. B., Pictet, O. V. & Von Weizsäcker, J. E. (1997). Volatilities of different time resolutions—analyzing the dynamics of market components. *Journal of Empirical Finance*, *4*(2–3), 213–239.
50. Nau, R. (2020). *Statistical forecasting: notes on regression and time series analysis.* Retrieved on 16 Januar 2020 from https://people.duke.edu/~rnau/411diff.htm
51. Neely, C. & Rapach, D. (2008). Real Interest Rate Persistence: Evidence and Implications. *Federal Reserve Bank of St. Louis Review, 90*(6), 609–642.
52. Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, *59*(2), 347–370.
53. Nguyen, D. B. B., Prokopczuk, M. & Sibbertsen, P. (2020). The memory of stock return volatility: Asset pricing implications. *Journal of Financial Markets*, *47*, 100487.
54. PennState. (2020). *Applied Time Series Analysis-Estimating the Spectral Density.* Retrieved on 1 May 2020 from https://online.stat.psu.edu/stat510/lesson/12/12.1
55. Perron, P. & Qu, Z. (2010). Long-Memory and Level Shifts in the Volatility of Stock Market Return Indices. *Journal of Business & Economic Statistics*, *28*(2), 275–290.
56. Pierse, R. G. (2020). *Financial Econometric Lecture 7: Long Memory and Fractional integration*. Retrieved on 19 Januar 2020 from http://rpierse.esy.es/rpierse/files/fe7.pdf

57. Poterba, J. M. & Summers, L. H. (1984). The Persistence of Volatility and Stock Market Fluctuations (September 1984). *NBER Working Paper No. w1462*. Cambridge; National Bureau of Economic Research.
58. Sadique, S. & Silvapulle, P. (2001). Long-term memory in stock market returns: international evidence. *International Journal of Finance & Economics, 6*(1), 59–67.
59. Schmitt, N. & Westerhoff, F. (2017). Herding behaviour and volatility clustering in financial markets. *Quantitative Finance, 17*(8), 1187–1203.
60. Shimotsu, K. & Phillips, P. C. B. (2004). Local Whittle estimation in nonstationary and unit root cases. *The Annals of Statistics*, *32*(2), 656–692.
61. Shimotsu, K. & Phillips, P. C. B. (2005). Exact local Whittle estimation of fractional integration. *The Annals of Statistics, 33*(4), 1890–1933.
62. Smith, D. R. (2008). Testing for structural breaks in GARCH models. *Applied Financial Economics, 18*(10), 845–862.
63. Stata. (2020). *Tests for structural breaks in time-series data*. Retrieved on 25 Januar 2020 from https://www.stata.com/features/overview/structural-breaks/
64. Tayefi, M. & Ramanathan, T. (2012). An overview of FIGARCH and related time series models. *Australian Journal of Statistics*, *41*(3), 175–196.
65. Teyssière, G. & Kirman, A. (2002). Microeconomic Models for Long-Memory in the Volatility of Financial Time Series, Universiteit van Amsterdam, Centre for Nonlinear Dynamics in Economics and Finance. *CeNDEF Workshop Papers, 5*(4), 281–302.
66. Volatility definition. (2019). In *Investopedia*. Retrieved on 17 July 2020 from https://www.investopedia.com/terms/v/volatility.asp
67. Voss, J. (2013). *Rescaled Range Analysis: A Method for Detecting Persistence, Randomness, or Mean Reversion in Financial Markets*. Retrieved on 25 Januar 2020 from https://blogs.cfainstitute.org/investor/2013/01/30/rescaled-range-analysis-a-method-for-detecting-persistence-randomness-or-mean-reversion-in-financial-markets/
68. Wenger, K., Leschinski, C. & Sibbertsen, P. (2018). A simple test on structural change in long-memory time series. *Economics Letters, 163*,

# APPENDICES

**Appendix 1: Slovenian summary of the Master's thesis.**

Vztrajnost volatilnosti je ena izmed najbolj preučevanih lastnosti volatilnosti, saj je bila prvič omenjena že v šestdesetih letih prejšnjega stoletja, ko je Mandelbrot (1963) na podlagi zgodovinskih podatkov delniških donosov opazil, da obdobjem z velikimi spremembami cen delnic prav tako sledijo obdobja z velikimi spremembami cen delnic. Zaradi pomembnih posledic, ki jih ima vztrajnost volatilnosti na napovedovanje volatilnosti v prihodnosti na podlagi zgodovinskih podatkov, je bila vztrajnost volatilnosti predmet številnih raziskav. Možnost napovedovanja volatilnosti na podlagi zgodovinskih podatkov je zanimiva tako za vlagatelje, ki lahko uporabijo preteke donose in gibanja cen sredstev za oblikovanje trgovalnih strategij v prihodnosti, kot tudi za akademike, saj vztrajnost volatilnosti potrjuje obstoj mehanizmov, ki omogočajo, da se sedanji šoki volatilnosti prenašajo v prihodnost.

Vztrajnost volatilnosti ni samo lastnost delniških trgov, ampak je bila povezana z različnimi vrstami sredstev, med drugim tudi deviznimi tečaji (Cheung, 1993), in prav tako tudi z makroekonomskimi spremenljivkami kot so realne obrestne mere (Neely and Rapach (2008). Pomembna ugotovitev empiričnih študij (Lamoureux & Lastrapes, 1990) je tudi, da lahko prisotnost strukturnih prelomov v volatilnosti vpliva na napačno zaznavo prisotnosti vztrajnosti volatilnosti, saj imajo stacionarne časovne vrste z vztrajnostjo v volatilnosti podobne vzorce obnašanja kot nestacionarne časovne vrste brez vztrajnosti v volatilnosti in statistične metode, ki se uporabljajo za merjenje vztrajnosti v volatilnosti ne morejo natančno razločiti med temi časovnimi vrstami, kar lahko pripelje do zmotnih zaključkov. Poterba in Summers (1984) sta tudi formalno dokazala, da bi cene delnice, katerih volatilnost izkazuje vztrajnost, morale vsebovati pozitiven pribitek za tveganje, saj imajo takšne delnice višji nivo volatilnosti v primerjavi z delnicami, katerih volatilnost ni vztrajna.

Glavni namen magistrskega dela je preučiti ali je vztrajnost statistično značilna lastnost volatilnosti delniškega indeksa S&P 500 in 50 naključno izbranih delnic iz tega indeksa v obdobju od januarja 2000 do maja 2020. To bo pripomoglo k spoznavanju vzorcev in lastnosti volatilnosti in bo privedlo do večjega razumevanja delniških trgov. Prav tako bo raziskano tudi kako strukturni prelomi vplivajo na vztrajnost volatilnosti in ali imajo globalni (strukturni prelomi v volatilnosti indeksa) in lokalni (strukturni prelomi v volatilnosti posameznih delnic) različen vpliv na vztrajnost. Prvi del magistrske naloge zajema teoretični del, kjer je predstavljena definicija vztrajnosti volatilnosti, njeno ekonometrično ozadje, vpliv na vrednotenje sredstev, ekonomske razlage, metode za merjenje in pregled že obstoječe literature. Drugi del magistrske naloge predstavlja empirična študija in njeni rezultati.

Glavna ugotovitev empiričnega dela je ta, da je vztrajnost statistično značilna lastnost vztrajnosti indeksa S&P 500 in 50 naključno izbranih delnic iz tega indeksa v obdobju od januarja 2000 do maja 2020. Upoštevanje strukturnih prelomov v volatilnosti indeksa in posameznih delnic ne vpliva na statistično značilnost, a po drugi strani vpliva na velikost vztrajnosti, saj le-ta variira skozi posamezna obdobja določena na podlagi strukturnih prelomov. Ena izmed ugotovitev je tudi ta, da so ocene vztrajnosti volatilnosti za indeks in posamezne delnice najvišje v obdobju, ki zajemo finančno krizo 2007–2008. Na podlagi rezultatov empirične analize, lahko zaključim, da se lahko pretekle in sedanje realizacije volatilnosti indeksa S&P 500 in njegovih komponent, uporabijo za napovedovanje volatilnosti le-teh v prihodnosti in da na delniških trgih obstajajo mehanizmi, ki omogočajo da se šoki v volatilnosti prenašajo skozi čas.