UNIVERSITY OF LJUBLJANA
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MASTER'S THESIS

CAPITAL BUDGETING DECISION-MAKING IN TELECOM SECTOR USING REAL OPTION ANALYSIS

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1.1 Problem description

Implementing risks techniques in capital budgeting decisions is becoming increasingly more important, especially concerning recourses that are getting scarcer while competition is becoming more intense. This is even more pronounced in the current telecom industry environment, where management decisions for large expenditures are made in uncertainty related to the advanced technology implementation, under regulated regimes, changing business models, service rates and cost levels (EURESCOM, 1999 p. iii). In search for additional revenue, large incumbent telecom operators enter areas where they are no longer market leaders[1]. In such ever evolving business environment, the need for additional information related to uncertainty is of a major importance for decision makers, as decisions made today will definitely impact the future of the firm.

Capital budgeting decisions in Makedonski Telekom (hereinafter: MKT) are made according to the Discounted Cash Flow (hereinafter: DCF) methodology, where Net Present Value (hereinafter: NPV), discounted payback period and Internal Rate of Return (hereinafter: IRR) are estimated. Further, for major investments (projects) risk estimation, a scenario analysis is made where standard deviation and probability distribution of the NPV are evaluated to serve as an indication of the project's riskiness. Multiple alternatives are analyzed with a Decision Tree Analysis (hereinafter: DTA). Even though scenario analysis and simulations increase the quality of the information presented to decision makers, this type of analysis does not indicate what might be the optimal implementation strategy in terms of timing. Also, DTA is a powerful tool for evaluating investments, when these are subject to firm – specific or private risks[2]; however, there is little direction what discount rate should be used inside the tree (Brigham & Ehrhardt, 2005, p.433; notwithstanding, Amram, 2002, Borrison, 2003, and others use risk free rate of return).

Given the fact that DCF analysis was primarily used and invented for evaluating financial assets, which are passive investments (Brigham, 2005, p.434), the question for adequacy is automatically raised. Namely, financial assets are considered static, because once purchased there are no possibilities for influencing and managing the desired objective.

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[1] In general Entertainment industry – TV, on-line games, internet content providers.

[2] The usual classification in finance theory divides risks into either market (systematic) or unique (diversifiable). The former, systematic risks are treated under risk neutral probabilities, while the latter according to subjective probabilities, when valuing the investment. See: James Smith and Robert Nau (1995). "Valuing risky projects: Option pricing theory and decision analysis" for "integrated approach" between DTA and option pricing.

Real assets, on the other hand, are different in terms of their dynamics as they can be managed by taking actions that could influence the objectives. In other words, the management has options to influence the project and thus alter the outcome of the investment.

Managerial flexibility or active management refers to the management's opportunity to take future actions as a response to the changing business environment but also to the internal environment of the firm. This opportunity to take action is in fact an **option** – a right, but not an obligation, which might change the outcome of the project, and therefore its value. In the quest to assessing the "true" project value, analysis of those options named **Real options**, as opposed to already known financial options, is of a major interest among academicians, practitioners and regulators.

Since the eighties when the term Real Options was coined by Stewart Myers, it is only in the last 10 – 15 years that this subject has received greater interest. This is due to the search for better tools in corporate finance, especially in capital budgeting decisions that are able to capture uncertainties but also recognize that there is an underling value in managerial ability to respond to those uncertainties (Alleman, 1999, p.XIII).

In the telecommunication industry, Real Option Analysis (hereinafter: ROA) emerged as a response to Regulators' policies, particularly during estimations of the Weighted Average Cost of Capital (hereinafter: WACC), employed in setting prices for network lease to competitors or alternative operators. The central role in ROA during consultation rounds with the telecom Regulators is their reversibility of the investments, characterized with sunk costs that are not accounted for in the WACC (Pindyck, 1990, 2004, 2008). Cash Flow (hereinafter: CF) constrains imposed by the Regulators, the inability to delay and abandon investments have significant costs, which lead to reduction in firms' valuations which in turn will lead to a reduction in economic welfare (Alleman and Rappaprot, 2002). There is now ample evidence that investment flexibility[3] has an economically significant impact on hurdle rates chosen by firms (Gurthrie, 2009, p.2), which are typically set above the WACC. This premium above the WACC can be attributed to the irreversibility of the firms' investment portfolio (Chirinko and Schaller, 2009). Also, there is evidence that access regulation, negatively affects both total telecom industry and individual operators, which hinders discretionary investment, hence the industry development (Grajek & Röller, 2010). However, we are all witnesses of the IT influence on the overall development of the economy, particularly the Internet. The slower development of the telecom sector would mean lowering the potential economic growth.

Modern literature of corporate finance acknowledges the limitations of the neoclassical approach of evaluating investments – projects under consideration, based on the DCF/NPV

[3] Guthrie, by "flexibility" is referring to ability to delay the investment.

rule, in that they cannot properly capture managerial flexibility to respond to the ever changing market environment by revising future decisions (Trigeorgis, 1999, p. 3). This holds true especially when attempt is made to incorporate the underlying external and internal risks and the management response to them, known as Risk management. When management creates a long-term strategic plan, they trace the road from where their firm currently is to where they envision the firm to be in the future. The assumption that this strategic path will take the exact routes and schedule is mindlessly accepted. As soon as the plan is made, the environment changes; the firm has learned something new and immediately undertakes some additional actions, while deliberately deferring others. This is where DCF fails to properly value the strategy, because it assumes that this predetermined path is going to be followed, regardless of how events develop. Moreover, the intense competitive environment drives a wedge between the planned and the actual results, while managerial investment decisions are made with explicit recognition that they may influence competitive reactions (Smith and Trigeorgis, 2004). ROA on the other hand, can accommodate both the uncertainty and the active decision making required for strategy to succeed (Luehrman, 1998).

The mechanic of DCF analysis involves forecasting the expected incremental net cash flows of the project and discounting them back to today with discount factor, usually the WACC. With this approach, risks are reflected in the discount rate, specifically market risk. The most widely used approach to arrive at the systematic/diversifiable risk is the Capital Asset Pricing Model (hereinafter: CAPM), which essentially involves estimating sensitivities of the project to the market, *i.e.* betas (β). Estimating β requires regressing the asset price and the market price as a whole or finding asset and market standard deviations, including their correlation. There are several problems with this approach. The first is that beta is estimated from past market performance, while its application throughout WACC is intended for the future project performance. The second is that the correlation between the asset and the market is diluting the discount factor especially with low correlated, but highly volatile asset because beta is measure of both the correlation and the volatility (McNulty & others, 2010). From a professional investors' perspective, such stock is used to hedge the equity portfolio. However, strategic investors and decision makers are more concerned with the total risk. The mainstream approach to capital budgeting focuses excessively on the special case where diversifiable risks do not influence the contribution of the project to the value of the firm, hence, ignores its impact on firms' total risk (Stulz, 1999). However, when frictions are present and as total risk increase, the ability of the firm to finance its growth opportunities becomes more costly, hence the WACC should include additional premium (Boyle P, 2002). The third is application of constant discount rate to non constant future cash flows. However, application of constant discount rate when uncertainties exist especially on more distant cash flows cannot appropriately account for the risks. The project riskiness is permanently evolving and changing during its useful life, hence the use of a constant discount rate is inappropriate. As mentioned previously, this is partially because the CF and WACC estimates are primarily based on recent past data and estimated to reflect future market conditions. Hence, they are somehow deterministic in a shorter timeframe horizon. However, as we forecast longer into the future the estimates became more unreliable and hard to estimate correctly. This implies that traditional techniques are best suited for shorter time horizons, particularly in the historical period and near term forecasted period usually not exceeding five years from now. Beyond the near term period, *i.e.* in the strategic period, deterministic valuation approach would not capture correctly the value of the project, hence probabilistic approaches would better do the job (Mun, 2006). An alternative approach would be to accommodate the risks in the expected cash flows, instead of in the WACC, by first transforming them into certainty equivalent cash flows, and then discounting them with risk free rate as to reflect the time – value of money (Schwartz and Trigeorgis, 2004). With this, it is recognized that the long term cash flows are not straight-line extrapolation but rather volatile with changes that might be mathematically described by some stochastic process. In addition, when managerial flexibility is incorporated into the asset valuation process, the value of the asset becomes contingent on the state of the market. Such projects are valued based on financial options valuation techniques due to the analogy of the pay-off characteristics (Amram, 2002, p.51-54; Damodaran P.26-27 & 53-54; Smit & Trigeorgis, 2004, p.11-12. etc.).

Companies in the telecom industry before 1990's were treated as public utility companies, often owned by governments, providing telephone services to the public. The business model was a simple one, with few revenue streams coming from one service – telephony or voice services with different rates for local, national and international calls. This was (is) enabled by a Public Switched Telephone Network or PSTN, which is a fixed line network where telephones are directly connected to a single telephone exchange or switch, usually via copper pair (cables). Copper pairs from the customer premises are concentrated into cables that are connected to the main distribution frame which is located in the central offices of the telecom provider and connected to the local switch. This first part of the layered architecture[4] is called access network, or in top down view (from the central office premises) it is called the last mile. Also, in regulatory context, this part of the network is called a local loop[5], because customers can only communicate within the local exchange where they are connected. In order for customers to be able to communicate outside their local loop, local exchanges are interconnected through transit exchanges (long distance) and international gateway exchanges, which handle national and international calls (traffic), respectively. The former part of the network that connects the national traffic is called backhaul, backbone network, or in layered architecture terms, transport network. Multiple network like fix and mobile or incumbents and competitors must be interconnected in order for customers to be able to communicate across those networks.

[4] The Layered architecture divides telecommunication network into parts representing different roles or functions.

[5] The physical wire connection between the local exchange and the customer is known as a "local loop", and is owned by the incumbent local exchange carrier or operator with SMP.

In general, three things reshaped the telecom business model – the commercialization of the Internet, particularly the World Wide Web, the invention of wireless networks, above all the mobile networks, and regulatory influence, aimed at liberalization of the industry. The latter divided the incumbents' public network into network elements, so as to bring competition by giving new entrants in the industry – competitive or alternative operators, access to customers. The regulatory process of allowing multiple telecommunications operators to use connections from the telephone exchange to the customers' premises is called unbundling of local loop[6].

Commercialization of the Internet during 1990's brought a new revenue stream to telecoms. At first, the service didn't require much changes in the existing network, due to low capacity demand (speed). However, as the content on the web increased, the demand for speed grew, thus the network had to be reshaped[7]. This, coupled with technology innovations, has enabled high speed, broadband services; hence, today voice can be transmitted over internet protocol ("IP"), including digital television (IPTV). The demand for bandwidth, lead to replacement of the copper network that has limitations, with superior fiber optics network. Nowadays, we are talking about new generation networks, where legacy PSTN are replaced with digital IP based networks. Also, in 1990's, the second generation of mobile phones system emerged, based on digital transmission as compared to the first generation of mobile phones. With that, new telecom operators focused on the mobile segment, while incumbents fixed line operators that had invested in mobile networks became combined operators. The demand for mobile services, particularly voice, was such that now in some developed, and even emerging countries, mobile penetration is well over 100%. However, development of mobile technology, especially the third generation of mobile phones/networks, enabled additional services such as: mobile internet, mobile TV, mobile payment, navigation services and other, all of them based on IP. Currently, the latest technology called Long Term Evolution (hereinafter: LTE), or fourth generation (4G) of mobile networks is being put into service in developed countries, aiming to replace the third generation (3G).

The implications of the above specified for telecom operators is that: **first**, large capital outlays are needed more frequently in order to maintain competitiveness. **Second**, broad flexibility is delivered by technology convergence, whereby the same services can be

[6] The individual parts of a network that the local, dominant telecom companies are required to share with their competitors. In U.S. the sharing requirement was made in the 1996 Telecom Act, which said that the local, dominant telecom companies are required to offer the individual parts of their network to competitors. The implementation of Local Loop Unbundling is a requirement of European Union policy on competition in the telecommunications sector and has been introduced, at various stages of development, in all member states.

[7] Practically, this means that the distance from customer premises to the central office (local exchange), has to be smaller in order to gain wider bandwidth and with that connection speed in case of copper network.

delivered through IP wireless or wire line networks. **Third**, entry barriers are diminished by regulators, hence competitors are emerging as network owners or lease-holders. And **finally**, new opportunities from various services delivered are establishing growth potential. In other words, the telecom business has changed dramatically. However, as almost in every industry; the main capital allocation tool remained the same – DCF/NPV analysis.

When do real options exist? As mentioned previously, when: the project payoff is uncertain, management can influence its outcome has significant flexibility, it is a long term capital intensive project, has proprietary rights. When projects are divisible in terms of time - implementation phases, the projects' initial or any other subsequent phase could be postponed in order to obtain more information about the uncertainty. This is so a called **option to differ** or **"wait and see",** which can be quite valuable if the uncertainty can be diminished by waiting (Trigeorgis, 1999, p.6, Brigham & Ehrhardt 2005, p.421). As most strategic projects in the Telecom Industry are long term projects, and can be characterized by the aforementioned specifics, ROA might be the tool that is needed for better recourse allocations. **Growth options**, especially the opportunity to add new products might be of interest when deciding on dedication of recourses (Brigham & Ehrhardt 2005, p.422). **Flexibility options** or sometimes called **switching options** permit the firm to alter operations depending on how conditions change during the life of the project, typically by changing the inputs, outputs or both (Trigeorgis, 1999, p.8). Due to technological convergence, where IP standard is used to deliver voice, internet and TV services, the flexibility option can be valuable in the Telecom Industry. This is valid for combined operators, fixed and mobile, where the same services can be delivered on different networks – wire line and wireless, so called fixed- mobile convergence[8].

The origins of incorporating ROA in the Telecom Industry date back to the nineties, when global telecommunication liberalization started. The basic idea was and still is deregulation, although it requires more regulation than a public monopoly. Incumbent telecom operators or operators with significant market power are constrained by the Regulators, which are setting prices for interconnection, directing network sharing rules on different levels, imposing obligations for creation of equivalent wholesale offers from retail product offer and limited to earn no more than the WACC on its cost allowance base.[9] However, Regulators are not taking into account actual costs incurred by the incumbent operator or historical costs, but rather so called forward looking costs of an

[8] Fixed Mobile Convergence is a transition point in the telecommunications industry that will finally remove the distinctions between fixed and mobile networks, providing a superior experience to customers by creating seamless services using a combination of fixed broadband and local access wireless technologies to meet their needs in homes, offices, other buildings and on the go (mobile).

[9] The cost allowance base should reflect part of the cost incurred by incumbent operator that are used for determining the price of the services (interconnection, network, duct or cables lease) given to the competitors.

most efficient operator[10], thus for any positive probability of disallowance, the incumbent would earn less than its WACC (Boyle, 2002). This Regulatory risk exists without considering any flexibility in incumbent's investments. As mentioned above, the Regulator mandates network, duct or cables lease to competitors, including regulated wholesale offers and interconnection prices. These are all are options given to the competitors which create asymmetries in risk sharing between the incumbent and competitive operators, because the competitor does not have to invest in building its own network, and can lease it in limited increments (part of the network) or time intervals. In other words, asymmetries are arising from the level of prices set, short term commitment for the network lease and option to access various points at incumbents' network (Alleman, Madden, Kim, 2009).

Hence, competitors are granted with:

- **option to choose**, as they can decide between rolling out network by their own or lease it from the incumbent operator;
- **flexibility option** as they can access at different interconnection network points and with that are flexible of creating the input mix[11].

In this context, the incumbent has the right but not the obligation to:

- **defer**[12] the investment in infrastructure or "wait and see" what would be potentially regulated in the future, prior to committing on large capital outlays;
- **contract** or **abandon** the project, should the market conditions turn out to be unfavorable.

The needs of the incumbent to respond to policy makers were the initiating factors for applying ROA in the Telecommunication Industry. Nearly all commissioned studies argued that incumbent operators are undercompensated for the investments made, as they are largely irreversible by nature. Only recently, and in fact only the most advanced regulators[13] are recognizing the existence and possibility of implementing ROA as a tool

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[10] This is in fact hypothetical operator that has most sophisticated, technologically efficient and large scale network already deployed. The Regulators rationale for setting forward looking cost is to eliminate any inefficiency caused by legacy equipment.

[11] Alternative operators are enabled by the regulation to choose where to connect to incumbents' network. For instance they can lease the local loop from incumbent operator, while build their own backbone network, or vice versa. Also, duct sharing or capacity sharing rules are available to alternative operators; hence they can optimize their costs.

[12] Depending on the regulatory regime, in some countries (*e.g.* USA), regulators are imposing the obligation to serve, since the operator do not have the choice – the option to defer, once the license for operation is obtained. EU is also taking measures for alleviating the digital gap, however differently. In Macedonia Universal service fund was established aiming to cover rural areas which are not profitable to be covered by operators. The fund is financed from operator's revenues.

[13] For instance UK regulator: Office of Communications (Ofcom). (2005). *Ofcom's approach to risk in the assessment of the cost of capital*

that will account for the value given to the competitors and correct the risk sharing asymmetry. In regulatory context and antitrust analysis, sunk costs and their distinction from fixed costs play a central role (Pindyck, 2008, p. 619), even though classical corporate finance does not consider the former in project evaluation. The rationale is that they are already committed hence cannot contribute to the incremental projects' revenues (Brigham & Ehrhardt, 2005, p.383). However, in *ex post* regulatory context those costs are yet to be sunk when deciding for the allowed investment base (Pindyck, 2004 p. 12).

The DCF analysis assumes that the investment environment is static, *i.e.*, investing today has no effect on the present value of the future investment opportunities. However, the value of the firm can be viewed as a sum of its existing assets in place and the value of its real growth options (Myers, 1977). Real growth options are in fact projects that the firm has not undertaken yet, but has the opportunity to exercise today or in the near future. Exercising them today means using up this value; hence the firms' value falls by the amount of the real growth option. Moreover, the market risk premium used in CAPM, and with that in the WACC, does not compensate shareholders for the fall in growth option's value caused by its exercise (Guthrie, 2009). The NPV rule also assumes that the project is a now-or-never proposition or there is no opportunity to wait and therefore no opportunity costs in investing today. In this case the WACC is the only cost of investment, with price of equity equal to financial asset - security of comparable risk. However, when there is an opportunity to wait, the cost of investing today is a call option on that security and since it is equivalent to leveraged position on the underlying asset it has to have higher expected return (Boyle, 2002). The latter also stems from the analogy between financial and real options, as every contract (option) represents a multiple number of shares, typically 100, of the underlying stock/asset while costing only a fraction of its price. Hence, an option's hedge ratio or option's delta[14], which is the price increase (change) of an option for a 1$ increase of the underlying asset is relatively small in dollar terms, however several times higher in percent terms as compared to the increase (change) of the underlying asset (Bodie, Kane & Marcus, 2009). So when deferral option exists, the cost of capital is not the WACC but rather a value that is higher than the WACC (Boyle, 2002). Similarly, the WACC computed using CAPM for the equity portion does not incorporate any adjustment for option value, hence it cannot be used as a hurdle rate or threshold expected return. So, when the investment is irreversible, an option to defer the investment is available, and is considered the "*hurdle rate must exceed this opportunity cost of capital"* (Pindyck, 2004).
But even investment projects with little or no uncertainty about their cash flows have option rights values. In an uncertain economy when interest rates are changing, nearly all projects are embedded with deferral option (Ingresoll, Ross, 1992). Hence, managerial

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[14] The option's delata is the slope of its value in relation to the current underlying asset price and typically it is in the range from 0.4 to 0.6. The delta is positive for calls while negative for puts. Further the so called option elasticity which is the ratio of percentage changes in option price per percentage change in stock price confirms the leveraged position. See Bodie, Kane and Marcus in Investments p.738.

flexibility or project with option-like futures is not an exclusive prerequisite for existence of the options.

In the constant search for additional revenue, firms are no longer exclusively focused on their core business but rather on cooperation with suppliers, distributors and even different industry sectors. With this, products are becoming more complicated, including the relations between entities in the value creation chain, which in some instances are becoming co-opetitive[15]. In such projects, often different sources of volatility exist and/or the volatility changes during the life of the project. These kinds of projects can be valued with advanced option techniques such as **rainbow**, **sequential options** and /or **non recombining trees[16].**

However, as in the classical DCF approach where real assets are valuated with financial asset valuation techniques, the same is done with ROA, where correspondence with financial options is recognized; hence same or similar assessment tools are used. This correspondence in terms of applicability of the valuation tools for real assets is of a major interest of academics but even more among practitioners who are trying to solve the problems of data unavailability or how close or far assumptions are from the financial markets.

In fact, the most criticized assumption in ROA is the market replicating portfolio or "twin-security" that is the link to financial markets, including the usage of option valuations tools, which are based on no arbitrage opportunities (Borrison, 2003). This is not a problem for existing future contracts on some commodities (e.g. oil, gas, copper), where prices are readily observable; however, for non-traded real assets, finding out a twin security or constructing replicating portfolio that mimics the value of the underlying asset is problematic due to different risk characteristics (Wang & Halal, 2010).

Even though it is recognized that there is a difference between real and financial options, still one of the most commonly used tools for their evaluation is the Black-Scholes (hereinafter: BS) model and lattices, particularly a binomial lattice. In addition, there is little direction of their applicability or best practice depending on the conditions. The latter is primarily related to the projects' distance from to the financial markets, or the extent to

[15] A blend of competition and cooperation. *Co-opetition* is a business strategy that goes beyond the old rules of competition and cooperation to combine the advantages of both. See Adam Brandenburger and Barry Nalebuff in Co-opetition (1998).

[16] Typically the volatility of the underlying asset is aggregate of several volatilities, however, when multiple volatilities exists and those are difficult to be correlated into a single one, the option becomes rainbow, as the asset can take plenty of values. For two different sources of volatility, the lattice is qudrinomial as the asset can take four values in each node. Compound options are option on a option, meaning that exercising one creates another option. A compound option derives its value from another option, not from the underlying asset.

which the real option can be clearly linked to an existing financial option or twin security; thus, arbitrage valuation can be used for assessing its value (Berk and Kase, 2009).

With the aim of trying to answer which approaches might be the best for typical telecom project, the applicability of different assessment tools in telecommunication investments are explored in this master thesis. Furthermore, the underlying assumptions of different approaches, and the difficulties in finding reliable data are discussed. The aim of this master thesis is to show that DCF/NPV analysis in some cases grossly undermines the potential value of the project, as it does not comprise the role of the management. The value of decision making or management's flexibility is indisputable; hence, it should be incorporated in the valuation process. It also limits the potential for broader strategic thinking and innovation related to timing, amount and scope of the project / investment. Hence, this master thesis will show that there are alternatives to the static view of the investments that bring additional value over and above the classic DCF/NPV analysis, should option like features of the investment be included in the analysis.

1.2 Methods of analytical approach

Deductive qualitative research is performed on the literature related to option pricing, ROA and calculation methodologies that are used for pricing options. The same is made for DCF/NPV analysis, in order to infer the differences in the assumptions and compare models and results. The concepts are explored using books, scientific journals, proceedings form conferences, telecommunication regulator's web sites etc.

ROA is applied to a typical, ongoing telecommunication project called: Fiber to the Home ("FTTH"). Several types of options are presented and discussed in order to create methodological knowledge base that will serve for future projects assessment. Different option assessment tools are used, depending on applicability, which are then compared and discussed.

2 REAL OPTION ANALYSIS

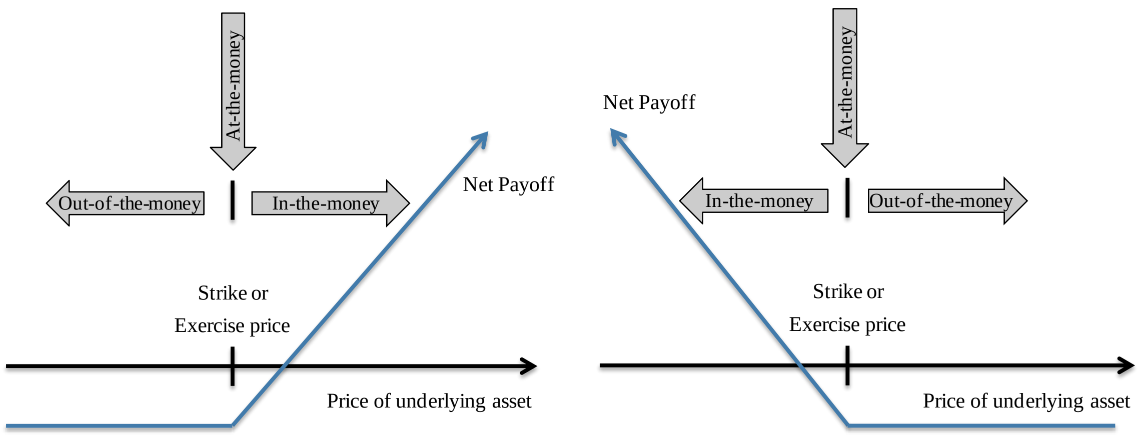
2.1 What are real options?

Prior to defining Real Options, it is reasonable to define financial options, as the assessment approach and their analogy stems from them.

An option is a contract that gives its holder the right to buy (or sell) an asset at some predetermined price within a specified period of time. The former is called call option, while the latter put option. If they are written against stock held, then they are covered, contrary to naked options which are sold without stock to back them up. There is the

American option that can be exercised at any time before they expire and the European option that can be exercised only on its expatriation date, which is the last day that the option can be exercised. The price of the stock written in the contract is called the strike or exercise price. The option is said to be in the money when its exercises would produce profit for its holder, while the opposite is described as out of the money (refer to Figure 1) (Brigham and Ehrhard, 2005).

Figure 1. Call (Left) and Put (Right) Option Payoff Diagrams



Source: A. Damodaran, *The Promise and Peril of Real Options*, 2005, p. 6 & 7.

The right, but not the obligation of the management, to buy operational assets or to take some actions, or to respond to changing circumstances related to projects is called management flexibility, strategic option or real option. They are called real options because they have an underlying asset which is real, unlike financial options where the underlying asset is stock or bond. Real options are not traded, as opposed to financial, while the underlying asset is usually real estate, project or intellectual property. There are several types of real options, however after framing and making parallel to financial options the process of calculation reduces to call or put options (or both at the same time) in the real option analysis. Some of them include (Brigham & Ehrhardt, 2005 Prasad, Trigeorgis, 1999 Amram, 2002 Mun, 2002):

- Option the differ or "wait and see",
- Growth (expansion) option,
- Option to abandon,
- Option to contract,
- Switching option,
- Option to choose,
- Compound options (option to stage),

- Sequential compound option (corporate growth option),
- Parallel compound option,
- Flexibility options (Rainbow option), *and*
- Learning option etc.

Although several types of options are listed above, in general the calculations and reasoning are deduced to two “generic” types: call and put financial options.

Option to defer

The option to defer, or sometimes called option to wait is one of the most used option in capital budgeting decisions. This is due to the fact that it is embedded in virtually every project, as the timing decision for the start of the project is purely private – decided/set by the management. However, investment delay should be carefully examined and compared to the benefits of first-mover advantage. (Damodaran, 2005, p.33) A project delay postpones potential revenues and/or loses revenues to the competition. Hence, in reality the deferral option is most valuable where owners have proprietary technology or exclusive ownership rights (patents, mineral leases) and in cases where the barriers of entry are high, regulated or capital intensive industry.

The rationale for project deferral and with that the value of the option stems from the resolution of uncertainty, which at the time makes the investment unacceptable, with a negative or marginal NPV. The uncertainty could be resolved by itself (passive learning) or cleared up by investing in research (active learning) (Amram, 2002). While the preceding learning approach is costless, the latter is not. Management would invest, or exercise the option only if the uncertainty in question has beneficial value at certain time period, but would not commit to the project if conditions turn out to be unfavorable, thus saving the planned capital outlays. The payoff characteristics of deferral option are equal to American call option; hence its value could be obtained using standard financial option pricing methodologies, using the investment as option’s striking or exercise price, the present value of project’s cash flows as the current price of the stock, while the time of deferral as expiration time. However, American type of options is difficult to be valued, at least without option valuation software tools; hence simplification can be made by valuing the real option with the European type which can be exercised only at maturity[17]. Also, as mentioned above, the timing of the exercise / investment is in management hands; hence the analytical tool can be the BS model, developed for valuing European type of options.
The defer option, as well as the option to expand are called upside potential options as they capture only the positive part of the variability or risk defined in broader context (Trigeorgis, 1999).

[17] Corresponding American option has higher value due to the possibility of early exercise; hence valuing the project with European type can be seen as conservative approach.

Option to expand

Option to expand is one of several possibilities to alter the operating scale of production. In this case, if market conditions turn out to be more favorable than expected the company can expand the scale of production, expand into new geographical markets or accelerate recourse utilization. The investment for expansion in option valuation is treated as the striking price, and the option would be exercised only if the expansion payoff is greater than the strike price, hence making it a call option.

The option to expand is seen by some authors as a part of the broader group of growth options. However, growth option can be viewed as a version of option to expand with considerable strategic importance to the company. In such cases the initial investment is a prerequisite for creating future interrelated projects or growth opportunities. Such options can be framed as option on an option, which makes them sequential compound options. These are contingent options, where exercising one generates another option, taking its value not from the underlying asset but from the preceding option.

Option to abandon (option to default)

Like deferral option, the abandonment option are common for literary every project, because operations may be stopped at any time should the conditions turn out to be unfavorable, while losses minimized by sales of the related assets. The decision to abandon the project is made in case the related payoff falls below the project's salvage value. The former is the underlying asset value, while the latter the strike price, making the payoff analogous to put option. However, the option has value even without considering the salvage value, as most projects are staged, the abandonment can occur in any phase during construction, while any subsequent investment outlays would be saved.

It has to be noted that often in practice, stakeholders are reluctant to make the decision to abandon a project. This is called "project stickiness", which refers to stakeholder psychological attachment to the project or political issues that may arise from closing down the project. Hence, abandonment decisions are not made completely on rational terms, *i.e.* once the asset value falls below the salvage value. Thus, within the organization a clear set of rules of selection and resource allocation (capital budgeting decisions) are essential for exploiting the flexibility of the project (Insead, 2004).

Option to contract

The option to contract can be viewed as a decision between status quo and abandonment of the project on the other extreme. Reducing the output, downsizing and gaining efficiencies throughout outsourcing is very common in today's market place, hence the option to contract can be very valuable. The flexibility to mitigate losses is analogous to put option as its value increases with the underlying asset decrease. *Ex post*, contracting the project is a very common management decision; however, it is rarely treated *ex ante* in project evaluation, since, as in case of abandonment decisions, the project manager is trying to "sell" the project to the decision makers, and his/her career dependents on it. Of course, project manager's commitment to the project increases the likelihood of its success, but should the outcome be negative, exercising the option to contract would protect further misallocation of recourses.

Switching option

When the project is such that it has the ability to alter the inputs, outputs or both of the product mix, including the scale of operations, then it can be valued using the switching option. Switching option includes both American put and call options (Copeland & Antikarov, 2001).

Option to choose (multiple real options)

A choosing option is a real option that gives the owner the right to choose between options to expand, contract, abandon and defer. It consists of multiple options that are pooled in one option, making it a combination of American calls and puts (Trigeorgis, 2006). The management has the possibility to choose from maintaining the status quo, expanding, contracting, abandoning and deferring the project depending on the conditions, similarly to a decision tree analysis. It has to be noted that framing such options is rather complicated, including the assessment during the backward induction process, as some options are mutually exclusive.

Compound options (option to stage)

Large projects are typically staged in several phases; hence, capital outlays occur in different time intervals. During each phase, management has the option to continue with the project, contract, abandon, expand or defer, depending on market conditions. Exercising one option creates another; therefore, compound options are contingent on one another, deriving its value from the previous one, not from the underlying asset (Trigeorgis 1999). The first investment can be seen as a prerequisite for the ensuing, and so on. This is called intra-project compoundness. However, in general, not only staged projects can be viewed as compound options, but any opportunity that creates another one. The latter represents inter-project compoundness, as several other discretionary investments stem from the initial investment.

Depending on the interrelation between phases, compound options can be sequential or parallel. If exercising one option is a requirement for the creation of the following option, then the compound option is sequential, while in case of simultaneous exercise, where both options are available at the same time, the compound option is called parallel (Kodikula & Papudesu, 2006).

Rainbow options

The volatility of the underlying asset in ROA is usually a single or consolidated figure that might be an aggregate of different sources of volatility. However, sometimes volatilities could differ significantly and cannot be easily incorporated into a single number. Rainbow options overcome this problem in option calculations, permitting usage of different volatilities for each source of uncertainty (Copeland & Antikarov, 2001). However, the more different volatilities are involved the more complex the tree becomes. If two volatilities are considered, the tree becomes qudrinomial, as the asset value in each node can take four values. Consequently, the second time step will have 16 asset values. Further increasing the time steps or longer life options makes the calculations cumbersome.

Other options

There are other options, such as barrier puts and calls, where a barrier price is subtracted or added to the strike price, respectively. This is made in order to account for the project's stickiness in case of abandonment, and in case of upside potential options (calls), in order to increase the margin of confidence that exercising would bring higher value (Damodaran, 2005).

2.2 Underlying assumptions behind ROA

Capital budgeting decisions are a process of deciding which projects from the portfolio of projects or growth opportunities, are candidates for implementation. In order to allocate those projects certain metrics must be applied and projects ranked accordingly, given the limited recourses of the firm. Assuming that the firm has all resources, such as human capital, know-how, technology and capabilities to implement the set of potential projects, except capital has to turn to suppliers of capital in order to pursue with project implementation. The suppliers of capital will have to assess the value of the firm by assessing the portfolio of projects that are supposed to be implemented, in order to determine the rate of return on which they are willing to give funds. So, first now it's clear that the metrics should be the value of the project or projects if there are several, and second the value should be assessed by the suppliers of capital. The value of the project or the firm considered as portfolio of projects is the value of its future cash flows generated by those projects, taking into account the related costs. However, included in this cost should be the costs of financing, which are usually expressed through the WACC as the firm can be financed with different instruments. Assessing WACC would again require obtaining rates or the return required by suppliers of capital. In case of equity, since rational well diversified investors are concerned only with systematic risk, they will assess the rate of return through asset pricing models (*i.e.* CAPM, APT, multi beta models etc.), specifically the expected rate of return which, in case of market equilibrium, should be the required rate of return. As for issuing new debt, usually the investment bankers value the rate of return. Nevertheless, a market referral is made for both rate of returns, and market equilibrium should exist in order to price the cost of capital and with that, the project or the firm. Methodologies for pricing financial assets are used for pricing real assets, arising from the assumption that the investor should be indifferent about the required rate of return from the "real" market and the financial market, should both be of equivalent risk. Here the risk is defined as variability of the future rates of return, or the width of their distribution. More specifically, less disperse return distribution compared to wide distribution, with the same expected mean value, means less and high risk respectively (Brigham & Ehrhardt, 2005).

The assumptions of the standard finance theory, e.g. the CAPM, assume that the average rational investor is a risk averse, particularly to systematic risk, as idiosyncratic or firm specific risk can be diversified. Naturally, it is also assumed that a rational investor prefers more to less wealth.

Here, introduction to the cost of capital is made because it is essential in DCF/NPV analysis, including the necessity of finding out the current value of the project that is one of the inputs in ROA. Hence, ROA is considered as an extension to the DCF/NPV analysis, not as a substitute (Kodikula & Papudesu, 2006, Trigeorgis, 1999 p.4).

In order to apply one of the option valuation tools, real options are framed to correspond to financial options payoff characteristics. Hence, the underlying assumptions valid for valuing financial options, including those that consider the differences of the assets – financial vs. real, are valid for ROA. All option pricing models are based on the concept of riskless hedge and no arbitrage opportunities. The latter condition is not just present in option pricing models but in all assets pricing models.

The most simple, however, most flexible in terms of adjustment and intuitive model is the binomial approach. It is a discrete time model, assuming that the underlying asset value follows binomial distribution. As mentioned above, the pricing of the option is made indirectly through forming riskless hedge or portfolio of an asset (”twin security”) and a riskless bond. With these assets it is possible to form a portfolio[18] that has the same value regardless of the value of the stock when the option expires. Since this portfolio is riskless, then its return should be the riskless rate, to keep the market in *equilibrium*. If the portfolio is offered at higher price than the riskless rate, *arbitrage* would occur, hence investors would buy the portfolio, push the price up and with that drive the return down. Given that the value of the portfolio is riskless, its current price is obtained by discounting with the riskless rate[19]. The current value of the portfolio is the current value of the stock in portfolio minus the current value of the call option; hence we can find the value of the option. In the binomial approach it is assumed that the stock price has binomial distribution, or that can take two values per time – higher and lower value than the current price. The grater the difference between those values the more risky the stock is[20].

Clearly, the assumption of only two possible states of the asset value is unrealistic, as assets can take an infinite number of values within a specified timeframe. A more realistic assumption would be that asset prices are stochastic or moving with respect of time, randomly. In dividing the timeframe on smaller portions, or equivalently increasing the number of steps *n* in the analysis, we are limiting the number of possible states of the asset values. With the increase in the number of steps *n* within a given timeframe, the stock price movements can be limited to small fluctuations (variances), and depending on the definitions on up u and down d factors in terms of *n* the process can be either continuous or a jump stochastic, as $n \rightarrow \infty$[21]. For a continuous[22] process the stock price will change ceaselessly, with small random variations, as the time interval gets smaller. The up and down factors can be calculated according to equations 10 and 11. However, it should be noted that these factors were derived using a logarithmic rate of return, hence there is a continuous compounded rate of return on the stock over each period[23]. Further with the use of the central limit theorem, as $n \rightarrow \infty$, the multiplicative binomial probability distribution of stock prices converges to lognormal distribution, which is the assumption made in deriving BS method (Cox, Ross Rubenstein, 1979). Both the binomial and lognormal distributions satisfy the condition that the asset value can increase to infinity, but only fall to zero (Benaroch and Kauffman, 1999).

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[18] The portfolio should equalize the range of payoffs of the stock and the option, so that the profits from the stock exactly offset the losses in satisfying the option holder. Can be calculated according to equation: $N = \frac{C_u - C_d}{P_u - P_d}$

[19] The discount rate should be continuously compounded, not discrete

[20] Further the variability of the asset values will be explained and finding out the up and down factors which are proportional to the standard deviation of the stock returns.

[21] Equations for the up and down factors are explained further (Equation 10 and 11)

[22] The binomial model can be used for continuous jump process, however the limiting theorem is not satisfied, while stock price movements converges to log - Poisson distribution.

[23] Due to the limited liability, the stock prices cannot drop below zero, hence they cannot be normally distributed as normal distribution requires some probability of infinitely negative values. However the log normal distribution can be applied, hence the natural logs of the stock prices are assumed. The variance / volatility is also the variance of natural log of stock prices.

What we have assumed up till now is that the market is efficient, there are no arbitrage opportunities and there is a twin security and a riskless bond, *i.e.* the market is complete.

A complete market is one in which the complete set of possible future states-of-the-world can be constructed with existing assets without friction (transaction costs), or any new cash flow stream (asset) can be replicated by some combination of securities that already exists in the financial markets. These not need be primary securities. (Arnold & Shockley, 2002) In other words, the market is complete if and only if the number of attainable unique securities equals the number of possible states. Therefore, in the binomial approach we have two future states – values of the stock, and two linearly independent securities – the "twin security" and a riskless bond; hence we can price the option (Smith & Nau, 1995).

Above, the changes in the interest rate, the risk – free rate were not considered. Clearly, if the risk – free rate changes, the option value will change. An increase in risk free will increase the call option value, as the PV of the portfolio will reduce. Also, dividends were not taken into account as this would imply changes in the asset value. However, as explained further, adjustment for dividends for both models exists.

Employing BS model in pricing options requires additional assumptions, beside the above mentioned (Brigham and Ehrhardt, 2005):

- There are no transaction costs.
- The investor may borrow any fraction of the purchase price at the short term risk free interest rate,
- Short selling is permitted,
- The call option can be exercised only on its expiration date,
- Trading of securities takes place continuously,
- The stock price (underlying asset) follows a geometric Brownian motion with constant drift and volatility, *and*
- The underlying security does not pay dividends.

The first assumption, however, also present in the CAPM, allows the market participant to form the portfolio without incurring costs, or to restructure it if needed, as the process is dynamic. Relaxing this assumption leads to making other assumptions and approximations in order to have an acceptable solution of the option value. There are different, rather complex, mathematical solutions; however, the influence of the friction is within the volatility factor, expressed through the level of the transaction costs (usually expressed as proportion of the stock) and a time measure related to number of re - hedging made to the portfolio. In general, the new volatility is slightly higher than the volatility without friction, which implies an increase of the call option value. However, recent research has shown that the changes in the option value are in rage of 5%, which in capital budgeting decisions is rather acceptable[24].

The BS model was derived also by a no-arbitrage concept and the concept of riskless hedge, hence the second and the third assumption above permit the hedge portfolio to be made as to exactly mimic the option's payoff. Early exercise in this model is not permitted; hence it can be used only for assessing European type of options.

The continuous trading assumption is necessary in order to secure market equilibrium of prices, hence, no-arbitrage opportunities. The assumed diffusion process in case of a BS model is continuous Geometric Brownian motion of the underlying asset prices, hence other stock price movements, such as jump processes cannot be accommodated. However, the diffusion process of real assets can be better approximated and explained with continuous stochastic process rather than with jump process. In pure jump process, each successive asset price is almost the same as the previous price, but only occasionally with low probability of occurrence significantly different – higher (CRR[25], 1979). Since the project (real asset) would be implemented only if it brings value to the firm, or it increases its value (cash flows) smoothly through time, it would be unreasonable to experience sudden discontinuing changes. Moreover, projects that might have high asset values accompanied with low probability of occurrence are very risky scenarios to invest in. Certainly, this depends highly on the project being analyzed. In this manner, contrary to the above, research and development projects might be better described by pure jump process as the new information arrives infrequently, and uncertainty reveals only after the project has delivered some findings (Haahtela, 2010). In financial markets, sudden and extreme price movements are rare and are usually present as a response to dramatic information, like a takeover attempt (Bodie, Kane & Marcus, 2009). Another alternative for the stochastic process is the assumption of mean reversion of the asset prices, which is commonly used when pricing derivatives on commodities (Dixit & Pindyck, 1994). In general, the investment decisions are influenced by uncertainties of different types, thus the selection of the stochastic process should be such that better describes the random behavior of the asset prices in time (Ozorio, Bastian & Brandão, 2012).

When the underlying asset/stock is paying dividends then the option value will change and there is potential for early exercise. Therefore BS used the assumption of non dividend payout option in order to find a closed form solution for the option value, while later this assumption was relaxed by Robert Merton for constant and continuous dividend payout.

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[24] For example: http://www2.math.uu.se/research/pub/Mawah1.pdf
[25] CRR stands for *Cox, Ross and Rubenstein*

Having indicated the underlying assumptions related to the most popular financial option pricing models, additional assumptions that are arising from the mapping the real assets with financial asset have to be included.

The most criticized aspect of the application of ROA is the link between the financial asset market and the “real asset market” or projects. Critics are pointing out that replicating portfolio assumption is difficult to accept based on principle or evidence. Basic arguments are that real assets are not traded as financial assets, and if similarity can be found, then they are far from the market or the correlation is weak, hence we cannot rely on arbitrage arguments when pricing the option (Borison, 2003). It’s true that not much research effort has been paid to co-variances between individual stocks[26] and even less between real and financial assets. However, ROA in capital decisions cannot be rejected based on those arguments as the well accepted DCF/NPV rule is based on the same assumption of tradability or market completeness. Also, it is reasonable to assume that the project, firm specific and finally market risk are highly correlated (Brigham & Ehrhardt, 2005); hence a traded firms stock may reveal the current market prices of the project. For DCF analysis to price correctly the new asset in the economy in addition to the above a proportionality restriction has to hold, such as that the payoff of the new security (the project for example) is proportional to the market portfolio (Arnold & Shockley, 2002). Moreover, when financial options written on the stock exist and are traded, as in our example, the future implied volatility on the underlying asset can be found straight from the market[27]. In case the current stock price including its volatility is subjectively assessed, then the value of the option can only serve as a yardstick for what might be the value of the project if traded in the financial market as we are getting far from the market. In other words, when assessing the volatility of the project through the same cash flows as some authors are suggesting (either through finding logarithmic cash flow returns or simulating), the correlation might be weak. In any case if we are willing to accept the DCF/NPV rule for valuing corporate static investment (without flexibility or option), we are not accepting additional assumption or the assumptions are not stronger when valuing flexibility (Arnold & Shockley, 2002).

The market replicating portfolio assumption as mentioned above is basically the link of the real asset with the financial asset, hence one of the most criticized. This approach assumes that a portfolio of traded assets can be constructed to replicate the returns of the project in question. The market replicating portfolio is created from the asset – a twin security that has the same (is perfectly correlated) or similar volatility as the underlying asset and riskless bond (Trigeorgis, 1999). In order to do so, the market has to be complete. What’s also assumed, and serves as a prerequisite for applying standard valuation tools like BS, is that the underlying asset price movements can be described with random walk, or geometric Brownian motion. If the market is complete, then the portfolio can be

[26] Because of the diversification effect, including CAPM estimates of the cost of equity
[27] Of course this would be the firm volatility not the projects. This is explained in details in section 4

constructed and unique risk neutral probabilities exist that can price the option by discounting with risk – free rate.

However if the markets are incomplete, as real asset markets usually are, the risk neutral distribution is not uniquely determent. In this case only lower and upper bounds of the asset prices can be determined (Smith and Nau, 1995). For lowering this gap, subjectively estimated probabilities have to be inserted in order to get closer to the asset value.

The market incompleteness assumption is partially solved by so called the integrated – two risks type approach, first implemented by Jim Smith and Bob Nau, which connects the benefits from the option pricing and decision analysis. The basic concept is disaggregation of risks to public and private, which are addressed separately. As for the former - the market uncertainty data are already available from the financial markets and the standard option valuating technique can be employed or they can be perfectly hedged with trading securities. The latter – private or firm/project specific can be addressed with DTA, employing subjective probabilities[28]. Hence, the assumptions / conditions for this so called "Partially complete market" (Smith & Nau, 1995, p.808) are:

- Securities depend only on market states,
- The market is complete with respect to the market uncertainties and
- Private events convey no information about future market events.

As before, the first two conditions enable unique risk neutral probability distribution, hence probabilities. However, private and market uncertainties may be dependent, and the third assumption might be relatively strong in modern markets. While this is not a perfect solution, it's usefully for the purpose of decision making.

The underlying asset value in ROA is the current value of the project, without investment costs. As usual, this is obtained through DCF analysis, and again, the market referral of the WACC – the discount rate, has to be made. Deducting the investment costs (in PV terms) from the current value of the project the NPV is obtained. More specifically, the passive NPV or NPV without management's flexibility is considered, as opposed to expanded NPV which includes the option value as well (Trigeorgis, 1999). The mechanics and the limitations of the DCF method applied to real assets assessment, specifically the passive NPV, were partially discussed in the problem description part of this thesis and partially introduced in this section. As the assumption of frictionless markets is also inherited in the cost of capital assessment, we have to explore the influence of these costs on the passive and expanded project value.

[28] Specifically they are using concave utility function to model time and risk preferences

When there are no transaction costs or other fees involved in raising capital, investors can easily, without incurring costs, switch their assets or reconstruct their portfolio. In the real world there are costs, and how they affect the NPV and the value of firm’s future opportunities, can be accessed through the firm’s ability to finance those opportunities. In special cases, when the firm is facing financial distress as seen by the market, the total risk matters (including firm specific) hence its increase would lead to difficulties in obtaining funds for financing firm’s current projects, but also future opportunities (Boyle G., 2002).

First, the immediate firm’s surrounding – suppliers and customers are less willing to be engaged with such firm, thus the former would charge additional price, while the latter would require less expensive output. Employees would also require “premium” for bearing the increased risk, which all leads to decrease in financial recourses available to the firm.

Second is the moral hazard incentive within firms with high probability of financial distress. While shareholders cannot lose more than 100% of their investment due to the limited liability, they are willing to go with risky decisions because they share the losses with debt holders in case of default, but not the reward. Thus, banks and other suppliers of debt capital are hesitant to lend money to highly risky firms.

Third, high total risk affects leveraged firms when they want to secure its debt by issuing equity. In this scenario, the old shareholders will be reluctant to issue new equity as they must bear the costs required by the new shareholders. To wrap up, when firm’s total risk is increased, its ability to fund current and future opportunities is limited; hence the WACC rises, while the NPV falls, but also the value of future investment opportunities. Assuming that markets are efficient, this would be reflected in investors required rate of return, the project execution is justified not only to cover this costs but also the fall in the value of the future investment opportunities or options[29] (Boyle G., 2002).

From the above it can be concluded that the discount factor might be different from the one adopted by the basic corporate finance rule for evaluating projects, the WACC. Typically, the discount factor is higher than the WACC, when options exist (Boyle, G., 2002). Even without existence of options, projects have to earn more than their WACC, as a result of the “lemons problem” - Akerlof (1970) and related information asymmetries. Namely, as investors and insiders have a different set of information for the project (the latter typically more complete), investors cannot distinguish weather the project is good or bad. The firm has an incentive to fund the project with new equity when they know that the existing securities are over-valued. When this is revealed to the investors they will require higher expected return for the investment, thus the project has to cover not just the WACC, but also the costs of issuing under-priced securities. This means that in a case of full-information state, the true cost of capital is the WACC. However, when information

[29] parallel can be made with reinvestment risk

asymmetry exists and the firm issues new equity in order to fund the project, the cost of capital rises due to decrease of the stock value.

And finally, this thesis will further explain that the WACC might not reflect the “true” cost of capital and that the NPV rule might not be the right tool for assessing projects with embedded options for regulated firms, as are large incumbent telecom operators.

National Regulators use the WACC to set price limits to interconnection agreements, apply the relevant costs base when network sharing arrangements are enforced, aiming to develop competition and protect customers from service overcharge. This means that regulated firms are constrained to earn no more than the WACC on allowed investment expenditure. The *ex ante* view is that the firm is uncertain about the regulator’s decision for the allowable cost base, so for any positive probability of disallowance, the firm is faced with expected rate of return lower than the WACC. In such cases the project will be rejected, while investors are going to be induced to invest only if there is additional premium to account for the uncertainty caused by the regulator. Here it is worth mentioning that one of the basic goals of the telecom’s regulatory body, apart from enforcing competition, is enabling future industry development, which is manly done by incumbents as they have the capacity for large cash outlays. Conversely, the aforementioned hinders discretionary investments, and with that development.

To retaliate, the friction influences the WACC, typically by increasing it, which lowers the projects value in the DCF/NPV analysis. However, it is influencing the growth opportunities as well, as the PV of the current projects value (or the static NPV) is used as the current asset price in the option valuation (or expanded NPV). The very special case of firm facing financial distress is not going to be explored as it is assumed the firm is in good financial condition. However, the regulatory constrains are explicitly modeled in the examples.

2.3 Methods for valuing real options

Real option analysis has its roots in derivative financial instruments, more specifically financial options, and is based on models developed for pricing financial options. The analogy, as described previously stems from options payoff characteristics, where the current value of the underlying asset flows in real option analysis is the project present value of its future cash, the strike price is the project cost, the volatility of the stock is the project cash flow volatility and time to expiration is the validity of the real option. Like every estimation method that forecasts future values, questions about inputs validity, their estimation and most of all theoretical foundations are constantly questioned and challenged among academicians and practitioners. Those frictions and different approaches are explained in the next headings through the examples, as the major goal of this master thesis is the practical considerations of valuing telecom projects with real option analysis.

Solution methods for valuating real options can be categorized in three broader valuation techniques:

- Partial differential equations
  - Black – Scholes as a closed – form solution
  - Analytical approximations
  - Numerical methods, such as finite difference method

- Simulations
  - Monte Carlo

- Lattices
  - Binomial
  - Trinomial
  - Quadrinomial
  - Multinomial

2.3.1 Black- Scholes option pricing model and its modifications

Black – Scholes equation is a closed form analytical solution to the partial differential equation, where the call option value is given by one equation:

$$C = N(d_1)S_0 - N(d_2)Xe^{-rT} \quad (1)$$

Where C is the value of the call option; S_0 is the current value of the underlying asset; X is the cost of investing or strike price; r risk free rate of return; T is the time to expiration or validity of the real option; $N(d_1)$ and $N(d_2)$ are values from normal standard distribution:

$$d_1 = \frac{ln\left(\frac{S_O}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (2)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (3)$$

The annual volatility of the future cash flows of the project is σ, however as mentioned before this volatility should be logarithmic or ln σ. The replicating portfolio strategy can be seen from the equation (1), as the first term is buy $N(d_1)S_0$ shares and the second, borrow $N(d_2)Xe^{-rT}$ amount.

Loosely speaking, the terms $N(d_1)$ and $N(d_2)$ represent the range of probabilities that the call will be in the money at expiration or $S_0 > X$. Since, $N(d_1) > N(d_2)$, as $d_1 > d_2$, for any $\sigma\sqrt{T} > 0$, $N(d_1)$ can be approximated with the upper end of the range.

The above equation of the BS model is for valuing options on no dividend paying stock. The effect of dividend payout reduces the value of the call, while increasing the value of the put option. One method for accounting the dividend payoff would be to estimate the PV of the expected dividends, and subtract it from the current value of the asset S_0, hence the term S_0 in Equation 1, should be replaced with $[S_0 - PV(dividend)]$.

Another approach for incorporating dividends into the calculation, assuming uniform dividends during the option life would be with dividend yield y, as shown below:

$$C = N(d_1)S_0e^{-yT} - N(d_2)Xe^{-rT} \quad (4) \text{ where}$$

$$d_1 = \left[ln\left(\frac{S_o}{X}\right) + \left(r - y + \frac{\sigma^2}{2}\right)T\right]\sigma\sqrt{T} \quad (5) \text{ and}$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (6) \text{ remains the same.}$$

From the above it can be seen that the stock is discounted by the dividend yield, while d_1 is falling, and with that $N(d_1)$ as well, which implies that the "upper probability limit" of the option to be exercised is lower (Damodaran, 2005). The above assumption for uniform dividend payout is reasonable for options written on relatively large stock index, where different stocks in the index pay dividends on different dates, hence forming continuous flow of income (Bodie, Kane & Marcus, 2009).

Early exercise or valuing American type of option is not possible with this equation. However, premature exercise of call option on non – dividend paying stock will not add value to the option owner as the time to maturity is shortened and with that the possibility of stock price to increase, thus the call option value. Though, for a dividend payout American call the premature exercise may be worth, as the underlying stock will certainly decline in value after the dividend payment. Hence, in case where the underlying asset pays large dividends, premature exercise of the call is likely if the time premium associated with the option is less than the expected decline of the asset value just before ex-dividend payout date (Damodaran, 2005).

The privilege of early exercise of non-dividend paying puts, in general is not significant, especially for $S_0/X>1$, and it increases with the increase of the strike price. On the other hand, a dividend paying American put is less likely to be prematurely exercised, since the stock is likely to decrease after dividend payout and with that increase the put option value.

Without any adjustment of the BS model, the value of the option can be viewed as a conservative estimate, as American type of option is worth at least as much as the European type. Another approach of considering the early exercise would be to value the option to each potential exercise date or to each ex-dividend day and chose the maximum of the estimated call values.

Valuing put options with the BS arises from the call-put parity with equal inputs: $C - P = S_0 - Xe^{-rT}$ which after subtitling for C the above equation (4), and rearranging, the put option value can be calculated[30] according to:

$$P = (N(d_1) - 1)S_0e^{-yT} - (N(d_2) - 1)Xe^{-rT} \qquad (7)$$

where d_1 and d_2 are obtained form (5) and (6) respectively.

The factors that affect the call option value can be seen from the equations above, e.g. BS equation (4). Larger variability of the underlying asset σ increases the call value as larger swings increase the likelihood that the option will be in the money. Also, the length of the option increases both calls and puts values, because the chances that the stock price would increase (decrease) above (below) the strike price are greater. The call option value is lower with higher striking prices, which can be seen from equation (5), and the ratio $\frac{S_o}{X}$ which decreases the upper probability limit of exercising. Dividends lower the call option value as they decrease the asset value, but also lower the probability the option to be in the money.

2.3.2 Simulations

Similar to Monte Carlo simulation, the value of the underlying asset is calculated thousands of times during the life of the option, within given boundaries as defined by the volatility. The asset values are calculated for each time step until the end of the option life according to the following equation:

$$S_t = S_{t-1} + S_{t-1}(r\,\delta t + \sigma\varepsilon\sqrt{\delta t}) \qquad (8)$$

where σ is the volatility, S_t and S_{t-1} are the values of the underlying asset at time *t* and *t-1*, δt is the time increment, r is the risk free rate of return, while ε is value from normal standard distribution with mean zero and a variance of 1.0. The underlying process of asset change according to the above equation is Geometric Brownian motion, with deterministic part $r\,\delta t$, where the drift grows at risk – free rate r, and stochastic part $\sigma\varepsilon\sqrt{\delta t}$, expressed through ε (Mun, 2002). Although the volatility factor σ is constant, the stochastic part

[30] Adjusted form of B-S model with dividends included

grows over the time by $\sqrt{\delta t}$, which is consistent with the notion that uncertainty increases as time passes. The asset values at the end of the option life are compared to the strike price, hence maximizing wealth decision rule is applied.

For a call option, if the strike price is lower than the end asset value, then the call is going to be exercised (investment made), hence the net payoff would be the difference between them. In the opposite case, the option would expire worthless and the net payoff or project value would be zero:

$$max[S_t - X, 0] \qquad (9)$$

For put option it would be vice versa. Each project value from the simulation is discounted back to today with risk free rate of return, while the option value for the project is the average of these values.

Simulation can additionally give us insight about the value of the probability that the option is exercised; however, with simulations it is difficult to value American options as the number of possible paths becomes enormous.

2.3.3 Lattices (Binomial model)

Although lattices are simple discrete time models for valuing options, they are open for adjustments that are either not possible in closed form solutions like BS or are difficult to be implemented. Also, lattices are more easily understandable and intuitive as the asset and option value, including the related decision, are presented for each time interval during the option life.

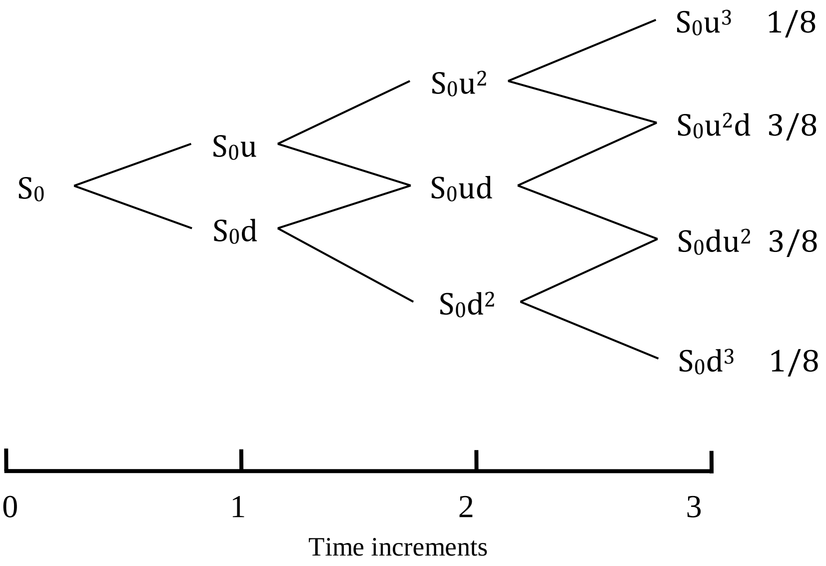
Lattices are like decision trees, with branches representing different paths that the underlying asset value may take in the nodes that are determined by time steps. Nodes are decision points where values of the asset are compared to the option of continuing or exercising. During the life of the option, the underlying asset value could go either up or down from its initial value or from every node, which is why it is called binomial. The up u and down d factors are determined by the volatility of the underlying asset, hence the possible values of the asset in the nodes. Their magnitude also depends on the time step chosen. The optimal solution (option value) is derived by backward induction process, optimizing future decision in each node.

The most commonly used lattice is the binomial one; however, if different volatility factors, rather than a single aggregate need to be incorporated into the calculation, the lattice becomes multinomial. Lattice with two sets of upward and downward factor is depicted in Figure 3 on page 31 making it a generic quadrinomial lattice.

Binomial lattice

Cox-Ross (1979), in their paper: "Option pricing: A simplified approach" have presented a simple discrete time formula for valuing options, as opposite to rather complex mathematical solution of the BS model. They have used the binomial tree for pricing options. A generic, recombining binominal lattice is presented in Figure 2, with three time period steps.

Figure 2. Generic, Recombining Binomial Lattice



Source: P. Kodikula & C. Papudesu, *Project Valuation Using Real Options*, 2006, p. 70.

Starting from today, when the current asset value is S_0, in the next time increment the asset value could take two possible values: go up by factor $u>1$ with probability p or go down by factor $d<1$ with probability $(1-p)$ becoming S_ou and S_0d, respectively. In the second time increment, the asset can take three possible values: S_ou^2, S_oud, and S_od^2, hence resulting in three nodes. The third time step results in four asset values: S_ou^3, S_ou^2d, S_oud^2, S_od^3, and so on. The up and down factors, which determine the possible asset values, are a function of the assets volatility σ and the time increments:

$$u = e^{(\sigma\sqrt{\delta t})} \qquad (10)$$

$$d = e^{-(\sigma\sqrt{\delta t})} \qquad (11), \text{ or}$$

$$u = \frac{1}{d} \qquad (12)$$

It has to be noted that the estimated asset volatility has to be consistent with the time step chosen. When volatility σ is high, the up factor will be larger, as it is exponentially dependent, while the down factor will be lower. This implies that the asset values in the first time increment, $S_o u$ and $S_0 d$ will diverge more largely with higher volatility asset/project. Since volatility is a measure of riskiness, while the asset values are project's cash flows in ROA, the greater the risk, the higher possible range of values of the project's cash flows.

As the time passes, the cash flows become more uncertain, which is represented by the square root of the time steps, and is consistent with the financial option characteristics. Forecasting cash flows is difficult, and the more distant they are, the more they become uncertain, hence their values could greatly differ from the one predicted. Going further in the second, third time step the difference between the top and bottom asset values in the tree becomes even more pronounced (Mun, 2002).

At the last nodes, the possible asset values can be presented and accompanied with a related frequency histogram or probability distribution. The total number of possible asset values, including the total number of paths is dependent on the time increments chosen. The latter is an exponential function of the time increments, and in this example with t=3 time increments the number of paths can be calculated as 2^t or $2^3=8$. The probability of occurrence of the asset value can be found by dividing all possible paths through branches and nodes of the tree from the beginning till the related end node and the total number of paths. For example the asset value at node $S_o u^3$ can be reached only through one path (through nodes: $S_o u^2$ and $S_o u$), which is valid for node $S_o d^3$ as the tree is symmetrical. For those nodes, the probability of occurrence is 1/8 or 12.5%.The middle end nodes $S_o u^2 d$ and $S_o u d^2$ have probability of 3/8 or 37.5%, as both can be reached through three paths ($S_o u$ and $S_o u^2$, $S_o u$ and $S_o u d$, $S_0 d$ and $S_o u d$). Further, the probability of exercising the option can be calculated depending on the total number of paths corresponding to the end nodes where option would be exercised. Assuming that in nodes $S_o u^3$ and $S_o u^2 d$ the asset value is such that the option is going to be exercised, then the total number of paths are 1+3=4, while the probability of exercising the option is 4/8 or 25%.

The solution to the binomial lattice and with that, the option value, is done using contingent claim analysis within a backward risk-neutral valuation process. When the markets are complete and free of arbitrage opportunities a unique probability distribution π exists under which the prices of all assets are proportional to their expected values. The risk neutral probability is used to adjust the cash flows or the asset values through the lattice as to account for the risk, while the time-value of money is resolved by discounting with risk-free rate, as the hedge or the portfolio is riskless. The risk neutral probability p can be obtained through the equation below:

$$p = \frac{e^{(r\delta t)} - d}{u - d} \quad (13), \text{ where}$$

u and d are up and down factors, r the risk free rate of return and δt time increment, calculated according to:

$$\delta t = \frac{T}{n} \quad (14)$$

with T being time to maturity and n number of steps in the lattice.

The value of the call option can be obtained through the process of backward induction and equation[31]:

$$C = \frac{\left[\sum_{j=0}^{n}\left(\frac{n!}{j!(n-j)!}\right)p^{j}(1-p)^{n-j}max\left[0,u^{j}d^{n-j}S_0-X\right]\right]}{e^{r\delta t}} \quad (15)$$

The risk neutral probability p is different from the actual probability q. In fact, p is the value q would have in equilibrium if investors were risk-neutral (CRR, 1979 p.7).

The risk neutral probability (non continuous compounding form) is defined as:

$$p = \frac{r-d}{u-d} \quad (16)$$

The absence of arbitrage requires that nether asset dominates the other or:

$$d < r < u \quad (17)$$

which from equation (16) implies that:

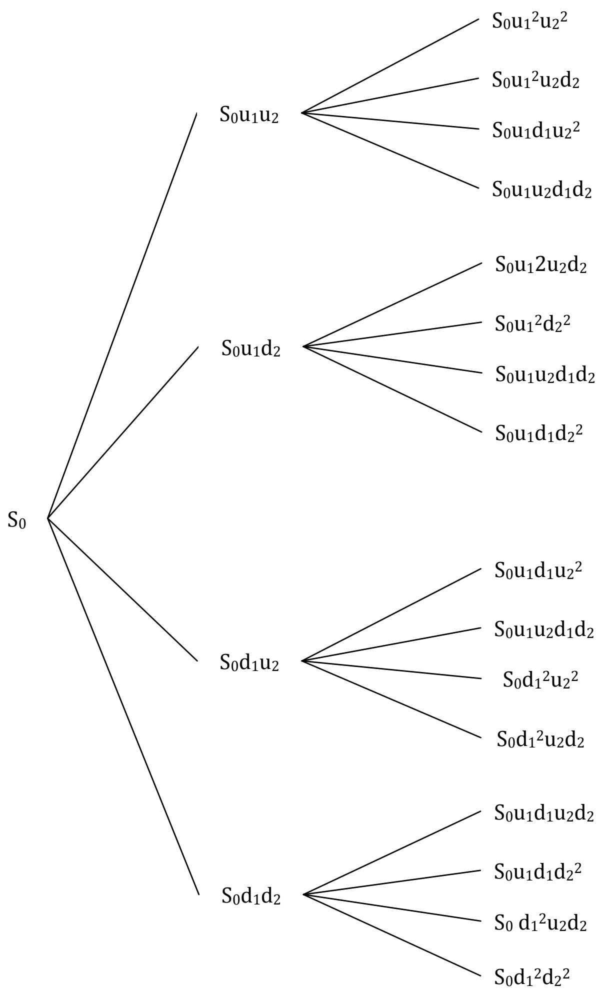
$$0 < p < 1$$

The parameterization of up u, down d factors, including the probability p, according to the equations 10, 11 and 13, are taken from CRR model. In fact they have been derived from system of three equations for the mean, variance and probabilities restriction with four unknowns. As n gets larger, and in search of realistic result – unlike zero or infinity, they set the mean and the variance of the continuously compound rate of return of the assumed stock price movement to correspond to that of the actual stock price. CRR set the product of up and down factors to be equal to one, in other to solve the system of equations (Haahtela, 2010), which gives central property in the lattice or in the second step the central node value of the asset is the same as in the beginning[32]. However, the

[31] Note here that compound interest is used.
[32] The proof and mathematical derivation of the parameters see: Chance, D. Proofs and derivations for binomial models

parameterizations given by CRR are approximate, for sufficiently low δt or large n, and as a result of this the consistency is not perfect, because the variance is slightly downward biased (Gamba & Trigerogis, 2001). The model is also unstable with small volatilities comparable with the risk free rate which can lead to one probability larger than one, hence the other, less than zero. There are several corrections to this[33], however, as the volatility in ROA is relatively high, there is no meaningful influence on the results[34], which is also confirmed in Chapter 3 through the project being analyzed.

Figure 3. Generic Qudrinomial Lattice



Source: P. Kodikula & C. Papudesu, *Project Valuation Using Real Options*, 2006, p. 74.

The diffusion process in the CRR binomial model is multiplicative, corresponding to possible asset - stock prices that can go to infinity, thought with very low probability, but

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[33] See Gamba & Trigeorgis 2001 or Haatela 2010

[34] See Gamba & Trigeorgis 2001, the comparison of CRR and log normal parameterization.

never become negative[35]. As described previously, in the limit as n gets very large, the multiplicative binomial distribution transforms to log – normal. However, the diffusion process in ROA should coincide with that of the real asset, which in some cases can be negative. In such cases it might be better to model the value of the underlying as arithmetic process (Copeland, 2001, p.122). The up and down movements are additive, hence the former is adding and the latter subtracting percent of the asset value. In the limit with equal objective probabilities for the up and down movements, the additive process approaches to normal distribution.

In the binomial approach, adjustment for constant dividends during the option life can be made by inclusion the dividend yield y or often called leakage in equation (13):

$$p = \frac{e^{(r-y)\delta t} - d}{u - d} \qquad (16)$$

If the dividend yield is at varying rates during the option life, than *p* has to be calculated and adjusted in the respective nodes. From the equation above (16), it can be seen that the dividends are lowering the risk neutral probability, and with that the value of the call option.

2.4 Discussion of input parameters, their assessment and applicability

2.4.1 Value of the underlying asset (value of the payoff)

In case of financial option the value of the underlying asset is the current stock price, which is readily available from the stock exchange where the asset is traded. Stock's current price, on the other hand, is the present value of its expected future cash flows, including those cash flows that are expected even if the option is not exercised (Brigham & Ehrhardt, 2005).

However, in case of real option, the asset value is the project itself and its current price is the present value of its expected future free cash flows. The present value is found by standard DCF analysis, discounting with the WACC as obtained from the capital markets. This is still valid for the passive component of the NPV, as capital markets are concerned only about the systematic risk, which on the other hand is defined as the width of the distribution of future returns. Typically, future returns are symmetrically distributed, hence in case of non-existence of options like payoffs, it is still considered as valid methodology for pricing the current value of the project.

[35] Since the starting value is positive, as $n \to \infty$, $\lim u^n \to \infty$, as $u>1$, however $p<1$ and $\lim p^n \to 0$, for the upper branch, while for the bottom branch, $\lim d^n \to 0$, as $d<1$, while $(1-p) \to 1$.

The value of the payoff can be calculated using ratios or valuation data on mature companies in the same industry (*e.g.* market-value-to-sales ratio) (Amram, 2002). Both methods are somehow close to the market; however, for the latter the peer company should have similar revenue stream structure and indebtedness, which might be difficult to be found.

2.4.2 Exercise (strike) price

In ROA the value of the exercise price is the costs of the project – capital outlay. Firms can estimate these costs fairly accurately, as they have frame contracts with suppliers for the equipment or services that are supplied from outsourced partners or managed by internal resources.

Very rarely disruption of the supply chain for equipment occurs, such as the last example in the IT industry where the floods in Thailand caused major hard disk producers to limit their output. IP TV set top boxes use hard disk drives for recording; hence the shortage increased their price as well.

In case of FTTH, budget overstep might only occur due to changes in cable routing, which are dependent on local authority building permissions. However, experienced project managers always estimate an extra budget, which on the other hand is insignificant compared to the overall costs.

The picture is rather different when technology risk exists, such as new competing technology that is not yet standardized or bidding on tenders for obtaining spectrum licenses for 4G cell network rollout. In such cases, exercise price could differ substantially; hence use of advanced chooser option would be more appropriate and/or option with varying strike price. However, those options are rather difficult to frame. In case of a FTTH project, a critical point was choosing the vendor for the IP TV platform, as this was new technology which had not been proven reliable at the time.

Such risks may be market or private, depending on where the decision is. Choosing the vendor of the IP TV equipment and platform is a private risk, as clearly it depends solely on internal firm decision. However, increased costs due to shortage of supply or price fluctuations of the inputs are market risks. If they are significant and uncorrelated to the volatility of the underlying asset, they have to be considered with multinomial lattices.

In case of option to defer, some authors are compounding the costs with risk-free rate in order to take account for the time value of money within the deferral period, as delayed costs are earning at least the risk free rate, however this is not crucial to the analysis (Trigerogis, 1999 p.12)

2.4.3 Time to maturity (expiration date)

The time until a financial option expires is clearly written in the contract and standardized. Exceptions are American type of options that can be exercised prior and on expiration. Conventional options are written on short time horizon – six months or less, except LEAPS[36] that have maturities up to two and a half years. Real options, on the other hand, have longer maturities, and also there is no exact exercise date. As the option value highly depends on the written timeframe, in case of a real option, early or late exercise would not exploit the value of the flexibility in full.

2.4.4 Risk - free rate

The risk-free rate is usually readily available, and can be taken from the current rate of Government bonds. However, the bond's maturity has to be consistent with the time increments in binomial model, or option time to maturity in case using BS model for option valuation. Also, some authors use continuous discounting in option valuation models; hence, converting discrete rates into continues is necessary which can be done according to:

$$r_f = \ln(1 + r_d) \qquad (17)$$

where r_f and r_d are continuous and discrete rates respectively.

In case of FTTH, the yield to maturity of denationalization bonds RM DEN is used as a risk-free rate. These are 10 year bonds, denominated in Euros, with par value of one euro. Since these are coupon paying bonds (10 equal coupons per annum), the current price is calculated from the cash flows of coupons and interest on the residual principal.

2.4.5 Variability of the underlying asset (volatility factor)

Estimating the volatility of the project is perhaps the greatest challenge in ROA. First uses of ROA were in exploiting natural recourses, for which traded financial options exist. Volatility of the underlying assets can be obtained from historical volatility of the stock, exchange prices of the commodities, or by reverse engineering form prices of calls and puts listed on option exchanges. Both methods as explained before are close to the market; the preceding method estimates historical volatility, while the latter, future volatility.
Stock market volatility can be calculated as Deutsche Telekom (hereinafter: DT) is a listed company; however, since DT is a combined telecom operator with several lines of business, the volatility obtained is not project specific, but rather corporate average or

[36] LEAPS stands for *Long-term Equity AnticiPation Securities*

diversified across the existing lines of businesses. Also, volatility can be estimated from the stock prices of MKT share, traded locally on the Macedonian Stock Exchange (hereinafter: MSE). This would be an appropriate estimate or representative volatility as it refers directly to the company "owning" the project and operating in the same market. However, since the MSE is small and immature market with relatively uneducated market participants, the volatility obtained should be treated with caution and compared to the volatility estimates through other sources. In addition, the traded volume of MKT stock is very low compared to total shares outstanding. Derivatives, including a corresponding market, do not exist in Macedonia, thus implied future volatility cannot be estimated from local data, which would be the most representative volatility.

Financial options written on DT's stock are traded on Eurex, hence implied volatility can be calculated and used as estimate for the volatility factor in ROA. However, there is additional challenge when calculating volatility in this way: the time to maturity. Although options with longer maturity exist, the trading volume is relatively low. Project proxy approach is a common way of estimating the volatility and application of ROA. In general, a traded twin security that has the same risk characteristic as the real project, such as stock price of unlevered company in the sector, can be used.

Given all of the above, the volatility of the project that contains only market risk can be spanned within the market, as the market is complete. In other words, a replicating portfolio can be made that completely mimics the project's cash flows and their variability.

Volatility can be estimated, based on the variability of the same cash flow estimates used when calculating the value of the project. The bottom line is: What is better correlated than the project itself? (Copeland & Antikarov, 2001 p.94) This is the *marketed asset disclaimer* assumption, which dismisses the need for a traded market replicating portfolio, but uses the project itself without flexibility or its NPV as a twin security. In other words, the NPV without option like features is used as an estimate of the price the project would have if it were a security traded in the open market, or the same underlying assumption as in the DCF/NPV analysis. This, so called direct method, uses the relative return of the cash flows, and their standard deviation represents the volatility factor. Since option pricing models require continuous variables, the standard deviation of logarithmic cash flow returns is calculated and annualized if necessary. Similarly, when several scenarios are made for the project with probabilities attached to each scenario, the volatility can be estimated as standard deviation from the returns. First, PV of each possible outcome is found until the option expires, and then the percentage return is calculated taking current "price" of the project, that is the expected present value of the cash inflows. The expected return is in fact weighted average return, with probabilities used as weights, which is then used for calculating the variance, standard deviation and CV of the project (Brigham &

Ehrhardt, 2005). Based on scenario data, the volatility can also be estimated using the equation:

$$\delta^2 = \frac{\ln(CV^2+1)}{t} \qquad (18)$$

were *CV* is a coefficient of variation and *t* options' time to expiration.

Of course, demand can take infinity number of outcomes and it is not limited to several as usually made in scenario analysis. Further, here we rely solely on estimates made within the firm, which involve use of empirical data and judgments that are a subjective educated guess (Cobb & Charnes, 2004, p.4).

Partly, the drawback of limited number of outcomes used in calculating the volatility can be solved by simulation (Harno et al, 2005, Copeland & Antikarov, 2001). This method can produce numerous cash flow profiles with different distributions attached to revenue streams and various operating costs. However, it is as good as the estimates of the distributions and input variables. Those methods include logarithmic present value approaches, simulating only future cash flows while keeping current cash flow constant, or simulating both, which in general overestimate the volatility[37] . The latter is because the volatility estimate is calculated for the total project period or it is outcome of all future uncertainties. However, the uncertainty partially clears out during the first year; hence, the first year's cash flow is stochastic, while the other subsequent cash flows are conditional on the outcome of the first's year cash flow. In this way, the correction of diminishing volatility with the pass of time is taken into account.

Estimating volatility with simulations can accommodate different volatilities for various input parameters, including use of the related correlation factors such as price and demand, serial price correlation or both, that can significantly affect project valuation. As the former correlation for internet subscription is found to be highly elastic[38], it can considerably decrease the estimated volatility (Cobb & Charnes 2004 p.14 & p.22).

The direct approaches for estimating volatility of the project itself are heuristic, *i.e.* they can be rather biased and involve subjectivity of the project leaders and management. Assumptions and the estimated value of the volatility should then be checked and compared how far or close they are from the market values.

[37] Refer to Haahtela T. (2007), for more details on different simulation methods.
[38] Se for example Ofcom: The communications market: Broadband, digital progress report research report, from http://stakeholders.ofcom.org.uk/binaries/research/cmr/broadband_rpt.pdf, or The consequences of net neutrality regulations on broadband investment and consumer welfare, from: http://www.theamericanconsumer.org/wp-content/uploads/2009/11/final-consequences-of-net-neutrality.pdf

Another method for estimating the volatility is the project proxy approach. Since FTTH is an ongoing project and has a "history", the volatility can be estimated from the project's behavior thus far. This estimate might be most representative as the most recent data can be used, and it can be calculated according to the variability of the actual projects' monthly revenues[39] or more precisely logarithmic returns according to the following equation:

$$\delta = \frac{\sum(lnR_t - lnR_a)^2}{n-1} \quad (19) \text{ where}$$

$$R_t = \frac{S_t}{S_{t-1}} \quad (20) \text{ and}$$

R_a - average return
S_t - cash flow / revenue/ stock price

All of the methods at certain points involve judgments; hence, the most appropriate one would depend on the availability of data, local and project specifics. However, applicability of the option methodology calculation requires tradability of the underlying asset or market referral, as all are derived on the no arbitrage assumption. Practitioners might consider all of them in order to derive a conclusion what might be the span of volatilities or even consider the average (Luehrman, 1998a).

Like the risk free rate, the timeframe of the volatility estimate should be consistent with the time step chosen in the option valuation tool. If transformation of the time period is needed, it can be made with the following equation:

$$\delta(T_2) = \delta(T_1)\sqrt{T_2/T_1} \quad (21)$$

In this master thesis the volatility was estimated employing reverse engineering with BS model and option prices traded on EUREX, from the project itself through the variability of the logarithmic cash flows and variability of stock prices traded on MSE. The calculations of the volatility estimates can be found in Appendix C, D and E and showed below on Table 1.

The volatility coming from the project returns (29.7%) is fairly close to the both market estimates – implied volatility from DT call option (27.47%) and stock price returns of MKT share (30.91%), which gives confidence in the reliability of the estimates.

[39] It would be better to employ the Operating income, however, this can get negative for which ln does not exists

Table 1: Estimates of Annual Volatilities

Method	annual volatility σ in %
Implied volatility B-S model	27.47
Logarithmic CF from project returns	29.70
Logarithmic stock price returns	30.91
Standard deviation in firm value[40]	40.64

2.4.6 Dividend yield

The owners of financial options have the exclusive rights of buying / selling the underlying asset at/on the pre -specified price / date. Real options in most cases are not proprietary (except the rights of patents), and the rights to undertaking the projects (buying/selling the underlying asset) are shared with competitors. Hence the cost of delay, expressed through dividend yield can be modeled and estimated directly from assuming market share loss due to the deferral or for uniform cash flows calculated as: $y = \frac{1}{n}$ where n is the life of the project. The logic behind this latter approach is that the project would bring excess profits (over and above the cost of capital) within its useful life, while after that those profits would disappear due to the competition. Thus, "….each year of delay translates into one less year of value creating cash flows" (Damodaran, 2005 p.30).

In case of FTTH, the cost of delay was estimated directly from the assumed market share loss within the year of deferral, and calculated from the cash flows with and without market share losses. This difference was expressed as a percentage of the total underlying asset value. An alternative would be to use directly the lower asset value (considering the competition effect). However, in this case the risk neutral probability in binomial approach and $N(d_1)$ in B-S would not be affected, hence the information of the probability of the option to be in-the-money at expiration would be missing.

3 REAL OPTION ANALYSIS OF A TYPICAL TELECOM PROJECT

The project chosen for application of ROA in this master thesis can be found in most of today's strategic portfolios of investment in fixed line telecom operators. Of course, ROA is applicable, and even more valuable in mobile businesses, especially for evaluating investments in new technology, like long term evolution (4G) or for finding out its license value. However, one of the purposes of this master thesis is to lay down methodological basis for application of ROA in capital decision-making, hence several different analyses

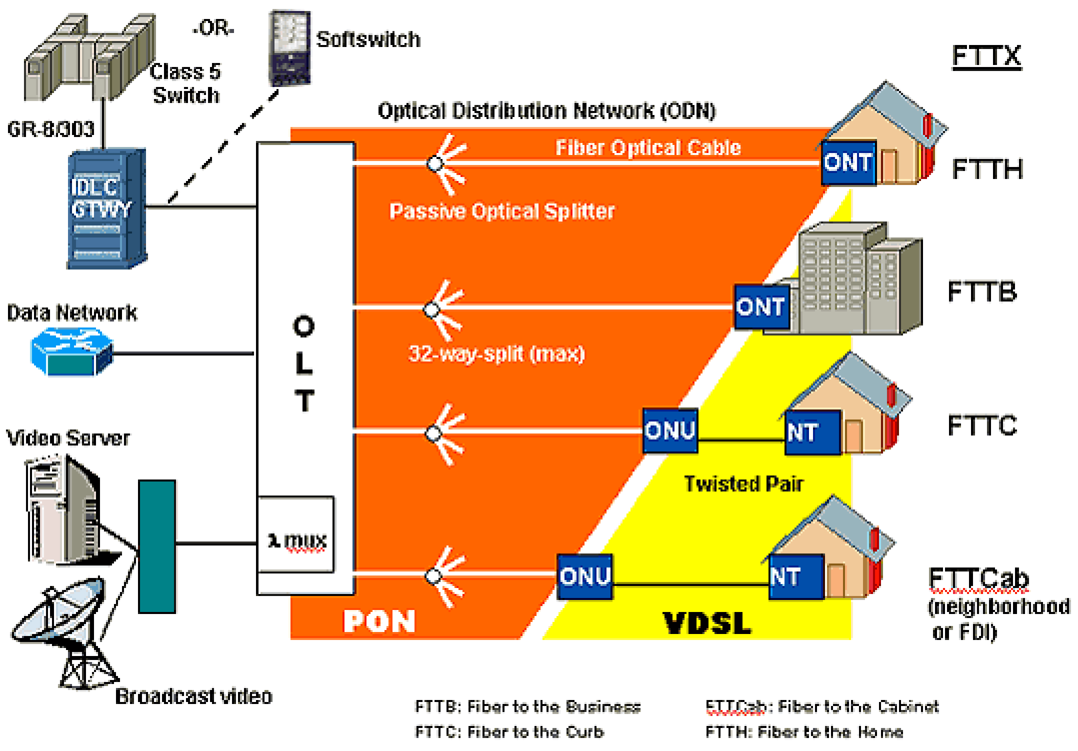
[40] See: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/optvar.html

are made on the same project as to explain the principles and challenges in the process. In this sense, different types of options are applied, even thought they might not be a perfect match for this project as explained further. Also, the project, particularly the DCF analysis was simplified due to the complexity of the related revenue streams, cost structure and capital expenditures, including the length. However, the simplification does not alter the dynamics or the related NPV of the project. One of the goals of this master thesis is to show the conceptual application of the ROA in the telecom sector, which is certainly not affected by the project simplification.

3.1 Project description – Fiber to the home (hereinafter: FTTH)

FTTH is an ongoing project in MKT, approved back in 2009. Although it is based on lukewarm NPV, it is qualified as a strategic project. It enables the support of customers' growing bandwidth requirements mainly driven by video, and also creates basis for future services that require ultra high speed.

Figure 4. Fiber Optical Network



Source: Telecordia, *Passive Optical Network Testing and Consulting, High-level PON Example*[41]

[41] Telecordia, *Passive Optical Network Testing and Consulting, High-level PON Example*, Retrieved May, 2013 from http://www.telcordia.com/services/testing/integrated-access/pon/

It's a long term and capital intensive project, hence qualifying for application of ROA (Amram, 2002 Trigeorgis, 1999 Pindyck, 1991 and others). As the name suggests, it is a fiber optic connection for the customers, enabling high speed digital connection as presented on Figure 4. There are several types of standardized fiber network topologies and technologies with different characteristics; however, the one chosen by MKT is Gigabit Passive Optical Network (hereinafter: GPON). The network consists of central office equipment, called optical line terminal, the optical distribution network consisting of fibers and splitters, and at the end, optical network terminals on customer premises. Being standardized, the technological risk is minimized; however, the risk of the competition developing and implementing better technology still exists.

The GPON is capable of delivering 2.5 Gbits/s bandwidth. Triple play services for residential customers, like broadband internet up to 100 Mbits/s, IP TV – HD and 3D Video on Demand ("VoD") with multiple streams including telephony that can be easily be accommodated with this bandwidth (speed). Business customers can benefit from ultra high speed internet connections, for intranets, VPN's, and most recently cloud computing.
Other benefits related to optical network implementation are efficient usage of the cable duct space, as fiber cables require less space compared to copper cables. Fiber is insensitive to electromagnetic influence and there is no crosstalk.

Building the network requires civil engineering works in urban areas, where cable ducts do not exist or are used, but also network layout inside residential buildings all the way to each customer premise. For this permissions from local authorities are needed, hence fast deployment is very difficult. Eventual deferral of the planned investment would mean not just loss of potential customers but also loss of competitive advantage, which will be elaborated further, when evaluating the option to differ. These are fixed costs, which can easily become sunk, if the demand turns out to be low, or competition takes much of the market share. Central office and customer premises equipment, including TV set top boxes are variable capital expenditures; hence their purchase can be optimized depending on the demand.

3.2 DCF/ NPV analysis

FTTH is an optical network till the last mile – customer. In fact, it is infrastructure that enables delivery of already existing services, such as Internet, telephony, digital television and video on demand. MKT holds the major market share for the former; however, it is now a entrant in the cable TV business. Moreover, cable TV operators are gaining significant market share in the Internet and telephony business. The competitive advantage of FTTH is in its speed, hence a premium over other similar products that are provisioned through ADSL / copper cables is charged.

3.2.1 Incremental revenues

Incremental revenues are comprised of two main revenue streams, the FTTH monthly fee, a "premium" above the current services, and green field customers. There are other revenues coming from externalities, like sales of High Density ("HD") TV sets in MKT shops, but these are insignificant compared to the former.

Also, wholesale revenues (revenue decrease) are forecasted from the second year, as it is highly likely that the FTTH is going to be regulated. It is assumed that 10% of total customers are going to be wholesale customers in the second year, while 20% in the next period. The assumed regulation is on retail minus[42] basis with price set to 30% less than the retail price. This was included as certainty in the base case scenario. However, in the second scenario, expected NPV was calculated assuming 90% probability of imposing regulation on FTTH as estimated by Regulatory area.

Price decrease is assumed as of second year, including discounts for new customers as part of the marketing campaign.

3.2.2 Incremental costs

As mentioned above, the major part of the CAPEX are costs related to building the transport and passive network. Other fixed capital expenditures comprise of adaptation of rooms, power supply equipment, air-conditioning and instruments for testing. The operational expenditures are HR related costs for installation, and standard marketing and sales expenditures. Major part of the CAPEX can be treated as variable, thus the capital layout can be adjusted according to the demand, but also staged in terms of timing. This characteristic also makes the project potential candidate for application of ROA (Trigerogis, 1999). Costs are also assumed to decrease with time, assuming that the prices of the equipment will decrease.

When examining the pattern of the cash flow over time, especially the CAPEX, we will recognize large uneven cash flows and spikes. These are surely discretionary cost, for which decision for spending is within the firm's competences, dependent on the market conditions. The later is typical for expansion or growth option (Luehrman, 1998a, p.10).

[42] The rate in retail minus regulatory approach depends on many factors one of being the technology in place and with that the related costs. However it is usually benchmark rate taken from other related markets. In this case it is on level 4, meaning that MKT has to secure everything from terminal equipment to the connection.

3.2.3 WACC

The post tax WACC of 13.6% is the corporate WACC used in the analysis, because this is considered as a core business project, hence it should reflect firm's average risk. Cost of equity is calculated according to CAPM[43], with historic rates of return for several similar integrated telecom operators in the region. Readily available data for the unlevered β of major integrated telecoms (British Telecom, Deutsche Telekom, Magyar Telekom etc.) are also included and then averaged.

The risk free rate is calculated with bond yield plus methodology. A premium obtained as a difference between long term inflation rates in Macedonia and EU is added to 10 year EU bond rate.

Market risk premium is usually taken from world robust data sets, from professors, analysts and companies. Methodologies for its estimate comprise: historical, required, expected or implied equity risk premium[44]. Also, studies commissioned by telecom regulators are a good source of the market risk premium[45].

Cost of debt is calculated adding premium on the risk-free rate, which reflects the country risk[46]. The latter is calculated as difference between averaged yield to maturity of local MK bonds and 3 year yield to maturity EU bond. Certainly, the market value of a corporate bond would be the most representative value of the cost of debt, however MKT do not use such an instrument. Another way of estimating the cost of debt is adding debt risk premium that is specific for the telecommunication sector to yield to maturity on government bonds. This debt risk premium is usually calculated as a spread between government bonds and telecommunication corporate bond in the same country[47].

The capital structure weights are obtained from debt weight based on book value and forecasted data. Total liabilities from the past and those estimated in the planning process are divided by total equity and liabilities, and then averaged.

As mentioned previously, WACC rate should include the riskiness of the project, assuming that the correlation between the systematic-market risk, corporate – firm specific and the project in question is high, which is typical for most projects (Brigham & Ehrhardt, 2005, p.406).

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[43] In fact modified CAPM is used - adding country risk premium and size risk premium to the cost of equity.
[44] See for example: http://www.iese.edu/research/pdfs/DI-0918-E.pdf
[45] See for example: http://www.aek.mk/, http://www.ofcom.org.uk/ or http://www.competition-commission.org.uk/
[46] Also, a credit spread between composite AAA and BB bonds can be used.

Much of the practitioner and academic discussions are focused on the discount rate. Should the same rate be applied to the revenues and costs? Typically, the revenues and the costs do not have the same risk characteristics. Costs can be estimated far more accurately than the demand, and with that the revenues. When the competition upstream within suppliers is high, then prices of the equipment are expected to be relatively stable and declining over time. Applying the same discount rate would lead to overestimating the NPV of the investment as costs, especially in the latter years, are over discounted; hence the NPV is skewed to the right (Amram, 2002). However, in the telecom industry there are a limited number of suppliers due to the specific usage of the assets, which cannot be used for other purposes. Hence, costs and revenues might not have the same risk characteristics, but are highly correlated in this case. Should different rate be used for discounting the costs, for instance the risk-free rate and the WACC for the revenues, the NPV becomes negative, and the project would be rejected.

3.2.4 Results

The NPV of 1.85 million € as of year six is relatively small compared to the capital outlay in PV terms of 34 million €. The discounted payback period is 5.8 years, while the internal rate of return is 15.7%, slightly above the WACC (Appendix B).

Terminal value is not calculated, however it doesn't mean that does not exists, but rather it is assumed that the industry is so dynamic that technological developments would depreciate the asset. This assumption is reasonable for the terminal equipment however; it might be too strong for the fiber network, which in turn makes the investment analysis rather conservative.

3.3 Identifying real options

The process of describing the problem, identifying the real option and its contingent decision is called framing. In contrast to financial options, real options can entail more than one decision or more than one option, depending on the specific problem and framing of the real option.

Embedded in most projects is the option to defer, or "wait and see", in case the firm has some exclusivity on the project, such as proprietary rights, high barriers to entry, or investment that requires high capital outlay. Otherwise, the competition can gain significant market share during the deferral. The idea is rather simple: if the market conditions turn out to be more favorable within the specified deferral period, then the option should be exercised or investment made. If not, the firm should wait for conditions to become more favorable or let the option expire worthless.

Specifically this project requires high financial recourses, which is typical for the industry. However, the competition is rather fierce, thus early entry could be more valuable than the option itself. This problem will be considered and evaluated further when applying option to defer to the project, including the techniques for adjustment of the real option value for the loss of first mover advantage. Since this project has already started, the option to defer is analyzed for establishing a methodological basis to be applied to other projects in the industry, which can have a greater value. However, similar and more applicable to this example would be the option to contract the operations.

The option to contract the operations is valuable when the demand is low; hence the output is reduced in order to save recourses. Should the market conditions improve, the investment would be adjusted accordingly. The extreme of the option to contract is the option to abandon the project. This would mean disinvestment and sale of the assets for their salvage value. However, much of the assets is industry specific, and cannot be easily sold, at least not for price comparable to the originally paid for. When the sector is experiencing unfavorable market conditions, other companies would not be interested in buying physical assets, hence the investment is largely irreversible (Pindyck, 2004 p.7).

Since FTTH is long term project, where investment occurs in every year, most appropriate is the use of advanced options for its analysis. As explained previously, staged investment can be framed as sequential compound option or parallel, depending weather executing the preceding option is necessary for creating another one, or whether they exist at the same time. Specifically in FTTH, the first investment or phase is a prerequisite for further investments, depending on the market conditions at the end of each phase. In this way a stream of options or sequence is obtained. The number of options in the sequence depends on the number of phases, which in case of FTTH is divided yearly. Hence, a six steps / period binomial tree is used for assessment. Compound options are option on option, where exercising one creates another option; hence they derive its value from another option, not from the value of the underlying asset.

3.3.1 Option to defer investment

In general, deferring the investment is made in order to gain additional knowledge and information for clearing out of the uncertainty related to the demand, technology or other risks. The uncertainty can be cleared out by itself, which is called passive learning, or the firm can invest in market research or establish a pilot project and with that actively learn about the uncertainty.

In this specific case, the technology risk is not taken into account as it is well proven and already chosen. Here the uncertainty about the demand is considered on whether the timing of the investment is appropriate. Having in mind that currently the project is somehow postponed due to management request for its reexamination, it confirms that the timing or scale of implementation was not adequate. This strengthens the conclusion that the DCF analysis is a now or never proposition, while the management is naturally responding to the altered conditions, exploiting the option to contract. If ROA had been performed initially, the strategy and potential implementation timing would have been set more properly.

Here, postponing the investment was assumed for one year, as longer periods would make the option worthless due to the competitive environment. The value of the underlying asset is the PV of the cash flows, while the cost of the investment is the option's exercise price. If the asset value is greater than the exercise price or costs, than the option will be exercised or investment made. The net payoff in this case would be the difference between the asset value and the exercise price, which reassembles payoff of a call option.

In the FTTH example the option to defer the investment was simplified and framed as a simple call option. Hence, the investment costs in later years were deduced on PV basis as if they appear at once in year 1, while the asset value is PV of the future cash flows. The investment costs as of now were compounded with a risk-free rate for two years, and because the investment is postponed, they should earn at least the risk free rate of return. Certainly this is oversimplification of the problem and more appropriate would be to use advanced options in this case compound option, which is presented further in Section 3.3.4.

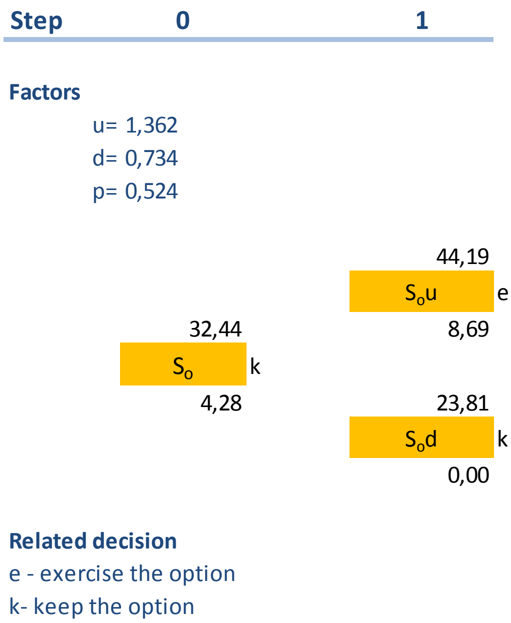
The input parameters (in million €) are as follows:

Stock price (PV of the cash flow as of now)	S_0= 32.4
Exercise price (PV of the investment as of Year 1)	X=35.5
Standard deviation (annual volatility of the asset)	σ=30.9%
Risk-free rate (continuous)	r_f=6.1%
Time to expiration	T=1 Year
Time step	δt=1

Input parameters for the binomial model, as calculated according to equations (10), (11) and (13) respectively, are shown with the binomial lattice on Figure 5.

As of now, the asset value $S_0 =32.4$ (upper figures), can take two values next year $S_0u=44.2$ and $S_0d=23.8$. Since the option has one year till expiration, at the last nodes we have to decide whether to exercise it (invest further) or terminate the project. However, as the asset value is greater than the investment $S_0u=44.2 > X=35.5$, a rational decision would be to exercise or invest, whereas the project value would be calculated according to equation (9) or the difference between S_0u and X, which is *8.69.*

Figure 5: Binomial Lattice for Option to Defer for One Year



At node S_0d maximizing the decision would give project value of zero, as the asset value would be less than the investment $S_0u=23.8< X=26.6$, hence the project would be left to expire as worthless. Moving one step back, the project value is weighted average of the values in nodes S_0u and S_0d with risk neutral probabilities p used as weights, and discounted back with risk free rate, according to the equation (15) which for $n=1$, becomes:

$$V = \frac{[p\, max[0, u\, S_0 - X] + (1-p) max[0, d\, S_0 - X]]}{e^{r\delta t}}$$ or, substituting V= 4.28 million €

The related decision (weather to exercise or keep the option) in this node is simply maximizing the outcome or

$$max[S_0 - X, V]$$

What does this project value or value of the call option represent? Since we have taken the market volatility σ of 30.9%, as obtained through logarithmic returns of MKT share price traded on MSE, while the PVs of the asset value and the investment using WACC is again linked with the market, this should be the value of the project as if traded in the market, inclusive the option to defer. In other words, this is the *expanded NPV*, that consists of *passive NPV* including the managerial flexibility or the option to defer or:

expanded NPV= passive NPV + value of the option (Trigeorgis, 1999, p.4)

So, it follows from the above that the flexibility or the additional value created by the deferral of the project is:

value of the option= expanded NPV- passive NPV=4.28-1.85=2.43 million € or 1.3 times the passive or static NPV.

Using the same inputs as above, the BS model would yield 3.58 million €, which substantially differs from the binomial model used with one step interval $\delta t=1$ or one year. Should we have divided the year in five steps *i.e.* $n=5$; $\delta t=1/5$ the binomial model (Figure 7) would have given a yield of 3.59 million €, which is a negligible difference as compared to the value obtained through BS model. This confirms that with increase of the time steps, the binomial model approaches or transforms to BS model.

The option to differ is also calculated using simulation as per equation (8) explained in section 2.3.4 with time step $\delta t=1/10$, but with low number of simulation trials[48], yielding project value of 3.98 million €. Of course, increasing the number of simulations made the value approach the value as calculated with the BS model. Simulations can give us additional insight for the project, which is the probability of exercise, calculated as the number of positive outcomes, divided by the total number of outcomes. In this case the probability that the option is going to be exercised is around 42%.

Obviously, flexibility has value, which in this case is quite substantial and mainly related to the fact that the option is close to exercising. This can be seen from the S_0/X ratio, which in this case is around 0.9. The underlying assumptions are completely justified with financial options, however unrealistic in this case. Namely, by oversimplifying we have assumed that the exercise would yield immediately all the CF from year two onwards, by investing the total amount X in year 1, which is impossible, as implementation of the project is within five years, as is acquirement of future customers. The most adequate approach would be to value the project as option to stage or "Time-to-build option" (Trigeorgis, 1999), which is an advanced option, particularly sequential compound. This approach is employed further in Section 3.3.4 of this master thesis.

In the example above the volatility used in estimating the option to defer is taken from the logarithmic stock price returns. If we use the volatility estimates as per Table 1, the option value would increase with the increase of the volatility, which is shown in Table 2, below, for different option pricing models.

From afore stated, it follows that the results from the option to differ are highly sensitive to the volatility of the underlying asset, but also dependent on the calculating method used.

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[48] Ten trails with 100 random number variables per time step

With annual volatility of around 31%, as used, the estimated option / project value can be considered as a conservative estimate.

Table 2. Option/Project Values With Different Volatility Estimates

Method	Annual volatility σ in %	Option value in 1.000.000 EUR Binomial 1 step	Binomial 5 steps	B-S model
Standard deviation in firm value	40,64	5,90	4,95	4,84
Logarithmic stock price returns	30,91	4,28	3,59	3,58
Logarithmic CF from project returns	29,70	4,07	3,59	3,58
Implied volatility B-S model	27,47	3,69	3,42	3,43

When comparing the models employed, the one step binomial yields highest values and the BS lower values for the project, while five step binomial model yield values in-between. As explained before, the binomial approach is not different from the BS but rather its simplification, which in its limiting case transforms to BS for continuous stochastic stock price process, when number of steps is $n\rightarrow\infty$ or adequately $\delta t\rightarrow 0$. Hence, whenever possible and especially when real options can be framed as simple, the BS model would be the preferred choice as it yields more accurate but also conservative results. Should the binomial approach be necessary, then it would be preferable to increase the number of steps as to minimize the "error" as measured through the difference of the results between the models. However, with the use of different (other than those proposed by CRR) parameterization for the up, down and probability factors, the binomial model faster converges to the value obtained by the BS model, and corrects the instability present in CRR parameters when the volatility factor is comparable to the risk – free rate or when the time step is too large.

For example, Trigerogis proposed a log-transformed binomial lattice approach, with improved qualities over the CRR, in stability – avoiding negative probabilities, and in efficiency – less time steps needed for convergence, "….. while keeping the appealing simplicity of the CRR approach." (Gamba & Trigeorgis, 2004. p4). The parameters for the jumps and the risk neutral probabilities according to the above log-transformed binomial approach are:

$$\mu = \frac{\alpha}{\sigma^2} - \frac{1}{2} \qquad (23)$$

$$k = \sigma\sqrt{\delta t} \qquad (24)$$

$$h = \sqrt{k^2 + (k^2\mu)^2} \qquad (25)$$

$$p = \frac{1}{2}(1 + \frac{k^2\mu}{h}) \qquad (26)$$

Where:

α is the risk neutral drift, which for dividend paying stock δ, is: α = r- δ, while r is the risk free interest rate. The up factor is $u = e^h$, and the down jump can be calculated as $d = e^{-h}$.

Rendelmann – Bartter[49] ("RB"), specify equal probabilities for the up and down jump factor of ½, however, unequal jump factors, hence the centrality is lost:

$$u = e^{\left(r-\frac{\sigma^2}{2}\right)\delta t+\sigma\sqrt{\delta t}} \quad (27)$$

$$d = e^{\left(r-\frac{\sigma^2}{2}\right)\delta t-\sigma\sqrt{\delta t}} \quad (28)$$

$$ud = e^{2\left(r-\frac{\sigma^2}{2}\right)\delta t} \quad (29)$$

The above parameterization has " …. perfect consistency so that the mean and the variance of the lognormal diffusion process are the same for any step size." (Haahtela, 2010 p.8).

A modification to the parameterizations for longer δt, as proposed by Hull, is instead of $\sigma\sqrt{\delta t}$ to use $\sqrt{e^{\sigma^2\delta t} - 1}$.

And finally, Haahtela proposes growing the central line of the lattice along risk free rate with the parameters as per below:

$$u = e^{\sqrt{e^{\sigma^2\delta t-1}+r}\delta t} \quad (30)$$

$$d = e^{-\sqrt{e^{\sigma^2\delta t-1}+r}\delta t} \quad (31)$$

$$p = \frac{e^{r\delta t}-d}{u-d} \quad (32)$$

claiming, that parameterizations are always stabile, regardless of the length of the time step.

Because in case of ROA, the step size corresponding to the number of time steps dividing the analyzed option time to expiration is relatively long, comparison of different parameterizations has been made to the B-S model as referent for the option to differ with single, two, five and ten steps, keeping the volatility constant at σ=30.9% (refer to Figure 6.)

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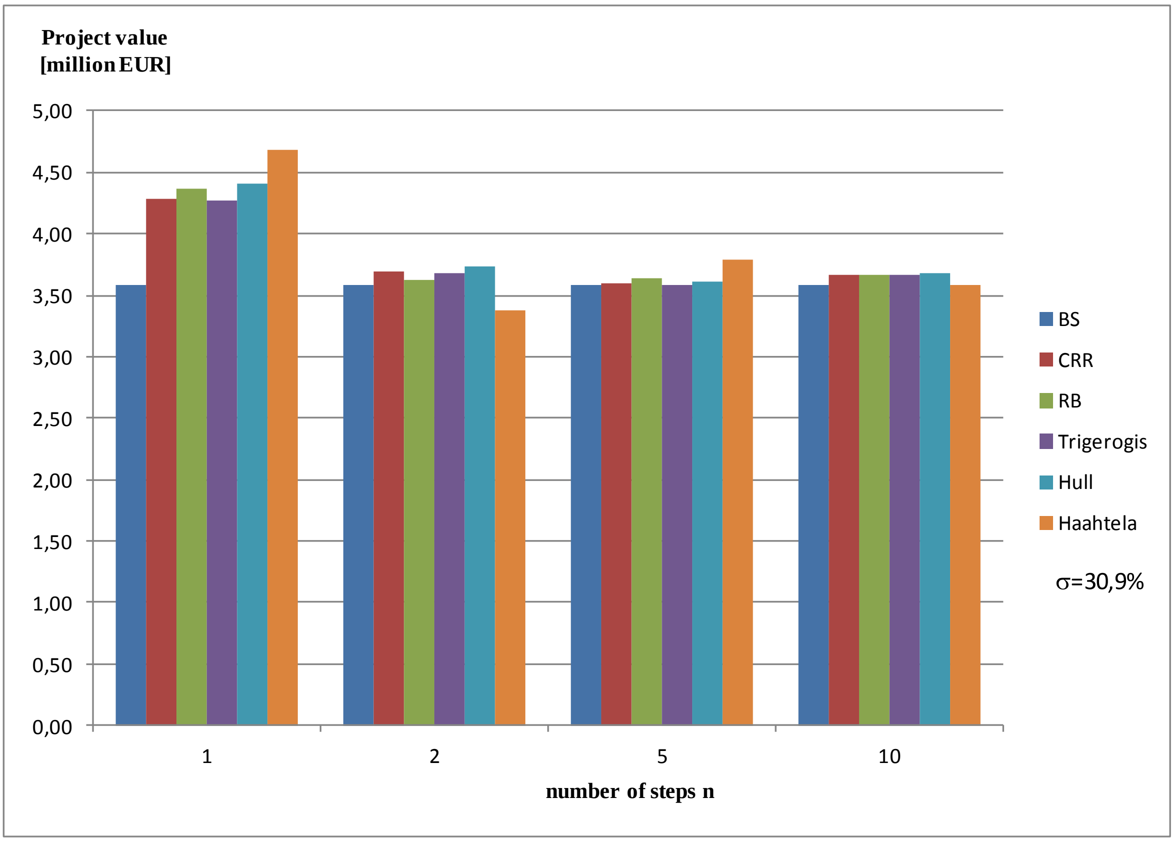
[49] Damodaran also uses this parameterization, refer to: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/spreadsh.htm

Figure 6. Option to Defer With Different Steps and Parameterizations for 30.9% Volatility



From Figure 6 it can be seen that with the increase of the time step, the option values under all parameterizations are converging to the BS value. With a single step the difference between BS and all other parameterizations is enormous. However, with the inclusion of just one step more, the values of the option are getting closer to the one obtained with the BS model. Increasing the number of steps to five and ten is further diminishing the difference with the BS value, but also in-between different parameterizations. It is interesting to note that Haahtela's parameterizations are getting closer to the BS model only with ten steps, at least for the given volatility. Hence, the steps in the binominal method, regardless of the parameterizations used, have to be greater than one in order the margin of error to be within reasonable values. Tables 3 and 4 show the relative error of each parameterization with the BS model used as reference, for single and two steps binomial lattice, respectively**.**

From Table 3 it is obvious that the time step is too small for any binomial model to give satisfactory results. In the range of low volatilities *i.e.* comparable to the risk free rate, all parameterizations are unstable, as pointed out by Trigeorgis. Within the higher volatiles (above 20%), typical for real options, all models are stabilizing, albeit overestimating the option value. The average relative errors within this range are from 17% up to 31% when compared to the BS.

With the inclusion of just one more step (n=2), the results are completely different (Table 4), showing greater stability and far closer values to the ones obtained by the BS model.

Table 3. Relative Error for Different Parameterizations and Volatilities as Compared to BS Model – Single Step Binomial Lattice

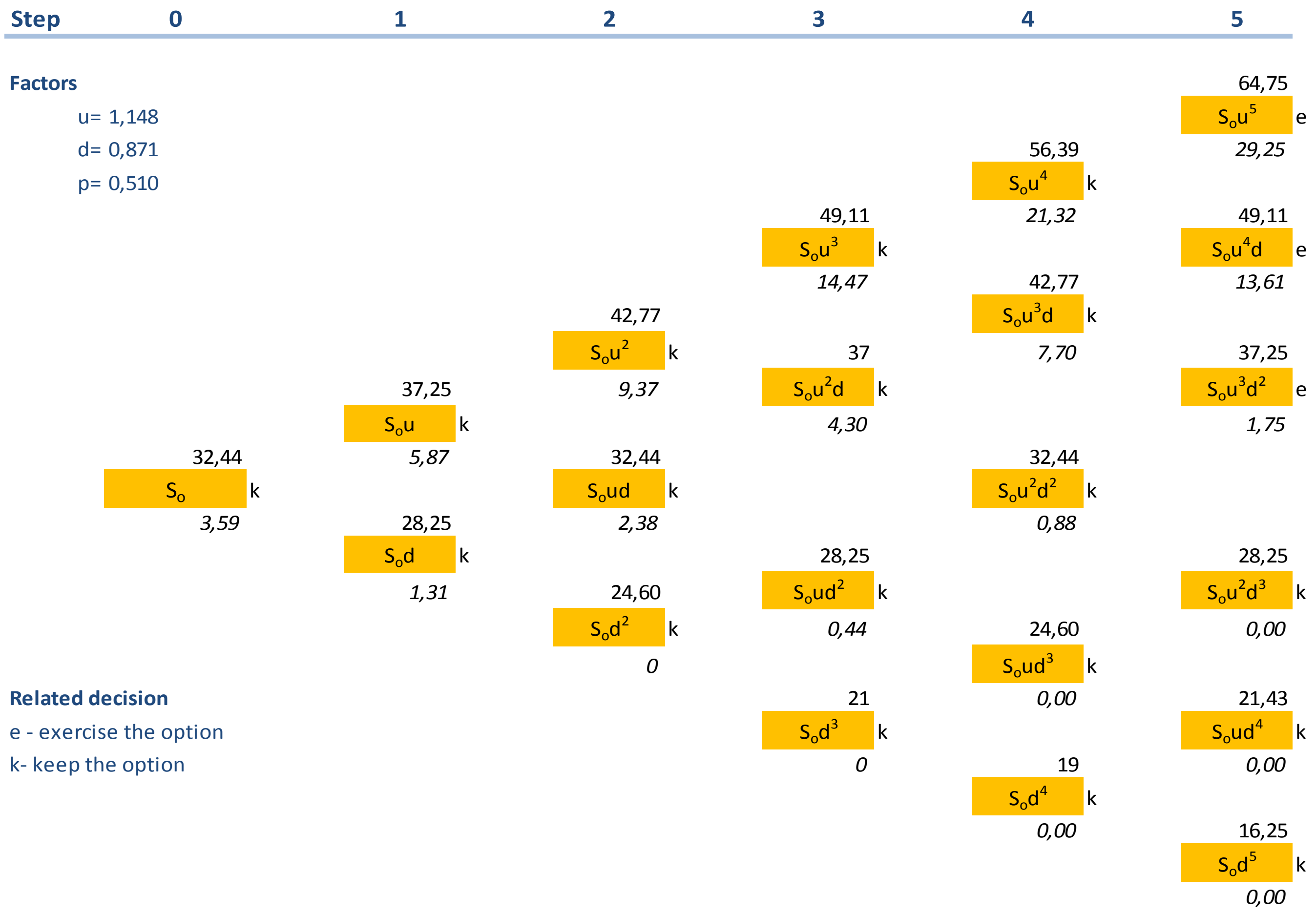
Method	Annual volatility σ in %	BS model in 1.000.000 EUR	Relative error between different parameterizations and B-S as reference in % CRR 1 step	RB 1 step	Trigerogis 1 step	Hull 1 step	Haahtela 1 step
Standard deviation in firm value	40,64	4,8347	22,07	18,38	21,92	27,93	32,23
Logarithmic stock price returns	30,91	3,5818	19,47	21,86	19,17	23,05	30,92
Logarithmic CF from project returns	29,70	3,4251	18,90	22,27	18,71	22,24	30,83
Implied volatility B-S model	27,47	3,1371	17,55	22,98	17,68	20,46	30,66
Assumed volatility	20,00	2,1707	8,32	25,29	11,57	10,04	30,59
Assumed volatility	10,00	0,8889	-70,76	27,97	-30,71	-70,04	31,59
Assumed volatility	7,00	0,5184	-100,00	26,27	-94,02	-100,00	30,04
Average of higher (20% and above) volatilities			17,26	22,16	17,81	20,74	31,04
Standard deviation of higher (20% and above) volatilities			5,26	2,50	3,83	6,59	0,68

Here the average relative errors in higher volatility range are between -5 and 5%, depending on the parameterizations used. Within lower volatilities, and especially the once comparable with the risk free rate, CRR's and Hull's parameterizations are not of any use as they produce huge errors. Indeed, Trigeorgies, RB, and Haahtela have solved the instability of the CRR binomial model in this range of volatilities. This can be seen on Figure 8, where the two step option values are presented in relation to the volatility.

Table 4. Relative Error for Different Parameterizations and Volatilities as Compared to BS Model – Two Step Binomial Lattice

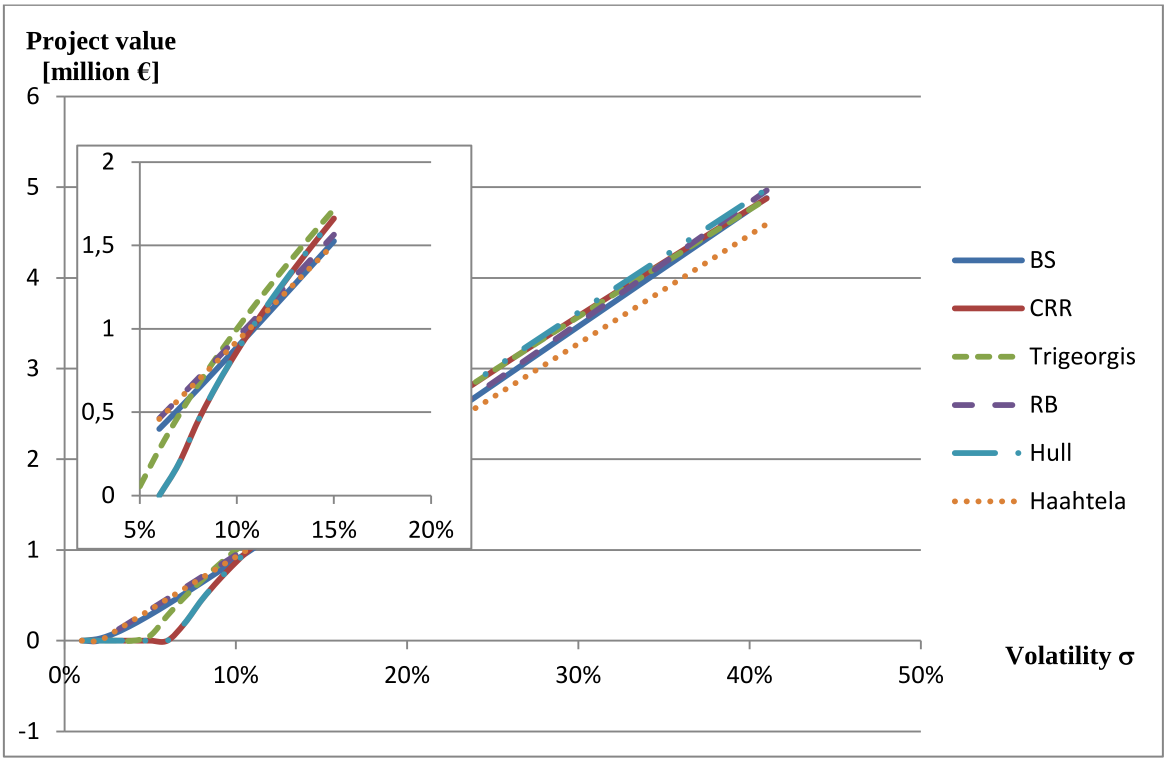
Method	Annual volatility σ in %	BS model in 1.000.000 EUR	Relative error between different parameterizations and B-S as reference in % CRR 2 steps	RB 2 steps	Trigerogis 2 steps	Hull 2 steps	Haahtela 2 steps
Standard deviation in firm value	40,64	4,8347	0,00	1,70	-0,13	-2,05	5,89
Logarithmic stock price returns	30,91	3,5818	2,94	1,09	2,72	-4,19	5,54
Logarithmic CF from project returns	29,70	3,4251	3,40	1,04	3,21	-4,56	5,42
Implied volatility B-S model	27,47	3,1371	4,29	0,97	4,22	-5,31	5,16
Assumed volatility	20,00	2,1707	7,69	1,28	8,71	-8,29	3,50
Assumed volatility	10,00	0,8889	-2,82	6,41	12,45	2,56	-4,06
Assumed volatility	7,00	0,5184	-63,46	12,37	-7,57	63,23	-10,99
Average of higher (20% and above) volatilities			3,67	1,21	3,75	-4,88	5,10
Standard deviation of higher (20% and above) volatilities			2,77	0,29	3,21	2,26	0,93

Figure 7: Binomial Lattice for Option to Defer for One Year With Five Steps



In the low volatility range (magnified graph), CRR's and Hull's parameterizations are far from the BS model, as pointed out previously. In the range from 10% to 15% volatility, Haahtela's parameterizations are giving the best results, while others are overvaluing the option value. However, above 15% volatility, Haahtela's parameterizations are undervaluing the option value. Parameterizations that can embody wide range of volatilities with low number of steps and still produce relatively low margin of error are the once of RB and Triegeorgis.

Figure 8. Option to Differ the Investment (Double Step) With Different Parameterizations in Relation to Volatility



With five steps binominal lattice (Table 5), the average relative errors are within lower limits, ranging from -1.2% up to 5.6%. For low volatilities the CRR's and Hull's parameterizations are still producing significant relative errors. Thus, if low number of steps is used in the analysis and the volatility is low, these parameterizations should not be employed. Related to volatility range between 10% and 40% the CRR's parameters give fairly good estimates.

With 10 step lattice, as presented on Table 6, the averages of the relative errors are similar in-between different parameterizations, in the higher volatility range, albeit overvaluing the option value. In this case, Haahtela's parameterizations are producing results that are closer to the once obtained with the BS model. Related to the lower volatility range *i.e.* from 10% and lower, the relative errors are far smaller than the once obtained with five steps.

Parameterizations as per RB, Trigeorgis and Haahtela are producing results that are within acceptable margin of error. However, CRR's and Hull's parameterizations are still giving relatively high errors in this range. Hence, if low volatility project is analyzed using ROA, the time steps chosen has to be high (five and above steps) accompanied by adequate parameterizations.

Table 5. Relative Error for Different Parameterizations and Volatilities as Compared to BS Model – Five Step Binomial Lattice

Method	Annual volatility σ in %	BS model in 1.000.000 EUR	Relative error between different parameterizations and B-S as reference in % CRR 5 step	RB 5 step	Trigerogis 5 step	Hull 5 step	Haahtela 5 step
Standard deviation in firm value	40,64	4,8347	2,40	0,56	2,35	3,36	6,09
Logarithmic stock price returns	30,91	3,5818	0,25	1,71	0,16	0,83	5,72
Logarithmic CF from project returns	29,70	3,4251	-0,20	1,82	-0,27	0,35	5,67
Implied volatility B-S model	27,47	3,1371	-1,19	1,99	-1,21	-0,71	5,58
Assumed volatility	20,00	2,1707	-7,28	2,23	-6,98	-7,09	5,09
Assumed volatility	10,00	0,8889	0,18	-1,18	4,89	0,26	1,39
Assumed volatility	7,00	0,5184	-23,24	-8,43	-5,48	-23,17	-6,02
Average of higher (20% and above) volatilities			-1,20	1,66	-1,19	-0,65	5,63
Standard deviation of higher (20% and above) volatilities			3,64	0,65	3,49	3,90	0,36

Table 6. Relative Error for Different Parameterizations and Volatilities as Compared to BS Model – 10 Step Binomial Lattice

Method	Annual volatility σ in %	BS model in 1.000.000 EUR	Relative error between different parameterizations and BS as reference in % CRR 10 step	RB 10 step	Trigerogis 10 step	Hull 10 step	Haahtela 10 step
Standard deviation in firm value	40,64	4,8347	2,02	2,23	1,99	2,47	-0,16
Logarithmic stock price returns	30,91	3,5818	2,46	2,29	2,42	2,74	0,23
Logarithmic CF from project returns	29,70	3,4251	2,48	2,30	2,45	2,74	0,31
Implied volatility B-S model	27,47	3,1371	2,47	2,32	2,45	2,69	0,47
Assumed volatility	20,00	2,1707	1,20	2,52	1,42	1,33	1,27
Assumed volatility	10,00	0,8889	-0,19	3,38	1,97	-0,16	3,43
Assumed volatility	7,00	0,5184	-11,94	2,53	-6,25	-11,93	3,28
Average of higher (20% and above) volatilities			2,13	2,33	2,15	2,39	0,42
Standard deviation of higher (20% and above) volatilities			0,56	0,11	0,45	0,61	0,52

Competition effects on option value

In the foregoing analysis, the project delay was assumed not to have any consequences on the project value or the underlying asset. This assumption for financial options is quite reasonable, especially for the European type, although it is too strong in case of a real option, particularly in the case where there are no proprietary rights or other competitive barriers on the project as such. It is very difficult to estimate what might be the loss of market share due to the delay, because it is hard to estimate the moves of the competition in such circumstances. However, we will assume a range from 30% to 100% loss from the total number of estimated customers within the first year, in order to estimate the range of the option value with assumptions relaxed[50]. This losses transformed into uniform dividend yield would be 0.58% and 0.96% respectively, which are insignificant as the option to differ has short time to expiration and the number of customers in the first year are relatively small compared to later years. Even with complete loss of the customers in the first year, the dividend yield is relatively small including the project / option value which for various valuation models is presented in Table 7.

Table 7. Option / Project Value With Dividends

Marke share loss first year in %	Dividend yield y in %	Option value in 1.000.000 EUR Binomial 1 step	Binomial 5 steps	BS model	p Binomial 1 step	Binomial 5 steps	B-S model
0	0,00	4,28	3,59	3,58	0,524	0,510	0,524
30	0,58	4,20	3,50	3,49	0,514	0,506	0,517
50	0,96	4,15	3,43	3,42	0,507	0,503	0,512
100	1,93	4,02	3,28	3,27	0,491	0,496	0,499

Hence, the preemption effect has relatively low impact on the project value, even when assumed that the delay would cause complete market share loss of the planned first year customers, which might be an overly "relaxed" assumption. In addition to lowering the call option value, dividends lower the probability of exercising or that the option will be in-the-money at expiration.

Again, in this particular case the change of the upper probability limit of the option to be in-the-money, is very low as compared to no-dividend paying asset (refer to Table 7).

Currently, there are no competitors which use the same (fiber optics) technology for the access part of the network, and it would be unreasonable to assume that they would implement it in such a short timeframe of one year, should they decided to. Thus, we should look at the incremental market share loss in areas where competition has already recourses in place, and not in the green field areas. In green filed areas, the competitors will rather wait for regulation to take place, and use the free option given to them by MKT, without bearing the risk of the business case (Hausman, p.193).

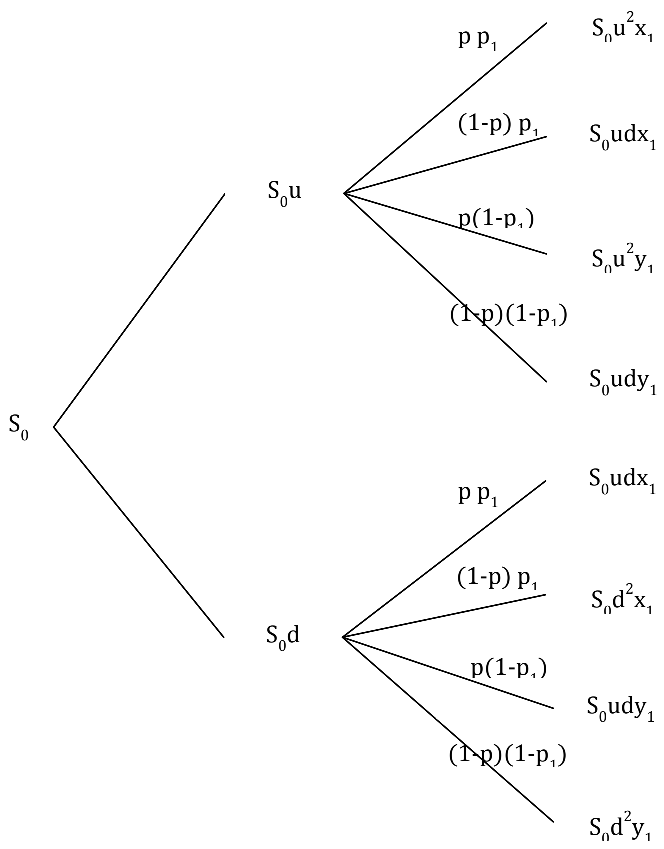
This example of application of ROA was performed on the base case scenario which treats or assumes the regulatory risk as certainty. However, if we want to include it in the ROA, the option to differ should be differently framed and analyzed.

[50] Certainly marketing area can estimate the customer loss due to the delay within a narrower range, which then can be used as input.

3.3.2 Implementing DTA into ROA

Private risks cannot be spanned within the markets since such securities do not exist. In this case we have the situation of incomplete markets, thus the risk neutral probabilities are not uniquely defined so as the project / option value. However, we can assume that the project value would be lying in the continuum between two limiting cases, treating the private risks as certainty on one extreme and neglecting them completely on the other extreme.

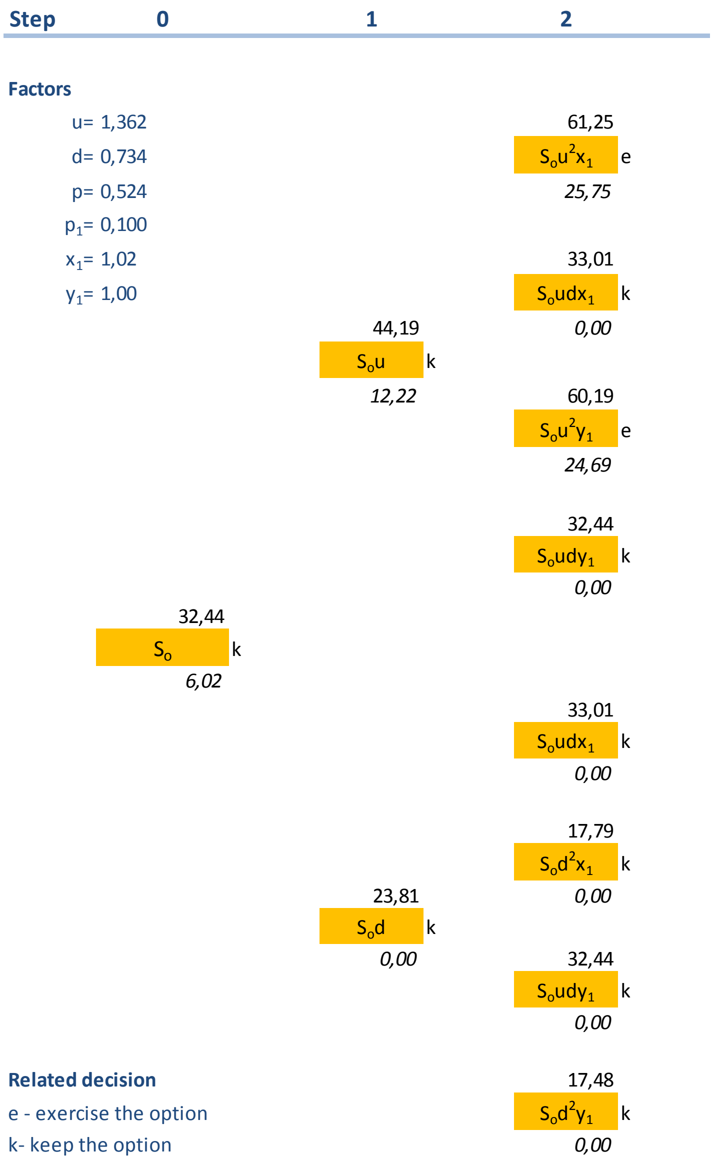
Figure 9. Non Recombining Lattice



Source: P. Kodikula & C. Papudesu, *Project Valuation Using Real Options*, 2006, p. 181.

Private risks by definition are endogenous to the firm and assumed to be manageable or within firm's competences. Here, in this master thesis, the regulatory risk as explained earlier is going to be modeled with DTA and treated as private. Although by its nature the regulatory uncertainty is exogenous to the firm, given the fact that the decision for taking the project is solely in the competence of the firm's top management, this risk can be considered as private. In addition to the preceding argument, it is highly unlikely that the market is efficient and has already incorporated this risk in today's prices, especially when the regulation has not been imposed to the FTTH yet.

Figure 10. Integration of DTA in Option Pricing



As mentioned before, the high probability of occurrence related to the imposing regulation, as estimated by the firm, was the reason for inclusion of the market share loss or wholesale product as certainty in the base case scenario. However, should we wish to analyze the project taking into account the real probabilities of possible outcomes of the regulation, the risks should be alienated and dealt with separately. In case of *ex post* regulatory regime, as in this case, first the market uncertainty is cleared out, and then the regulatory risk is resolved. Hence, in the first year the lattice is binomial, but in the second year it becomes a non-recombining qudrinomial lattice, due to inclusion of additional probabilities as presented in Figure 9. The latter are objective probabilities p_1 of the related regulatory outcome, while the factors x_1 and y_1 are asset correction factors. As the base case scenario is calculated with regulatory constrains as certainty, $y_1=1$, $x_1=S_1/S_0$ would be the correction factor of the asset value reflecting the scenario without regulatory constrain.

The corresponding lattice with calculation of the option value, accounting for the regulatory uncertainty is presented in Figure 10. When compared to the base case scenario (V= 4.28 million €), the option/project value is significantly higher in this case (V= 6.02 million €), as expected. Also, there is only 25% probability that the option/project is going to be exercised, if we look at the related decision at the end nodes of the lattice, which explains the impact of the regulation imposed on the development of the sector.

3.3.3 Option to contract the investment / project

Option to contract the project is the most relevant option for this project, as currently, without any application of ROA, just based on the actual results, project implementation is postponed. Top management has requested further analysis, prior to committing on continuing with investments.

As noted before, the goal of abandoning or contracting the project is to mitigate the potential losses. Such downside protection of the project has payoff characteristics of a put option, whose value increases as the asset value decreases.

In this case a scenario of contracting 50% of the operations is presented, resulting in around 60% cost reduction or 16.24 million €.[51] It is also assumed that the contraction can happen any time within the next three years. As this real option assembles put option, the related maximizing decision during the backward induction process for the last node is: $\max(S_n, cf * S_n + cs)$ where S_n is the asset value at that node. The intermediate nodes are calculated using risk neutral probabilities, and discounted back with the risk free rate.

The input parameters (in million €) are as follows:

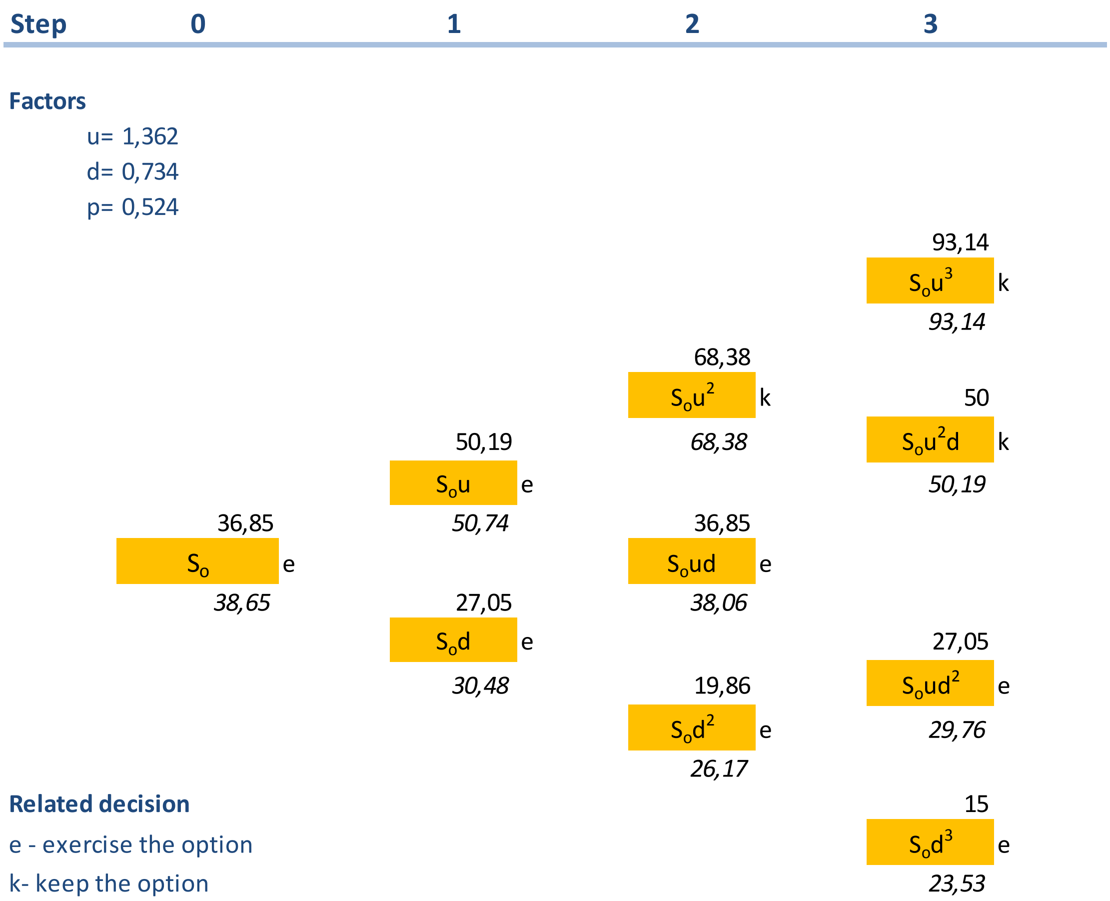
Stock price (PV of the cash flow as of now)	S_0= 36.85
Savings from contraction	cs=16.24
Contraction factor	cf = 50%
Standard deviation (annual volatility of the asset)	σ=30.9%
Risk-free rate (continuous)	r_f=6.1%
Time to expiration	T=3 Year
Time step	δt=1

Related results and the binomial lattice is shown in Figure 11, from which it can be seen that the project value, including the flexibility to contract the project, is 38.65 million €,

[51] This is due to the investment in transport network.

while the option itself is 1.8 million €, calculated as difference between the project value with the option and without. The Black Scholes with the same inputs would yield 1.45 million € for this put option, which represents an insignificant difference as compared to the binomial method calculation.

Figure 11. Binomial Lattice for Option to Contract the Project



3.3.4 Advanced options – sequential compound

Investments in infrastructure are projects that require large capital outlays within long time intervals. Just as in case of FTTH, it is impossible, even with unlimited financial recourses, to build the network within a short time frame. In the previous section, the investment outlay was considered as one time lump sum payment, which is certainly oversimplification of the problem. Rather than seeing the project as single stage, it would be more adequate be to treat it as a multi stage project, with sequence of investment costs or "installments", starting immediately and extending through the life of the project (Trigeorgis, 1999 p.19). In case of FTTH, we are talking about Intra – project compoundness, as it is within the same multi staged project. The initial investment costs are treated as a prerequisite for the following investment or option for further investment –

"installment". As the preceding investment is a requirement for the next, they represent an option on an option, where exercising one creates another in a row or sequence.

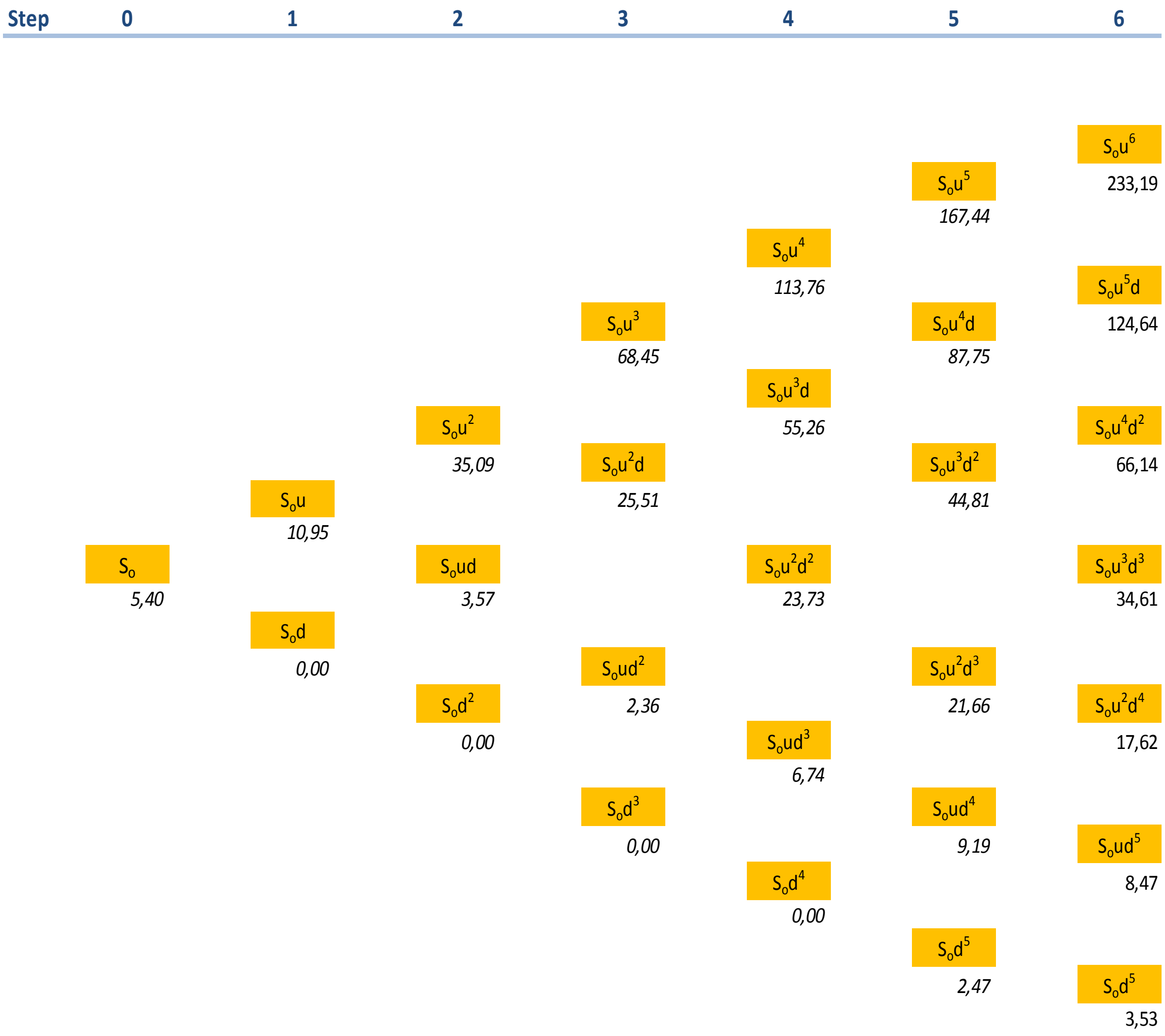
As per the DCF analysis (Appendix B), the FTTH project is evaluated within a six time period and accordingly with six installments. The input parameters are the same, except the installments which are the capital expenditures, occurring as of now till the fifth time interval, treated as the exercise price:

Stock price (PV of the cash flow as of now)	S_0= 36.85
Exercise prices	X_5= 2.24
	X_4= 3.28
	X_3= 8.05
	X_2= 12.35
	X_1= 10.06
	X_0= 7.93
Standard deviation (annual volatility of the asset)	σ=30.9%
Risk-free rate (continuous)	r_f=6.1%
Time to expiration	T=6 Year
Time step	δt=1

As usual, the procedure for calculating the option/project value with the binomial method starts from the longest option, on which we construct a six step lattice, with each step representing one year (Appendix F). The up and down, including the risk neutral probability are the same, while the only difference at this stage is the use of the previous year's CAPEX as exercise price (X_5), during the backward induction process. In the following calculation stage, which represents a five step lattice, the asset values are replaced by the option values of the preceding lattice, because the previous option creates the following one. In this second calculating stage the strike price is year four CAPEX (X_4). The same principles are used to calculate the option/ project value as of now in the remaining four lattices. The resulting lattice is obtained by replacing option values from last year's nodes from each lattice, and it is shown in Figure 12 on page 61.

It is interesting to note that the project value or expanded NPV is V=5.4 million €, which is almost three times the passive NPV of 1.85 million €, without any intervention, like deferral, contracting or else. Just the fact that the management has the option to invest in installments, given the asset values in each period, creates additional (option) value expressed as the difference between the expanded and passive NPV or 3.55 million €. This is due to the fact that the claim is asymmetrical and potential negative outcomes are limited to zero, as in those points of time the management has the flexibility to exercise or keep the option open.

Figure 12. Resulting Lattice for Sequential Compound Option



The binomial lattice can be viewed as a strategic roadmap that enables the firm to take an adequate - maximizing decision in each time step, and with that optimize the project. If several investment scenarios are made, then choosing the optimal would enable most efficient use of the resources, and with that maximization of the firm's profit. In general, investments in the telecom sector are divisible and separate entities – unbundled network elements can be used for different services or by different firms.

Specifically in FTTH case, the investments in the transport part of the network can be optimized not only to serve the project but both the project and the wholesale partners. In fact, the relatively low passive NPV is mainly due to the high home passed and home connected ratio, or under utilization of the transport network in the first several years. Had the project costs been estimated slightly higher than those assumed here, the project would have had a negative NPV and consequently it would have been rejected.

As mentioned before, this is intra-project compoundness, as opposed to inter-project compoudness, where one project can create an option to invest in other related projects or externalities. Intra – project compoundness can be found mainly within research and development investments. However, should the FTTH be reframed by separating voice and internet broadband from IPTV, then the latter can be seen as additional opportunity created by investing in former.

3.3.5 Option to defer the investment / project with changing volatilities

The volatility factor in option pricing models represents the changes in the underlying asset value or the project payoff in case of real options, throughout its life, reflecting the uncertainty. While it is reasonable to use single or constant volatility in pricing financial options, in the case of ROA this might mean oversimplification of the problem. This is because real options have usually longer maturities than financial options, thus the volatility factor is reasonable to change during the lifetime of the project, even though, the maturities of the estimated market volatility should match the option lifetime. Particularly, the volatility in investment projects decline over time because of the obtained new information and knowledge (Haahtela, 2010).

In the integrated approach elaborated previously, it was assumed that the regulatory uncertainty is going to be resolved during the second year. Due to this, it is reasonable to assume that the volatility of the underlying asset during the first and second year might be different, typically higher in the former case and lower in latter. When the regulatory uncertainty is resolved, the market uncertainty is assumed to be lower, thus the volatility. However, it is not just the regulatory influence that might change market volatility. This could often happen with new technology products in their introductory phase, when market acceptance is uncertain in the beginning, while later when the technology has proven to be reliable[52], the uncertainty and with that the volatility decreases. Another argument for declining volatility over the time is that in case of ROA the underlying asset value is an estimate with uncertainty[53] not well known in the beginning. After introduction of the service or conducting a pilot project more information can be gathered, hence more reliable estimate can be made for the uncertainty of the underlying asset including its value.

In this section, an example of option to differ for two years will be presented, with changing volatility throughout the time. Specifically higher volatility is going to be used in

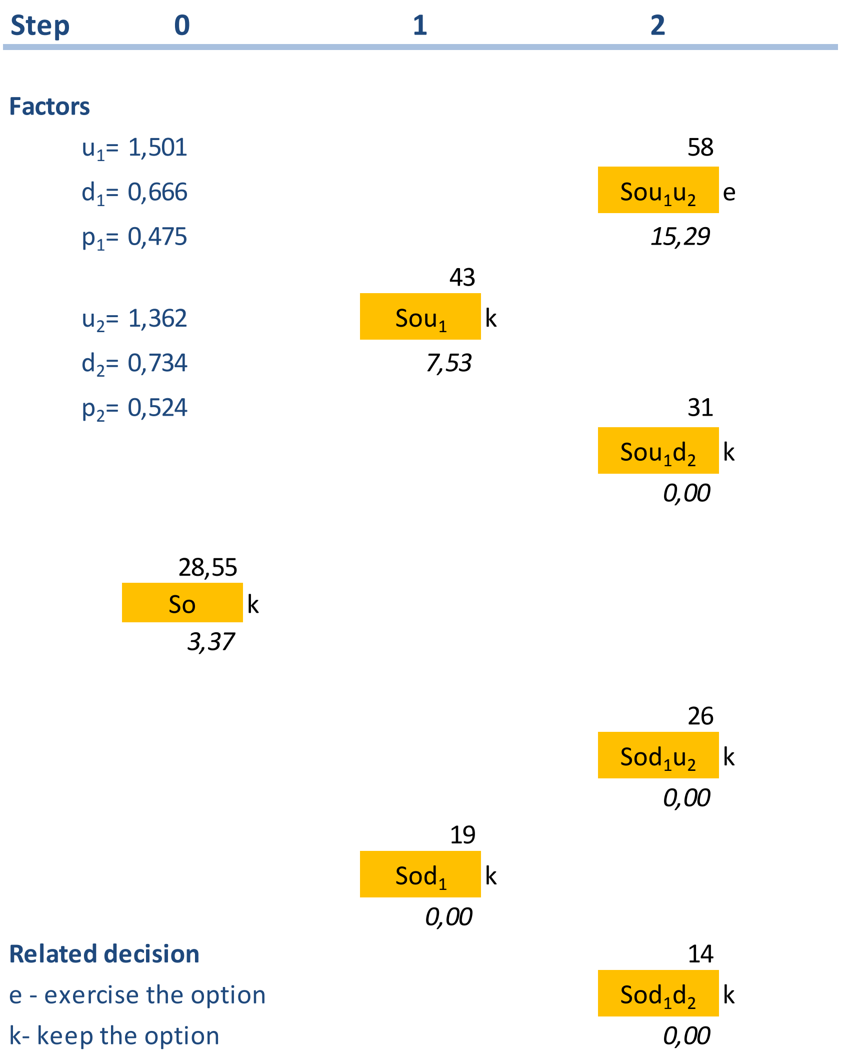
[52] Implementation of the FTTH has required PSTN lines to be migrated to VoIP, in which process service interruptions were very common.

[53] This might be even uncertainty of second order or ambiguity, for which we cannot specify probability distributions.

the first year and lower in the second year, in order to show the calculation technique[54]. The input values for the binomial model, adjusted for the two years delay are as follows:

Stock price (PV of the cash flow as of now)	S_0= 28.55
Exercise price (PV of CAPEX as of Y2)	X=43.10
Standard deviation (annual Volatility year 1)	σ_1=40.64%
Standard deviation (annual Volatility year 2)	σ_2=30.91%
Risk-free rate (continuous yearly)	r_f=6.1%
Time to expiration	T=2 Years
Time step	δt=1

Figure 13. Option to Defer the Project for Two Years With Changing Volatilities



54 There are other techniques for incorporating changing volatilities through option life – see for example Haahtela T. (2010 & 2011)

The binomial lattice, which is becoming non-recombining in the second year due to the inclusion of additional up and down factors, is shown on Figure 13. The up and down factors, as well as the risk neutral probabilities are calculated in the same manner, using the respective volatilities. Calculation of the project value during the backward induction process is essentially the same, with the only difference being the use of probability p_2 and p_1 for weighting the project values in the second and first year respectively.

The project value of 3.37 million €, is significantly higher than the passive NPV of 1.85 million €, hence the option to defer gives additional value of 1.52 million €. Had we used single volatility factor *i.e.* σ_1 and σ_2, the lattice would have been recombining, yielding 4.8 and 2.71 million €, respectively.

One of the limitations of the Black Scholes option pricing model is that it cannot accommodate different volatilities for calculating the project value. The only solution would be to use an aggregate volatility factor, which for uncorrelated uncertainties can be calculated from:

$$\sigma_r^2 = \sigma_1^2 + \sigma_2^2 \quad (33)$$

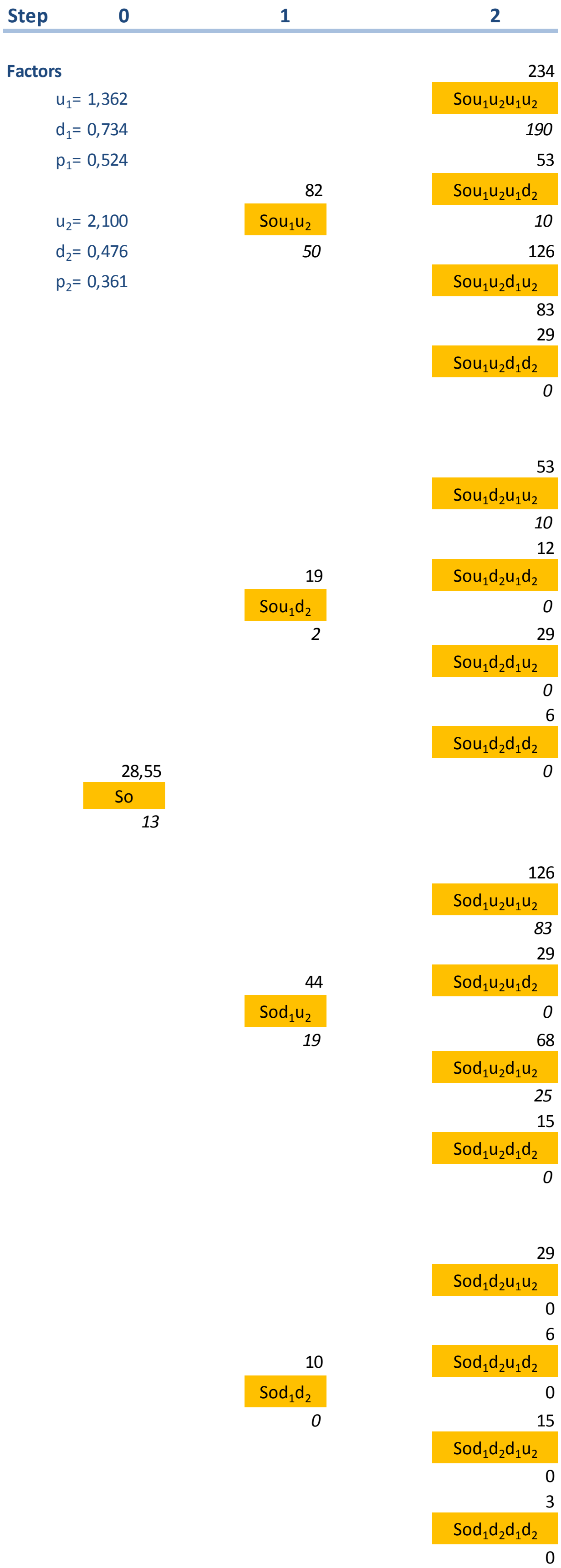
In the above example two market data volatility factors were used (refer to Table 1), and both refer to the same source of uncertainty, *i.e.* market risk, hence the real option value can be considered as reliable estimate.

3.3.6 Option to defer investment / project with different volatilities

In the previous example, volatility changed in relation of time - one volatility was used in the first year and another in the second. In general, multiple uncertainties influence the volatility of the underlying asset and with that the project payoff. As the project payoff is the present value of its cash flows, it is subject to several sources of uncertainties, such as: volatility of market demand, material and labor costs including their supply, etc. In case that one source of uncertainty has significant impact on the project value, adjustments can be made by treating and incorporating them separately in the analysis. In this example two uncorrelated sources of uncertainty are going to be introduced: market demand volatility and volatility of one of the inputs (costs) for the project – the internet and TV content providers. The latter volatility is taken from the entertainment sector, particularly the standard deviation in the firm's value calculated for larger number of related firms[55]. The two volatility factors make the lattice qudrinomial non-recombining or a rainbow lattice as shown previously on Figure 3.

[55] These can be found at Prof. Damodaran's page: http://pages.stern.nyu.edu/~adamodar/ One of the TV content provider to MKT is within the group of 77 firms.

Figure 14. Option to Defer the Project for Two Years with Different Volatilities



During the backward induction process, the same maximizing decision rule is applied in the last nodes. However in intermediate nodes, calculating the expected asset value for keeping the option open and account for the downstream optimal decision involves two risk neutral probabilities. For example the value in the node $S_0u_1u_2$ is weighted average (with risk neutral probabilities) of the potential future option values, discounted at the risk-free rate or:

$$[p_1p_2S_0u_1u_2u_1u_2 + p_1(1-p_2)S_0u_1u_2u_1d_2 + (1-p_1)p_2S_0u_1u_2d_1u_2 + (1-p_1)(1-p_2)S_0u_1u_2d_1d_2]e^{-r\delta t}$$

For the other intermediate nodes (top – down view), the same combination of risk neutral probabilities is applied to calculate the asset value for keeping the option open or exercising.

The lattice for the option to defer with different volatilities is shown on figure 14, as obtained with the same inputs as in the previous example, except for the volatility factors σ_1 and σ_2, for which values of 30.91% and 74.17% were used respectively. The project value of 12.6 million €, is significantly higher than any value calculated previously in this master thesis. This is obviously due to the very high volatility factor σ_2, which increases the option value. However, comparing the risk neutral probabilities p_1 and p_2, the increased volatility σ_2 decreases the probability of option's exercise or introducing more uncertainty into the exercise decision (Damodaran, 2005 p.43). Specifically if we use only σ_1, with related risk neutral probability p_1=0.524, the lattice would collapse to binomial recombining, yielding project value of 2.4 million €. However, if we use the higher volatility σ_2, which yields project value of 9.57 million €, the related risk neutral probability would be p_2=0.361, smaller than p_1=0.524, which reflects the probability of option exercise. Since those uncertainties – volatilities are uncorrelated, the resulting volatility or aggregate can be found according to equation (33) yielding σ_r=80.35%. In this case, as we have a single volatility, we can price the project with binomial recombining lattice, which would yield 10.4 million €, with p_r=0.345. The latter project value is relatively lower than that calculated with the use of two volatilities; however, the resulting risk neutral probability p_r is almost the same as p_2. With this resulting single volatility σ_r, we can also value the project with the BS model, which would result in 9.96 million €.

3.4 Discussion of the results

In this section, several type of real options and methodologies were presented that are typical for capital decision making in the telecom sector. Without exception, all project values are significantly higher than compared to the passive DCF/NPV value.

The option to defer the investment, presented in section 3.3.1, is a simple one, analyzed with the so called "revised classic approach", which makes a distinction between private and public risks. In this case the project has mainly public risks, hence uses volatility as obtained from market data, while the value of the underlying asset is subjectively estimated, with use of market data for the discount rate. The "classic (no-arbitrage, market data) approach" requires sizing the asset value relative to similar firm in the industry, which would practically mean making a "mark to market" with a firm from abroad in case of FTTH. This would most probably distort the estimate of the underlying asset, as there are significant differences related to the market size, income of the population, competition etc. Hence, the firm's subjectively estimated asset value might be the best estimate.

Regarding the methods used, there is a substantial difference in the real option value when comparing the binomial one step lattice with the binomial two, five step lattice or BS model. The former is due to the long time frame chosen for the δt, particularly one year. For example the CRR's parameterizations (Table 3,4,5 and 6) with volatility of σ=31% are giving 19,5%, 2,9%, 0,3 and 2,5% relative error for single, double, five and ten step respectively. There is no correction for this, because all lattice models assume low δt (or zero in the limit), in order the adequately transform the process of future price changes of the underlying asset from continuous to discrete. Neither of the proposed parameterization can alleviate the difference with the BS with single step as shown in Table 3. The biggest difference between time steps chosen is between a single and double step in higher range of volatilities. For instance, with single step binomial the range of relative errors are from 8% to 32%, while in double step -8% up to 7% depending on the parameterizations (Table 3 and 4). This is for higher volatilities *i.e.* 20% and above, while for lower volatilities single and double step models are not of use as the relative errors are very high.

Five step parameterizations are further lowering the span of relative errors, ranging from -7% up to 6%, in higher volatility range. Here the biggest benefit is within lower volatilities where parameterizations according to RB, Trigeorgis and Haahtela are giving fairly good estimates, producing relative errors ranging from -8% up to 1% (Table 5).

Ten step parameterizations are increasing the precision, however with smaller difference when compared to the difference between single and double, and double and five step models. Here the relative errors and in the range from -0,2 up to 2,5 %, depending on the volatility and parameterizations used.

Hence, with the increase of the number of steps, the precision is increasing regardless of the parameterizations used. This difference is more pronounced between single and double step models and less between double and five steps, which in latter case is becoming sufficiently precise for valuing real options. This is valid within the higher volatilities, while for lower, suitable parameterizations have to be employed and at least five steps models. Certainly ten step models are adding precision, however at double computational effort. Since the option to differ the project analyzed on the onset is simple one *i.e.* with single and constant volatility, employing BS model is simplest and the most precise solution. The only disadvantage would be that the process is not visible as in case of lattice models, hence difficult to be explained to the management.

The telecommunication sector has become a competitive industry in the last two decades, partially naturally and in part due to the regulation. In such environment, the option to differ the project is not a proprietary, but rather a shared option (Tirgeorgis, 1999); hence, the assumptions of no competition effect are too strong. In this respect, the most convenient way of adjusting ROA for the loss due to the competition, is by lowering the underlying asset value by assuming dividend payout (Damodaran, 2005). Both models (BS and binomial), can accommodate dividends or convenience yield, as to reflect relaxing assumptions and include the first time mover advantage in the analysis. In the FTTH case, the competition does not have a significant impact on the option value; however, this is due to the relatively low market share loss or dividend payout assumed (Table 8). Cost of delay or dividends also lower the probability of the option to be in-the-money or make its exercise less uncertain.

Integration of decision tree analysis into real option valuation is necessary when markets are incomplete or private risks exist. Of course, the solution is not perfect; however, for decision making purposes it is useful, hence it was presented herewith in other to establish and show the technique. In the FTTH case presented in section 3.3.2, the value of the project, including the option to defer, has significant value. However, this is solely due to the up jump factor, regardless of the asset value with and without imposing regulation (x_1), as it is too small to have any effect on the exercise (Figure 10). Hence, the value of the project including flexibility is mainly derived from the deferral, and not from the private risk. This is a result of the assumptions made in the analysis.

The option to contract the operations has also significant value, especially in strategic investments where the uncertainty is high, when a decision for implementing the project has to be made even with a negative or marginal NPV, due to technological changes. This is exactly what has happened with the FTTH project, as the new services demand higher speed that cannot be met by the copper network, the decision for transfer to fiber optic network technology had to be made on boundary NPV. Hence, contracting is an option that has substantial value of around 1.8 million € that could be saved, should the market conditions remain the same. This put option has a small difference according to the BS (1.45 million €) and binomial model (1.8 million €), and it is mainly due to the low number of steps chosen in the binomial model.

Table 8. Summary of the Results

1.	Option to differ the investment for one year											in 1.000.000 EUR	
				Calculation model									
				B-S		CRR 1 step		CRR 5 steps		Simulation		Average	Average
	Method	Annual volatility σ in %	Passive NPV	Option Value	Expanded NPV	Option Value	Expanded NPV	Option Value	Expanded NPV	Option Value	Expanded NPV	Option value	Expanded NPV
	Standard deviation in firm value	40,64	1,85	2,98	4,83	4,05	5,90	3,10	4,95	2,90	4,75	3,26	5,11
	Logarithmic stock price returns	30,91	1,85	1,73	3,58	2,43	4,28	1,74	3,59	2,13	3,98	2,01	3,86
	Logarithmic CF from project returns	29,70	1,85	1,58	3,43	2,22	4,07	1,57	3,42	1,39	3,24	1,69	3,54
	Implied volatility B-S model	27,47	1,85	1,29	3,14	1,84	3,69	1,25	3,10	1,32	3,17	1,42	3,27
	Option to differ the investment for one year with competition inclouded (dividend yield y=1,93%)											in 1.000.000 EUR	
	Logarithmic stock price returns	30,91	1,85	1,42	3,27	2,17	4,02	1,43	3,28	1,12	2,97	1,54	3,39
2.	Option to differ the investment for two years with private risks											in 1.000.000 EUR	
						CRR 1 step + DTA						Average	
	Method	Annual volatility σ in %	Passive NPV			Option Value	Expanded NPV					Option value	Expanded NPV
	Logarithmic stock price returns	30,91	1,85			4,17	6,02					4,17	6,02
3.	Option to contract the investment in the following three years											in 1.000.000 EUR	
				B-S		CRR 3 steps						Average	Average
	Method	Annual volatility σ in %	Passive NPV	Option Value	Expanded NPV	Option Value	Expanded NPV					Option value	Expanded NPV
	Logarithmic stock price returns	30,91	1,85	1,45	3,30	1,80	3,65					1,63	3,48
4.	Sequential compound option											in 1.000.000 EUR	
						CRR 6 steps						Average	
	Method	Annual volatility σ in %	Passive NPV			Option Value	Expanded NPV					Option value	Expanded NPV
	Logarithmic stock price returns	30,91	1,85			3,55	5,40					3,55	5,40
5.	Option to differ the investment for two years with changing volatilities											in 1.000.000 EUR	
						CRR 2 steps						Average	
	Method	Annual volatility σ in %	Passive NPV			Option Value	Expanded NPV					Option value	Expanded NPV
	Standard deviation in firm value	40,64	1,85			1,52	3,37					1,52	3,37
	Logarithmic stock price returns	30,91											
6.	Option to differ the investment for two years with differnet volatilities											in 1.000.000 EUR	
				B-S		non recomb. 2 steps		CRR 2 steps				Average	Average
	Method	Annual volatility σ in %	Passive NPV	Option Value	Expanded NPV	Option Value	Expanded NPV	Option Value	Expanded NPV			Option value	Expanded NPV
	Logarithmic stock price returns	74,17	1,85			10,75	12,60					10,75	12,60
	Logarithmic stock price returns	30,91											
	$\sigma_r^2=\sigma_1^2+\sigma_2^2$	80,35	1,85	8,11	9,96			8,55	10,40			8,33	10,18

Projects requiring substantial investment needed in infrastructure or large fixed costs over the long time and that have high operating leverage will gain more from options created by taking the project in multiply stages (Damodaran, 2005). Hence, long term investments most adequately can be framed as sequential compound real option (Kodikula & Papudesu, 2006). In reality, capital outlays occur gradually over the years, while in each period the management has the decision power to alter the investment schedule, should there is need for that. Indeed, as in the case of FTTH, the option value of 3.55 million €, is not just a decisive factor, but rather assurance that the project is adding value to firm's shareholders.

New technology entry changes market participant perception and thus makes industry prospects uncertain. However, as time passes, the outcome of the technology change becomes less uncertain. Hence, it is reasonable to assume that uncertainty of the project is likely to be different in the early phase and in its maturity. Hence, the option to differ the FTTH project for two years was analyzed with non recombining binomial lattice with two different volatilities per year. The option value of 2 million € is a result of the two volatility factors σ_1=45.66% and σ_2=30.91%. The former volatility is higher than the latter used in deferral option in section 3.3.1, hence it would be expected that the option value would be higher too. However the option value in this case is lower than in section 3.3.1 (2 vs. 2.43 million €), which is due to the lower underlying asset value and higher CAPEX as a result of the two year deferral period. Namely, the ratio S_0/X, which represents the return to immediately exercising the option, in this case is 0.66, while in section 3.3.1 it is 0.9. In the financial markets, an option with S_0/X less than 1 is "out-of-the-money", while above 1 is known as "in-the-money" option. Hence, the greater volatility in this case is offset by the lower S_0/X ratio, thus the lower option value.

4 RECOMMENDATIONS

4.1 Recommendations for input variables

The assessment and the applicability of the input variables were discussed in Section 2.4 of this master thesis. As mentioned, previously the most difficult variable to asses was the volatility of the underlying asset, which reflects its risk. However, in case of real assets, the risks can be market priced risk or private, but more typically they contain mixture of both (Amram. 2002, p.104). In case of market priced risk alone and existence of traded financial assets (stock or option written on that stock), the volatility should be taken from the market data. Otherwise the applicability of the models which all rely on riskless hedge is questioned. Should both market assessed volatilities exist[56] - as in this case, it is recommended to use the one that is more close to the market where the firm operates. Also, it is better to opt for conservative estimates of the option value; hence, a lower value of the volatility would be more appropriate. In case of FTTH, the volatility obtained from the project itself is almost the same as the volatility obtained from the stock prices of the firm[57], which confirms that the cash flows are fairly estimated and/or the market has priced the firm adequately.

In case of market incompleteness or existence of private risks, there is no alternative; the use of heuristic approach is inevitable. For this, it is recommendable to have several experts involved in the estimation of the probabilities, taking into account all relevant experience. In the telecom sector, the highest private uncertainty is the implementation of new technology. Since in case of FTTH the technology is proven to be operational, in this master thesis the regulatory influence was modeled and considered as private risk. The expert judgment has been made according to the latest trends in EU regulatory regimes, local regulatory announcements and competition requests for wholesale products.

[56] In fact for this asset (FTTH), four market assessed volatilities can be found that can be used to replicate its payoff: logarithmic stock returns of Deutche Telekom (DT), Magyar Telekom and Makedonski Telekom, including implied volatility of the option prices written on DT's stock.
[57] Refer to Table 1.

4.2 Recommendations of ROA model

The purpose of employing ROA is to capture the intrinsic management value of managing projects, with the arrival of new information. The ultimate goal is making better capital decisions, in a sense of efficient and effective resource allocation, which would lead to increase in shareholder value. From this perspective, a simple, intuitive and transparent model is preferred to complicated but on the other hand more precise ones. Another aspect that has to be satisfied when choosing the model is its flexibility. The telecommunication industry has transformed from "connecting A to B points" to service and content driven. This is reflected in complicated technology interactions, convergence and revenue streams from vast of services, all with different characteristics. Hence, the chosen model has to be flexible to accommodate different volatilities, changes in asset values through time, or even assessment of several types of options at the same time.

Lattices are the only choice for satisfying the above requirements in performing ROA. Of course, if the BS model is applicable to the problem/project analyzed, it should be used to cross check of the final value of the option/project, although in most cases the BS applicability is limited only to simple applications. As it was shown in case of deferral option, the lattice method is highly sensitive to the number of steps chosen. However, even with relatively low number of steps (five in this case), the binomial method converges relatively fast to the BS model.

Another advantage of lattice methods is that the asset value and the related decisions across time are visible (the diffusion process). This provides the possibility of early identification of deviations of the actual results from the planned, and with that possibility for reaction.

On the other hand, the BS model gives a solution instantaneously, making it superior during identification of the option and in checking whether there is real option value at all. Also, sensitivity analysis to every single input is effortless, which can give additional insight of the option value range, with all possible states of the variables.

CONCLUSION

There is no doubt that management actions and decisions differentiate good from great firms. Very important parts of those are capital budgeting decisions, which include the three dimensions of allocation decisions: which project to invest in, how much and when?

As it is shown in this master thesis, future management response to altered future business environment conditions expands an investment opportunity value by improving its upside potential or by limiting its downside losses, relative to the passive operating strategy, *i.e.* the classical DCF/NPV analysis, and decisions resulting there from. The examples in this master thesis summarized in Table 8 have revealed significant value, over and above the passive NPV.

In general, three types of real options were applied to a typical telecommunication project (FTTH): option to differ, option to contract and growth option. The first was calculated using several parameterizations in order to explore which of them might best describe the approximation of the continuous process of the BS model, with the discrete one, as used in the binomial approach. The conclusions are twofold:

- For low volatilities *i.e.* comparable with the risk free rate, combination of specifically designed parameterizations and increased number of steps must be used in order to have results that are with low margin of error and
- Within higher volatilities, all parameterizations are producing fairly good estimates as long as the number of steps is greater than two.

Option to differ the investment is also analyzed with inclusion of private risks, dividend yield or inclusion of market loss as a result of the deferral, with different and changing volatilities. Limiting the potential downside of the project is presented by the option to contract the project, whereas staged growth through the sequential compound option.

The average option values, when averaged across different models, for same variability (*e.g.* 30.91%) are in the range of 1.63 up to 3.55 million €, for option to contract and growth option, respectively. In the middle of those values is the option to differ the investment, yielding 2.01 million €. Even with inclusion of the preemption effect (losing the first mover advantage due to the deferral), the option to differ has considerable value, in this example 1.54 million €. Option to differ the investment with changing volatilities with respect to time has also a significant value of 1.54 million €, but also the option to differ with different (disaggregated) volatilities, which according to the example used, is in the range from 10.18 up to 12.6 million €. From the above it can be concluded that Real options have significant value, which when attributed to the passive NPV increase the potential value of the investment.

Not considering the options embedded in the investment may lead to underinvestment if decisions are made only relaying to DCF/NPV analysis, hence the static DCF/NPV analysis grossly undermines the value of the investment as shown in this master thesis, thus the model for valuing future opportunities has to be amended in order to reflect its true potential. Recognizing real option in projects is finding alternatives to the initial operating strategy, making efficient and effective use of the recourses or maximizing the value of the investment.

Certainly, valuing real options is a non-trivial task. However, just the process of recognizing the capital budgeting project as a real option has value. First, it distinguishes that there are more alternatives to single operating / implementation strategy. Their identification makes broader scope of project uses and it increases choices and innovation within the firm. Second, it creates a clear framework for managing the project and related decision making. Once the firm has taken a "go" decision, ROA, or particularly the lattice explicitly, shows the predicted asset values and the related decision as compared to the actual project results. Third, it reveals not just in which project to invest, but also when to invest. Looking at the firm's future growth opportunities as a portfolio of projects analyzed by employing ROA would reveal the timing of exercising a decision, hence maximizing its enlargement, with optimal recourses utilization.

In this Master thesis it is assumed that the underlying asset follows a Brownian motion, which in general might not be the adequate approximation when valuing options. The other extreme would be assuming a pure jump process, as discussed in this work. However, since a particular industry application of ROA is analyzed, the assumed process of future asset prices must correspond to the actual project analyzed. Most, if not all, telecommunication projects[58] involve long term investments, that contribute gradually with cash flows, thus changing the asset value. In general, the dynamic of real asset's changes in respect of time is far slower than for financial assets. Hence, the closest approximation of future movements of the underlying assets prices in telecommunication sector, particularly telecommunication utility, is continuous Brownian motion.

Regarding to the model for valuing real assets explored in this master thesis, and given the limitations of their applicability, both the BS and the lattice methods have advantages and disadvantages. When both methods can be employed, the BS model is superior in fast convergence and precision However, the lattice method is more intuitive in its appeal as the price path of the underlying is visible and more easily explainable to the management; it is also more flexible in the adjustment needed for particular applications when framing the option. The recommended model for valuing real options, particularly in the telecommunication sector therefore is the lattice method, specifically the binomial.

Certainly, there is a tradeoff between the precision and appeal / flexibility of these two methods, arising from the mathematical approximation in discretization of the dynamic process of asset prices. In order to solve the problem of finding the parameters uniquely, an additional equation is required, which is somehow arbitrary. Theoretically, there are an infinite number of choices without obvious criteria. Ideally the fourth equation should be such as to enable fast convergence of the binomial approximation procedure. Thus, several different parameterizations for the binomial approach were explored in this master thesis

[58] Within telecom service providers, not telecommunication equipment producers, where new product / technology might change the stock price value dramatically, and with that the diffusion process.

and compared to the BS. The findings presented in Tables 4 to 6, for double, five and ten steps binomial model, compared to BS as a reference reveal an insignificant difference in option prices within the higher volatility range. Hence I can conclude that the choice of parameterization in ROA when using lattice methods is not necessarily crucial to the analysis as long as both the volatility and the time steps are relatively high.

Asset pricing theories and models are not perfect. They all relay on critical assumptions, which in some cases are too strong and make their generalization difficult or even impossible. Contingent claim analysis or pricing by arbitrage is not an exception. It doesn't give the answer where the prices come from, but merely specifies what the prices should be when the market is complete, free from arbitrage, and thus efficient. Investors do not have the same (complete) information, nor do they have same risk preferences or tolerance. Further, they do not use and cannot agree on the same models for valuing assets, regardless whether they are real (*i.e.* firm or project) or financial. Hence, in reality investors do not have heterogeneous expectations. In the real world the investors do not hold just "primary" securities (*i.e.* stocks and bonds), but also nonlinear derivatives such as options.

Apart from the imperfection of the models and theories related to pricing of assets, rational investors would choose the model that would yield the best approximate results and fit the purpose, *i.e.* analyzing the specific type of the asset. Real assets are dynamic in their nature, in a sense that management is continuously modifying them as to bring out the most of them. This is the basic role of management governing the firm. Neglecting this role in capital budgeting decisions is like admitting that there is no value in the management *per se*. The tool – ROA, might be imperfect, but it is the best that fits the real world in pricing real assets when the uncertainty is high and there are alternatives for modifying the initial operating strategy. By separating time from risk preferences, this method acceptably values downstream management influence or flexibility. Unlike in classical DCF analysis where the discount rate, particularly the cost of equity, is unobservable, in ROA the discount factor, which is the risk free rate, is observable. Also, the implied volatility of the derivative (the option on the particular asset) is observable, as derived by its current and future price written in the contract. Hence, the risk neutral distribution or probabilities might be the best unbiased estimate of the changes of assets prices.

Having choices is like owning options, and options, as it is presented in this master thesis, are valuable.

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APPENDICES

LIST OF APPENDICES

Appendix A: List of Abbreviations

Abbreviation	Meaning
ADSL	Asymmetric Digital Subscriber Line
APT	Arbitrage Pricing Model
BS	Black - Scholes
CAPM	Capital Asset Pricing Model
CF	Cash Flow
CRR	Cox, Ross, Rubenstein
DCF	Discounted Cash Flow
DTA	Decision Tree Analysis
FTTH	Fiber to the Home
GPON	Gigabit Passive Optical Network
HD	High Density
IP	Internet Protocol
IPTV	Internet Protocol Television
IRR	Internal Rate of Return
IT	Information Technology
LTE	Long-term Evolution
MKT	Makedonski Telekomunikacii A.D. Skopje
NPV	Net Present Value
PSTN	Public Switched Telephone Network
ROA	Real Option Analysis
TV	Television
VoD	Video on Demand
VPN	Virtual Private Network
WACC	Weighted Average Cost of Capital

Appendix B: FTTH DCF Analysis

FTTH DCF Analysis

Year	0	1	2	3	4	5	6
FTTH Residential & Business Fast Internet customers - new	100%	8700	20000	28000	44000	10000	6000
WS participation in total sales			10%	20%	20%	20%	20%
Distribution of FTTH Fast Internet customers							
Residential		59%	50%	50%	50%	50%	50%
Residential +HD TV		32%	41%	41%	41%	41%	41%
Business		10%	10%	10%	10%	10%	10%
FTTH incremental monthly fee							
Residential monthly fee - FTTH Fast Internet		€ 4,5	€ 4,2	€ 3,9	€ 3,9	€ 3,9	€ 3,9
Residential monthly fee - FTTH Fast Internet and HD TV		€ 10,0	€ 9,3	€ 8,6	€ 8,6	€ 8,6	€ 8,6
Business monthly fee - FTTH Fast Internet		€ 11,0	€ 10,2	€ 9,5	€ 9,5	€ 9,5	€ 9,5
Average Rate Per User (ARPU)		€ 24,0	€ 24,0	€ 23,0	€ 22,0	€ 22,0	€ 22,0
Revenues							
Residential monthly fee - FTTH Fast Internet		€ 137.417	€ 464.861	€ 781.585	€ 1.149.390	€ 1.126.402	€ 367.805
Residential monthly fee - FTTH Fast Internet and HD TV		€ 164.430	€ 845.203	€ 1.421.064	€ 2.089.800	€ 2.048.004	€ 668.736
Business monthly fee - FTTH Fast Internet		€ 57.420	€ 229.561	€ 385.968	€ 567.600	€ 556.248	€ 181.632
ARPU revenues		€ 1.252.800	€ 5.385.600	€ 11.785.200	€ 20.776.800	€ 27.904.800	€ 30.808.800
Total Revenues		€ 1.612.067	€ 6.925.225	€ 14.373.817	€ 24.583.590	€ 31.635.454	€ 32.026.973
Cost margin as % of sales		50%	40%	39%	38%	37%	36%
Total operating costs		€ 806.033	€ 2.770.090	€ 5.605.789	€ 9.341.764	€ 11.705.118	€ 11.529.710
EBITDA		€ 806.033	€ 4.155.135	€ 8.768.028	€ 15.241.826	€ 19.930.336	€ 20.497.263
Depreciation		€ 1.089.866	€ 2.858.296	€ 4.950.663	€ 6.670.126	€ 6.399.402	€ 5.556.420
EBIT		-€ 283.833	€ 1.296.839	€ 3.817.366	€ 8.571.700	€ 13.530.934	€ 14.940.842
Tax @ 10%		€ 0	€ 129.684	€ 381.737	€ 857.170	€ 1.353.093	€ 1.494.084
Net income		-€ 283.833	€ 1.167.155	€ 3.435.629	€ 7.714.530	€ 12.177.841	€ 13.446.758
Operating cash flow (CF)		€ 806.033	€ 4.025.451	€ 8.386.292	€ 14.384.656	€ 18.577.243	€ 19.003.178
Present value (PV) CF @ 13.6% WACC		€ 709.536	€ 3.119.305	€ 5.720.513	€ 8.637.462	€ 9.819.505	€ 8.842.117
CF PV cumulative		€ 709.536	€ 3.828.842	€ 9.549.355	€ 18.186.817	€ 28.006.322	€ 36.848.439
Capital expenditures (CAPEX)	€ 7.933.494	€ 10.055.587	€ 12.351.415	€ 8.046.443	€ 3.283.084	€ 2.236.696	€ 0
CAPEX PV @ 13.6% WACC	€ 7.933.494	€ 8.851.749	€ 9.571.060	€ 5.488.693	€ 1.971.373	€ 1.182.267	€ 0
Total CAPEX PV	€ 34.998.635						
CF PV cummulative - CAPEX PV cummulative	-€ 34.998.635	-€ 34.289.099	-€ 31.169.793	-€ 25.449.280	-€ 16.811.818	-€ 6.992.313	€ 1.849.804

Net Present Value (NPV) as of Y6	€ 1.849.804
Discounted payback period (Years)	5,79
Internal rate of return (IRR)	15,79%

Appendix C: Calculation of Implied Volatility

Inputs			Date	
Stock price	€ 8,68		05.03.2012	
Dividend	€ 0,70			
Dividend yield	8,06%			
Call option value	0,9		05.03.2012	
Exercise price	8		21.06.2013	
Risk - free rate (annual) dis.	0,13%			
Risk - free rate (annual) con.	0,13%			
Black Scholes calculator				
INPUTS			OUTPUTS	
Standard deviation (annual)	27,47%		d1	0,088264
Maturity (in years)	1,30		d2	-0,22446
Risk - free rate (annual)	0,13%		N(d1)	0,535167
Stock price	€ 8,68		N(d2)	0,4112
Exercise price	€ 8,00		B-S call	0,900052
Divident yield (annual)	8%		B-S put	1,068341

Strike Price	Vers. Num.	Opening Price	High	Low	Bid Vol.	Bid Price	Ask Price	Ask Vol.	Diff. to previous day	Last Price	Date	Time	Daily Settlem. Price	Traded Contr.	Open Interest (adj.)	Open Interest Date
16,00	0	n/a	n/a	n/a	0	n/a	n/a	0	0,00% →	n/a	n/a	n/a	0,01	0	0	05.03.2012
14,00	0	n/a	n/a	n/a	2	0,01	n/a	0	0,00% →	n/a	n/a	n/a	0,01	0	3671	05.03.2012
13,50	0	n/a	n/a	n/a	0	n/a	n/a	0	0,00% →	n/a	n/a	n/a	0,02	0	3620	05.03.2012
13,00	0	n/a	n/a	n/a	0	n/a	n/a	0	0,00% →	n/a	n/a	n/a	0,02	0	15	05.03.2012
12,50	0	n/a	n/a	n/a	0	n/a	n/a	0	0,00% →	n/a	n/a	n/a	0,03	0	0	05.03.2012
12,00	0	n/a	n/a	n/a	2000	0,01	0,06	1000	0,00% →	n/a	n/a	n/a	0,05	0	25442	05.03.2012
11,50	0	n/a	n/a	n/a	500	0,02	0,07	500	0,00% →	n/a	n/a	n/a	0,07	0	80010	05.03.2012
11,00	0	n/a	n/a	n/a	500	0,05	0,10	1000	0,00% →	n/a	n/a	n/a	0,10	0	100723	05.03.2012
10,50	0	n/a	n/a	n/a	500	0,09	0,14	1000	0,00% →	n/a	n/a	n/a	0,14	0	7518	05.03.2012
10,00	0	0,20	0,20	0,20	500	0,15	0,20	1500	-4,76% ↘	0.20	06.03.2012	09:50:06	0,21	100	296245	05.03.2012
9,50	0	n/a	n/a	n/a	500	0,23	0,28	1000	0,00% →	n/a	n/a	n/a	0,30	0	64890	05.03.2012
9,00	0	0,38	0,38	0,38	500	0,34	0,40	1000	-15,56% ↘	0.38	06.03.2012	13:42:11	0,43	500	8532	05.03.2012
8,50	0	0,60	0,60	0,60	2000	0,51	0,57	500	-7,69% ↘	0.60	06.03.2012	12:38:07	0,62	30	7807	05.03.2012
8,00	0	0,86	0,87	0,86	500	0,77	0,83	1000	-14,71% ↘	0.87	06.03.2012	13:43:09	0,90	1631	4830	05.03.2012
7,50	0	n/a	n/a	n/a	500	1,11	1,18	1500	0,00% →	n/a	n/a	n/a	1,26	0	3126	05.03.2012
7,20	0	n/a	n/a	n/a	1000	1,34	1,43	500	0,00% →	n/a	n/a	n/a	1,52	0	515	05.03.2012
7,00	0	n/a	n/a	n/a	500	1,52	1,62	1000	0,00% →	n/a	n/a	n/a	1,70	0	350	05.03.2012
6,40	0	n/a	n/a	n/a	500	2,08	2,18	500	0,00% →	n/a	n/a	n/a	2,28	0	0	05.03.2012
6,00	0	n/a	n/a	n/a	500	2,47	2,58	500	0,00% →	n/a	n/a	n/a	2,68	0	0	05.03.2012
5,60	0	n/a	n/a	n/a	500	2,81	3,05	500	0,00% →	n/a	n/a	n/a	3,08	0	0	05.03.2012
4,80	0	n/a	n/a	n/a	500	3,54	3,89	500	0,00% →	n/a	n/a	n/a	3,88	0	0	05.03.2012
Total														**2261**	**607294**	

Appendix D: Calculation of Project's Cash Flow Volatility

Year	Cash flow (Eur)	$ln(S_t/S_{t-1})$	Variance
	S_t	R_t	$(R_t-R_a)^2$
1	806.033	1,608267296	0,953005469
2	4.025.451	0,733961445	0,010386492
3	8.386.292	0,539563613	0,008553234
4	14.384.656	0,255775253	0,141580658
5	18.577.243	0,022668923	0,371342014
6	19.003.178		
Average	R_a=	0,632047306	
Total			1,48
	Volatility σ=	29,70%	

Appendix E: Calculation of Stock Price Volatility[59]

Week	Date	S_t	R_t	$(R_t-R_a)^2$
1	09.04.2012-12.04.2012	466	-0,157	0,023034672
2	02.04.2012-06.04.2012	545	0,028	0,001071722
3	26.03.2012-30.03.2012	530	0,009	0,000204694
4	19.03.2012-23.03.2012	525	-0,009	2,16258E-05
5	12.03.2012-16.03.2012	530	0,038	0,001874428
6	05.03.2012-09.03.2012	510	-0,025	0,000413811
7	27.02.2012-02.03.2012	523	0,006	0,000111958
8	20.02.2012-24.02.2012	520	0,012	0,000270074
9	13.02.2012-17.02.2012	514	0,008	0,000159793
10	06.02.2012-10.02.2012	510	-0,008	8,90519E-06
11	30.01.2012-03.02.2012	514	0,006	0,000114106
12	23.01.2012-27.01.2012	511	-0,004	8,50322E-07
13	16.01.2012-20.01.2012	513	-0,004	8,78585E-07
14	09.01.2012-13.01.2012	515	-0,002	8,34354E-06
15	03.01.2012-05.01.2012	516	-0,008	8,37328E-06
16	26.12.2011-29.12.2011	520	0,000	2,33133E-05
17	19.12.2011-23.12.2011	520	-0,010	2,24777E-05
18	12.12.2011-16.12.2011	525	0,006	0,000111494
19	05.12.2011-09.12.2011	522	-0,015	0,000107766
20	28.11.2011-02.12.2011	530	0,019	0,000570091
21	21.11.2011-25.11.2011	520	0,016	0,000413413
22	14.11.2011-18.11.2011	512	0,010	0,000214388
23	07.11.2011-11.11.2011	507	0,002	4,6277E-05
24	31.10.2011-04.11.2011	506	0,002	4,63301E-05
25	24.10.2011-28.10.2011	505	0,000	2,33133E-05
26	17.10.2011-21.10.2011	505	0,000	2,33133E-05
27	10.10.2011-14.10.2011	505	0,000	2,33133E-05
28	03.10.2011-07.10.2011	505	-0,002	8,12332E-06
29	26.09.2011-30.09.2011	506	-0,004	7,80759E-07
30	19.09.2011-23.09.2011	508	0,006	0,000115593
31	12.09.2011-16.09.2011	505	-0,010	2,52397E-05
32	05.09.2011-09.09.2011	510	-0,004	8,36282E-07
33	29.08.2011-02.09.2011	512	0,004	7,64275E-05
34	22.08.2011-26.08.2011	510	0,016	0,000425968
35	15.08.2011-19.08.2011	502	-0,006	1,27673E-06
36	08.08.2011-12.08.2011	505	0,006	0,000116353
37	01.08.2011-05.08.2011	502	0,002	4,65452E-05
38	25.07.2011-29.07.2011	501	0,000	2,33133E-05
39	18.07.2011-22.07.2011	501	0,000	2,33133E-05
40	11.07.2011-15.07.2011	501	-0,004	7,12868E-07
41	04.07.2011-08.07.2011	503	0,006	0,000116866
42	27.06.2011-01.07.2011	500	0,000	2,33133E-05
43	20.06.2011-24.06.2011	500	-0,002	8,01106E-06
44	13.06.2011-17.06.2011	501	-0,002	8,03363E-06
45	06.06.2011-09.06.2011	502	-0,006	1,27673E-06
46	30.05.2011-03.06.2011	505	0,012	0,000281592
47	23.05.2011-27.05.2011	499	0,018	0,000530339
48	16.05.2011-20.05.2011	490	-0,002	7,78213E-06
49	09.05.2011-13.05.2011	491	-0,008	1,07929E-05
50	03.05.2011-06.05.2011	495	0,031	0,001267363
51	26.04.2011-29.04.2011	480	0,043	0,002245622
52	18.04.2011-21.04.2011	460	-0,264	0,067188126
53	11.04.2011-15.04.2011	599	/	/
Average			-0,005	
Total				0,101483
	Weekly volatility σ=	4,46%		
	Yearly volatility σ=	30,91%		

59 According to equation (19) and (20)

Appendix F: Sequential Compound Option

Factors

u= 1,362

d= 0,734

p= 0,524

X_5= 2,237

Step 0	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
						235,43 S_ou^6 e *233,19*
					172,83 S_ou^5 e *170,59*	
				126,88 S_ou^4 e *124,83*		126,88 S_ou^5d e *124,64*
			93,14 S_ou^3 e *91,25*		93,14 S_ou^4d e *91,04*	
		68,38 S_ou^2 e *68,38*		68,38 S_ou^3d e *66,40*		68,38 $S_ou^4d^2$ e *66,14*
	50,19 S_ou e *50,19*		50 S_ou^2d e *48,33*		50,19 $S_ou^3d^2$ e *48,09*	
36,85 S_o e *36,50*		36,85 S_oud e *36,85*		36,85 $S_ou^2d^2$ e *34,87*		36,85 $S_ou^3d^3$ e *34,61*
	27,05 S_od e *26,27*		27,05 S_oud^2 e *25,19*		27,05 $S_ou^2d^3$ e *24,95*	
		19,86 S_od^2 e *18,11*		19,86 S_oud^3 e *17,88*		19,86 $S_ou^2d^4$ e *17,62*
			15 S_od^3 e *13*		14,58 S_oud^4 e *12,47*	
						10,70 S_oud^5 e *8,47*
				11 S_od^4 e *8,72*		
					7,86 S_od^5 e *5,75*	
						5,77 S_od^5 e *3,53*

Related decision

e - exercise the option

k- keep the option

Appendix E: Sequential Compound Option Cont'd.

Factors

u= 1,362
d= 0,734
p= 0,524
X_4= 3,283

Step 0	1	2	3	4	5
					170,73 $S_o u^5$ e *167,44*
				124,90 $S_o u^4$ e *121,81*	
			91,28 $S_o u^3$ e *88,37*		91,04 $S_o u^4 d$ e *87,75*
		68,38 $S_o u^2$ e *65,09*		66,40 $S_o u^3 d$ e *63,31*	
	50,19 $S_o u$ e *47,11*		48,33 $S_o u^2 d$ e *45,43*		48,09 $S_o u^3 d^2$ e *44,81*
36,50 S_o e *33,99*		36,85 $S_o ud$ e *33,57*		34,87 $S_o u^2 d^2$ e *31,78*	
	26,27 $S_o d$ e *24,06*		25,19 $S_o ud^2$ e *22,28*		24,95 $S_o u^2 d^3$ e *21,66*
		18,11 $S_o d^2$ e *16,80*		17,88 $S_o ud^3$ e *14,79*	
			12,72 $S_o d^3$ e *9,81*		12,47 $S_o ud^4$ e *9,19*
				8,72 $S_o d^4$ e *5,63*	
					5,75 $S_o d^5$ e *2,47*

Related decision

e - exercise the option
k- keep the option

Appendix E: Sequential compound option cont'd.

X_3= 8,046

Step 0	1	2	3	4
				121,81 $S_o u^4$ e *113,76*
			88,37 $S_o u^3$ e *80,80*	
		65,09 $S_o u^2$ e *57,05*		63,31 $S_o u^3 d$ e *55,26*
	47,11 $S_o u$ e *39,54*		45,43 $S_o u^2 d$ e *37,86*	
33,99 S_o e *26,87*		33,57 $S_o ud$ e *25,52*		31,78 $S_o u^2 d^2$ e *23,73*
	24,06 $S_o d$ e *16,50*		22,28 $S_o ud^2$ e *14,71*	
		16,80 $S_o d^2$ e *8,76*		14,79 $S_o ud^3$ e *6,74*
			9,81 $S_o d^3$ e *3,32*	
				5,63 $S_o d^4$ k *0,00*

Related decision

e - exercise the option

k- keep the option

Appendix E: Sequential compound option cont'd.

Step	0	1	2	3
X_2= 12,351				80,80
				$S_o u^3$ e
				68,45
			57,05	
			$S_o u^2$ e	37,86
		39,54	*45,15*	$S_o u^2 d$ e
		$S_o u$ e		*25,51*
	26,87	*28,35*	25,52	
	S_o e		$S_o ud$ e	
	17,20	16,50	*13,62*	
		$S_o d$ e		14,71
		7,23	8,76	$S_o ud^2$ e
			$S_o d^2$ e	*2,36*
			1,16	
Related decision				3,32
e - exercise the option				$S_o d^3$ k
k- keep the option				*0,00*

Appendix E: Sequential compound option cont'd.